| | | |
|---|---|---|
| 09:15 | Ralf Borndörfer | Design of Public Transit Systems |
| 10:00 | Niels Lindner | Periodic timetable optimization in public transport |
| 11:15 | Daniel Rehfeldt | Optimizing vehicle and crew schedules in public transport |
| 12:00 | Daniel Roth | Using airline planning software to plan ICU personnel |
| 14:15-<br>17:45 | Milena Petkovic | Computational Challenge Day 4 |

# CO@Work 2024

## Traffic Day

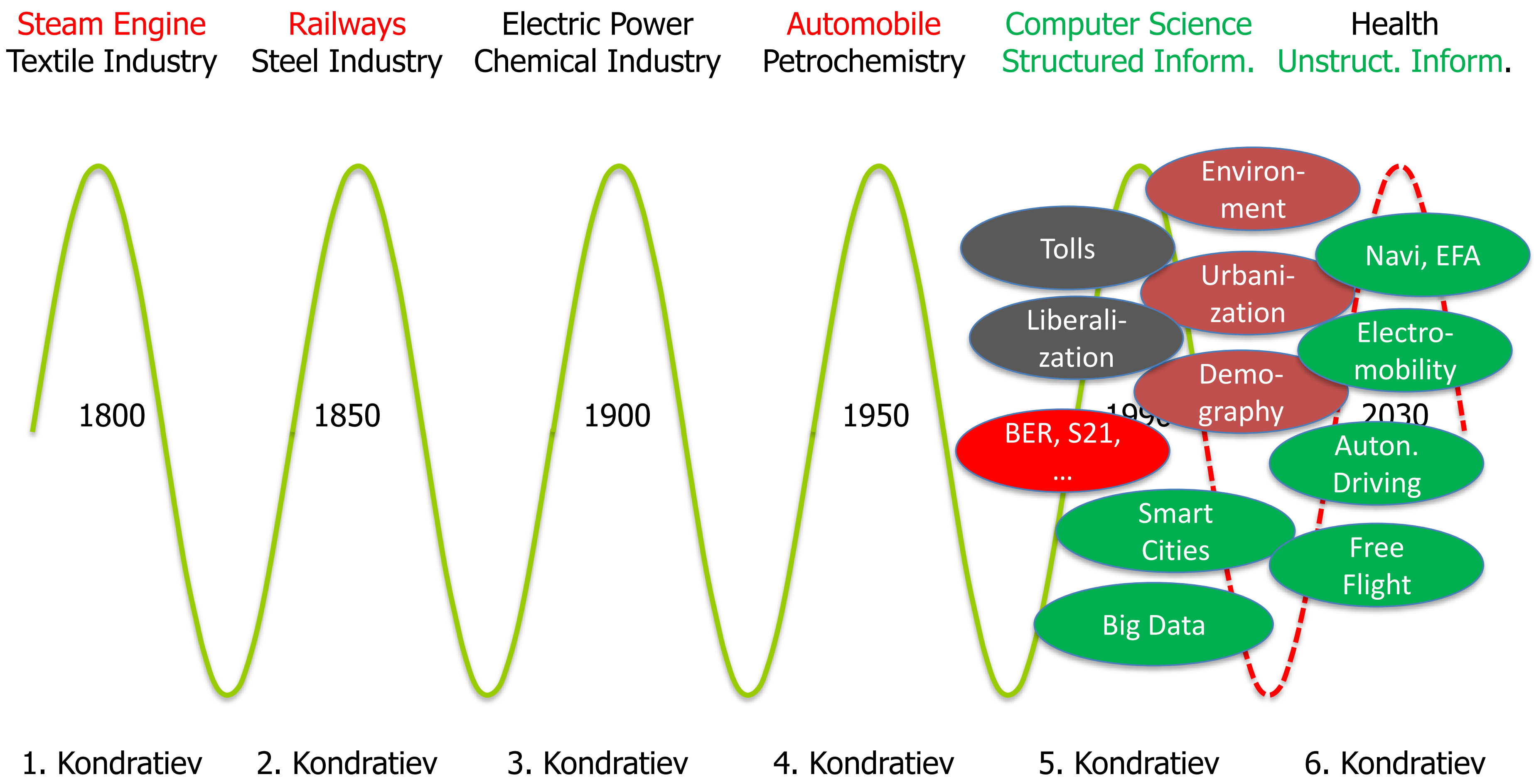
# Ralf Borndörfer
# Design of Public Transit Systems

# Long Economic Waves & Basic Innovations

Nikolai D. Kondratieff

Joseph Schumpeter

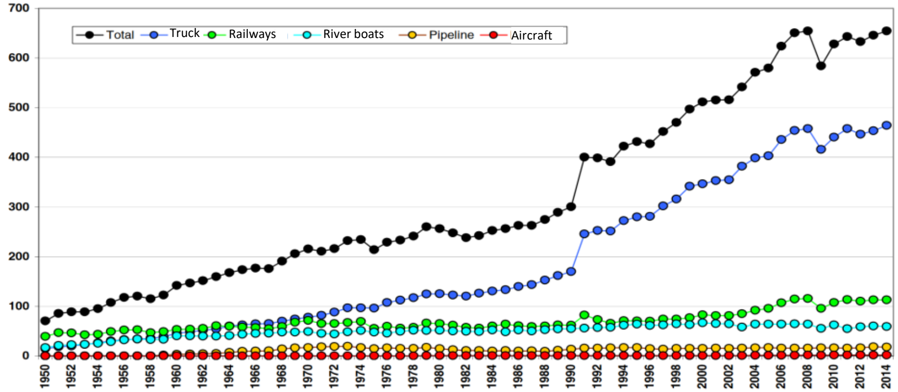
# What significance has mobility?



# What is happening?

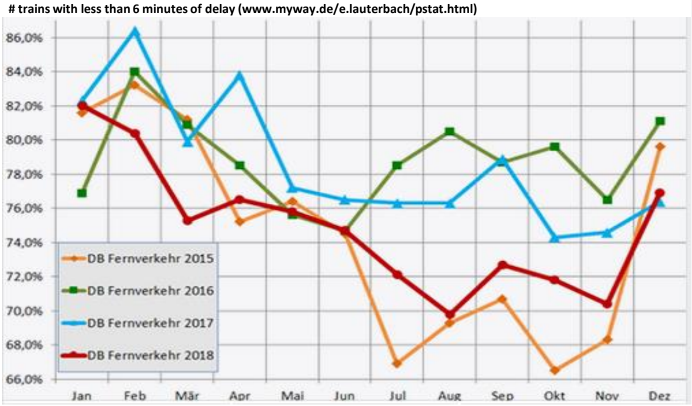
▶ Volume

Billions of ton kilometers, FIS Mobilität und Verkehr (www.forschungsinformationssystem.de)




▶ Complexity

# trains with less than 6 minutes of delay (www.myway.de/e.lauterbach/pstat.html)



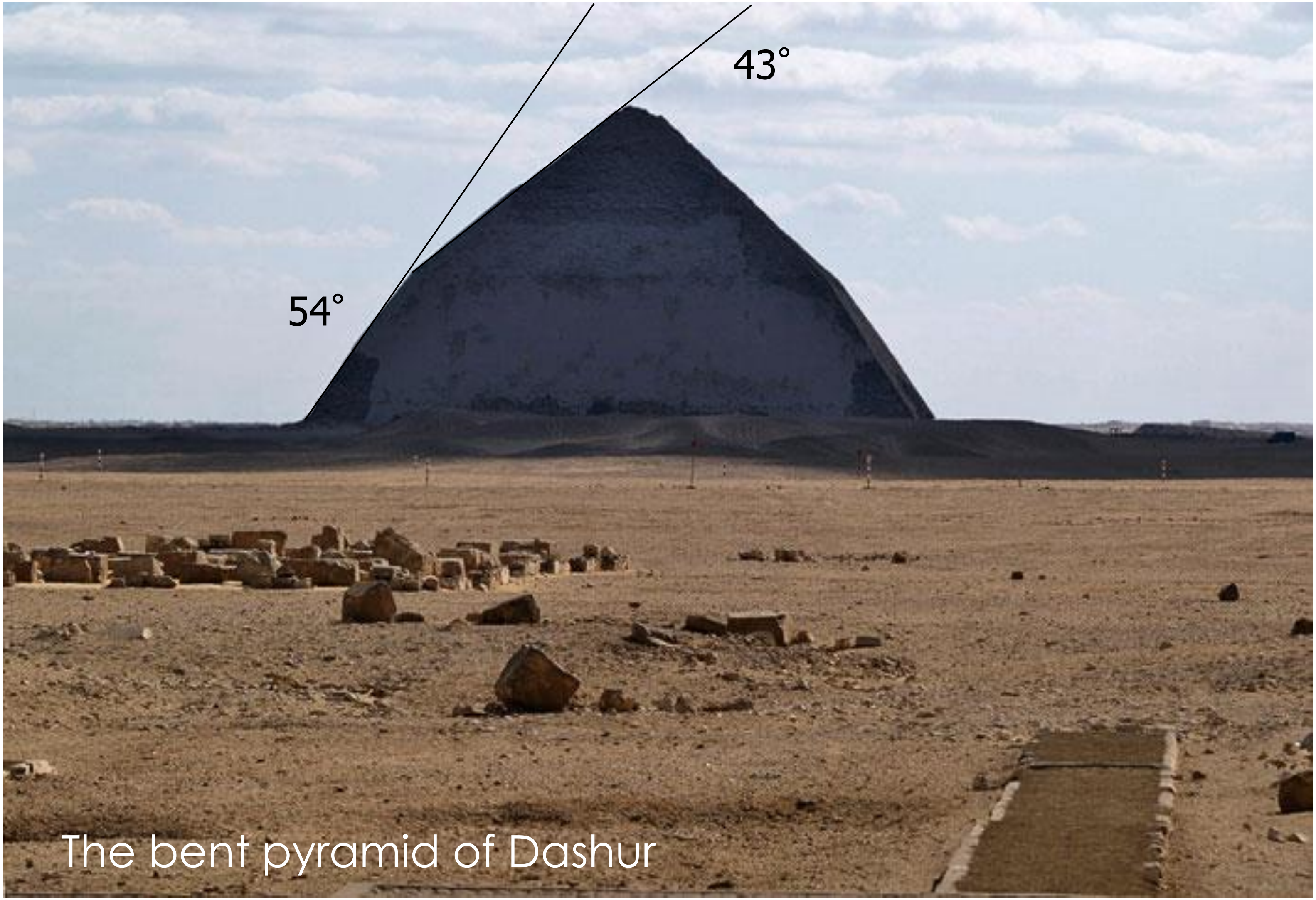

▶ Urbanization

Volkswagen Group Italy S.P.A. (modo.volkswagengroup.it)

▶ Digitization

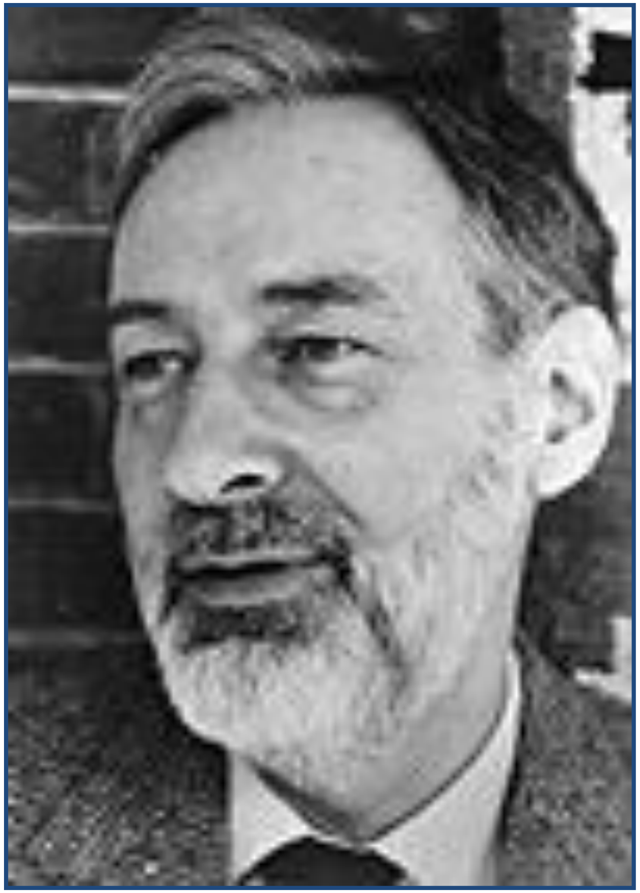
Digitale Schiene Deutschland, www.digitaleschiene.de

# Do we need mathematics?



# Smart City = Netzworks + Data + Math + …

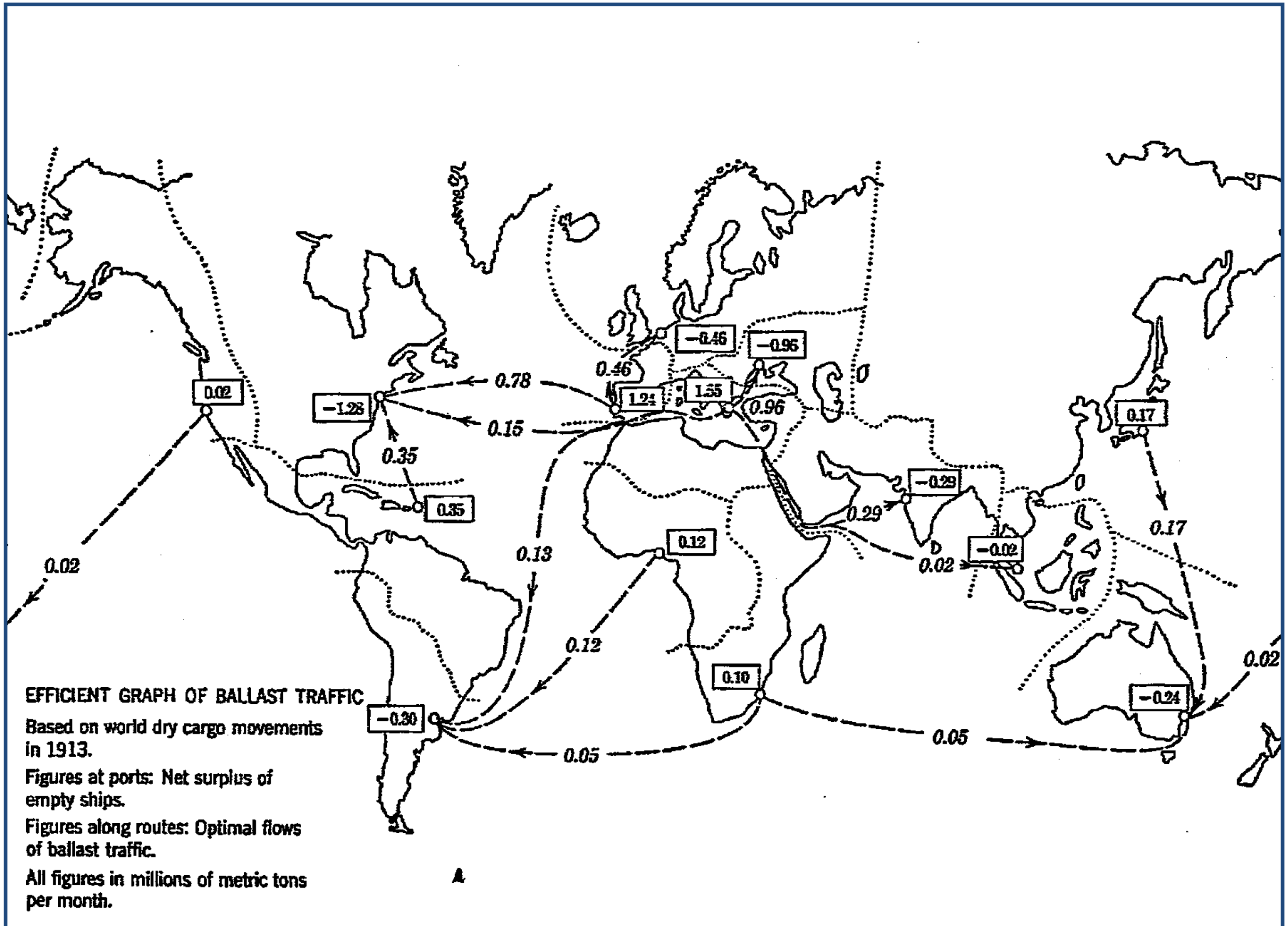
# Mathematics & Mobility



- Leonid V. Kantorovich
  Nobel prize for Economics 1975
- Tjalling C. Koopmans
  Nobel prize for Economics 1975

# Resource Allocation: Sea Freight

**(Koopmans [1965], 7 Sources, 7 Sinks, all Sea Links)**



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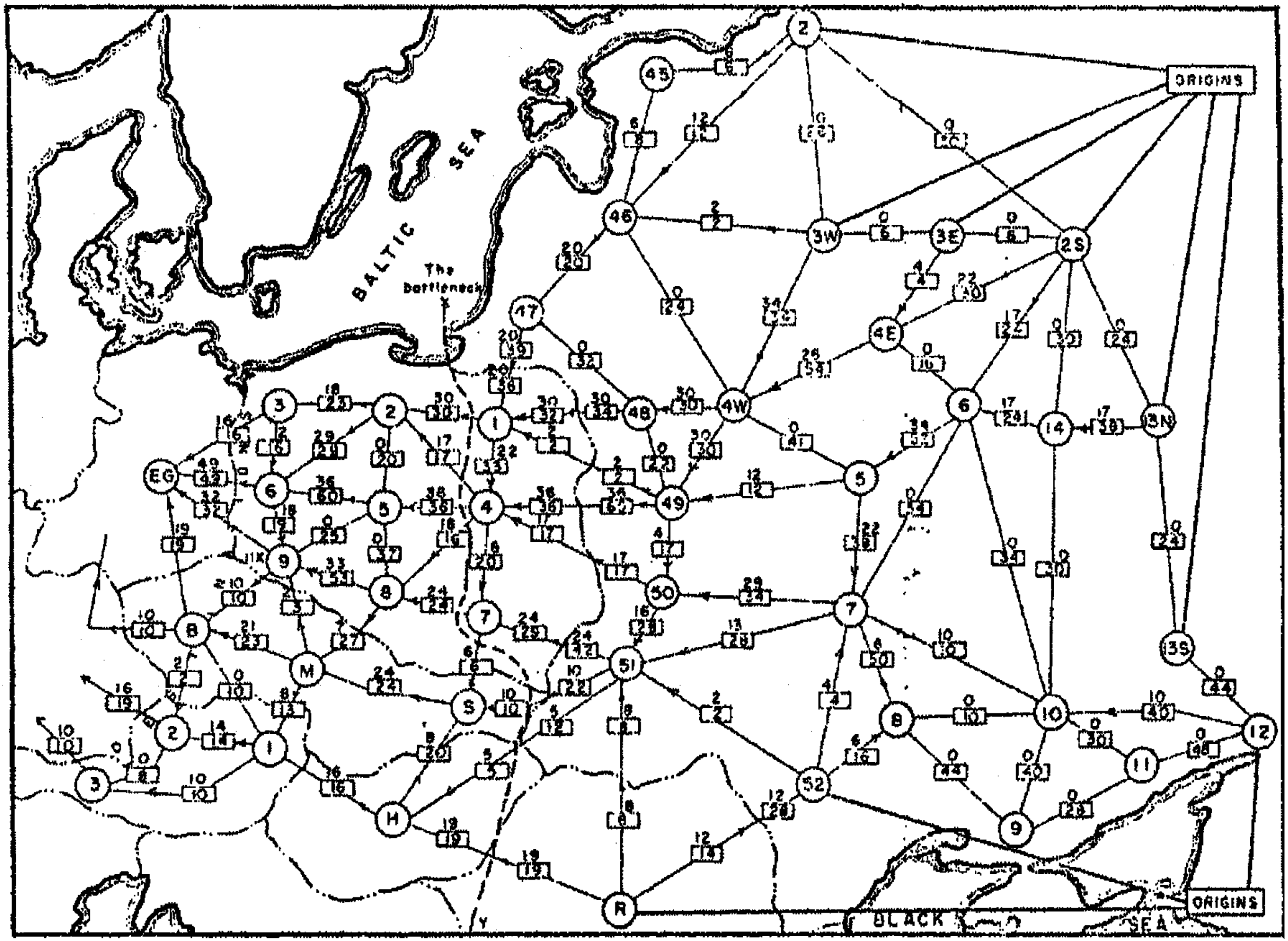
# Mathematics & Mobility

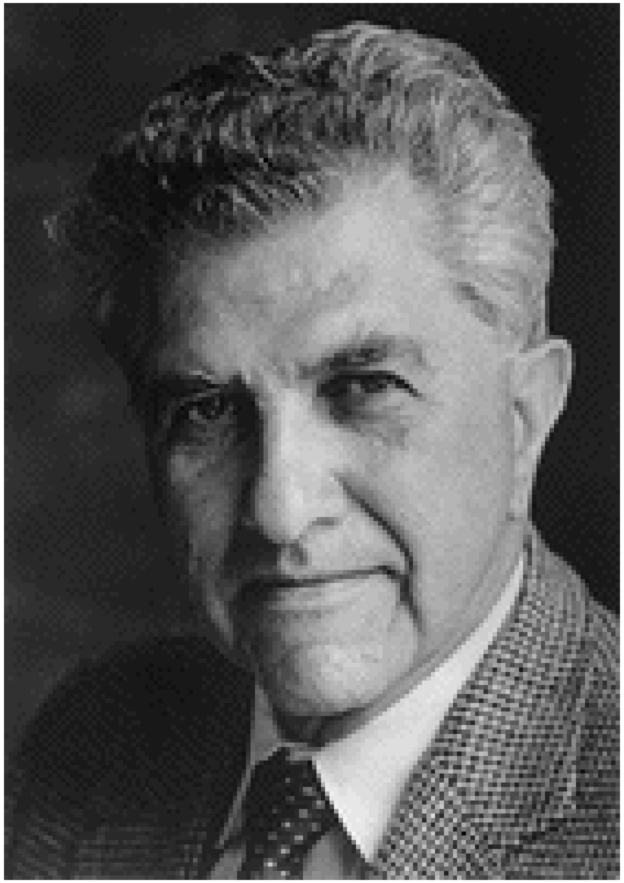
- D. Ray Fulkerson
- L. Randolph Ford Jr.

# Network Flows: Military Logistics

**(Ford & Fulkerson [1955], Schrijver [2002])**



# Mathematics & Mobility



- **Abraham Charnes**
  Finalist for the Nobel prize in Economics 1975
- **Merton H. Miller**
  Nobelprize for Economics 1990 mit Markowitz & Sharpe

Management Science vol. 3, No. 1, 1956

# A MODEL FOR THE OPTIMAL PROGRAMMING OF RAILWAY FREIGHT TRAIN MOVEMENTS*

A. CHARNES AND M. H. MILLER

*Purdue University and Carnegie Institute of Technology*

The structure shown in Table 1 can be translated into equation form by moving a row of λ's, one for each column, up through the rows and inserting the equal sign to the right of the $P_0$ column. The first two equations, for example, would be:

$$4 = 1\lambda_1 + 1\lambda_4 - 1\lambda_6 + 1\lambda_{12}$$

$$1 = 1\lambda_1 + 1\lambda_5 - 1\lambda_7 + 1\lambda_{13}$$

With the addition of the variables, the problem has been reduced to a standard simplex problem of the form:

$$\text{Min.} \sum_{i=1}^{n} \lambda_i c_i$$

subject to:

$$\sum_{i=1}^{n} \lambda_i P_i = P_0$$

$$\lambda_i \geqq 0$$

and can be solved by the simplex technique.

More fundamentally, the train-scheduling problem will be seen to possess certain striking structural features which may merit its inclusion among the basic model types of linear programming.[2] The background necessary for an understanding

* The research underlying this paper was supported, in part, by a grant to the Graduate School of Industrial Administration, Carnegie Institute of Technology by the Westinghouse Air Brake Corp. for fundamental research on problems of the transportation industries and in part by the Office of Naval Research.

The authors wish to acknowledge the many contributions made to the study by their colleague, W. W. Cooper; and by their co-workers at the railroad which served as the focus of the study, Messrs. John Cunningham, Robert Lake, Harold Soyster and Glenn Squibb. We also wish to thank Miss Suzanne Levin, Mr. Kenneth Kretschmer and Mr. Richard Poulin for assistance and advice on the computations during the research phase of the project; and the other members of the Westinghouse Air Brake Project, Messrs. Frank Brown, Edwin Mansfield and Harold Wein for many helpful suggestions made throughout the course of the investigation.

[1] In 1952, there were some 230 companies classified as terminal railroads with roughly 7500 miles of track and a total investment in railway property of over $1 billion [8]. Total revenues from handling some 20 million freight cars were in excess of 250 million dollars. These figures are conservative. They understate considerably the size of the terminal switching operation since they do not include the essentially similar services undertaken directly by the trunklines and consolidated in their regular accounts.

[2] For a discussion of L.P. model types and their significance for management science: See A. Charnes and W. W. Cooper [1].





TABLE 1

*Structural tableau of train-scheduling model*

| $c_j \rightarrow$ | | | 1.0 | 1.0 | 1.0 | 1.2 | 1.2 | 0 | 0 | 0 | 0 | 0 | 0 | M | M | M | M | M | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Shipment Requirements | Routes | | | | | Surplus Vectors (light moves) | | | | | | Artificial Vectors (legs) | | | | | |
| From | To | | 1,2 | 1,3 | 2,3 | 1,2,3 | 1,3,2 | 1–2 | 2–1 | 1–3 | 3–1 | 2–3 | 3–2 | 1–2 | 2–1 | 1–3 | 3–1 | 2–3 | 3–2 |
| | | $P_0$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $P_{13}$ | $P_{14}$ | $P_{15}$ | $P_{16}$ | $P_{17}$ |
| 1 | 2 | 4 | 1 | | | 1 | | −1 | | | | | | 1 | | | | | |
| 2 | 1 | 1 | 1 | | | | 1 | | −1 | | | | | | 1 | | | | |
| 1 | 3 | 9 | | 1 | | | 1 | | | −1 | | | | | | 1 | | | |
| 3 | 1 | 5 | | 1 | | 1 | | | | | −1 | | | | | | 1 | | |
| 2 | 3 | 6 | | | 1 | 1 | | | | | | −1 | | | | | | 1 | |
| 3 | 2 | 3 | | | 1 | | 1 | | | | | | −1 | | | | | | 1 |

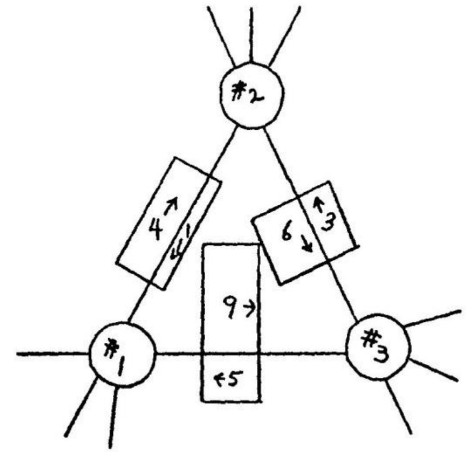



CHART 1. Simplified map of terminal switching railroad, showing connections with trunklines, major interchange and customer yard areas, and traffic requirements (in trainloads) between major points.

postponed until the description of the model and the computational routine has been completed.

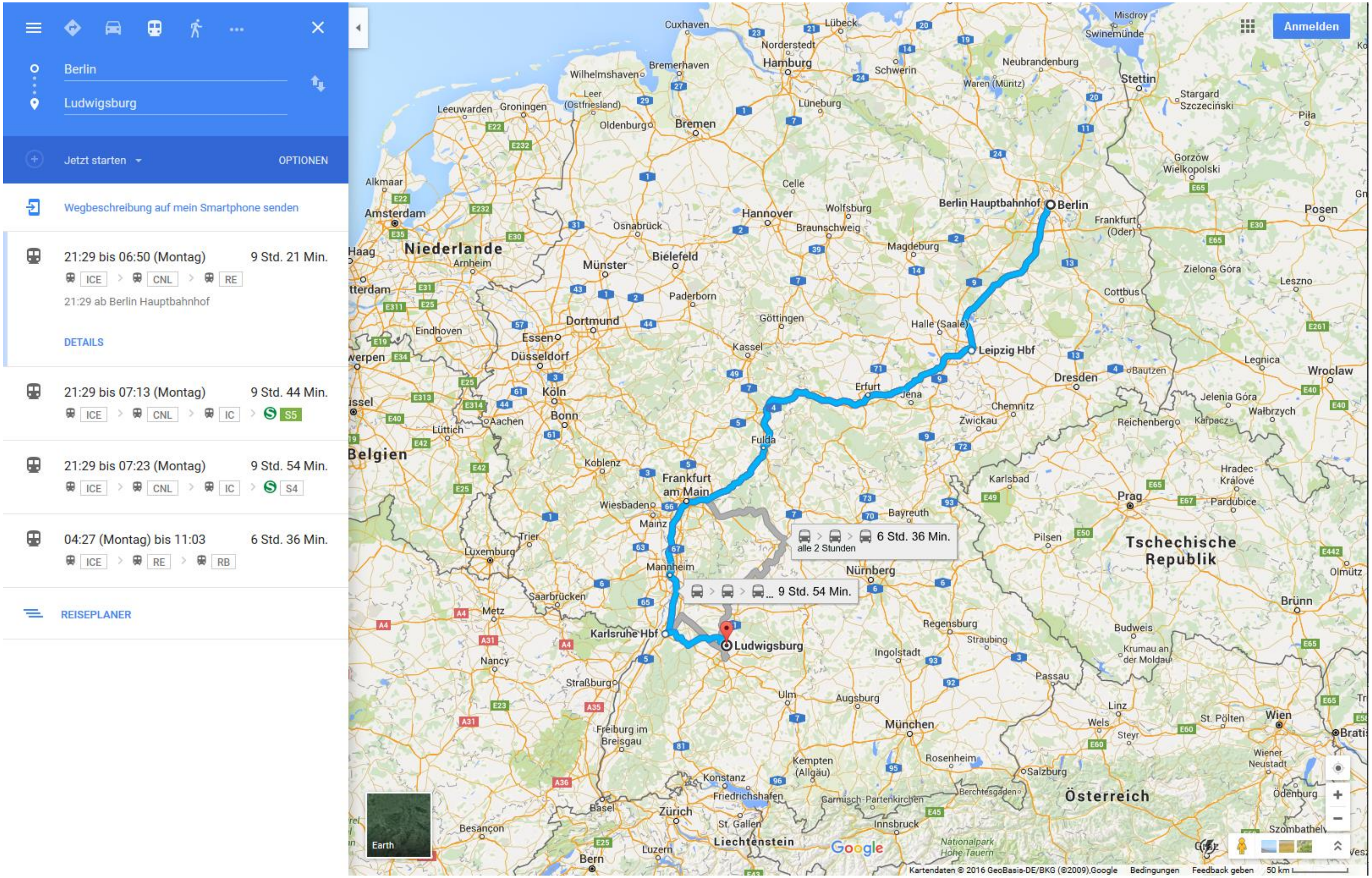
Above the routes, in the row labeled $c_j$, are entered the costs of assigning a single crew and engine package to the route in question. These costs may be stated either as the standard crew and engine expense, or as the expected costs reflecting the fact that on longer runs there is a greater probability of running into overtime. We constructed working models both ways and found, that optimal programs were not particularly sensitive to variations in the cost of crews. In fact, it was usually possible to simplify the calculation by minimizing the number of crews, that is treating the cost of each crew as 1.

$P_6$ to $P_{11}$ in the tableau are overfulfillment slack vectors. In the train scheduling context they correspond to "light moves", or trips by a crew and engine without cars. If, for example, four crews should be assigned to the route $P_1$—which runs

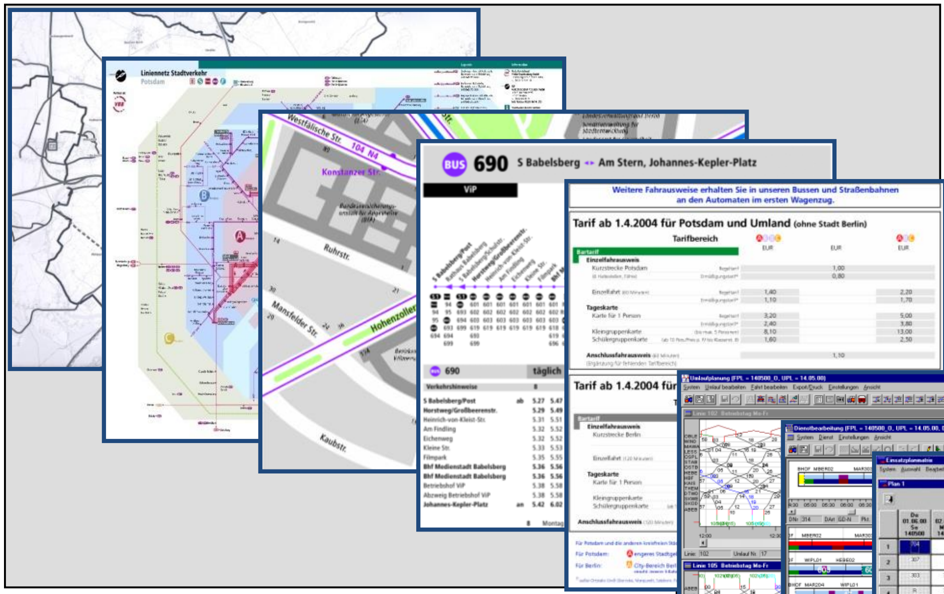
# Mathematics & Mobility

Edsger W. Dijkstra

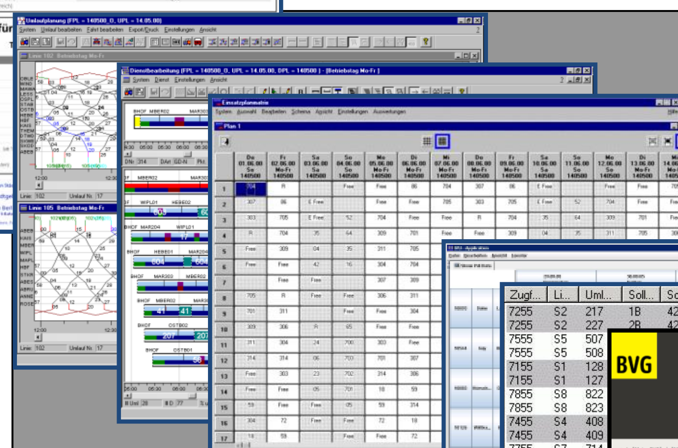
# Shortest-Path-Algorithms: Route Planning



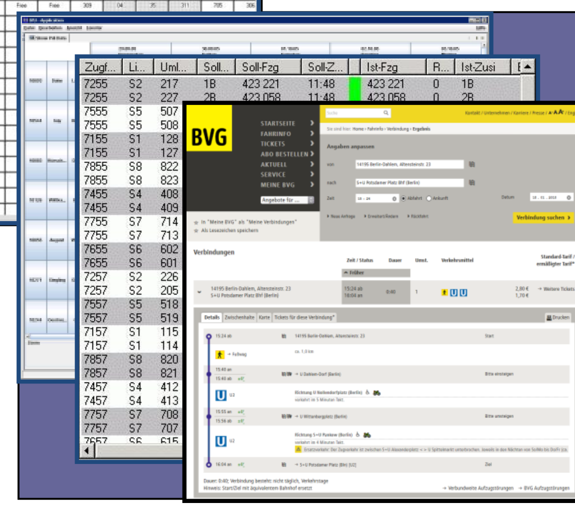
# Planning Problems in Public Transport




Strategic Planning

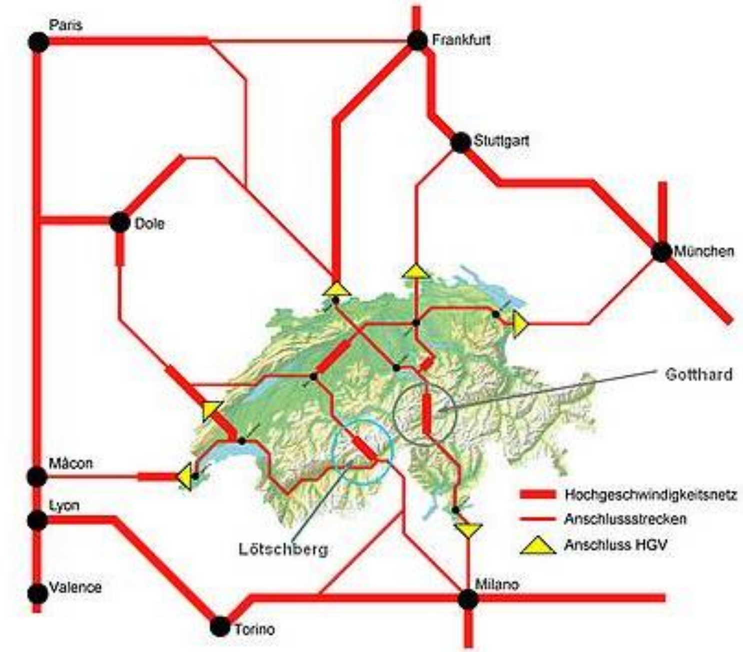


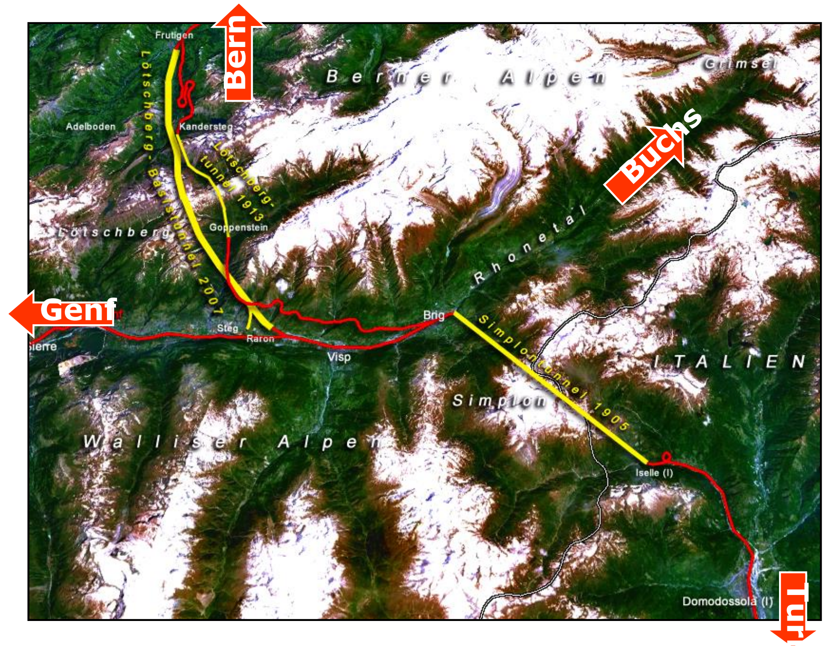
Operational Planning



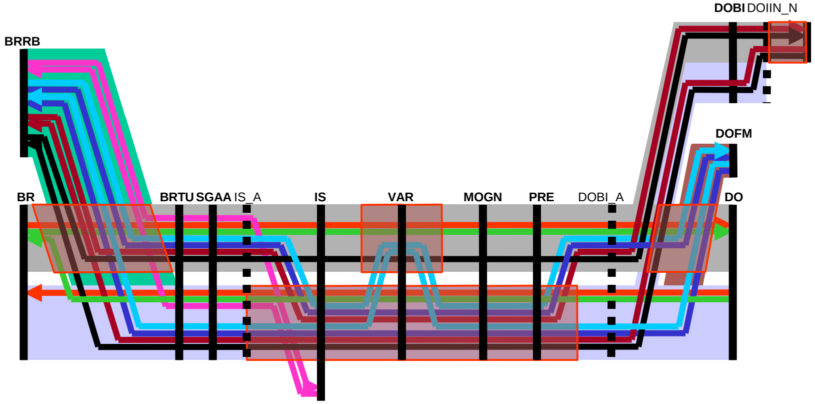

Operations Control

# Track Capacity

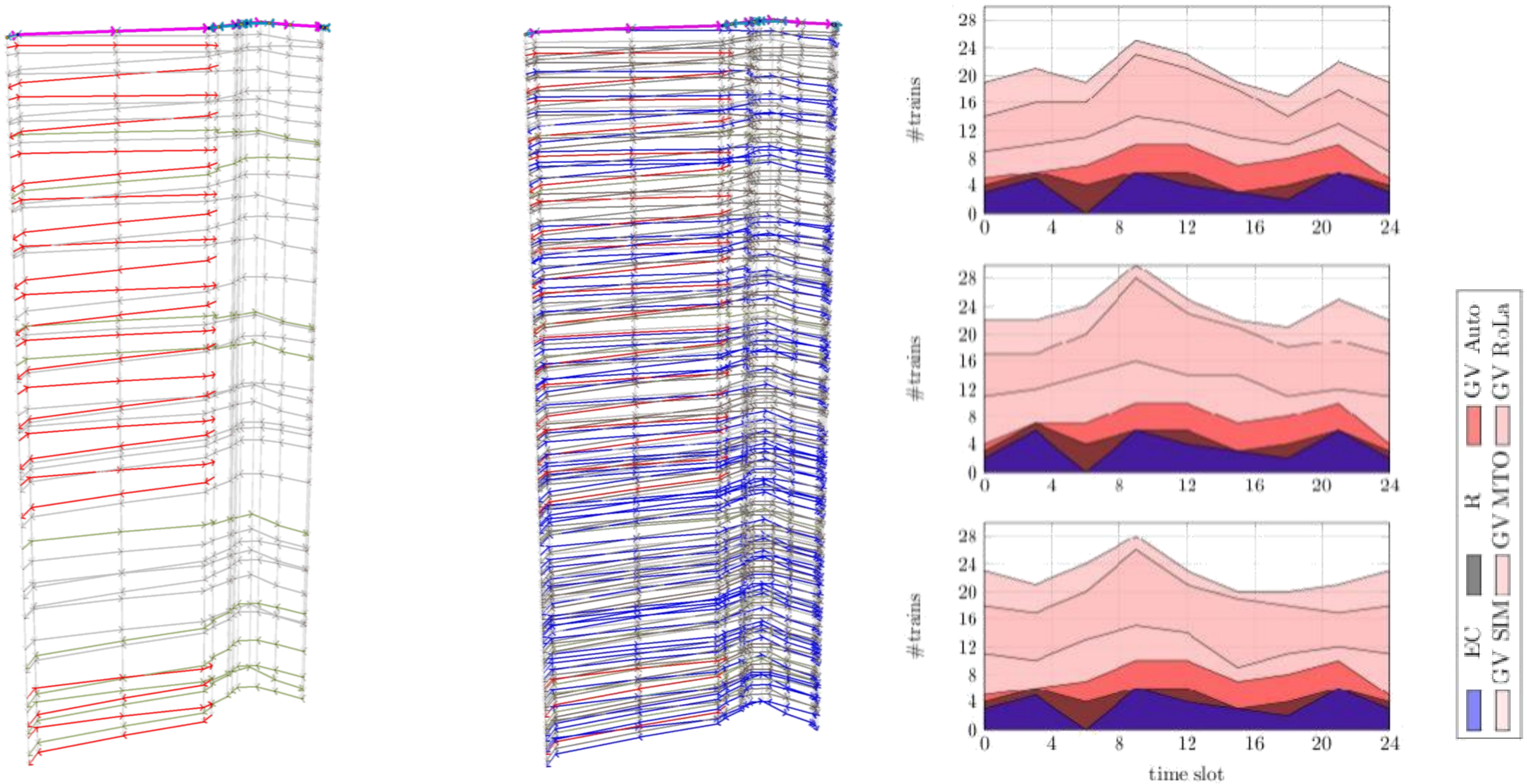




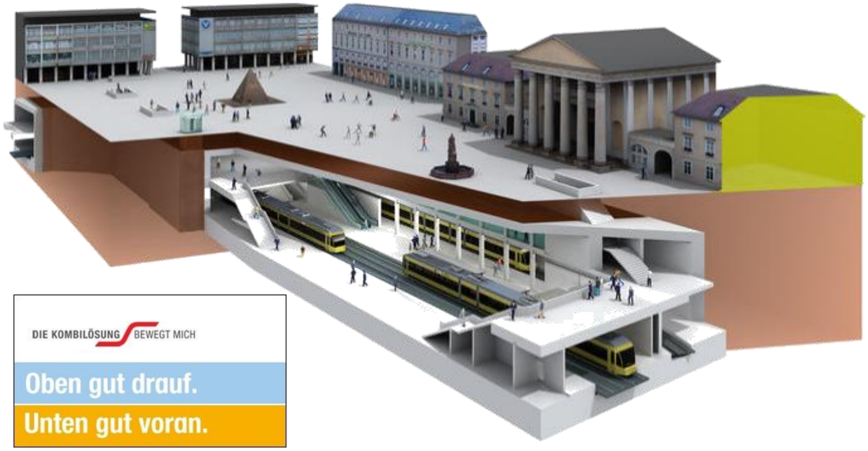


# The corridor capacity can be explored.

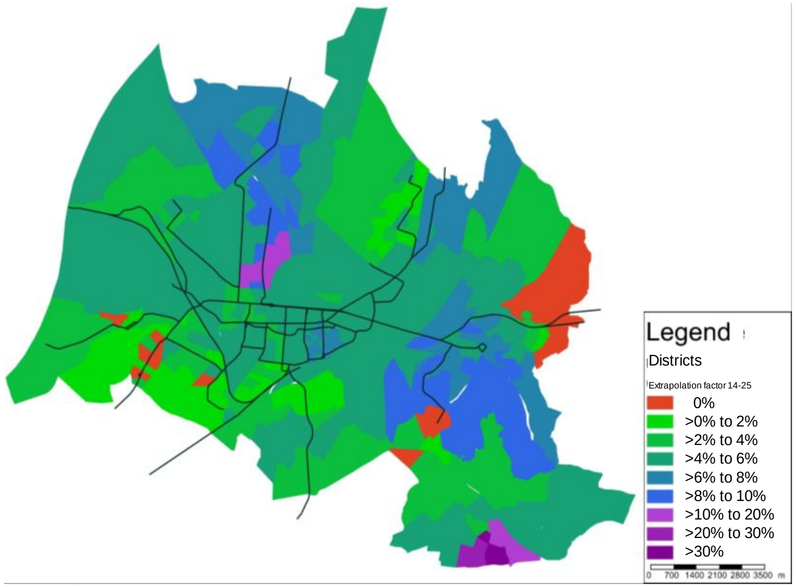


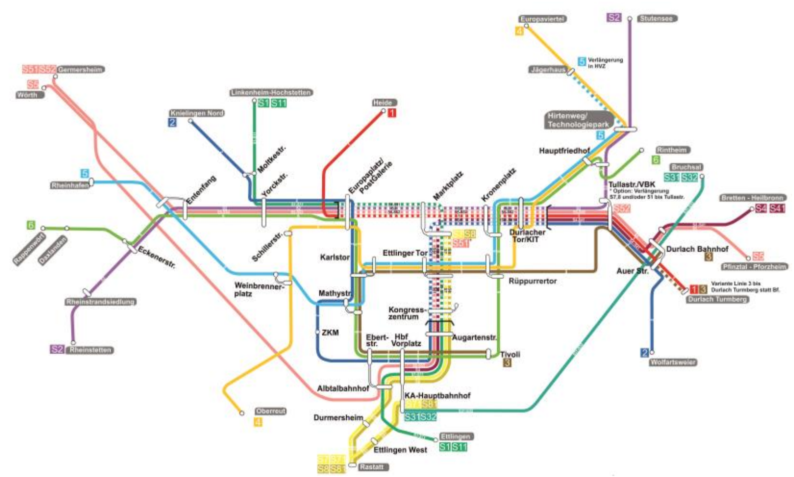

- 180 trains for network small (no station routing, no buffer times)
- 196 trains for network big with precise routing through stations (no buffer times)
- 175 trains for network big with precise routing through stations and buffer times

# Line Planning @ Karlsruhe

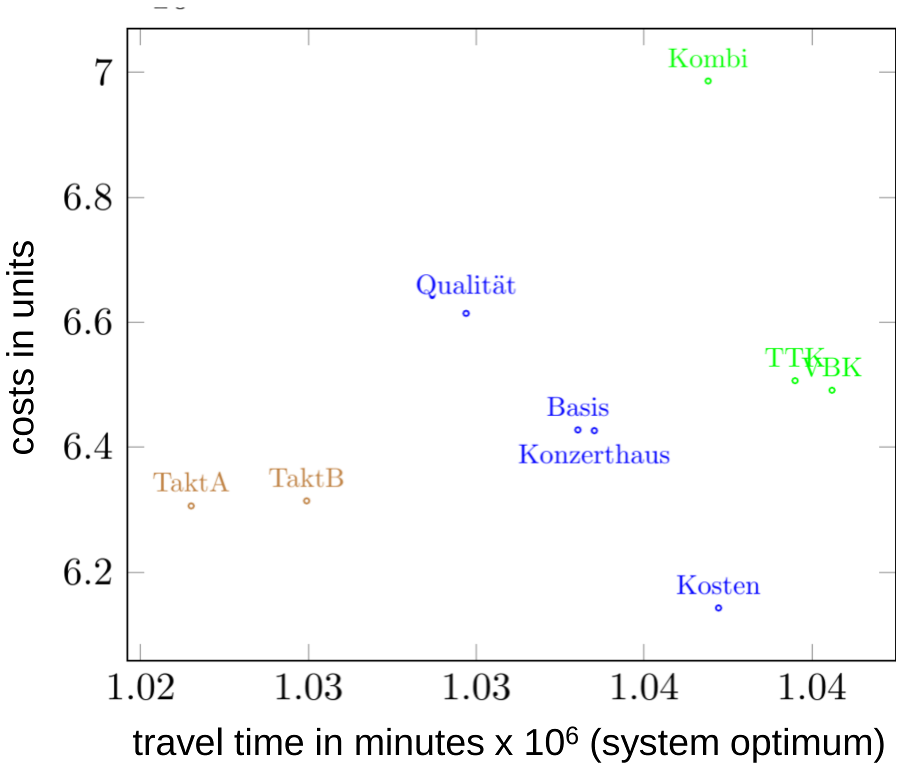


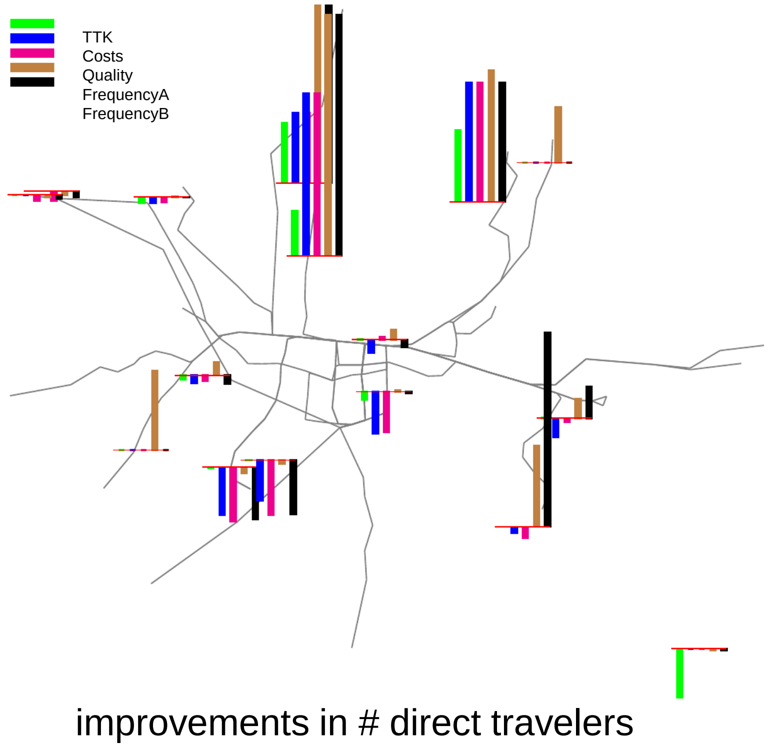




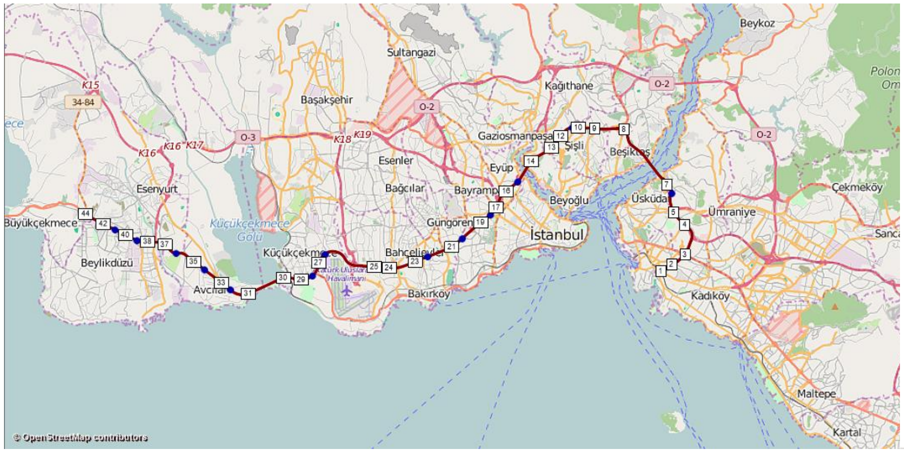
# Substantial improvements are possible.

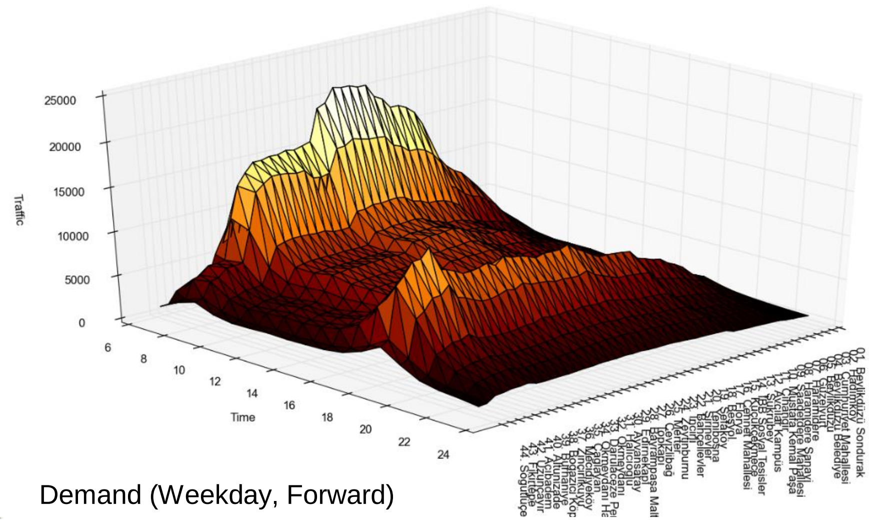


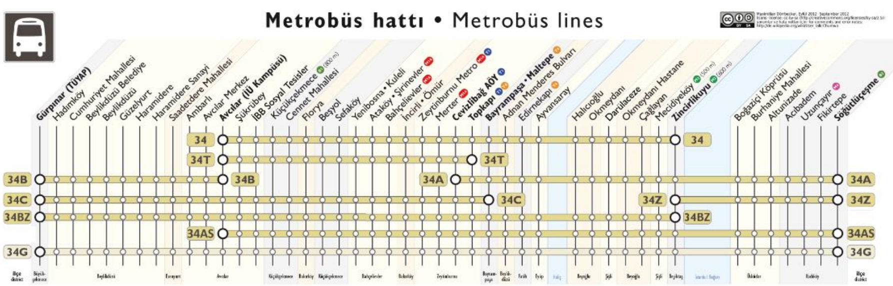

| Scenario | Average Perceived TT | ∅ perceived TT [% plan VBK] | ∅ # transfers | ∅ transfer freq. [% plan VBK] | Operation costs [% plan VBK] |
|---|---|---|---|---|---|
| Reference case | 26.771 | 99.4% | 0.3959 | 98.9% | 108.6% |
| Plan VBK | 26.922 | 100.0% | 0.4004 | 100.0% | 100.0% |
| Quality | 26.601 | 98.8% | 0.3958 | 98.9% | 102.1% |
| Costs | 26.695 | 99.2% | 0.3972 | 99.2% | 90.4% |

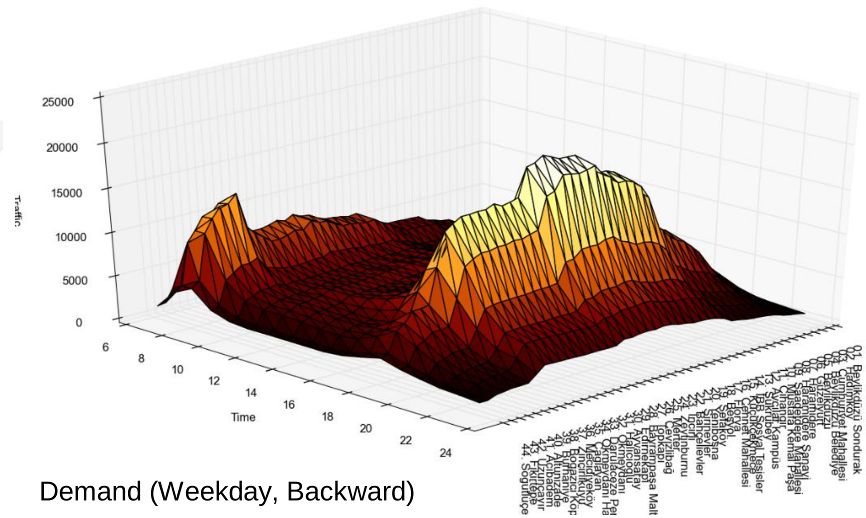
# Line Planning @ Istanbul




Demand (Weekday, Forward)

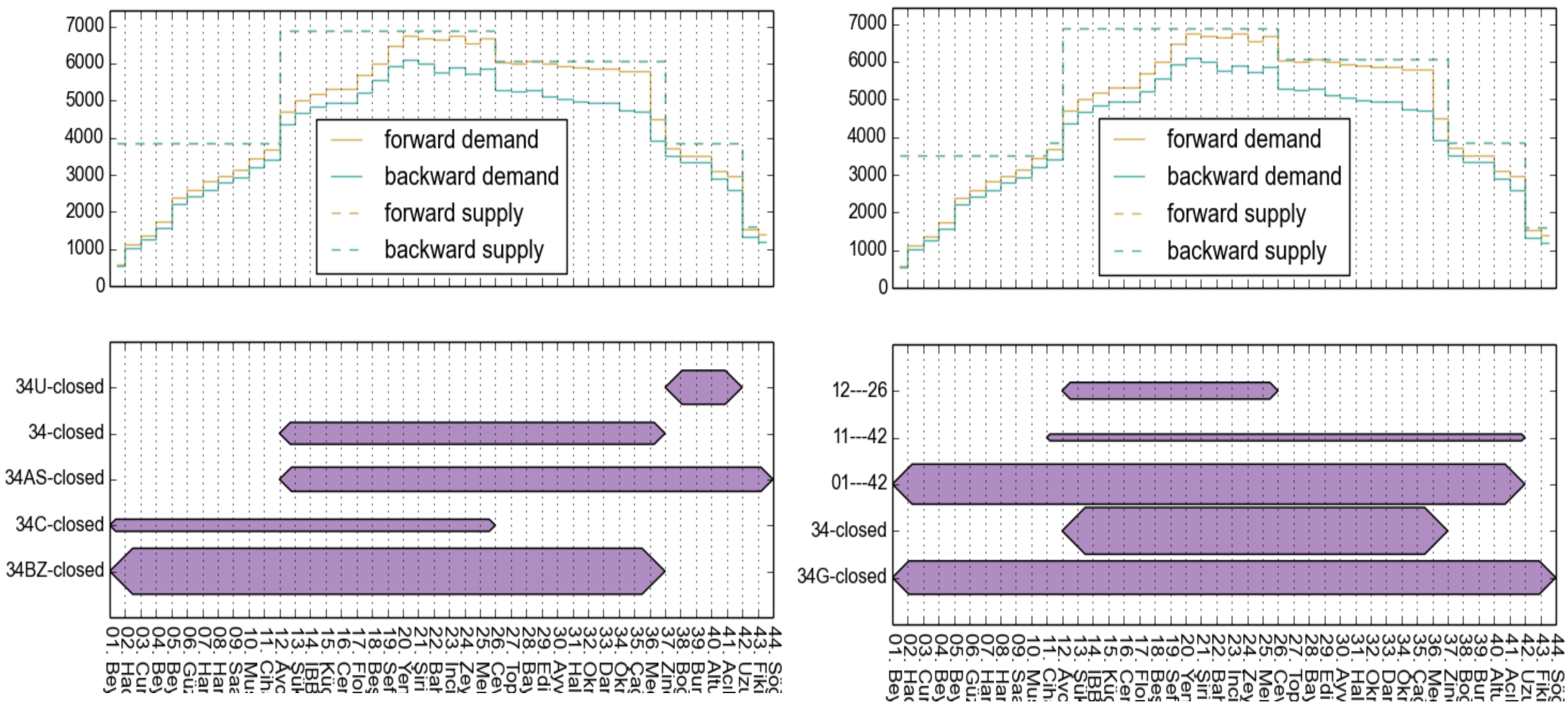



Demand (Weekday, Backward)

- Metrobüs BRT System

# Mathematics can explore new ideas.




- Demand & Status Quo vs. Unimodal Timetable (14:00)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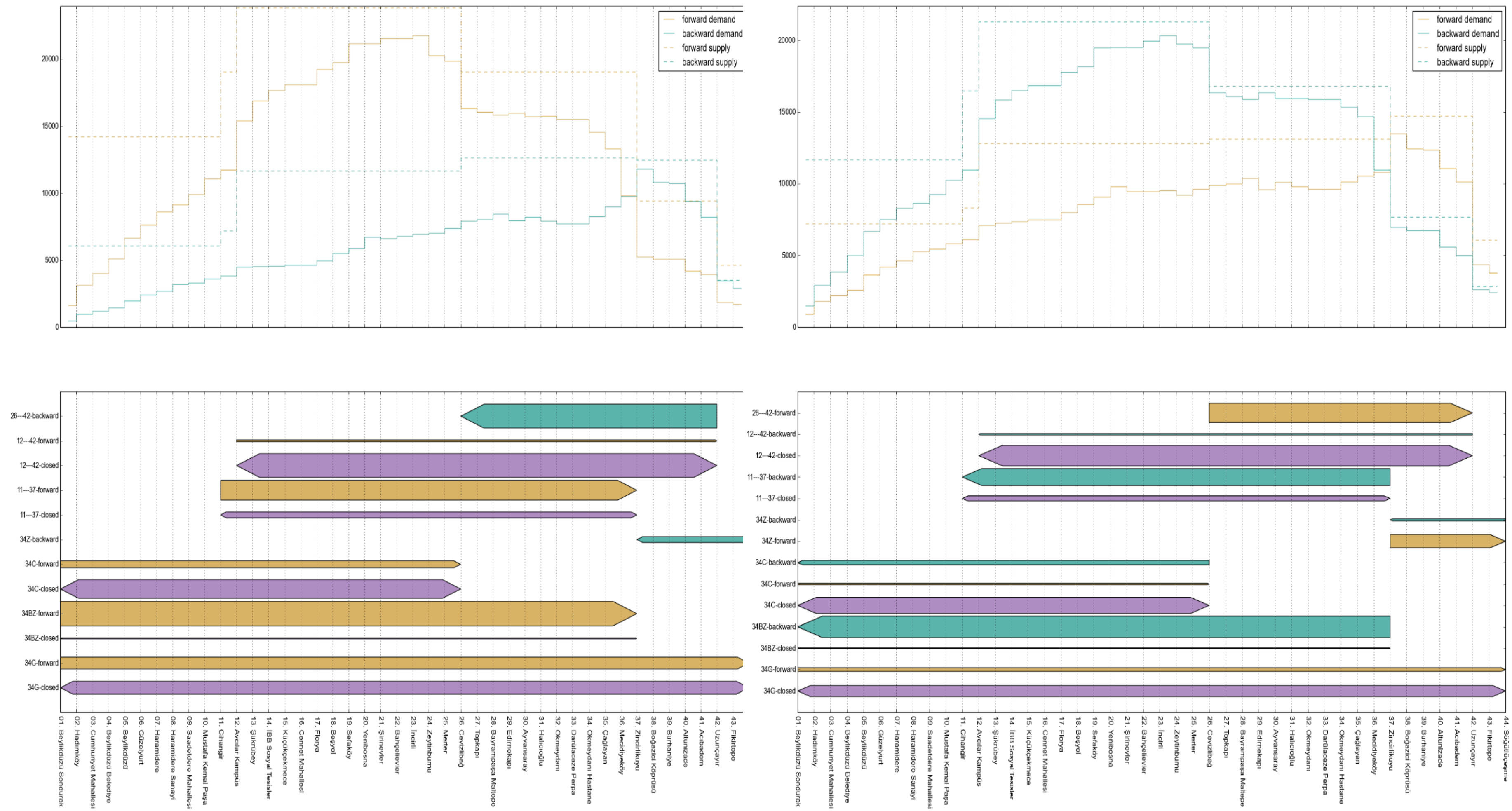
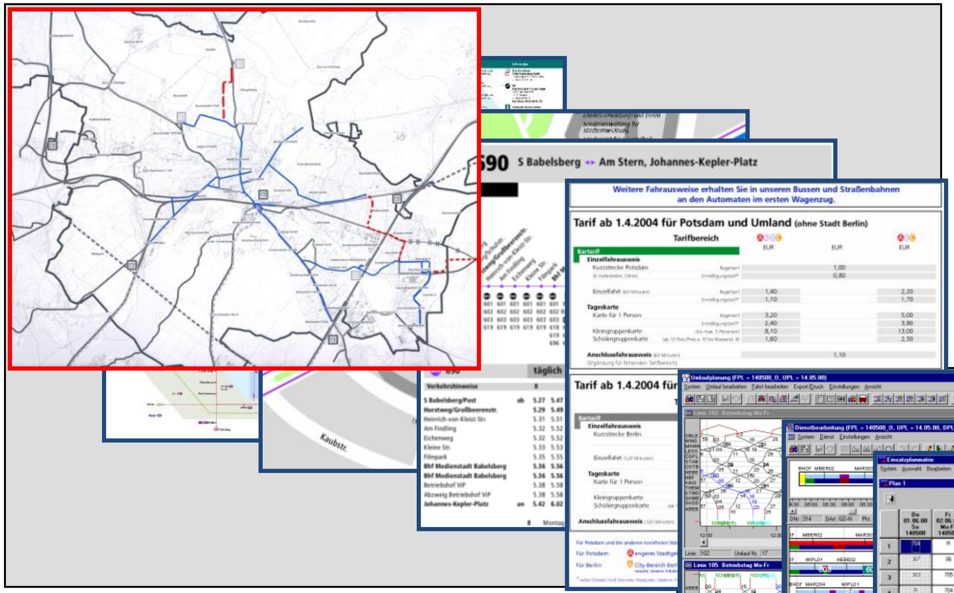
# Mathematics can explore new ideas.

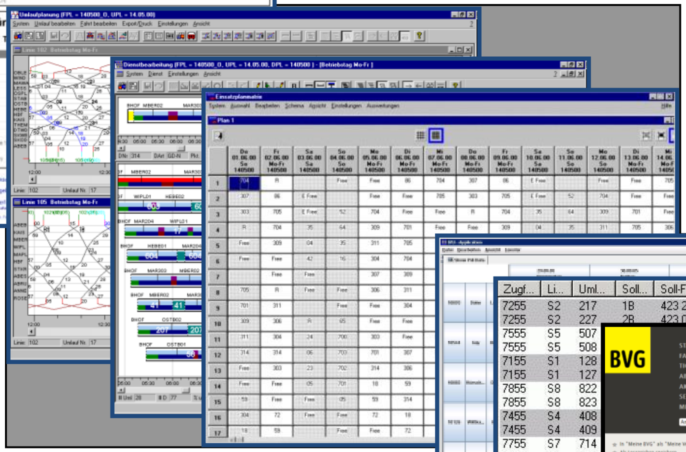



- Morning & Evening Peak Timetable (Open Lines)

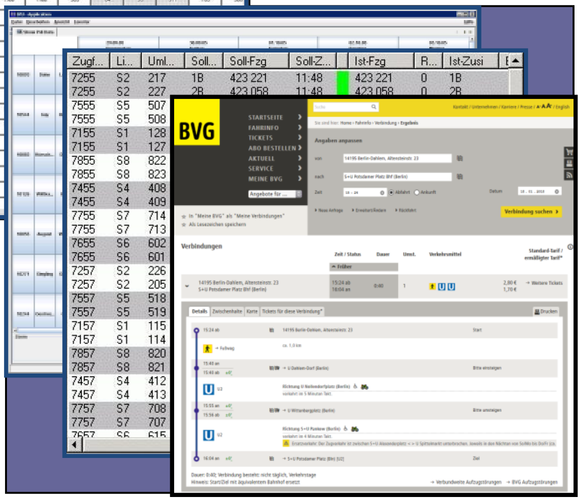
# Planning Problems in Public Transport




Strategic Planning



Operational Planning



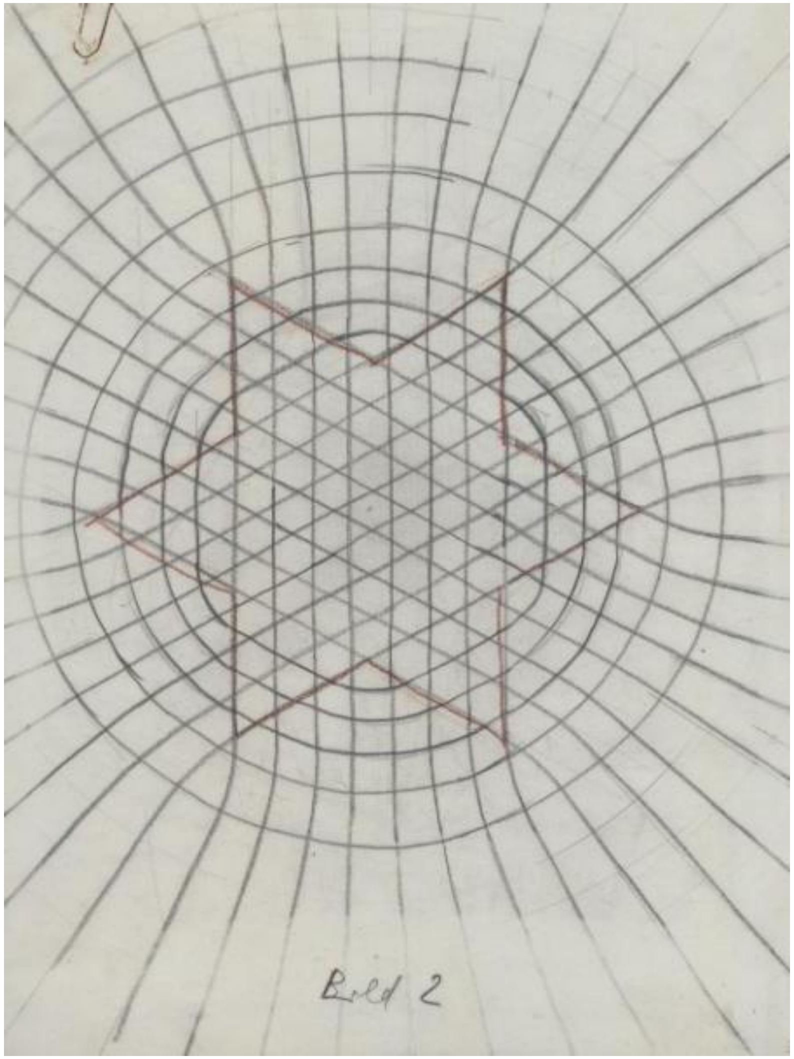

Operations Control

# Konrad Zuse


Metropolis


Konrad Zuse and his Z1

# Metropolis (Δ-Grid with Branches & Rings)




„In those days, either the Am-

60° system in the center."



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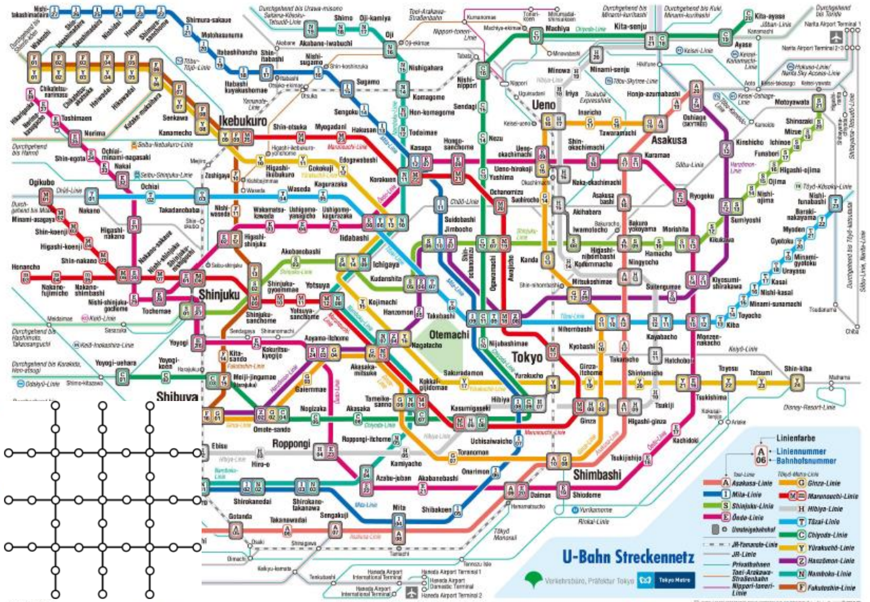
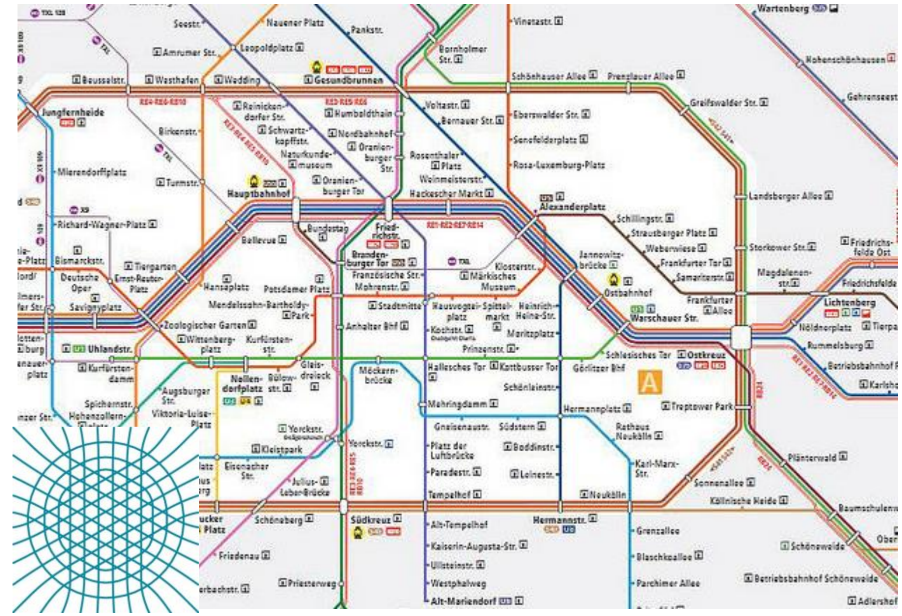
# Once built, networks persist.



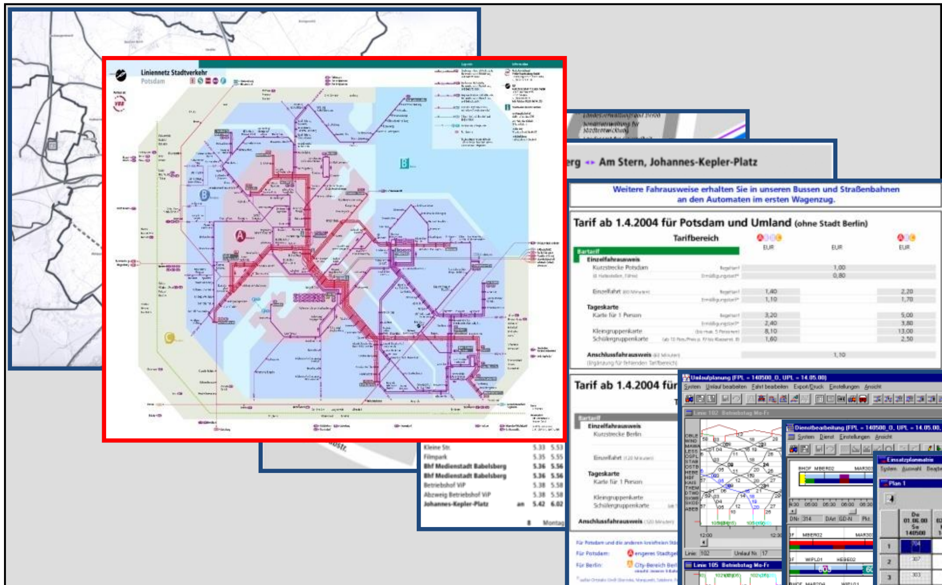

Grid / Tokyo



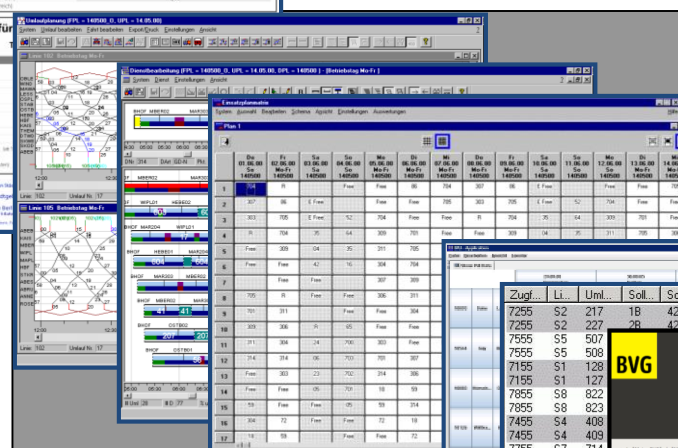

Metropolis / Berlin

| Thoma [2016] **Scenario** | $p$ | $\|V\|$ | $\|E\|$ | $\kappa$ relative cost | $\nu$ pax utility | $q^*$ pax-operator efficiency |
|---|---|---|---|---|---|---|
| 10x10,1,0 | 3 | 121 | 220 | 1.8334 | 0.6001 | 0.5952 |
| Metropolis | 3 | 252 | 450 | 1.9736 | 0.7121 | 0.6443 |
| Berlin | 3 | 122 | 157 | 1.3039 | 0.5365 | 0.4115 |
| Tokyo | 3 | 125 | 183 | 1.6668 | 0.5424 | 0.3254 |

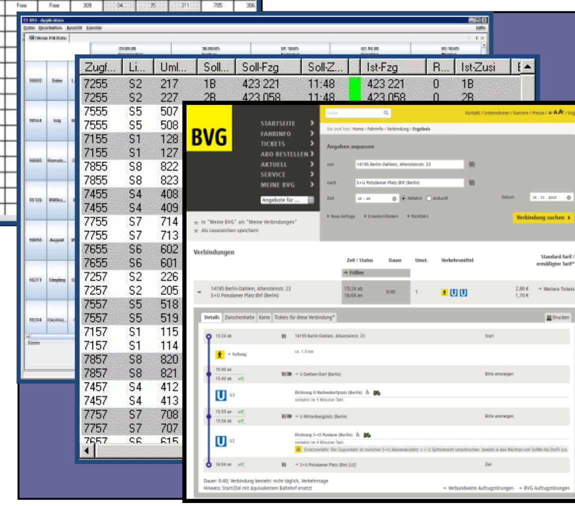
# Planning Problems in Public Transport




Strategic Planning



Operational Planning




Operations Control



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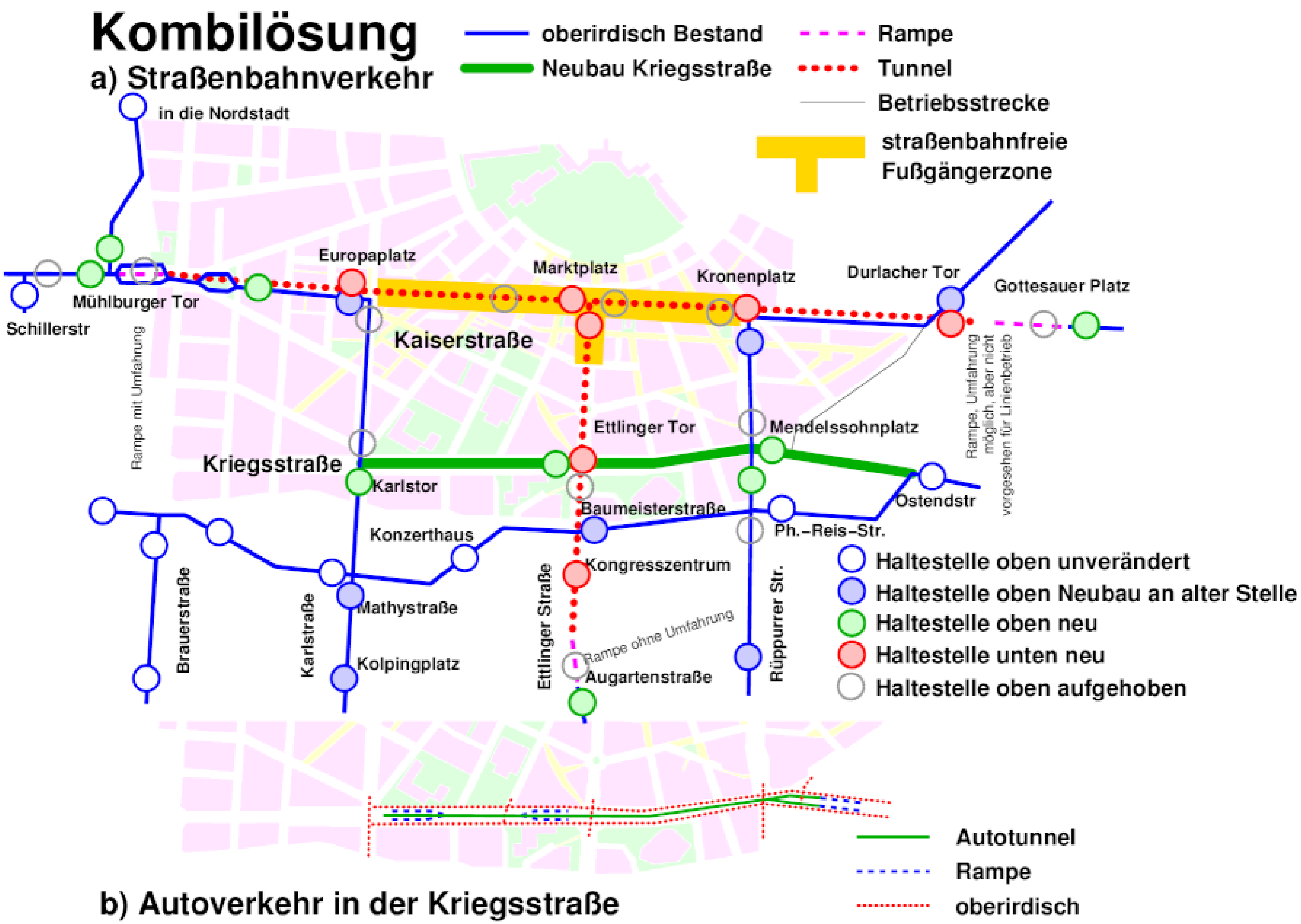
# "Kombiverkehr" (Combined Traffic) @ Karlsruhe



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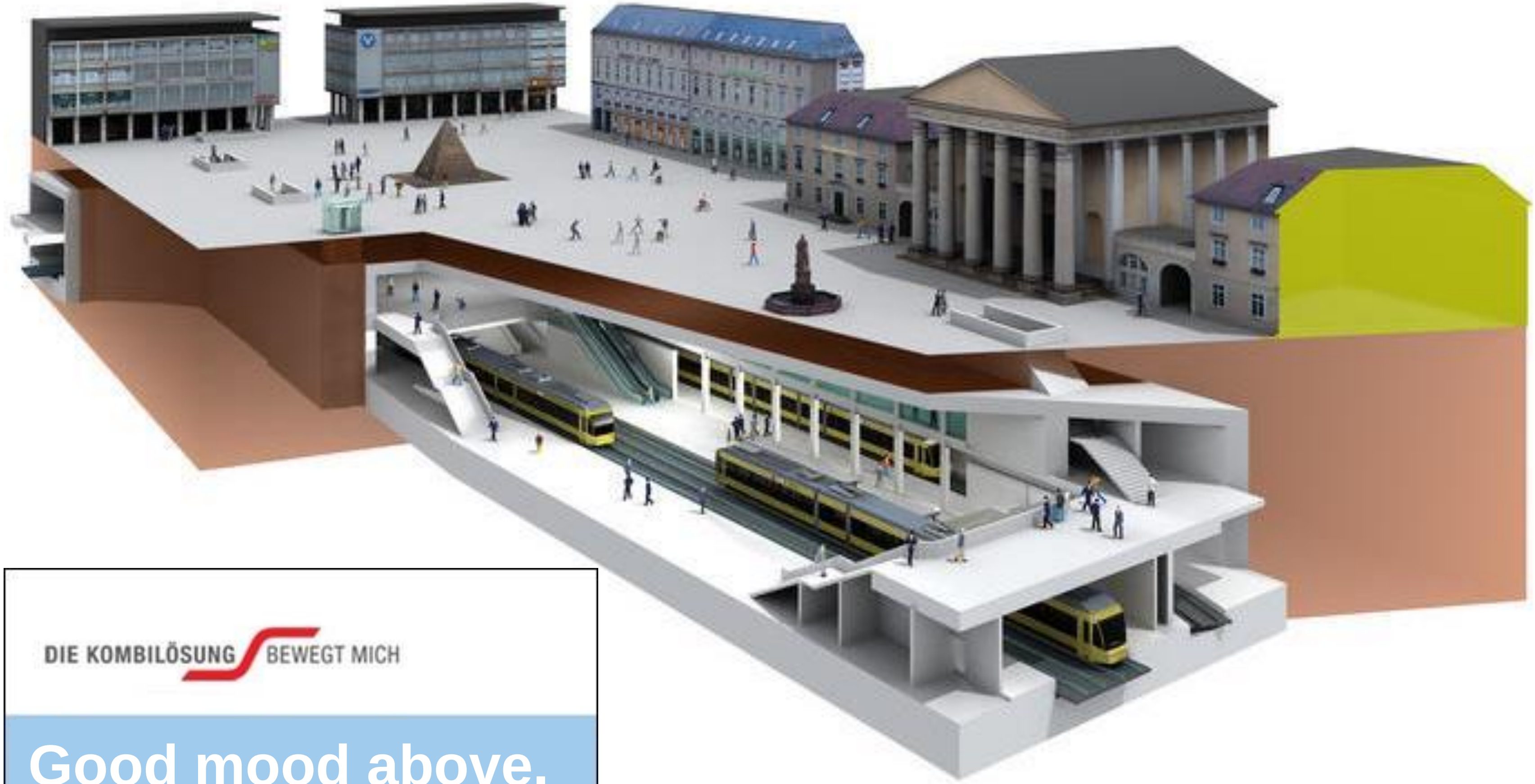
# "Kombiverkehr" (Combined Traffic) @ Karlsruhe



DIE KOMBILÖSUNG BEWEGT MICH

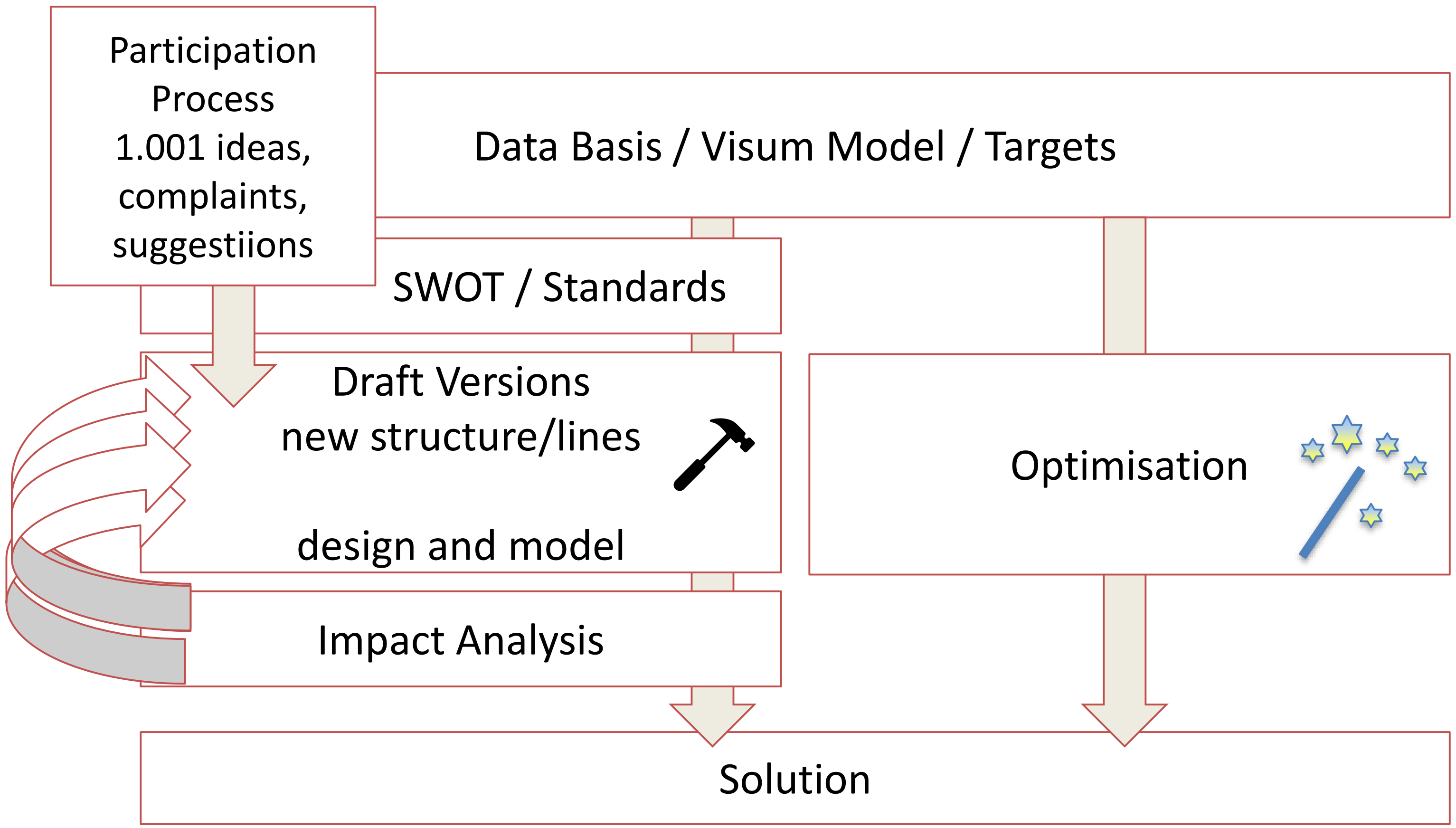
**Good mood above.**

**Good move below.**

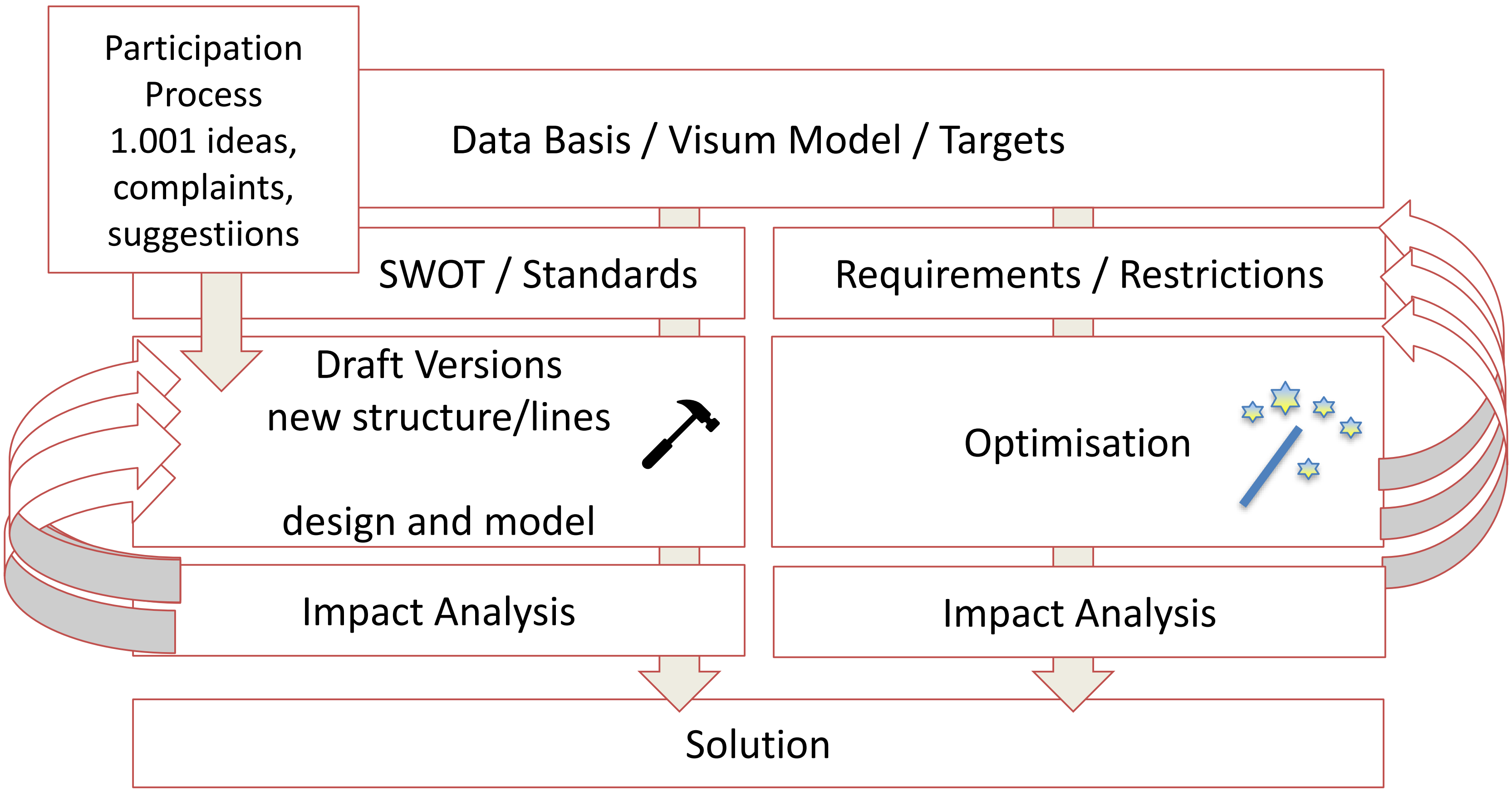
# "Kombiverkehr" Construction Site


Ka-news.de

# Planning Process



# Planning Process



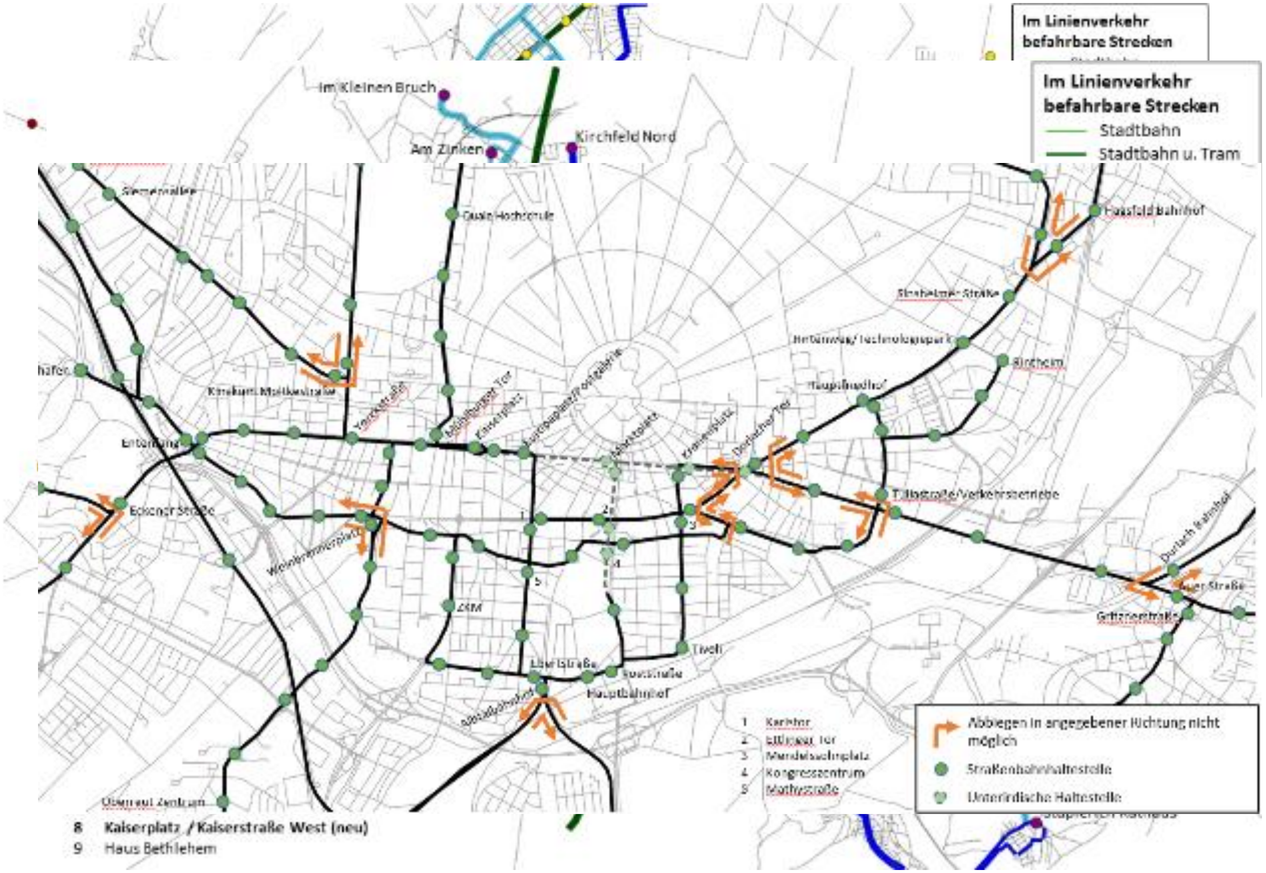
# Data to set up Visum Model

routes/stops: appropriate/not appropriate for tram / light rail / bus

stops: appropriate/not appropriate for double traction

define potential terminal stops/stations

turns for trams/LRT possible, not possible



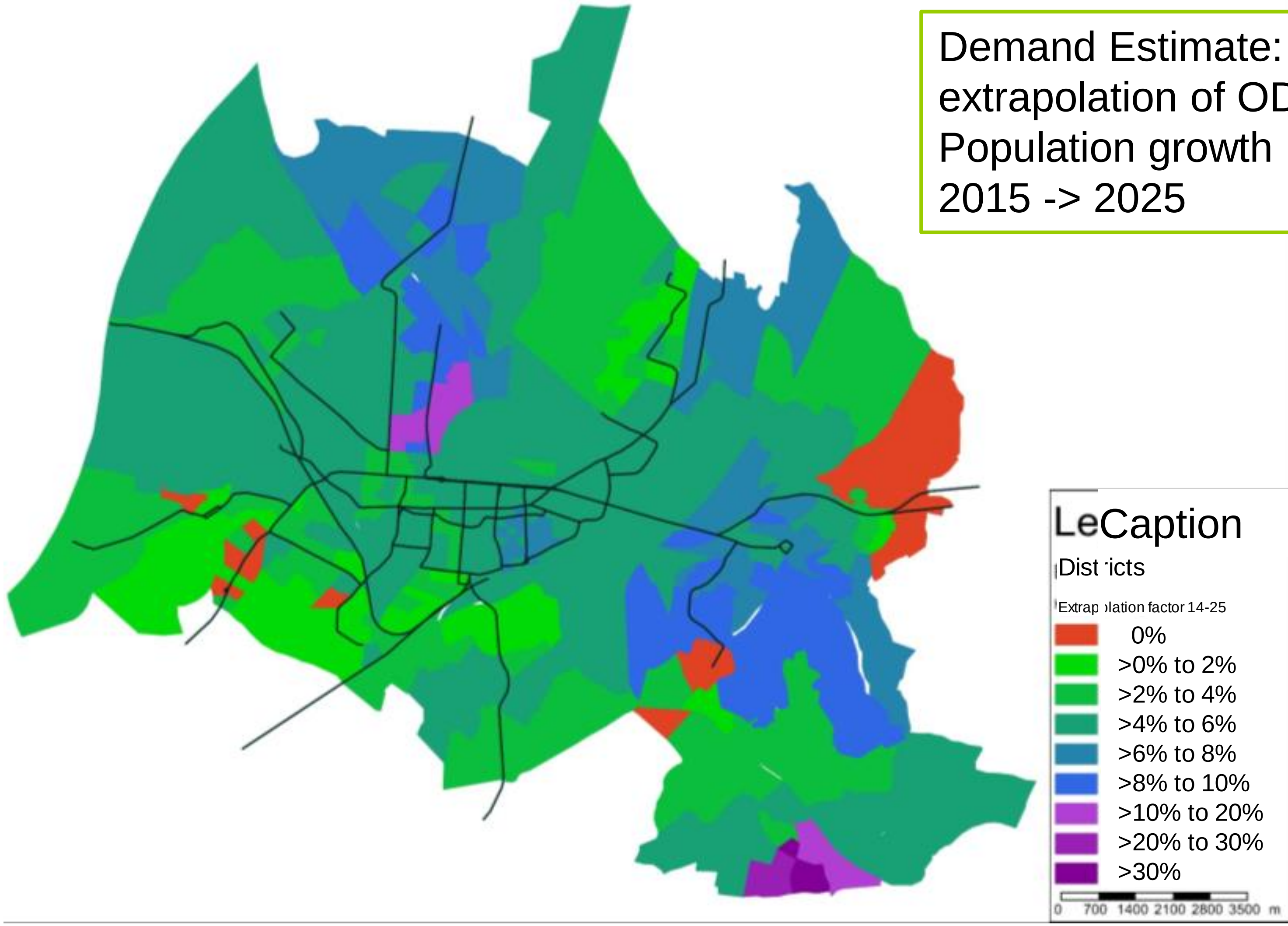

Public transport network including lengths and travel times (bus, tram, light rail)

# 2015 PT Demand Data


Demand Estimate:
OD-Data (KVV inquiry 2008, pax counts 2015)

# 2025 PT Demand Forecast

Demand Estimate:
extrapolation of OD-data
Population growth
2015 -> 2025

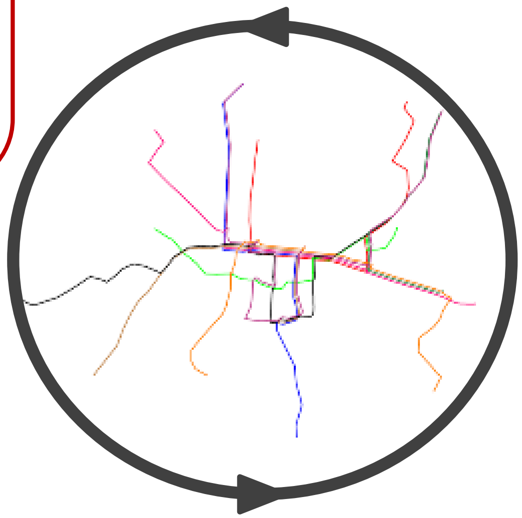


# Planning Process: Definitions

**Reference Case**:
The overall costs should not exceed the „Standi“ solution, constructed for a cost benefit-study in 2003

transfer **penalty**:
8 minutes

small penalties on overloaded segments



fixed passenger demand (2025)

determine **vehicle capacities and costs** per kilometer taking fleet development into account

Operation costs = line length (to+fro) ·frequenz ·cost rate

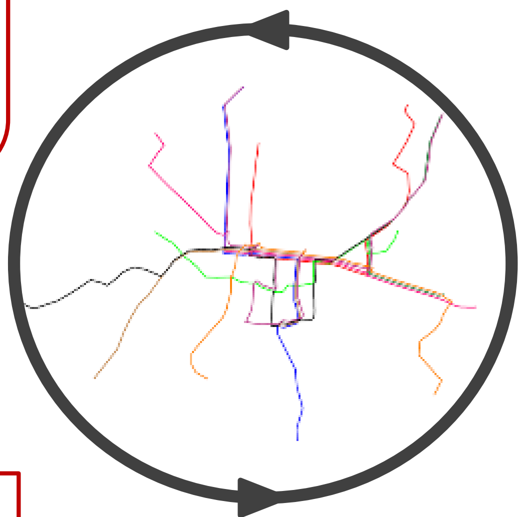
Vehicle costs = ⌈"line travel time (to+fro)" /“frequency"  " " ⌉·fixed cost rate

# Planning Process: Restrictions / Iterations

permit only 10 minutes **headway** or superposition of at least 2 frequencies of 20 minutes

**fixed** bus **lines**

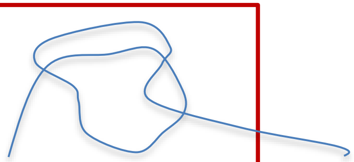
fixed LRT lines



maximum capacity of the tunnel

forbid some branch-combinations: weakly vs. highly capacitated

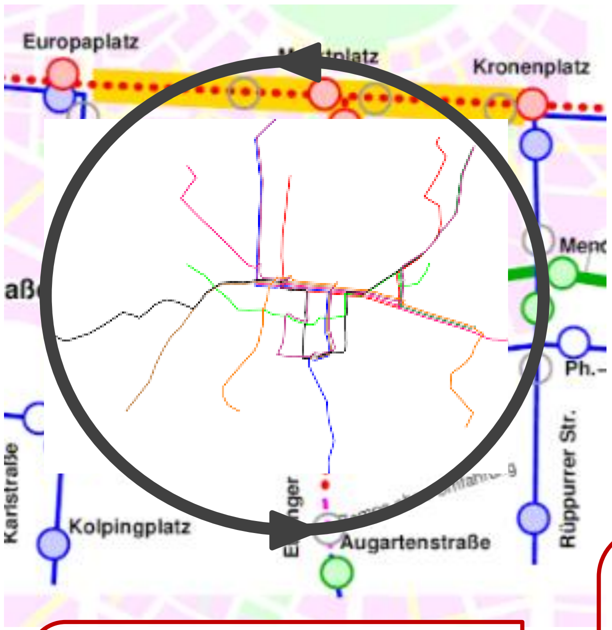
lines should not cross themselves



# Planning Process: “Specific” Restrictions

one direct connection via Karlsstr. / via Ettlinger Str. / via Rüppurer Str. to Karlsruhe main station

special effect: Konzerthaus



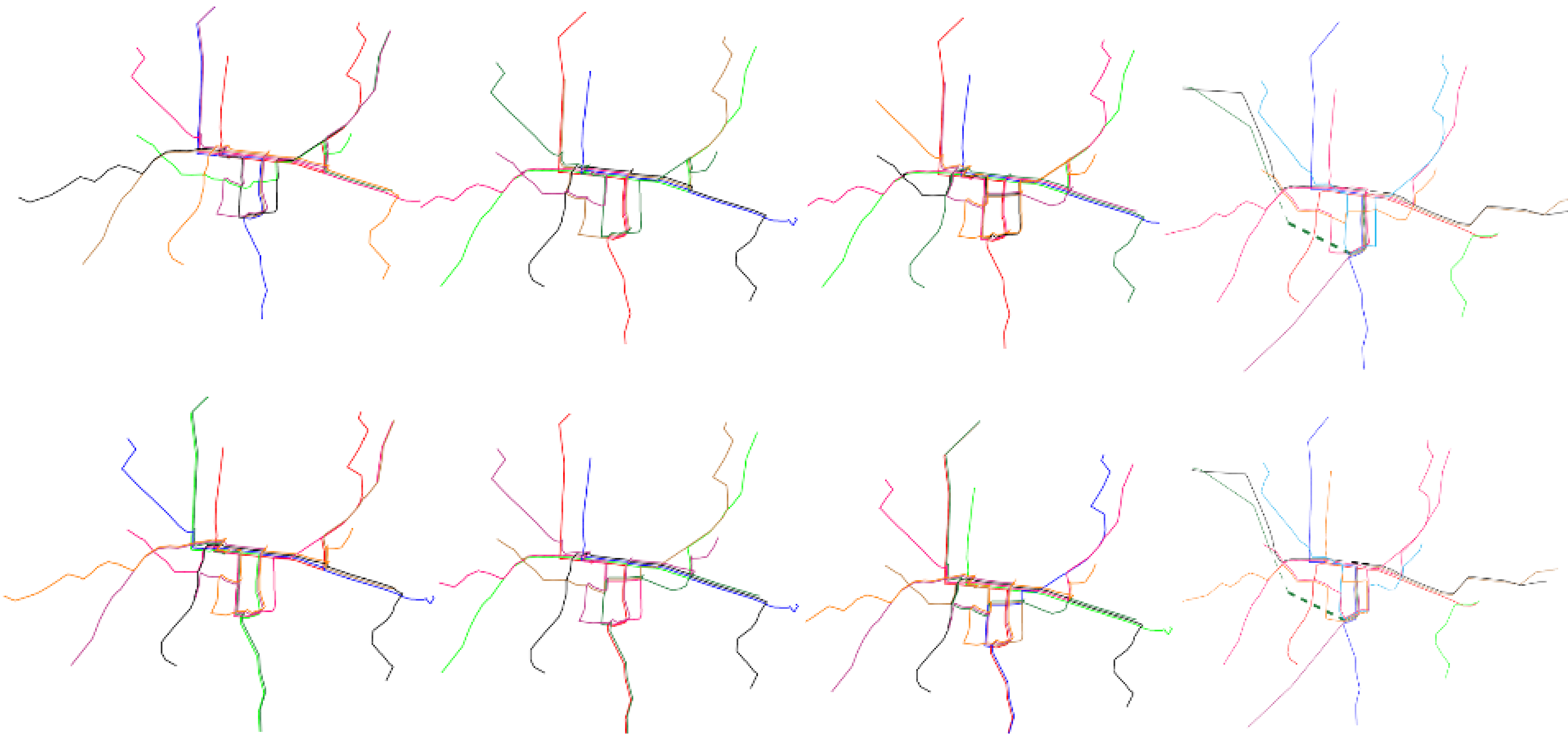

all tracks must be covered (except Kapellenstr.)

at most 6 lines via Gottesauer Platz

e.g. S2 generally considered as operating in single traction

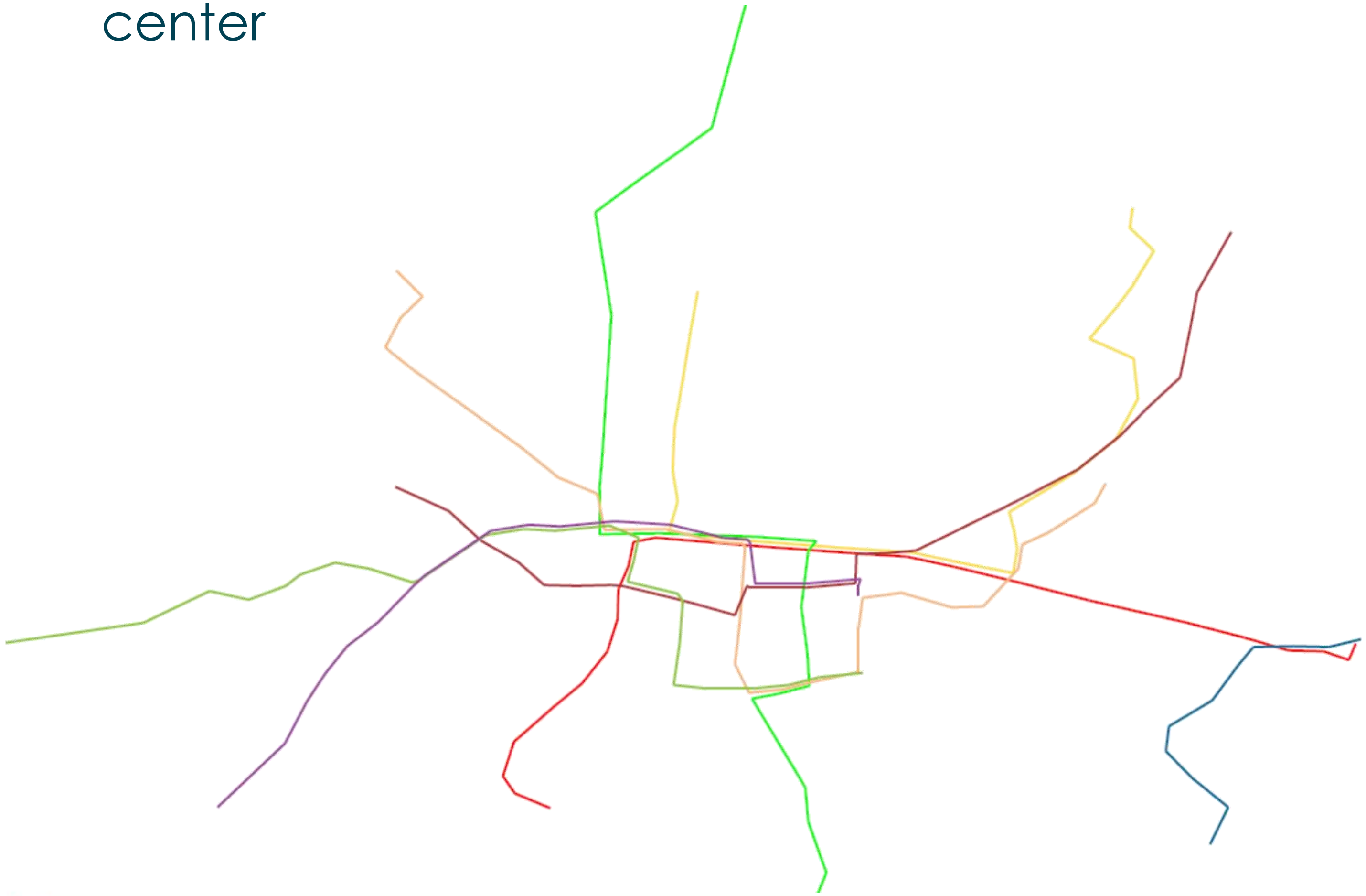
# Computing Solutions

- Various solutions computed with varied restrictions
- Travel time improvements vs. cost reductions



# Extreme "Costs" Optimization

- Capacity utilization at maximum
- no direct connection between Wolfartsweier and center

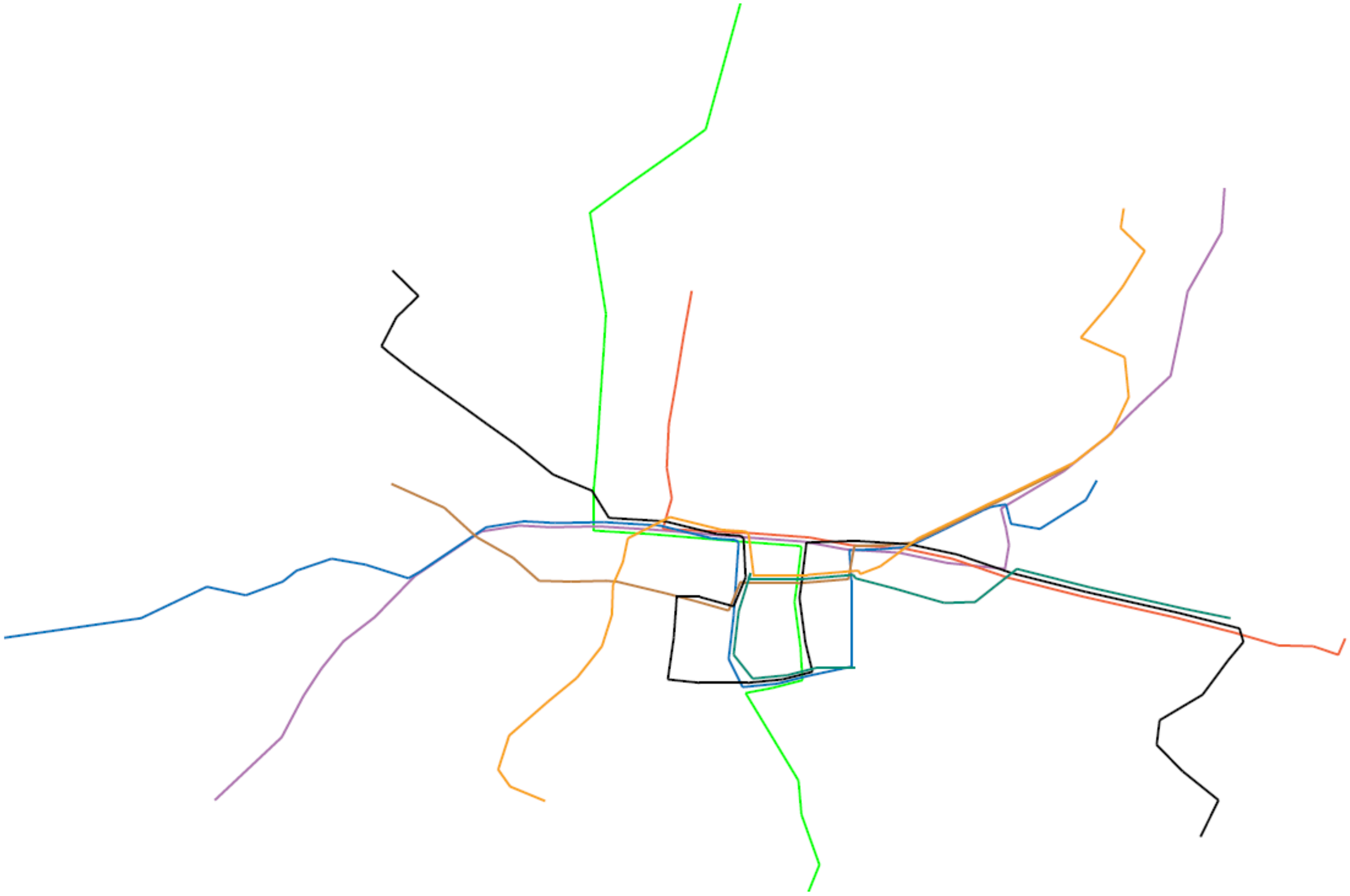


# Extreme Travel Time Optimization

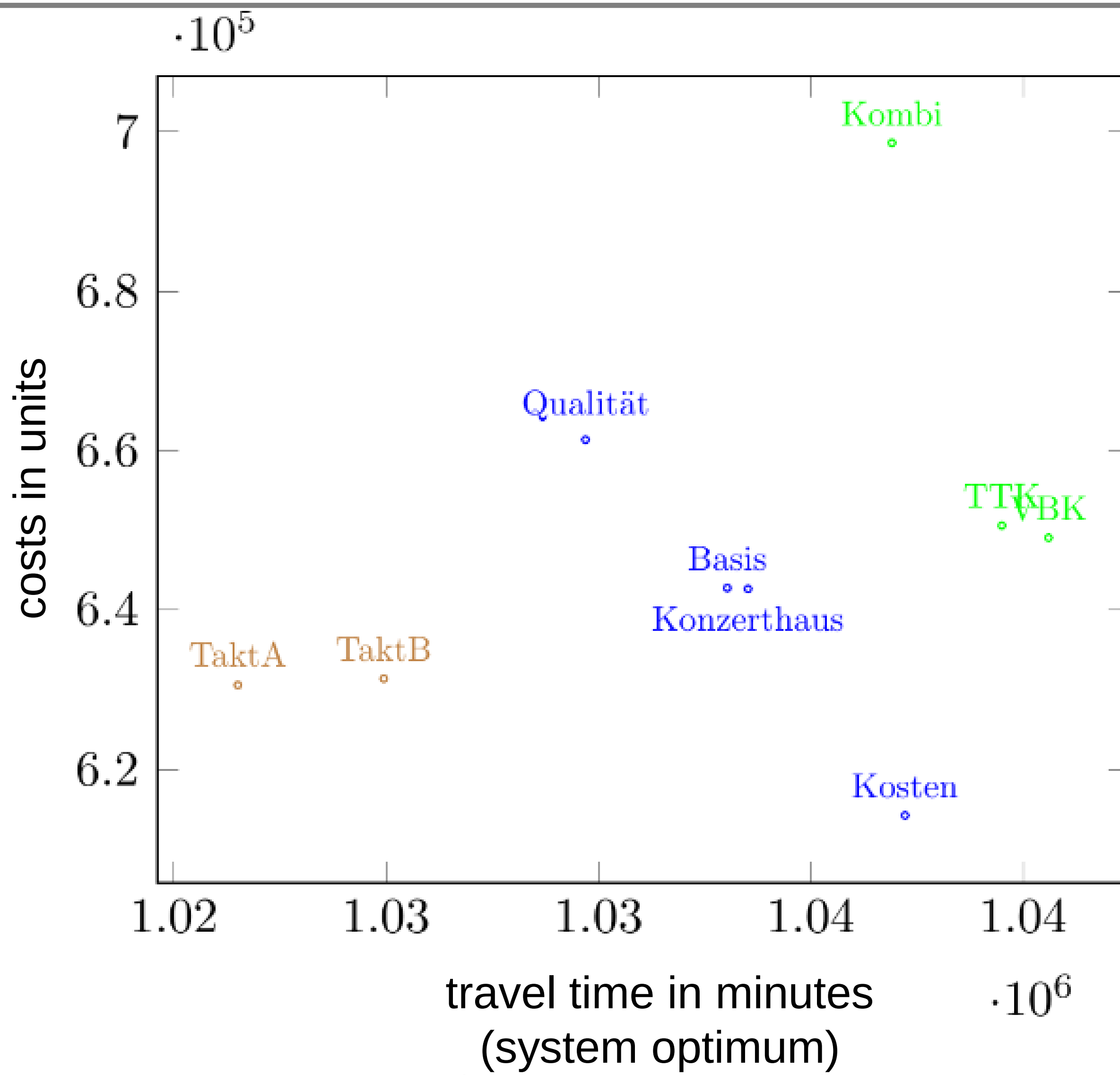
- Tunnel capacity and costs at maximum

# "Quality" Solution

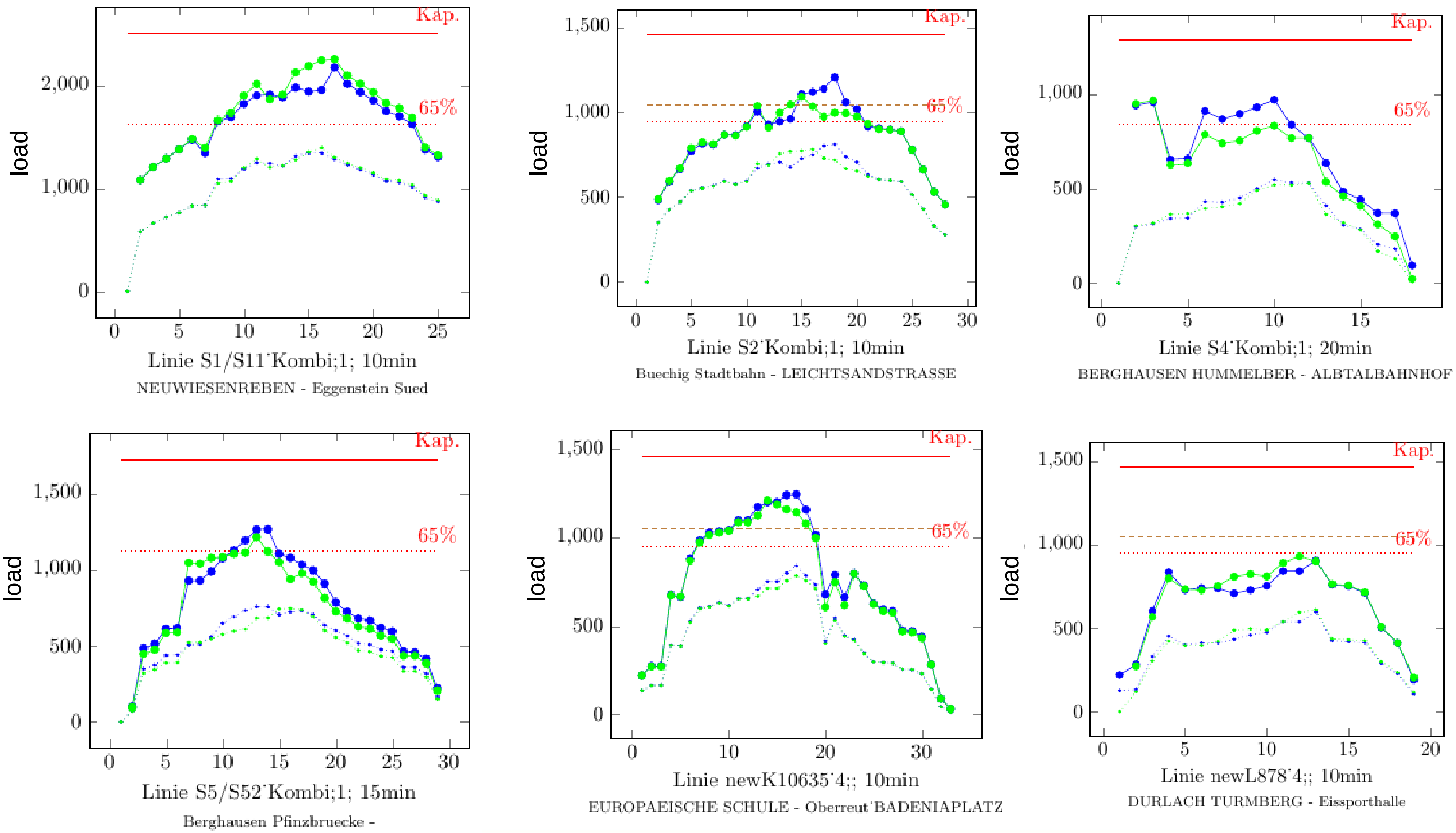
- 8 lines (reference case: 9)



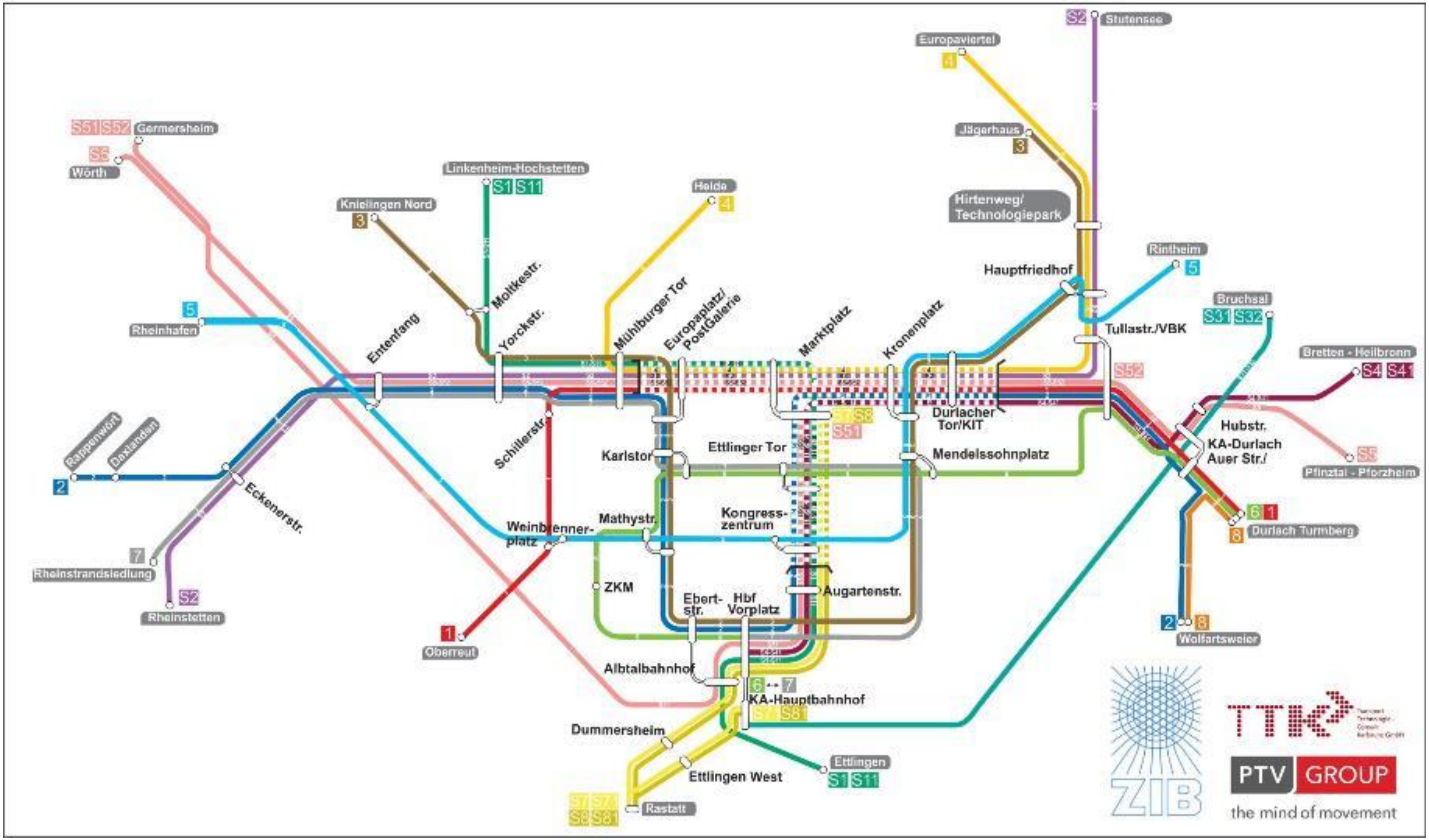
# Solution Overview



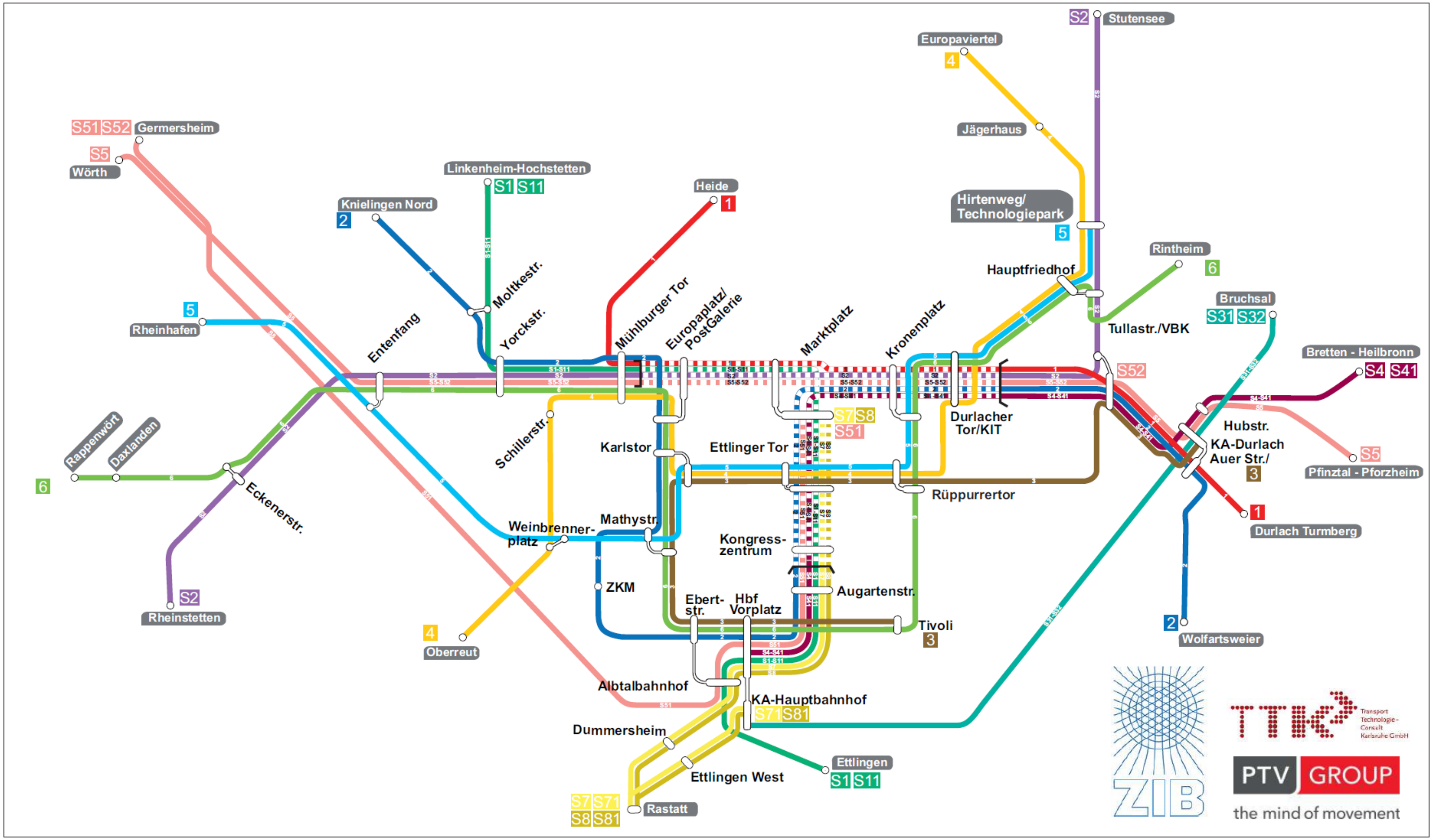
# Scenario "Costs" (6 Lines of Maximum Load)



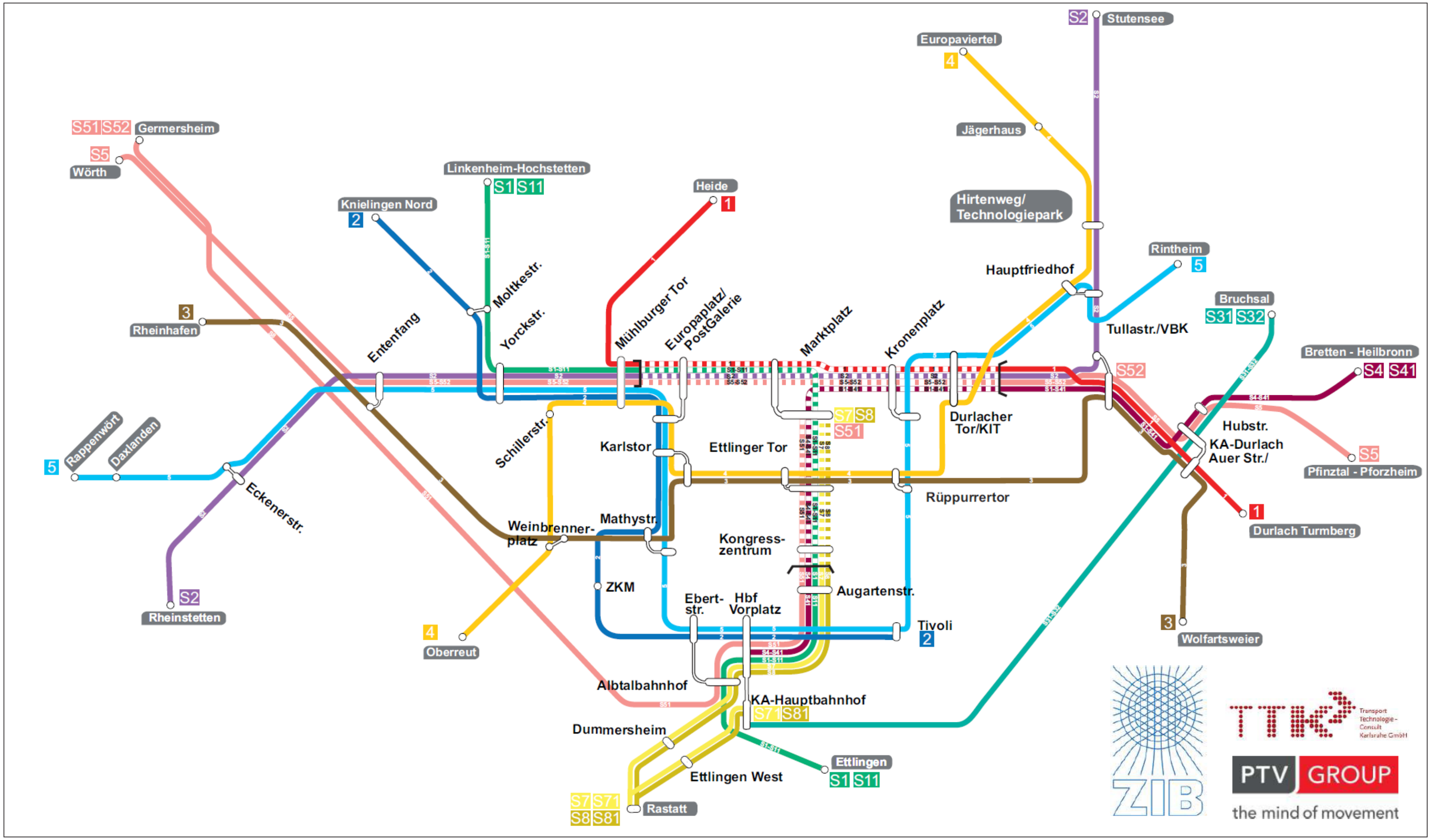
# Reference Case



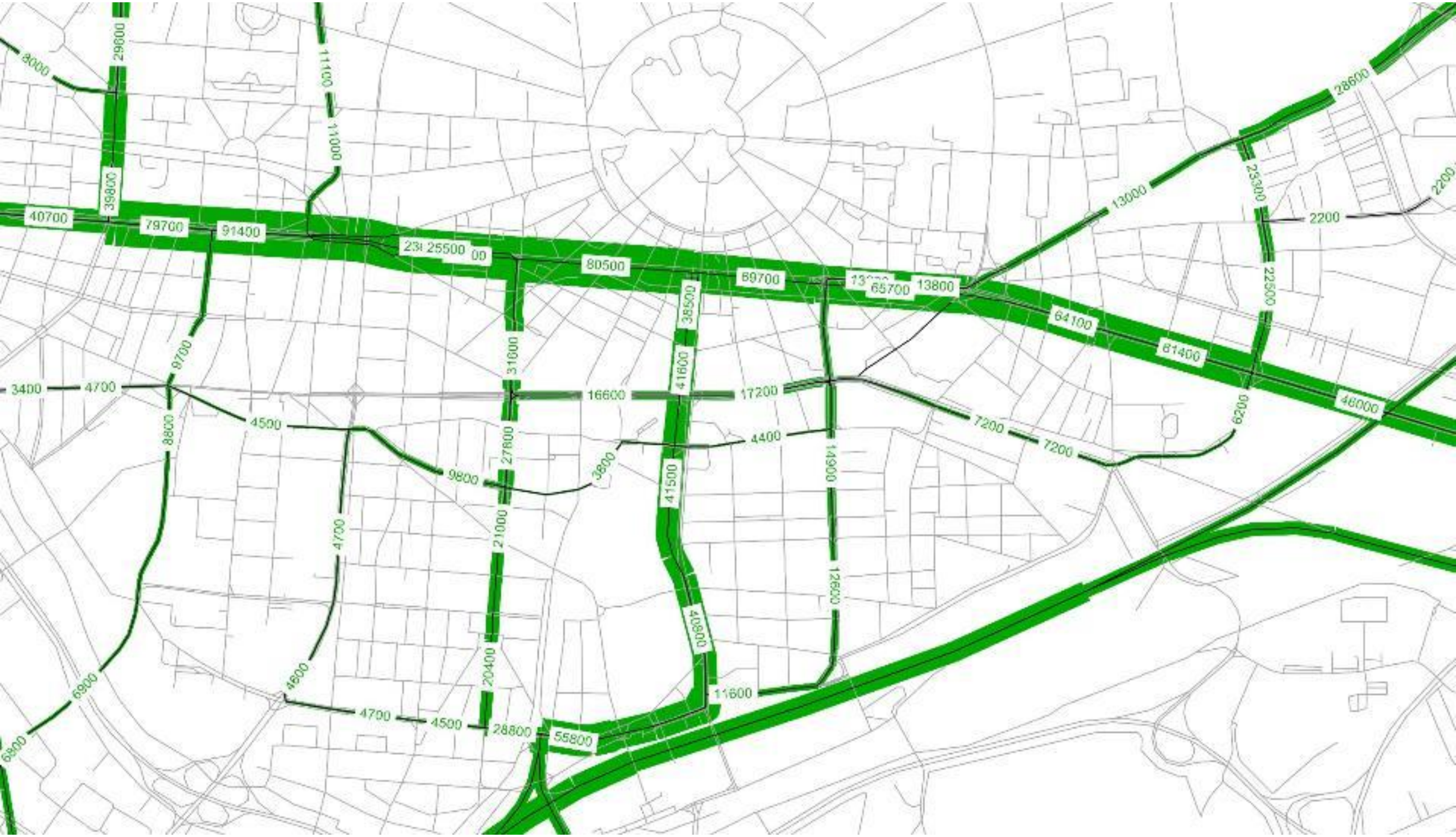
# Solution "Quality"



# Solution "Costs"



# Reference Case: Visum Pax Routing



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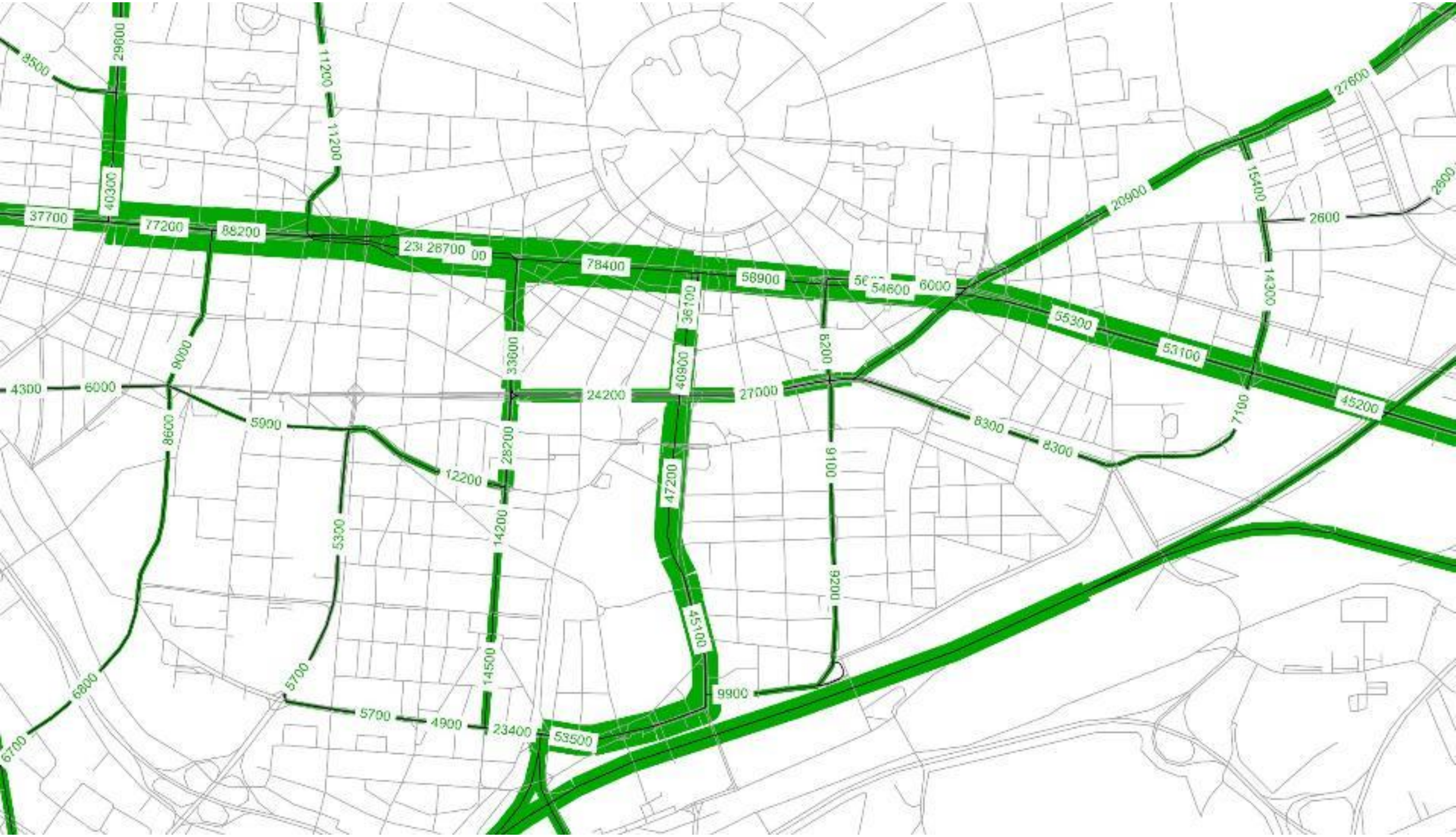
# Solution "Quality": Visum Pax Routing

# Solution "Costs": Visum Pax Routing



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# Visum Pax Routing: Comparison of KPIs

- Scenario "Costs": -10% costs compared to status quo ante
- Quality: - 6% costs compared to reference case

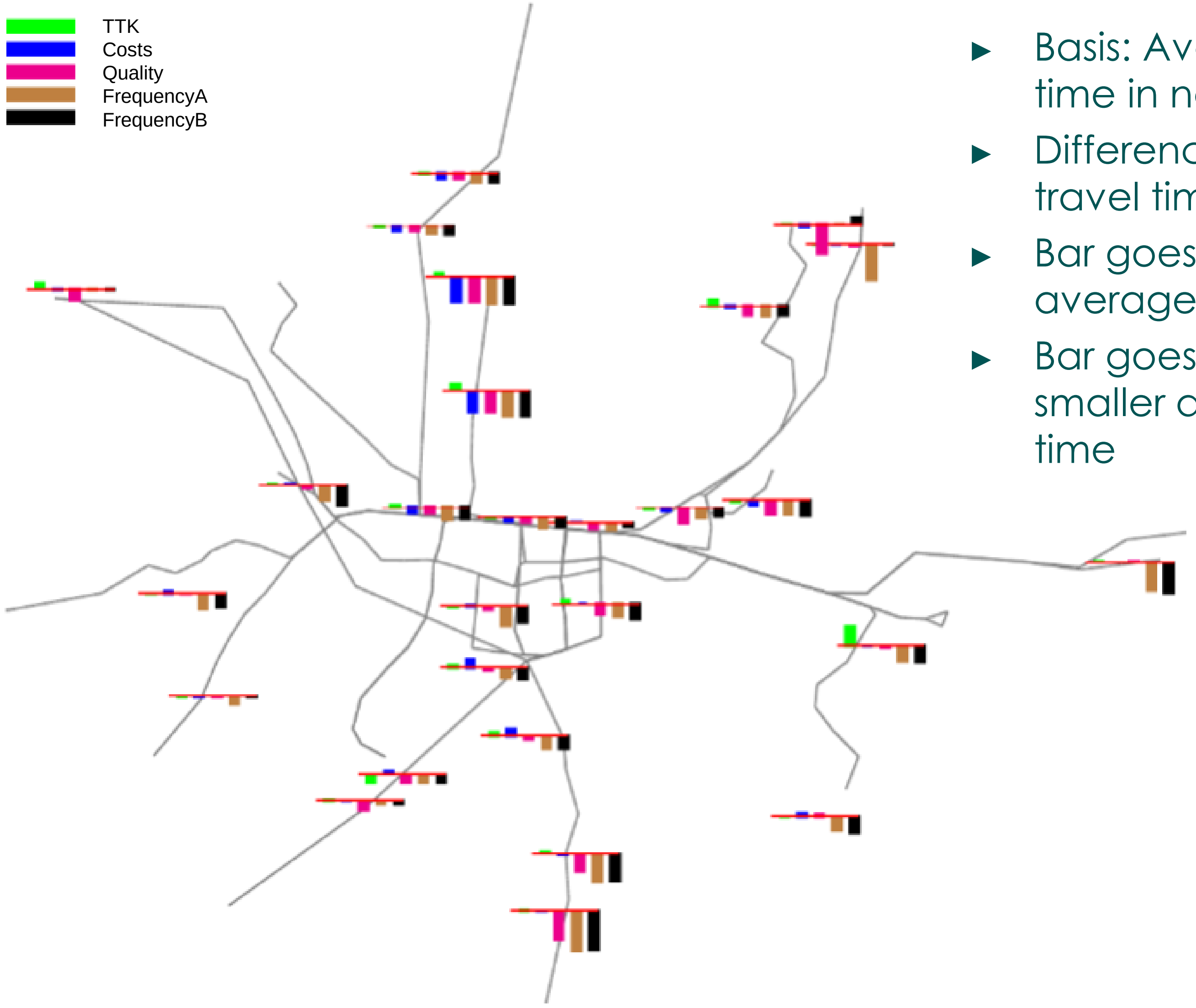
| Scenario | Average perceived TT [min] | Average perceived TT [% plan VBK] | Average # of transfers | Average transfer frequency [% plan VBK] | Operation costs [% plan VBK] |
|---|---|---|---|---|---|
| Reference case: Stand. Bewertung | 26 771 | 99.4% | 0.3959 | 98.9% | 108.6% |
| Current plan VBK | 26 922 | 100.0% | 0.4004 | 100.0% | 100.0% |
| Network TTK 2025 updated | 27 200 | 101.0% | 0.4005 | 100.0% | 100.1% |
| ZIB scenario basis | 26 627 | 98.9% | 0.3947 | 89.6% | 97.9% |
| ZIB scenario quality | 26 601 | 98.8% | 0.3958 | 98.9% | 102.1% |
| ZIB scenario costs | 26 695 | 99.2% | 0.3972 | 99.2% | 90.4% |
| ZIB scenario frequency A | 26 434 | 98.2% | 0.3814 | 95.3% | 95.0% |

# Vsisum Pax Routing: Comparison of Loads

- Tunnel (East-West): 4 instead of 5 lines (10-mins headway)

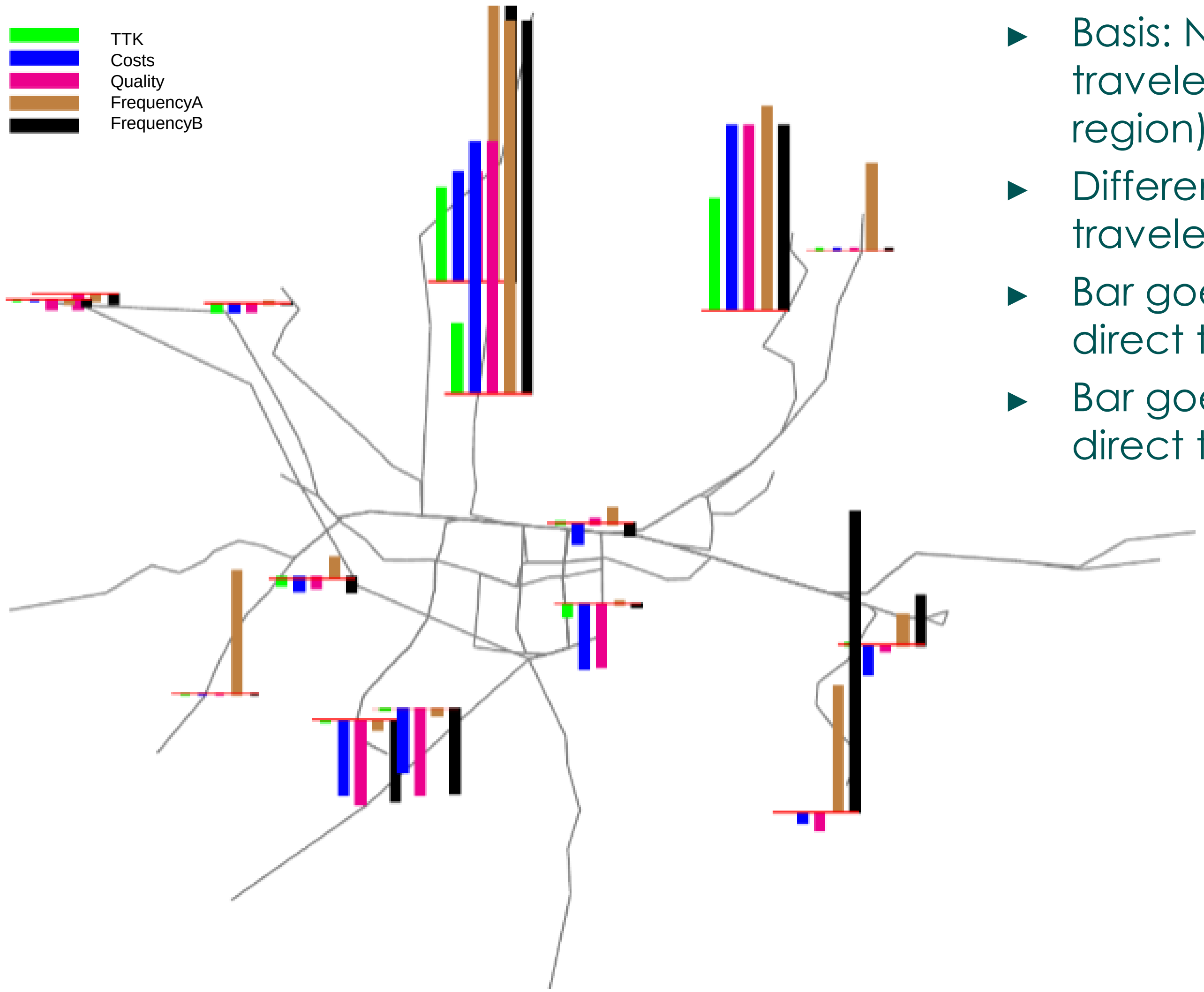
| Scenario | Load Kaiserstr. West [pax/weekday] | Load Kriegsstr. West [pax/weekday] | Load Kriesstr. West [percentage] |
|---|---|---|---|
| Reference case: Stand. Bewertung | 80 500 | 16 600 | 17.1% |
| Current plan VBK | 78 300 | 16 700 | 17.6% |
| Network TTK 2025 updated | 81 400 | 25 200 | 23.6% |
| ZIB scenario basis | 77 100 | 28 900 | 27.3% |
| ZIB scenario quality | 76 200 | 29 900 | 28.2% |
| ZIB scenario costs | 78 400 | 24 200 | 23.6% |
| ZIB scenario frequency A | 72 800 | 32 700 | 31.0% |

# Average Travel Time (Comparison to "VBK")




- Basis: Average travel time in network VBK
- Difference of average travel time in percent
- Bar goes up = higher average travel time
- Bar goes down = smaller average travel time

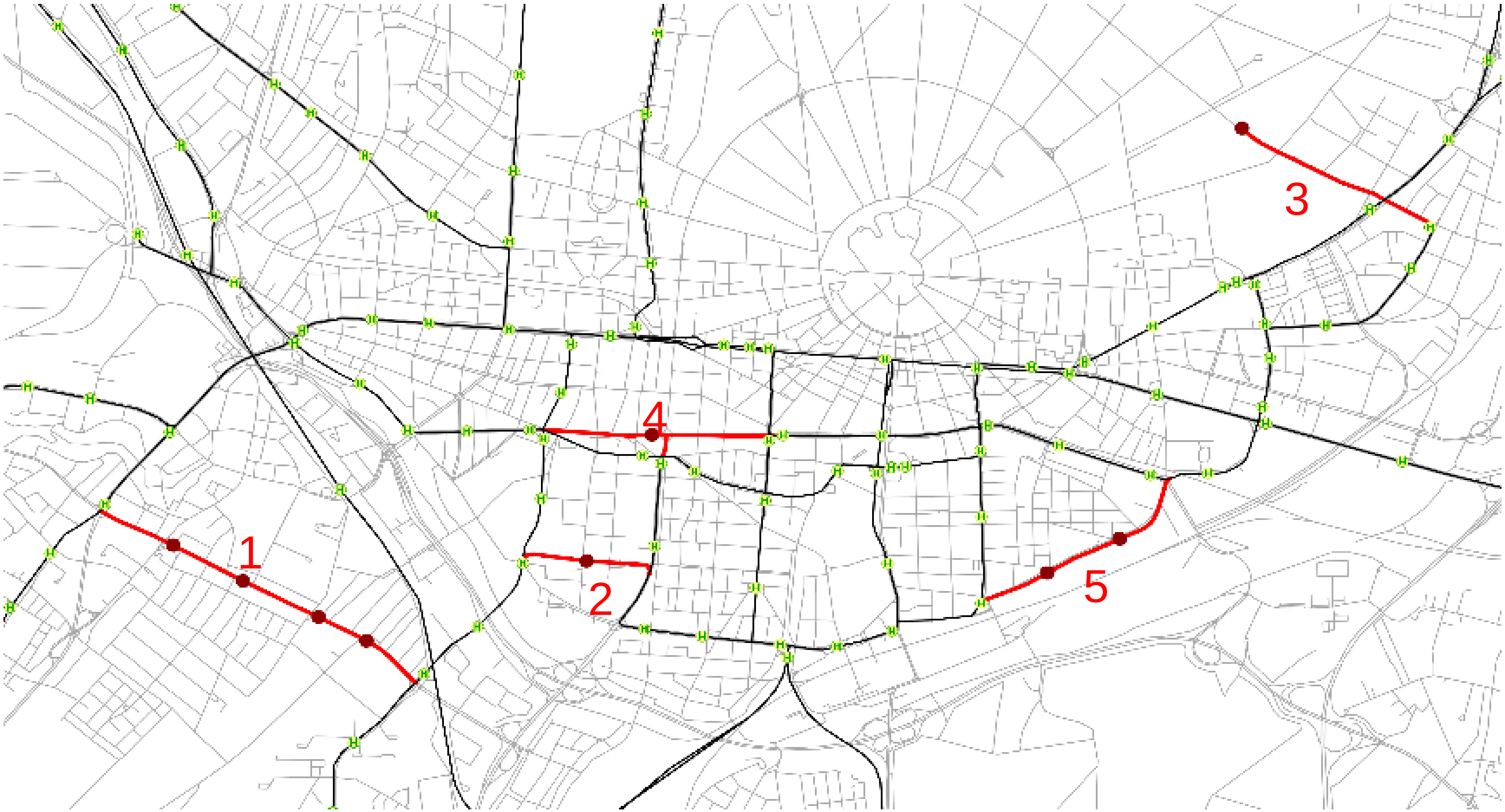
# Direct Travelers (Comparison to "VBK")




- Basis: Number of direct travelers (starting in region) in network VBK
- Difference of direct travelers in percent
- Bar goes up = more direct travelers
- Bar goes down = less direct travelers

# Network Extensions

- The demand justifies only the extension on Kriegsstraße.



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# Conclusions

- A high level of accuracy is required regarding the modelling of infrastructural and operational parameters.
- Optimization needs high quality OD-data.
- Restrictions can be standardized to some extent, but some requirements will be specific in each case/town.
- Discussions in the planning process focus on restrictions, not on quality measures/solutions.
- An iterative planning process is essential to improve solutions.

# Line Planning and Steiner Path Connectivity

**Line Planning Problem**

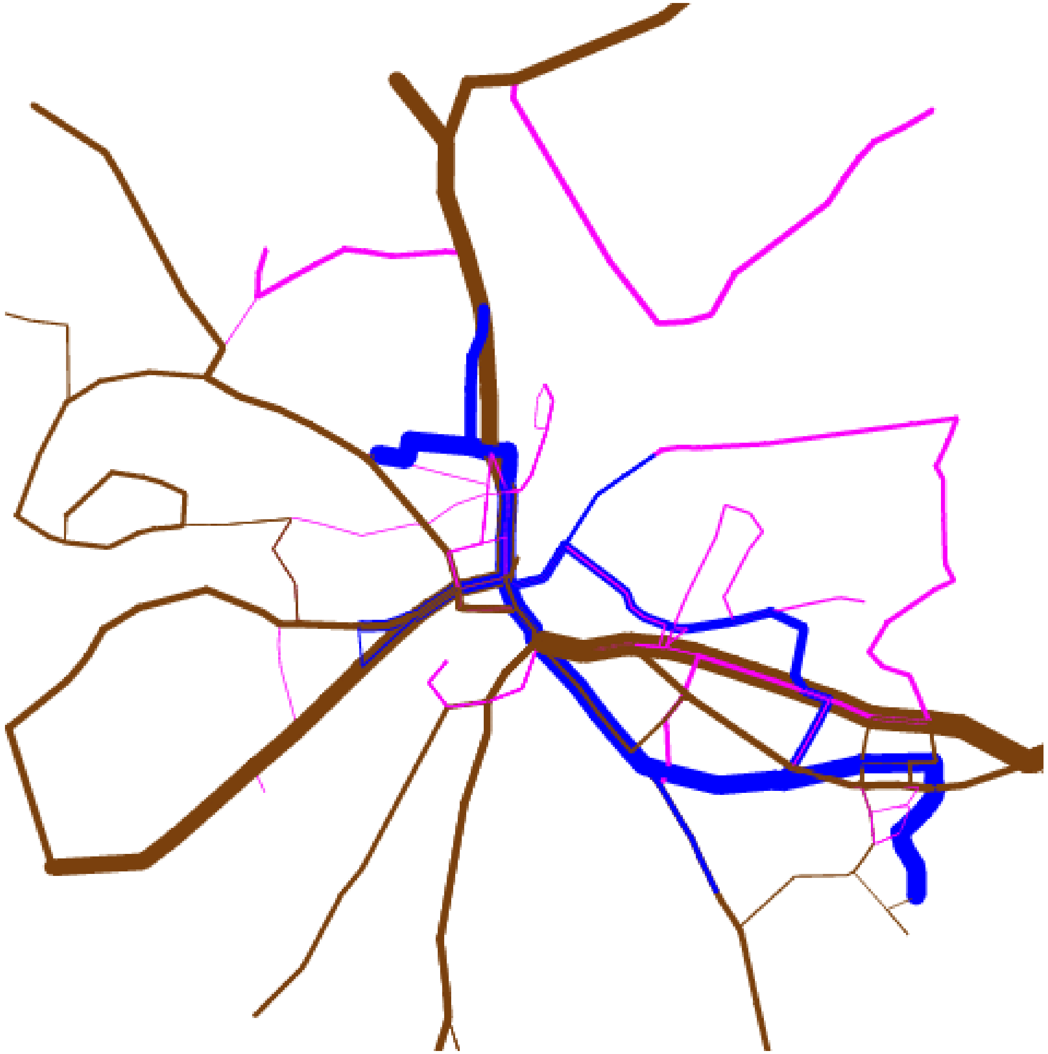
Find a cost minimal set of lines and associated frequencies, s.t. a given set of travel demands can be transported in minimal time.

**Steiner Path Connectivity Problem**

Find a cost minimal set of paths that provide enough capacity to route a fastest multi-commodity flow.

Features

- Bicriteria problem (cost vs. quality)
- Passenger behavior (transfers)



Graphics: JavaView, F4

# Steiner (Path) Connectivity Problem

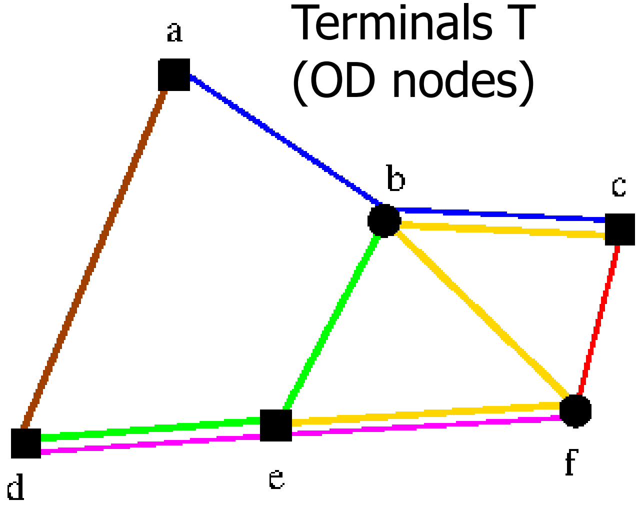
$$\min \sum_{\ell \in \mathcal{L}} c_\ell x_\ell$$

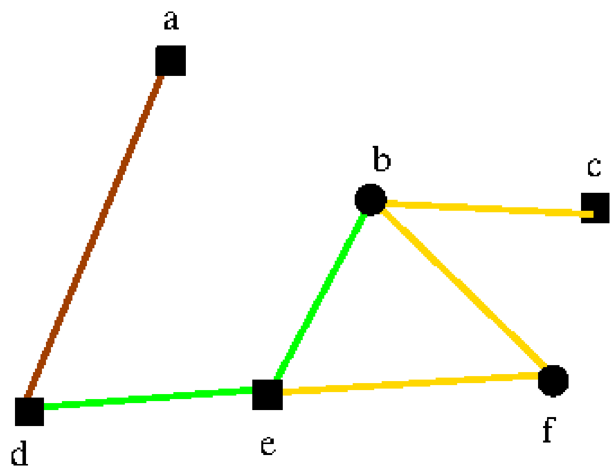
Minimize cost

$$\text{s.t.} \quad \sum_{\ell \in \mathcal{L}_{\delta(W)}} x_\ell \geq 1 \qquad \emptyset \neq W \cap T \neq V$$

Connect all OD nodes

$$x_\ell \in \{0, 1\}$$



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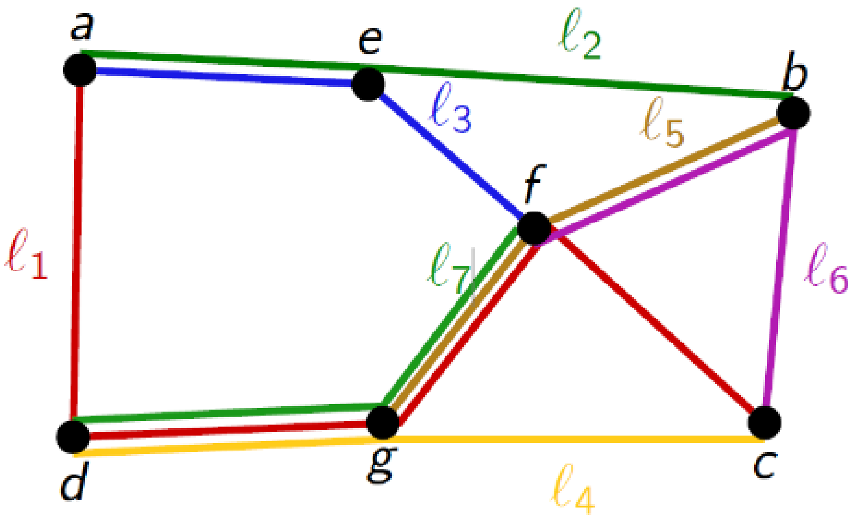
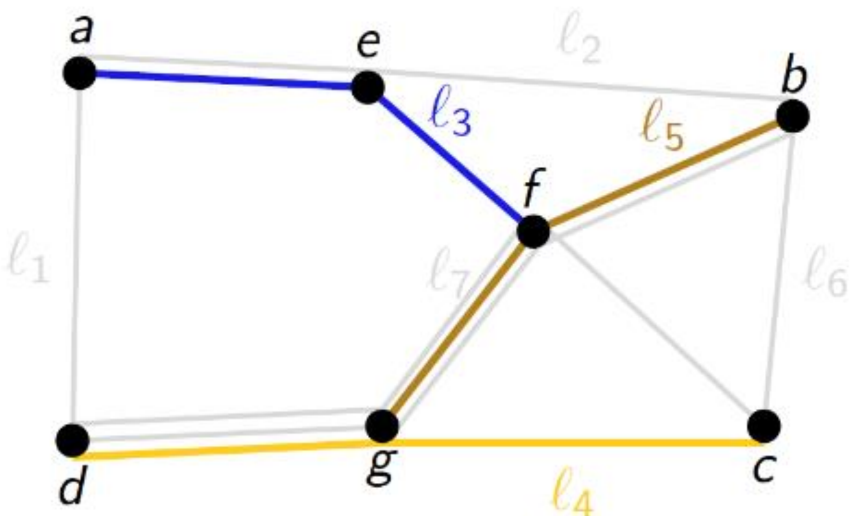
# Mathematical Line Planning Example

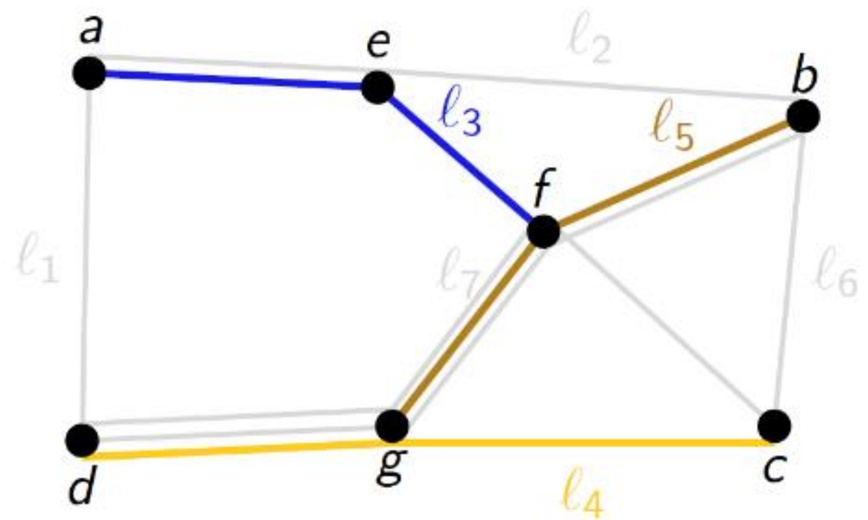



- line capacity 50
- demands:
  $a \to f$: 50; $a \to b$: 50;
  $d \to f$: 20; $d \to c$: 80




- feasible solution:
  lines $\ell_3, \ell_4$ at frequency 2
  line $\ell_5$ at frequency 1

# Basic Line Planning Model




- feasible solution:
  lines $\ell_3, \ell_4$ at frequency 2
  line $\ell_5$ at frequency 1

- travel time on path = sum of travel times on edges

  $p_1 = (a, e, f)\ \tau_{p_1} = \tau_{ae} + \tau_{ef}$

  $p_2 = (a, e, f, b)\ \tau_{p_2} = \tau_{ae} + \tau_{ef} + \tau_{fb}$

  $p_3 = (d, g, f)\ \tau_{p_3} = \tau_{df} + \tau_{gf}$

  $p_4 = (d, g, c)\ \tau_{p_4} = \tau_{dg} + \tau_{gc}$

- direct connections are not distinguished from non-direct connections, transfer times (within a mode) are ignored

# Basic Line Planning Model

$$\min \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in \mathcal{F}} c_{\ell,f} x_{\ell,f} + (1-\lambda) \sum_{p \in \mathcal{P}} \tau_p y_p$$

Minimize cost and travel time

$$\sum_{p \in \mathcal{P}_{st}} y_p = d_{st} \qquad \forall\, s,t \in D$$

Transport all demand

$$\sum_{p \ni a} y_p \leq \sum_{\ell \in \mathcal{L} \ni a} \sum_{f \in \mathcal{F}} \kappa_{\ell,f}\, x_{\ell,f} \qquad \forall\, a \in A$$

Capacity constraints

$$\sum_{f \in \mathcal{F}} x_{\ell,f} \leq 1 \qquad \forall\, \ell \in \mathcal{L}$$

One frequency per line

$$y_p \geq 0,\; x_{\ell,f} \in \{0,1\}$$

Features

- Complete line pool
- Multi-criteria objective
- Integrated passenger routing

Disadvantage

- No transfers (within a mode)

# Literature (with Passenger Routing)

**Maximize direct travelers**

Bussieck, Kreuzer & Zimmermann [1997], Bussieck [1997]

- System split (a priori pax routing)

**Minimize transfers/transfer time**

Scholl [2005]; Schöbel & Scholl [2005]; Schmidt [2012]

- detailed treatment of transfers
- *change-&-go-graph* on the basis of all lines; large scale model

**Maximize travel quality**

Nachtigall & Jerosch [2008]

- utility for each path including all transfers
- capacity constraint for each partial route and line; large scale model

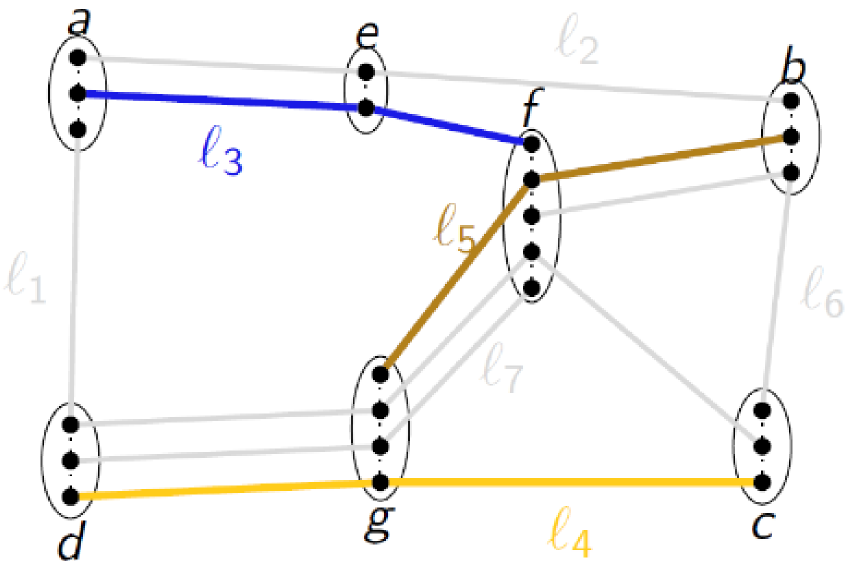
**Minimize pareto function of line cost and travel times**

B., Grötschel & Pfetsch [2007]; B., Neumann & Pfetsch [2008]

- allows line pricing; computationally tractable
- ignores transfers within same transportation mode

# Change-and-Go Model

**(Schöbel & Scholl [2005])**




- change-and-go graph
- each node/edge is copied for each line covering it
  $\mathcal{V} = \{(v,\ell): v \in V, \ell \in \mathcal{L}, v \in V(\ell)\}$
- complete graph (of transfers)
  $\big((v,\ell),(v,\ell')\big) \quad \forall \ell,\ell' \in \mathcal{L}$
- travel time on path = sum of travel times on edges
  $p_1 = (a,e,f)\ \tau_{p_1} = \tau_{(a,\ell_3)(e,\ell_3)} + \tau_{(e,\ell_3)(f,\ell_3)}$
  $p_2 = (a,e,f,b) \quad \tau_{p_2} = \tau_{(a,\ell_3)(e,\ell_3)} + \tau_{(e,\ell_3)(f,\ell_3)} + \tau_{(f,\ell_3)(f,\ell_5)} + \tau_{(f,\ell_5)(b,\ell_5)}$
  $p_3 = (d,g,f)\ \tau_{p_3} = \tau_{(d,\ell_4)(g,\ell_4)} + \tau_{(g,\ell_4)(g,\ell_5)} + \tau_{(g,\ell_5)(f,\ell_5)}$
  $p_4 = (d,g,c)\ \tau_{p_4} = \tau_{(d,\ell_4)(g,\ell_5)} + \tau_{(g,\ell_4)(c,\ell_5)}$
- all transfers are considered

# Change-and-Go Model
**(Schöbel & Scholl [2005])**

$$\min \lambda \sum_{\ell\in\mathcal{L}} \sum_{f\in\mathcal{F}} c_{\ell,f}\, x_{\ell,f} + (1-\lambda) \sum_{p\in\mathcal{P}} \tau_p\, y_p$$ Minimize cost and travel time

$$\sum_{p\in\mathcal{P}_{st}} y_p = d_{st} \quad \forall\,(s,t)\in D$$ Transport all demand

$$\sum_{p\in\mathcal{P}:a\in p} y_p \leqslant \sum_{f\in\mathcal{F}} \kappa_{\ell,f}\, x_{\ell,f} \quad \forall\,(a,\ell)\in\mathcal{A}_{\mathcal{L}}$$ Capacity constraints

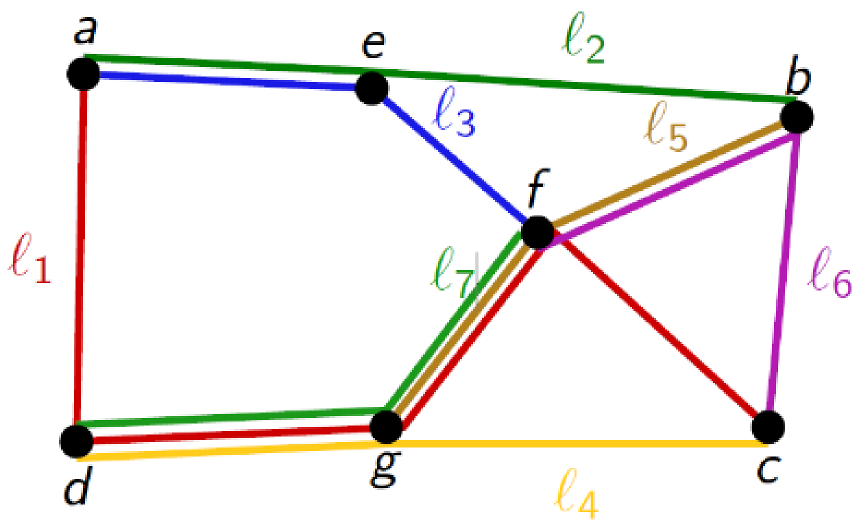
$$\sum_{f\in\mathcal{F}} x_{\ell,f} \leqslant 1 \quad \forall\,\ell\in\mathcal{L}$$ One frequency per line

Variables: $x_{\ell,f} = 1$ if line $\ell \in \mathcal{L}$ is chosen with frequency $f \in \mathcal{F}$; $x_{\ell,f} = 0$ otherwise
$y_p \geq 0$ passenger flow on path $p \in \mathcal{P}$

Features
- (Complete) line pool
- Multi-criteria objective
- Integrated passenger routing with transfers

Disadvantage
- Very large scale (needs enumeration of all possible lines)

# Idea of the Direct Connection Model




- Idea: Associate a passenger path either with a *direct connection line* or with a transfer penalty

$z^{\ell}_{p,0}$ # passengers on path $p$ traveling directly with line $\ell$

$y_{p,1}$ # passengers on path $p$ traveling with $\geq 1$ transfer

- add transfer penalty $\sigma$ on non-direct connections

$p_1 = (a, e, f)\ z^{\ell_3}_{p_1,0} = 50 \quad \tau_{p_1} = \tau_{ae} + \tau_{ef}$

$p_2 = (a, e, f, b) \quad y_{p_2,1} = 50 \quad \tau_{p_2} = \tau_{ae} + \tau_{ef} + \tau_{fb} + \sigma$

$p_3 = (d, g, f)\ y_{p_3,1} = 20 \quad \tau_{p_3} = \tau_{df} + \tau_{gf} + \sigma$

$p_4 = (d, g, c)\ z^{\ell_4}_{p_4,0} = 80 \quad \tau_{p_4} = \tau_{dg} + \tau_{gc}$

- transfer times for $\geq 2$ transfers are underestimated

# Direct Path Connection Model

$$\text{(DPC)} \quad \min \lambda \sum_{\ell\in\mathcal{L}} \sum_{f\in F} c_{\ell,f}\, x_{\ell,f} + (1-\lambda)\Big( \sum_{p\in\mathcal{P}^0} \tau_{p,0} \sum_{\ell\in\mathcal{L}(p)} z^{\ell}_{p,0} + \sum_{p\in\mathcal{P}} \tau_{p,1}\, y_{p,1} \Big)$$

$$\sum_{p\in\mathcal{P}^0_{st}} \sum_{\ell\in\mathcal{L}(p)} z^{\ell}_{p,0} + \sum_{p\in\mathcal{P}_{st}} y_{p,1} = d_{st} \qquad \forall\,(s,t)\in D$$

$$\sum_{p\in\mathcal{P}^0(a)} \sum_{\ell\in\mathcal{L}(p)} z^{\ell}_{p,0} + \sum_{p\in\mathcal{P}(a)} y_{p,1} \le \sum_{\ell\in\mathcal{L}(a)} \sum_{f\in F} \kappa_{\ell,f} x_{\ell,f} \qquad \forall\, a\in A$$

$$\sum_{p\in\mathcal{P}^0(a):\ell\in\mathcal{L}(p)} z^{\ell}_{p,0} \le \sum_{f\in F} \kappa_{\ell,f} x_{\ell,f} \qquad \forall\, a\in A,\ \forall\,\ell\in\mathcal{L}(a)$$

$$\sum_{f\in F} x_{\ell,f} \le 1 \qquad \forall\,\ell\in\mathcal{L}$$

$$x_{\ell,f}\in\{0,1\} \qquad \forall\,\ell\in\mathcal{L},\ \forall\, f\in F$$

$$z^{\ell}_{p,0}\ge 0 \qquad \forall\, p\in\mathcal{P}^0,\ \forall\,\ell\in\mathcal{L}(p)$$

$$y_{p,1}\ge 0 \qquad \forall\, p\in\mathcal{P}$$

## Problem

- Still many variables
- Primal degeneracy (pax paths can be assigned to many lines)

## Idea

- Line independent aggregation of direct connections as $y_{p,0} = \sum_{\ell\in\mathcal{L}(p)} z^{\ell}_{p,0}$

# "Skeleton" Direct Connection Model

$$
\begin{aligned}
\text{(DC-skeleton)} \quad \min\ & \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in F} c_{\ell,f}\, x_{\ell,f} + (1-\lambda)\Big( \sum_{p \in \mathcal{P}^0} \tau_{p,0}\, y_{p,0} + \sum_{p \in \mathcal{P}} \tau_{p,1}\, y_{p,1} \Big) \\
& \sum_{p \in \mathcal{P}^0_{st}} y_{p,0} + \sum_{p \in \mathcal{P}_{st}} y_{p,1} = d_{st} && \forall (s,t) \in D \\
& \sum_{p \in \mathcal{P}^0(a)} y_{p,0} + \sum_{p \in \mathcal{P}(a)} y_{p,1} \le \sum_{\ell \in \mathcal{L}(a)} \sum_{f \in F} \kappa_{\ell,f} x_{\ell,f} && \forall a \in A \\
& \sum_{f \in F} x_{\ell,f} \le 1 && \forall \ell \in \mathcal{L} \\
& x_{\ell,f} \in \{0,1\} && \forall \ell \in \mathcal{L}, \forall f \in F \\
& y_{p,0} \ge 0 && \forall p \in \mathcal{P}^0 \\
& y_{p,1} \ge 0 && \forall p \in \mathcal{P}
\end{aligned}
$$

## Properties

- Only necessary variables
- Treatment of direct connections needs to be added

## Idea

- Line independent aggregation of direct connections as $y_{p,0} = \sum_{\ell \in \mathcal{L}(p)} z^{\ell}_{p,0}$



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# What about the direct connection capacities?

Either the aggregated direct connection flow can be split …

$$(\mathrm{C}) \qquad \sum_{p \in \mathcal{P}^0(a):\ell \in \mathcal{L}(p)} z_{p,0}^{\ell} \leq c^{\ell} (:= \sum_{f \in F} \kappa_{\ell,f} x_{\ell,f}^{*}) \qquad \forall\, a \in A, \forall \ell \in \mathcal{L}\ (a \in \ell) \qquad (\mu_a^{\ell})$$

$$\sum_{\ell \in \mathcal{L}(p)} z_{p,0}^{\ell} = y_{p,0}^{*} \qquad \forall\, p \in \mathcal{P}^0 \qquad (\omega_p)$$

$$z_{p,0}^{\ell} \geq 0 \qquad \forall p \in \mathcal{P}^0,\ \forall \ell \in \mathcal{L}(p)$$

… or the Farkas dual solves:

$$(\overline{\mathrm{C}}) \qquad \sum_{\ell \in \mathcal{L}} c^{\ell} \sum_{a \in \ell} \mu_a^{\ell} + \sum_{p \in \mathcal{P}^0} \omega_p y_{p,0}^{*} < 0$$

$$\sum_{a \in p} \mu_a^{\ell} + \omega_p \geq 0 \qquad \forall\, p \in \mathcal{P}^0,\ \forall\, \ell \in \mathcal{L}(p)$$

$$\mu_a^{\ell} \geq 0 \qquad \forall \ell \in \mathcal{L},\ \forall a \in A$$

# What about direct connection capacities?

$$(\overline{\mathrm{C}}) \qquad \sum_{\ell \in \mathcal{L}} c^\ell \sum_{a \in \ell} \mu_a^\ell + \sum_{p \in \mathcal{P}^0} \omega_p y_{p,0}^* < 0$$

$$\sum_{a \in p} \mu_a^\ell + \omega_p \geq 0 \qquad \forall\, p \in \mathcal{P}^0, \forall\, \ell \in \mathcal{L}(p)$$

$$\mu_a^\ell \geq 0 \qquad \forall \ell \in \mathcal{L}, \forall a \in A$$

- Consider a solution of the dual:
  - W.l.o.g. $-\omega_p = \min_{\ell \in \mathcal{L}(p)} \{\sum_{a \in p} \mu_a^\ell\} \quad (=: \mathrm{dist}_\mu^{\mathcal{L}}(p))$

$(\overline{\mathrm{C}})$ has a solution if and only if there exists $\mu \in [0,1]^{\mathcal{L} \times A}$ s.t.

$$\sum_{\ell \in \mathcal{L}} c^\ell \sum_{a \in A} \mu_a^\ell < -\sum_{p \in \mathcal{P}^0} \omega_p\, y_{p,0}^* = \sum_{p \in \mathcal{P}^0} \mathrm{dist}_\mu^{\mathcal{L}}(p)\, y_{p,0}^*$$

# The Direct Connection Metric Inequalities

**Theorem (Direct Connection Metric Inequalities):**

A capacity vector $c \in \mathbb{R}_+^{\mathcal{L}}$ supports a direct connection routing $y_{p,0}^*$ if and only if

$$\sum_{\ell \in \mathcal{L}} c^\ell \sum_{a \in A} \mu_a^\ell \geq \sum_{p \in \mathcal{P}^0} \mathrm{dist}_\mu^{\mathcal{L}}(p)\, y_{p,0}^* \quad \forall \mu \in [0,1]^{\mathcal{L} \times A}$$

- Characterization of path capacities that support a direct connection routing
- Can be generalized to more than one transfer
- Relation to multicommodity flow results of Iri [1971] & Kakusho & Onaga [1971]
  - Characterize arc capacities that support a multicommodity flow by metric inequalities
  - Paths are more general than arcs
  - Direct connection routing is more restrictive than gen. routing

# Direct Connection Model

$$(\text{DC-complete}) \quad \min \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in F} c_{\ell,f}\, x_{\ell,f} + (1-\lambda)\Big( \sum_{p \in \mathcal{P}^0} \tau_{p,0}\, y_{p,0} + \sum_{p \in \mathcal{P}} \tau_{p,1}\, y_{p,1} \Big)$$

$$\sum_{p \in \mathcal{P}^0_{st}} y_{p,0} + \sum_{p \in \mathcal{P}_{st}} y_{p,1} = d_{st} \qquad \forall\, (s,t) \in D$$

$$\sum_{p \in \mathcal{P}^0(a)} y_{p,0} + \sum_{p \in \mathcal{P}(a)} y_{p,1} \leq \sum_{\ell \in \mathcal{L}(a)} \sum_{f \in F} \kappa_{\ell,f} x_{\ell,f} \qquad \forall\, a \in A$$

$$\sum_{\ell \in \mathcal{L}} \sum_{a \in A} \mu_a^\ell \sum_{f \in F} \kappa_{\ell,f} x_{\ell,f} \geq \sum_{(s,t) \in D} \sum_{p \in \mathcal{P}^0_{st}} \operatorname{dist}_\mu^{\mathcal{L}}(p)\, y_{p,0} \qquad \forall \mu \in [0,1]^{\mathcal{L} \times A}$$

$$\sum_{f \in F} x_{\ell,f} \leq 1 \qquad \forall\, \ell \in \mathcal{L}$$

$$x_{\ell,f} \in \{0,1\} \qquad \forall\, \ell \in \mathcal{L}, \forall\, f \in F$$

$$y_{p,0} \geq 0 \qquad \forall\, p \in \mathcal{P}^0$$

$$y_{p,1} \geq 0 \qquad \forall\, p \in \mathcal{P}$$

- Equivalent to basic DC model
- Algorithmically tractable?

# Separating the DC Metric Inequalities

$$(\overline{\mathrm{C}}) \qquad \sum_{\ell \in \mathcal{L}} c^{\ell} \sum_{a \in \ell} \mu_a^{\ell} + \sum_{p \in \mathcal{P}^0} \omega_p y^*_{p,0} < 0$$

$$\sum_{a \in p} \mu_a^{\ell} + \omega_p \geq 0 \qquad \forall\, p \in \mathcal{P}^0,\ \forall\, \ell \in \mathcal{L}(p)$$

$$\mu_a^{\ell} \geq 0 \qquad \forall \ell \in \mathcal{L},\ \forall a \in A$$

Restating the Farkas dual as an optimization problem:

$$(S) \quad \min \sum_{\ell \in \mathcal{L}} \sum_{a \in \ell} \mu_a^{\ell} \sum_{f \in F} \kappa_{\ell,f} x^*_{\ell,f} - \sum_{p \in \mathcal{P}^0} \omega_p y^*_{p,0}$$

$$\text{s.t.} \qquad \sum_{a \in p} \mu_a^{\ell} - \omega_p \geq 0 \qquad \forall\, p \in \mathcal{P}^0,\ \forall\, \ell \in \mathcal{L}(p)$$

$$\mu_a^{\ell} \geq 0 \qquad \forall \ell \in \mathcal{L},\ \forall a \in A$$

$$1 \geq \omega_p \geq 0 \qquad \forall p \in \mathcal{P}^0$$

# Separating the DC Metric Inequalities

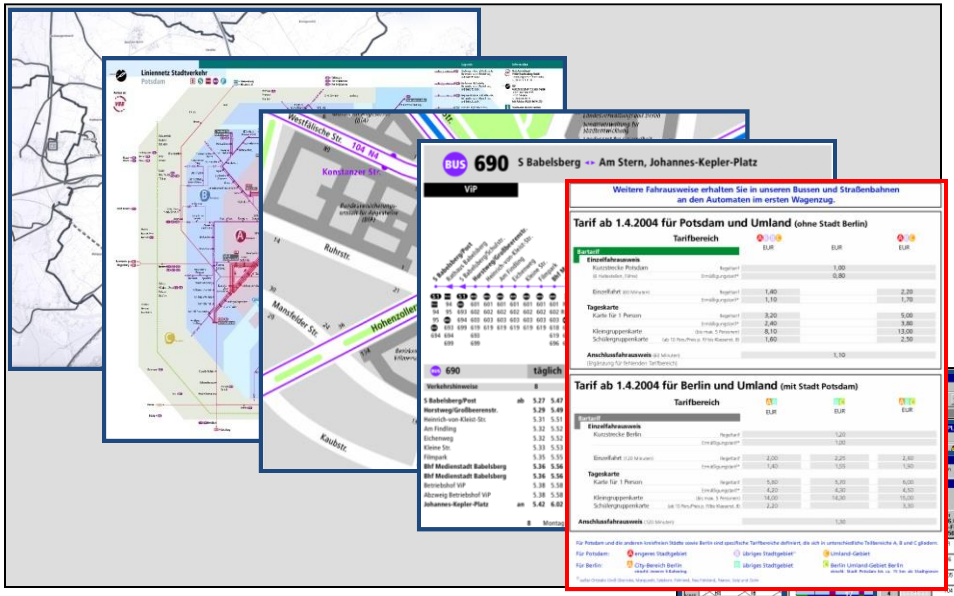
$$
\begin{aligned}
(S) \quad \min \sum_{\ell\in\mathcal{L}} \sum_{a\in\ell} \mu_a^\ell \sum_{f\in F} \kappa_{\ell,f} x^*_{\ell,f} - \sum_{p\in\mathcal{P}^0} \omega_p y^*_{p,0} & \\
\text{s.t.} \quad \sum_{a\in p} \mu_a^\ell - \omega_p \geq 0 & \quad \forall\, p\in\mathcal{P}^0,\ \forall\, \ell\in\mathcal{L}(p) \\
\mu_a^\ell \geq 0 & \quad \forall \ell\in\mathcal{L},\ \forall a\in A \\
1 \geq \omega_p \geq 0 & \quad \forall p\in\mathcal{P}^0
\end{aligned}
$$

- Feasible region does not depend on flow
- Polyhedron, polynomial number of (explicitly known) constraints

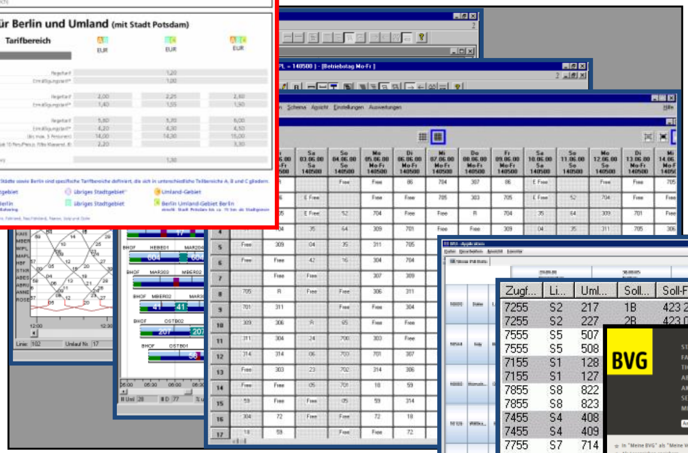
**Proposition (Separation of Direct Connection Metric Inequalities):**

The dcmetric inequalities can be separated in polynomial time. Hence, the LP relaxation of the direct connection model can be solved in polynomial time (non-direct connection paths can be priced in polynomial time).

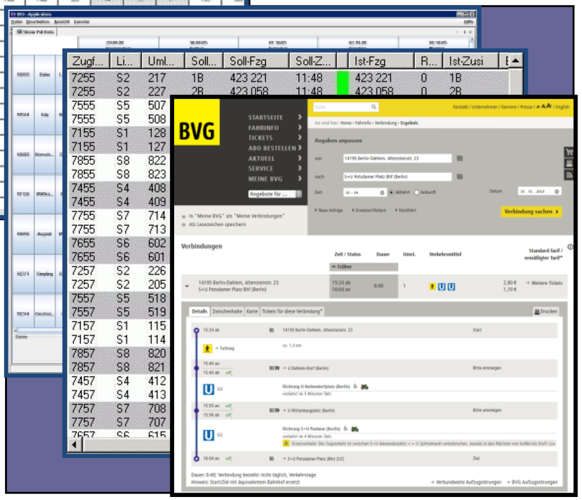
# Planning Problems in Public Transport




Strategic Planning



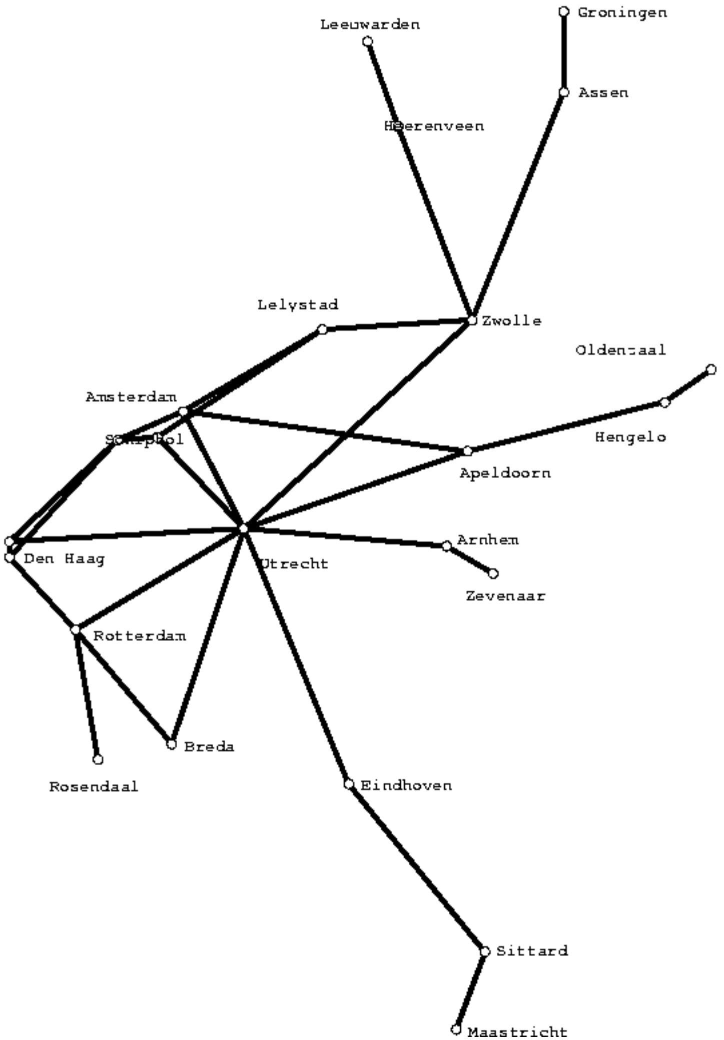
Operational Planning




Operations Control

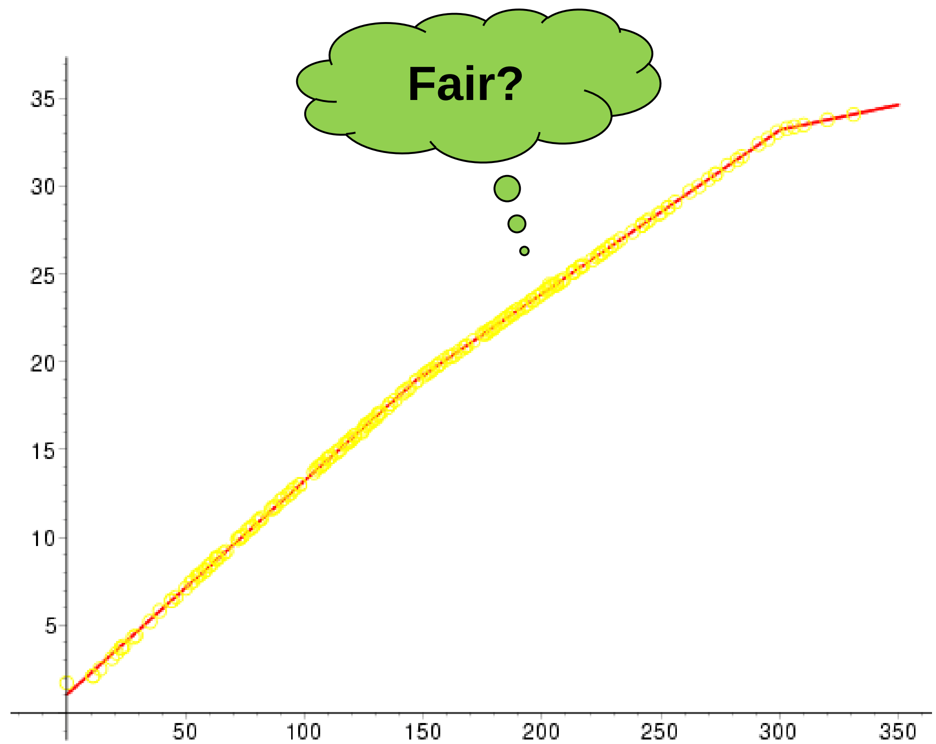
# Dutch Intercity Network

(Bussieck [1998], Bussieck, Kreuzer, Zimmermann [1996], Claessens, van Dijk, Zwaneveld [1998])




| | Hr | Asn | Zl | Hgl | Ah | Ut | Shl | Asdz | Asd | Gv | Gvc | Rtd | Bd | Ehv | Std | Mt | Lls | Rsdg | Zvg | Odzg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lw | 478 | | 380 | | 13 | 145 | 20 | 21 | 90 | 6 | 26 | 36 | 14 | 9 | 9 | 4 | 77 | 7 | 14 | |
| Gn | | 1720 | 720 | | | 331 | 48 | 88 | 205 | 12 | 73 | 75 | 34 | 28 | 29 | 13 | 200 | 33 | 14 | |
| Hr | | | 511 | | 11 | 209 | 20 | 16 | 115 | 10 | 48 | 58 | 16 | 11 | 8 | 4 | 77 | 10 | 19 | |
| Asn | | | 854 | | 16 | 502 | 32 | 58 | 235 | 13 | 117 | 125 | 42 | 33 | 28 | 14 | 152 | 48 | 19 | |
| Zl | | | | | 56 | 1112 | 64 | 171 | 400 | 33 | 163 | 182 | 79 | 47 | 46 | 21 | 390 | 100 | 32 | |
| Apd | | | | 468 | | 1160 | 32 | 76 | 917 | 21 | 202 | 143 | 57 | 62 | 10 | 5 | | 47 | 83 | 71 |
| Hgl | | | | | | 422 | 11 | 24 | 287 | 20 | 81 | 52 | 39 | 28 | 20 | 12 | | 24 | | 75 |
| Ah | | | | | | 4244 | 60 | 721 | 726 | 109 | 741 | 180 | 136 | 101 | | | 8 | 320 | 602 | |
| Ut | | | | | | | 278 | 5826 | 4919 | 225 | 3138 | 2260 | 1165 | 3109 | 720 | 359 | 89 | 325 | 996 | 21 |
| Shl | | | | | | | | 1456 | 6469 | 1339 | 1503 | 509 | 7 | 99 | 44 | 29 | 103 | 164 | | |
| Asdz | | | | | | | | | | 461 | 207 | 369 | 138 | 542 | 203 | 149 | 819 | 6 | 155 | |
| Asd | | | | | | | | | | 730 | 2540 | 1756 | 154 | 437 | 155 | 37 | 2783 | 2258 | 489 | 22 |
| Gv | | | | | | | | | | | 785 | 4586 | 531 | 35 | 22 | 8 | 29 | 890 | | |
| Gvc | | | | | | | | | | | | 2829 | 228 | 335 | 104 | 41 | 31 | 3 | 229 | 7 |
| Rtd | | | | | | | | | | | | | 1829 | 569 | 179 | 73 | 46 | 1077 | 157 | 11 |
| Bd | | | | | | | | | | | | | | 950 | 157 | 79 | 6 | 329 | 14 | 5 |
| Ehv | | | | | | | | | | | | | | | 936 | 404 | 8 | 75 | 11 | 3 |
| Std | | | | | | | | | | | | | | | | 863 | 2 | 19 | | |
| Mt | | | | | | | | | | | | | | | | | 1 | 22 | | |
| Lls | | | | | | | | | | | | | | | | | | 15 | | |

| | Hr | Asn | Zl | Hgl | Ut | Shl | Asdz | Asd | Gv | Gvc | Rtd | Bd | Ehv | Std | Mt | Lls | Rsdg | Zvg | Odzg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lw | 29 | | | | | | | | | | | | | | | | | | |
| Gn | | 28 | | | | | | | | | | | | | | | | | |
| Hr | | | 66 | | | | | | | | | | | | | | | | |
| Asn | | | 78 | | | | | | | | | | | | | | | | |
| Zl | | | | | 85 | | | | | | | | | | | 50 | | | |
| Apd | | | | 69 | 64 | | | 89 | | | | | | | | | | | |
| Hgl | | | | | | | | | | | | | | | | | | | 18 |
| Ah | | | | | 58 | | | | | | | | | | | | | 19 | |
| Ut | | | | | | | 34 | 39 | | 61 | 57 | 92 | 81 | | | | | | |
| Shl | | | | | | | 9 | 19 | 43 | 43 | | | | | | | | | |
| Asdz | | | | | | 9 | | | | | | | | | | 56 | | | |
| Asd | | | | | | | | | | | | | | | | 54 | | | |
| Gv | | | | | | | | | | 1 | 23 | | | | | | | | |
| Rtd | | | | | | | | | | | | 49 | | | | | 18 | | |
| Ehv | | | | | | | | | | | | | | 78 | | | | | |
| Std | | | | | | | | | | | | | | | 21 | | | | |



# Cost Allocation Games

- $N = [n]$ players
- $N$ grand coalition
- $\Sigma \subseteq 2^N, \Sigma^+ = \Sigma \setminus \emptyset$ coalitions
- $c: \Sigma^+ \to \mathbb{R}_{\geq 0}$ cost function
- $P \subseteq \mathbb{R}^N_{\geq 0}$ feasible prices (polyhedron)
- $\Gamma = (N, c, P, \Sigma)$ cost allocation game
- $f: \Sigma^+ \to \mathbb{R}_{>0}$ weight function $(1, |\cdot|, c)$
- $e_f(S, x) = \frac{c(S) - x(S)}{f(S)}, S \in \Sigma^+, x \in P$ $f$-excess of $S$ at price $x$
- $\mathcal{X}(\Gamma) = \{x \in P: x(N) = c(N)\}$ imputation set
- $\mathcal{C} := \{x \in \mathcal{X}: e_f(\cdot, x) \geq 0\}$ core
- $\mathcal{C}_{\epsilon,f} := \{x \in \mathcal{X}: e_f(\cdot, x) \geq \epsilon\}$ $(\epsilon, f)$-core
- $\epsilon_f := \max\ \epsilon: \mathcal{C}_{\epsilon,f} \neq \emptyset$ $f$-least core radius
- $\mathcal{LC}_f := \mathcal{C}_{\epsilon_f,f}$ $f$-least core
- $\mathcal{N}_f := \text{lexmax}\, \mathcal{LC}_f$ $f$-nucleolus
- $\phi: \Gamma \to P$ cost allocation method

# Desirable Properties

1. $\phi(\Gamma)(N) = c(N)$ — efficiency
2. $\phi(\Gamma)(S) \leq \phi(\Gamma_S)(S) \;\; \forall S \in \Sigma^+$ — coalitional stability
3. $\phi(\Gamma) \in \mathcal{C}$ — core price
4. $\exists K, \alpha > 0 \colon |\tilde{c}(\cdot) - c(\cdot)| \leq \alpha c(\cdot)$
   $\Longrightarrow |\phi(\tilde{c})_i - \phi(c)_i| \leq K\alpha\phi(c)_i \forall i$ — bounded variation

Cost allocation methods

- $\phi(\Gamma)_i = c(i) \;\; \forall i \in N$ — fixed-price $(\neg 1, \neg 3)$
- $\phi(\Gamma)_i = c(i) \cdot \frac{c(N)}{\sum_{j \in N} c(j)} \;\; \forall i \in N$ — proportional $(\neg 2, \neg 3)$
- $\phi(\Gamma) = \mathcal{LC}_f$ — $f$-least core $(\neg 2, \neg 4)$

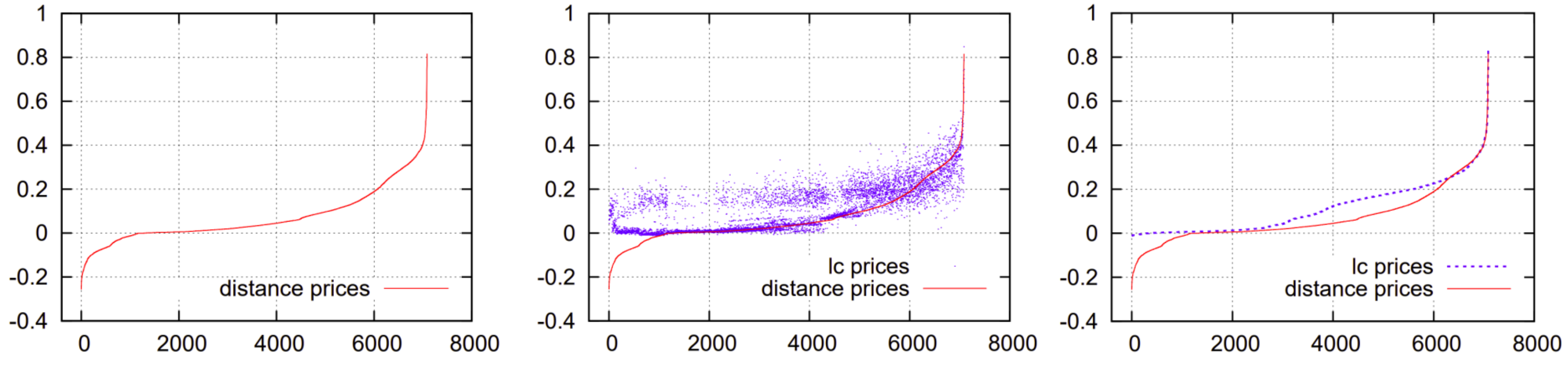
$$= \operatorname{argmax}_x \epsilon$$
$$x(S) + \epsilon f(S) \leq c(S) \;\; \forall S \in \Sigma^+ \setminus N$$
$$x(N) = c(N)$$
$$x \in P$$

# Desirable Properties

1. $\phi(\Gamma)(N) = c(N)$ efficiency
2. $\phi(\Gamma)(S) \leq \phi(\Gamma_S)(S) \ \ \forall S \in \Sigma^+$ coalitional stability
3. $\phi(\Gamma) \in \mathcal{C}$ core price
4. $\exists K, \alpha > 0 \colon |\tilde{c}(\cdot) - c(\cdot)| \leq \alpha c(\cdot)$
   $\Longrightarrow |\phi(\tilde{c})_i - \phi(c)_i| \leq K\alpha\phi(c)_i \forall i$ bounded variation

- **Proposition (Hoang [2010]):** There is no (general) cost allocation method that can guarantee more than 2 out of the above 4 properties (for all games), even for cost allocation games with monotone, subadditive cost functions.
- **Theorem (Hoang [2010]):** For weight functions $f = \alpha g + \beta c$, where $\alpha, \beta \in \mathbb{Q}, \alpha + \beta > 0, g \colon \Sigma \to \mathbb{Q}$ modular and positive on $\Sigma^+$, the $f$-nucleolus of a "strongly bounded" cost allocation game can be computed in time that is polynomial in oracles for computing $c(S)$, $g(S)$, membership for $\Sigma$, and separation for $P$.

# Fairness Diagram




- $\Lambda = \{\{i\}: i \in N\} \cup N$
- $\mathrm{R}: \Lambda \to \mathbb{R}_{>0}, \sum_i R(\{i\}) = c(N)$ reference price function
- $r_i = R(\{i\}), i \in N$ reference price for player $i$
- $\mathcal{LC}_{f,r} := \mathcal{N}_R(N, R, \mathcal{LC}_f(\Gamma), \Lambda)$ $(f, r)$-least core

$$= \operatorname{arglexmin}_{x \in \mathcal{LC}_f} \left(\frac{x_i}{r_i}\right)_{i \in N}$$ as $e_R(\{i\}, x) = \frac{r_i - x_i}{r_i} = 1 - \frac{x_i}{r_i}$

# Cost Function (Line Planning Problem)

$$c(S) = \min \sum_{(r,f)\in\mathcal{R}\times\mathcal{F}} \left(c^1_{r,f}\xi_{r,f} + c^2_{r,f}\rho_{r,f}\right)$$

arc capacities for shortest path routing

$$\text{s.t.} \quad \sum_{e\in r\in\mathcal{R}} \sum_{f\in\mathcal{F}} c_{cap} f\left(m\xi_{r,f} + \rho_{r,f}\right) \geq \sum_{i\in S} P^i_e \qquad \forall e \in E$$

min frequencies for shortest path routing

$$\sum_{e\in r\in\mathcal{R}} \sum_{f\in\mathcal{F}} f\xi_{r,f} \geq F^i_e \qquad \forall (e,i) \in E \times S$$

$$\rho_{r,f} - (M-m)\xi_{r,f} \leq 0 \qquad \forall (r,r) \in \mathcal{R}\times\mathcal{F}$$

≤ 1 frequencies per route

$$\sum_{f\in\mathcal{F}} \xi_{r,f} \leq 1 \qquad \forall r \in \mathcal{R}$$

$$\xi \in \{0,1\}^{\mathcal{R}\times\mathcal{F}}, \rho \in \mathbb{Z}^{\mathcal{R}\times\mathcal{F}}_{\geq 0}$$

route $r$ has frequency $f$

route $r$ with frequency $f$ has $m + \rho_{r,f}$ coaches

Separation problem extends to OD-pair choice plus objective change to excess.

# Setup

- Players

  $n = 81$ OD-pairs of highest demand, $2^{81} - 1$ coalitions
- Reference price

$$P_{st} = \frac{c(N)}{\sum_{st \in N} d_{st}} \cdot d_{st}$$

- Monotonicity

$$0 \leq \frac{x_{uv}}{P_{uv}} \leq \frac{x_{st}}{P_{st}} \leq \sum_{uv \in \mathcal{P}_{st}} \frac{x_{uv}}{P_{uv}} \qquad \forall st \in S, uv \in \mathcal{P}_{st}$$

- Distance-likeness

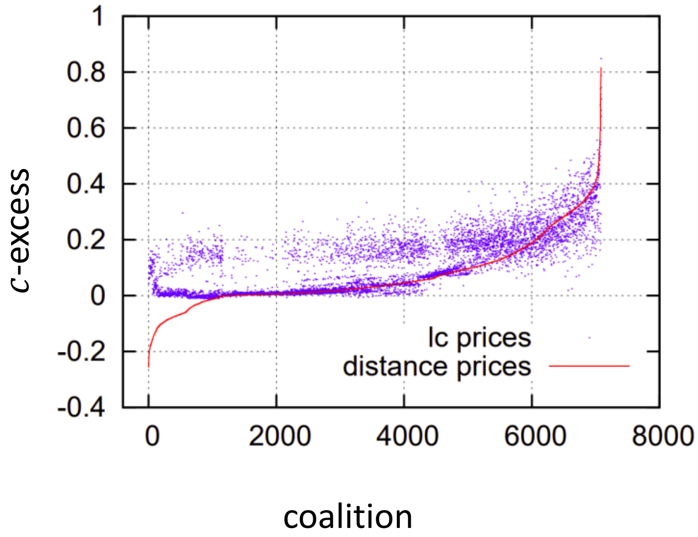
$$\max_{st} \frac{x_{st}}{d_{st} P_{st}} \leq K \min_{st} \frac{x_{st}}{d_{st} P_{st}} \qquad \forall st \in S$$

- Weight function

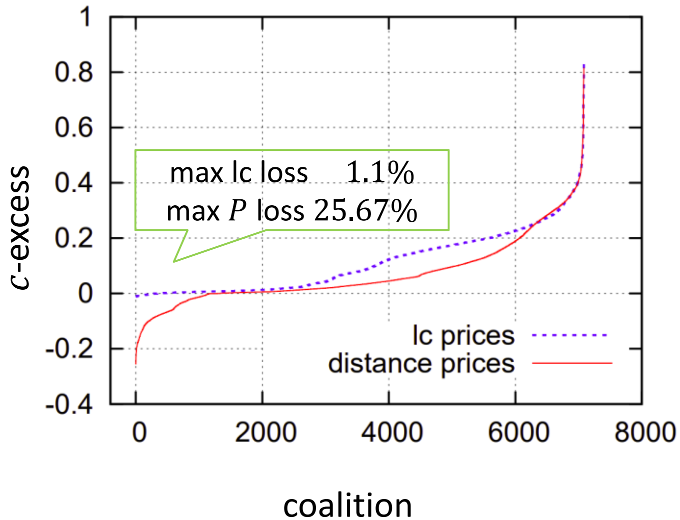
$$f = c,$$

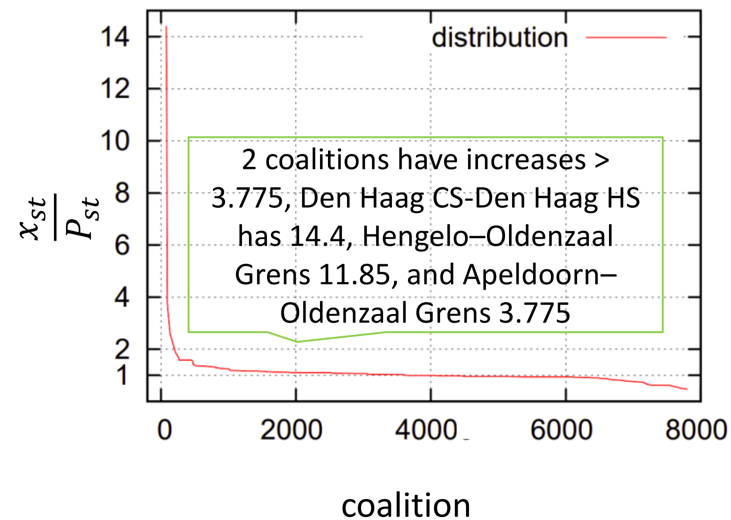
  i.e, $e_f(S,x) = e_c(S,x) = \frac{c(S) - x(S)}{c(S)}$ = relative savings

# Least-core Prices ($K = +\infty$)







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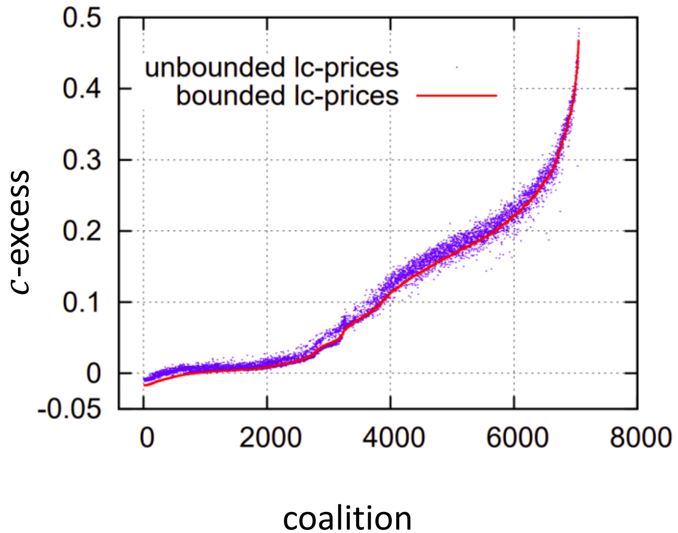
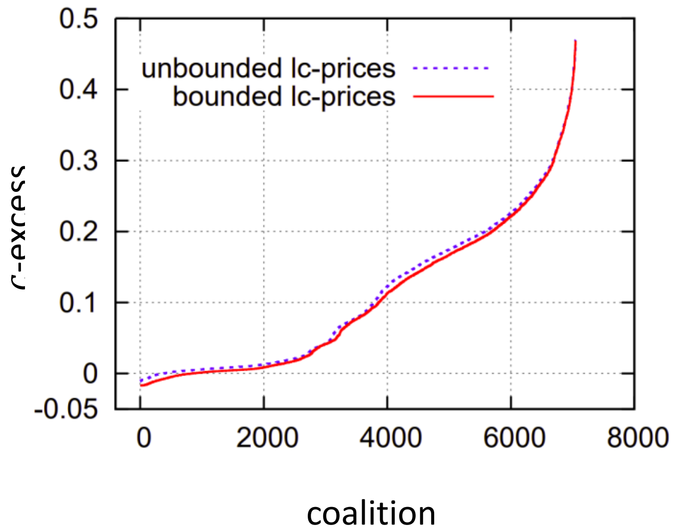
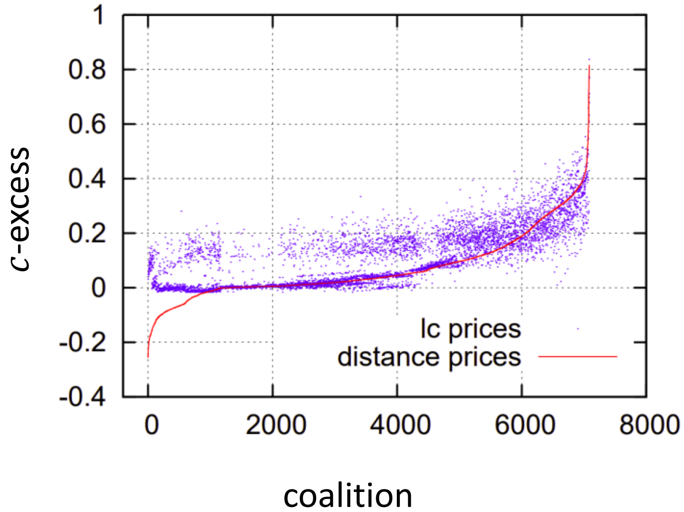
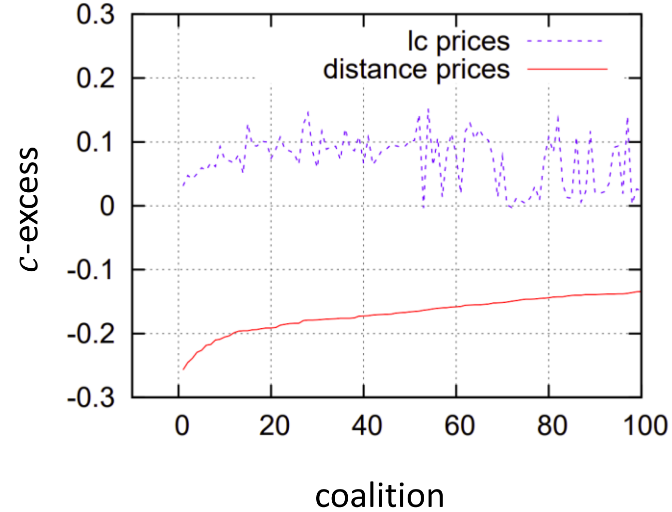
# Least-core Prices ($K = +\infty$ vs. $K = 3$)

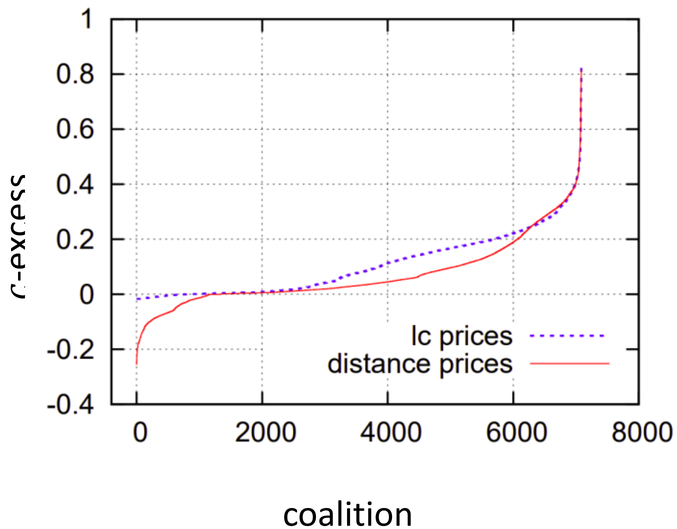


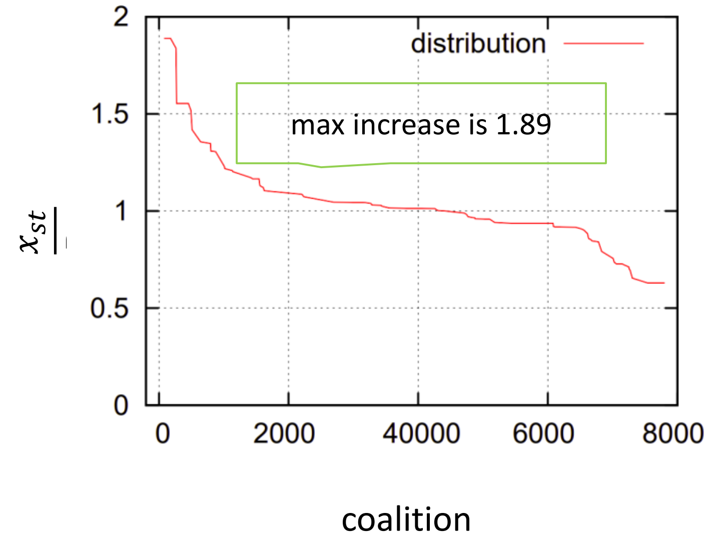



# Least-core Prices ($K = 3$)



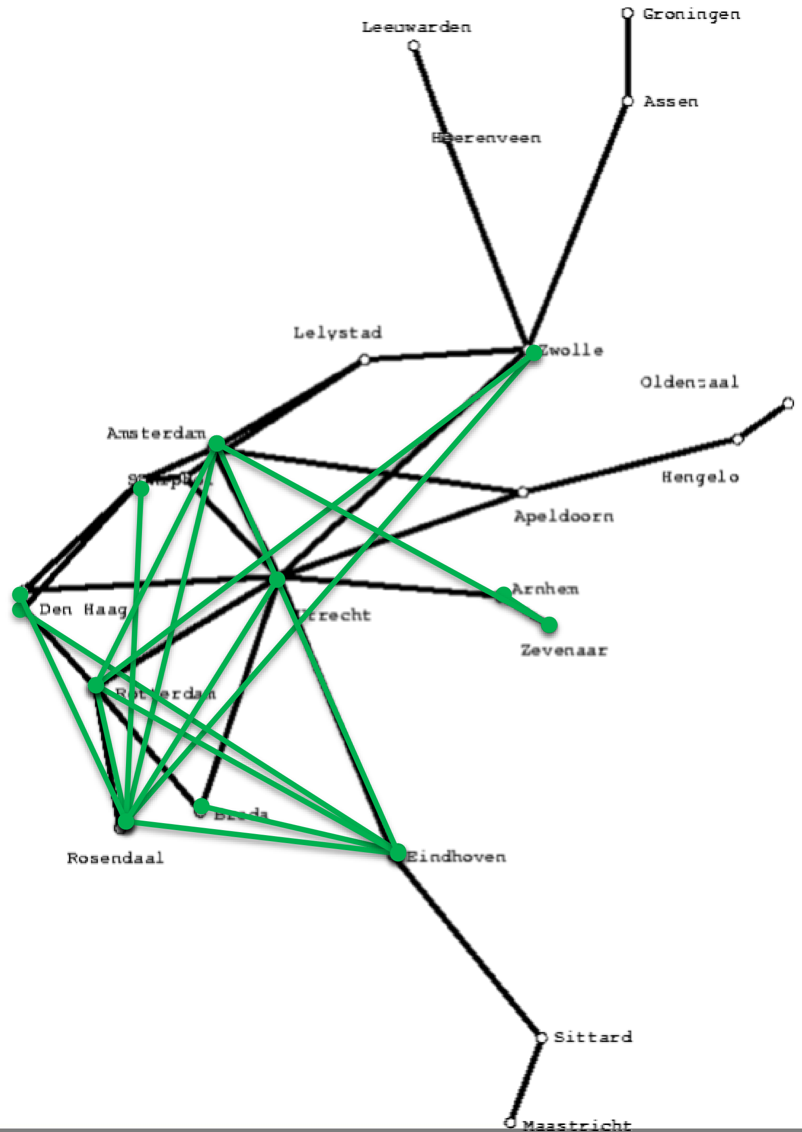



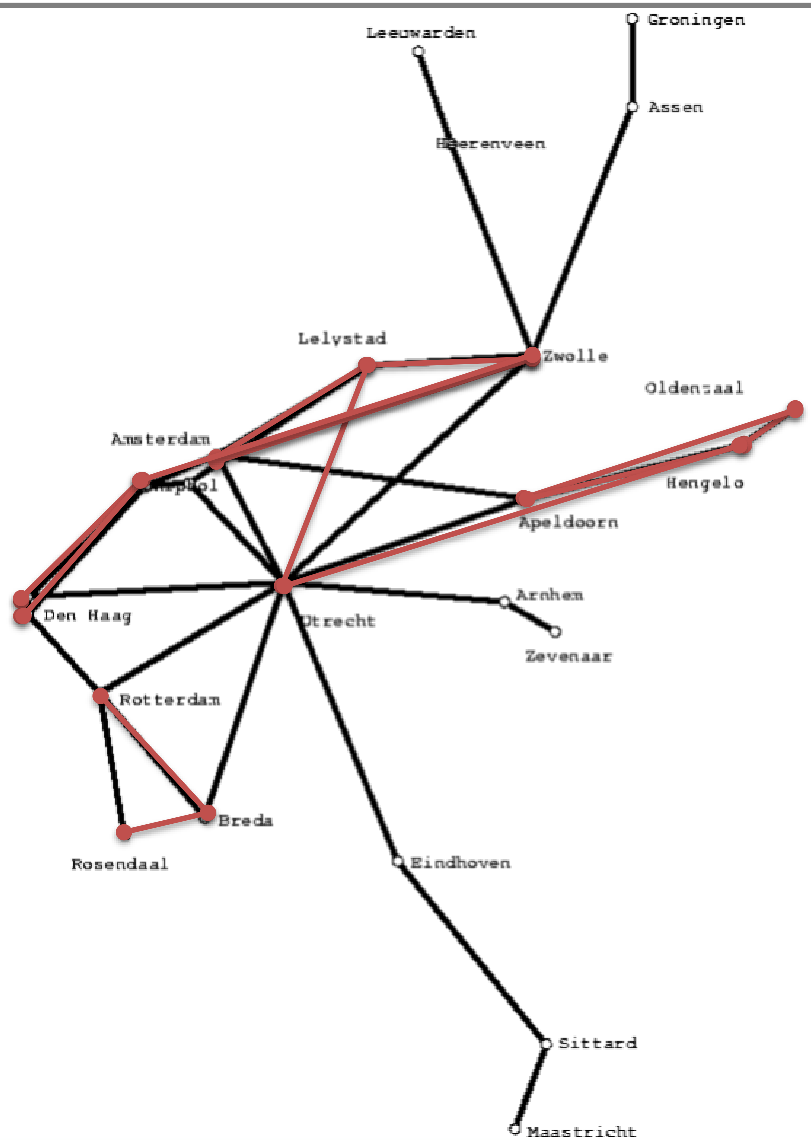
# Best 15 ($x_{st}/P_{st}$ for $K = 3$ and $K = +\infty$)

| | |
|---|---|
| Arnhem-Zevenaar Grens | 0.63 |
| Amsterdam CS-Roosendaal | 0.63 |
| Breda-Eindhoven | 0.63 |
| Roosendaal-Rotterdam CS | 0.63 |
| Roosendaal-Zwolle | 0.65 |
| Eindhoven-Den Haag CS | 0.65 |
| Roosendaal-Schiphol | 0.69 |
| Eindhoven-Roosendaal | 0.70 |
| Eindhoven-Rotterdam CS | 0.71 |
| Amsterdam CS-Eindhoven | 0.72 |
| Den Haag HS-Roosendaal | 0.73 |
| Roosendaal-Utrecht CS | 0.73 |
| Rotterdam CS-Zwolle | 0.74 |
| Amsterdam CS-Rotterdam CS | 0.76 |
| Amsterdam CS-Zevenaar | 0.79 |

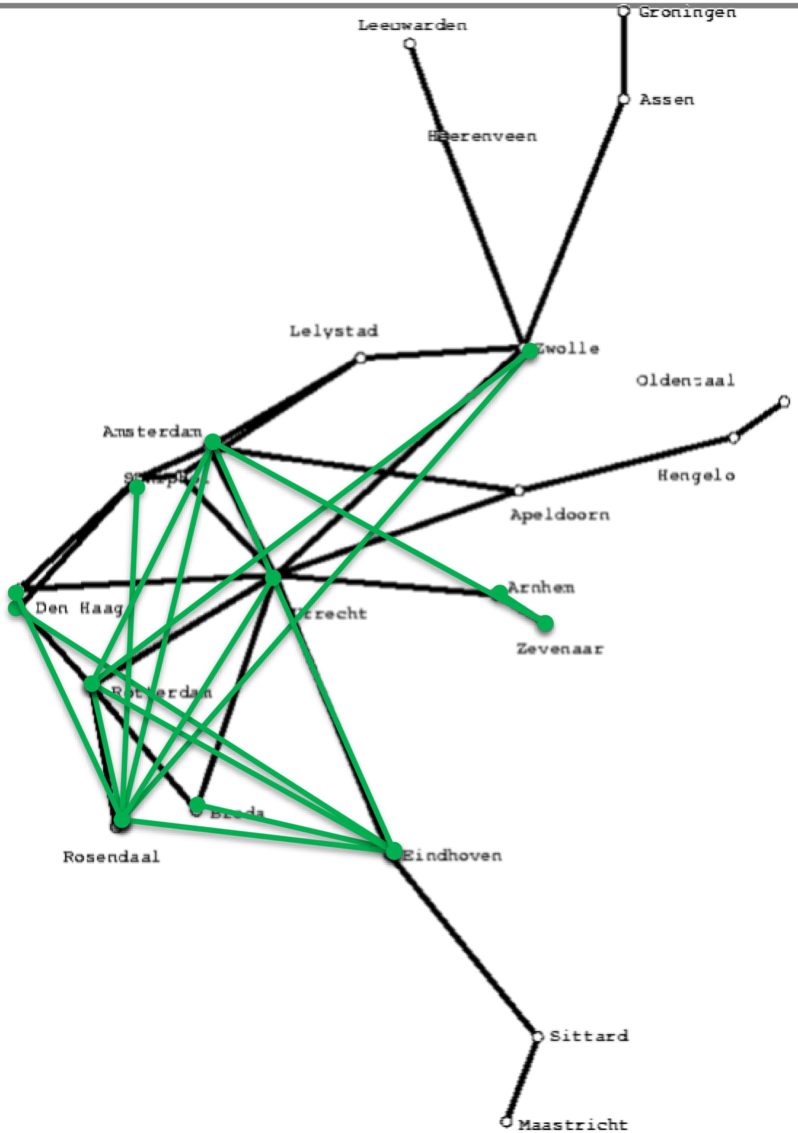


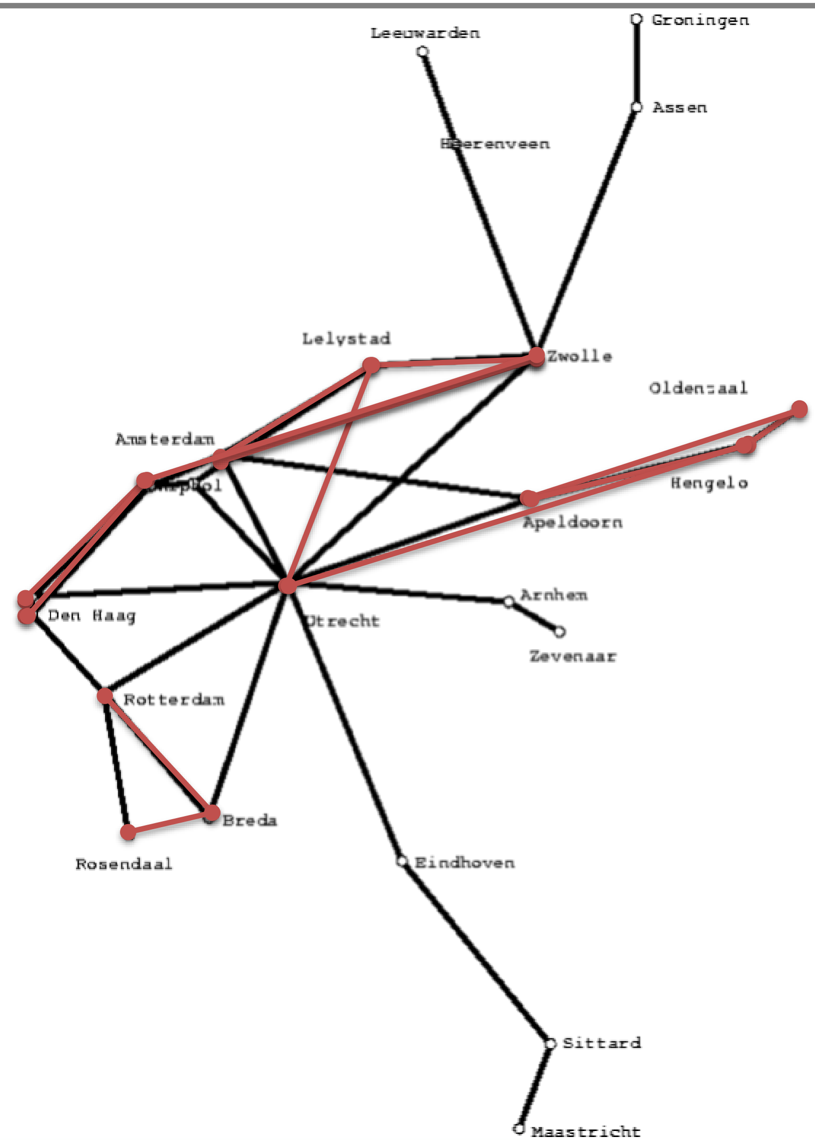
# Worst 15 ($x_{st}/P_{st}$ for $K = 3$ and $K = +\infty$)

| | |
|---|---|
| Breda-Roosendaal | 1.30 |
| Hengelo-Utrecht CS | 1.30 |
| Schiphol-Zwolle | 1.30 |
| Den Haag CS-Schiphol | 1.35 |
| Den Haag HS-Schiphol | 1.36 |
| Amsterdam Zuid-Zwolle | 1.42 |
| Amsterdam CS-Zwolle | 1.52 |
| Breda-Rotterdam CS | 1.55 |
| Leylstad-Utrecht CS | 1.55 |
| Amsterdam Zuid-Leylstadt | 1.84 |
| Apeldoorn-Hengelo | 1.89 |
| Apeldoorn-Oldenzaal | 1.89 |
| Den Haag HS-Den Haag CS | 1.89 |
| Hengelo-Oldenzaal | 1.89 |
| Leylstad-Zwolle | 1.89 |



# Worst 15 ($x_{st}/P_{st}$ for $K = 3$ and $K = +\infty$)



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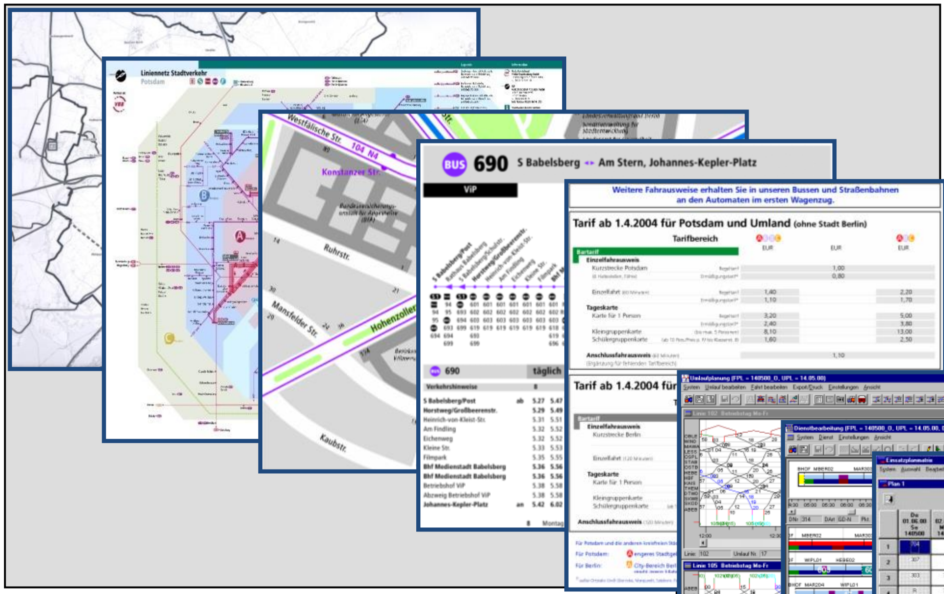
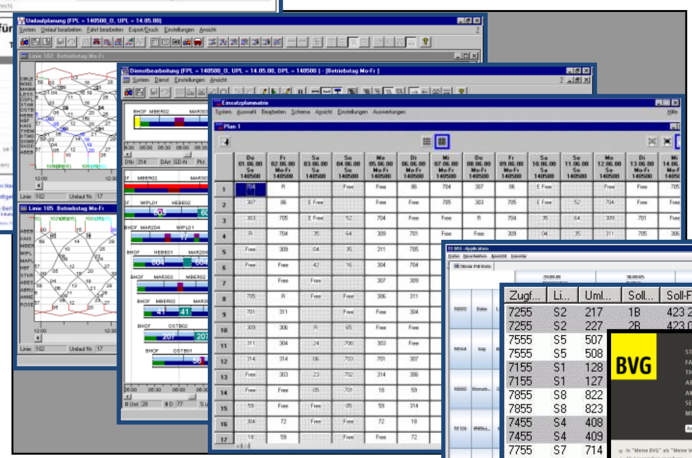
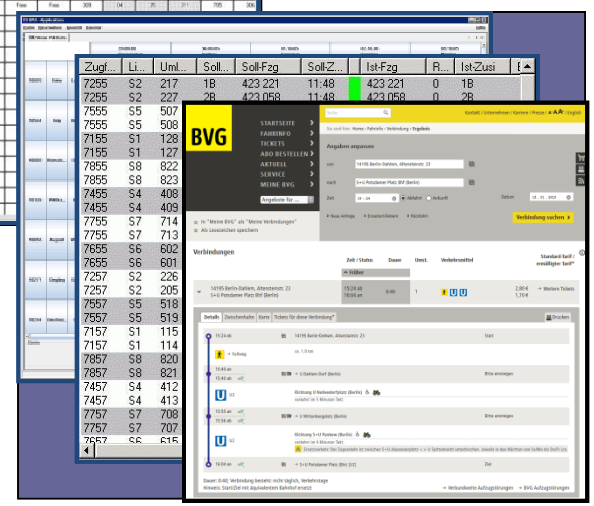
# Thank you very much for your attention!




Strategic Planning



Operational Planning




Operations Control

# Beijing (□-Grid with Ls & Rings)