Constraint Integer Programming

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Outline

Constraint Integer Programming

SCIP's Design

The Solving Process of SCIP

http://scipopt.org

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What is a Constraint Integer Program?

Constraint Integer Program

Objective function:

- linear function

Feasible set:

- described by arbitrary constraints

Variable domains:

- real or integer values

Restriction:

- When all integer variables are fixed, remaining subproblem is LP or NLP

$$
\begin{aligned}
\min \quad & c^T x \\
s.t. \quad & x \in F \\
& (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C
\end{aligned}
$$

Remark:

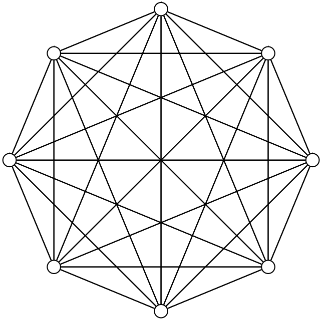
- arbitrary objective or variables modeled by constraints

An Example: the Traveling Salesman Problem

Definition (TSP)

Given a complete graph $G = (V, E)$ and distances d_e for all $e \in E$:

Find a Hamiltonian cycle (cycle containing all nodes, tour) of minimum length.



K_8

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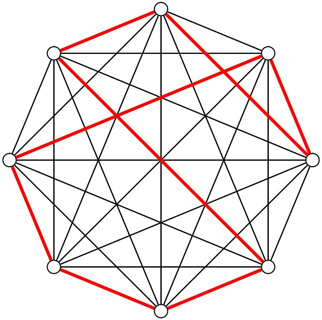
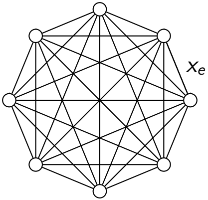


K_8

TSP – Integer Programming Formulation



K_8

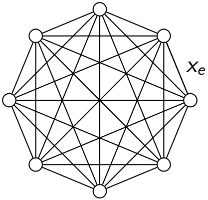
Given

- complete graph $G = (V, E)$
- distances $d_e > 0$ for all $e \in E$

Binary variables

- $x_e = 1$ if edge e is used

TSP – Integer Programming Formulation



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$$
\begin{aligned}
\min \quad & \sum_{e \in E} d_e \, x_e & \\
\text{subject to} \quad & \sum_{e \in \delta(v)} x_e = 2 & \forall v \in V \\
& \sum_{e \in \delta(S)} x_e \geq 2 & \forall S \subset V, S \neq \varnothing \\
& x_e \in \{0, 1\} & \forall e \in E
\end{aligned}
$$

TSP – Integer Programming Formulation

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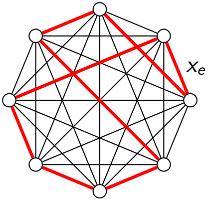

Binary variables

- $x_e = 1$ if edge e is used

$$
\begin{array}{llll}
\min & \sum_{e \in E} d_e \, x_e & & \\
\text{subject to} & \sum_{e \in \delta(v)} x_e = 2 & \forall v \in V & \text{node degree} \\
& \sum_{e \in \delta(S)} x_e \geq 2 & \forall S \subset V, S \neq \varnothing & \\
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\end{array}
$$

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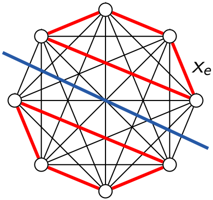

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$$\begin{aligned}
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& \sum_{e \in \delta(S)} x_e \geq 2 && \forall S \subset V, S \neq \emptyset \quad \text{subtour elimination} \\
& x_e \in \{0, 1\} && \forall e \in E
\end{aligned}$$

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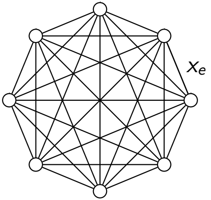

K_8

Binary variables

- $x_e = 1$ if edge e is used

$$\min \sum_{e \in E} d_e \, x_e \qquad \text{distance}$$

$$
\begin{aligned}
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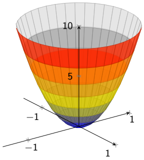
s.t. & \sum_{e \in \delta(v)} x_e & = 2 & \forall \, v \in V \\
 & \text{nosubtour}(x) & & \\
 & x_e & \in \{0, 1\} & \forall \, e \in E
\end{array}
$$

(CIP formulation of TSP)

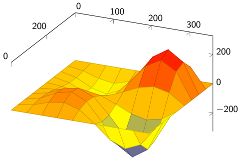
Single nosubtour constraint rules out subtours (e.g. by domain propagation). It may also separate subtour elimination inequalities.

$$\begin{aligned} \min\ & c^{\mathsf{T}} x \\ \text{s.t. } & g_k(x) \leq 0 && \forall k \in [m] \\ & x_i \in \mathbb{Z} && \forall i \in \mathcal{I} \subseteq [n] \\ & x_i \in [\ell_i, u_i] && \forall i \in [n] \end{aligned}$$

The functions $g_k \in C^1([\ell, u], \mathbb{R})$ can be

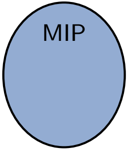


convex or



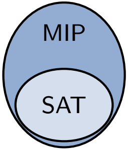
nonconvex

- Mixed Integer Programs



Constraint Integer Programming

- Mixed Integer Programs
- SATisfiability problems



Constraint Integer Programming

- Mixed Integer Programs
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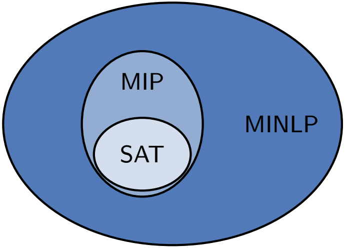
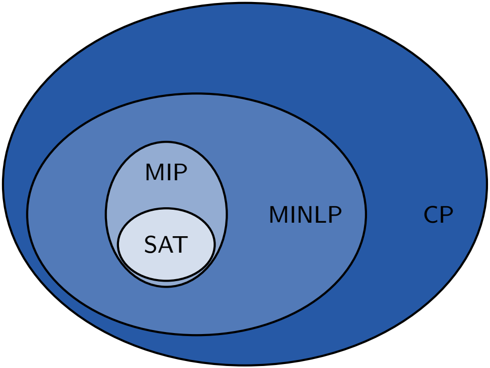
- Mixed Integer Nonlinear Programs



Constraint Integer Programming

- Mixed Integer Programs
- SATisfiability problems
- Mixed Integer Nonlinear Programs
- Constraint Programming



Constraint Integer Programming

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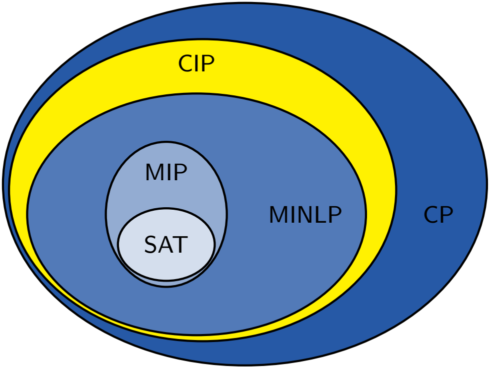

Quiz - Part I

- What is the key restriction that defines a CIP and why is it necessary?

- Think of a different problem that could benefit from a formulation as a CIP

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  Possibilities: Linear ordering problem, Steiner tree problem, scheduling, ...

Outline

Constraint Integer Programming

SCIP's Design

The Solving Process of SCIP

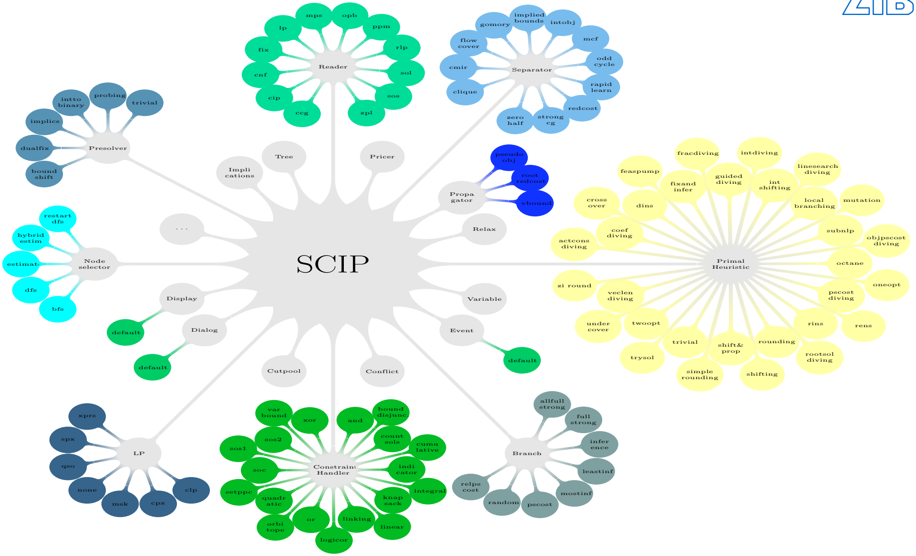
http://scipopt.org

Different Tasks – Different Plugins

Different plugin classes are responsible of the following tasks.

1. Presolving and node propagation
   - **Constraint handlers**
   - Presolvers
   - Propagators
2. Separation
   - **Constraint handlers**
   - Separators
3. Improving solutions
   - Primal heuristics
4. Branching
   - **Constraint handlers**
   - Branching rules
5. Node selection
   - Node selectors

Structure of SCIP



Plugin based design

SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- ...

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Plugins

- **external callback objects**
- interact with the framework through a **very detailed interface**

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Plugins

- **external callback objects**
- interact with the framework through a **very detailed interface**
- SCIP knows for each plugin type:
  - the number of available plugins
  - priority defining the calling order (usually)
- SCIP does not know any structure behind a plugin

$\Rightarrow$ **plugins are black boxes** for the SCIP core

Plugin based design

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$\Rightarrow$ **plugins are black boxes** for the SCIP core

$\Rightarrow$ **Very flexible branch-and-bound based search algorithm**

Constraint Handlers

Constraint handlers

- most powerful plugins in SCIP
- define the feasible region
- a single constraint may represent a whole set of inequalities

Functions

- check and enforce feasibility of solutions
- can add linear representation to LP relaxation
- constraint-specific presolving, domain propagation, separation

Result

- SCIP is constraint based
  - Advantage: flexibility
  - Disadvantage: limited global view: A constraint knows its variables but a variable does not know the constraints it appears in.
  - LP relaxation is an add-in and handled by external software via LP interface.

Types of Plugins

- Constraint handler: assures feasibility, strengthens formulation
- Separator: adds cuts, improves dual bound
- Pricer: allows dynamic generation of variables
- Heuristic: searches solutions, improves primal bound
- Branching rule: how to divide the problem?
- Node selection: which subproblem should be regarded next?
- Presolver: simplifies the problem in advance, strengthens structure
- Propagator: simplifies problem, improves dual bound locally
- Reader: reads problems from different formats
- Event handler: catches events (e.g., bound changes, new solutions)
- Display: allows modification of output
- Relaxation handlers: custom relaxations
- ...

What does SCIP know about plugins?

- interactive shell shows the information SCIP has for a plugin type

```
SCIP> display {branching | conshdlrs | heuristics | ...}
```

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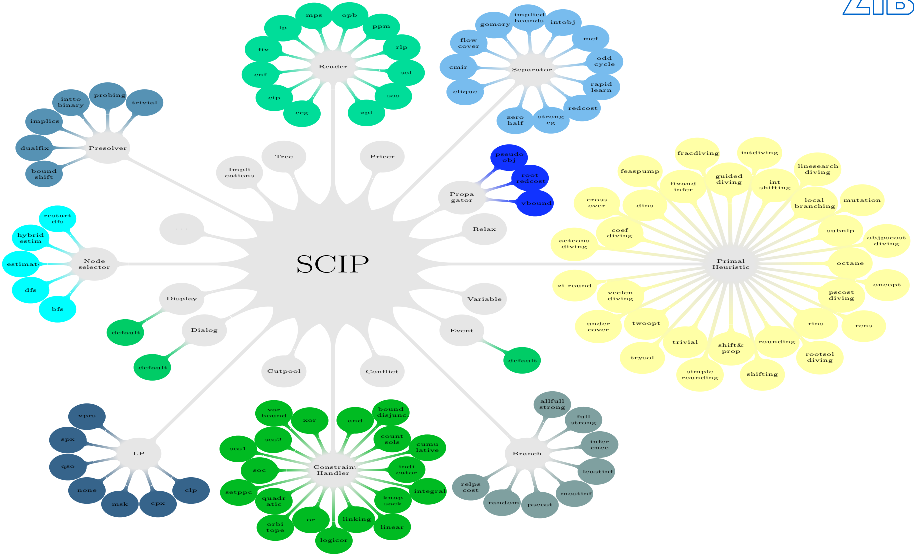
```

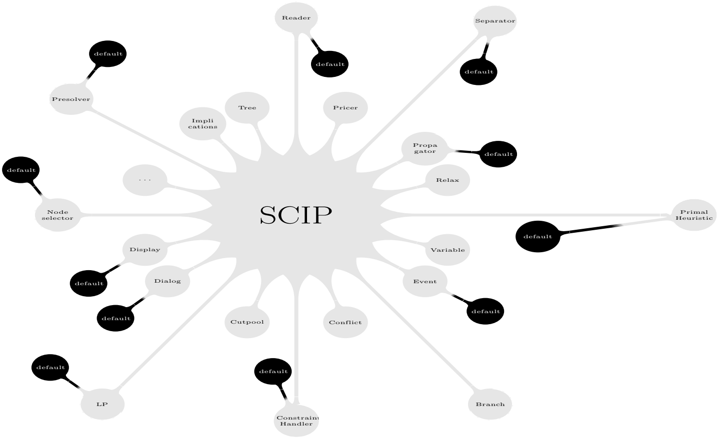
```
SCIP> display branching

branching rule priority maxdepth maxbddist description
-------------- -------- -------- --------- -----------
relpscost         10000       -1    100.0% reliability branching on pseudo cost values
pscost             2000       -1    100.0% branching on pseudo cost values
inference          1000       -1    100.0% inference history branching
mostinf             100       -1    100.0% most infeasible branching
leastinf             50       -1    100.0% least infeasible branching
distribution          0       -1    100.0% branching rule based on variable influence on cumu
fullstrong            0       -1    100.0% full strong branching
cloud                 0       -1    100.0% branching rule that considers several alternative
allfullstrong     -1000       -1    100.0% all variables full strong branching
random          -100000       -1    100.0% random variable branching
```

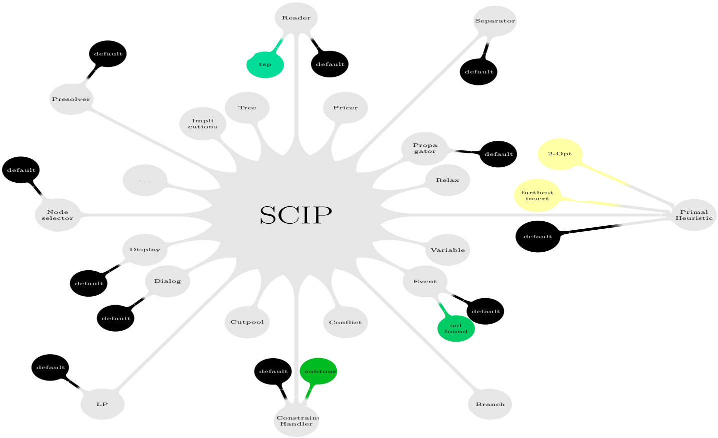
Extending SCIP



Extending SCIP



Extending SCIP: TSP



Summary

Plugins for the SCIP core

- are black boxes for SCIP
- perform all problem specific actions
- interact through a **very detailed interface**

Plugins

- can have private data
- can access global information of the SCIP core
- should not access data of other plugins

Result

- SCIP is **constraint based**
- new types of constraints can easily be defined
- easy to add problem specific algorithms

Advantages and Disadvantages of SCIP

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- Can be used as a black box MIP solver

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- **Extendable in any direction since constraint based**
  - Adding global constraints
  - Adding problem specific plugins

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- Hundreds of parameters to play with
- Robust, fast, and well documented
- **Extendable in any direction since constraint based**
  - Adding global constraints
  - Adding problem specific plugins

Disadvantages

- Hundreds of parameters to play with
- In general variables do not know in which constraint they appear since constraint based

Quiz - Part II

- Think of 3 different types of plugins within SCIP
- Why can't plugins directly communicate with each other?
- Recall your own CIP formulation from Quiz - Part I.
  What plugin types would you need for it?

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  Constraint handler, event handler, separator, reader, ...

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  Very likely: Constraint handler, reader, heuristic

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SCIP Interactive Shell Basics

Basic Workflow

```
read ../check/instances/MIP/bell5.mps
optimize
write solution mysolution.sol
quit
```

Displaying information

Use the `display ...` command to enter the menu and

- obtain `solution` information
- print the current `transproblem` to the console
- display plugin information, e.g., list all available branching rules

Changing Settings

Use the `set ...` command to list the settings menu.

Important Parameters

Numerical parameters

These must be set **before** reading a problem.

- `numerics/feastol`, default 10^{-6}
- `numerics/epsilon`, default 10^{-9}
- `numerics/infinity`, default 10^{20}

Limits

- `limits/time`
- `limits/nodes`
- `limits/gap`
- ...

Randomization

- `randomization/randomseedshift`
- `randomization/lpseed`
- `randomization/permutationseed`

Interfaces to SCIP

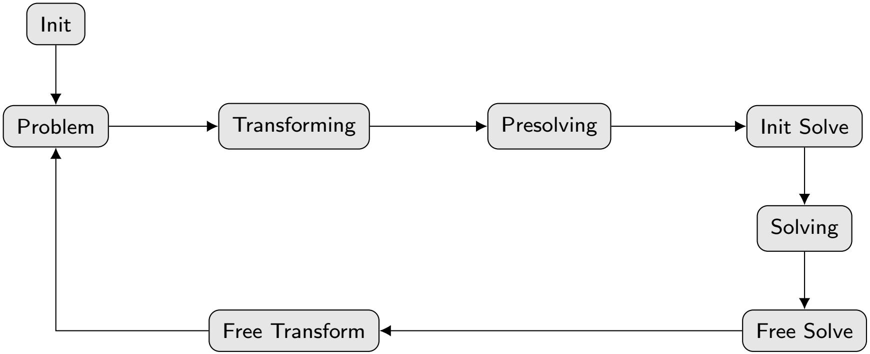
- interactive shell supports 11 different input formats
  → cip, cnf, flatzinc, rlp, lp, mps, opb, pip, wbo, zimpl, smps
- C API/callable library
- C++ wrapper classes
- Python interface
  - New Dockerized SCIP
  - conda integration coming soon
- Java JNI interface
- Julia interface
- AMPL
- GAMS
- Matlab (see also OPTI toolbox, http://www.i2c2.aut.ac.nz/Wiki/OPTI/)

Getting help

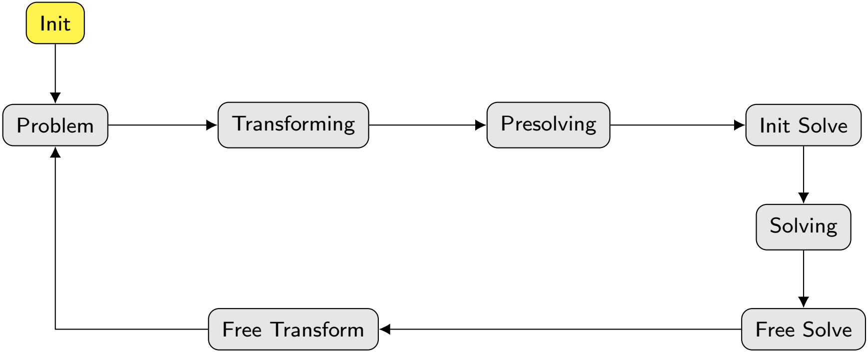
If you should ever get stuck, you can . . .

1. type `help` in the interactive shell
2. read the documentation `http://scipopt.org/doc/html`
   → FAQ, HowTos for each plugin type, debugging, automatic testing, . . .
3. search or post on Stack Overflow using the tag `scip` (more than 100 questions already answered)
4. active mailing list scip@zib.de (350+ members)
   - search the mailing list archive (append site:listserv/pipermail/scip)
   - register `http://listserv.zib.de/mailman/listinfo/scip/` and post

Operational Stages

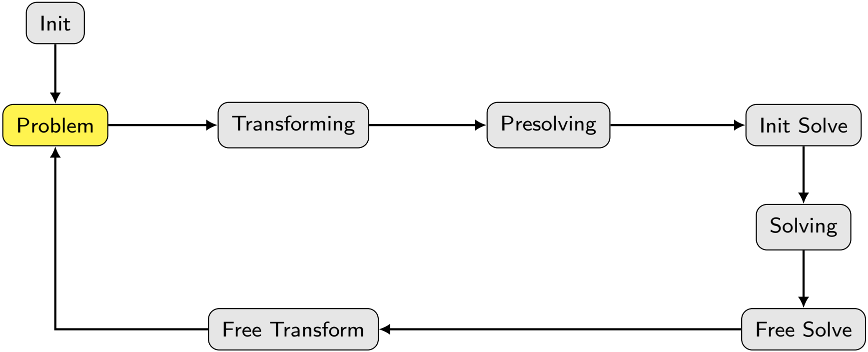


Operational Stages



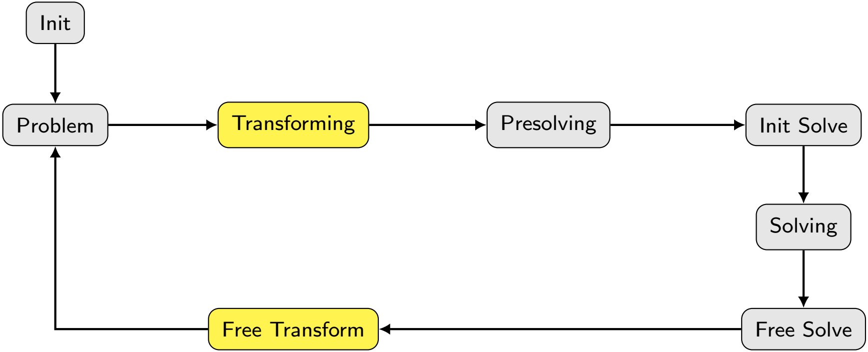
- Basic data structures are allocated and initialized.
- User includes required plugins (or just takes default plugins).

Problem Specification



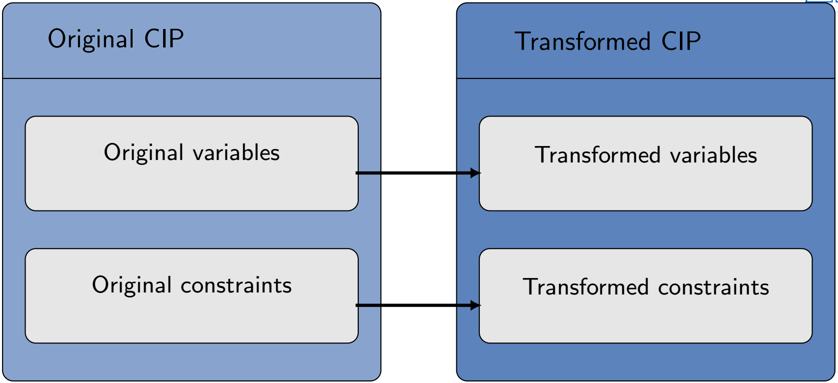
- User creates and modifies the original problem instance.
- Problem creation is usually done in file readers.

Transformation



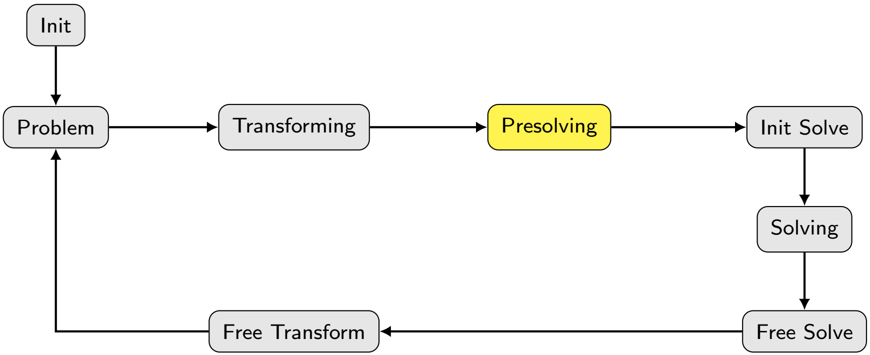
- Creates a working copy of the original problem.

Original and Transformed Problem



- data is copied into separate memory area
- presolving and solving operate on transformed problem
- original data can only be modified in problem modification stage

Presolving


Init
Problem
Transforming
Presolving
Init Solve
Solving
Free Solve
Free Transform

Presolving Tips and Parameters

Use `display presolvers` to **list** all presolvers of SCIP.

Disable Presolving

Disable all presolving for a model

```
set presolving emphasis off
```

Deactivate single techniques

```
set presolving tworowbnd maxrounds 0
set propagating probing maxprerounds 0
set constraints components advanced maxprerounds 0
```

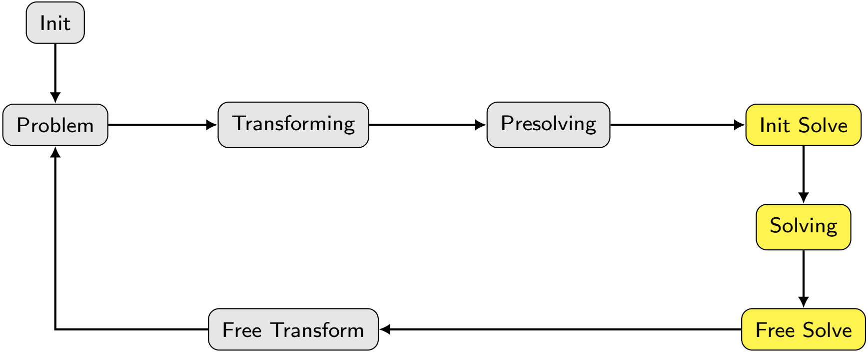
Aggressive Presolving

```
set presolving emphasis aggressive
```

General Rule of Thumb

Only deactivate single presolving techniques if you encounter performance problems.

Solving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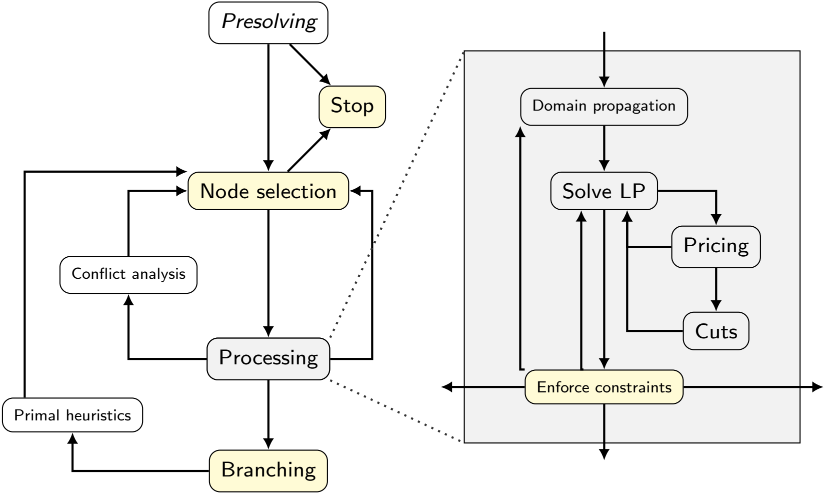
Flow Chart SCIP



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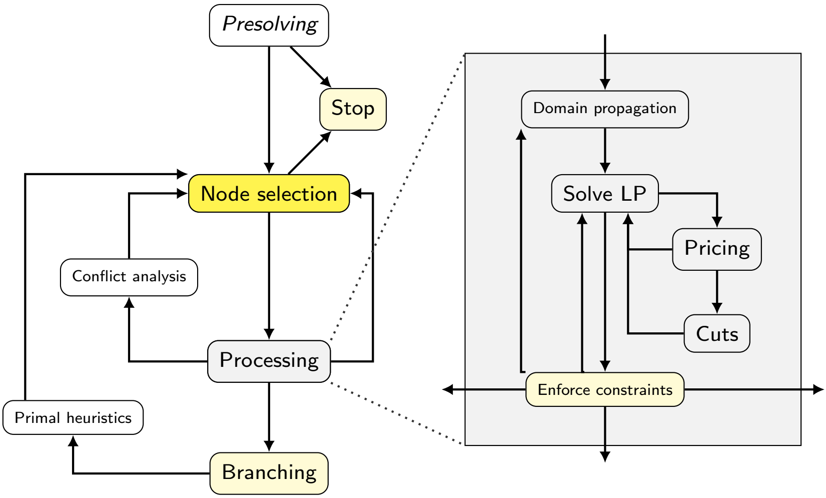


Node Selection Tips and Parameters

Available Node Selectors

```
display nodeselectors

node selector std priority memsave prio  description
------------- ------------ ------------  -----------
estimate            200000          100  best estimate search
bfs                 100000            0  best first search
...
dfs                      0       100000  depth first search
```

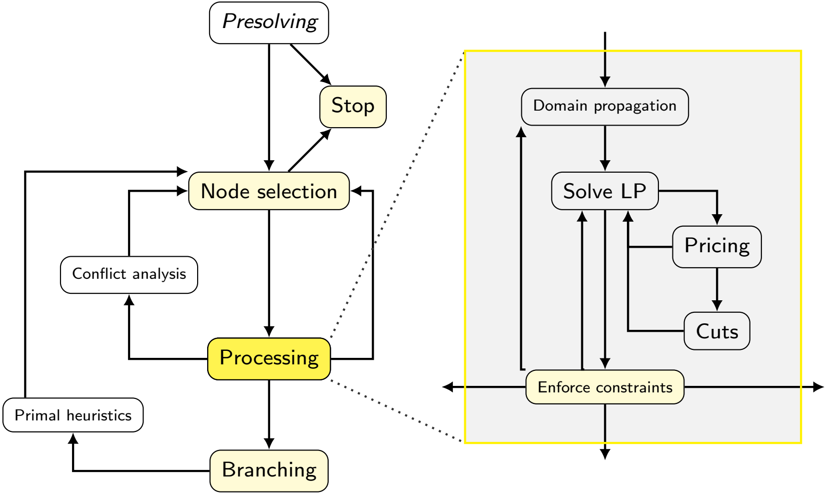
Switching Node Selectors

Only the node selector with highest standard priority is active. Use

```
set nodeselection dfs stdpriority 1000000
```

to activate depth first search also in non-memsave mode.

Flow Chart SCIP



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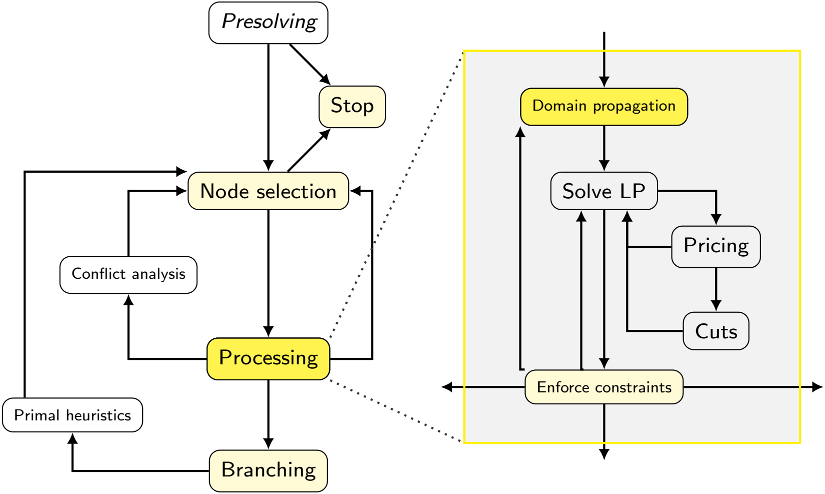
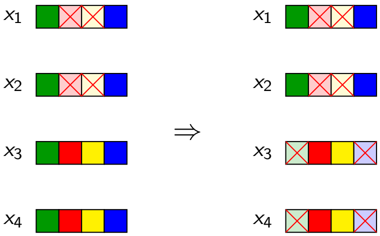


Domain Propagation



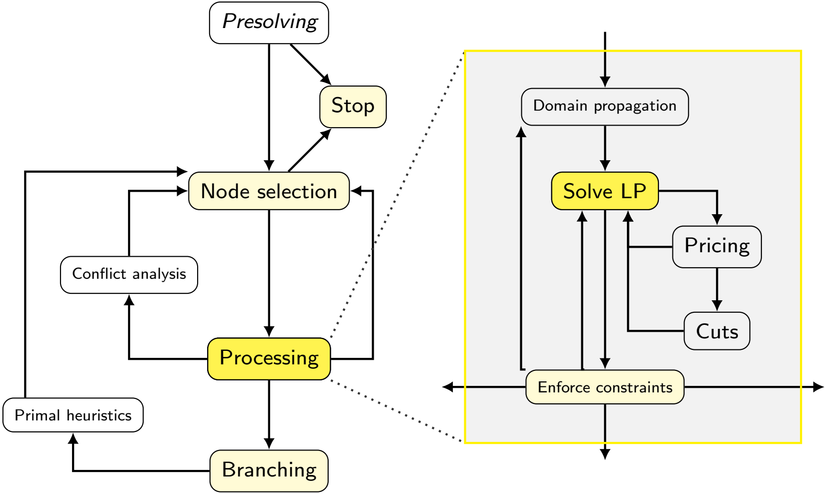
Task

- simplify model locally
- improve local dual bound
- detect infeasibility

Techniques

- constraint specific
  - each cons handler may provide a propagation routine
  - reduced presolving (usually)
- dual propagation
  - root reduced cost strengthening
  - objective function
- special structures
  - variable bounds

Flow Chart SCIP



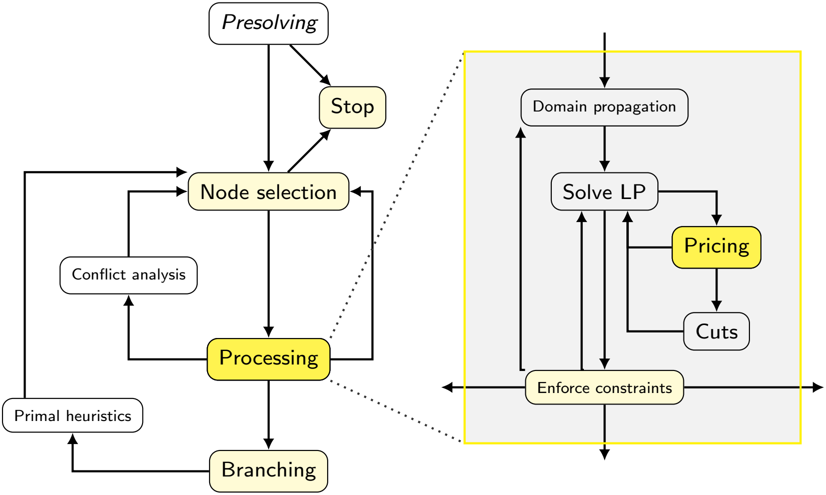
LP Solving Tips and Parameters

Most Important LP Parameters

- `lp/initalgorithm`, `lp/resolvealgorithm`
  - Primal/Dual Simplex Algorithm
  - Barrier w and w/o crossover
- `lp/pricing`
  - normally LP solver specific default
  - Devex
  - Steepest edge
  - Quick start steepest edge
- `lp/threads`

Slow LP performance is a blocker for the solving process and can sometimes be manually tuned significantly.

Flow Chart SCIP



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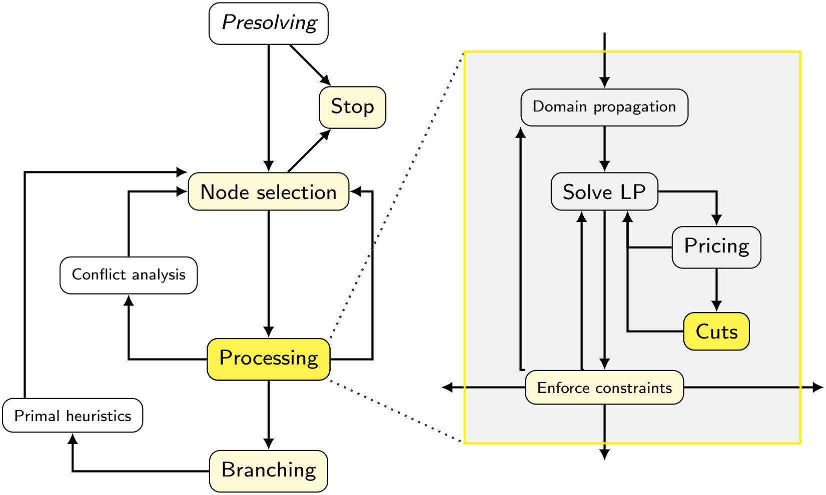


Separation Tips and Parameters

Disable/Speed up/Emphasize All Separation

```
set separating emphasis off/fast/aggressive
```

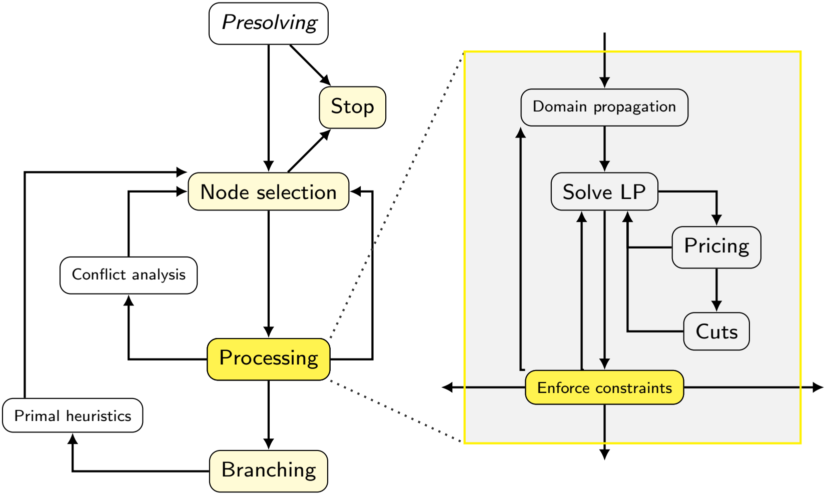
Disable Single Separation Techniques

```
set separating clique freq -1
set constraints cardinality sepafreq -1
```

Some Important Parameters

- `separating/maxcuts`, `separating/maxcutsroot`
- `separating/maxrounds`, `separating/maxroundsroot`
- `separating/maxstallrounds`, `separating/maxstallroundsroot`

Flow Chart SCIP



Constraint Enforcement

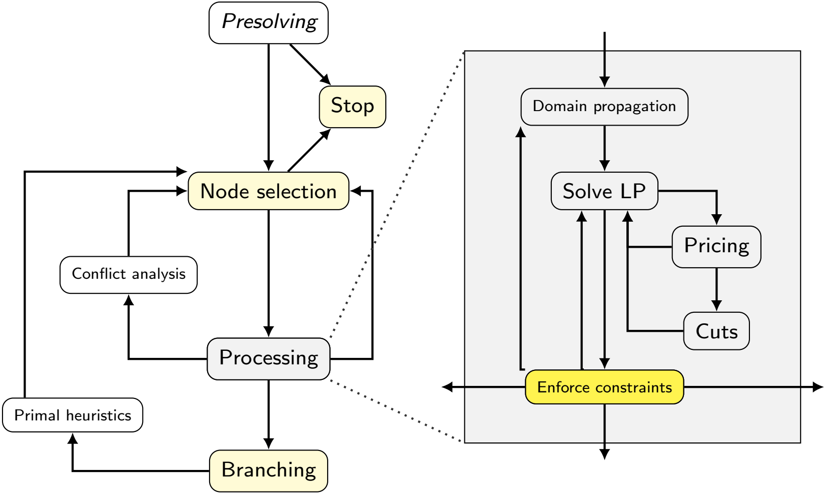
LP solution may violate a constraint not contained in the relaxation.

Enforcing is necessary for a correct implementation!

Constraint handler resolves the infeasibility by ...

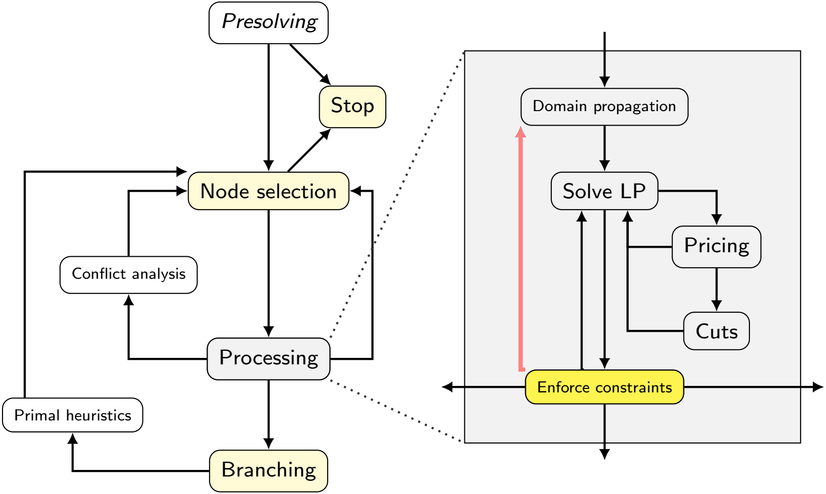
- Reducing a variable's domain,
- Separating a cutting plane (may use integrality),
- Adding a (local) constraint,
- Creating a branching,
- Concluding that the subproblem is infeasible and can be cut off, or
- Just saying "solution infeasible".

Constraint Enforcement



- Reduced domain
- Added constraint
- Added cut
- Branched
- Cutoff
- Infeasible
- Feasible

Constraint Enforcement



- **Reduced domain**
- **Added constraint**
- Added cut
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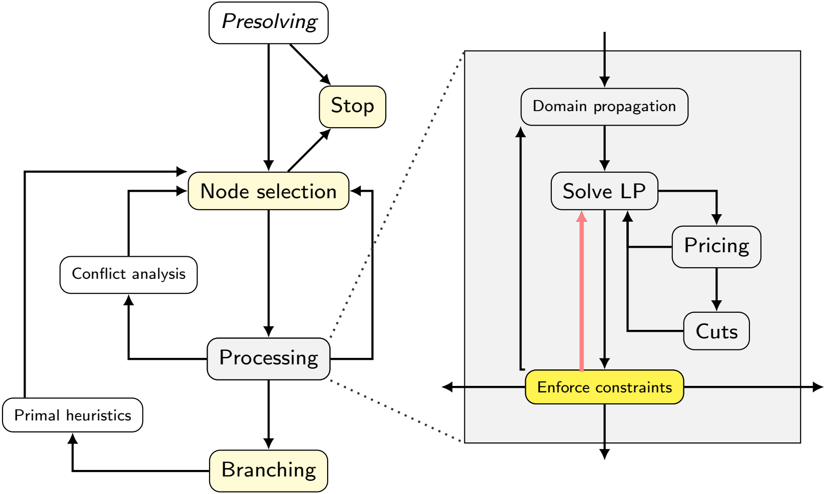
Constraint Enforcement



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- Added constraint
- **Added cut**
- Branched
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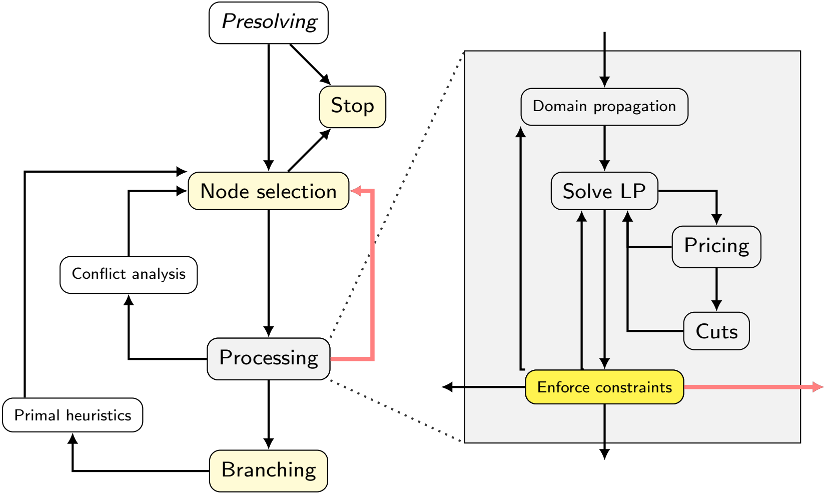
- Infeasible
- Feasible

Constraint Enforcement



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- Added cut
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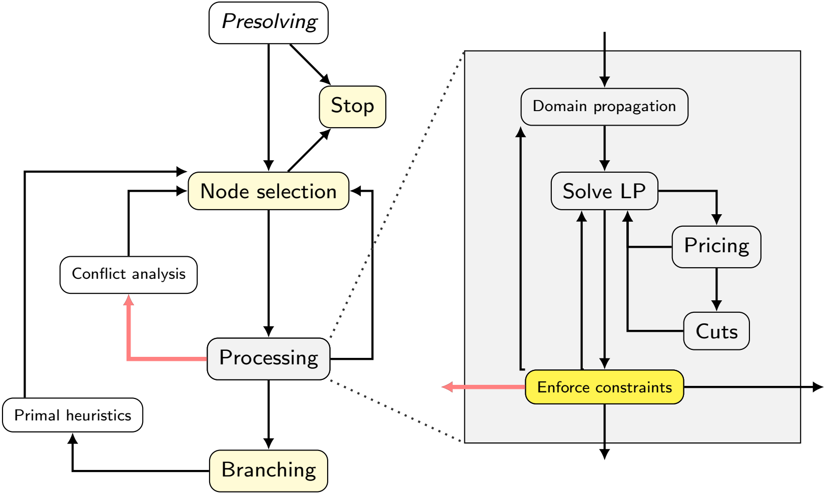
Constraint Enforcement



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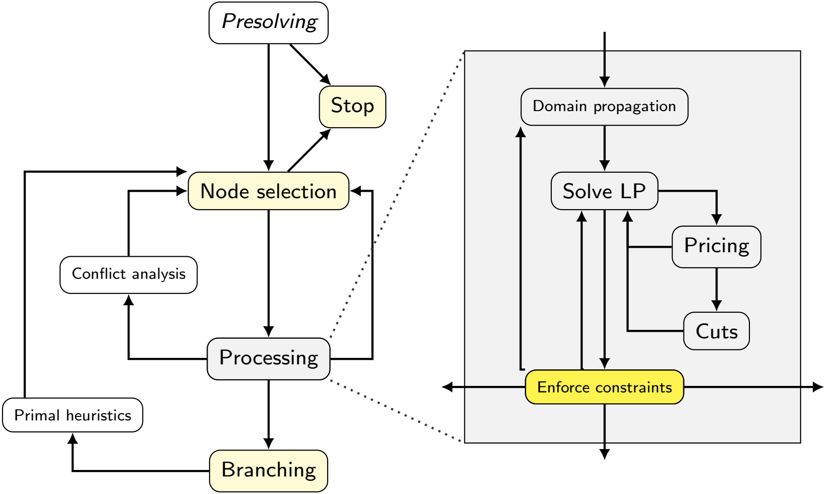
Constraint Enforcement



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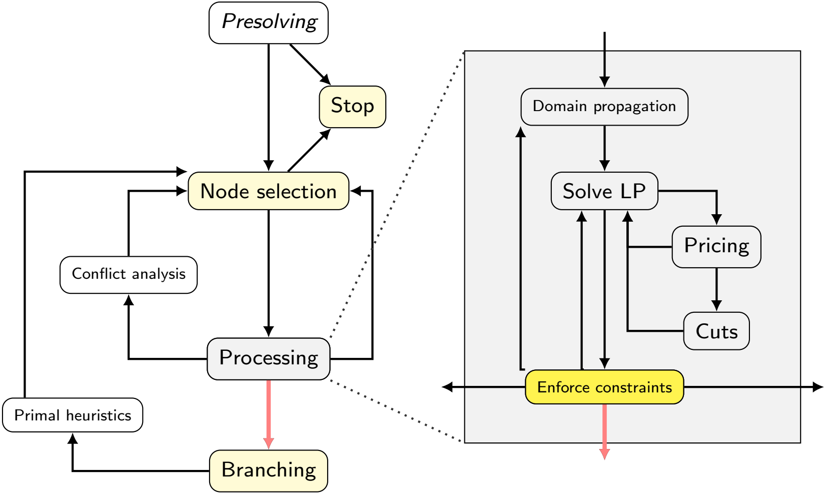
Constraint Enforcement



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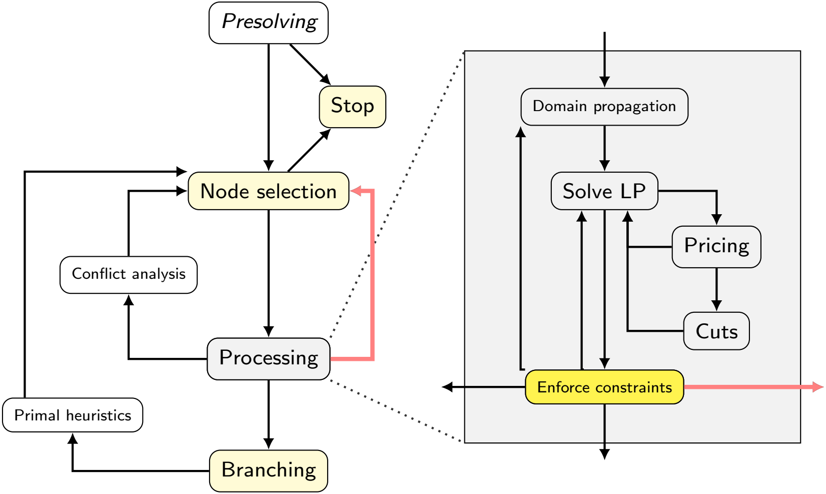
Constraint Enforcement



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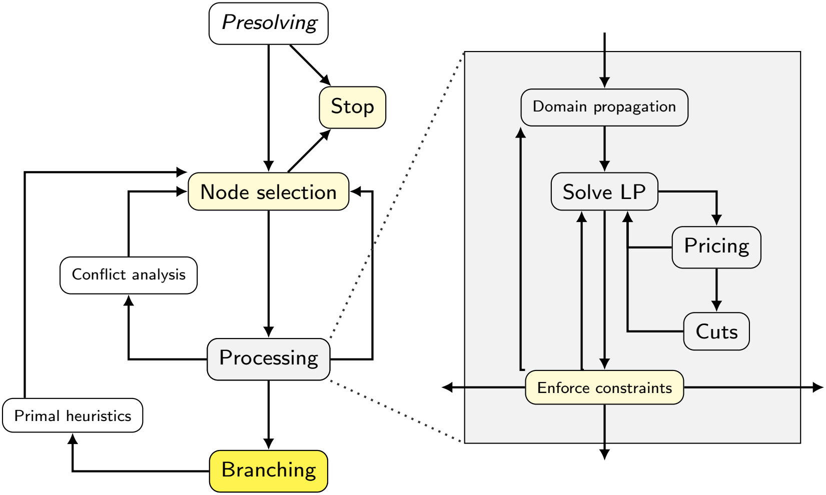
Flow Chart SCIP



Branching Rule Tips and Parameters

Branching Rule Selection

Branching rules are applied in decreasing order of priority.

```
SCIP> display branching

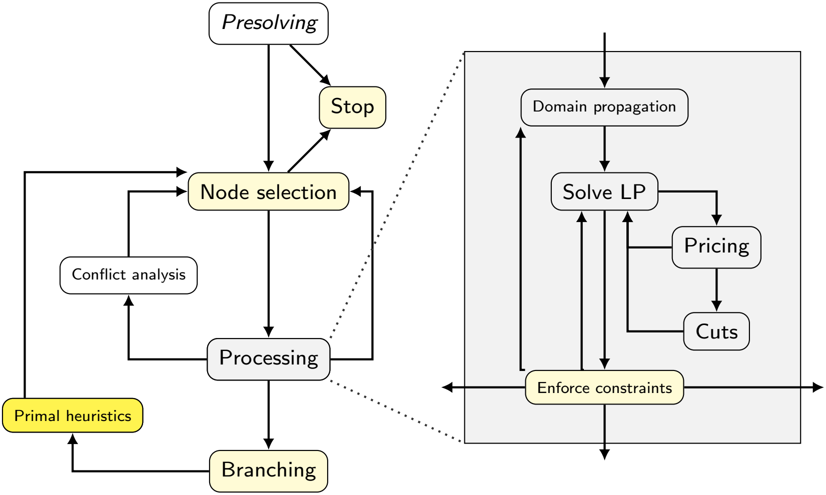
 branching rule          priority  maxdepth  maxbddist
 ----------------        --------  --------  ---------
 relpscost                  10000        -1     100.0%
 pscost                      2000        -1     100.0%
 inference                   1000        -1     100.0%
 mostinf                      100        -1     100.0%
```

Reliability Branching Parameters

All parameters prefixed with `branching/relpscost/`

- `sbiterquot`, `sbiterofs` to increase the budget for strong branching
- `minreliable` $(= 1)$, `maxreliable` $(= 5)$ to increase threshold to consider pseudo costs as reliable

Flow Chart SCIP



Primal Heuristics Tips and Parameters

Disable/Speed Up/Emphasize Heuristics

```
set heuristics emphasis off/fast/aggressive
```

Disable an individual heuristic via

```
set heuristics feaspump freq -1
```

Important Parameters

- `heuristics/alns/nodesofs`, `heuristics/alns/nodesquot` to increase the computational budget of this LNS technique
- `heuristics/guideddiving/... lpsolvefreq`, `maxlpiterofs` `maxlpiterquot` to control the LP solving during this diving technique

Advice

Use emphasis settings. Do not attempt to individually tune heuristics by hand.

Quiz - Part III

- Why does SCIP create the transformed problem?
- What is the difference between enforcing and checking a solution?
- During which stage do you have to include user-created plugins?

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- During which stage do you have to include user-created plugins?

  The init stage

Advanced Topics And Further Reading

Important SCIP topics not covered in this talk:

- more Details in Documentation
- branch-and-price: SCIP can be extended to a problem-specific branch-cut-and-price solver
  - Q&A on youtube channel
  - see The Bin Packing Example in C
  - see also GCG
- allows for Benders decomposition since version 6.0
  - Q&A on youtube channel
- browse technical reports for details on recently added cutting plane selection, primal heuristics, symmetry breaking, and much more

Thank you for your attention