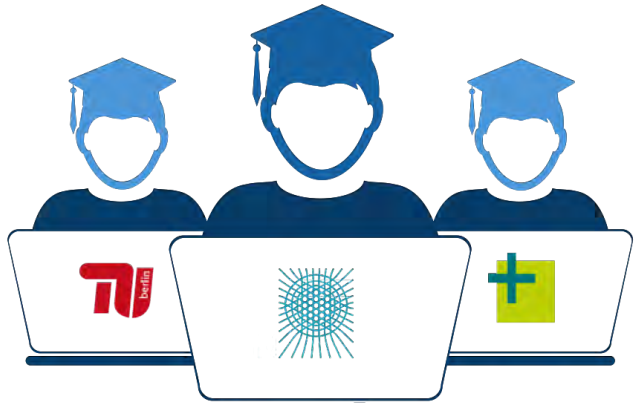


# CO@Work 2020

## Gas Networks Introduction



# CO@Work 2020

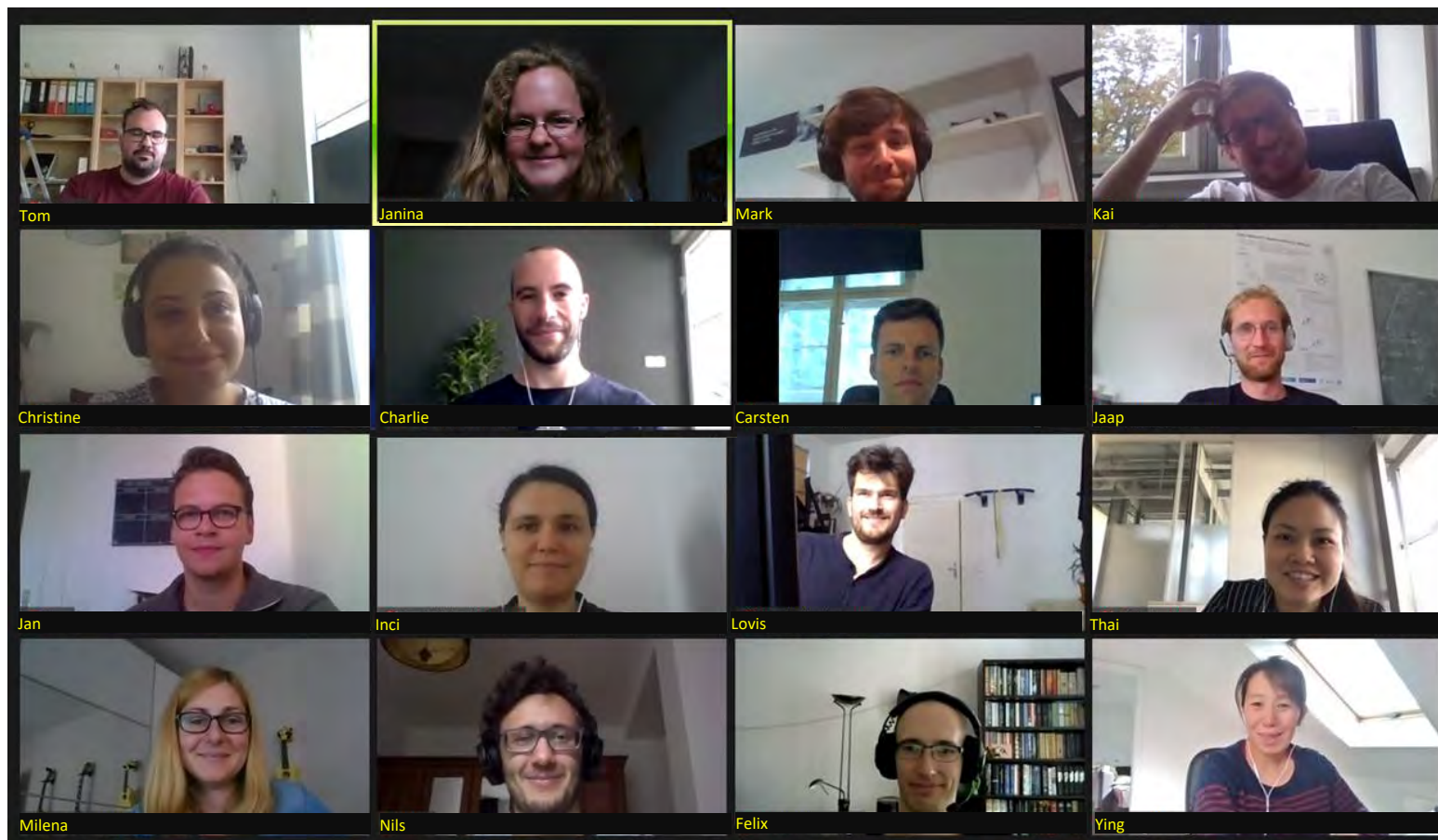
**September 2020**

**Online**

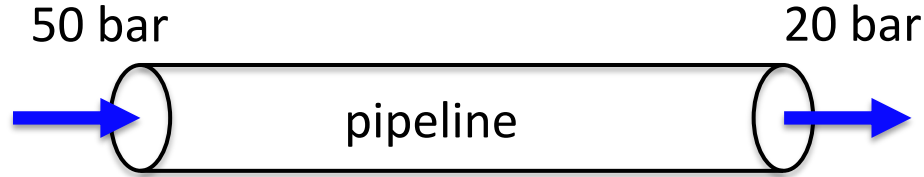
<http://co-at-work.zib.de>

Topics are:

1. European Regulations
2. Gas Network Basics
3. Network Design
4. Gas Network Capacity
5. Gas Network Control



... and me



$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} &= 0 \\ \frac{\partial(\rho v)}{\partial t} + \frac{\partial(p + \rho v^2)}{\partial x} + \frac{\lambda}{2D}|v|v\rho + g s \rho &= 0 \\ \frac{\partial}{\partial t} \left( \rho \left( \frac{1}{2} v^2 + e \right) \right) + \frac{\partial}{\partial x} \left( \rho v \left( \frac{1}{2} v^2 + e \right) + p v \right) + \frac{k_w}{D}(T - T_w) &= 0\end{aligned}$$



$$\alpha |q| q = p_{out}^2 - p_{in}^2$$

$\alpha$  depending on dimension and inclination of the pipeline, the friction in the pipeline, gas temperature, gas composition, outside temperature, and more.

Given a graph  $G = (V, A)$   
with pressure  $p_u$ , and flow  $q_a$ ,  
for  $u \in V, a \in A$  and  $\pi_u = p_u^2$ .

Exists  $q, \pi$

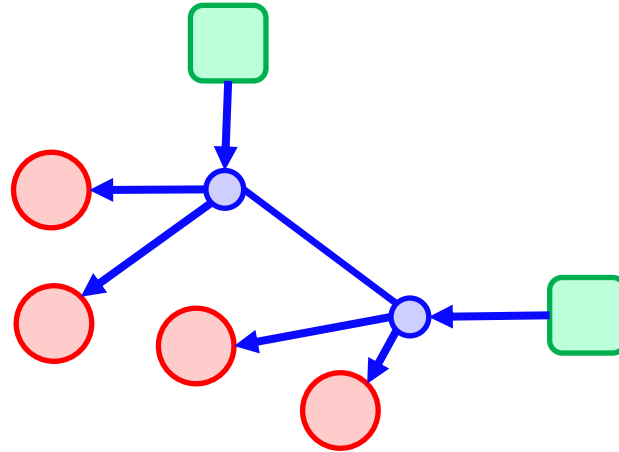
subject to

$$\sum_{a \in \delta^+(u)} q_a - \sum_{a \in \delta^-(u)} q_a = d_u \quad \text{for all } u \in V$$

$$\alpha_a | q_a | \beta_a = \pi_u - \pi_v \quad \text{for all } a = (u, v) \in A$$

$$\underline{\pi}_u \leq \pi_u \leq \bar{\pi}_u \quad \text{for all } u \in V$$

$$\underline{q}_a \leq q_a \leq \bar{q}_a \quad \text{for all } a \in A$$



**Theorem** (Maugis, 1977, Collins at al, 1978, Humpola, K., et al, 2013)

Let  $d \in \mathbb{R}^V$  be a balanced demand and  $\Phi_a$  strictly increasing function.

Then the solution space of

*exists  $q, \pi$  subject to*

$$\begin{aligned} \sum_{a \in \delta^+(u)} q_a - \sum_{a \in \delta^-(u)} q_a &= d_u && \text{for all } u \in V \\ \Phi_a(q_a) &= \pi_u - \pi_v && \text{for all } a = (u, v) \in A \\ \underline{\pi}_u \leq \pi_u \leq \bar{\pi}_u &&& \text{for all } u \in V \\ \underline{q}_a \leq q_a \leq \bar{q}_a &&& \text{for all } a \in A \end{aligned}$$

is either empty or fulfills the conditions:

1. The flow is unique
2. The squared pressure component  $\pi$  has the form

$$\{\pi^* + \eta^1 \mid \underline{\eta} \leq \eta \leq \bar{\eta}\}$$

for some  $\pi^*, \underline{\eta}, \bar{\eta}$ .

- ▷ Flow is unique and  $\Phi_a$  is strictly increasing.
- ▷  $\pi_u - \pi_v$  are uniquely determined for all arcs.
- ▷ Shifting all values by a constant is feasible.
- ▷ Constant shift is the only possible source of difference.

Consider two feasible squared pressure vectors:  $\pi'$  und  $\pi''$ . Both are shifted in such a way that they coincide in the value at  $u$ .

The difference is constant which implies

$$\pi'_u - \pi'_v = \phi_a(q_a) = \pi''_u - \pi''_v.$$

Since  $\pi'_u = \pi''_u$ , we also have  $\pi'_v = \pi''_v$ .

The solution space of the problem

$$\min \sum_{u \in V} \Delta_u + \sum_{a \in A} \Delta_a$$

subject to

$$\sum_{a \in \delta^+(u)} q_a - \sum_{a \in \delta^-(u)} q_a = d_u \quad \text{for all } u \in V$$

$$\Phi_a(q_a) = \pi_u - \pi_v \quad \text{for all } a = (u, v)$$

$$\underline{\pi}_u - \Delta_u \leq \bar{\pi}_u \quad \text{for all } u \in V$$

$$\underline{\pi}_u + \Delta_u \geq \underline{\pi}_u \quad \text{for all } u \in V$$


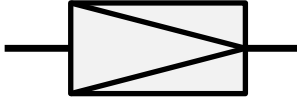
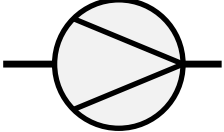
$$\Delta_u \geq 0 \quad \text{for all } u \in V$$

$$q_a - \Delta_a \leq \bar{q}_a \quad \text{for all } a \in A$$

$$q_a + \Delta_a \geq \underline{q}_a \quad \text{for all } a \in A$$

$$\Delta_a \geq 0 \quad \text{for all } a \in A$$

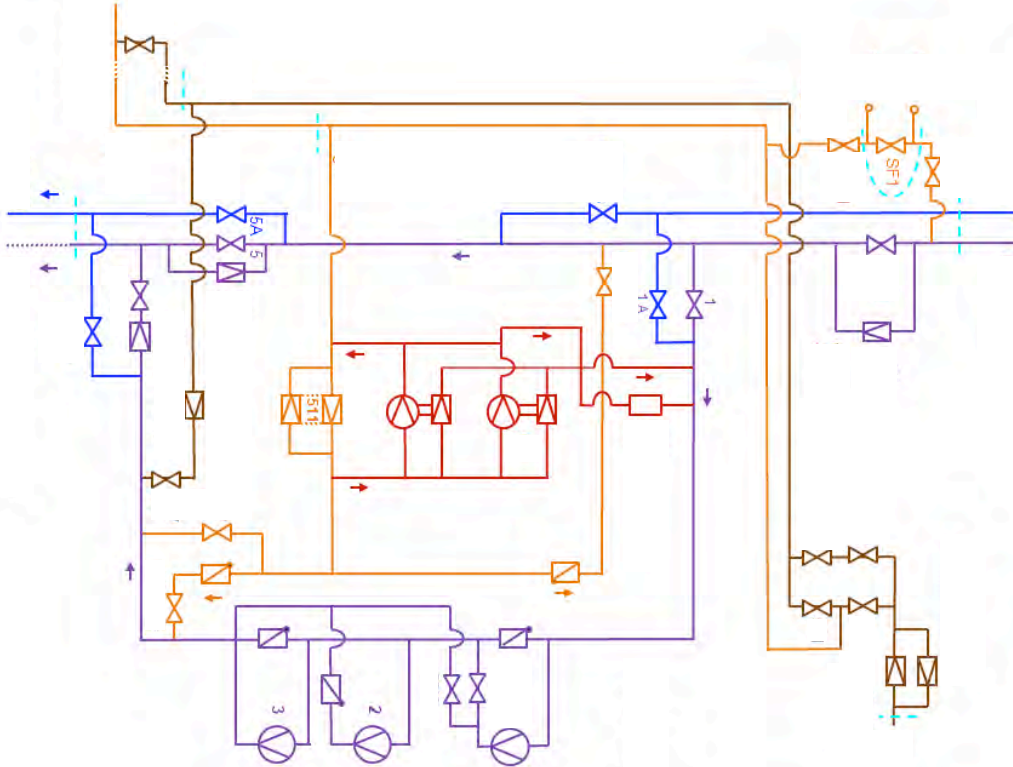
is non-empty and is convex.

| Element    | Function          | Symbol  |
|------------|-------------------|---|
| Valve      | Switch on/off     |  |
| Regulator  | Decrease pressure |  |
| Compressor | Increase pressure |  |





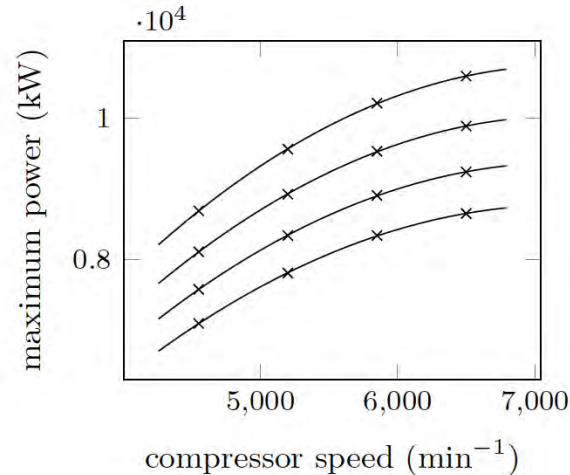
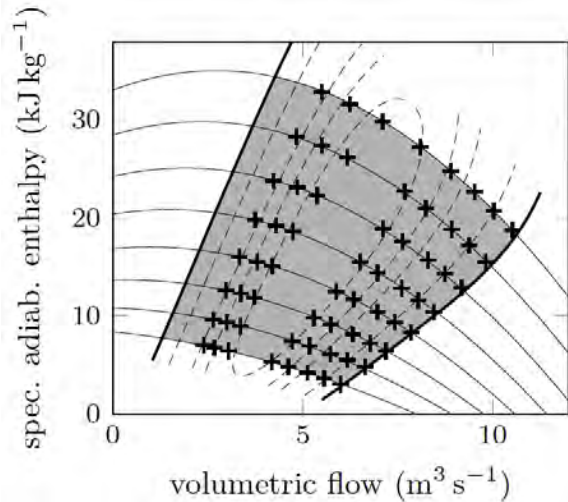
This is an example of just a compressor station



Compressor performance depends on input pressure, output pressure, flow, temperature, composition, compressor power.

Thorsten Koch

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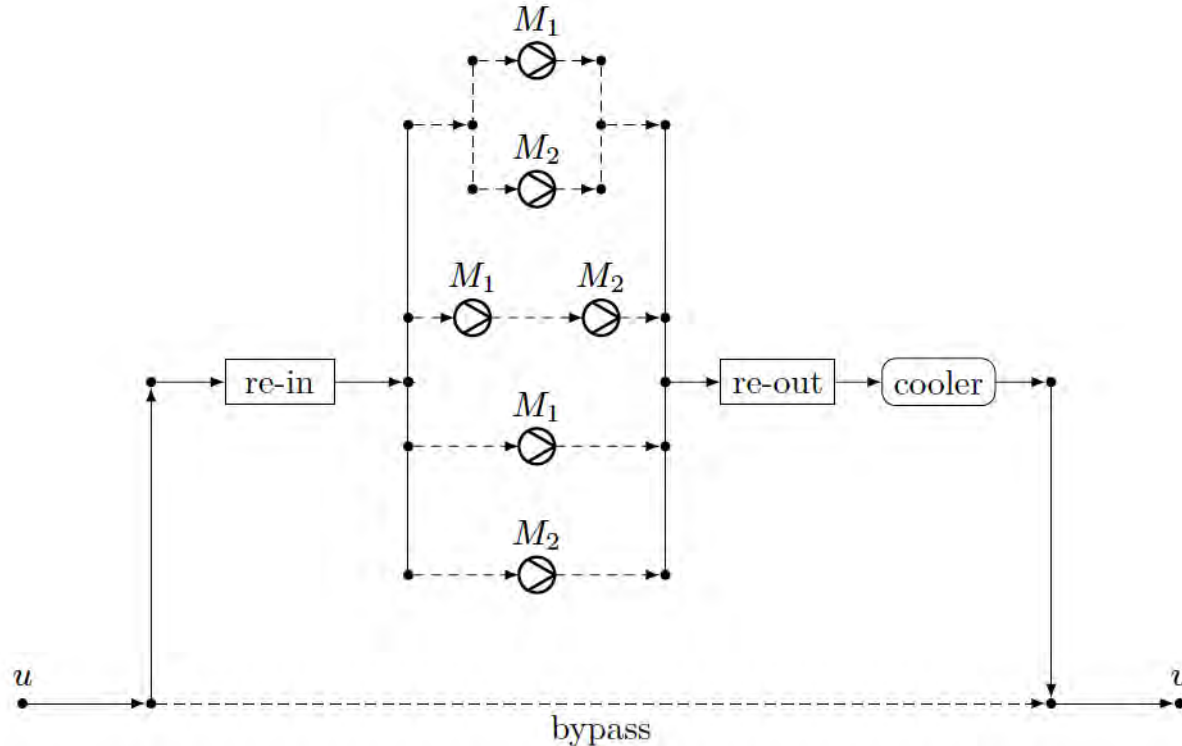


How precise can we model a compressor?

Let us assume we assume the gas temperature  $10^{\circ}\text{C}$  too low.

This gives about 3% more power to the compressor station.

This might be enough to get another 1500 MW gas through.



- ▶ Design a new network  
(or extend an existing one)
- ▶ Determine the capacity  
of a given network
- ▶ Control a network to achieve  
maximum efficiency

# Network Design

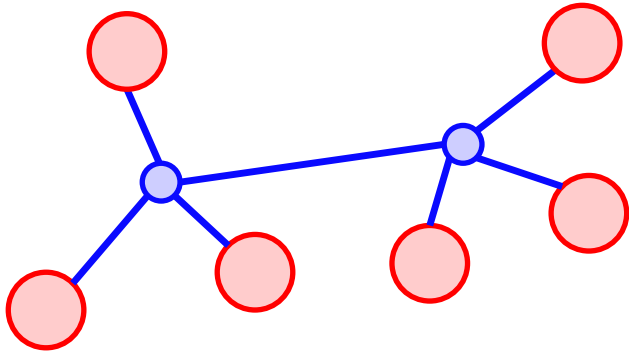
(Just a teaser)

Thorsten Koch

CO@Work

Online, September 2020

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## The Steiner tree problem in graphs (STP)

Given an undirected connected graph  $G = (V, E)$ ,  
costs  $c: E \rightarrow \mathbb{Q}^+$  and a set  $T \subset V$  of *terminals*,  
find a minimum weight tree  $S \subset G$  which spans  $T$ .

The STP is one of the classical 21 **NP**-hard problems.

$$\min c^T y$$

subject to

$$y(\delta_W^+) \geq 1, \quad \text{for all } W \subset V, r \in W, (V \setminus W) \cap T \neq \emptyset$$

$$y(\delta_v^-) \begin{cases} = 0, & \text{if } v = r; \\ = 1, & \text{if } v \in T \setminus r; \\ \leq 1, & \text{if } v \in N \end{cases} \quad \text{for all } v \in V$$

$$y(\delta_v^-) \leq y(\delta_v^+), \quad \text{for all } v \in N;$$

$$y(\delta_v^-) \geq y_a, \quad \text{for all } a \in \delta_v^+, v \in N;$$

$$0 \leq y_a \leq 1, \quad \text{for all } a \in A;$$

$$y_a \in \{0,1\}, \quad \text{for all } a \in A,$$

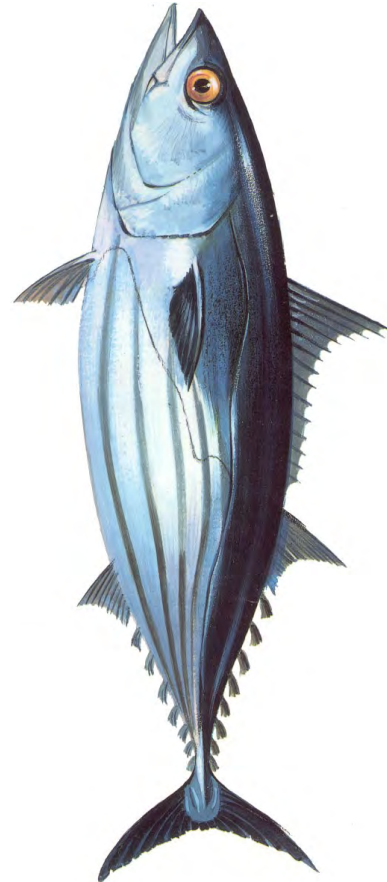
where

$$N = V \setminus T, \delta_X^+ := \{(u, v) \in A \mid u \in X, v \in V \setminus X\}, \delta_X^- := \delta_{V \setminus X}^+ \text{ for } X \subset V.$$

See, e.g., Koch, Martin, *Solving Steiner tree problems in graphs to optimality*, *Networks* (1998)  
Polzin, *Algorithms for the Steiner problem in networks*, Uni Saarland, 2004,  
Rehfeldt, Koch, Combining NP-Hard Reduction Techniques and Strong Heuristics in an Exact  
Algorithm for the Maximum-Weight Connected Subgraph Problem, *SIAMOPT* (2019),  
Shinano, Rehfeldt, Koch, *Building Optimal Steiner Trees on Supercomputers by Using up to 43,000  
Cores*, CPAIOR 2019, LNCS 11494, and the references there in.

It is part of the SCIP Optimization Suite and can solve:

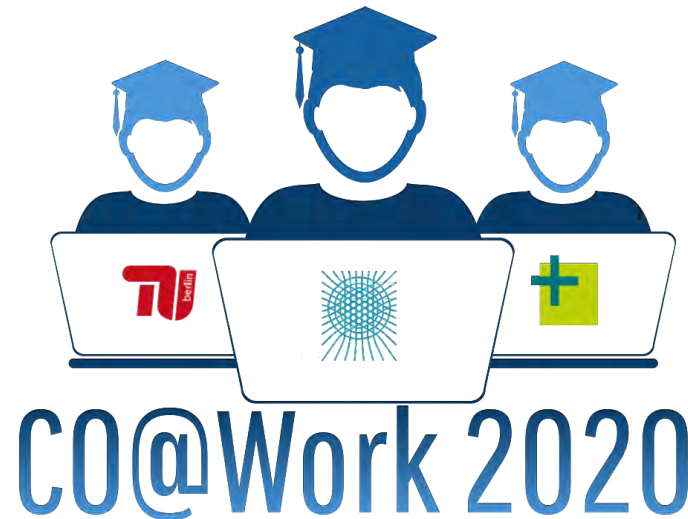
- ▷ Steiner Tree Problem in Graphs (STP)
- ▷ Steiner Arborescence Problems in Graphs (SAP)
- ▷ Rectilinear Steiner Minimum Tree (RSMTP)
- ▷ Node-weighted Steiner Tree (NWSTP)
- ▷ Prize-collecting Steiner Tree (PCSTP)
- ▷ Rooted Prize-collecting Steiner Tree (RPCSTP)
- ▷ Maximum-weight Connected Subgraph (MWCSP)
- ▷ Degree-constrained Steiner Tree (DCSTP)
- ▷ Group Steiner Tree (GSTP)
- ▷ Hop-constrained directed Steiner Tree (HCDSTP)



<http://scipopt.org>



# Please continue with the lecture on Gas Network Capacity



# Gas Network Capacity

Thorsten Koch

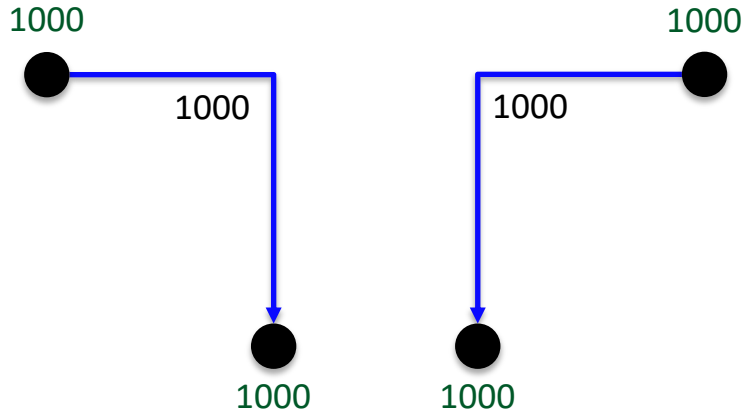
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# What does “Capacity of the Network” mean?

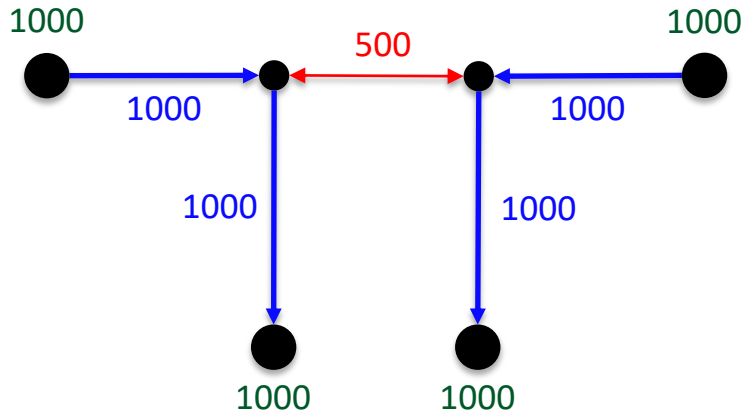
The *Technical Capacity* is defined as the maximum flow bounds at the entry- and exit-nodes, such that any possible balanced demand scenario within these bounds can be fulfilled by the network.



# What does “Capacity of the Network” mean?

The *Technical Capacity* is defined as the maximum flow bounds at the entry- and exit-nodes, such that any possible balanced demand scenario within these bounds can be fulfilled by the network.

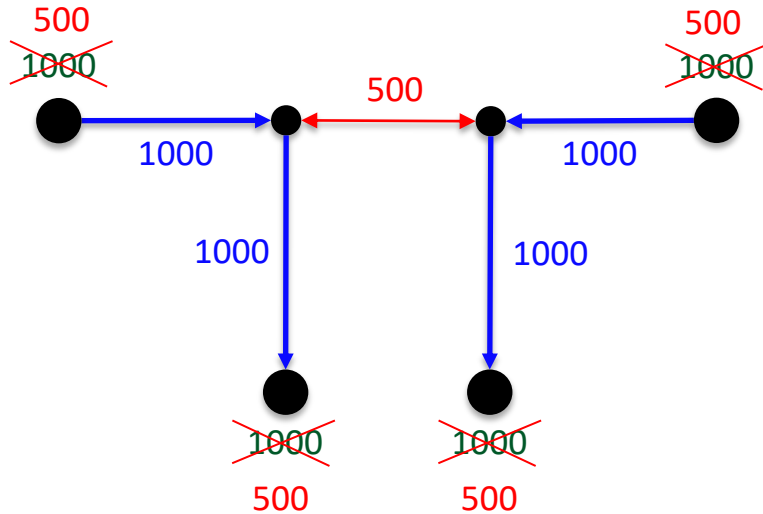
Now adding a connection...



# What does “Capacity of the Network” mean?

The *Technical Capacity* is defined as the maximum flow bounds at the entry- and exit-nodes, such that any possible balanced demand scenario within these bounds can be fulfilled by the network.

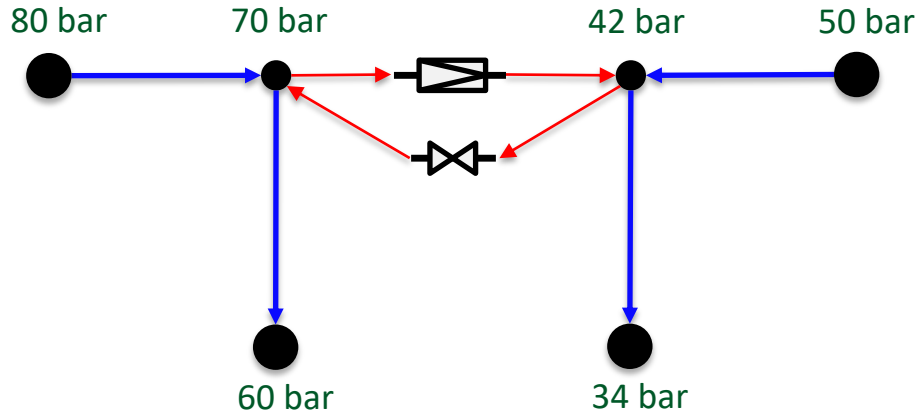
Now adding a connection ... does not necessarily increase it.



# What does “Capacity of the Network” mean?

The new connection also adds restrictions due to pressure coupling. Not all splits of the inflow between the two entries are easily possible anymore.

See also [Braess's paradox](#).



Transient models describe the network state over time.

## Advantages

- ▷ **This is quite realistic** (depending on the time step size)

## Disadvantages

- ▷ Can only be computed over a finite time horizon
- ▷ Requires a forecast of the in- and outflow over time
- ▷ Requires a start state, which is not known for planning
- ▷ Deviations between the predicted and the physical network state grow over time

If we want to decide the feasibility of a future demand scenario should we test against:

- ▷ A worst case start state? **Far too pessimistic**
- ▷ All possible start states? **Infinitely many**
- ▷ A suitable start state? **Likely overly optimistic**

Stationary models describe a (timeless) equilibrium network state.

## Advantages

- ▷ Stable situation (by definition)  
modelling an “average network” state
- ▷ No start state needed, no time horizon to consider
- ▷ Ensures that the situation is sustainable  
(we cannot paint ourselves easily into a corner)
- ▷ Much less data requirements, simpler physics

## Disadvantages

- ▷ Using pipes as gas storage (linepack) cannot be modelled
- ▷ Transition between states cannot be modelled
- ▷ Too pessimistic, especially regarding short-term peak situations

Often better suited for medium and long-term planning.



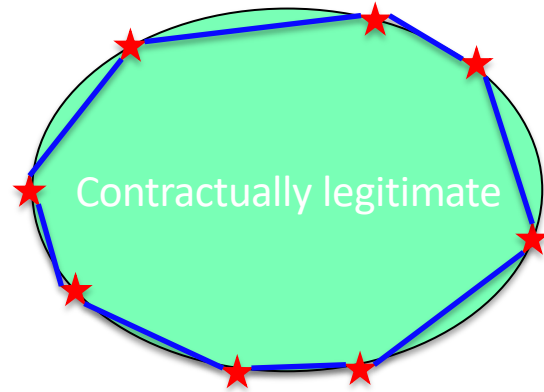
Given a directed graph  $G = (V, A)$  that models a **gas network**. The arcs represent the elements of the network. We distinguish between **passive network elements** (pipes and resistors), whose behavior cannot be influenced, and **active network elements** (valves, control valves, and compressors), which allow to control the network. The active and passive elements are collected in the arc sets  $A_{active}$  and  $A_{passive}$ , respectively.

## Demand Scenario

$d$  specifies for each  $u \in V$  the amount of flow that enters ( $d_u \geq 0$ ) or leaves ( $d_u \leq 0$ ) the network at  $u$ . The scenario  $d$  is balanced, i.e., we have

$$\sum_{u \in V} d_u = 0.$$

The task is to decide, whether or not the scenario  $d$  can be realized in the network by controlling the active elements.

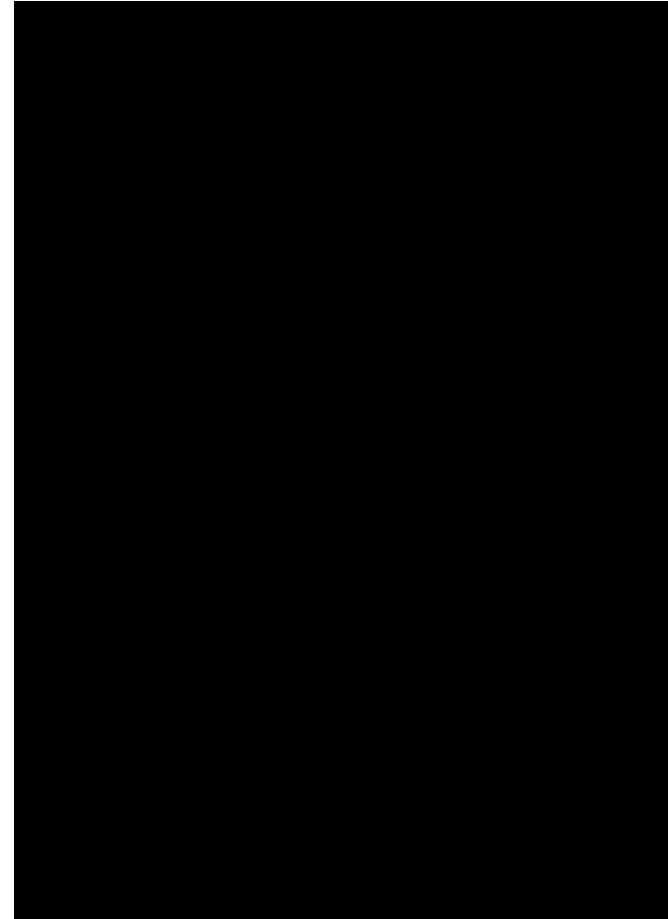
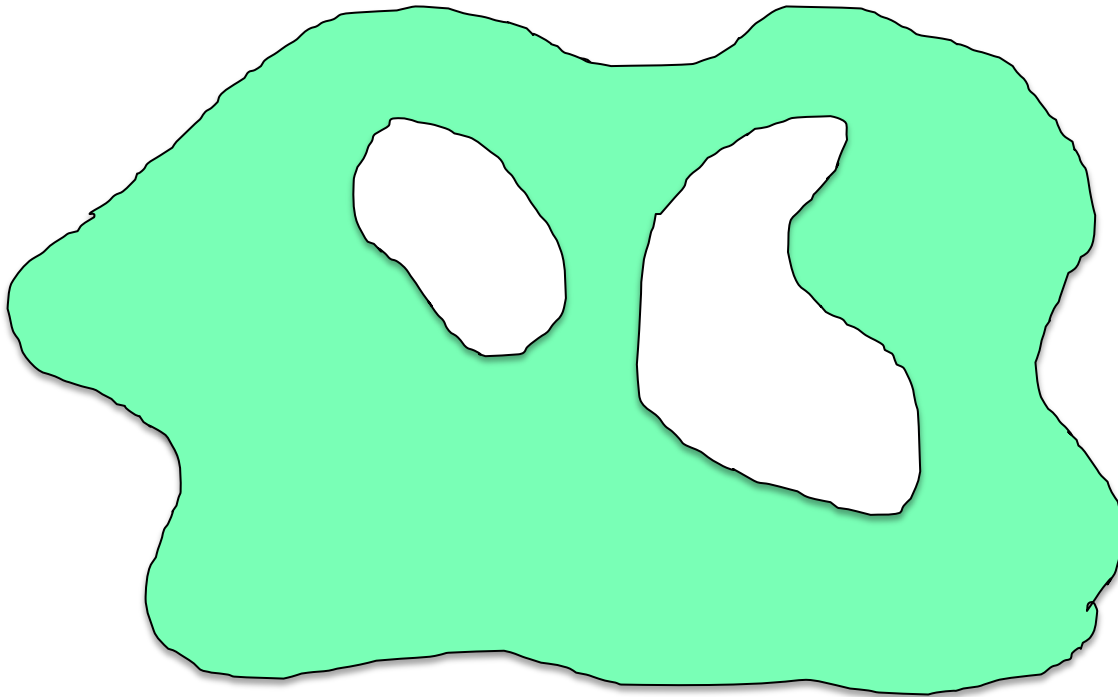


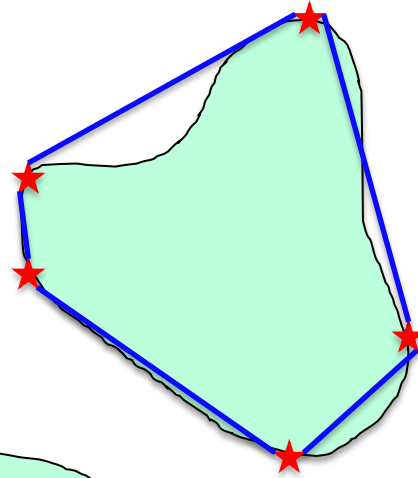
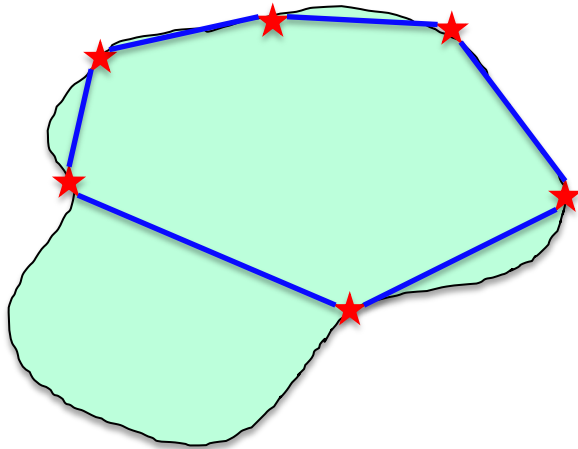
Contractually illegitimate

## A possible approach:

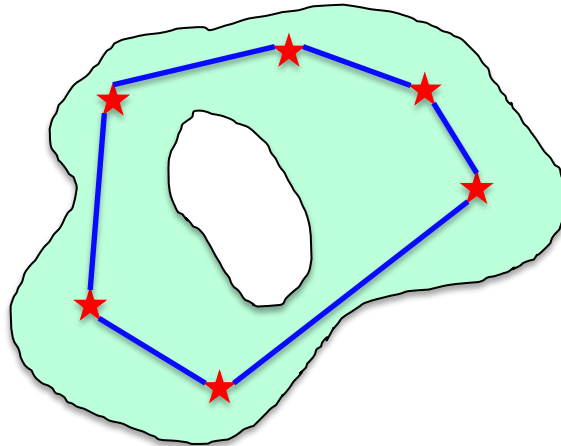
1. Experts derive scenarios that are on the border of feasibility.
2. These scenarios are checked using stationary models.
3. If they are feasible, it is concluded that also all other scenarios are feasible.

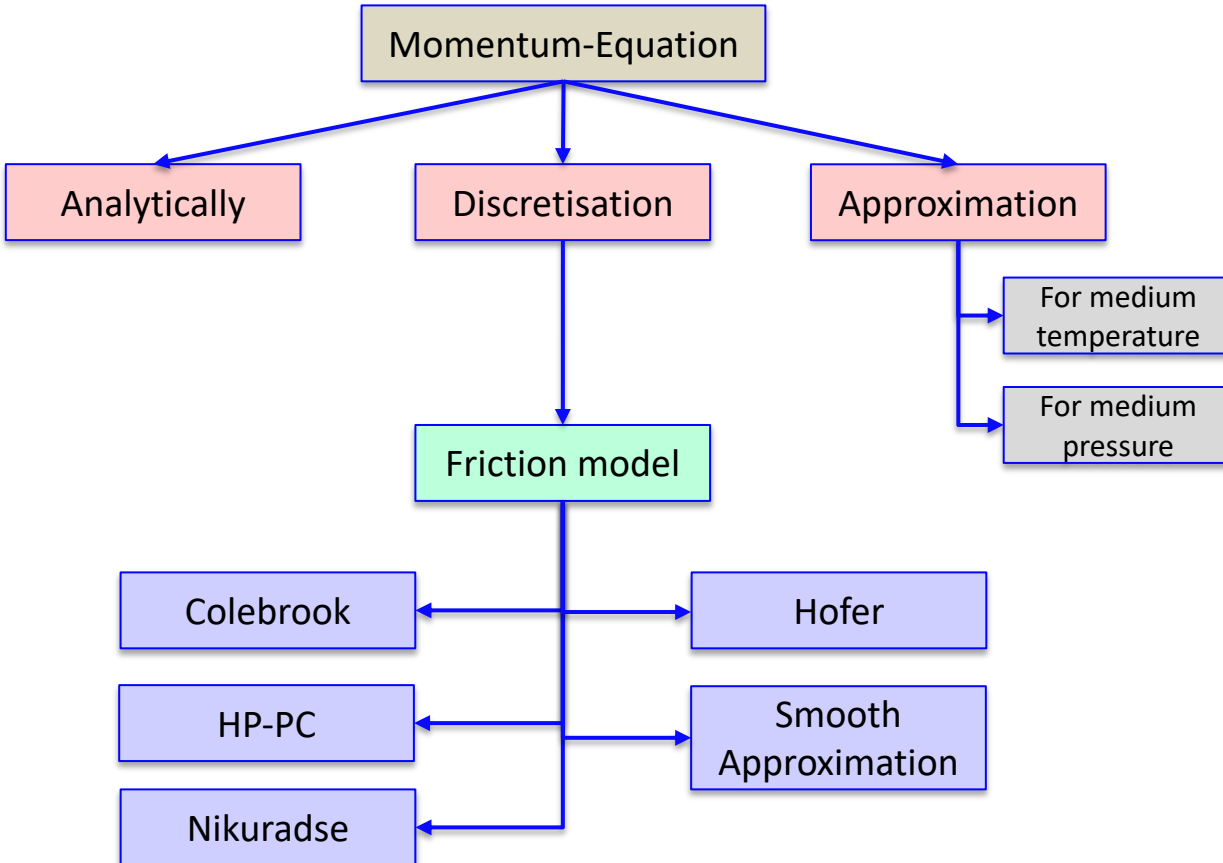
Non-smooth  
Non-convex  
Non-compact

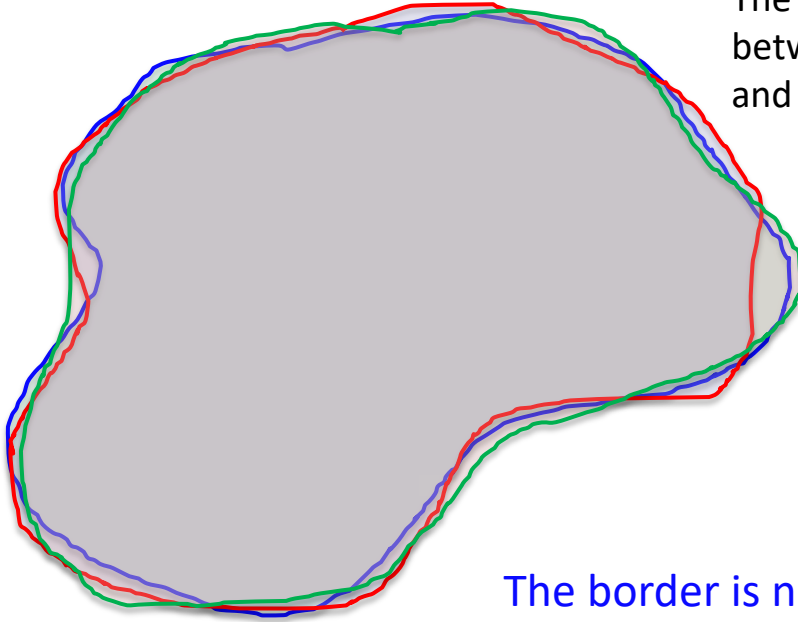




- ▷ Span the set
- ▷ Convexity
- ▷ Sweep the border
- ▷ **Monotonicity**



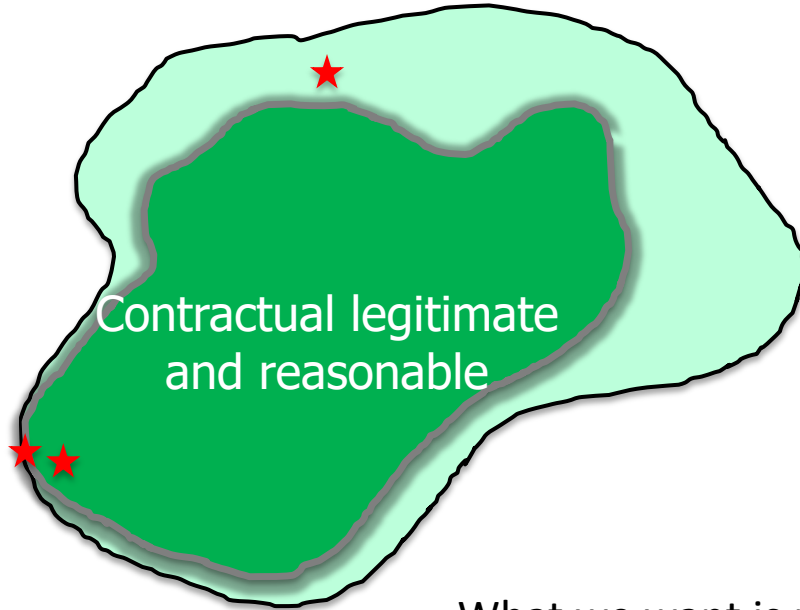




The border is the frontier between technical feasible and technical infeasible.

The border is not unique

- ▷ different **Models/Methods**
- ▷ different **Parameters**
- ▷ different Implementations



What we want is not the contractual legitimate area, but the **contractual legitimate** and **reasonable** area.

**This area is not well defined.**

# Shit!

[www.kika.de](http://www.kika.de)



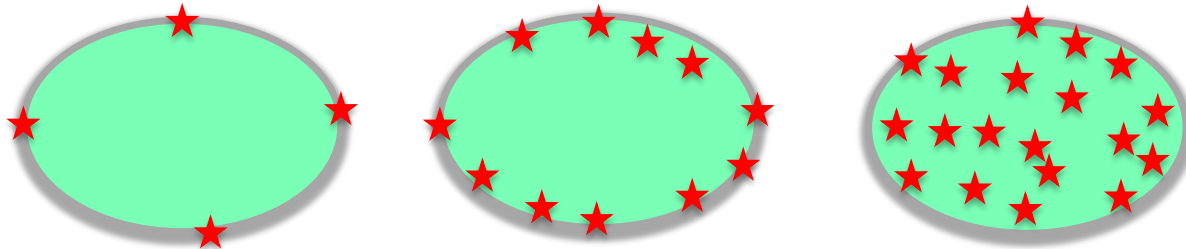
Thorsten Koch



**We are able to generate scenarios and test their technical feasibility**  
(with some limitations).

## Generation of scenarios:

- ▷ Manual generation of **expert scenarios**
  - limited number,
  - + high quality
- ▷ Automatic generation of **extreme value scenarios**
  - difficult to distinguish between contractual legitimate and reasonable,
  - + high number
- ▷ Automatic generation of **stochastic scenarios**
  - seldom extreme,
  - + reasonable, + high number, + probable distribution



## Advantages:

- ▶ **High Quality**
- ▶ Very well understood

## Problems:

- ▶ Risk of being too much tuned towards a particular model, parameter setting, or implementation  
*„let us see how far we can drive this“*

## Disadvantages:

- ▶ **Small number**
- ▶ Complete coverage of the feasible set is hard to ensure
- ▶ Only extreme value scenarios, i.e., the probability for the scenarios to happen in reality is nearly zero

## Advantages:

- ▶ **High number**
- ▶ Can be generated with different methods

## Problems:

- ▶ **Hard to detect *contractual legitimate but unreasonable***

## Disadvantages:

- ▶ Only the border of the feasible set is covered
- ▶ Only extreme value scenarios, i.e. the probability for the scenarios to happen in reality is nearly zero

## Advantages:

- ▶ High number
- ▶ Can be generated with different methods
- ▶ Realistic
- ▶ Coverage of the inner part of the feasible set
- ▶ Can be attributed with probabilities

## Problems:

- ▶ Methods for generation are involved and need much data
- ▶ Refer to the past

## Disadvantages:

- ▶ Extreme values occur seldom since they are not probable to happen

- ▶ **If all three methods are combined, there are nearly no disadvantages left.**
- ▶ This may lead to a huge number of scenarios of which many are possibly similar.
- ▶ This can be countered by methods for scenario reduction.
- ▶ **Testing the technical feasibility of the scenarios has necessarily to happen automatically.**
- ▶ This can be troublesome regarding extremal value scenarios.
- ▶ **Now it is possible to automatically validate scenarios.**
- ▶ This can be extended to compute capacities.
- ▶ It is possible to extend these concepts to answer more sophisticated questions, e.g. looking at probabilities for buy back happening when overbooking.

## Simulation:

- ▶ allows very accurate gas physics models
- ▶ Relies on human experience to decide feasibility
- ▶ Therefore cannot determine infeasibility

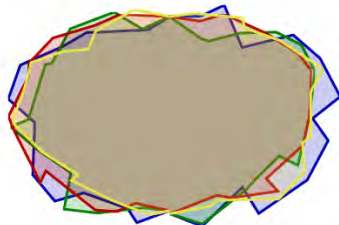
## Optimization:

- ▶ Works on simplified models of gas physics
- ▶ Automatically finds settings for active elements
- ▶ Eventually can prove infeasibility of a scenario

**Beware: different solution spaces due to different modeling**

simulation A

simulation B



optimization A

optimization B



Zuse-Institute Berlin  
Mathematical Optimization Methods  
Prof. Dr. Thorsten Koch



Technische Universität Darmstadt  
Discrete Optimization  
Prof. Dr. Marc Pfetsch



Humboldt-Universität zu Berlin  
Department of Mathematics  
Prof. Dr. Werner Römisch



Weierstraß-Institut für Angewandte  
Analysis und Stochastik (WIAS)  
Nonlinear Optimization and Inverse  
Problems  
PD Dr. René Henrion



BMWi  
Projekträger Jülich  
Energy Business Area



develOPT GmbH  
Big Data Optimization  
Dr. Antonio Morsi, Dr. Björn Geißler



Friedrich-Alexander Universität  
Erlangen-Nürnberg  
Economics · Discrete Optimization ·  
Mathematics  
Prof. Dr. Alexander Martin



Universität Duisburg-Essen  
Department of Mathematics  
Prof. Dr. Rüdiger Schultz



Leibniz-Universität Hannover  
Institute for Applied Mathematics  
Prof. Dr. Marc Steinbach



Open Grid Europe GmbH  
Netzplanung und -steuerung /  
Netzoptimierung  
Klaus Spreckelsen



atesio GmbH  
Dr. Andreas Eisenblätter,  
Dr. Benjamin Hiller

## Goal of the project:

Build an automatic system that given a network and a set of supplies and demands at the border points computes settings for the active elements of the network such that the resulting stationary scenario is feasible.





# Nova Progress

Start of project



Subject: E.ON  
 Date: 19.05.2008 14:14  
 To: <koch@zib.de>

Dear Thorsten,

Here is the phone number of Mr. xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx  
 Would you please call him back!

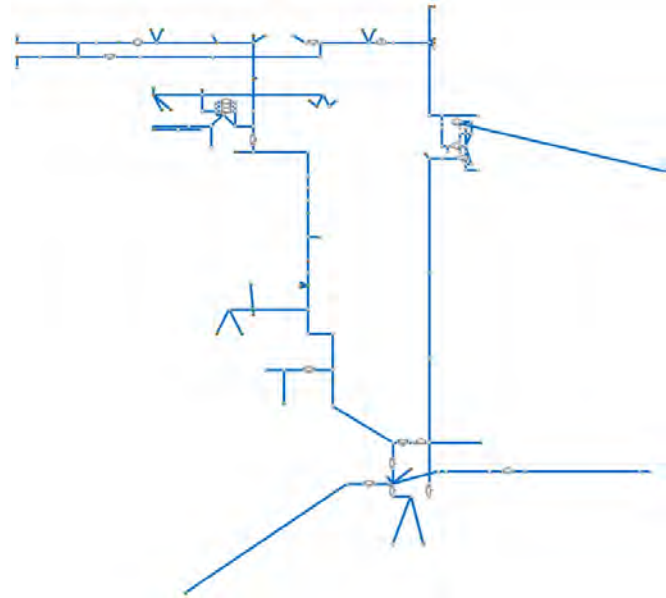
Best,  
 Bettina



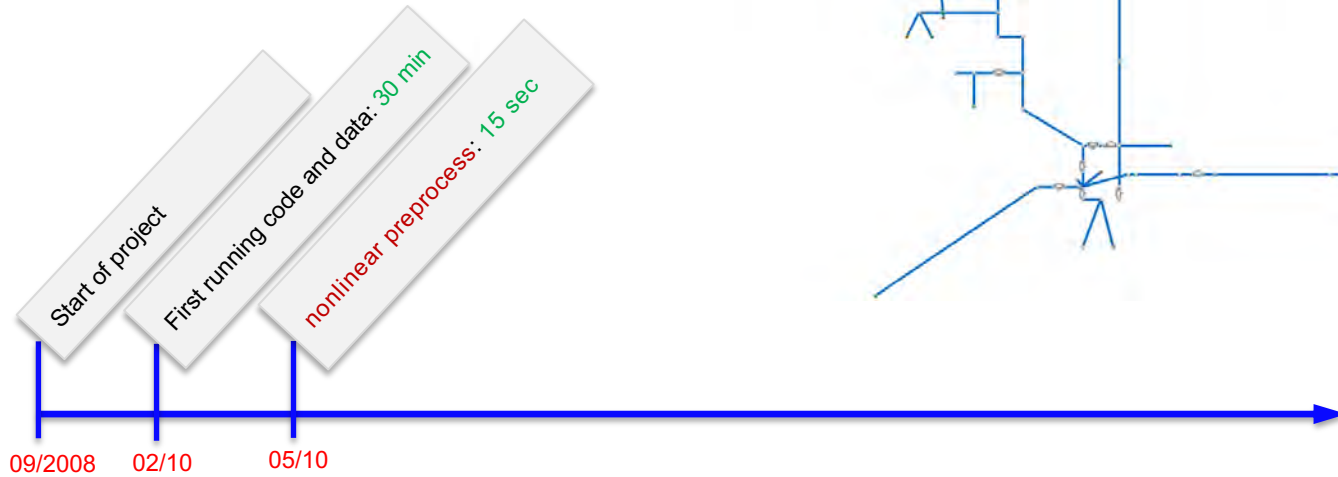
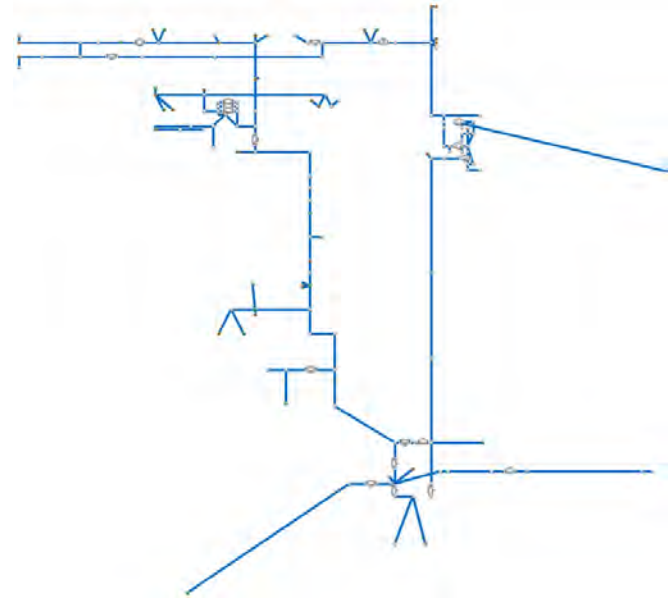
09/2008

07/2016

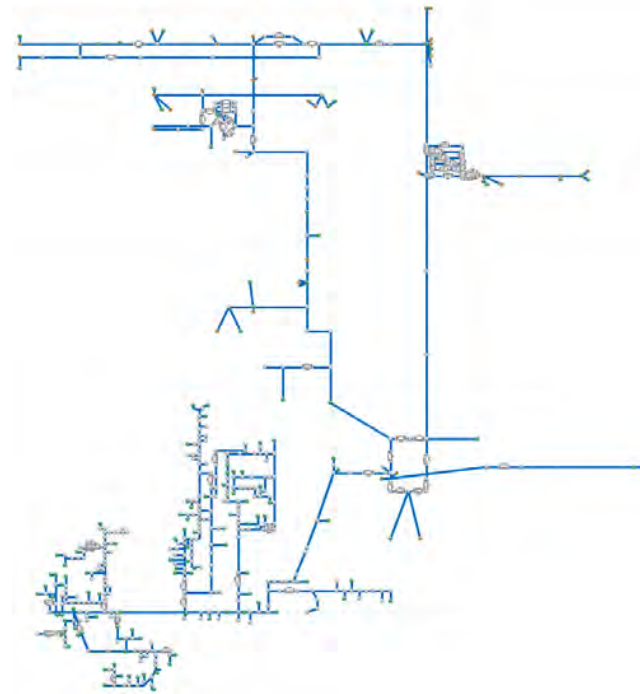
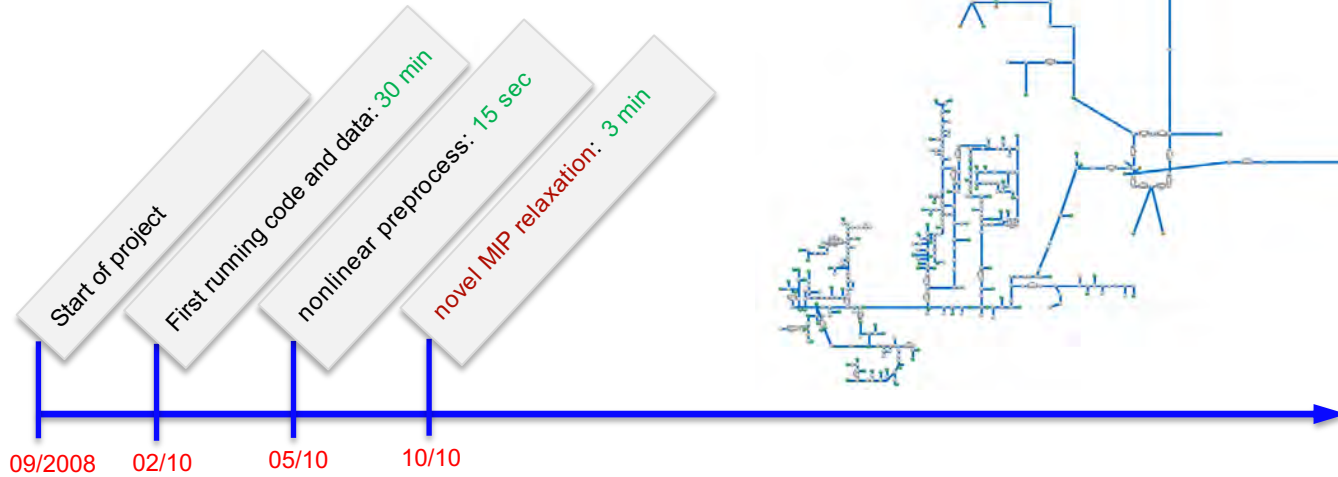
| Element           | Count |
|-------------------|-------|
| Pipes             | 136   |
| Compressor groups | 3     |
| Resistors         | 8     |
| Control valves    | 7     |
| Valves            | 1     |
| Entries           | 26    |
| Exits             | 14    |



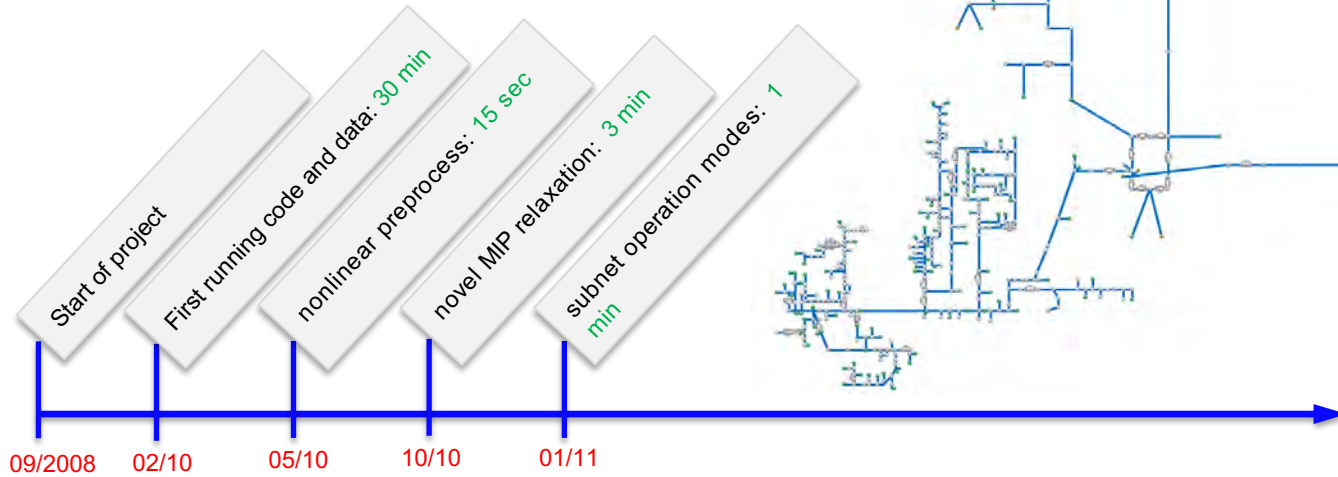
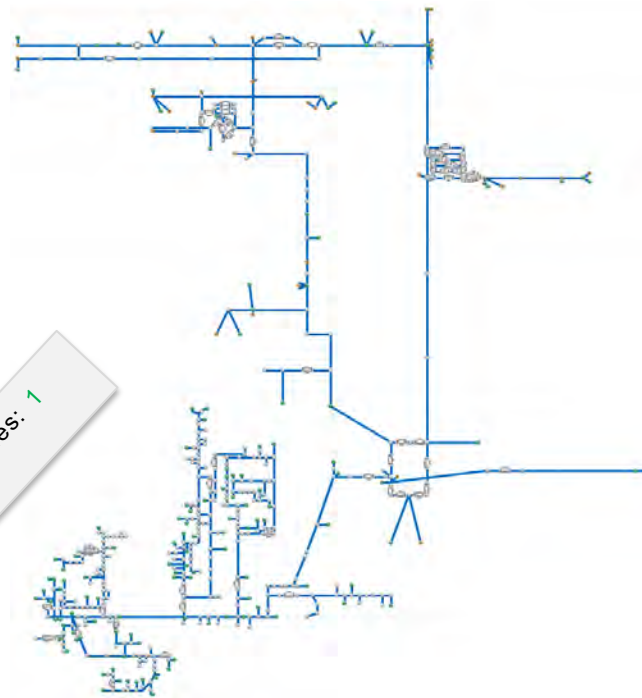
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| Entries           | 26    |
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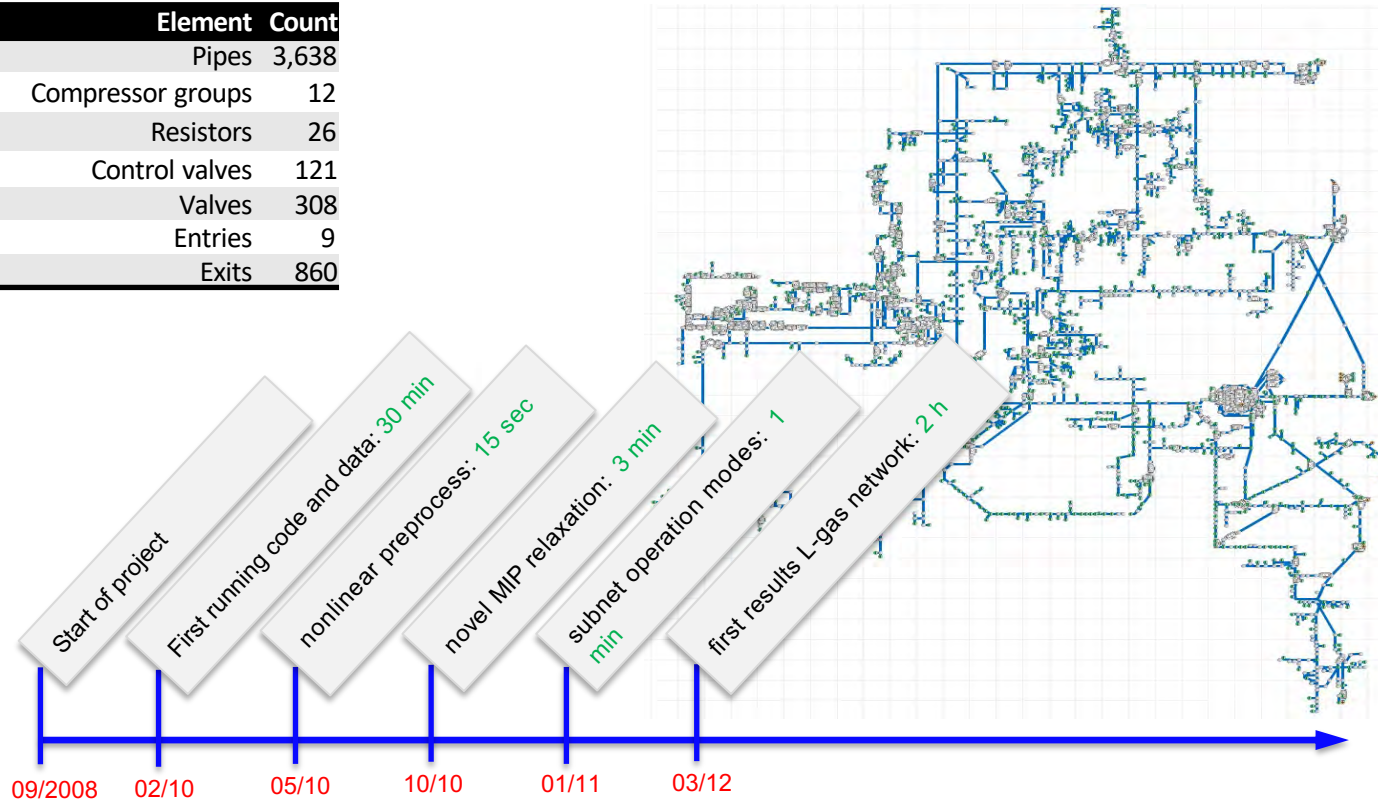
| Element           | Count |
|-------------------|-------|
| Pipes             | 470   |
| Compressor groups | 6     |
| Resistors         | 8     |
| Control valves    | 23    |
| Valves            | 34    |
| Entries           | 26    |
| Exits             | 87    |



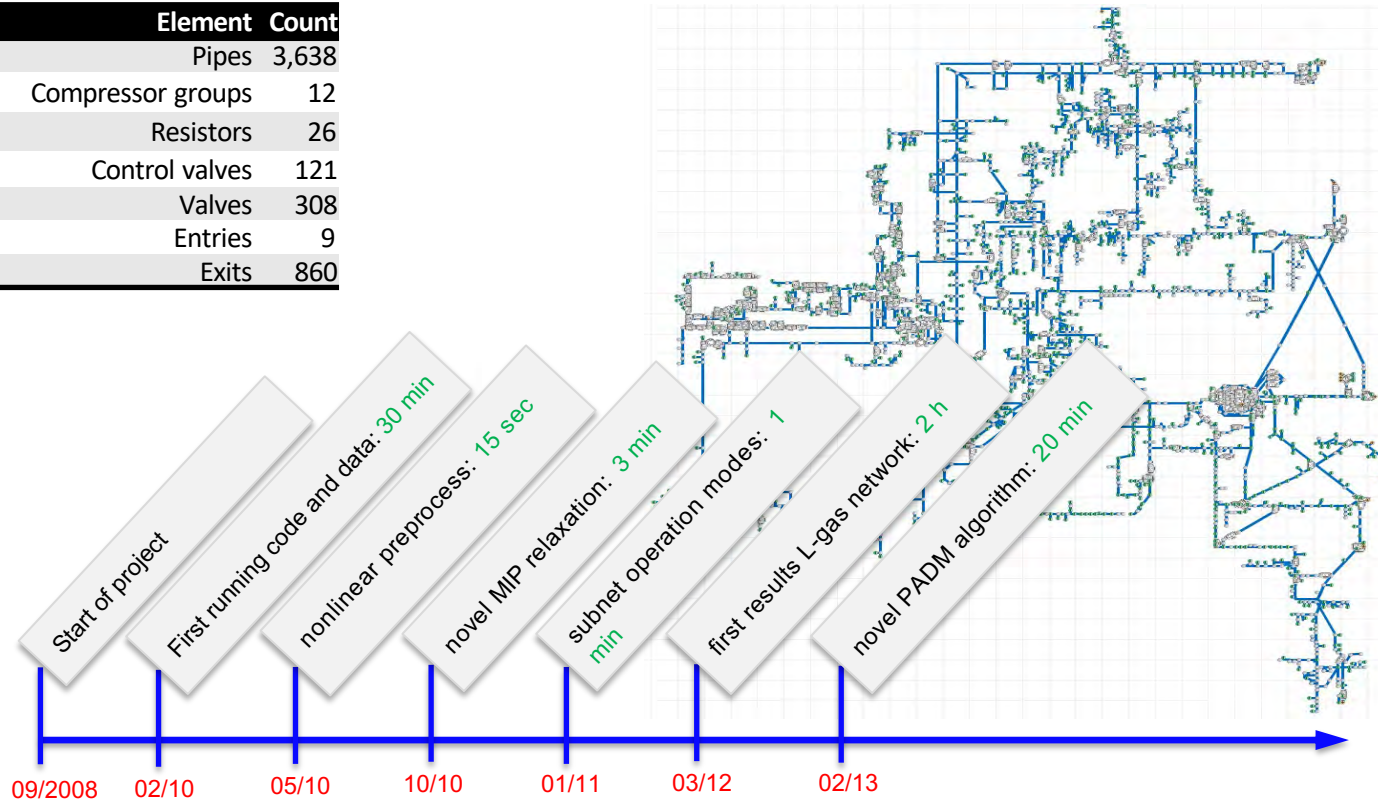
| Element           | Count |
|-------------------|-------|
| Pipes             | 470   |
| Compressor groups | 6     |
| Resistors         | 8     |
| Control valves    | 23    |
| Valves            | 34    |
| Entries           | 26    |
| Exits             | 87    |



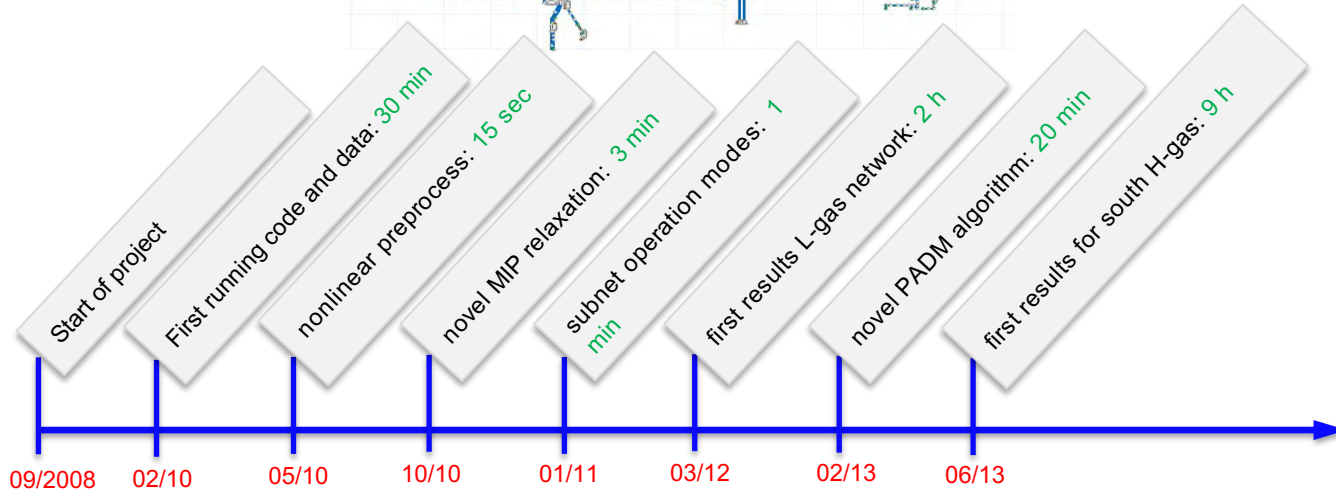
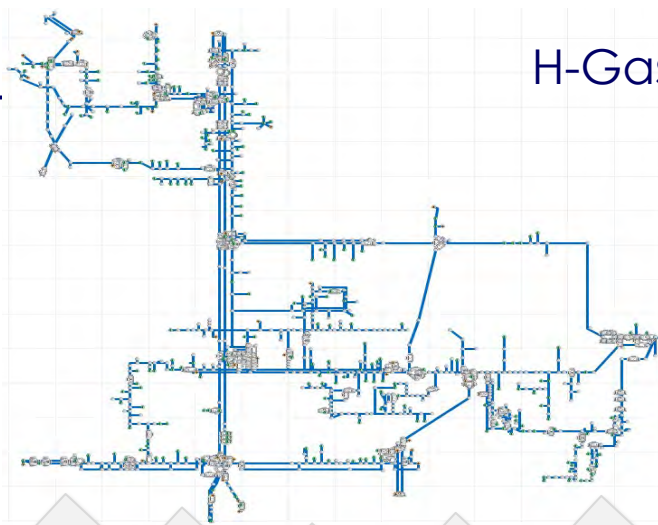
| Element           | Count |
|-------------------|-------|
| Pipes             | 3,638 |
| Compressor groups | 12    |
| Resistors         | 26    |
| Control valves    | 121   |
| Valves            | 308   |
| Entries           | 9     |
| Exits             | 860   |



| Element           | Count |
|-------------------|-------|
| Pipes             | 3,638 |
| Compressor groups | 12    |
| Resistors         | 26    |
| Control valves    | 121   |
| Valves            | 308   |
| Entries           | 9     |
| Exits             | 860   |

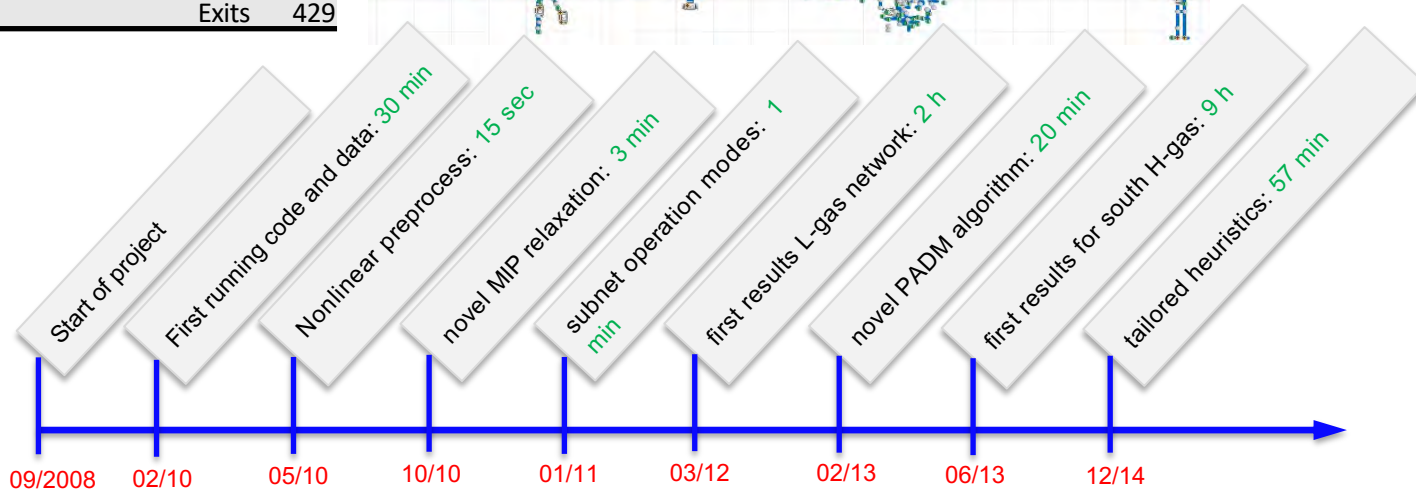
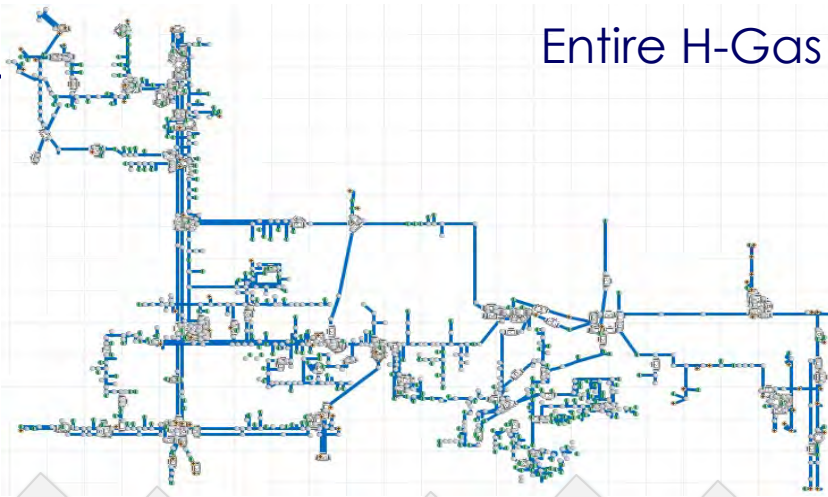


| Element           | Count |
|-------------------|-------|
| Pipes             | 1,218 |
| Compressor groups | 34    |
| Resistors         | 47    |
| Control valves    | 115   |
| Valves            | 473   |
| Entries           | 17    |
| Exits             | 286   |



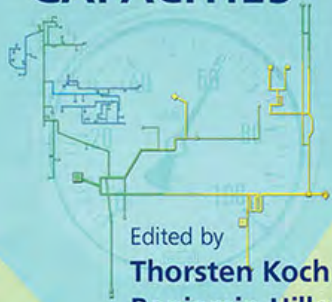


| Element           | Count |
|-------------------|-------|
| Pipes             | 1,747 |
| Compressor groups | 41    |
| Resistors         | 85    |
| Control valves    | 145   |
| Valves            | 545   |
| Entries           | 45    |
| Exits             | 429   |





## EVALUATING GAS NETWORK CAPACITIES



Edited by  
**Thorsten Koch**  
**Benjamin Hiller**  
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**Lars Schewe**

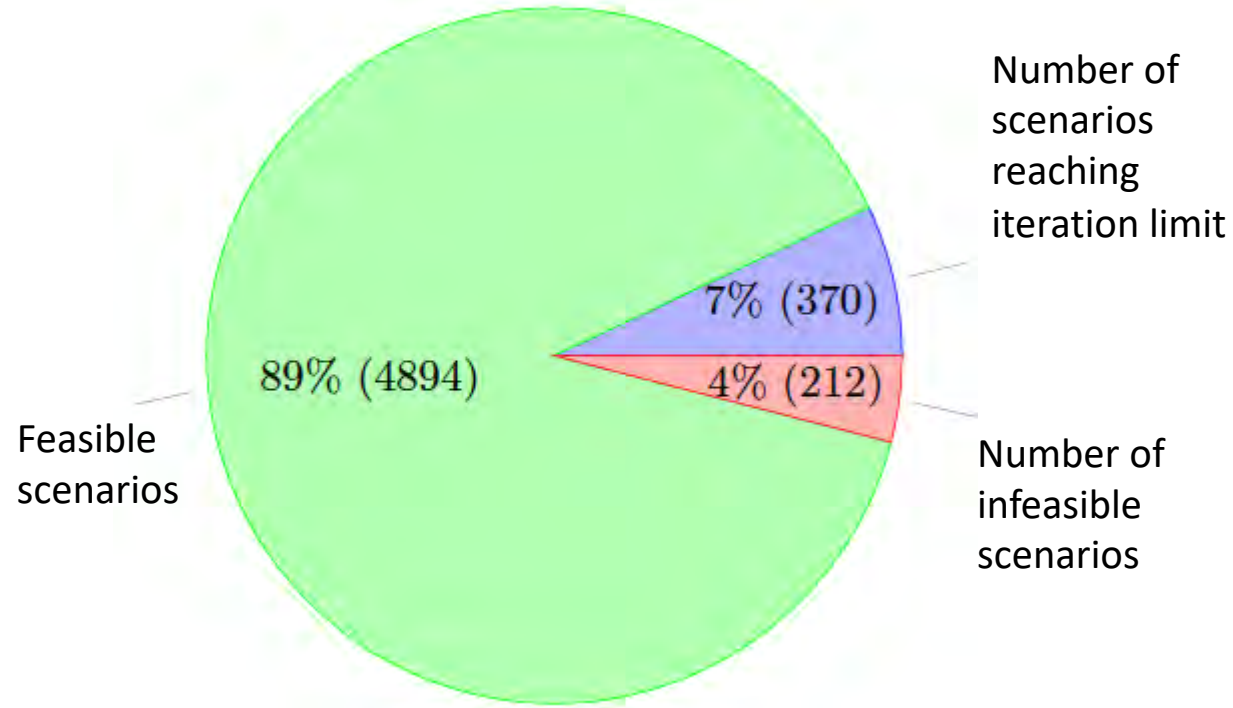
MOS-SIAM Series on Optimization

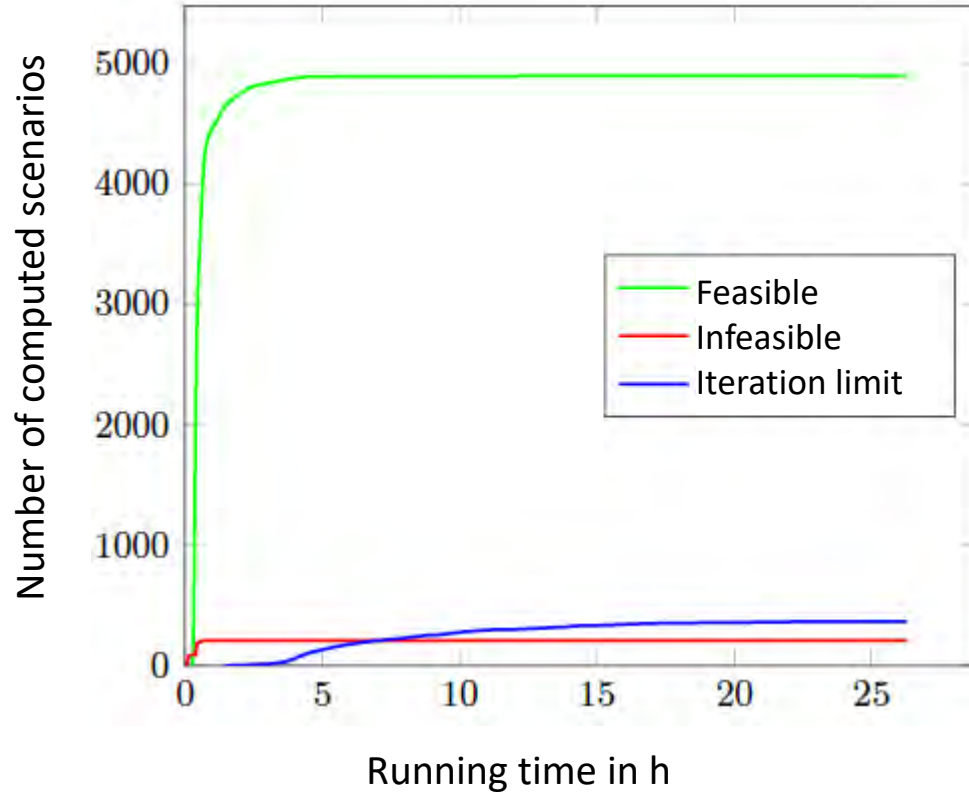
### The **Research Cooperation Network Optimization ForNe**

ran for 6 years and involved more than 30 people from around 10 universities and institutes along with more than 10 employees from Germany's largest gas network system operator OGE.

← Here are the results.







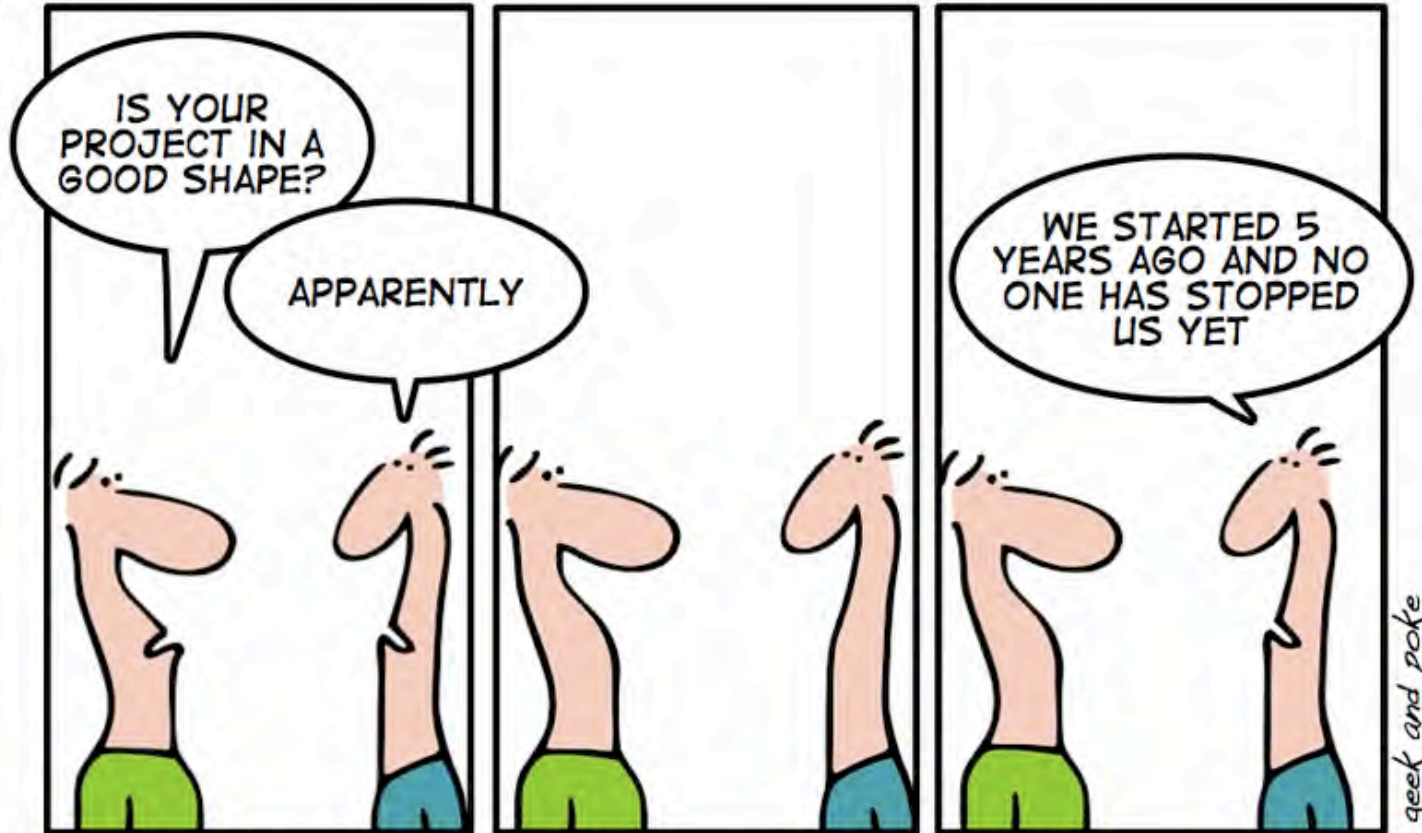
- ▶ Technology is changing all the time, new systems mix with existing ones
- ▶ **Often hard to determine actual system limits**  
(many assumptions are necessary for optimization)
- ▶ Optimization drives errors to cluster in one direction
- ▶ For larger models/scenarios hard to check against reality  
(also very hard to find data errors/inconsistencies)
- ▶ Marketization makes it difficult to predict behavior of participants, since the objectives of the individual players are unknown, and the outcome is result of a game (negotiation).

Relevant real-world questions can be tackled efficiently with mathematical optimization and algorithmic intelligence.

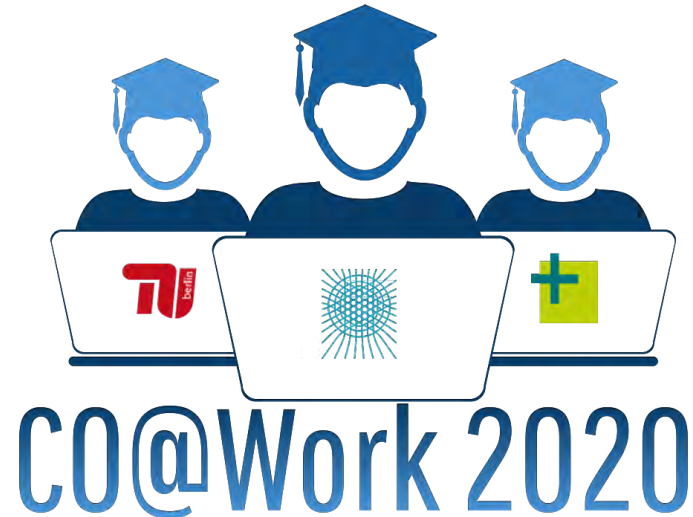
## However,

- ▶ a substantial effort is needed to succeed,
- ▶ the setup cost is high compared to pure research,
- ▶ close cooperation with practitioners is indispensable,
- ▶ different disciplines need to collaborate,
- ▶ access to and curation of data is essential.

Good luck!



# Thank you very much





# Please continue with the lecture on Gas Network Control

