Algorithms with Predictions

Learning-Augmented Algorithms for Scheduling Problems

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Faculty of Mathematics and Computer Science
University of Bremen

CO@Work 2024, Berlin

Optimziation under Uncertain Inputs

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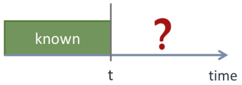
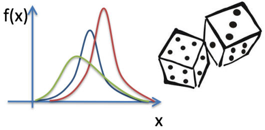
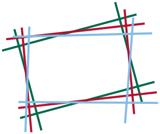
Different models for uncertain input

Online Information	Stochastic Information	Uncertainty sets







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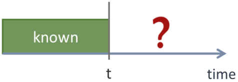	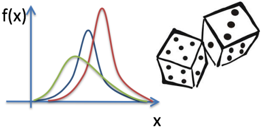	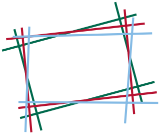

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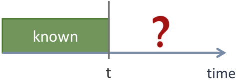	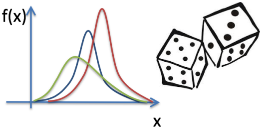	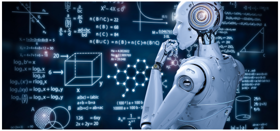

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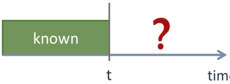

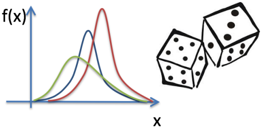

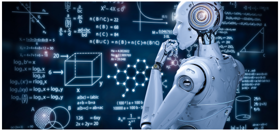

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Can imperfect predictions improve rigorous performance guarantees?

Motivating Example: Binary Search [Kraska et al. SIGMOD 2018]

n elements

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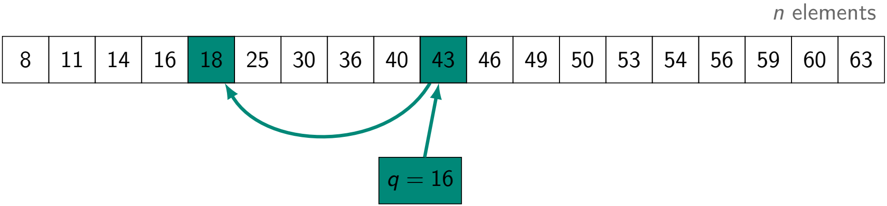

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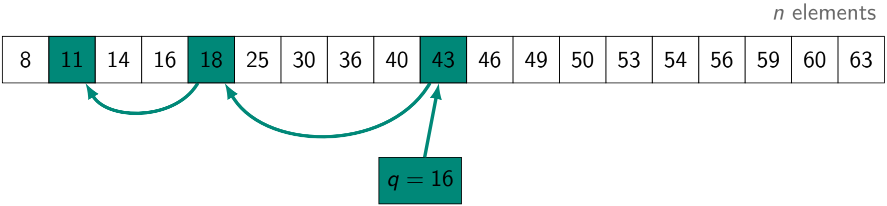

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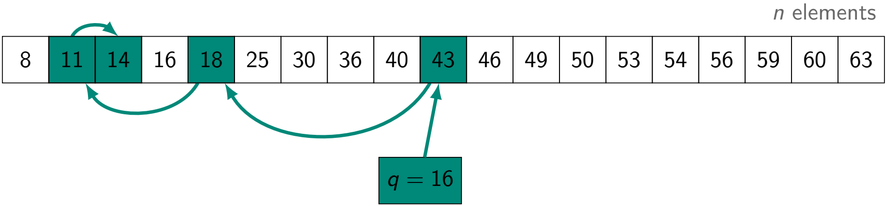

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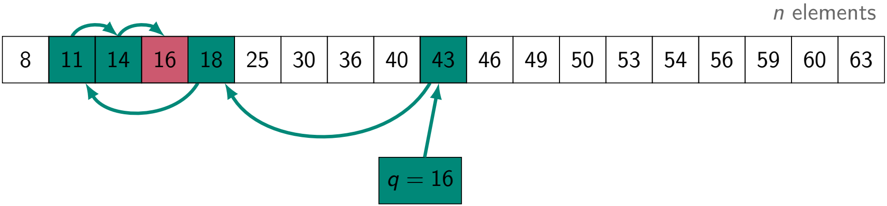

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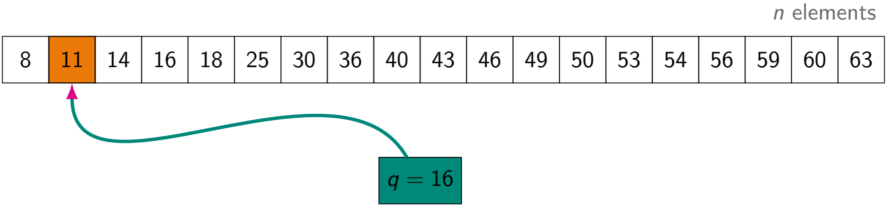

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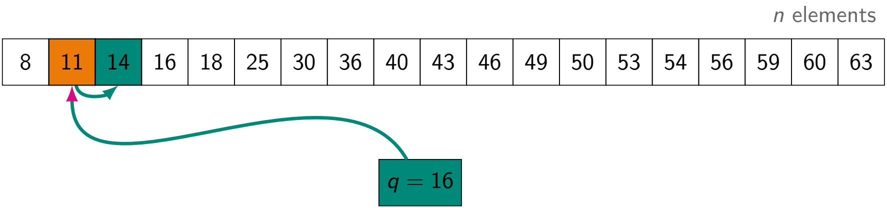

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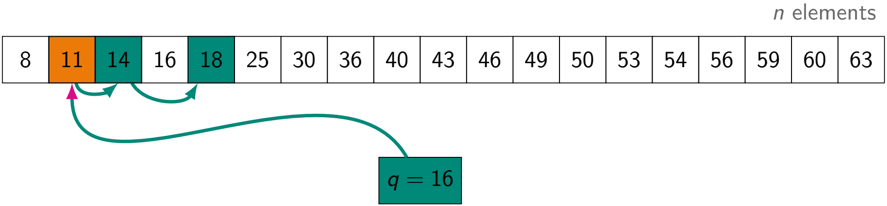

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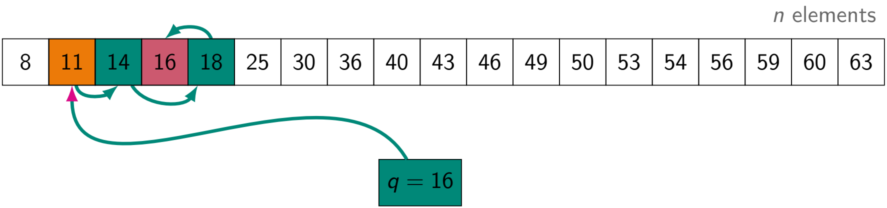

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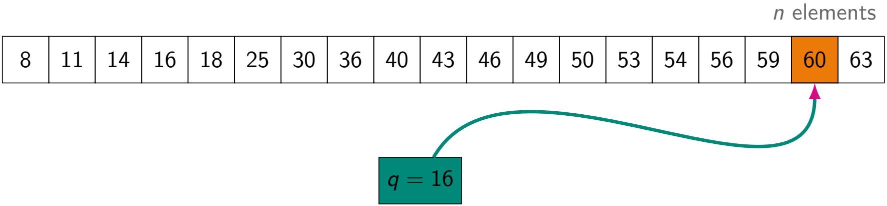

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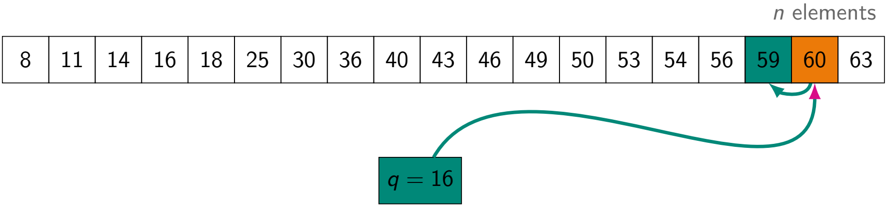

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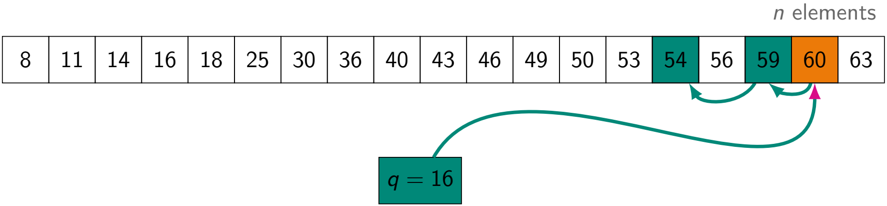

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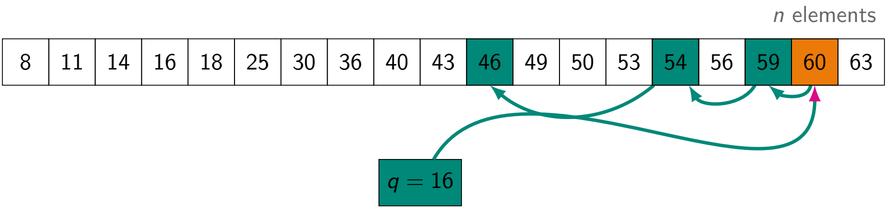

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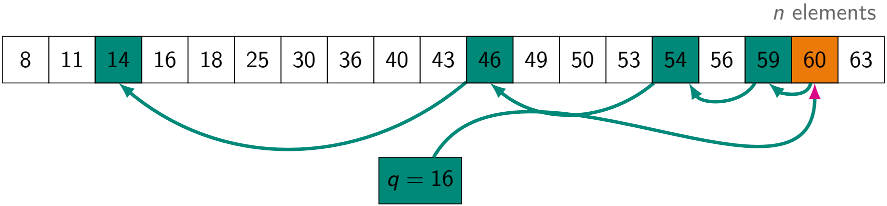

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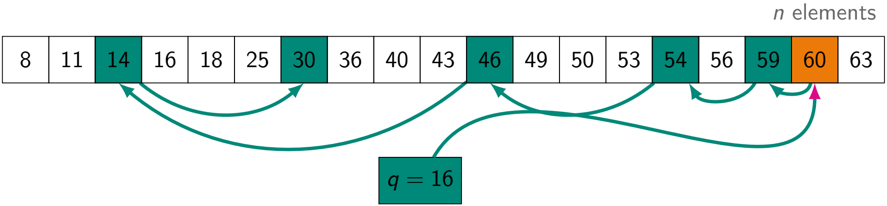

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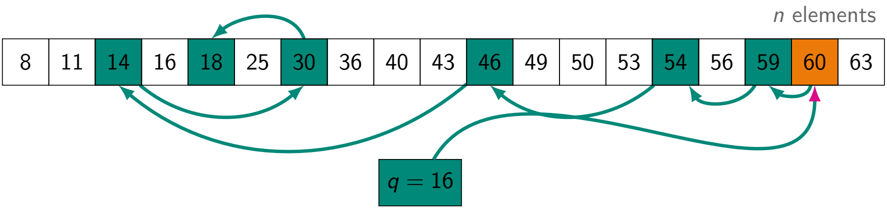

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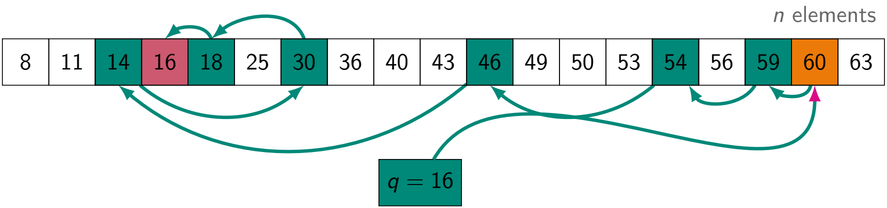

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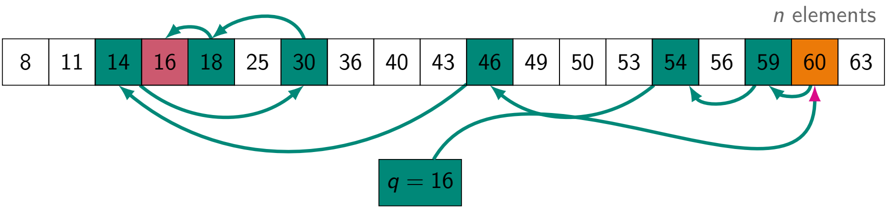

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$$\boxed{\Theta(\log n)} \xrightarrow{\text{predictions}} \boxed{\Theta(\log \eta)}$$

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- Prediction is imperfect
- No information about their quality

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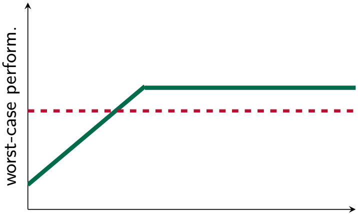
Desired algorithm properties

- Consistency: better than worst case if the prediction errors are small
- Robustness: bounded worst case for arbitrary predictions
- Error-dependency: graceful degradation with the error



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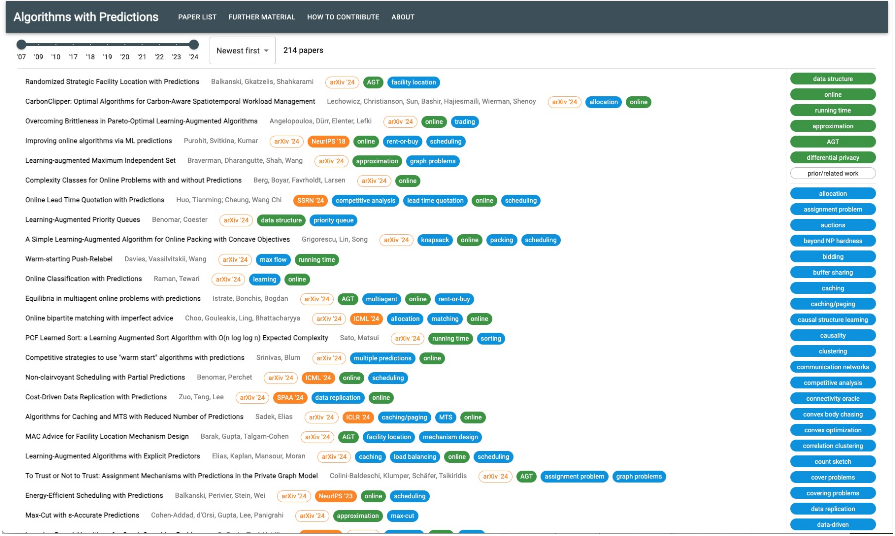
Line of research (re)initiated by [Lykouris, Vassilvitskii (ICML 2018)], [Kraska et al. (SIGMOD 2018)] – became an extremely vibrant area

Paper Repository https://algorithms-with-predictions.github.io/



hosted by Alex Lindermayr and M.

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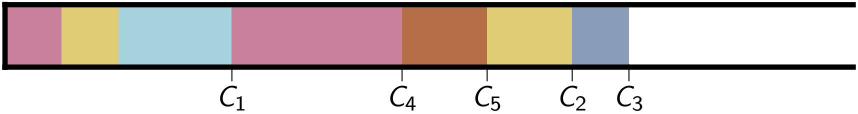
Goal: schedule jobs (preemptively) on a single machine



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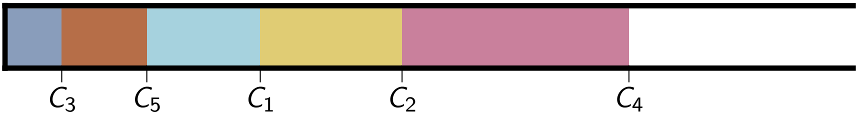

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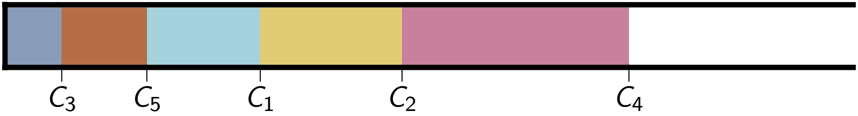

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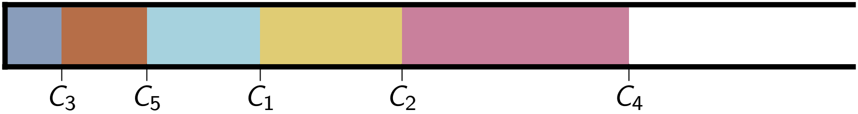

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Unrelated machines $(R|r_j, pmtn|\sum w_j C_j)$: 1.99-approximation [Im, Li 2016]

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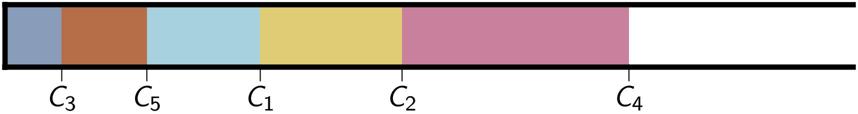

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Suppose processing times are unknown! (non-clairvoyant scheduling)

Non-Clairvoyant Min-Sum Scheduling

Processing times are unknown.

We cannot expect to find the optimal solution.

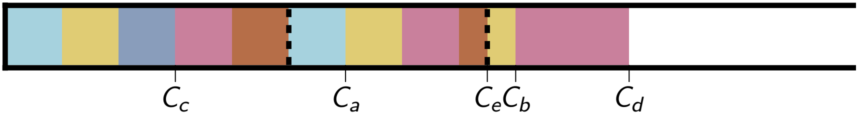
Competitive analysis (worst-case analysis)

An online algorithm is ρ-competitive if it achieves, for any input instance, a solution of cost within a factor ρ of the optimal cost:

$$\textsc{Alg}(I) \leq \rho \cdot \textsc{Opt}(I), \quad \text{for any input } I.$$

Non-Clairvoyant Min-Sum Scheduling

Round-Robin (RR) is 2-competitive for minimizing $\sum_j C_j$ on a single machine, and this is best-possible. [Motwani, Phillips, Torng 1994]



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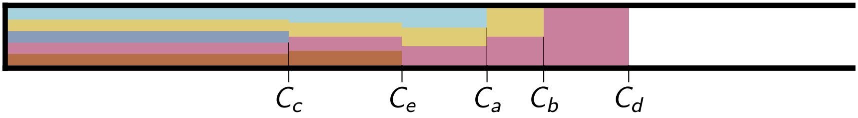

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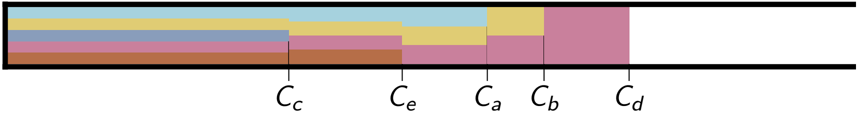

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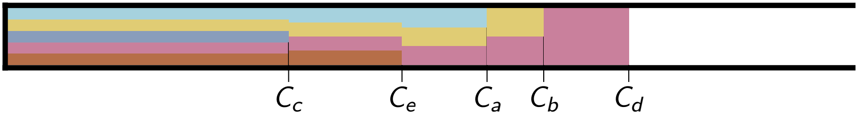

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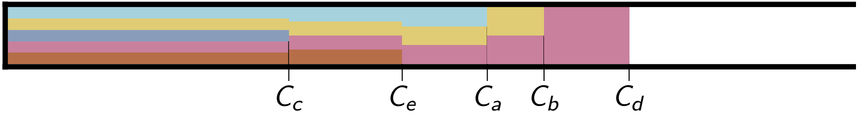

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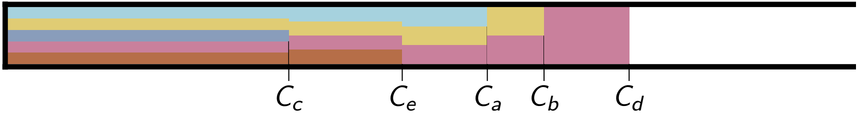

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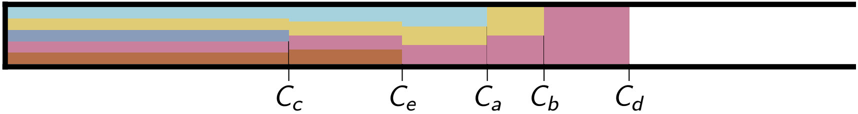
$$C_3 = n \cdot p_1 + (n-1) \cdot (p_2 - p_1) + (n-2) \cdot (p_3 - p_2 - p_1) \leq p_1 + p_2 + (n-2) \cdot p_3$$

Non-Clairvoyant Min-Sum Scheduling

Round-Robin (RR) is 2-competitive for minimizing $\sum_j C_j$ on a single machine, and this is best-possible. [Motwani, Phillips, Torng 1994]



Proof. Let $p_1 \leq p_2 \leq \ldots \leq p_n$.

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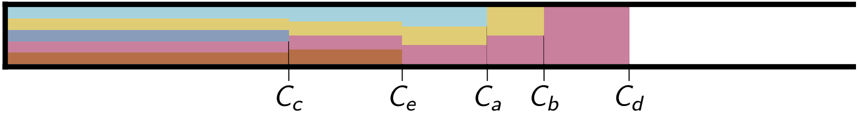

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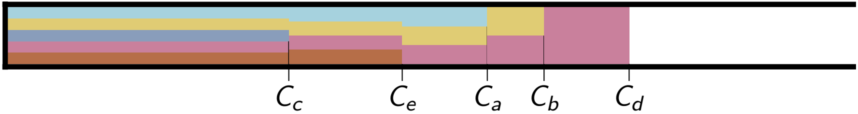

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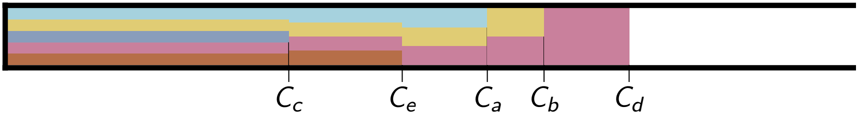

Further **Time-Sharing** algorithms for more general problems:

- Individual job weights: Weighted Round-Robin (2-competitive) [Kim, Chwa 2003]
- Identical machines: Weighted Dynamic Equipartition (2-comp.) [Beaumont, Bonichon, Eyraud-Dubois, Marchal 2012]
- Unrelated machines: Proportional Fairness (128-comp., 4.62-comp.) [Im, Kulkarni, Munagala 2018], [Lindermayr, **M.**, Jäger 2024]

Prediction Model

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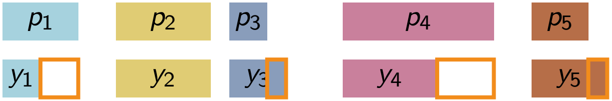
Predict job lengths y_j [Kumar, Purohit, Svitkina, NIPS 2018]



ℓ_1-error

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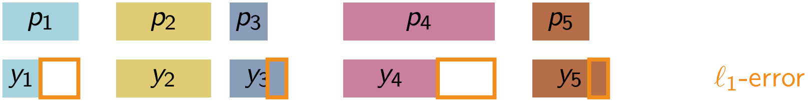

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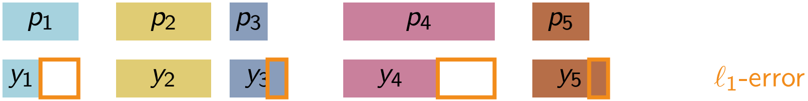

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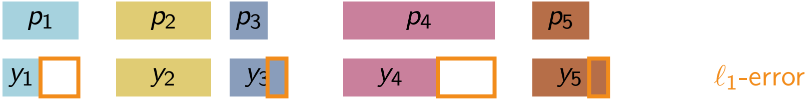

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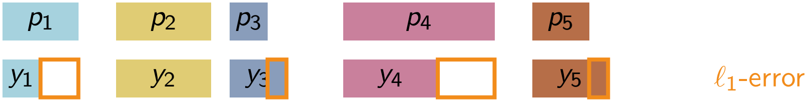

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Consistent but not robust (against bad predictions).

Preferential Time Sharing Framework (PTS)

[Kumar, Purohit, Svitkina 2018], [Lindermayr, **M.** 2022]

Input:

- prediction-clairvoyant alg. $\mathcal{A}^C$ ("follow the prediction") with some error-dependent competitive ratio
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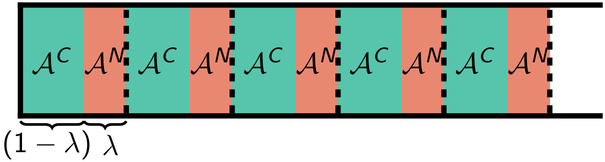

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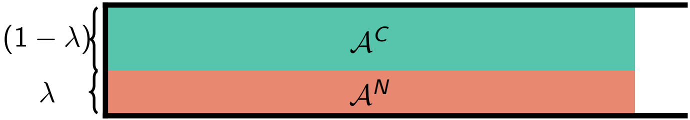

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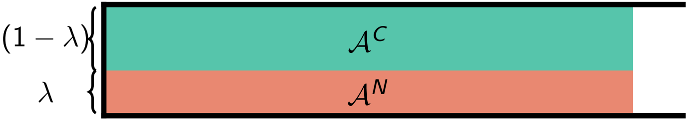

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Analysis: slowed down execution by factors $\frac{1}{1-\lambda}$ resp. $\frac{1}{\lambda}$

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PTS($\lambda, \mathcal{A}^C, \mathcal{A}^N$) has competitive ratio $\min\left\{\frac{1}{1-\lambda}\left(\alpha + \frac{\eta}{\textsc{Opt}}\right), \frac{\beta}{\lambda}\right\}$, if

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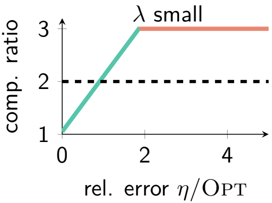

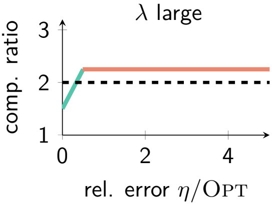

Powerful Framework?!

Yes, works for more general scheduling problems [Lindermayr, **M.** 2022]

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- Do we really need to predict all the job lengths?

Permutation Predictions

Lindermayr and **M.** 2022

Permutation predictions: predict an order of jobs: $\hat{\sigma} : [n] \to [n]$

Motivation: knowing WSPT order is often sufficient for good approximations:

- optimal for $1|(pmtn)|\sum w_j C_j$ [Smith 1956]
- 2-competitive for $P|r_j, pmtn|\sum w_j C_j$ [**M**. & Schulz 2004]
- 5.83-competitive for $R|r_j, pmtn|\sum w_j C_j$ [Lindermayr & **M**. 2022]

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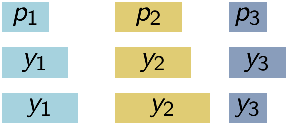

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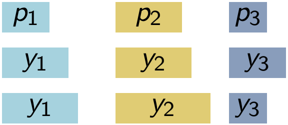

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$$\eta^S = \sum_{(i,j)\in\mathcal{I}} (w_i p_j - w_j p_i)$$

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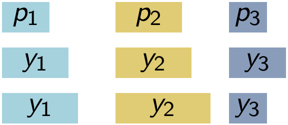

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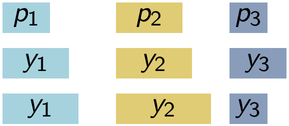

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- For $1||\sum w_j C_j$ this is exactly $\eta^S = \textsc{Opt}(\hat{\sigma}) - \textsc{Opt}(\sigma)$.
- η^S captures structure instead of irrelevant numerical values.

Scheduling on a Single Machine

PTS for weighted jobs on a single machine $1|pmtn|\sum w_j C_j$

1. prediction-clairvoyant $\mathcal{A}^C$: WSPT (optimal) [Smith56]

Schedule jobs in WSPT order

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WSPT order also useful for more general scheduling settings.

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PTS for multiple machines and release dates $P|r_j, pmtn|\sum w_j C_j$

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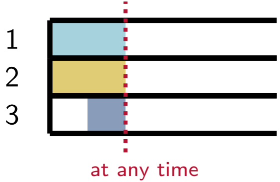

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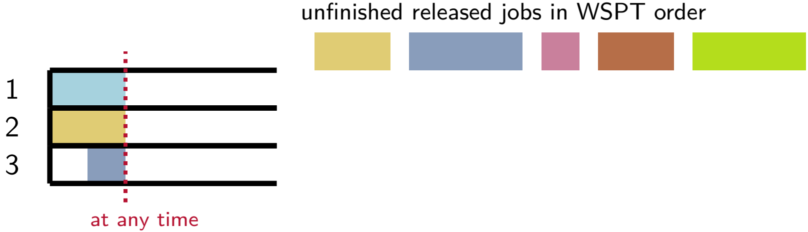

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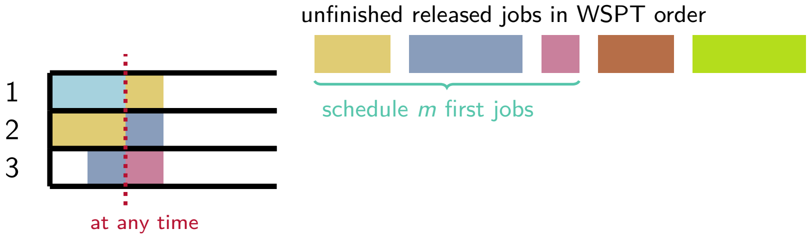

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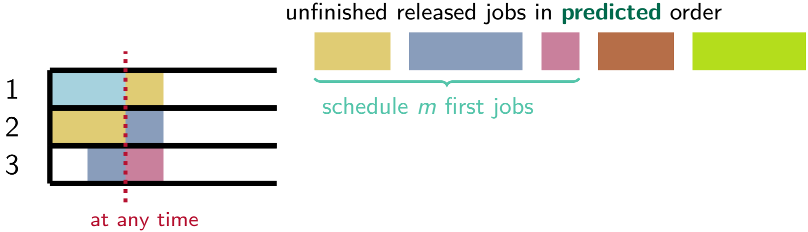

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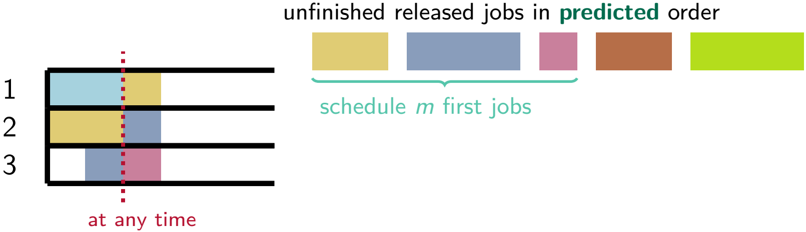

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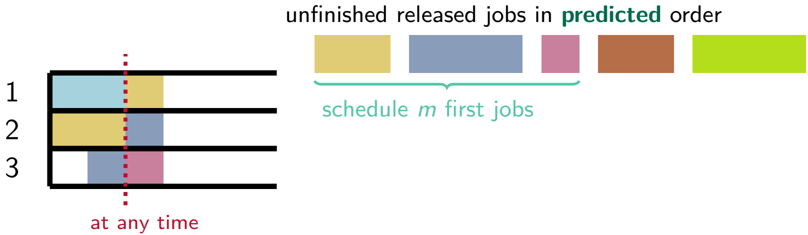

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PTS on unrelated machines $R|r_j, pmtn|\sum w_j C_j$

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   Known algorithms may not always work!

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   We can design such algorithm, but it is worse than the non-clairvoyant algorithm Proportional Fairness (PF).

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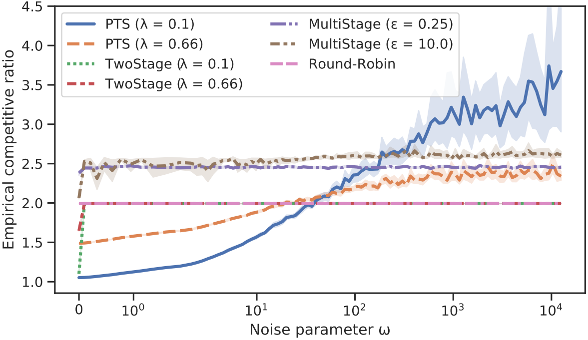
2. non-clairvoyant $\mathcal{A}^N$: Proportional Fairness (4.62-competitive)

[Lindermayr, **M.**, Jäger 2024]

Sensitivity Experiments

- Single machine, unweighted jobs
- Synthetic instances sampled from Pareto-distribution with shape 1.1
  Many small jobs and few very large jobs! (common)



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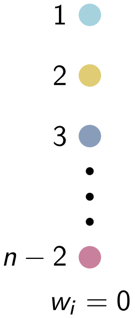

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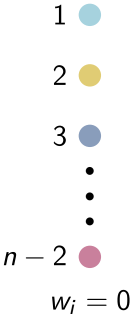

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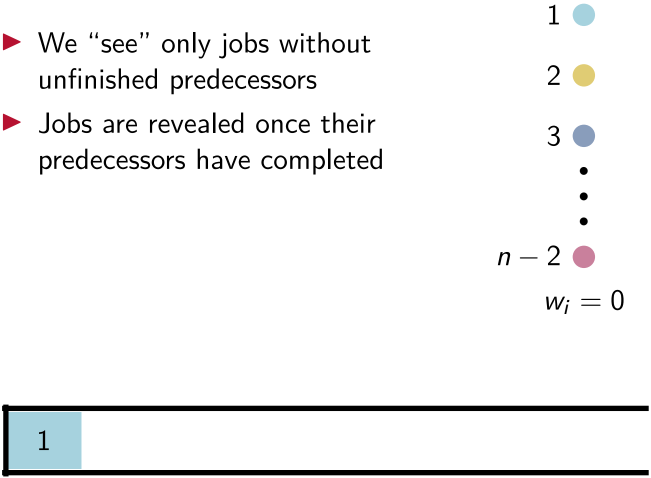

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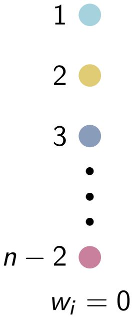

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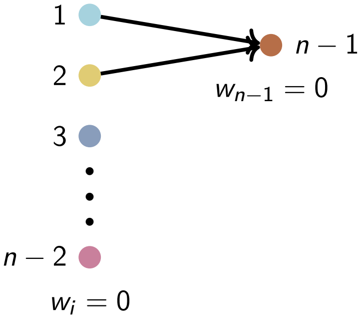

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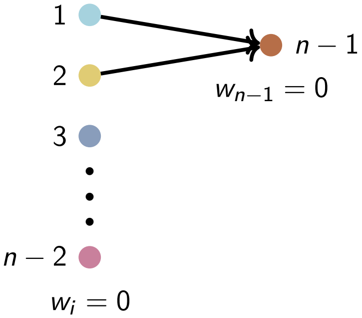

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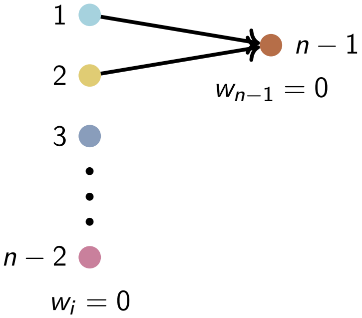

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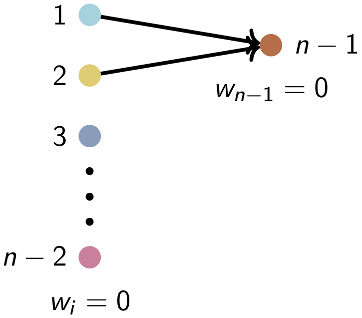

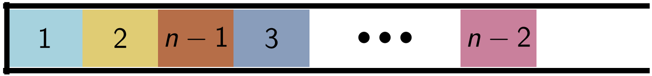

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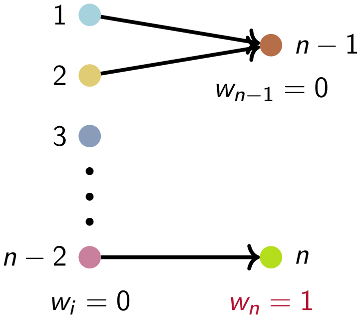

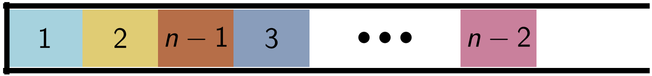

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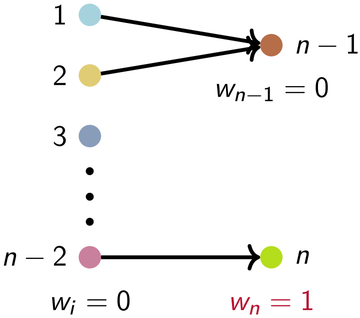

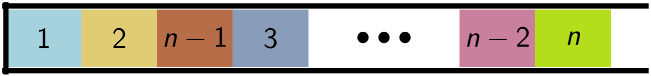

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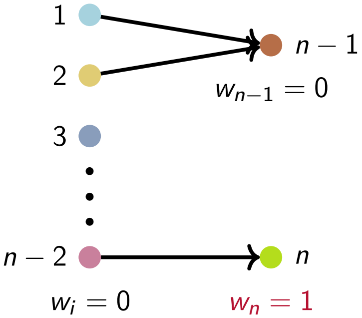

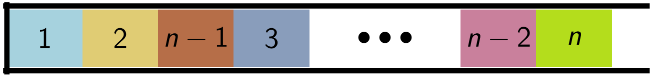

$\sum_{j=1}^{n} w_j C_j = n$

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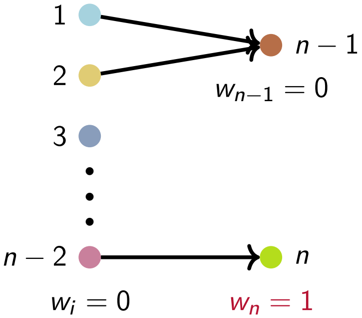

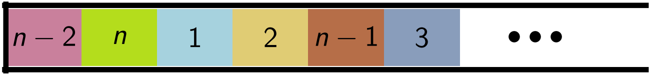

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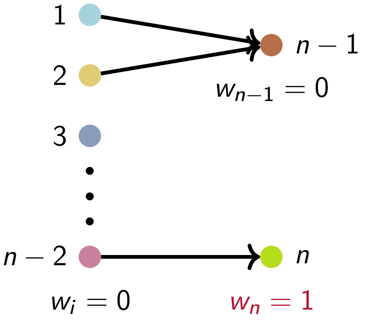

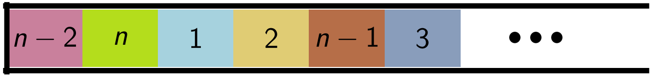

$\sum_{j=1}^{n} w_j C_j = 2$

Online Precedence Constraints (min $\sum_j w_j C_j$)

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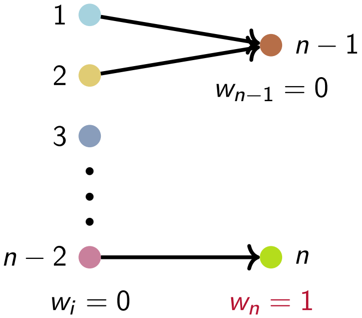
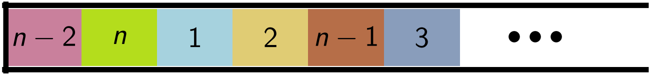
Any online algorithm has a competitive ratio $\Omega(n)$!





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Full Input Prediction – PTS Framework

[Lassota, Lindermayr, **M.**, Schlöter, ICML 2023]

Predict the full instance or a permutation of jobs

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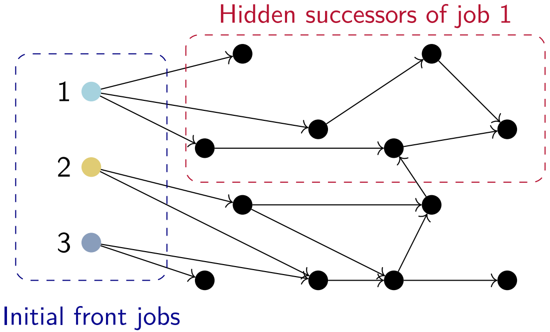

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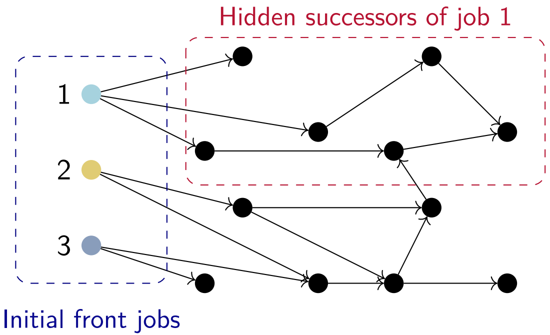
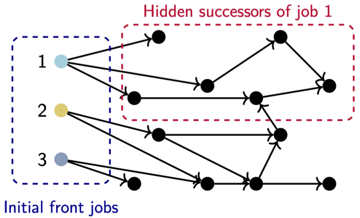


Theorem. Preferential Time Sharing is $\mathcal{O}(\min\{1+\eta, \omega\})$-competitive.

Minimalistic Input Prediction

What additional information is needed to improve upon lower bound?

$\rightarrow$ exact information!



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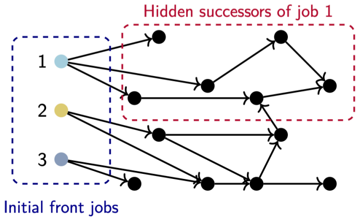

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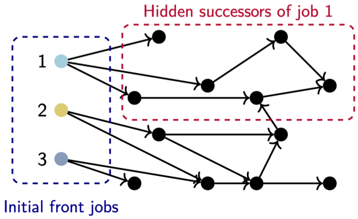

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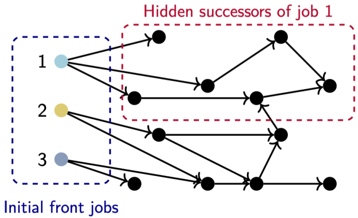

Full input

Total successor weight

Avg. successor weight

Weight order

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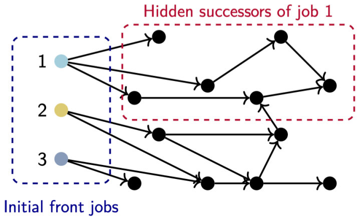

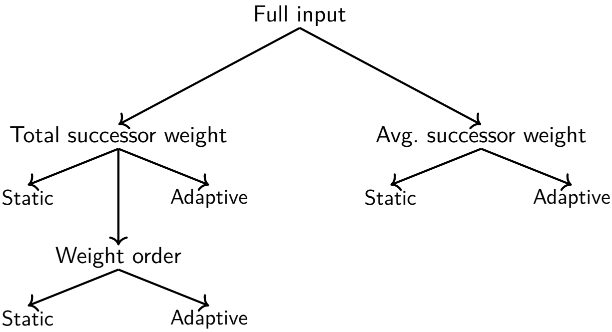

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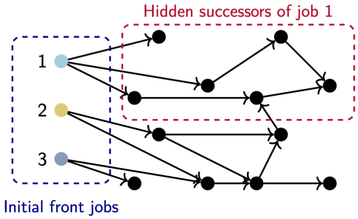
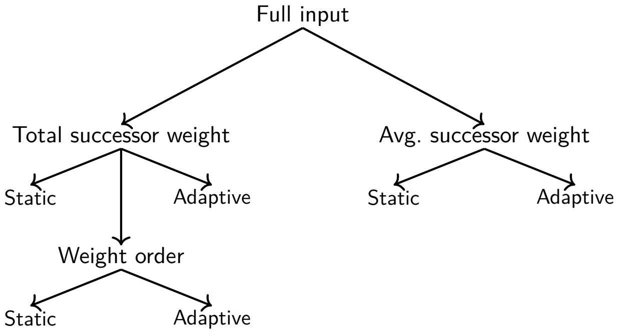
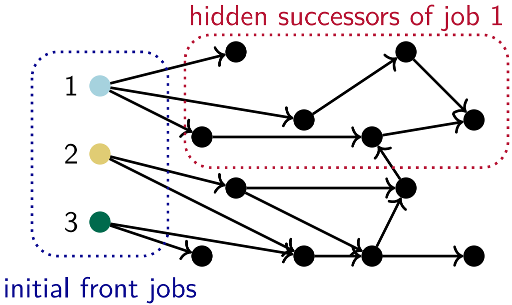




… results for different topologies – not today.

Weight Prediction based on Graph Decomposition

[Jäger & Warode, 2024]



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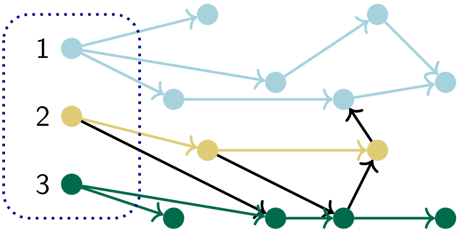

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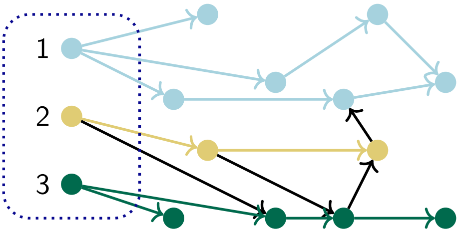

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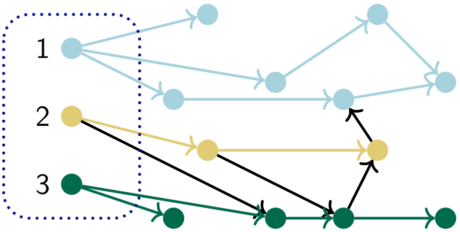

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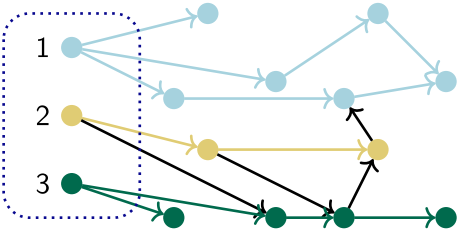

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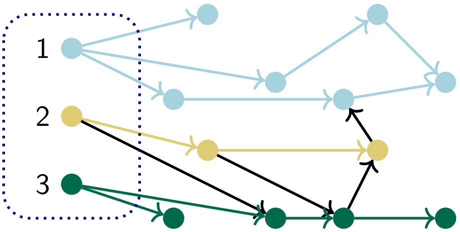

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Theorem. The time sharing framework is $\mathcal{O}(\min\{2+\eta, \omega\}$-competitive.

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