Optimization and OR:
A Sketch of Historical Developments

Lecture at CO@Work 2024
Computational Optimization at Work
September 16, 2024

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- Institut für Mathematik, Technische Universität Berlin (TUB) (1991-2015, im Ruhestand)
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- DFG-Forschungszentrum "Mathematik für Schlüsseltechnologien" (MATHEON) (2002-2014)
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Introductory Remark

If one looks back into recorded history or at the analysis of archaeological findings, it is clear that humans have always tried to act, build, and work efficiently – subject to available knowledge and technology.

OR and optimization seem to be genuine human features. I plan to briefly sketch some of these historical developments starting in antiquity, but my focus will be on the details of the advancements in optimization (in particular linear and integer programming) in the last 70 years.

One part of my lecture will be staged as a quiz. Some of the answers can be found in the book "Optimization Stories" that I edited in 2012. The PDF of this book can be downloaded from my homepage at https://www.zib.de/userpage//groetschel/publications/OptimizationStories.pdf

or from the Webpage of Documenta Mathematica: https://www.elibm.org/article/10011477

Operations Research (OR) is a difficult discipline

- OR is interdisciplinary.
- In the academic world, OR may be located in departments or faculties of mathematics, management science, economics, computer science, industrial or other types of engineering.
- In industry the positioning of OR specialists is similarly fuzzy.
- There is almost no situation (and there are very few companies) where OR forms the core of some organization.
- Companies rarely advertise OR jobs or claim that they are OR specialists.
- All this makes it difficult to pursue OR careers in industry or academia.

EURO Gold Medal winners on Operational Research

- S. Vajda, (article on EURO Gold Medal Ceremony)
  "The three ages of Operational Research",
  European Journal of Operational Research 45(1990)131-134
  "I believe that the term fits awkwardly those activities which OR comprises now, but it is too late to change."
- J. Krarup, "EURO Gold Medal 1986: A parable on two-level parallelism",
  European Journal of Operational Research 38(1989)274-276
  "…an interdisciplinary bastard like operational research…"
- R. Burkard, "OR Utopia"
  European Journal of Operational Research 119(1999)224-234
  "The borders of OR Utopia have yet another quality: people can come and go, without passport. There is no quota for foreigners…
  OR Utopia…is a peaceful border between OR, mathematics, and computer science, …management science, economy, logistics, … "

Operations Research: approaching it as a subject

- OR as a topic in mathematics
  - optimization
  - mathematical programming
- OR as a topic in management science / business administration
  - operations management
  - management engineering
- OR as a topic in engineering
  - industrial engineering
  - supply chain management/flexible manufacturing
- OR as a topic in computer science
- OR as a topic in psychology/sociology
- Systems Theory/Cybernetics
- Decision Sciences, Decision Aid
- …

The name

- What makes it also bad is that the name is unstable.
- It is changing over time and from country to country.
- Even worse, the various names for OR are not well understood – not at all by the general public, but even people close to the methodology of OR do not know OR and what it does.

What names do we know for OR?

English:

- Operations Research (North America and most other countries)
- Operational Research (United Kingdom)
- OR: The science of better (INFORMS)
- INFORMS even coined the name Analytics some years ago

German:

- Unternehmensforschung (West Germany)
- Unternehmungsforschung (West Germany)
- Operationsforschung (East Germany)
- now mostly Operations Research
- and many more names elsewhere

Google Search on September 14, 2024

- "**operations research**"
  about 27.400.000 results
- "**operational research**"
  about 11.100.000 results
- **"analytics"**
  about 4.200.000 results
  (and almost no entries related)
- **"Unternehmensforschung"**
  about 123.000 results
- **"Unternehmungsforschung"**
  about 123.000 results
- **"Operationsforschung"**
  about 11.300 results
  (Mostly from medicine, though)

Name consultants are needed

My name proposal: Switch to Chinese

运筹学

My proposal: Switch to Chinese

运筹学

- First character means: move.
  (It is also part of the words transportation and logistics.)
- Second character means: plan or prepare.
  (Chinese military 2000 years ago: Army may win by good planning. I can win the war 10.000 miles away.)
- The first two characters together literally mean:
  „moving the bamboo sticks".
  The actual meaning is „making a plan" „making decisions".
- Last character means: study or subject or research.
  (Last character plus person means student.)

My proposal: Switch to Chinese

The Chinese name for OR (in my translation) means:
One has to consider various options to make good decisions.

Latin spelling: YUN CHOU XUE
Pronunciation: Üen tschou schüe

The Operational Research Society

History of OR (the British version)

Operational research (OR), also known as operations research, has its roots in the early 20th century, particularly during the World Wars. It emerged as a response to the complex logistical and strategic problems faced by military organisations. Here's a brief history:

World Wars: The origins of OR can be traced back to the British military efforts during the Great War and the Second World War. There are many examples, such as how the British Royal Air Force established the Operational Research Section to analyse and optimise military operations. Teams of scientists, mathematicians and engineers were tasked with solving various military problems, such as improving aircraft performance, optimising bomber tactics, and minimising losses.

The birth of operational research: The term "operational research" was coined in 1940 by A.P. Rowe, a British Air Ministry scientist. The phrase referred to the application of scientific methods to analyse and improve the effectiveness of military operations.

American Version of the History of OR

History and Traditions Committee of INFORMS

(Contact with Linus Schrage)

Current activities

The committee has undertaken a multi year project to collect historical materials about operations research and management science. The History and Traditions web site will be enhanced to assist in accessing these materials directly or via links or guidance to external locations. Watch for changes to the History and Traditions web site in 2020 and beyond.

Oral Histories

Many oral interviews of American OR persons, (last entries 2021, 2019)

My advice

- Let us define the field OR .
  (OR stands for itself, without interpretation of the abbreviation)

- Don’t care about definitions of OR.
- Be flexible.
- Enjoy the stuff that you do.
- Position yourself depending on your own needs, goals, and wishes and that of your company or academic institution.

- It is a miracle that OR has survived through the last 80 years, having undergone many battles, splits, (re-)unifications and name changes. This is good sign for robust health and indicator for longevity of the field – independent of the name mutations.

What is good OR?

- In OR you are not supposed to work in isolation.
- OR is a team effort relying on the knowledge of many persons working in different subjects and having a multitude of experiences.

- Important: Stay in touch with everybody.

https://www.zib.de/userpage//groetschel/publications/mybooks.html

Documenta Mathematica,
Extra Vol.,
Optimization Stories (2012)
https://elibm.org/article/10011477

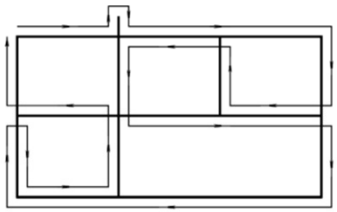
DOCUMENTA MATHEMATICA

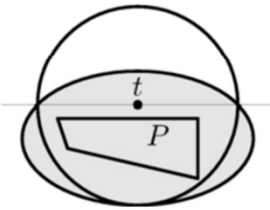
JOURNAL DER
DEUTSCHEN MATHEMATIKER-VEREINIGUNG
GEGRÜNDET 1996

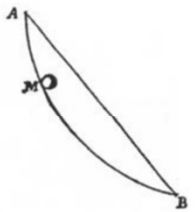
EXTRA VOLUME

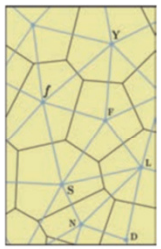
OPTIMIZATION STORIES

21ST INTERNATIONAL SYMPOSIUM ON MATHEMATICAL PROGRAMMING
BERLIN, AUGUST 19–24, 2012





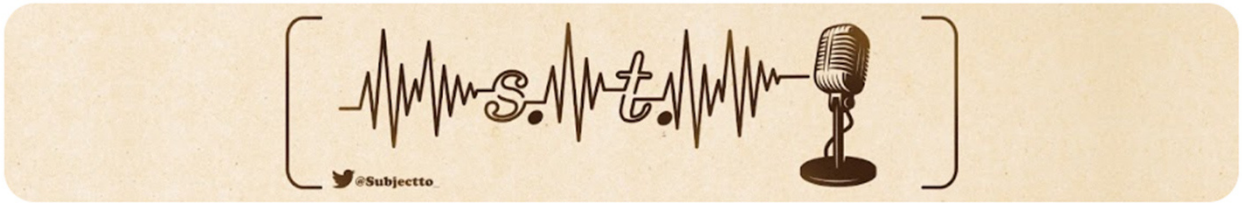




EDITOR:
MARTIN GRÖTSCHEL

Advertisement: Anand Subramanian's s.t.

"Subject to" offers a series of informal conversations with relevant figures in the fields of Operations Research, Combinatorial Optimization and Logistics, they are hosted by Anand Subramanian, an Associate Professor at Universidade Federal da Paraíba (UFPB), Brazil.



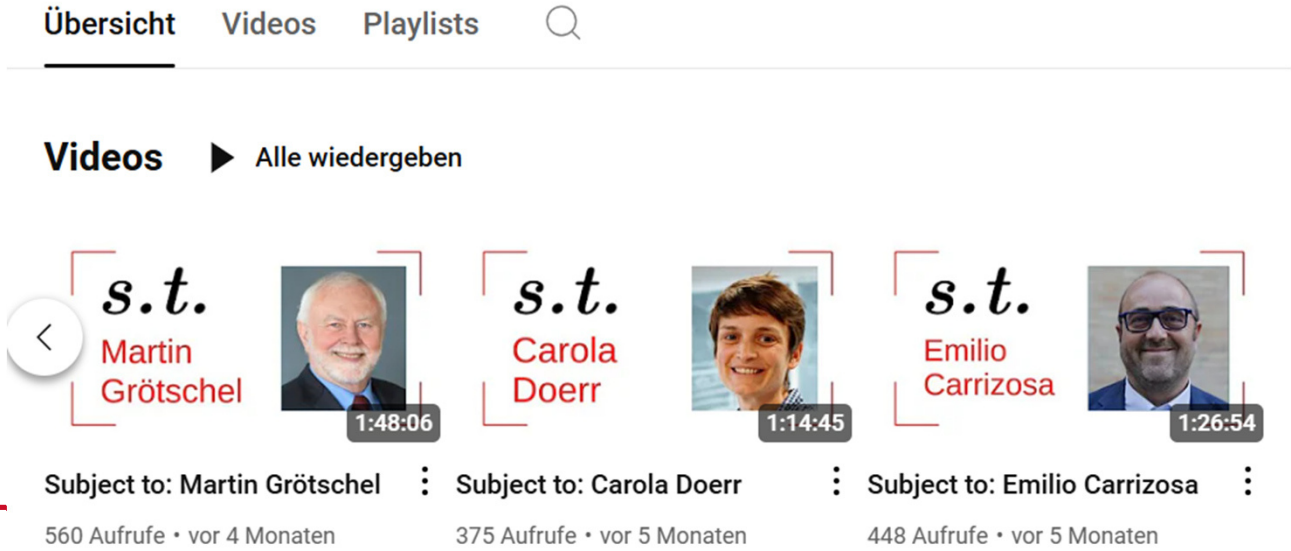
s.t. **Subject to**

@Subjectto_ · 2950 Abonnenten · 95 Videos

"Subject to" offers a series of informal conversations with relevant figures in the fields o ...mehr

twitter.com/Subjectto_ und 4 weitere Links

https://www.youtube.com/@Subjectto_/about



Programming vs Optimization (Optimisation)

In German:

Programmierung vs Optimierung

Conflicts in Libraries (for instance)

MPS 2007

mps
Mathematical Programming Society

Home | Meetings | Join MPS | Members | Contact
About MPS | Publications | Prizes | Links | Album

Welcome to the website of the Mathematical Programming Society

Good Morning!
Today is
Thursday, September 12.

■ Search the Member Directory

What is mathematical programming

News...

■ **Election Results 2006**

- *Chair-Elect: Steve Wright*
- *Treasurer: David Gay*
- *Council Members-at-Large: Frédéric Bonnans, Alberto Caprara, Andreas Schulz, Shuzhong Zhang*

Recent and upcoming meetings:

ICCOPT 2007
McMaster University, Hamilton, Ontario, Canada, August 12-17, 2007

IPCO 2007
Cornell University, Ithaca, NY, USA, June 25-27, 2007

ISMP 2006
Federal University of Rio de Janeiro in Praia Vermelha, Rio de Janeiro, Brazil, July 30 to August 4, 2006

Further Meetings ...

Publications

SERIES A
SERIES B

Coming soon: an MPS members TSP!

top of page

Meet MPS Chair Rolf Möhring, and all of the MPS officers.

The MPS online photo and history album: Contributors from all across optimization.

Optimization Online
mps an eprint site for the optimization community

More links related to mathematical programming.

Mathematical Programming Society | IEEE | GML | Webmaster | Contact

MOS 2024

Home | Contact | About MOS | Membership | Publications | Prizes | Meetings | IPCO | ISMP | Links | Album | Jobs

Mathematical Optimization Society

News

Inaugural International MIP Workshop to be held at ITT Mumbai, India (02/2024)

ICCOPT 2025 will be held at USC (01/2024)

Call for nominations: Fulkerson Prize 2024 (11/2023)

Call for site preproposals ISMP 2027 (11/2023)

More News

Welcome to the website of the Mathematical Optimization Society

The Mathematical Optimization Society (MOS), founded in 1973, is an international organization dedicated to the promotion and the maintenance of high professional standards in the subject of mathematical optimization. Up to 2010 its name was "Mathematical Programming Society (MPS)".

The Society

What is the Mathematical Optimization Society?

Meet MOS chair Miguel Anjos, and all of the MOS officers.

What is mathematical optimization?

More links related to mathematical optimization.

Follow us on X (formerly known as Twitter)

Recent and upcoming meetings:

- **MIP 2024**
  University of Kentucky, USA, June 3-6, 2024
- **IPCO 2024**
  Wrocław, Poland, July 3-5, 2024
- **ISMP 2024**
  Montréal, Canada, July 21-26, 2024
- **International MIP 2024**
  Mumbai, India, December 2-6, 2024
- **ICCOPT 2025**
  University of Southern California, USA, July 21-24, 2025
- **Further Meetings ...**

MOS Today

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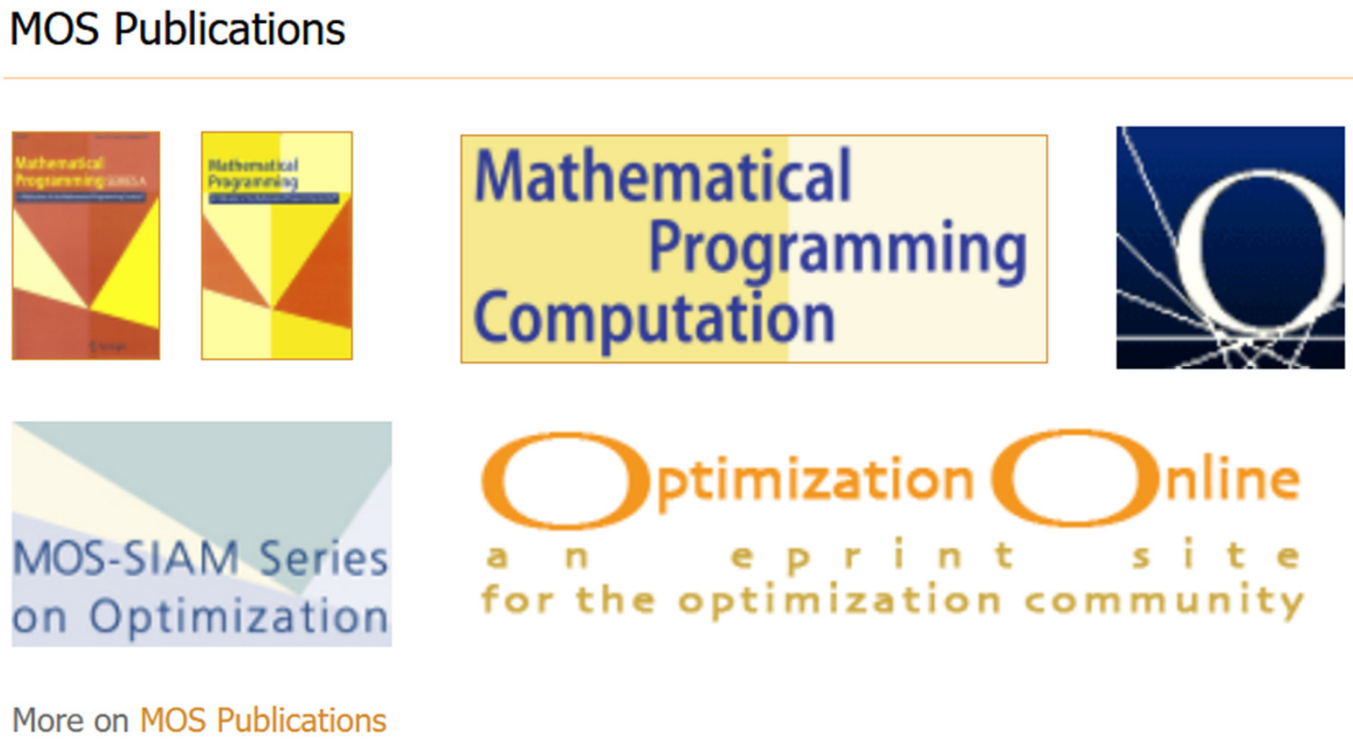

ZIB's MOS Involvement

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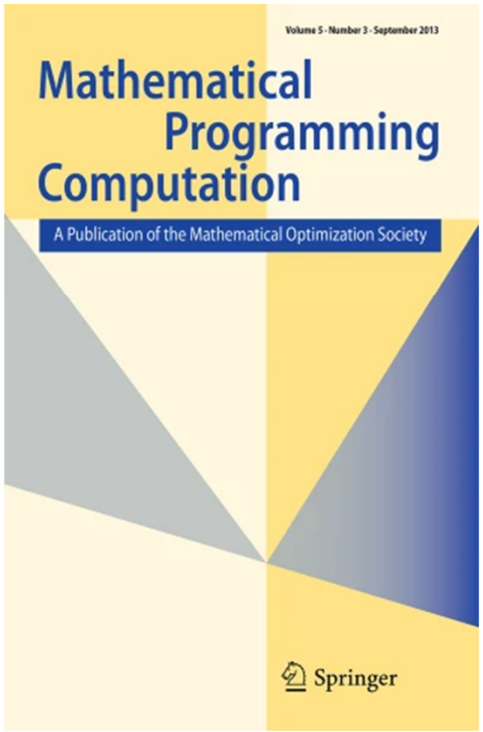

This Journal was conceived at ZIB with Thorsten Koch as driving force.

Many current and former members of ZIB act on various branches of the Editorial Board.

Mathematical Programming Computation (MPC) publishes original research articles advancing the state of the art of practical computation in Mathematical Optimization and closely related fields. Authors are required to submit software source code and data along with their manuscripts (while open-source software is encouraged, it is not required). Where applicable, the review process will aim for verification of reported computational results.

Question 0 of our History Quiz

Where does the term algorithm come from?

Musa al-Khwarizmi (~780 – ~ 835/850)

Monument for Abu Ja'far Muhammad ibn Musa al-Khwarizmi in Khiva/Xiva/Chiwa (Uzbekistan)

First systematic solution theory for linear and quadratic equations!

Algebra

Algorithmus

I was in Khiva in May 2024. This al-Khwarizmi monument has disappeared. Nobody could tell me to which other place the monument has been dislocated.

Picture by Manuel Cohen taken on June 6, 2010
https://www.manuelcohen.com/image/I00005WA.rb0ScBE

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The historical LP development

1. Linear equations: First appearance?

Linear equations: First appearance?

- It is impossible to find out who knew what when first.
- Just two "references":
- Egyptians and Babylonians considered about 2000 B.C. the solution of special linear equations.
  But, of course, they described examples and did not describe the methods in "today's style".
- What we call "Gaussian elimination" today has been explicitely described in the Chinese "Nine Chapters on the Mathematical Art" which is a compendium written in the period 10th century B.C. to 2nd century A.D., but the methods were probably known long before that.

- Carl Friedrich Gauß, by the way, never described: "Gaussian elimination".
  He just used it and stated that linear equations can be solved "per eliminationem vulgarem".

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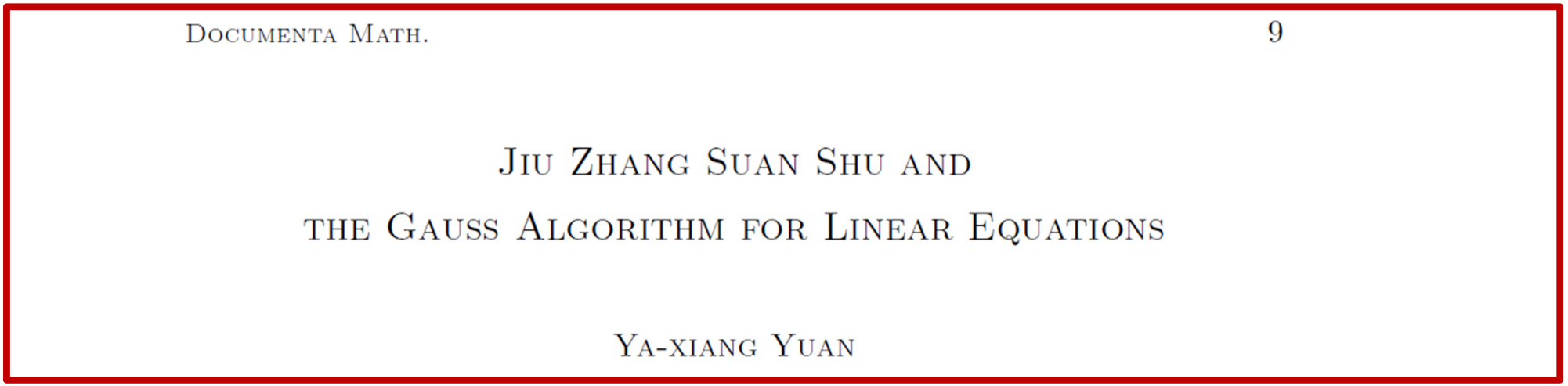

Documenta Math. 9

Jiu Zhang Suan Shu and
the Gauss Algorithm for Linear Equations

Ya-xiang Yuan

- Carl Friedrich Gauß, by the way, never described: "Gaussian elimination".
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The historical LP development

1. Linear equations: First appearance?
2. First formulation of a linear program instance

Who formulated the first LP instance?

This is, again, unknown, but the usual credit goes to George J. Stigler

Min x1 + x2	costs
2x1 + x2 ≥ 3	protein
x1 + 2x2 ≥ 3	carbohydrates
x1 ≥ 0	potatoes
x2 ≥ 0	beans

minimizing the cost of food

Stigler's „Diet Problem":
„The first linear program"
published in 1945

George J. Stigler
Nobel Prize in economics 1982
https://en.wikipedia.org/wiki/George_Stigler

Sets n nutrients / calorie thousands , protein grams , calcium grams , iron milligrams vitamin-a thousand ius, vitamin-b1 milligrams, vitamin-b2 milligrams, niacin milligrams , vitamin-c milligrams /

f foods / wheat , cornmeal , cannedmilk, margarine , cheese , peanut-b , lard liver , porkroast, salmon , greenbeans, cabbage , onions , potatoes spinach, sweet-pot, peaches , prunes , limabeans, navybeans /

Parameter b(n) required daily allowances of nutrients / calorie 3, protein 70 , calcium .8 , iron 12 vitamin-a 5, vitamin-b1 1.8, vitamin-b2 2.7, niacin 18, vitamin-c 75 /

Table a(f,n) nutritive value of foods (per dollar spent)

	calorie (1000)	protein (g)	calcium (g)	iron (mg)	vitamin-a (1000iu)	vitamin-b1 (mg)	vitamin-b2 (mg)	niacin (mg)	vitamin-c (mg)
wheat	44.7	1411	2.0	365		55.4	33.3	441	
cornmeal	36	897	1.7	99	30.9	17.4	7.9	106	
cannedmilk	8.4	422	15.1	9	26	3	23.5	11	60
margarine	20.6	17	.6	6	55.8	.2			
cheese	7.4	448	16.4	19	28.1	.8	10.3	4	
peanut-b	15.7	661	1	48		9.6	8.1	471	
lard	41.7				.2		.5	5	
liver	2.2	333	.2	139	169.2	6.4	50.8	316	525
porkroast	4.4	249	.3	37		18.2	3.6	79	
salmon	5.8	705	6.8	45	3.5	1	4.9	209	
greenbeans	2.4	138	3.7	80	69	4.3	5.8	37	862
cabbage	2.6	125	4	36	7.2	9	4.5	26	5369
onions	5.8	166	3.8	59	16.6	4.7	5.9	21	1184
potatoes	14.3	336	1.8	118	6.7	29.4	7.1	198	2522
spinach	1.1	106		138	918.4	5.7	13.8	33	2755
sweet-pot	9.6	138	2.7	54	290.7	8.4	5.4	83	1912
peaches	8.5	87	1.7	173	86.8	1.2	4.3	55	57
prunes	12.8	99	2.5	154	85.7	3.9	4.3	65	257
limabeans	17.4	1055	3.7	459	5.1	26.9	38.2	93	
navybeans	26.9	1691	11.4	792		38.4	24.6	217	

Positive Variable x(f) dollars of food f to be purchased daily (dollars)

Free Variable cost total food bill (dollars)

Equations nb(n) nutrient balance (units), cb cost balance (dollars) ;

nb(n).. sum(f, a(f,n)*x(f)) =g= b(n); cb.. cost=e= sum(f, x(f));

Model diet stiglers diet problem / nb,cb /;

http://www.gams.com/modlib/libhtml/diet.htm

The historical LP development

1. Linear equations: First appearance?
2. First formulation of a linear program instance
3. First "method" in the literature that solves linear programs

Who had the first LP solver (without knowing it)?

- In 1827 Fourier described a variable elimination method for linear inequalities, today often called Fourier-Motzkin elimination (Motzkin 1936). Three variables!?
- By adding one variable and one inequality, Fourier-Motzkin elimination can be turned into an LP solver.

The historical LP development

1. Linear equations: First appearance?
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3. First "method" in the literature that solves linear programs
4. First "algorithm" in the literature that solves linear programs

Who described the first LP solver?

This is a subtle case.

- 1826/1827 Jean Baptiste Joseph Fourier (1786-1830):
  rudimentary form of the simplex method in 3 dimensions.
- 1939 L. V. Kantorovitch (1912-1986):
  Foundations of linear programming (Nobel Prize 1975)
- 1947 G. B. Dantzig (1914-2005):
  Invention of the (primal) simplex algorithm

The historical LP development

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5. First LP solver with impact in practice

Who described the first LP solver with impact in practice?

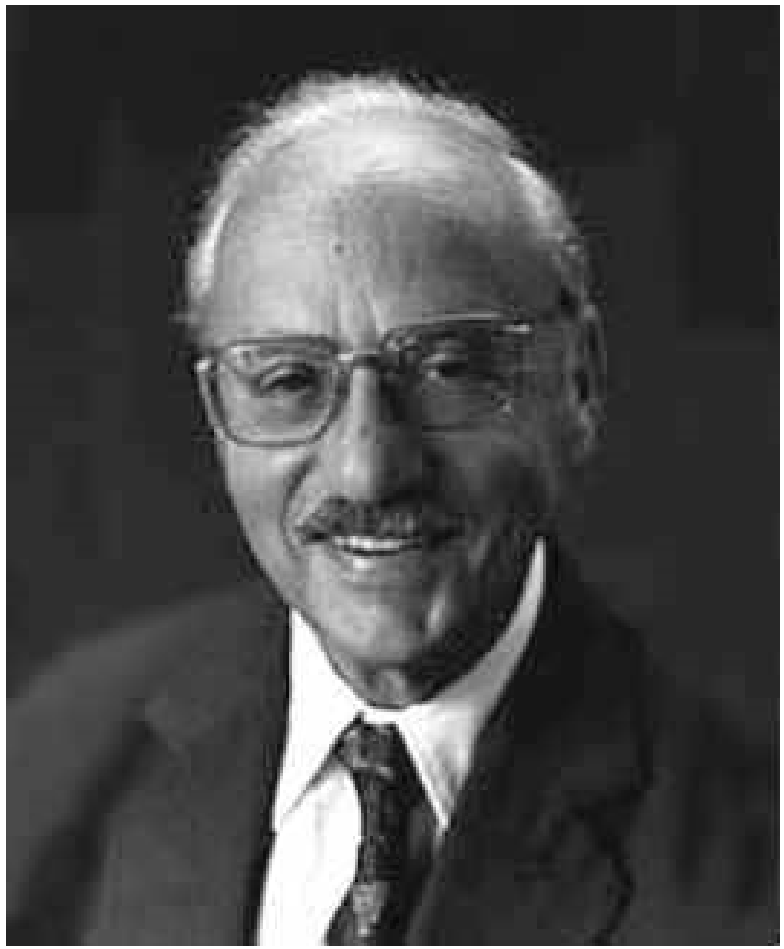
- 1947 G. B. Dantzig (1914-2005):

  Invention of the (primal) simplex algorithm

$$\max c^T x$$

$$Ax = b$$

$$x \geq 0$$



https://news.stanford.edu/stories/2005/05/george-b-dantzig-operations-research-professor-dies-90

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4. First "algorithm" in the literature that solves linear programs
5. First LP solver with impact in practice
6. (Currently) most frequently used LP solver?

Who described the currently most frequently used LP solver?

- 1954 C.E. Lemke:
  Dual simplex algorithm

- 1953 G.B. Dantzig,
  1954 W. Orchard Hays, and
  1954 G. B. Dantzig & W. Orchard Hays:
  Revised simplex algorithm

The historical LP development

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5. First LP solver with impact in practice
6. (Currently) most frequently used LP solver?
7. First commercial LP code?

Who implemented the first commercial LP code?

William Orchard-Hayes (in the period 1953-1954)

The first commercial LP-Code was on the market in 1954 and available on an IBM CPC (card programmable calculator).

An entire iteration could not be finished without manual intervention: the CPC had to be fed more card decks.

In early 1953, we solved few actual models with the second CPC setup. The largest was about 26 × 71. As I recall, it took eight hours of hard work feeding decks to the hoppers but the principle was clear—and indelibly impressed on my mind. We started on a true set of programs for the forthcoming 701, IBM's first real scientific computer.

embellishments. Most of my mnemonics have persisted: FTRAN, BTRAN, PRICE, UPDATE, and so forth. (One bad one, CHUZR for "choose r," I later replaced with PIVOT but references to CHUZR still crop up.)

Who implemented the first commercial LP code?

William Orchard-Hayes (in the period 1953-1954)

The first commercial LP-Code was on the market in 1954 and available on an IBM CPC (card programmable calculator):
Code: Simplex Algorithm with explicit basis inverse, that was recomputed in each step.

Shortly after, Orchard-Hayes implemented a version with product form of the inverse (idea of A. Orden)

Record: 71 variables, 26 constraints, 8 h running time

About 1960: LP became commercially viable, used largely by oil companies.

History of the Development of LP Solvers
Author(s): William Orchard-Hays
Source: *Interfaces*, Vol. 20, No. 4, The Practice of Mathematical Programming (Jul. - Aug., 1990), pp. 61-73
Published by: INFORMS
Stable URL: http://www.jstor.org/stable/25061371

IBM

- MPS

  MPS file format:
  MPS data files are analogous to a deck of computer input cards: each line of the **MPS** file represents a single card record. …
- MPSX, MPSX/370, MPSX-MIP/370 (The 370/145 introduced in 1970)

  IBM Mathematical Programming System Extended /370 (MPSX/370), Mixed Integer Programming/370 (MIP/370) Program Reference Manual
  Reference Number / ISBN : SH19-1099-1
  Date Published : November 1975
  Publisher : IBM
- My own experiences!

The historical LP development

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5. First LP solver with impact in practice
6. (Currently) most frequently used LP solver?
7. First commercial LP code?
8. Nobel Prize for linear programming

Who received a Nobel Prize for LP?

Optimal use of scarce ressources: foundation and economic interpretation of LP

Leonid V. Kantorovich Tjalling C. Koopmans

Nobel Prize for Economics 1975

https://www.nobelprize.org/prizes/economic-sciences/1975/kantorovich/facts/

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7. First commercial LP code?
8. Nobel Prize for linear programming
9. Expected polynomial running time of the simplex method

Who proved expected polynomial running time of the simplex method first?

- Borgwardt (1977) Schatteneckenalgorithmus
- Borgwardt (1982)

- Survey (in German):
  Wie schnell arbeitet das Simplexverfahren normalerweise?
  Oder: Das Streben nach (stochastischer) Unabhängigkeit
  Karl Heinz Borgwardt
- DOI 10.1515/dmvm-2014-0037
  MDMV 22 / 2014 | 81–93

The historical LP development

1. Linear equations: First appearance?
2. First formulation of a linear program instance
3. First "method" in the literature that solves linear programs
4. First "algorithm" in the literature that solves linear programs
5. First LP solver with impact in practice
6. (Currently) most frequently used LP solver?
7. First commercial LP code?
8. Nobel Prize for linear programming
9. Expected polynomial running time of the simplex method
10. First polynomial time linear equations solver

Who described the first polynomial time linear equations solver?

- Gaussian elimination (and other linear equations solvers) may lead, if not implemented with care, to exponential size rational numbers!

- See GLS, page 37-38

$$\begin{pmatrix} 2\,0\,0\,0\,\ldots\ldots\ldots\,0 \\ 1\,2\,0\,0\,\ldots\ldots\ldots\,0 \\ 1\,1\,2\,0\,\ldots\ldots\ldots\,0 \\ 1\,1\,1\,2\,0\,\ldots\ldots\ldots\,0 \\ . \\ . \\ 1\,1\,1\,1\,1\,1\,1\,\ldots\ldots\,2 \end{pmatrix}$$

- Edmonds (1967) showed how to compute "with care".

The historical LP development

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8. Nobel Prize for linear programming
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10. First polynomial time linear equations solver
11. First polynomial time LP solver

Who had the first polynomial time LP solver (without knowing it)?

- Fiacco & McCormick (1968)

- A.V. Fiacco, G.P. McCormick, *Nonlinear Programming: Sequential Unconstrained Minimization Techniques*, John Willey & Sons, 1968.

- Reprinted as *Classics* in Applied Mathematics 4, *SIAM*, 1990.

Who described the first polynomial time LP solver?

Доклады Академии наук СССР
1979. Том 244, № 5

УДК 519.95

МАТЕМАТИКА

Л. Г. ХАЧИЯН

ПОЛИНОМИАЛЬНЫЙ АЛГОРИТМ В ЛИНЕЙНОМ ПРОГРАММИРОВАНИИ

(Представлено академиком А. А. Дородницыным 4 X 1978)

Рассмотрим систему из $m \geqslant 2$ линейных неравенств относительно $n \geqslant 2$ вещественных переменных $x_1, \ldots, x_j, \ldots, x_n$

$$a_{i1}x_1 + \ldots + a_{in}x_n \leqslant b_i, \quad i = 1, 2, \ldots, m, \tag{1}$$

с целыми коэффициентами a_{ij}, b_i. Пусть

$$L = \left[\sum_{i,j=1}^{m,n} \log_2(|a_{ij}|+1) + \sum_{i=1}^{m} \log_2(|b_i|+1) + \log_2 nm \right] + 1 \tag{2}$$

есть длина входа системы, т. е. число символов 0 и 1, необходимых для записи (1) в двоичной системе счисления.

Mathematical Programming Study 14 (1981) 61–68.
North-Holland Publishing Company

KHACHIYAN'S ALGORITHM FOR LINEAR PROGRAMMING*

Peter GÁCS and Laszlo LOVÁSZ

Computer Science Department, Stanford University, Stanford, CA 94305, U.S.A.

Received 10 October 1979

L.G. Khachiyan's algorithm to check the solvability of a system of linear inequalities with integral coefficients is described. The running time of the algorithm is polynomial in the number of digits of the coefficients. It can be applied to solve linear programs in polynomial time.

Key Words: Linear Programming, Inequalities, Complexity, Polynomial Algorithms.

0. Introduction

L.G. Khachiyan [1, cf. also 2, 3] published a polynomial-bounded algorithm to solve linear programming. These are some notes on this paper. We have ignored his considerations which concern the precision of real computations in order to make the underlying idea clearer; on the other hand, proofs which are missing from his paper are given in Section 2. Let

$$a_i x < b_i \quad (i = 1, \dots, m,\ a_i \in Z^n,\ b_i \in Z) \tag{1}$$

be a system of *strict* linear inequalities with integral coefficients. We present an algorithm which decides whether or not (1) is solvable, and yields a solution if it is. Define

$$L = \sum_{i,j} \log(|a_{ij}| + 1) + \sum_i \log(|b_i| + 1) + \log nm + 1.$$

L is a lower bound on the space needed to state the problem.

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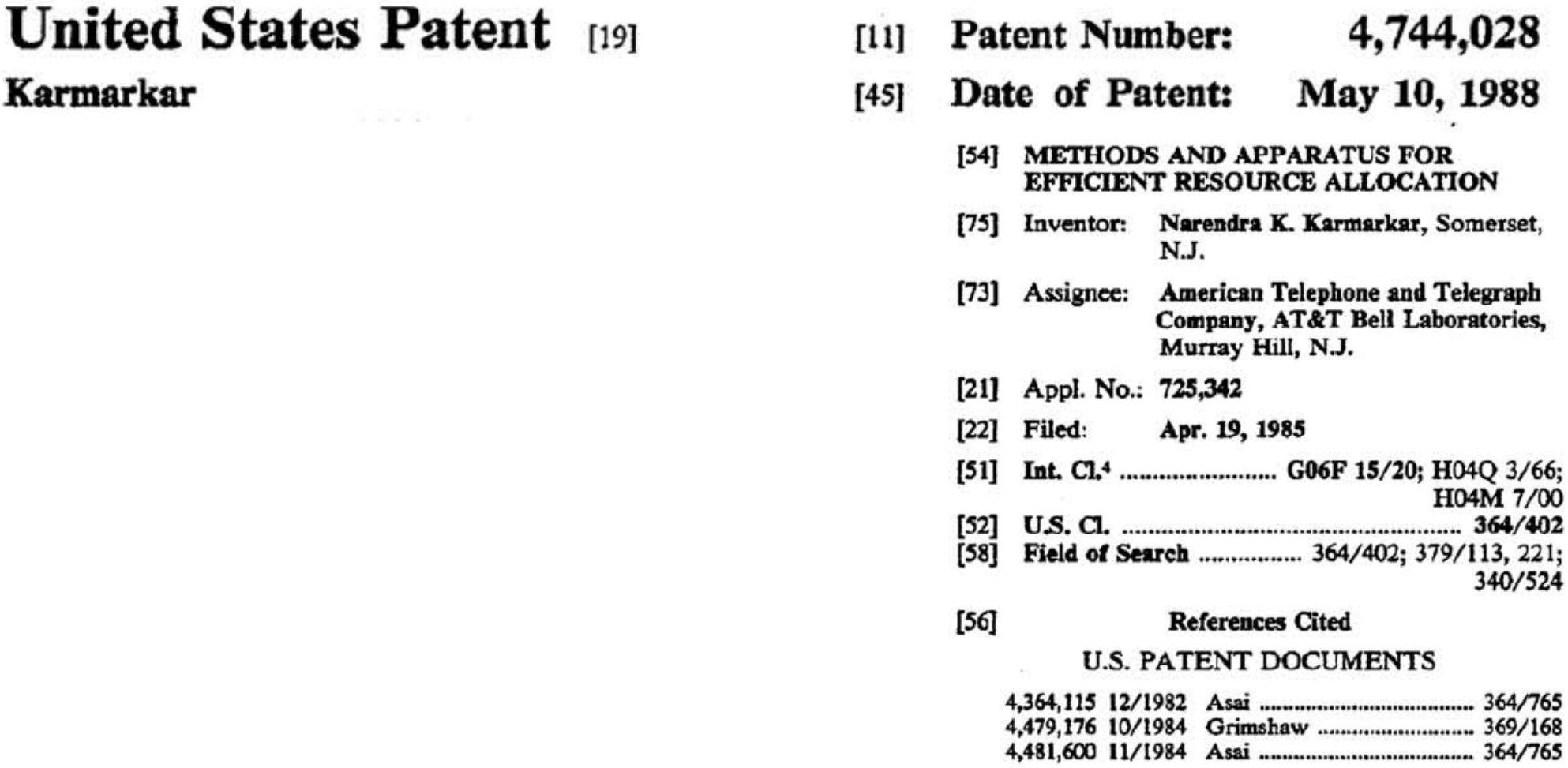
9. Expected polynomial running time of the simplex method
10. First polynomial time linear equations solver
11. First polynomial time LP solver
12. Second polynomial time LP solver and a patent

Who described the second polynomial time LP solver?

- N. Karmarkar,
  A new polynomial-time algorithm for linear programming.
  Combinatorica 4 (1984), no. 4, 373--395.



United States Patent [19]

Karmarkar

[11] **Patent Number: 4,744,028**

[45] **Date of Patent: May 10, 1988**

[54] **METHODS AND APPARATUS FOR EFFICIENT RESOURCE ALLOCATION**

[75] Inventor: **Narendra K. Karmarkar**, Somerset, N.J.

[73] Assignee: **American Telephone and Telegraph Company, AT&T Bell Laboratories, Murray Hill, N.J.**

[21] Appl. No.: **725,342**

[22] Filed: **Apr. 19, 1985**

[51] **Int. Cl.**[4] **G06F 15/20; H04Q 3/66; H04M 7/00**

[52] **U.S. Cl.** **364/402**

[58] **Field of Search** 364/402; 379/113, 221; 340/524

[56] **References Cited**

U.S. PATENT DOCUMENTS

4,364,115	12/1982	Asai	364/765
4,479,176	10/1984	Grimshaw	369/168
4,481,600	11/1984	Asai	364/765

Breakthrough in Problem Solving

By JAMES GLEICK

A 28-year-old mathematician at A.T.&T. Bell Laboratories has made a startling theoretical breakthrough in the solving of systems of equations that often grow too vast and complex for the most powerful computers.

The discovery, which is to be formally published next month, is already circulating rapidly through the mathematical world. It has also set off a deluge of inquiries from brokerage houses, oil companies and airlines, industries with millions of dollars at stake in problems known as linear programming.

Faster Solutions Seen

These problems are fiendishly complicated systems, often with thousands of variables. They arise in a variety of commercial and government applications, ranging from allocating time on a communications satellite to routing millions of telephone calls over long distances, or whenever a limited, expensive resource must be spread most efficiently among competing users. And investment companies use them in creating portfolios with the best mix of stocks and bonds.

The Bell Labs mathematician, Dr. Narendra Karmarkar, has devised a radically new procedure that may speed the routine handling of such problems by businesses and Government agencies and also make it possible to tackle problems that are now far out of reach.

"This is a path-breaking result," said Dr. Ronald L. Graham, director of mathematical sciences for Bell Labs in Murray Hill, N.J.

"Science has its moments of great progress, and this may well be one of them."

Because problems in linear programming can have billions or more possible answers, even high-speed computers cannot check every one. So computers must use a special procedure, an algorithm, to examine as few answers as possible before finding the best one — typically the one that minimizes cost or maximizes efficiency.

A procedure devised in 1947, the simplex method, is now used for such problems,

Continued on Page A19, Column 1

THE NEW YORK TIMES, November 19, 1984

Karmarkar at Bell Labs: an equation to find a new way through the maze

Folding the Perfect Corner

A young Bell scientist makes a major math breakthrough

Every day 1,200 American Airlines jets crisscross the U.S., Mexico, Canada and the Caribbean, stopping in 110 cities and bearing over 80,000 passengers. More than 4,000 pilots, copilots, flight personnel, maintenance workers and baggage carriers are shuffled among the flights; a total of 3.6 million gal. of high-octane fuel is burned. Nuts, bolts, altimeters, landing gears and the like must be checked at each destination. And while performing these scheduling gymnastics, the company must keep a close eye on costs, projected revenue and profits.

Like American Airlines, thousands of companies must routinely untangle the myriad variables that complicate the efficient distribution of their resources. Solving such monstrous problems requires the use of an abstruse branch of mathematics known as linear programming. It is the kind of math that has frustrated theoreticians for years, and even the fastest and most powerful computers have had great difficulty juggling the bits and pieces of data. Now Narendra Karmarkar, a 28-year-old Indian-born mathematician at Bell Laboratories in Murray Hill, N.J., after only a years' work has cracked the puzzle of linear programming by devising a new algorithm, a step-by-step mathematical formula. He has translated the procedure into a program that should allow computers to track a greater combination of tasks than ever before and in a fraction of the time.

Unlike most advances in theoretical mathematics, Karmarkar's work will have an immediate and major impact on the real world. "Breakthrough is one of the most abused words in science," says Ronald Graham, director of mathematical sciences at Bell Labs. "But this is one situation where it is truly appropriate."

Before the Karmarkar method, linear equations could be solved only in a cumbersome fashion, ironically known as the simplex method, devised by Mathematician George Dantzig in 1947. Problems are conceived of as giant geodesic domes with thousands of sides. Each corner of a facet on the dome

TIME MAGAZINE, December 3, 1984

Patent on interior point methods for linear programming (Karmarkar, AT&T)

United States Patent [19]

Karmarkar

[11] **Patent Number: 4,744,028**

[45] **Date of Patent: May 10, 1988**

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4,481,600	11/1984	Asai	364/765

OTHER PUBLICATIONS

Linear Programming and Extensions, G. B. Dantzig, 1963.

Hacijan, L. G., A Polynomiar Algorithm ia Linear Programming, Soviet Math. Dokl, vol. 20, #1, 1979.

Bland, R. G. et al., The Elipsoid Method: A Survey, vol. 29, No. 6, Operations Research 1981.

A New Polynomial–Time Algorithm Ser Linear Programming, N. Karmarkar, ACM 1984.

Design and Optimization of Networks with Dynamic Routing, G. R. Ash et al., The Bell System Tech. Journal, vol. 60, No. 8, 10/1981, p. 1787.

G. R. Ash et al., Servicing and Real–Time Control of Networks with Dynamic Routing, The Bell System Tech. Journal, vol. 60, No. 8, 10/1981, p. 1821.

"The Ellipsoid Method and its Consequences in Combinatorial Optimization," Combinatorica 1(2), Grotschel et al., 1981, pp. 169–197.

Primary Examiner—Joseph Ruggiero

Assistant Examiner—Charles B. Meyer

Attorney, Agent, or Firm—Robert O. Nimtz; Henry T. Brendzel

[57] **ABSTRACT**

A method and apparatus for optimizing resource allocations is disclosed which proceeds in the interior of the solution space polytope instead of on the surface (as does the simplex method), and instead of exterior to the polytope (as does the ellipsoid method). Each successive approximation of the solution point, and the polytope, are normalized such that the solution point is at the center of the normalized polytope. The objective function is then projected into the normalized space and the next step is taken in the interior of the polytope, in the direction of steepest-descent of the objective function gradient and of such a magnitude as to remain within the interior of the polytope. The process is repeated until the optimum solution is closely approximated. The optimization method is sufficiently fast to be useful in real time control systems requiring more or less continual allocation optimization in a changing environment, and in allocation systems heretofore too large for practical implementation by linear programming methods.

36 Claims, 5 Drawing Sheets

The trouble with the patent

- Company founded by Marsten, Lustig and Shanno:

- AT&T patent attorneys
- Patent invalid due to prior published work of Fiacco&McCormick
- Warnings to other companies
- AT&T offered deal
- Marsten, Lustig and Shanno closed their business

- E.M.L. Beale - W.B. Orchard-Hayes Prize for Excellence in Computational Mathematical Programming, Mathematical Programming Society, 1991, (I.J. Lustig, R.E. Marsten, D. Shanno).

- See David Shanno: "Who invented the Interior-Point Method" in "Optimization Stories", (M. Grötschel, ed.) 2012, p. 55-64.

The historical LP development

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Who described the first polynomial time LP solver with practical impact?

- N. Karmarkar,
  A new polynomial-time algorithm for linear programming.
  Combinatorica 4 (1984), no. 4, 373--395.

- But indirect. Many others tried to improve the method, and some had success.

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12. Second polynomial time LP solver and a patent
13. First polynomial time LP solver with practical impact
14. (Currently) most frequently used barrier LP solver?

Who described the currently most frequently used barrier (interior point) LP solver?

- Masakazu Kojima (1989): primal-dual interior point algorithm

- dominant since then (together with additional ideas from others)

The historical LP development

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13. First polynomial time LP solver with practical impact
14. (Currently) most frequently used barrier LP solver?
15. What is the state of the art in LP solving?

Contents

Historical IP Questions (Persons and Time)

Historical IP Questions (Persons and Time)

1. Which is the most important paper on integer programming?

Which is the most important paper on integer programming?

- G.B. Dantzig, D.R. Fulkerson, and S. Johnson (1954), *Solution of a Large Scale Traveling Salesman Problem*,
  P-510, The Rand Corporation.
- G.B. Dantzig, D.R. Fulkerson, and S. Johnson (1954). *Solution of a Large Scale Traveling Salesman Problem*,
  Operations Research 2, 393-410.

- Explanation of the importance of the work above:
  Martin Grötschel, George L. Nemhauser, *George Dantzig's contributions to integer programming*.
  Discrete Optimization 5(2): 168-173 (2008)

On the DFJ (1954) paper

These three researchers had laid out a versatile scheme to address general integer programs successfully, namely:

- preprocessing,
- warm start,
- variable fixing,
- reduced cost exploitation,
- cutting plane recognition, and
- elements of branch-and-bound.

As it turns out, this is the basic strategy that today almost all optimization codes relevant in practice employ. DFJ have not received the due recognition for this work, I believe.

Quote from DFJ (1954)

"It is clear that we have left unanswered practically any question one might pose of a theoretical nature concerning the traveling-salesman problem; however, we hope that the feasibility of attacking problems involving a moderate number of points has been successfully demonstrated, and that perhaps some of the ideas can be used in problems of similar nature."

Historical IP Questions (Persons and Time)

1. Which is the most important paper on integer programming?
2. Which is the most important paper on combinatorial algorithms?

Which is the most important paper on combinatorial algorithms?

- H.W. Kuhn, *The Hungarian Method for the assignment problem*, Naval Research Logistic Quarterly, 2 (1955) 83-97.
- Was voted in 2004 as most important paper that ever appeared in Naval Research Logistic Quarterly.
- András Frank, *On Kuhn's Hungarian Method – A tribute from Hungary,*
  Naval Research Logistic Quarterly, 22 (2005) 2-5.
- Munkres (1957): strongly polynomial time

- E-mail from Harold Kuhn of March 9, 2006:
- Dear Friends:
- As participants in the 50th Birthday celebration of the Hungarian Method, you should be among the first to know that Jacobi discovered an algorithm that includes both Koenig's Theorem and the Egervary step. I was told about Jacobi's paper by Francois Ollivier who has a website with the original papers and French and English translations. They were published in Latin after his death and so the work was done prior to 1851!!!

Historical IP Questions (Persons and Time)

1. Which is the most important paper on integer programming?
2. Which is the most important paper on combinatorial algorithms?
3. Who described the first cutting plane algorithm?

The first cutting plane algorithm

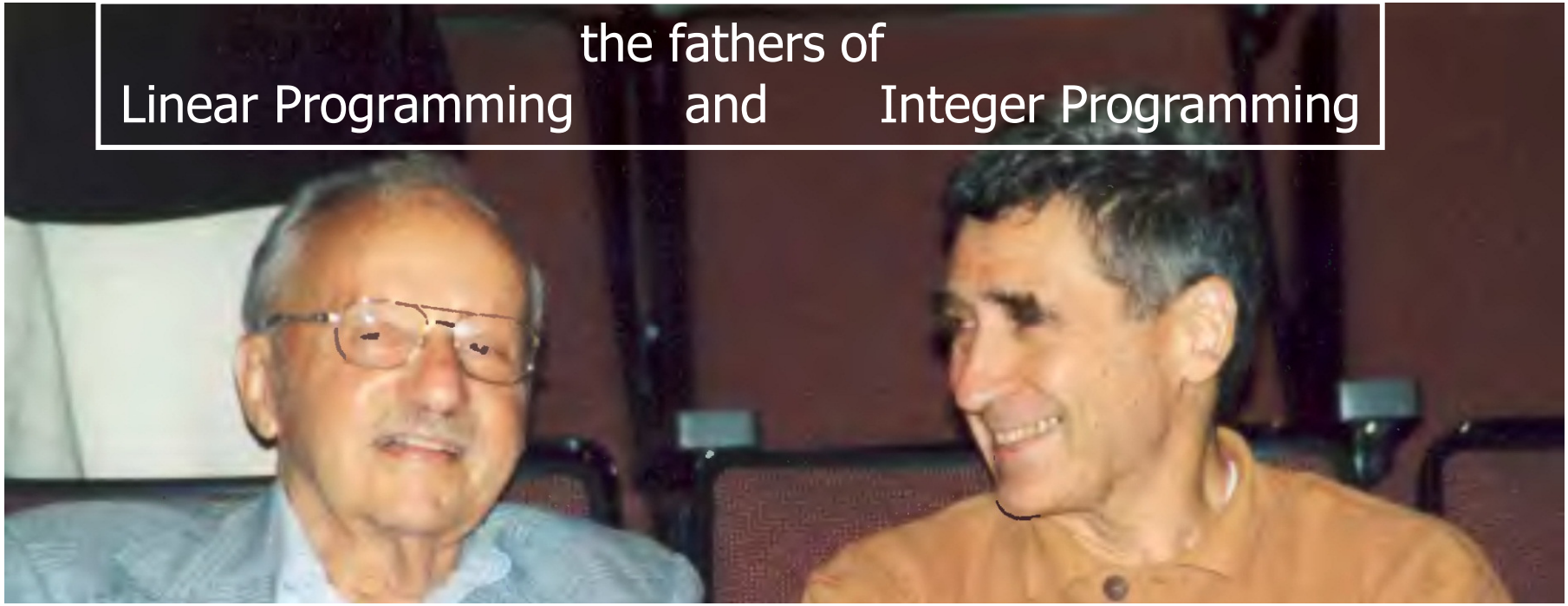
- Gomory, R. E. (1958), *Outline of an algorithm for integer solutions to linear programs*,
  Bull. AMS 64, 275 – 278.

- Not the first cutting plane algorithm, though, but the first one with a convergence proof.

- G.B. Dantzig, D.R. Fulkerson, and S. Johnson (1954), "Solution of a Large Scale Traveling Salesman Problem", described a cutting plane method with manual addition of cutting planes.
- G.B. Dantzig (1960), "On the Significance of Solving Linear Programs with some Integer Variables," Econometrica 28, 30-34. described an automated cutting plane method which was provably non-convergent, as shown by Gomory and Hoffman.

George Dantzig (1914-2005) and Ralph Gomory (1929 -)

ISMP Atlanta 2000



Ralph joined the Research Division of IBM in 1959. There, while continuing his significant mathematical work, he also launched a career that helped to establish that company as one of the major research institutions in the world. After eleven years at IBM, he was named director of research and immediately began leading the company in the development of some of the world's most exciting new products and technologies. He continued to play a leadership role for 20 years, eventually being promoted to the position of IBM senior vice president for Science and Technology.
http://en.wikipedia.org/wiki/Ralph_E._Gomory

Equivalence of Optimization and Separation

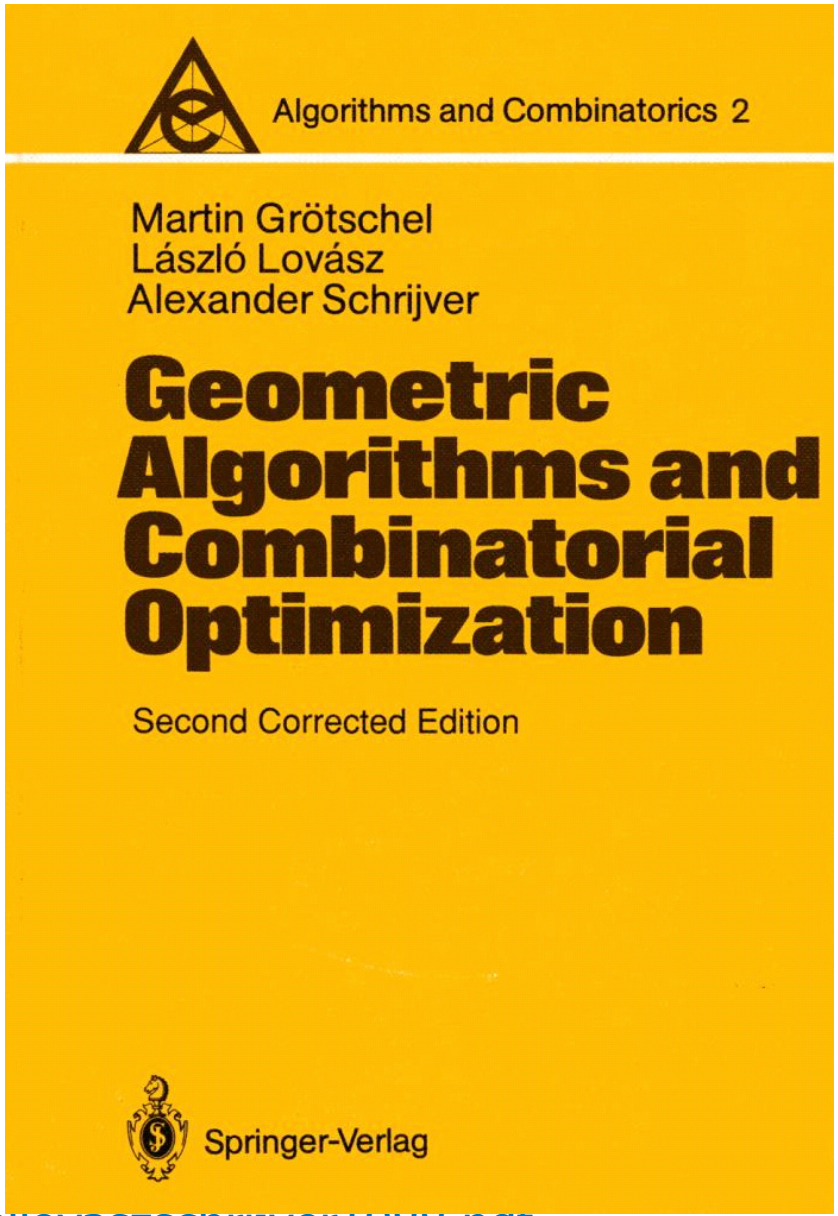
The ellipsoid method as a theoretical tool:

- Shor, 1970 - 1979
- Yudin & Nemirovskii, 1976
- Khachiyan, 1979

The applicability of the ellipsoid method to combinatorial optimization via the oracle approach was discovered independently by

KARP and PAPADIMITRIOU (1980),
PADBERG and RAO (1981),
GRÖTSCHEL, LOVÁSZ and SCHRIJVER (1981).

The book (GLS) can be downloaded from my Homepage:

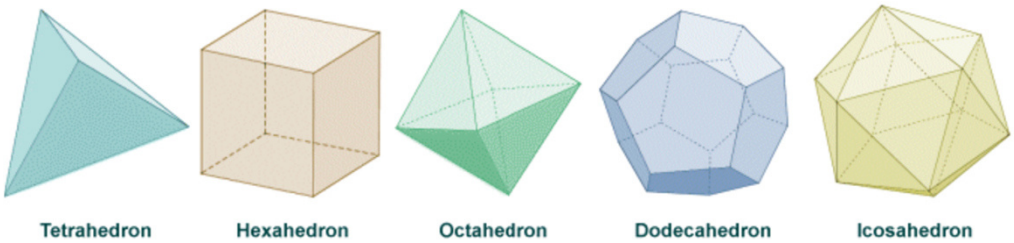


http://www.zib.de/groetschel/pubnew/paper/groetschellovaszschrijver1988.pdf

History of Convexity

- Many sources can be found throughout the ages which have some flavour of convexity. One interesting topic, e.g., are
  - regular polytopes

https://www.technologyuk.net/mathematics/geometry/platonic-solids.shtml



- Key contributions by: Hermann Minkowski (1864-1909) in hiswork on "geometry of numbers"
- No time to discuss that here.
- And there is no time to sketch the work with convexity contents of
  - Kepler (1571-1630)
  - Descartes (1596-1650)
  - Euler (1707-1783)
  - Lagrange (1736-1813)
  - Legendre (1752-1833)
  - Fourier (1768-1830)
  - Poinsot (1777-1859)
  - Cauchy (1798-1857)
  - Gauss (1777-1855)
  - Steiner (1796-1863)
  - Catalan (1814-1894)
  - and many other authors of the 19th century

GLS (1988)

(2.1.22) The Weak Constrained Convex Function Minimization Problem. *Given a compact convex set* $K \subseteq \mathbb{R}^n$, *a convex function* $f : \mathbb{R}^n \to \mathbb{R}$, *and a rational number* $\varepsilon > 0$, *find a vector* $y \in \mathbb{Q}^n \cap S(K, \varepsilon)$ *such that* $f(y) \leq f(x) + \varepsilon$ *for all* $x \in S(K, -\varepsilon)$.

(4.3.13) Theorem. *There exists an oracle-polynomial time algorithm that solves the following problem:*

Input: *A rational number* $\varepsilon > 0$, *a centered convex body* $(K; n, R, r, a_0)$ *given by a weak membership oracle, and a convex function* $f : \mathbb{R}^n \to \mathbb{R}$ *given by an oracle that, for every* $x \in \mathbb{Q}^n$ *and* $\delta > 0$, *returns a rational number* t *such that* $|f(x) - t| \leq \delta$.

Output: *A vector* $y \in S(K, \varepsilon)$ *such that* $f(y) \leq f(x) + \varepsilon$ *for all* $x \in S(K, -\varepsilon)$.

Briefly: Convex functions can be optimized over convex bodies (sets) in polynomial time.

Historical IP Questions (Persons and Time)

1. Which is the most important paper on integer programming?
2. Which is the most important paper on combinatorial algorithms?
3. Who described the first cutting plane algorithm?
4. Who described the first branch&bound algorithm?

The first branch&bound algorithm

- A. H. Land, A. G. Doig, *An automatic method of solving discrete programming problems*.
  Econometrica 28, 1960, S. 497–520
- R. J. Dakin, *A tree-search algorithm for mixed integer programming problems*.
  The Computer Journal, Volume 8, 1965, S. 250–255
- J. D. C. Little, K. G. Murty, D. W. Sweeney, C. Karel, *An algorithm for the traveling salesman problem*. Operations Research 11, 1963, 972–989.
- and many predecessors, such as
  W. L. Eastman, *Linear Programming with Pattern Constraints*.
  Ph.D. Thesis. Department of Economics, Harvard University, Cambridge, Massachusetts, USA, 1958.
  and DFJ (1954)

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5. Who described the first branch&cut algorithm?

The first branch&cut algorithm

This is difficult to pin down.

- Grötschel, Jünger, Reinelt (1984, 1985, 1987)
- Padberg, Rinaldi (1991), the name was coined.

- Branch&Cut&Price

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4. Who described the first branch&bound algorithm?
5. Who described the first branch&cut algorithm?
6. Who described the first column generation algorithm?

The first column generation algorithm

- L. R. Ford, Jr. and D. R. Fulkerson, *A Suggested Computation for Maximal Multi-Commodity Network Flows*
  Management Science, Vol. 5, No. 1, 1958, pp. 97-101
- A. Charnes and M. H. Miller (1956)

Ambros Gleixner made me aware of the following paper that can be found in *Optimization Online* (2024/01/06 updated):

Kantorovich and Zalgaller (1951): the 0-th Column Generation Algorithm by Eduardo Uchoa and Ruslan Sadykov:

This article delves into the early development of the Column Generation technique. […] Then, it brings to light Kantorovich and Zalgaller's lesser-known 1951 book, which is revealed to contain a complete Column Generation algorithm.
https://optimization-online.org/2024/01/kantorovich-and-zalgaller-1951-the-0-th-column-generation-algorithm/

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5. Who described the first branch&cut algorithm?
6. Who described the first column generation algorithm?
7. Who implemented the first commercial IP code?

From Orchard-Hays 1990

The real trouble in LP-III, however, was the third extended feature. This was a reimplementation of the Dzielinski-Gomory [1965] algorithm for discrete production-resource allocation. Mead Paper Company was interested in this technique, and IBM hoped to provide them with an efficient implementation. At best, this algorithm was a development effort, not a proven technique.

value. MPSX/370 is still IBM's current system and includes several new features, including a MIP (mixed-integer) extension and a means of utilizing major components under PL/I. MIP extensions have been added to most systems with varying degrees of success. The ability to utilize components to tailor a system to special problems was a feature long sought.

The first commercial IP code?

- ???

- MPSX? UMPIRE?

NIC 10424 Karl Kelley
NWG/RFC 345 University of Illinois
May 26, 1972

INTEREST IN MIXED INTEGER PROGRAMMING (MPSX ON 360/91 AT CCN)

MPSX is a newer version of the IBM project MPS, used for integer programming. From what I've been told, MPSX outperforms the previous package. In addition, it has available a feature of mixed integer programming.

Who implemented the first commercial IP code?

- From: Martin Groetschel [mailto:groetschel@zib.de]
  Sent: Wednesday, May 25, 2011 5:42 PM
  To: Robert E. Bixby
  Subject: Question

Bob, who had/developed the first commercial IP and/or MIP code?
Martin

- Von: Robert Bixby [mailto:bixby@gurobi.com]
  Gesendet: Donnerstag, 26. Mai 2011 00:52
  An: 'Martin Groetschel'
  Betreff: RE: Question

Boy, I don't think I know the answer to that one. For LP I could probably find it, but I don't recall seeing statements about that for IP/MIP.
My suggestion is to ask Alan Hoffman. He certainly knew about early LP codes and corrected some of my misinformation.
Bob

Historical IP Questions (Persons and Time)

1. Which is the most important paper on integer programming?
2. Which is the most important paper on combinatorial algorithms?
3. Who described the first cutting plane algorithm?
4. Who described the first branch&bound algorithm?
5. Who described the first branch&cut algorithm?
6. Who described the first column generation algorithm?
7. Who implemented the first commercial IP code?
8. Who described the first polynomial time IP solver?

Who described the first polynomial time IP solver?

- Well, IP is NP hard!

- But
- H.E. Scarf, Production sets with indivisibilities—
  Part I: Generalities, Econometrica 49 (1981) 1–32.
- Part II: The case of two activities, Econometrica 49 (1981) 395-423.
- H. W. Lenstra. *Integer programming with a fixed number of variables.*
  Mathematics of Operations Research, 8(4):538 – 548, 1983.

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9. What is the state of the art in MIP solving?

Question

Compared to the best MIP codes of 1991, how much faster are today's best MIP codes, not including the speed-ups of computers or super computers?

A. Less than 100,000 times faster
B. 100,000 to 1,000,000 times faster
C. 1,000,000 to 5,000,000 times faster
D. More than 5,000,000 times faster

MIP-Codes are today 6.5 million times faster than
32 years ago – without including computer speed-ups.

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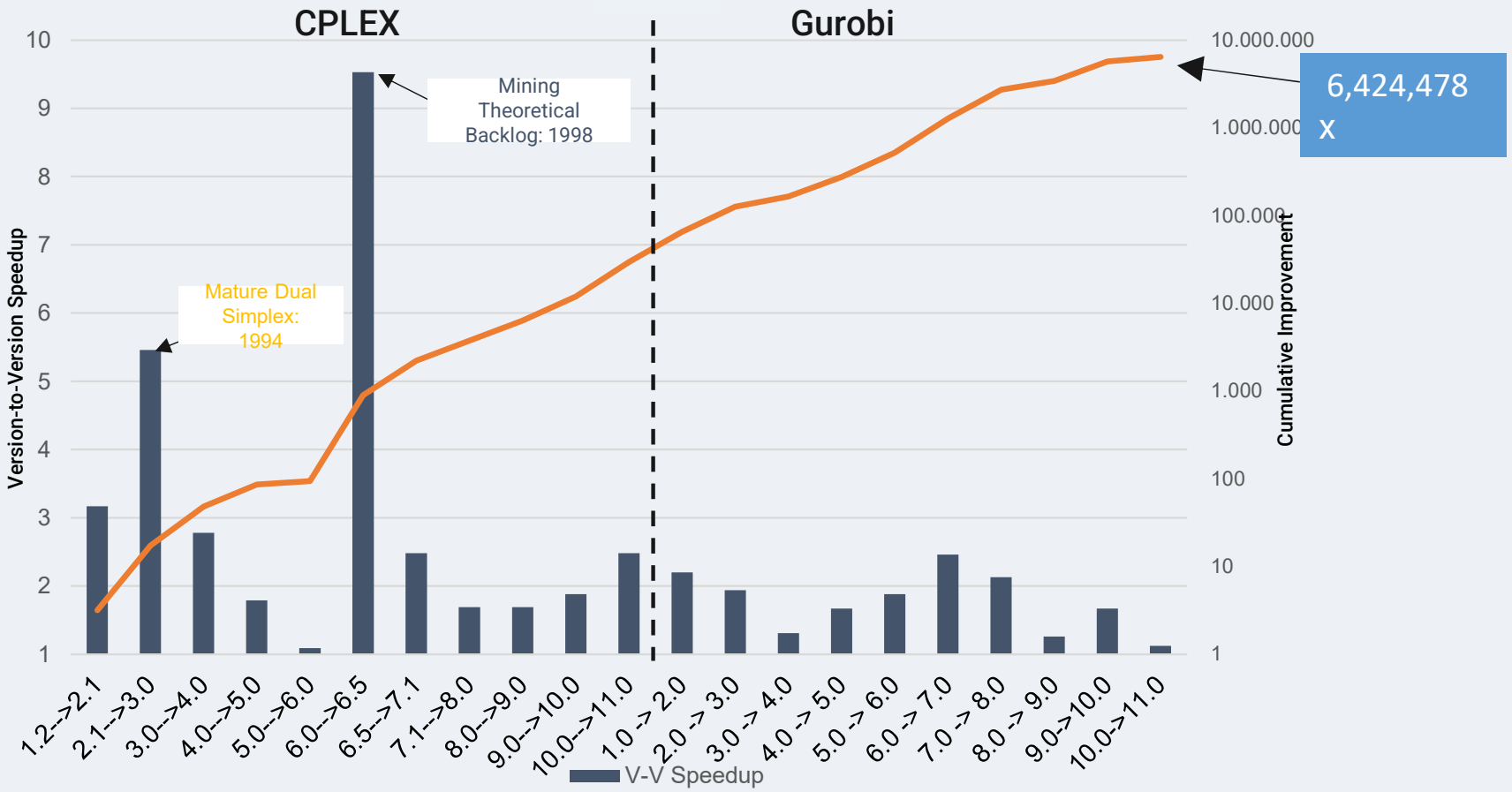
GUROBI OPTIMIZATION

MIP Performance Improvements

1991-2023



From the GOR-Vortrag of Bob Bixby on 6.9.2024 in München

Further Questions

1. The first recorded TSP World record: 49 cities, by DFJ (1954)
   The current TSP World record: How many cities?

Some TSP World Records

2006
pla 85,900
solved
3,646,412,050
variables

number of cities
2000x
increase

4,000,000
times
problem size
increase

in 52
years

year	authors	# cities	# variables
1954	DFJ	42/49	820/1146
1977	G	120	7140
1987	PR	532	141,246
1988	GH	666	221,445
1991	PR	2,392	2,859,636
1992	ABCC	3,038	4,613,203
1994	ABCC	7,397	27,354,106
1998	ABCC	13,509	91,239,786
2001	ABCC	15,112	114,178,716
2004	ABCC	24,978	311,937,753
2005 W. Cook, D. Epsinoza, M. Goycoolea		33,810	571,541,145

hyg109399

https://www.math.uwaterloo.ca/tsp/star/hyg.html

The 109,399-star tour, shown in the moving image, is the shortest possible route through the core stars of the HYG collection, where point-to-point travel is measured by straight-line 3D Euclidean distance, rounded to the nearest 1/10th parsec.

Research team

- David Applegate, Google Research
- Robert Bixby, Gurobi Optimization and Rice University
- Vašek Chvátal, Charles University
- William Cook, University of Waterloo and Johns Hopkins University
- Daniel Espinoza, Google Inc.
- Marcos Goycoolea, Universidad Adolfo Ibanez
- Keld Helsgaun, Roskilde University

1,904,711-city instance (World TSP)

The current best lower bound on the length of a tour for the World TSP is 7,512,218,268. This bound was established by the Concorde TSP code (June 5, 2007), using CPLEX as a linear-programming solver. The bound shows that Keld Helsgaun's tour has length at most 0.0471% greater than the length of an optimal tour.

https://www.math.uwaterloo.ca/tsp/world/

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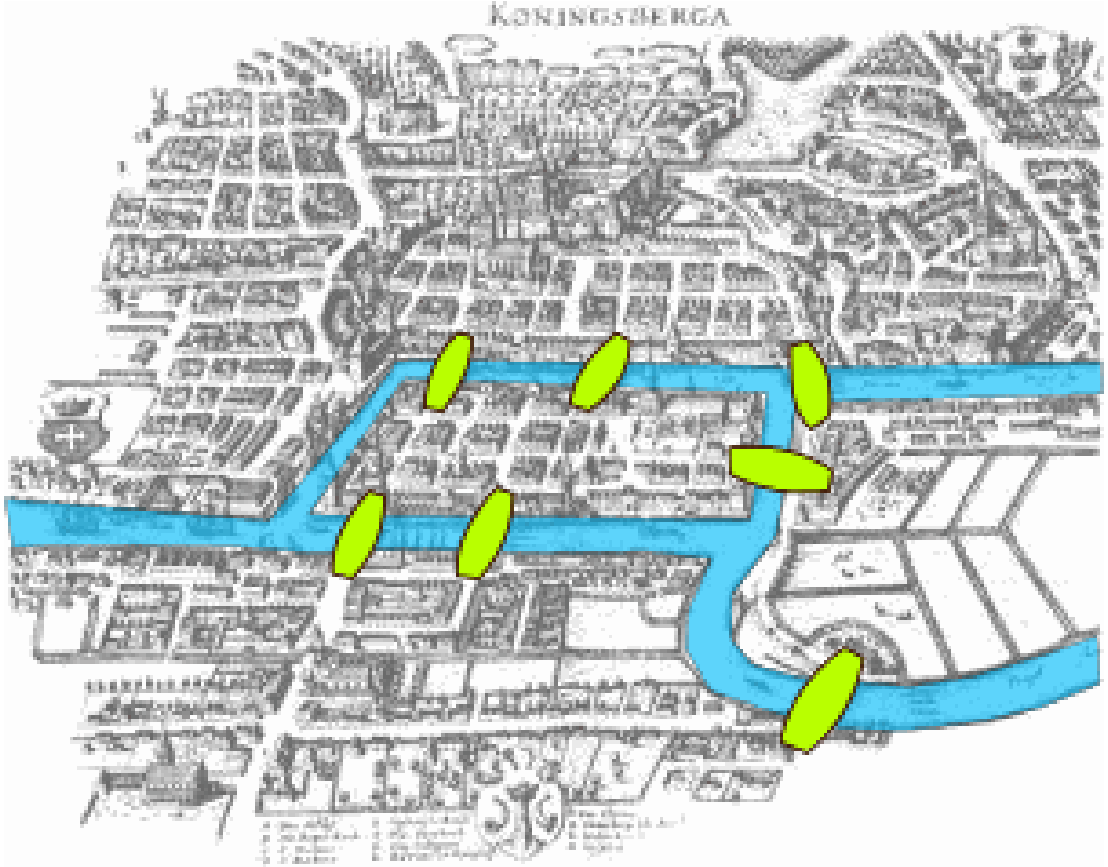
2. Which of the following persons has never been in Königsberg:
   Euler, Hilbert, Kant, Minkowski?



https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg#/media/File:Konigsberg_bridges.png

Königsberg (Prussia), today Kaliningrad (Russia)

- David Hilbert (* 23.01.1862 in Königsberg; † 14.02.1943 in Göttingen)
- Immanuel Kant (* 22.04.1724 in Königsberg; † 12.02.1804 in Königsberg)
- Hermann Minkowski (* 22.06.1864 in Aleksotas, Russia, today Kaunas, Lithuania; † 12.01.1909 in Göttingen), 1872-1880 Altstädtisches Gymnasium Königsberg.

SOLUTIO PROBLEMATIS
AD GEOMETRIAM SITUS PERTINENTIS

Commentatio 53 indicis Enestroemiani
Commentarii academiae scientiarum Petropolitanae 8 (1736), 1741, p. 128—140

1. Praeter illam geometriae partem, quae circa quantitates versatur et omni tempore summo studio est exculta, alterius partis etiamnum admodum ignotae primus mentionem fecit Leibnitzius[1]), quam *Geometriam situs* vocavit.

Where did Euler live?

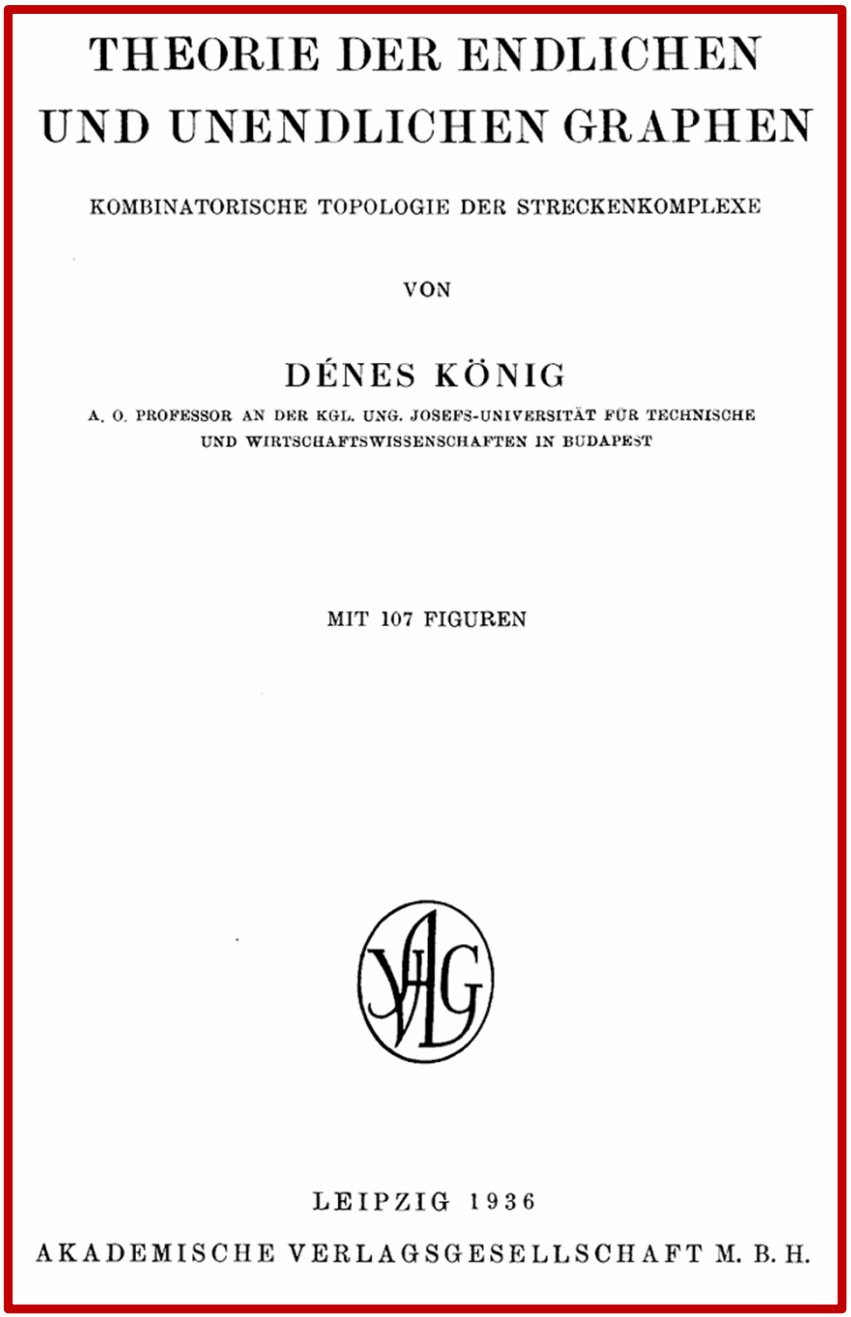
Born 15.04.1707 in Basel; Died 7.09./18.09.1783 in Sankt Petersburg

- Basel (1707-1726)
- St. Petersburg (1726-1741)
- Berlin (1741-1766)
- St. Petersburg (1766-1783)

Dénes König: the first book on graph theory

200 years later!

Today, graph theory permeates almost every field of the sciences and humanities as a modelling tool, and coupled with optimization and computer science, as a solution technology.


THEORIE DER ENDLICHEN
UND UNENDLICHEN GRAPHEN

KOMBINATORISCHE TOPOLOGIE DER STRECKENKOMPLEXE

VON

DÉNES KÖNIG

A. O. PROFESSOR AN DER KGL. UNG. JOSEFS-UNIVERSITÄT FÜR TECHNISCHE UND WIRTSCHAFTSWISSENSCHAFTEN IN BUDAPEST

MIT 107 FIGUREN

LEIPZIG 1936

AKADEMISCHE VERLAGSGESELLSCHAFT M. B. H.

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   Euler, Hilbert, Kant, Minkowski?
3. What happened in combinatorial optimization in 1965?

1965: Matching and Polyhedral Combinatorics

William Cook University of Waterloo, Canada
Martin Grötschel Zuse Institute and TU Berlin, Germany
Alexander Schrijver CWI and University of Amsterdam, Netherlands

[4] Edmonds, J. 1965. Maximum matching and a polyhedron with 0,1-vertices. Journal of Research of the National Bureau of Standards 69B, 125–130.

[5] Edmonds, J. 1965. Paths, trees, and flowers. Canadian Journal of Mathematics 17, 449–467.

The Year Combinatorics Blossomed

One summer in the mid 1980s, Jack Edmonds stopped by the Research Institute for Discrete Mathematics in Bonn for an extended visit. As usual, the institute administrator asked Professor Edmonds for a curriculum vitae to complete the university paperwork. The conversation took place in the library, so Edmonds pulled from a nearby shelf a text in combinatorial optimization: *"Here is my CV."*

Much has been written about linear programming, including several hundred texts bearing the title. Dantzig's creation of the model and the simplex algorithm for its solution is rightly viewed as one of the greatest contributions of applied mathematics in the past century. For our purposes, it will suffice to give the briefest of descriptions.

Matching: Polyhedra and Algorithms

Jack Edmonds (1965) proved that linear programs can be solved over the matching polytope in polynomial time, although the number of facets of the polytope is exponential in the number of nodes of the underlying graph.

The matching polytope has exponential extension complexity

- **DOI:** https://doi.org/10.48550/arXiv.1311.2369
- **Authors:** Thomas Rothvoss
- **Veröffentlicht:** 2013/11/11
- **RelatedEntitiesURL:** http://arxiv.org/pdf/1311.2369.pdf
- **Subject:** Computational Complexity (cs.CC)
- **Cite As:** arXiv:1311.2369 [cs.CC]

Thanks for your Attention

Optimization and OR:
A Sketch of Historical Developments

Lecture at CO@Work 2024
Computational Optimization at Work
September 16, 2024

Martin Grötschel

- Institut für Mathematik, Technische Universität Berlin (TUB) (1991-2015, im Ruhestand)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB) (1991-2015)
- DFG-Forschungszentrum "Mathematik für Schlüsseltechnologien" (MATHEON) (2002-2014)
- Berlin-Brandenburgische Akademie der Wissenschaften (2015-2020)

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