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Bundesministerium für Bildung und Forschung

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Quota Steiner Tree Problem and its application on Wind Farm Planning

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CO@Work 2024 - Computational Optimization at Work

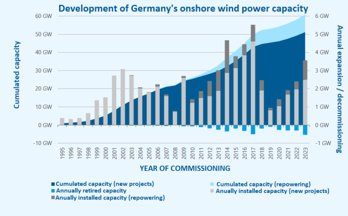
ZIB
ZUSE INSTITUTE BERLIN

What to expect

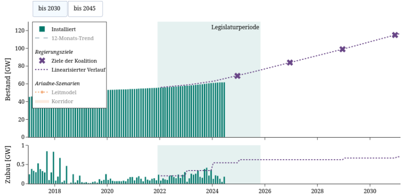
- Steps in designing wind farms
- Steiner tree problem and its quota-constrained variant
- Advantages of a problem-specific approach
- Open problems

Current trends in wind power

- Onshore wind low-cost renewable electricity option
- In recent years targets have not been reached
- Need for accelerating wind power expansion
- The planning involves many decisions



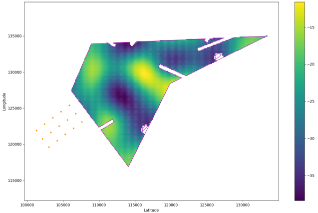
https://www.cleanenergywire.org/factsheets/german-onshore-wind-power-output-business-and-perspectives



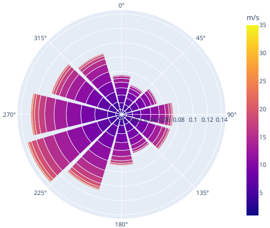
https://www.diw.de/de/diw_01.c.841560.de/ampel-monitor_energiewende.html

Large-scale Wind Farm Planning

1. Finding suitable areas
   - Onshore vs. offshore
   - Detailed wind analysis
   - Distances to, e.g., settlements or other wind farms
   - Acceptance of the population
   - . . .
2. Designing the wind farm itself:
   - How many turbines? costs vs. revenue or epxansion targets
   - Where to place the turbines? Interference problem, minimum distance
   - How to route the cables to connect the turbines?
   - . . .
3. Building and operating the wind farm



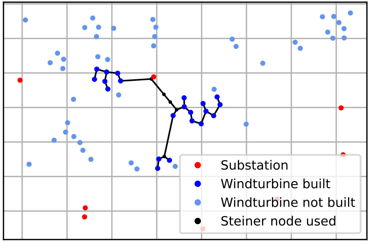
Example region [Cazzaro and Pisinger, 2022]



Example wind rose

Large-scale Wind Farm Planning

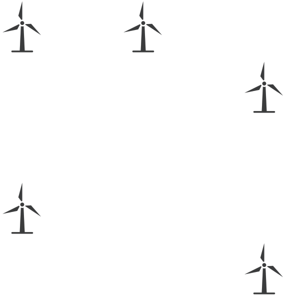
- Usually a two step approach:
  1. Wind farm layout problem: Placing the turbines
  2. Wind farm cable routing: Routing the power cables
- A good overview can be found in [Fischetti, 2021]
- Single wind farm: exact and highly detailed, but small [Fischetti and Pisinger, 2018], or large size, but heuristically [Cazzaro et al., 2023]
- Regional planning: large size, but no routing [Weinand et al., 2022]
- Expansion targets instead of only maximizing profits
- To increase acceptance other objectives than only costs are of interest, e.g., visual impact



Goal: Choose a subset of potential wind turbines to fulfill an expansion target while minimizing costs and visual impact of the turbines and the cable routing

Modelling

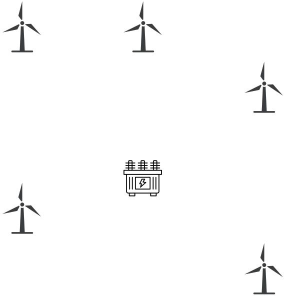
- Set of potential turbines



icons: Flaticon.com

Modelling

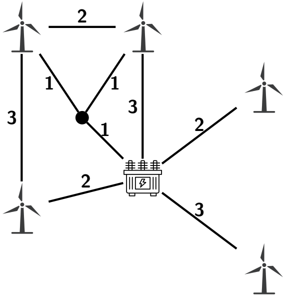
- Set of potential turbines
- Set of substations



icons: Flaticon.com

Modelling

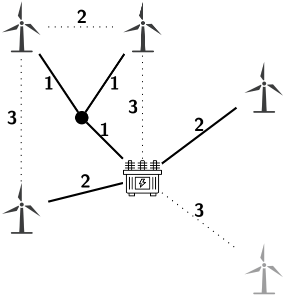
- Set of potential turbines
- Set of substations
- Set of intermediate nodes
- Set of cable connections



icons: Flaticon.com

Modelling

- Set of potential turbines
- Set of substations
- Set of intermediate nodes
- Set of cable connections
- Find cheapest connection between a subset of turbines and the substation to fulfill expansion target



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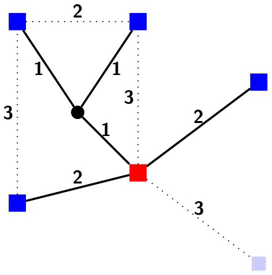

- Set of potential terminals
- Set of fixed terminals
- Set of Steiner nodes
- Set of edges
- Find a tree connecting all fixed terminals and a subset of potential terminals to fulfill a quota with minimal costs

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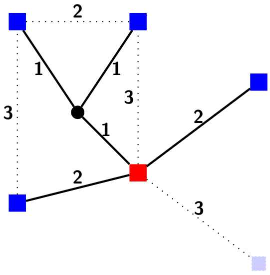

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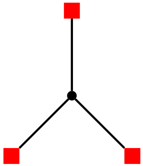
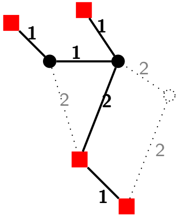
This problem is called the Quota Steiner tree problem in graphs (QSTP)

What is the Steiner Tree Problems in Graphs?

- The Steiner Tree Problem in Graphs (**STP**) is a classical combinatorial optimization problem
- Some history: The **STP** goes back to the 17th century, when Pierre de Fermat formulated the problem:
  Given 3 points in the plane, find the intermediate node such that the interconnecting distance is minimal
- The STP is formulated as follows:
  Given an undirected graph $G = (V, E)$, with edge costs $c : E \to \mathbb{R}_{\geq 0}$, and terminal nodes $T \subseteq V$, find a tree $S = (V', E') \subseteq G$ that contains all terminals such that the costs $C(S) := \sum_{e \in E} c_e$ are minimized



Fermat-Torrecilli Problem



Steiner Tree Problem in Graphs

Complexity

The STP is *NP-hard* in general. [Karp, 1972]

Steiner tree problems in graphs

- Special cases of $|T| = 2$ and $|T| = V$ are the *shortest-path problem* and *minimum-spanning tree problem*, which are solvable in polynomial time
- There exists a number of variants to the STP, e.g.:
  - Euclidean Steiner tree problem
  - node-weighted Steiner tree problem
  - prize-collecting Steiner tree problem
  - maximum-weight connected subgraph problem
  - ...
  - and the **quota Steiner tree problem**
- Recent surveys can be found in [Ljubić, 2021] and [Rehfeldt, 2021]
- SCIP-Jack [Rehfeldt and Koch, 2023]:
  - **world-wide fastest** exact STP solver
  - Extension to open-source solver **SCIP**
  - **branch-and-cut** to deal with the exponential number of constraint
  - **16 variations** of STP
- The **directed cut formulation** is most-used in practical exact solving

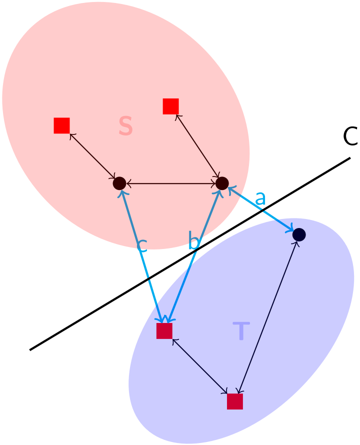
Reminder: What is a cut?

A **cut** $C = (S, T)$ is a partition of V of a graph $G = (V, E)$ into two subsets S and T.

The **cut-set** of a cut $C = (S, T)$ is the set $\{(u, v) \in E | u \in S, v \in T\}$

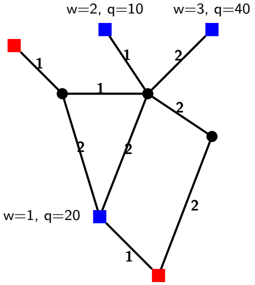
In an directed graph $G = (V, A)$, let the set of incoming arcs into $T \subset V$ be denoted as:

$$\delta^{-}(T) := \{(u, v) \in A \mid u \notin T, v \in T\}$$



Quota Steiner tree problem

- Given a graph $G = (V, E)$
- A set of existing substations $\Rightarrow$ set of **fixed terminals** $T_f \subset V$
- A set of potential wind turbines $\Rightarrow$ set of **potential terminals** $T_p \subset V$
- Each turbine is associated with costs $w > 0$ and an energy yield (**quota profit**) $q > 0$
- A set of possible cable connection $\Rightarrow$ set of edges E
- Each edge is associated with costs $c > 0$
- An expansion target $\Rightarrow$ a quota $Q > 0$



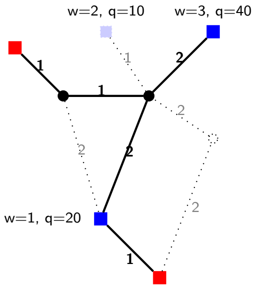
Quota Steiner tree problem

Given an undirected graph $G = (V, E)$, a set of **fixed terminals** $T_f \subset V$, and a set of **potential terminals** $T_p \subset V$ with $T_f \cap T_p = \emptyset$, where each edge $(i, j) \in E$ is associated with costs $c : E \to \mathbb{R}_{\geq 0}$, and each potential terminal $v \in T_p$ with costs $w : T_p \to \mathbb{R}_{>0}$ and **quota profits** $q : T_p \to \mathbb{R}_{>0}$.

The goal is to find a Steiner tree $S = (E', V') \subseteq G$ that contains all terminals T_f such that the **total cost**

$$C(S) = \sum_{e \in E'} c(e) + \sum_{v \in T_p \cap V'} w(v)$$

is minimized



Minimum weighted Steiner tree fulfilling a quota of $Q = 40$; terminals in red, potential terminals in blue

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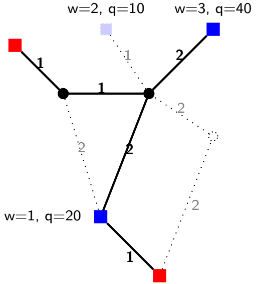
is minimized and a given **quota** $Q \in \mathbb{R}_{>0}$ is fulfilled

$$Q(S) = \sum_{i \in T_p \cap V'} q_i \geq Q$$

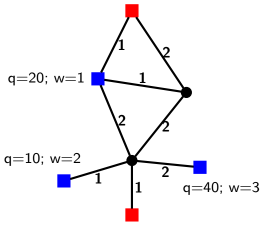


Minimum weighted Steiner tree fulfilling a quota of $Q = 40$; terminals in red, potential terminals in blue

Integer programming formulation of the QSTP

- Transform original graph G into a **directed graph** $D = (V, A)$
- **Shifting the costs**[1] of a vertex v onto the costs of its incoming arcs

$$c(i,j) = \begin{cases} c_e + w_j & \text{if } j \in T_p, \\ c_e & \text{otherwise} \end{cases} \qquad \forall a = (i,j) \in A$$



[1] [Ljubić et al., 2006]

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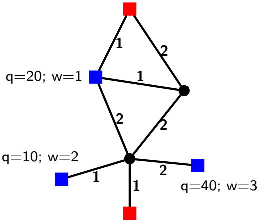

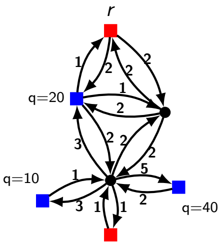

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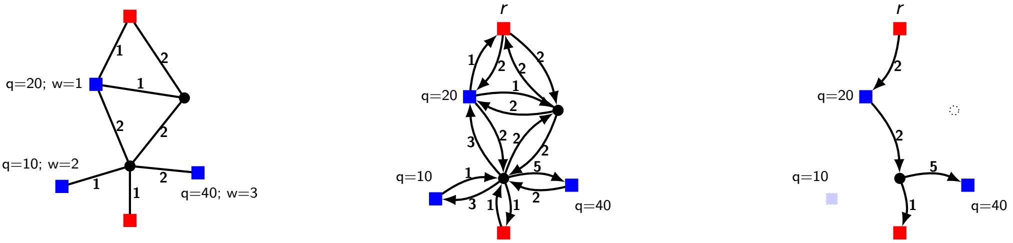
- Find a tree rooted at r that minimizes the costs and fulfills the quota



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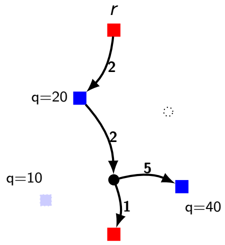

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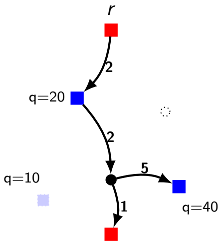

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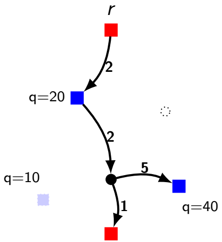

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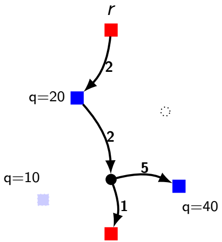

- Exponentially many constraints due to Steiner-cut constraints
- Cut-separation one of essential features in SCIP-Jack [Rehfeldt, 2021]
- Cut inequalities are separated using a maximum-flow algorithm

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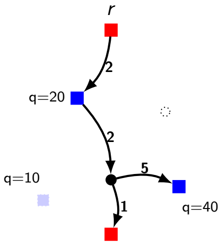
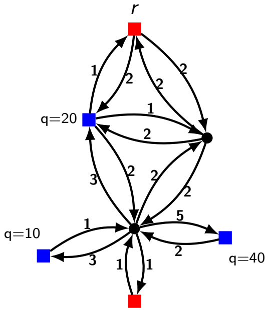


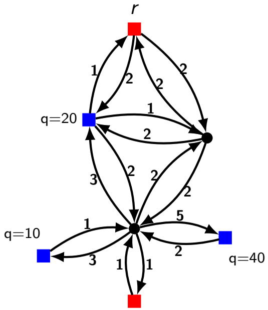
- Exponentially many constraints due to Steiner-cut constraints
- Cut-separation one of essential features in SCIP-Jack [Rehfeldt, 2021]
- Cut inequalities are separated using a maximum-flow algorithm
- **Problem**: Binaries y prevent the direct usage
- **Solution**: Transform the problem

Transformation



Original

Transformation



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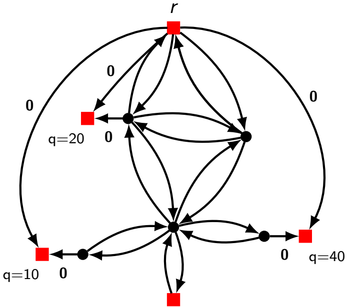

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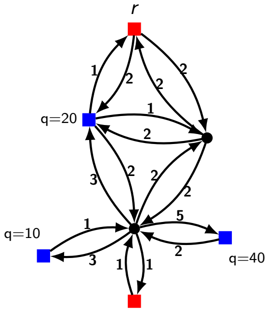

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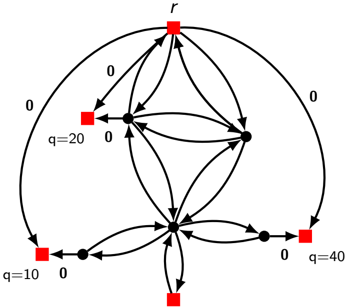

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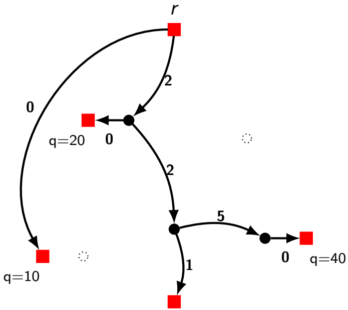

Solution

Directed cut formulation of the transformed QSTP

$$I_{\mathrm{QT}}: \min \quad c^T x$$

$$\text{s.t.} \quad x(\delta^-(W)) \geq 1 \qquad \forall W \subset V', r \notin W, |W \cap T'| \geq 1$$

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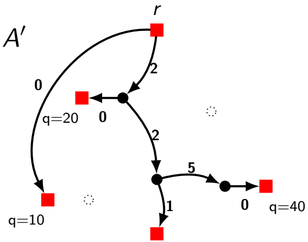

Directed cut formulation of the transformed QSTP

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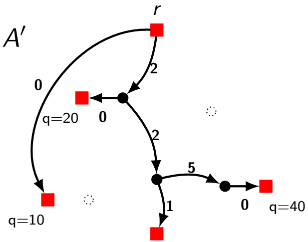

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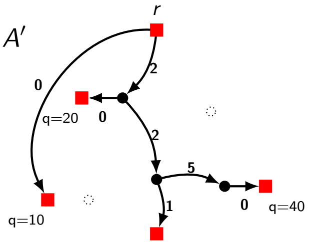


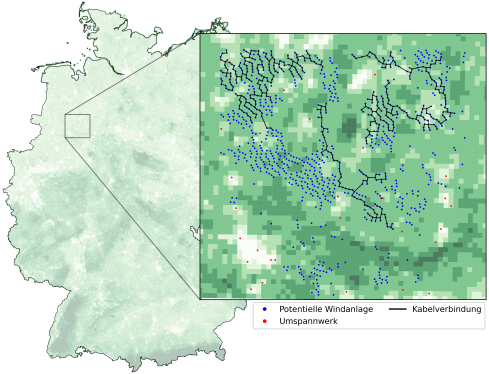
Proposition [**P.**, Weinand, Syranidou, Rehfeldt (2024)]

$$
proj_{xy}(\mathcal{P}_{LP}(\overline{I_{\mathrm{QT}}})) = \mathcal{P}_{LP}(I_{\mathrm{Q}}).
$$

Back to Onshore Wind Farm Planning

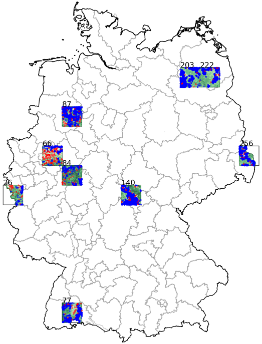
Goal

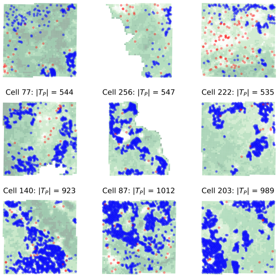
Install a subset of possible wind turbines to fulfill a given expansion target with minimum costs of turbine layout and cable routing on regional level.



Computational Study - Input

- Open-source German data by [Ryberg et al., 2019, Roth, 2018]
- Potential positions based on state-of-the-art methods
- Energy yield covers stochastic effects, such as wake effects and turbine availability
- Additional regions A and B with a high number of Steiner nodes
- Objective based on costs and visual impact using weighted-sum approach



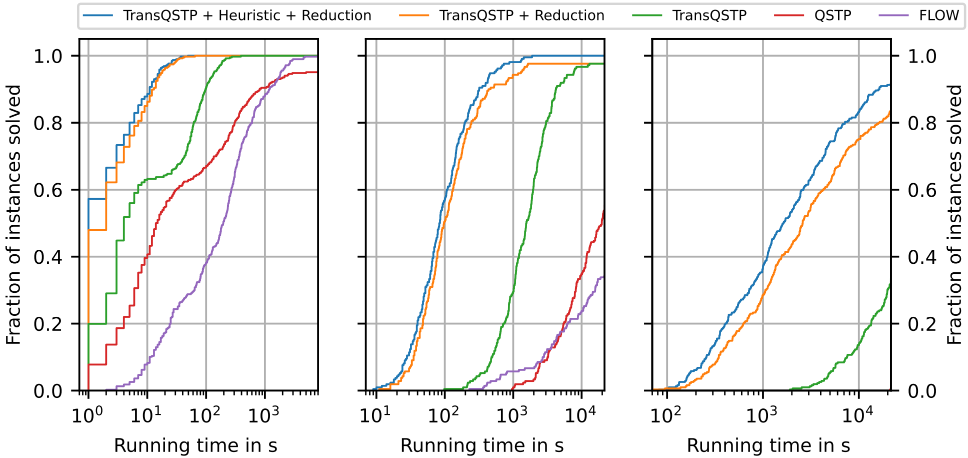


Computation Study - Setup

Name	Instances	$\lvert T_p \rvert$	$\lvert V \rvert$	$\lvert E \rvert$	Description
Small	210	92, 201, 245	117 - 515	6786 - 46056	Areas of 50 km x 50 km chosen by number of potential turbines and value of scenicness, for each area: 5x Q, 7x α, without Steiner nodes and with Steiner nodes for every turbine
Medium	210	535, 544, 547	547 - 1114	149331 - 162709	
Large	210	923, 989, 1012	944 - 2057	445096 - 546502	
Region A	176	22	147	10731	Steiner nodes on a grid of 1 km, 16x Q, 11x α
Region B	99	65	1639	1342341	Steiner nodes on a grid of 0.5 km, 9x Q, 11x α

- Integrate transformed QSTP (**TransQSTP**) into SCIP-JACK
- Shortest path reduction and shortest path heuristic (**TransQSTP+** and **TransQSTP++**)
- Verify against the flow-based MIP formulation (**FLOW**) solved by GUROBI 9.5
- We use SCIP-JACK in SCIP 8.0.1 using CPLEX 12.10 as LP solver.
- SCIP-JACK is run **single-threaded**, GUROBI 9.5 is run **32-threaded** *Intel XeonGold 6342* CPUs running at 2.8 GHz, where five CPUs and 32 GB of RAM are reserved, time limit of six hours

Computational Study - Results



(a) small + region A **(b)** medium **(c)** large + region B

Computational Study - Results

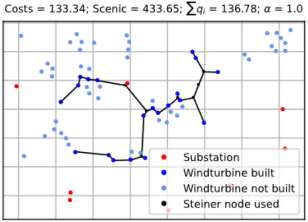
Advantages of a problem-specific solver:

- Problem-specific reduction techniques using the structure of the underlying graph
- Problem-related cut-separation algorithm depending on the structure of the combinatorial problem
- Problem-specific primal heuristic to find fast and good solutions
- Efficient usage of memory due to construct cutting planes on the fly

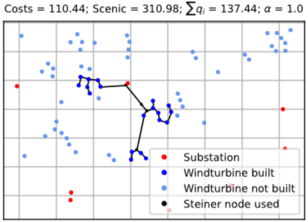
Computational Study - Result

- Comparison of sequential to integrated approach
- Previous Germany-wide study for optimal location to reach target of 200 GW in 2050
- Expansion target for a selected region by Germany-wide study
- Compare position assigned by that study with our integrated approach
- Compare costs and visual impact of these

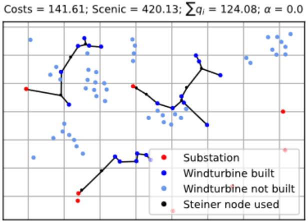
Computational Study - Results



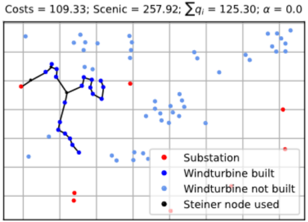
(a) Minimize costs: Sequential



(b) Minimize costs: Combined



(c) Minimize landscape impact: Sequential



(d) Minimize landscape impact: Combined

P., J.-M. Weinand, C. Syranidou, D. Rehfeldt (2024); "An efficient solver for large-scale onshore wind farm siting including cable routing", European Journal of Operational Research, `https://doi.org/10.1016/j.ejor.2024.04.026`

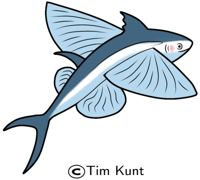
SCIP-Jack is getting wings

Lessons learned:

- Glimpse into designing wind farms
- STP and QSTP Maybe think a little about how these problems relate to each other...
- Using specialized QSTP approach highly effecient for large-scale onshore wind farm planning
- Integrated approach is vital to avoid excessive costs and landscape impact

What next?

- Preprocessing is vital for STP-related problems. Investigate techniques in terms of the QSTP.
- Extending classical QSTP towards single wind farm planning by introducing interference constraint:



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$$\sum_{i \in T_p} (q_i - \sum_{\mathbf{j} \neq \mathbf{i} \in \mathbf{T_p}} \mathbf{l_{ij} y_j}) y_i \geq Q$$

Photo: Vattenfall

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