Optimal Operation of Transient Gas Transport Networks

Kai Hoppmann-Baum

Combinatorial Optimization @ Work 2020

Gas Network Control Problem

General Description

- Optimizing the short-term transient control of large real-world gas transport networks
- "Navigation system" for gas network operators

Source: Open Grid Europe

Problem

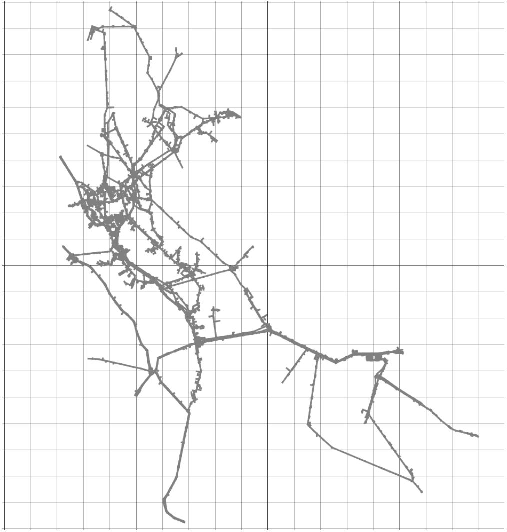
Given

- Network topology
- Initial network state
- Short-term supply/demand and pressure forecast, e.g., 12–24 hours

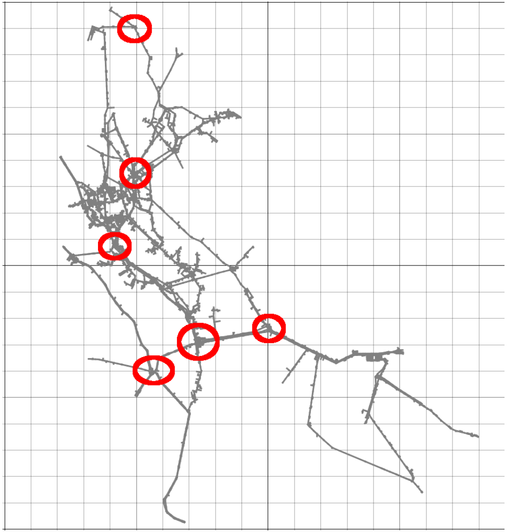
Goal

- Control each element such that the network is operated "best"
- Good control here means: Satisfy all supplies and demands while changing network control as little as possible

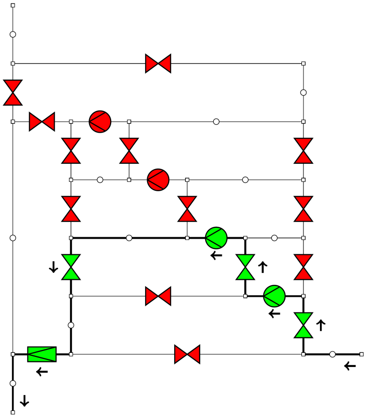
Example Gas Grid



Example Gas Grid - Network Stations



Two Main Sources of Complexity



Combinatorics of Network Stations

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial p}{\partial x} + \frac{\partial(\rho v^2)}{\partial x} + \frac{\lambda_a}{2D_a}|v|v\rho + g s_a \rho = 0$$

Transient Gas Flow in Pipelines
Isothermal Euler Equations

KOMPASS - Basic Algorithmic Framework

1. Replace network stations by simplified graph representation

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2. Further network simplifications:

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   - Merge pipes (parallel, sequential)

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5. Solve transient operation problem for original network stations (Station Model Algorithm)
6. Retrieve control suggestions for dispatchers, i.e., operation modes, target values,...

Outside Network Stations - Pipelines

Gasflow in a pipe (u, v) between timesteps t_i and t_{i+1} can be described by

$$\frac{p_{u,t_{i+1}} - p_{u,t_i}}{2} + \frac{p_{v,t_{i+1}} - p_{v,t_i}}{2} + \frac{R_s\, T\, z\, \Delta t}{L\, A} \left(q_{v,t_{i+1}} - q_{u,t_{i+1}}\right) = 0$$

$$\frac{\lambda\, R_s\, T\, z\, L}{4\, A^2\, D} \left(\frac{\left|q_{u,t_i}\right| q_{u,t_i}}{p_{u,t_i}} + \frac{\left|q_{v,t_i}\right| q_{v,t_i}}{p_{v,t_i}} \right)$$

$$+ \frac{g\, s\, L}{2\, R_s\, T\, z} \left(p_{u,t_i} + p_{v,t_i}\right) + p_{v,t_i} - p_{u,t_i} = 0$$

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$$\frac{\lambda\, R_s\, T\, z\, L}{4\, A^2\, D} \left(\frac{|q_{u,t_i}|\, q_{u,t_i}}{p_{u,t_i}} + \frac{|q_{v,t_i}|\, q_{v,t_i}}{p_{v,t_i}} \right)$$

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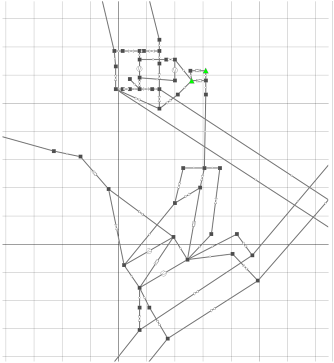
Fixing absolute velocity:

$$\frac{\lambda\, L}{4\, A\, D} \left(|v_{u,0}|\, q_{u,t_i} + |v_{v,0}|\, q_{v,t_i} \right) + \frac{g\, s\, L}{2\, R_s\, T\, z} \left(p_{u,t_i} + p_{v,t_i}\right) + p_{v,t_i} - p_{u,t_i} = 0$$

Simplifying Network Stations

- Network stations are bounded by fence nodes



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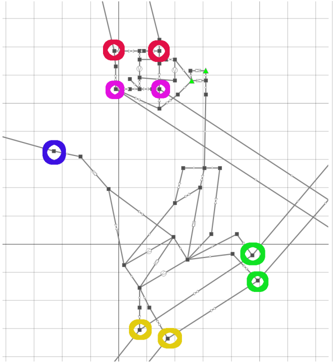

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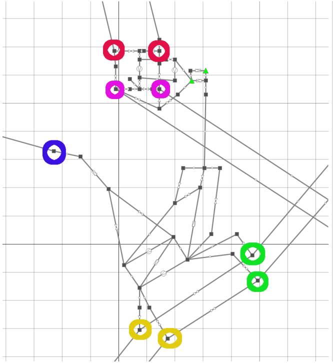

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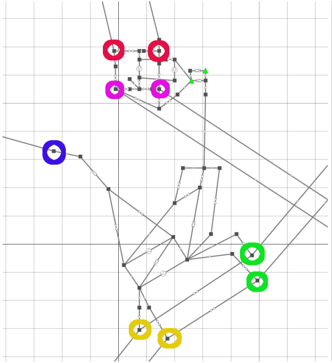

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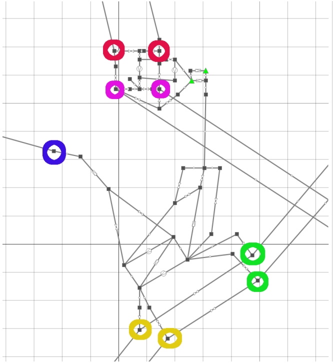

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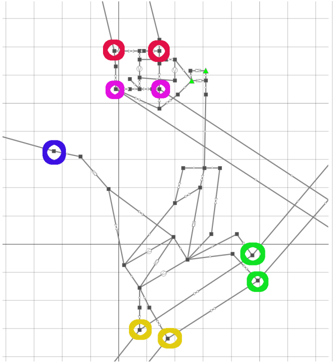

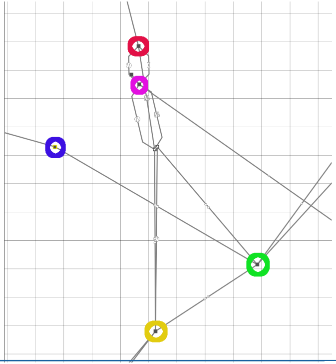

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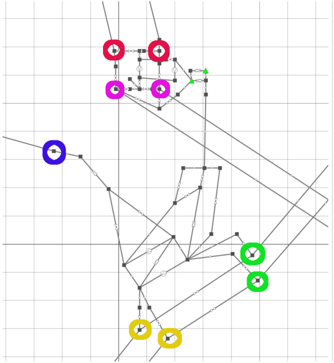

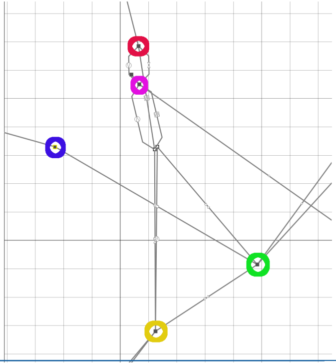

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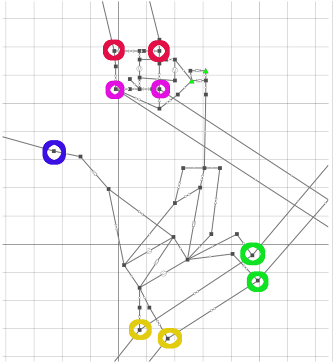

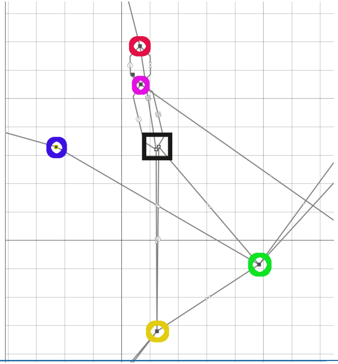

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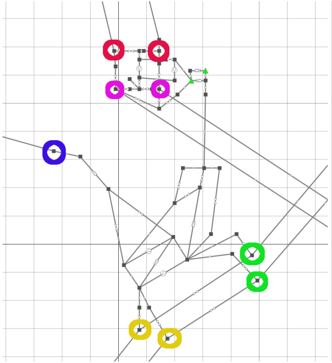
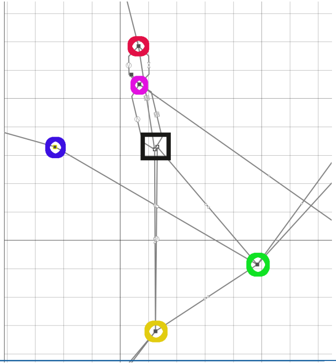
- Auxiliary nodes (for modelling purposes) may be introduced
- Auxiliary links represent the capabilities of a network station



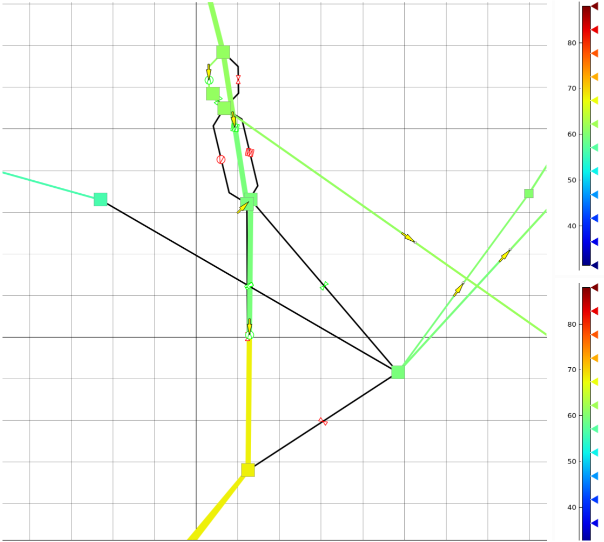


Flow Directions and Simple States

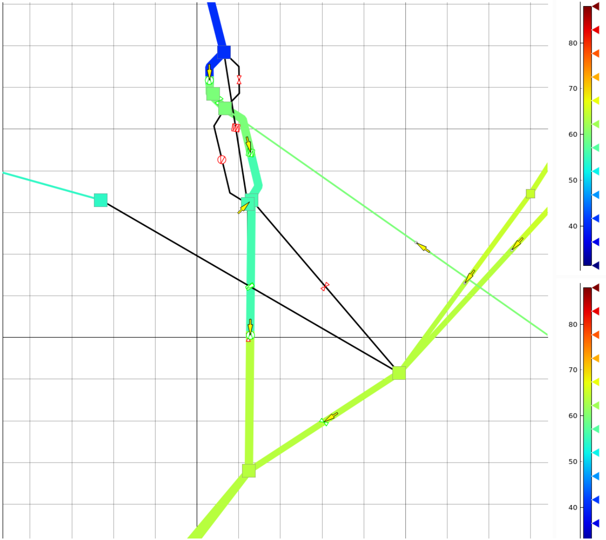
For each network station (V, A) we are given

- Flow directions $\mathcal{F} \subseteq \mathcal{P}(V) \times \mathcal{P}(V)$ with $f = (f^+, f^-) \in \mathcal{F}$
- Simple states $\mathcal{S} \subseteq \mathcal{P}(\mathcal{F}) \times \mathcal{P}(A) \times \mathcal{P}(A)$ with $s = (s_f, s_a^{on}, s_a^{off}) \in \mathcal{S}$

Example I



Example II



Flow Directions and Simple States

For each network station (V, A) we are given

- Flow directions $\mathcal{F} \subseteq \mathcal{P}(V) \times \mathcal{P}(V_i)$ (example: (f^+, f^-))
- Simple states $\mathcal{S} \subseteq \mathcal{P}(\mathcal{F}) \times \mathcal{P}(A) \times \mathcal{P}(A)$ (example: $(s_f, s_a^{on}, s_a^{off})$)
- $x_{f,t} \in \{0,1\}$ for flow direction $f \in \mathcal{F}$ and time step $t \in T$
- $x_{s,t} \in \{0,1\}$ for simple state $s \in \mathcal{S}$ and time step $t \in T$
- $x_{a,t} \in \{0,1\}$ for artificial arc $a \in A$ and time step $t \in T$

$$
\begin{aligned}
\sum\nolimits_{f \in \mathcal{F}} x_{f,t} &= 1 && \forall t \in T \\
\sum\nolimits_{f \in s_f} x_{f,t} &\geq x_{s,t} && \forall s \in \mathcal{S}, \forall t \in T \\
\sum\nolimits_{s \in \mathcal{S}} x_{s,t} &= 1 && \forall t \in T \\
x_{s,t} &\leq x_{a,t} && \forall s \in S, \forall a \in s_a^{on}, \forall t \in T \\
1 - x_{s,t} &\geq x_{a,t} && \forall s \in S, \forall a \in s_a^{off}, \forall t \in T
\end{aligned}
$$

... additional flow direction related constraints ...

Shortcuts

For a shortcut $a = (u, v)$ and each $t \in T$:

Not Active ($x_{a,t} = 0$):

- Decoupled pressure values
- No flow allowed

Active ($x_{a,t} = 1$):

- Coupled pressure values
- Bidirectional flow up to an amount of $\overline{q}_a$ (Big-M).

$$
\begin{aligned}
p_{u,t} - p_{v,t} &\leq (1 - x_{a,t})(\overline{p}_v - \underline{p}_u) \\
p_{u,t} - p_{v,t} &\geq (1 - x_{a,t})(\underline{p}_v - \overline{p}_u) \\
q^{\rightarrow}_{a,t} &\leq x_{a,t}\overline{q}_a \\
q^{\leftarrow}_{a,t} &\leq x_{a,t}\overline{q}_a.
\end{aligned}
$$

Regulating Arcs

For a regulating arc $a = (u, v)$ and each $t \in T$:

Not Active ($x_{a,t} = 0$):

- Decoupled pressure values
- No flow allowed

Active ($x_{a,t} = 1$):

- Pressure at u not smaller than pressure at v
- Unidirectional flow up to an amount of $\overline{q}_a$ (Big-M).

$$p_{u,t} - p_{v,t} \geq (1 - x_{a,t})(\underline{p}_v - \overline{p}_u)$$
$$q^{\rightarrow}_{a,t} \leq x_{a,t}\overline{q}_a.$$

Compressing Arcs

Not Active ($x_{a,t} = 0$):

- No machine assigned
- Decoupled pressure values
- No flow allowed

Active ($x_{a,t} = 1$):

- Assign machines to compressing arc
- Pressure at v not smaller than pressure at u
- Pressure at v at most r_a times greater than $p_{u,0}$
- Flow limited by sum of max flows of assigned machines
- Respect approximated power bound equation

Compressing Arcs - MILP Model

For each machine $i \in M$ and for each timestep $t \in T$ we have

$$\sum_{a \in A: i \in M_a} y^i_{a,t} \leq 1$$

$$y^i_{a,t} \leq x_{a,t}$$

For each compressing arc a and for each timestep $t \in T$ we have

$$q^{\rightarrow}_{a,t} \leq \sum_{i \in M_a} F^i y^i_{a,t}$$

$$r_{a,t} = 1 + \sum_{i \in M_a} (1 - R^i) y^i_{a,t}$$

$$\pi_{a,t} \leq \sum_{i \in M_a} P^i y^i_{a,t}$$

$$p_{u,t} - p_{v,t} \leq (1 - x_{a,t})(\overline{p}_v - \underline{p}_u)$$

$$r_a p_{u,0} - p_{v,t} \geq (1 - x_{a,t})(p_{u,0} - \overline{p}_{v,t})$$

$$\alpha_1 p_{u,t} + \alpha_2 p_{v,t} + \alpha_3 q^{\rightarrow}_{a,t} + \alpha_4 \pi_{a,t} \leq \beta x_{a,t} + (1 - x_{a,t})(\alpha_1 \underline{p}_u + \alpha_2 \overline{p}_v + \alpha_3 \overline{q}_a)$$

$$\alpha_1 p_{u,t} + \alpha_2 p_{v,t} + \alpha_3 q^{\rightarrow}_{a,t} + \alpha_4 \pi_{a,t} \geq \beta x_{a,t} + (1 - x_{a,t})(\alpha_1 \overline{p}_u + \alpha_2 \underline{p}_v + \alpha_4 \overline{\pi}_a)$$

Combined Arcs

Introduce binary variables $x^r_{a,t}, x^c_{a,t} \in \{0,1\}$ indicating whether the arc is regulating or compressing:

$$x^r_{a,t} + x^c_{a,t} = x_{a,t}$$

Not Active ($x_{a,t} = 0$):

- No machine assigned
- Decoupled pressure values
- No flow allowed

Active and regulating ($x_{a,t} = 1$ and $x^r_{a,t} = 1$):

- Like regulating arc

Active and compressing ($x_{a,t} = 1$ and $x^c_{a,t} = 1$):

- Like compressing arc

Coupling Pipelines and Network Stations & Objective

Flow conservation holds at all nodes in the network

$$\sum \text{outgoing flow} - \sum \text{ingoing flow} = b_{v,t}$$

where $b_{v,t} = 0$ for inner nodes, $b_{v,t} \geq 0$ for entries, and $b_{v,t} \leq 0$ for exits.

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The (current) objective of Netmodel-MILP is to minimize the number of

1. flow direction changes,
2. simple state changes,
3. and artificial link switches.

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The (current) objective of Netmodel-MILP is to minimize the number of

1. flow direction changes,
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Currently, we discuss to additionally penalize

- compressor/combined links being active,
- assigning machines,
- power used for compression,
- ...

3-Stage Approach and Solution Smoothening

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To avoid infeasibility, we have a 3-Stage Approach

1. Initial MILP

3-Stage Approach and Solution Smoothening

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2. Stage 1 infeasible $\Rightarrow$ add (expensive) slack on supplies/demands

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For a Netmodel-MILP solution we solve a smoothening given by

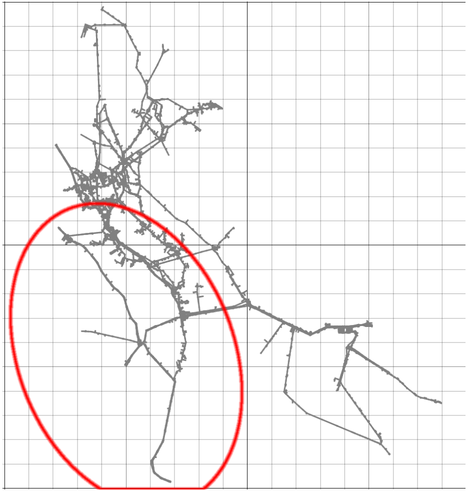
1. Fix all binary variables in Netmodel-MILP
2. Introduce variables and constraints accounting for pressure and flow differences at boundaries between t and $t - i$
3. Minimize the sum of these variables

Netmodel-Algorithm

Solve MILP
if MILP is infeasible **then**
    Add slack on supply/demands and resolve
    if MILP is infeasible **then**
        Add slack on pressure bounds and resolve
$\text{sol}_0 \leftarrow$ smoothened solution of MILP

Future Work and Challenges

- Automatization of simplified graph representation



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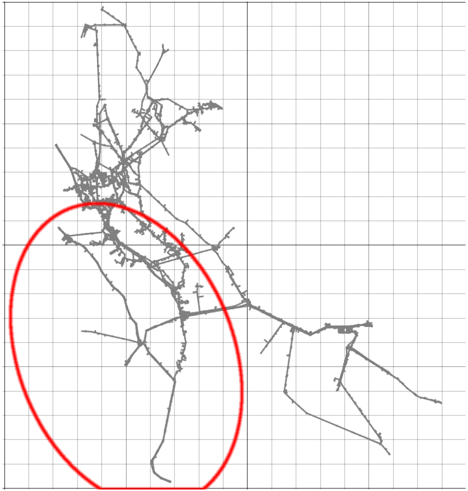
- Automatization of simplified graph representation
- Solution for non-linear Momentum Equations



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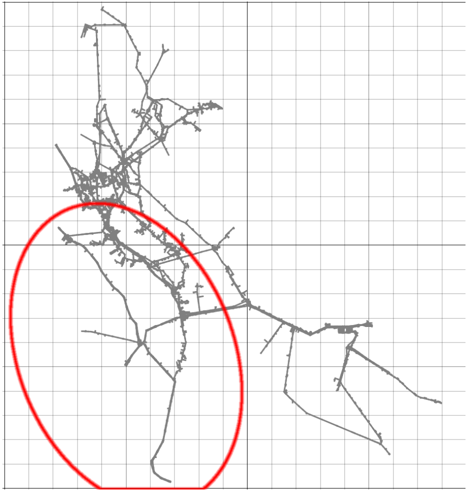

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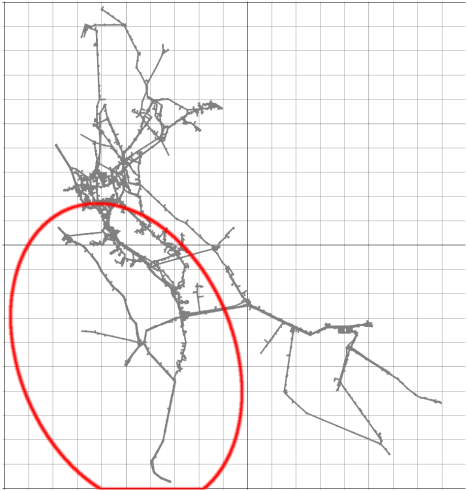

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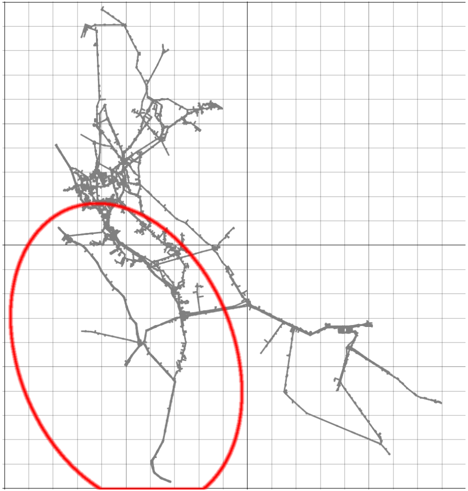

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- Solution for non-linear Momentum Equations
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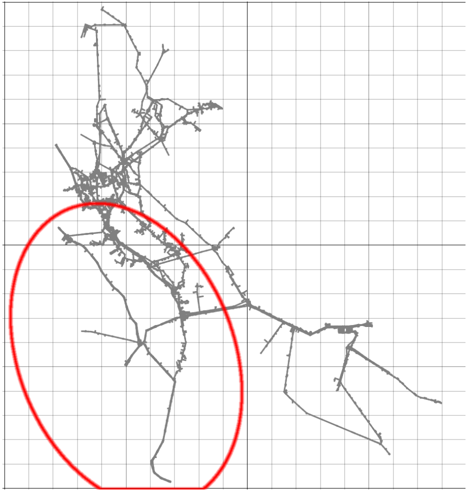
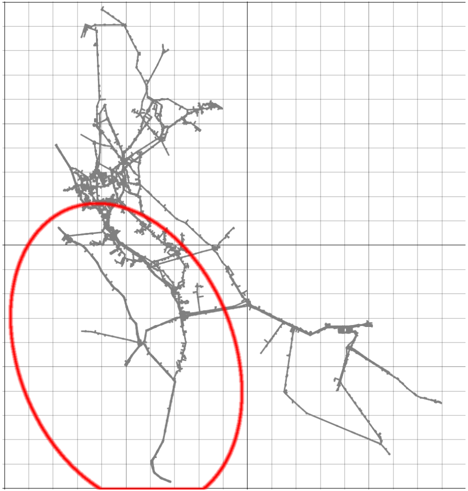
- More realistic element modelling
  - Air temperature dependent compression power bound
  - Semi-fixed elements



Future Work and Challenges

- Automatization of simplified graph representation
- Solution for non-linear Momentum Equations
- Feedback from Station Model to Netmodel
- More realistic element modelling
  - Air temperature dependent compression power bound
  - Semi-fixed elements
  - Single special network elements



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CO@Work 2020

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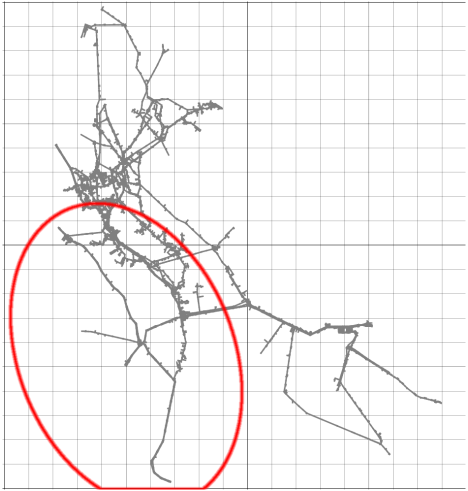
    - Single special network elements
    - Reduce simplifications made in the Netmodel
- Stable solutions over sequential runs



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CO@Work 2020

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- Solution for non-linear Momentum Equations
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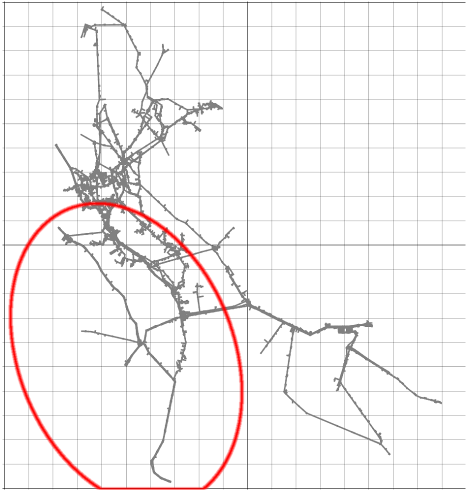
  - Single special network elements
  - Reduce simplifications made in the Netmodel
- Stable solutions over sequential runs
- Increase network size



The Last Slide

Thanks for watching!

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Combinatorial Optimization @ Work 2020