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Solving Mixed-Integer SDPs

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based on work together with
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Main source: Dissertation of Tristan Gally, 2019

Discrete Optimization

Mixed-Integer Semidefinite Programming

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Mixed-integer semidefinite program (MISDP)

$$
\begin{aligned}
\sup \quad & b^T y \\
\text{s.t.} \quad & C - \sum_{i=1}^{m} A_i y_i \succeq 0, \\
& \qquad\qquad y_i \in \mathbb{Z} \quad \forall\, i \in \mathcal{I}
\end{aligned}
$$

where A_i, $C \in \mathbb{R}^{n \times n}$ are symmetric, $b \in \mathbb{R}^m$, $\mathcal{I} \subseteq \{1, \dots, m\}$.

- Linear constraints, bounds, multiple blocks possible within SDP-constraint.

Overview

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Robust Truss Topology Design

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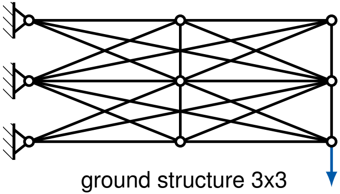
- n nodes $V \subset \mathbb{R}^d$
- n_f free nodes $V_f \subset V$
- m possible bars E
- force $f \in \mathbb{R}^{d_f}$ for $d_f = d \cdot n_f$



ground structure 3x3

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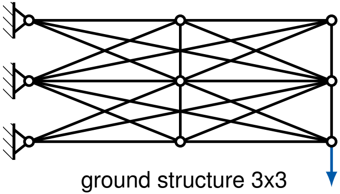
- Cross-sectional areas $x \in \mathbb{R}^m_+$ for bars minimizing volume while creating a “stable” truss
- Stability is measured by the compliance $\frac{1}{2} f^T u$ with node displacements u.



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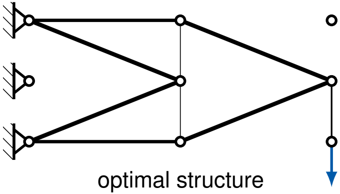

optimal structure

Robust Truss Topology Design

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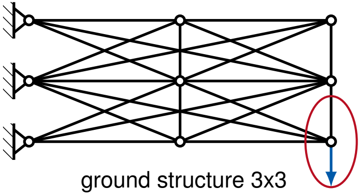

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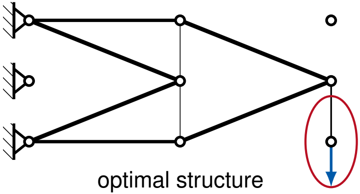

optimal structure

- Use uncertainty set $\{f \in \mathbb{R}^{d_f} \;:\; f = Qg : \|g\|_2 \leq 1\}$ instead of single force f.
- Instead of arbitrary cross-sections $x \in \mathbb{R}^m_+$ restrict them to discrete set $\mathcal{A}$.

Robust Truss Topology Design

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Elliptic Robust Discrete TTD [Ben-Tal/Nemirovski 1997; Mars 2013]

$$
\begin{aligned}
\inf \quad & \sum_{e \in E} \ell_e \sum_{a \in \mathcal{A}} a\, x_e^a \\
\text{s.t.} \quad & \begin{pmatrix} 2C_{\max} I & Q^T \\ Q & A(x) \end{pmatrix} \succeq 0, \\
& \sum_{a \in \mathcal{A}} x_e^a \leq 1 && \forall e \in E, \\
& x_e^a \in \{0, 1\} && \forall e \in E, a \in \mathcal{A},
\end{aligned}
$$

with bar lengths ℓ_e, upper bound $C_{\max}$ on compliance and stiffness matrix

$$A(x) = \sum_{e \in E} \sum_{a \in \mathcal{A}} A_e\, a\, x_e^a$$

for positive semidefinite, rank-one single bar stiffness matrices A_e.

Cardinality Constrained Least Squares

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- Sample points as rows of $A \in \mathbb{R}^{m \times d}$ with measurements $b_1, \ldots, b_m \in \mathbb{R}$
- Find $x \in \mathbb{R}^d$ minimizing $\frac{1}{2}\|Ax - b\|_2^2 + \frac{\rho}{2}\|x\|_2^2$ for a regularization parameter ρ.
- Further restrict x to at most k non-zero components.

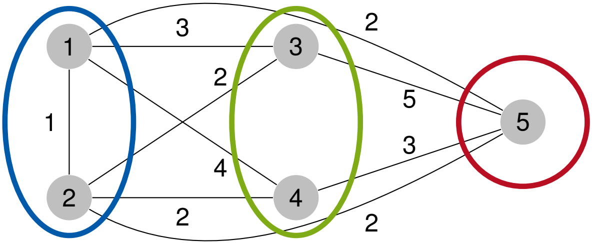
Cardinality Constrained Least Squares [Pilanci/Wainwright/El Ghaoui 2015]

$$
\begin{aligned}
\inf \quad & \tau \\
\text{s.t.} \quad & \begin{pmatrix} I + \frac{1}{\rho} A \operatorname{Diag}(z) A^\top & b \\ b^\top & \tau \end{pmatrix} \succeq 0, \\
& \sum_{j=1}^{d} z_j \leq k, \quad z \in \{0,1\}^d.
\end{aligned}
$$

Minimum k-Partitioning

- Given undirected graph $G = (V, E)$, edge costs c and number of parts $k \in \mathbb{N}$.
- Find partitioning of V into k disjoint sets $V_1, \dots, V_k$ minimizing the total cost within the parts

$$\sum_{i=1}^{k} \sum_{e \in E[V_i]} c(e).$$



- Applications in, e.g., frequency planning and layout of electronic circuits.

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Minimum k-Partitioning [Eisenblätter 2001]

$$\begin{aligned} \inf \quad & \sum_{1 \le i < j \le n} c_{ij}\, Y_{ij} \\ \text{s.t.} \quad & \tfrac{-1}{k-1}\, J + \tfrac{k}{k-1}\, Y \succeq 0, \\ & Y_{ii} = 1,\ \ Y_{ij} \in \{0, 1\}, \end{aligned}$$

where J is the all-one matrix.

Constraints on the size of the partitions can be added as

$$\ell \le \sum_{j=1}^{n} w_j\, Y_{ij} \le u \qquad \forall i \le n,$$

with w_j weight of node j and ℓ and u bounds on total weight of each partition.

Further Applications

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- Computing restricted isometry constants in compressed sensing
- Optimal transmission switching problem in AC power flow
- Robustification of physical parameters in gas networks
- Subset selection for eliminating multicollinearity
- . . .

Overview

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Outer Approximation / Cutting Planes

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- Idea: Solve LP/MIP and enforce SDP-constraint via linear cuts
- Cutting plane approach [Kelley 1960]:
  - Solve a single MIP.
  - In each node add cuts to enforce nonlinear constraints and resolve LP.
- Outer Approximation [Quesada/Grossmann 1992]:
  - Solve MIP (without nonlinear constraints) to optimality.
  - Solve continuous relaxation for fixed integer variables.
  - If objectives do not agree, update polyhedral approximation.
  - Resolve MIP and continue iterating.

Enforcing the SDP-Constraint

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- For convex MINLP one usually uses gradient cuts

$$g_j(\overline{x}) + \nabla g_j(\overline{x})^\top (x - \overline{x}) \leq 0.$$

- But function of smallest eigenvalue is not differentiable everywhere.
- Instead use characterization $X \succeq 0 \quad \Leftrightarrow \quad u^\top X\, u \geq 0$ for all $u \in \mathbb{R}^n$
- If $Z := C - \sum_{i=1}^m A_i\, y_i^* \not\succeq 0$, compute eigenvector v to smallest eigenvalue. Then

$$v^\top Z\, v \geq 0$$

is valid and cuts off y^*.

Cutting Planes: MISOCP vs. MISDP

- Cutting planes often used by solvers for mixed-integer second-order cone problems.
- Outer approximation for SOCPs possible with polynomial number of cuts [Ben-Tal/Nemirovski 2001].
- Outer approximation for SDPs needs exponential number of cuts [Braun et al. 2015].

SDP-based Branch-and-Bound

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- Relax integrality instead of SDP-constraint.
- Branch on y-variables.
- Need to solve a continuous SDP in each branch-and-bound node.
- Relaxations can be solved by problem-specific approaches (e.g. conic bundle or low-rank methods) or interior-point.
- Need to satisfy convergence assumptions of SDP-solvers.

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Strong Duality in SDP

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Dual SDP (D)

$$\begin{aligned} \sup \quad & b^T y \\ \text{s.t.} \quad & C - \sum_{i=1}^{m} A_i y_i \succeq 0, \\ & y \in \mathbb{R}^m. \end{aligned}$$

Primal SDP (P)

$$\begin{aligned} \inf \quad & C \bullet X \\ \text{s.t.} \quad & A_i \bullet X = b_i \quad \forall\, i \leq m, \\ & X \succeq 0. \end{aligned}$$

where $A \bullet B = \mathrm{Tr}(AB) = \sum_{ij} A_{ij} B_{ij}$.

Strong Duality in SDP

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$$
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\begin{aligned}
\inf \quad & C \bullet X \\
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& X \succeq 0.
\end{aligned}
$$

where $A \bullet B = \mathrm{Tr}(AB) = \sum_{ij} A_{ij} B_{ij}$.

- Strong Duality holds if Slater condition holds for (P) or (D): $\exists\, X \succ 0$ feasible for (P) or y such that $C - \sum_{i=1}^{m} A_i y_i \succ 0$ in (D).
- If Slater holds for (P), optimal objective of (D) is attained and vice versa.
- Existence of a KKT-point is guaranteed if Slater holds for both, this is assumed by most interior-point SDP-solvers.
- Can these assumptions be lost through branching?

Strong Duality in Branch-and-Bound

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Theorem [Gally, P., Ulbrich 2016]

Let (D_+) be the problem formed by adding a linear constraint to (D). If

- strong duality holds for (P) and (D),
- the set of optimal $Z := C - \sum_{i=1}^{m} A_i y_i$ in (D) is compact and nonempty,
- problem (D_+) is feasible,

then strong duality also holds for (D_+) and (P_+) and the set of optimal Z for (D_+) is compact and nonempty.

- Compactness of set of optimal Z also necessary for strong duality [Friberg 2016].
- Analogous result for adding linear constraints to (P) with set of optimal X compact and nonempty and (P_+) feasible.

Slater Condition in Branch-and-Bound

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Proposition [Gally, P., Ulbrich 2016]

After adding a linear constraint $\sum_{i=1}^{m} a_i y_i \geq c$ (or $\leq$ or $=$) to (D), if (P) satisfies the Slater condition and the coefficient vector a satisfies $a \in \text{Range}(\mathcal{A})$, for $\mathcal{A} : S_n \to \mathbb{R}^m$, $X \mapsto (A_i \bullet X)_{i \in [m]}$, then the Slater condition also holds for (P_+).

- $a \in \text{Range}(\mathcal{A})$ is implied by linear independence of A_i.
- Dual Slater condition is preserved after adding linear constraint to (P) (without additional assumptions on the coefficients).

KKT-condition in Branch-and-Bound

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KKT-points may get lost after branching, for example:

(D)

$$\begin{aligned} \sup \quad & 2\,y_1 - y_2 \\ \text{s.t.} \quad & \begin{pmatrix} 0.5 & -y_1 \\ -y_1 & y_2 \end{pmatrix} \succeq 0. \end{aligned}$$

(P)

$$\begin{aligned} \inf \quad & 0.5\,X_{11} \\ \text{s.t.} \quad & \begin{pmatrix} X_{11} & 1 \\ 1 & 1 \end{pmatrix} \succeq 0. \end{aligned}$$

- Strictly feasible solutions given by $y = (0, 0.5)$, $X_{11} = 2$.
- Optimal objective of 0.5 attained (only) for $y = (0.5, 0.5)$, $X_{11} = 1$.

KKT-condition in Branch-and-Bound

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After branching on y_2 and adding cut $y_2 \leq 0$:

(D_+)

$$\begin{array}{ll} \sup & 2\,y_1 - y_2 \\ \text{s.t.} & \begin{pmatrix} 0.5 & -y_1 & 0 \\ -y_1 & y_2 & 0 \\ 0 & 0 & -y_2 \end{pmatrix} \succeq 0, \end{array}$$

(P_+)

$$\begin{array}{ll} \inf & 0.5\,X_{11} \\ \text{s.t.} & \begin{pmatrix} X_{11} & 1 & X_{13} \\ 1 & X_{22} & X_{23} \\ X_{13} & X_{23} & X_{22} - 1 \end{pmatrix} \succeq 0. \end{array}$$

- Optimal objective 0 attained for $y = (0, 0)$.
- Relative interior of (D_+) is empty.
- (P_+) still has strictly feasible solution $X_{11} = X_{22} = 2$, $X_{13} = X_{23} = 0$.
- (P_+) has minimizing sequence $X_{11} = 1/k$, $X_{22} = k$, $X_{13} = X_{23} = 0$.
- No longer satisfies assumptions for convergence of interior-point solvers.

Slater Condition in Practice

	Dual Slater				Primal Slater		
application	✓	✗	infeas	?	✓	✗	?
TTD	83.22 %	5.82 %	10.96 %	0.00 %	99.99 %	0.00 %	0.01 %
CLS	56.26 %	3.44 %	40.30 %	0.00 %	100.00 %	0.00 %	0.00 %
M*k*P	3.66 %	62.93 %	33.41 %	0.00 %	100.00 %	0.00 %	0.00 %
overall	45.89 %	25.33 %	28.78 %	0.00 %	100.00 %	0.00 %	0.00 %

run on cluster of 64-bit Intel Xeon E5-1620 CPUs running at 3.50 GHz with 32 GB RAM using SCIP-SDP 3.1.1, SCIP 6.0.0, and MOSEK 8.1.0.54 on test set of 194 CBLIB instances

Checking Infeasibility

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If interior-point solver did not converge for original formulation, solve

Feasibility Check [Mars 2013]

$$\begin{aligned} \inf \quad & r \\ \text{s.t.} \quad & C - \sum_{i=1}^{m} A_i y_i + I\, r \succeq 0. \end{aligned}$$

If optimum $r^* > 0$, original problem is infeasible and node can be cut off.

Handling Failure of the Dual Slater Condition

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If problem is not infeasible, solve

Penalty Formulation [Benson/Ye 2008]

$$\begin{aligned} \sup \quad & b^\top y - \Gamma\, r \\ \text{s.t.} \quad & C - \sum_{i=1}^{m} A_i y_i + I\, r \succeq 0, \\ & r \geq 0 \end{aligned}$$

for sufficiently large Γ to compute an upper bound.

- If optimal $r^* = 0$, then solution is also optimal for original problem.
- Adds constraint $\mathrm{Tr}(X) \leq \Gamma$ to primal problem, for large enough Γ also preserves primal Slater condition.

SDP-Solvers depending on Slater Condition

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Behavior if Slater condition holds for (P) and (D)

solver	default	penalty	bound	unsucc
SDPA	90.78 %	5.50 %	0.00 %	3.73 %
DSDP	99.68 %	0.32 %	0.00 %	0.00 %
MOSEK	99.51 %	0.49 %	0.00 %	0.00 %

Behavior if Slater condition fails for (P) or (D)

solver	default	penalty	bound	unsucc
SDPA	56.15 %	1.14 %	13.00 %	29.71 %
DSDP	99.81 %	0.13 %	0.00 %	0.05 %
MOSEK	99.20 %	0.79 %	0.01 %	0.00 %

Behavior if problem is infeasible

solver	default	penalty	bound	unsucc
SDPA	46.99 %	39.46 %	4.88 %	8.67 %
DSDP	92.44 %	2.23 %	1.39 %	3.94 %
MOSEK	88.42 %	10.36 %	1.22 %	0.00 %

Overview

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SCIP-SDP

- Based on SCIP framework.
- Supports both nonlinear B&B and LP-based branch-and-cut.
- Two file-readers
  - CBF
  - SDPA with added integrality information
- Constraint handler for SDP-constraints
- Interfaces to three SDP solvers
  - DSDP
  - SDPA
  - MOSEK
- Two additional heuristics
  - SDP-based diving, SDP-based randomized rounding
- Two additional propagators
  - SDP-based OBBT, SDP-based dual fixing
- Parallelized version available as UG-MISDP.
- Supports rank 1 constraints (implemented together with Frederic Matter).

Constraint Handler

- Handles SDP-constraints in dual form

$$C - \sum_{i=1}^{m} A_i y_i \succeq 0.$$

- For branch & cut separate eigenvector cuts.
- Adds linear constraints implied by SDP-constraint during presolving (e.g., non-negativity of diagonal entries).
  - Redundant for nonlinear branch-and-bound, but can be used by SCIP during presolving for fixing variables.
  - Still lead to speedup of 6% even for nonlinear branch-and-bound.

Relaxator and SDPI

- Relaxator solves trivial relaxations (e.g., all variables fixed), otherwise calls SDP interface (SDPI).
- Upper level SDPI does some local presolving important for SDP-solvers, e.g.,
  - removing fixed variables,
  - removing zero rows/columns.
- Lower level SDPI brings SDP into the form needed by the solver (e.g., primal instead of dual SDP for MOSEK) and solves it.
- In case SDP-solver failed to converge (e.g., because of failure of constraint qualification), upper level SDPI can apply penalty formulation and call lower level SDPI for adjusted problem.

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Dual Fixing

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- Extension of reduced-cost fixing to general MINLPs by [Ryoo and Sahinidis 1996] and primal MISDPs by [Helmberg 2000].
- Our approach uses conic duality and only requires feasibility.

Theorem [Gally, P., Ulbrich 2018]

- (X, W, V): Primal feasible solution, where W, V are primal variables corresponding to variable bounds ℓ, u in the dual,
- f: Corresponding primal objective value,
- L: Lower bound on the optimal objective value of the MISDP.

Then for every optimal solution $y^\star$ of the MISDP

$$y_j^\star \leq \ell_j + \frac{f - L}{W_{jj}} \text{ if } \ell_j > -\infty \quad \text{and} \quad y_j^\star \geq u_j - \frac{f - L}{V_{jj}} \text{ if } u_j < \infty.$$

- If $f - L < W_{jj}$ for binary y_j, then $y_j^\star = 0$, if $f - L < V_{jj}$, then $y_j^\star = 1$.
- 9% reduction of B&B-nodes, 23% speedup.

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Warmstarts

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- MIP: Large savings by starting dual simplex from optimal basis of parent node.
- Interior-point solvers: Need $X \succ 0$ and $Z := C - \sum_{i=1}^{m} A_i y_i \succ 0$ for initial point.
- Not satisfied by optimal solution of parent node, which will be on boundary.
- Infeasible-interior-point methods update Z and y separately, so Z does not necessarily need to be updated after branching, but has to be positive definite.

$\Rightarrow$ Cannot easily warmstart with unadjusted solution of parent node.

Warmstarting Techniques

- Starting from earlier iterates
  - Proposed by [Gondzio 1998] for MIP.
  - Store previous iterate, further away from optimum but still sufficiently interior.
- Convex combination with strictly feasible solution
  - Due to [Helmberg and Rendl 1998]
  - Convex combination with scaled identity matrix or analytic center of root node.
- Projection onto positive definite matrices
  - Project onto set of matrices with $\lambda_{\text{min}} \geq \underline{\lambda} > 0$.
  - Can be computed explicitly from eigendecomposition.
- Rounding problems
  - Proposed by [Çay, Pólik and Terlaky 2017] for MISOCP.
  - Compute feasible solutions for adjusted problems by fixing eigenvectors of parent node and optimizing over eigenvalues as LP.
  - Can prove optimality/suboptimality/infeasibility by linear programming only.
  - Solution still needs to be adjusted for strict feasibility.

Comparison of Warmstarting Techniques

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settings	solved	time	sdpiter
`no warmstart`	288	117.47	18,957.04
`simple warmstart`	127	794.32	–
`preoptgap 0.01`	191	349.88	–
`preoptgap 0.5`	243	232.49	22,830.56
`0.01 id pdsame`	288	110.63	16,172.51
`0.5 id pddiff`	288	105.36	15,125.79
`0.5 id pdsame`	290	105.56	16,362.67
`0.5 anacent`	286	140.88	20,463.24
`proj minev 0.1`	285	112.74	16,277.19
`roundingprob 0.5 id`	282	174.83	13,952.38
`roundingprob inf only`	287	155.19	15,282.34

run on cluster of 64-bit Intel Xeon E5-1620 CPUs with 3.50 GHz and 32 GB RAM using SCIP-SDP 3.1.1, SCIP 6.0.0, and SDPA 7.4.0 on test set of 194 CBLIB instances and 126 compressed sensing instances; times (and iterations) as shifted geometric means (over instances solved by all settings except unadjusted warmstart and preoptimal)

Comparison of Warmstarting Techniques

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Speedup for conv 0.01 pdsame

application	solved	time	sdpiter
TTD	−1	+22.4	+10.6
CLS	0	−9.1	−13.0
M*k*P	+1	−16.0	−21.0
RIP	0	−9.7	−18.1

Speedup for conv 0.5 pddiff

application	solved	time	sdpiter
TTD	-1	+15.7	−10.4
CLS	0	−5.2	−4.0
M*k*P	+1	+0.1	−9.8
RIP	0	−27.9	−31.2

Speedup for conv 0.5 pdsame

application	solved	time	sdpiter
TTD	+1	−8.7	−26.3
CLS	−1	−8.7	−11.1
M*k*P	+2	−8.6	+0.6
RIP	0	−12.6	−17.4

Speedup for projection

application	solved	time	sdpiter
TTD	−3	+10.4	−21.7
CLS	−1	+0.7	−5.5
M*k*P	+1	+5.7	+12.1
RIP	0	−17.3	−24.1

run on cluster of 64-bit Intel Xeon E5-1620 CPUs running at 3.50 GHz with 32 GB RAM using SCIP-SDP 3.1.1, SCIP 6.0.0, and SDPA 7.4.0 on test set of 194 CBLIB instances and 126 compressed sensing instances; times (and iterations) as shifted geometric means (over instances solved by all settings except unadjusted warmstart and preoptimal)

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MISDP Solvers

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Nonlinear branch-and-bound

- SCIP-SDP 3.1.1 (nonlinear branch-and-bound)
  Our implementation, using SCIP as B&B-framework
- YALMIP-BNB R20180926
  MATLAB toolbox for rapid prototyping

Cutting plane / outer approximation approaches

- SCIP-SDP 3.1.1 (LP-based cutting planes)
- YALMIP-CUTSDP R20180926
- Pajarito 0.5.0
  - Julia implementation for mixed-integer convex including MISDP
  - MIP-solver-drives version (single MIP with SDP solves for stronger cuts)

Comparison of MISDP Solvers I

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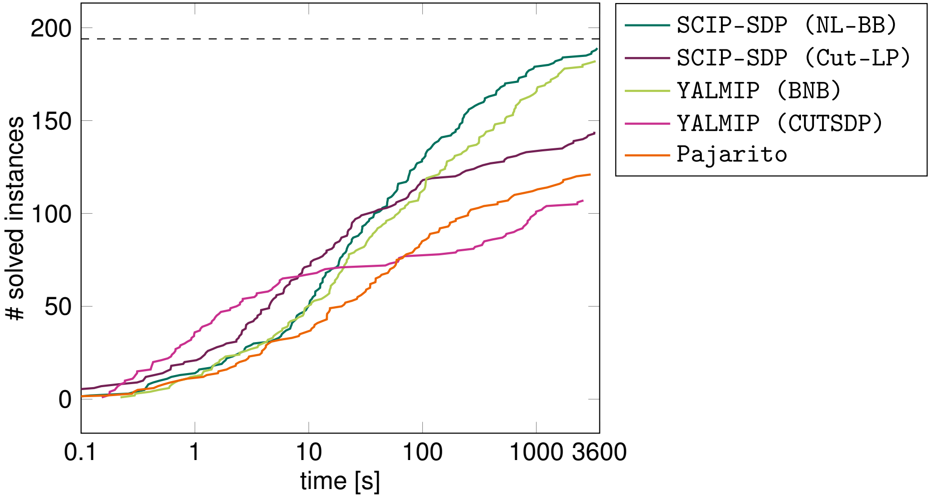

Comparison of MISDP Solvers II

solver	TTD		CLS		Mk-P		Total	
	opt	time	opt	time	opt	time	opt	time
`SCIP-SDP (NL-BB)`	57	64.4	63	94.3	69	36.4	189	60.4
`SCIP-SDP (Cut-LP)`	44	143.6	65	9.0	35	640.3	144	117.5
`YALMIP (BNB)`	52	203.0	62	132.0	68	25.2	182	88.1
`YALMIP (CUTSDP)`	22	1026.8	58	33.1	27	657.2	107	295.5
`Pajarito`	43	190.9	65	54.3	13	1503.5	121	271.2

run on 8-core Intel i7-4770 CPU with 3.4 GHz and 16GB memory over 196 instances of CBLIB; time limit of 3600 seconds, times as shifted geometric means, SDPs solved using MOSEK 8.1.0.54, MIPs/LPs using CPLEX 12.6.1; all solvers single-threaded

Overview

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Parallelization

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- ▷ Based on UG framework.
- ▷ Can either parallelize on SDP or on MIP side.
- ▷ Parallel Cholesky for SDPs depends on sparsity pattern and usually only efficient for larger SDPs.
- ▷ Solving subtrees of branch and bound tree in parallel not possible for root node.

$\Rightarrow$ Use racing ramp-up in root node to decide between different settings, in particular LP vs. SDP.

- ▷ Start a number of SCIP-SDP instances in parallel with half of them using LP-based and the other half SDP-based settings.
- ▷ After enough nodes have been generated, decide on "best" solver and distribute this solver's tree.

Numerical Results for Parallelization

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solver / # threads	TTD opt	TTD time	CLS opt	CLS time	Mk-P opt	Mk-P time	Total opt	Total time
`SCIP-SDP`	55	84.01	62	142.19	67	54.44	184	86.59
`UG-MISDP 1`	54	107.49	62	156.70	58	107.81	174	122.23
`UG-MISDP 2`	56	64.93	64	23.31	56	92.25	176	53.79
`UG-MISDP 4`	58	39.76	65	18.48	60	85.61	183	42.07
`UG-MISDP 8`	58	32.07	65	14.51	60	72.35	183	34.57
`UG-MISDP 16`	59	21.03	65	16.37	59	78.46	183	32.65
`UG-MISDP 32`	59	21.27	65	18.38	56	92.14	180	36.11

run on Intel Xeon E5-4650 CPUs running at 2.70 GHz with 512 GB of shared RAM; time limit of 3600 seconds, times as shifted geometric means; using developer versions of SCIP 6.0.0, SCIP-SDP 3.1.1, UG 0.8.6, SDPs solved using MOSEK 8.1.0.54, LPs using CPLEX 12.6.3; instances from CBLIB

Overview

Conclusion & Outlook

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- Framework for solving general MISDPs
- Several methods help to improve performance.
- Solving SDPs is still one bottleneck, but often yields strong bounds.
- Future: follow development path for MIP-solvers for MISDP-solvers as well.

SCIP-SDP is available in source code at

`http://www.opt.tu-darmstadt.de/scipsdp/`

Thank you for your attention!

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