Solving Heated Oil Pipeline Problems Via Mixed Integer Nonlinear Programming Approach

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Overview

Overview

Crude Oil and Pipeline Transport

Figure: Pipes and stations

Crude oil

- is one of the most important resources
- can produce many kinds of fuel and chemical products
- is usually (**51%** around the world) transported by **pipelines**

Energy Loss in Pipeline

Pressure P and **head** H of the oil

$$P = \rho g H$$

where ρ is the density of the oil, g is gravity acceleration

- **Head (pressure) loss**: friction and elevation difference
- **Temperature loss**: dissipation

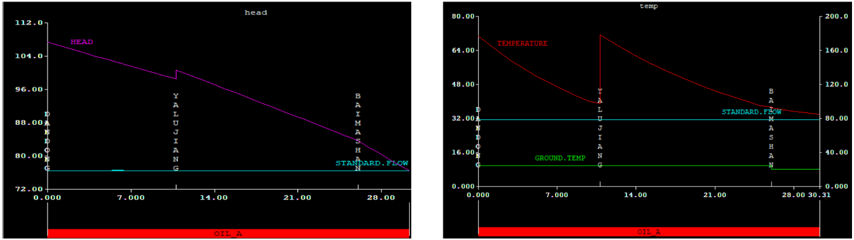
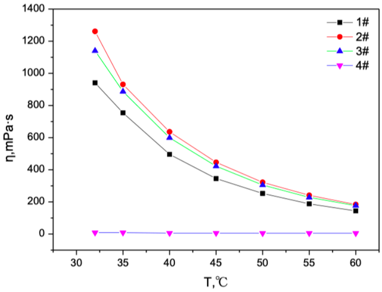


Figure: Mileage-head curve (left) and mileage-temperature curve (right)

Without **proper transport temperature**, some crude oil may

- dramatically increase the viscosity (**high friction**)
- precipitate wax
- freeze (Some oil freezes under **32°C**)

Normal temperature pipelines are incapable!



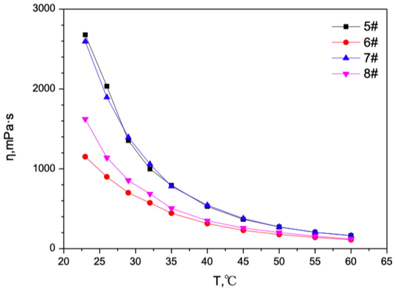


Figure: Viscosity-temperature curves

The Heated Oil Pipeline (HOP)

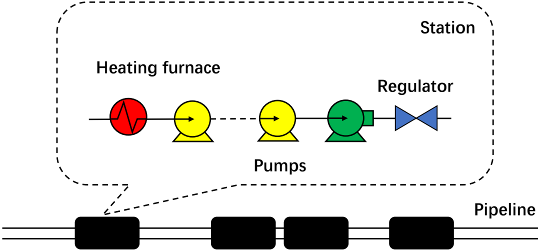


Figure: Stations with pumps and furnaces

In each station

- **Heating furnace**: variable $\Delta T \in \mathbb{R}_+$
- **Constant speed pump (CSP)**: constant H^{CP}
- **Shifted speed pump (SSP)**: variable $\Delta H^{SP} \in \left[\underline{H}^{SP}, \overline{H}^{SP}\right]$
- **Regulator**: head restriction

Safety requirements

- Inlet, outlet head and temperature bounds in each station
- Head bounds in the pipeline

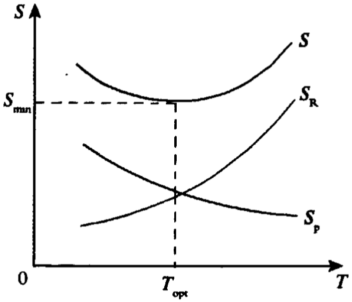


Figure: Temperature (T) and transport cost (S)

- Lots of feasible schemes with huge cost differences (vary over **50,000** yuan/d)
- High heating consumption (consumes fuel equivalent to **1%** transported oil)
- Huge rate of flow per day (about **72,000** m^3/d)

The optimal scheme will save a lot!

MINLP Implementation and the Nonconvexity

- **(Nonconvex) MINLP Model**
  - **Integer (binary) variables:** on-off status of pumps
  - **Continuous variables:** temperature rise comes from heating furnaces
- **Nonconvexity: Friction loss** (hydraulic friction based on **Darcy-Weisbach formula** and **Reynold numbers**)

$$\mathrm{HF}(T, Q, D, L) = \beta(T)\frac{Q^{2-m(T)}}{D^{5-m(T)}}\nu(T)^{m(T)}L$$

L is the pipe length, T is the oil temperature, Q is the volume flow of oil, D is the inner diameter of the pipe, $\beta(\cdot)$ and $m(\cdot)$ are **piecewise constant functions**, $\nu(\cdot)$ is the kinematic viscosity of oil.

  - HF is **nonconvex** or even **discontinuous** about T in general

Motivation and Contribution

M Literatures focus on approximation or meta-heuristics

- Two-cycle strategy based on model decomposition [Wu and Yan, 1989]
- Improved genetic algorithm [Liu et al., 2015]
- Differetial evolution and particle swarm optimization [Zhou et al., 2015]
- Linear approximation [Li et al., 2011]
- Simulated annealing algorithm [Song and Yang, 2007]

C Consider using deterministic global optimization methods

M Lack detailed and general mathematical model

C Consider different kinds of pumps and a general formulation of hydraulic frictions

M General MINLP solvers may not efficient on the HOP problems

C Design an efficient specific algorithm for HOP problems

Overview

Notations

Suppose

- there are N^S stations in the pipeline;
- the pipe between stations j and $j+1$ is divided into N_j^P segments, $j = 1, \ldots, N^S - 1$;
- there are N_j^{CP} CSPs and N_j^{SP} SSPs in station j.

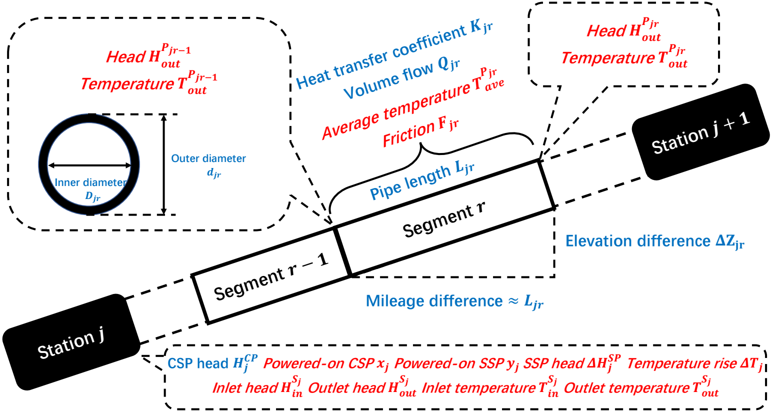


Figure: Constants (blue) and variables (red) in the pipe between stations j and $j+1$

MINLP Model for HOP

Pipe calculation constraints

- **Head loss**

$$H_{out}^{P_{jr}} = H_{out}^{P_{j,r-1}} - F_{jr} - \Delta Z_{jr},\ j = 1, ..., N^S - 1,\ r = 1, ..., N_j^P. \quad (1)$$

- **Friction**

$$F_{jr} = f\left(T_{ave}^{P_{jr}}, Q_{jr}, D_{jr}\right) L_{jr},\ j = 1, ..., N^S - 1,\ r = 1, ..., N_j^P. \quad (2)$$

- **Average temperature** (based on **axial temperature drop formula** and **empirical formula**)

$$T_{out}^{P_{jr}} = T_g^{P_{jr}} + T_f^{P_{jr}} + \left[T_{out}^{P_{j,r-1}} - \left(T_g^{P_{jr}} + T_f^{P_{jr}}\right)\right] e^{-\alpha_{jr} L_{jr}}, \quad (3)$$

$$T_{ave}^{P_{jr}} = \frac{1}{3} T_{out}^{P_{j,r-1}} + \frac{2}{3} T_{out}^{P_{jr}},\ j = 1, ..., N^S - 1,\ r = 1, ..., N_j^P. \quad (4)$$

T_g is the ground temperature, T_f is the environment temperature variation caused by friction heat, $\alpha = (K\pi d)/(\rho Q c)$ is a parameter, c is the specific heat of the oil.

Heating station calculation constraints

- **Outlet value**

$$H_{in}^{S_j} + \left(x_j H_j^{CP} + \Delta H_j^{SP}\right) \geq H_{out}^{S_j},\ j = 1, ..., N^S - 1, \quad (5)$$

$$T_{in}^{S_j} + \Delta T_j = T_{out}^{S_j},\ j = 1, ..., N^S - 1. \quad (6)$$

Constraints (5) are **inequalities** due to the **regulators**

- **SSP head bound**

$$y_j \underline{H}_j^{SP} \leq \Delta H_j^{SP} \leq y_j \overline{H}_j^{SP},\ j = 1, \ldots, N^S - 1. \quad (7)$$

Connection constraints

- **Pipe station connection**

$$H_{out}^{P_{j0}} = H_{out}^{S_j},\ H_{out}^{P_{jN_j^P}} = H_{in}^{S_{j+1}},\ j = 1, ..., N^S - 1, \quad (8)$$

$$T_{out}^{P_{j0}} = T_{out}^{S_j},\ T_{out}^{P_{jN_j^P}} = T_{in}^{S_{j+1}},\ j = 1, ..., N^S - 1. \quad (9)$$

MINLP Model for HOP (cont'd)

Bound constraints

- **Number of powered-on CSPs and SSPs bounds**

$$x_j \in \{\underline{x}, ..., \overline{x}\},\ j = 1, ..., N^S - 1, \tag{10}$$
$$y_j \in \{\underline{y}, ..., \overline{y}\},\ j = 1, ..., N^S - 1. \tag{11}$$

- **Temperature rise bounds**

$$\Delta T_j \geq 0, \qquad j = 1, ..., N^S - 1. \tag{12}$$

- **Inlet and outlet value bounds**

$$\underline{H}^{S_j}_{in} \leq H^{S_j}_{in} \leq \overline{H}^{S_j}_{in}, \qquad j = 1, ..., N^S, \tag{13}$$
$$\underline{H}^{S_j}_{out} \leq H^{S_j}_{out} \leq \overline{H}^{S_j}_{out}, \qquad j = 1, ..., N^S - 1, \tag{14}$$
$$\underline{T}^{S_j}_{in} \leq T^{S_j}_{in} \leq \overline{T}^{S_j}_{in}, \qquad j = 1, ..., N^S, \tag{15}$$
$$\underline{T}^{S_j}_{out} \leq T^{S_j}_{out} \leq \overline{T}^{S_j}_{out}, \qquad j = 1, ..., N^S - 1. \tag{16}$$

- **Head bounds at transition point**

$$\underline{H}^{P_{jr}}_{out} \leq H^{P_{jr}}_{out} \leq \overline{H}^{P_{jr}}_{out},\ j = 1, ..., N^S - 1,\ r = 1, ..., N^P_j. \tag{17}$$

Transport cost (pump cost and furnace cost) objective function

$$C(x, \Delta H^{SP}, \Delta T) = \sum_{j=1}^{N^S-1} \left[C_p \rho Q_{j0} g \left(\frac{x_j H_j^{CP}}{\xi_j^{CP}} + \frac{\Delta H_j^{SP}}{\xi_j^{SP}} \right) + C_f c \rho Q_{j0} \frac{\Delta T_j}{\eta_j V_c} \right].$$

C_p and C_f are the unit price of electricity and fuel, respectively, ξ^{CP} and ξ^{SP} are the efficiency of CSP and SSP, respectively, η is the efficiency of furnace, V_c is the heating value of the fuel.

Integer variables vector and scheme vector

$$z := (x, y), \quad \Psi := \left(z, \Delta H^{SP}, \Delta T, H^S_{in}, H^S_{out}, T^S_{in}, T^S_{out}, H^P_{out}, T^P_{out}, T^P_{ave}, F\right).$$

MINLP Model for HOP

$$\begin{aligned} \min_{\Psi} \quad & C(x, \Delta H^{SP}, \Delta T) \\ \text{s.t.} \quad & (1) - (17) \end{aligned} \qquad \text{(HOP)}$$

Difficulties and Ideas

(HOP) is $\mathcal{NP}$-hard even if we only consider the pump combination.

Difficulties

- **Discrete variables**: powered on CSPs and SSPs x, y
- **Nonconvex constraints**: **friction** F

Idea

- Find appropriate relaxation problems
  - **continuous relaxation**
  - **convex relaxation**
- Implement **branch-and-bound** techniques

Overview

Continuous Relaxation

Relax integer variables x, y

$$x_j \in \left[\underline{x}_j, \bar{x}_j\right], \; j = 1, \ldots, N^S - 1, \tag{18}$$

$$y_j \in \left[\underline{y}_j, \bar{y}_j\right], \; j = 1, \ldots, N^S - 1. \tag{19}$$

Continuous relaxation of (HOP)

$$\begin{aligned} \min_{\Psi} \quad & C(x, \Delta H^{SP}, \Delta T) \\ \text{s.t.} \quad & (1) - (9), (12) - (19). \end{aligned} \tag{HOPnr1}$$

- Nonlinear Programming (NLP) problem
- Nonconvex (hard to obtain the optimal solution)

Upper Bound

If $\check{x}$ and $\check{y}$ are linear relaxation solution, then

$$\hat{H}_{in}^{S_j} + \lceil x_j \rceil H_j^{CP} + \lceil y_j \rceil \overline{H}_j^{SP} \geq \check{H}_{in}^{S_j} + \check{x}_j H_j^{CP} + \Delta \check{H}_j^{SP} \geq \check{H}_{out}^{S_j} = \hat{H}_{out}^{S_j},\ j = 1, ..., N^S - 1.$$

That is, $\lceil \check{x} \rceil$ and $\lceil \check{y} \rceil$ consist of a feasible pump combination scheme.

Each feasible solution of (HOPnr1) derives an upper bound for (HOP)

Assumption on Friction

Friction

$$\mathrm{HF}\,(T, Q, D, L) = \beta(T)\frac{Q^{2-m(T)}}{D^{5-m(T)}}\nu(T)^{m(T)}L.$$

Observation

- **Discontinuity**: hard to handle.
- **Viscosity-temperature curve**: nearly **convex** and **monotonically decreasing** about the temperature;
- **Friction**: same convexity and monotonicity with viscosity in the **hydraulic smooth** case ($\beta \equiv 0.0246$, $m \equiv 0.25$) with positive Q, D and L.

Assumption 1

Given $Q > 0$, $D > 0$ and $L > 0$, the function f is convex and monotonically decreasing about $T > 0$.

Convex Relaxation and Lower Bound

Relax nonconvex equality constraints (2)

$$F_{jr} \geq f\left(T_{ave}^{P_{jr}}, Q_{jr}, D_{jr}\right) L_{jr} \; j = 1, \ldots, N^S - 1, \;\; r = 1, \ldots, N_j^P. \tag{20}$$

Convex relaxation of (HOP)

$$\begin{aligned} \min_{\Psi} \quad & C(x, \Delta H^{SP}, \Delta T) \\ \text{s.t.} \quad & (1), (3) - (9), (12) - (20). \end{aligned} \tag{HOPnr2}$$

- Convex NLP problem

Each local optimum of (HOPnr2) is a lower bound of (HOP)

(HOPnr1) and (HOPnr2)

- (HOPnr1) and (HOPnr2) are equivalent if constraints (20) are active at the optimal solution of (HOPnr2).
- Otherwise, this is not true.

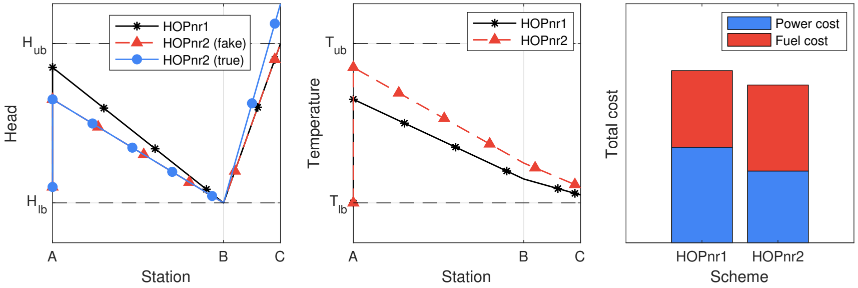


Figure: An example with inactive constraints (20)

- The blue curve is infeasible due to the upper bound violation of temperature
- The relaxation on nonconvex constraints break the hidden limitations on the upper bound of the temperature

Equivalence Between (HOPnr1) and (HOPnr2)

Lemma 1

Suppose Assumption 1 holds. For each $j = 1, ..., N^S - 1$*, if there exists a feasible solution* $\tilde{\Psi}$ *of (HOPnr1) such that the* $\tilde{T}^S_{out}$ *satisfies*

$$\tilde{T}^{S_j}_{out} = \overline{T}^{S_j}_{out},$$

then (HOPnr2) is feasible. Moreover, for each feasible solution $\check{\Psi}$ *of (HOPnr2) , there exists a feasible solution* $\hat{\Psi}$ *of (HOPnr1) such that*

$$C(\check{x}, \Delta\check{H}^{SP}, \Delta\check{T}) \geq C(\hat{x}, \Delta\hat{H}^{SP}, \Delta\hat{T}).$$

Theorem 2

Under the conditions of Lemma 1, (HOPnr1) and (HOPnr2) have the same optimal objective value.

A feasible (HOPnr1) is equivalent to (HOPnr2) as long as the **upper bounds** of variables $T^{S_j}_{out}$, $j = 1, ..., N^S - 1$ are **exact**!

Overview

Bounds for Branch-and-Bound Framework

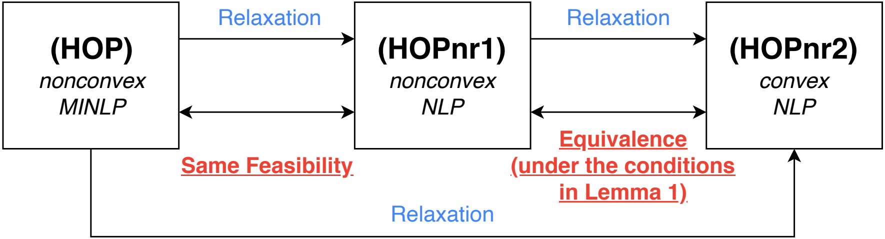


Figure: Relation among (HOP) and relaxations

Local optimal solution of (HOPnr2) (Lemma 1 holds)

- the global optimal solution of (HOPnr2) $\Rightarrow$ **lower bound**
- the global optimum of (HOPnr1) $\Rightarrow$ **upper bound**

HOP Branch-and-Bound (HOPBB) Method

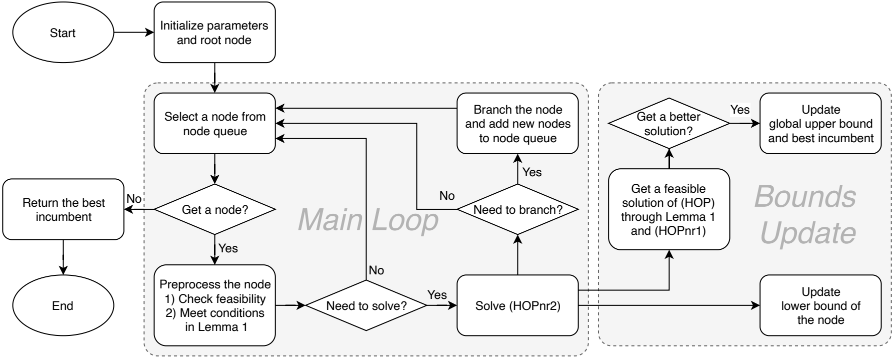


Figure: HOPBB method

- Finite termination and global optimality

Check whether Lemma 1 holds

- **Feasibility** of (HOPnr1)
- Get the **exact bound** of $T_{out}^{S_j}$, $j = 1, \ldots, N^S - 1$

Idea

- Stations can be decoupled since the exact upper bound of outlet temperature and lower bound of outlet head are not influenced by inlet values
- For each station, a feasible solution reaches the above two bounds is derived by constantly coordinating the head and temperature value

Find the exact upper bound of outlet temperature at station A

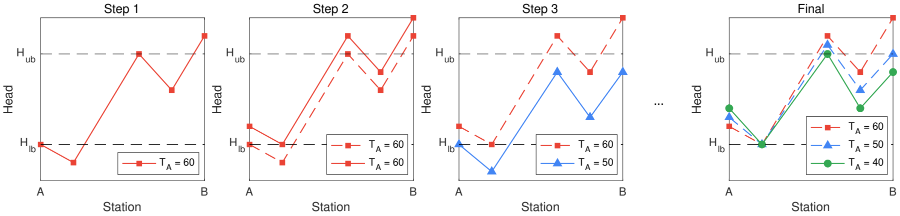


Figure: An example for illustrating preprocessing in HOPBB

1. Initialize $(H_A, T_A) := (H_{lb}, 60)$, calculate the heads between stations A and B;
2. Find out the maximal violation point to H_{lb}, increase H_A until the head at this point satisfies the lower bound constraint;
3. Find out the maximal violation point to H_{ub}, decrease T_A to some appropriate value (solving a nonlinear equation), reset $H_A := H_{lb}$ and update heads in pipeline;
4. Repeat steps 2 and 3 until there exists no violation point, return (H_A, T_A).

Overview

Test Problem: Pipeline

Q-T pipeline

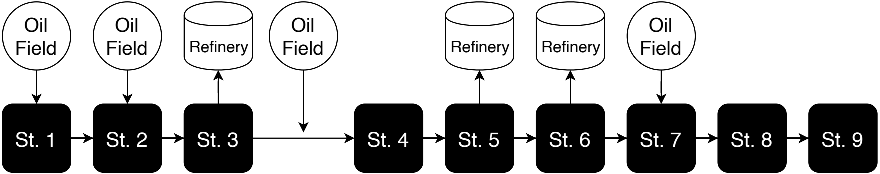


Figure: Schematic diagram of the Q-T pipeline layout

- **Total mileage**: 548.54 km
- **Pipe segments (up to 5000 m)**: 131
- **Pumps (CSPs and SSPs)**: 24 (20, 4)

Test Problem: Oil Property

- **Dynamic viscosity** ($\mu = 1000\rho\nu$):

$$\mu(T) = 8.166 \times 10^6 \exp(-0.3302\,T) + 77.04 \exp(-0.02882\,T)$$

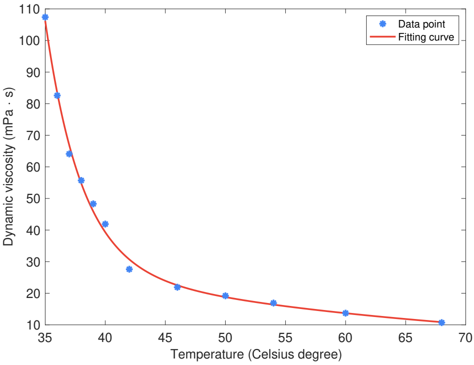


Figure: Viscosity-temperature curve fitting

Results

Station	Practical scheme					Optimal scheme				
	x	ΔH^{SP}	C_{power}	ΔT	C_{fuel}	x	ΔH^{SP}	C_{power}	ΔT	C_{fuel}
1	3	0	72510.05	3.00	**14047.23**	3	0	72510.05	7.30	34181.59
2	0	0	0	3.50	20818.94	0	0	0.00	1.24	**7391.16**
3	1	219.96	**47578.26**	2.90	**13597.40**	1	244.24	50020.79	10.13	47496.64
4	3	0	85751.64	2.80	**17431.60**	1	0	**28583.88**	3.83	23825.27
5	2	104.33	67879.53	4.10	23875.84	1	231.35	**55390.45**	0.00	**0**
6	1	170.27	**39140.92**	6.48	**27735.78**	2	224.68	67530.40	6.84	29293.14
7	1	203.94	43330.03	8.80	39156.12	0	175.05	**16858.35**	7.46	**33178.22**
8	0	0	0	6.90	**29898.62**	0	0	0.00	9.10	39416.83

Table: Practical and optimal operation scheme comparison (cost unit: yuan/d)

Total cost

- **Practical scheme**: 542751.95 (yuan/d)
- **Optimal scheme**: **505676.76 (yuan/d), 6.83% improvement**

Results (cont'd)

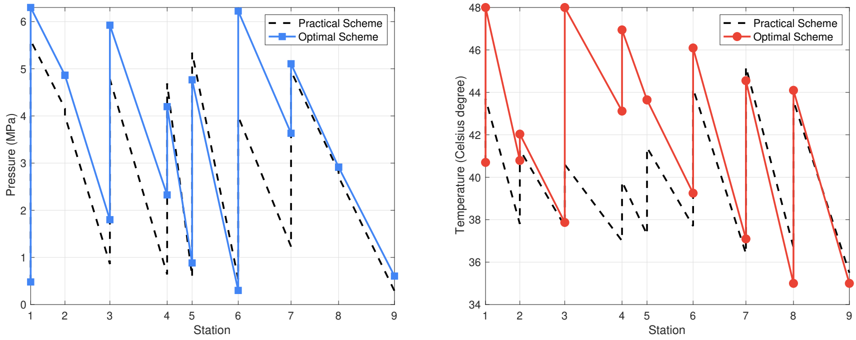


Figure: Pressure and temperature curves of the practical scheme and the optimal scheme

- Advice from the optimal scheme: higher transport temperature may lead to lower total transport cost

Efficiency of HOPBB

Methods in the comparison

- Two versions of HOPBB method
  - **HOPBB-IPM**: (HOPnr2) solved by IPM (IPOPT [Wächter and Biegler, 2006])
  - **HOPBB-OAP**: (HOPnr2) solved by outer approximation
- Nonconvex MINLP solvers aim at global optimal solution
  - **BARON** [Tawarmalani and Sahinidis, 2005]
  - **ANTIGONE** [Misener and Floudas, 2014]
  - **LINDOGlobal** [Lin and Schrage, 2009]

Table: Performances comparison of different methods on solving the Q-T HOP problem

Method	Best solution	Relative gap	Feasibility	Iterations	Time (sec)
LINDOGlobal	21069.87	1.11×10^{-8}	8.38×10^{-13}	845	1047
ANTIGONE	21069.87	1.00×10^{-5}	5.84×10^{-8}	34439	651
BARON	21069.87	9.85×10^{-3}	2.55×10^{-10}	28007	541
HOPBB-IPM	21069.86	0	7.48×10^{-6}	25	41
HOPBB-OAP	21069.85	0	8.59×10^{-7}	25	9

Efficiency of HOPBB (cont'd)

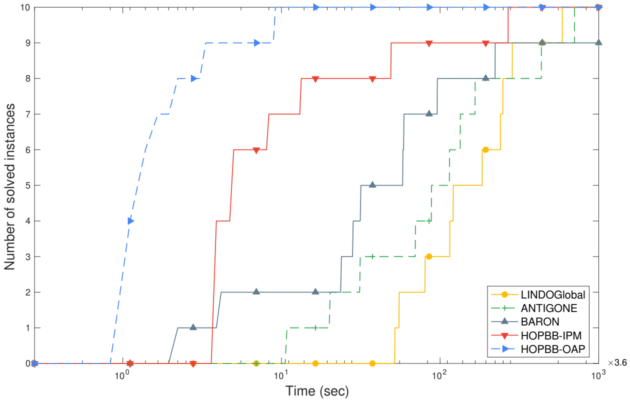


Figure: Performances of different methods on 10 HOP instances

Overview

Conclusions

- Proposed an MINLP model for the HOP problem considering different kinds of pumps and a general formulation of frictions
- Found the equivalence condition on two relaxation problems
- Implemented the branch-and-bound framework on proposed MINLP model and obtained the global optimal solution
- Designed a preprocessing algorithm to guaranteed the equivalence condition
- Achieved a considerable improvement compared with a practical operation scheme
- Showed the high efficiency of HOPBB methods on HOP problems comparing with general MINLP solvers

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Thanks for watching!