Integrated pollster and vehicle routing

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CO@Work 2020

TU Berlin - Berlin Mathematical School, September 14th - 25th, 2020

Outline



Motivation and problem definition

- Modeling via mixed integer programming
- Computational results



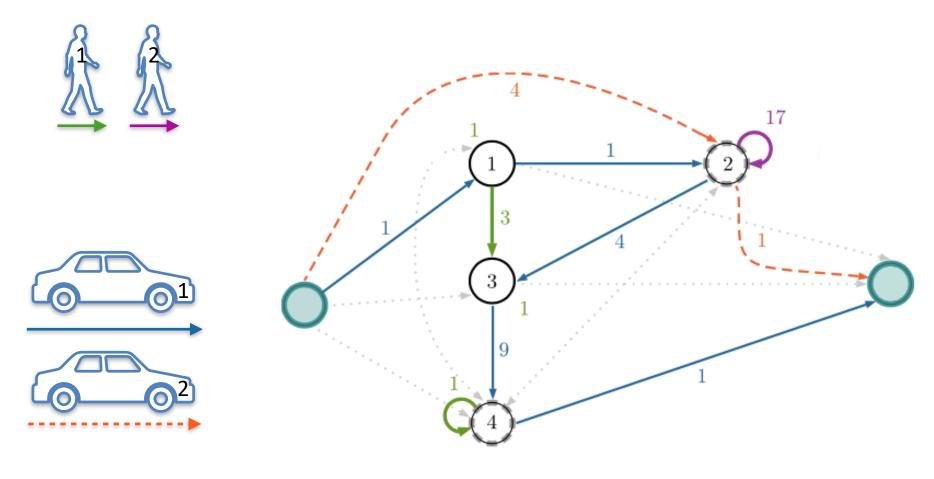
The problem



- The National Statistics Bureau of Ecuador (INEC) is responsible for constructing the Consumer Price Index.
- To collect information a sample set of stores must be visited monthly.
- Stores are visited by a set of pollsters, transported by a fleet of hired vehicles. Pollsters also walk between stores.
- \star Tasks:
 - * schedule visits to stores within time horizon
 - * schedule daily service duties for pollsters
 - * define daily routes for vehicles

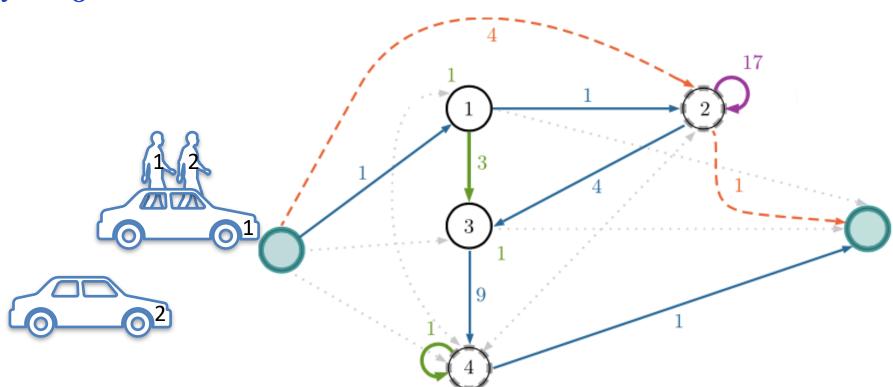
A Mixed Linear and Integer Programming Problem





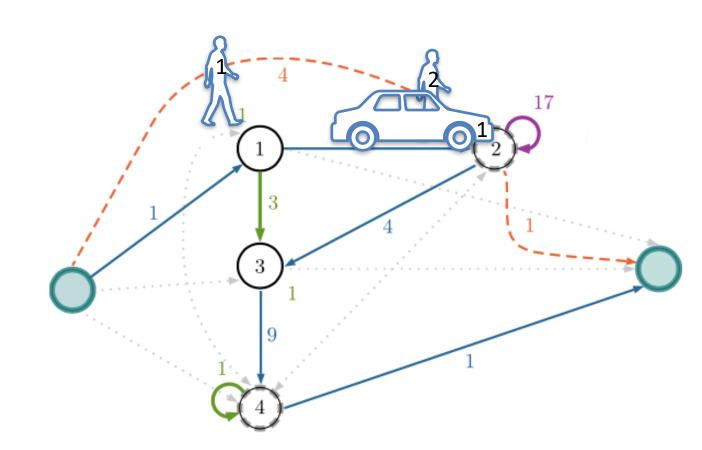
A Mixed Linear and Integer Programming Problem





A Mixed Linear and Integer Programming Problem







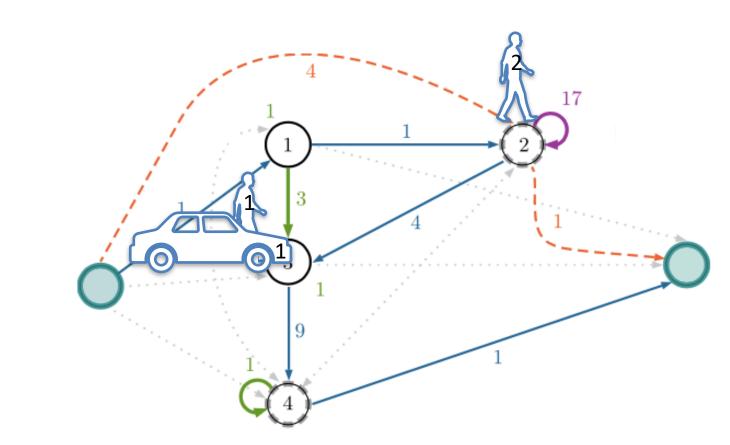
A Mixed Linear and Integer Programming Problem



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A Mixed Linear and Integer Programming Problem

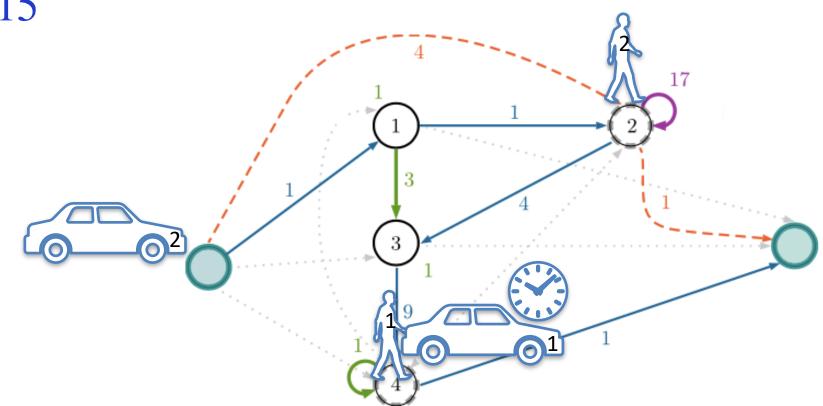






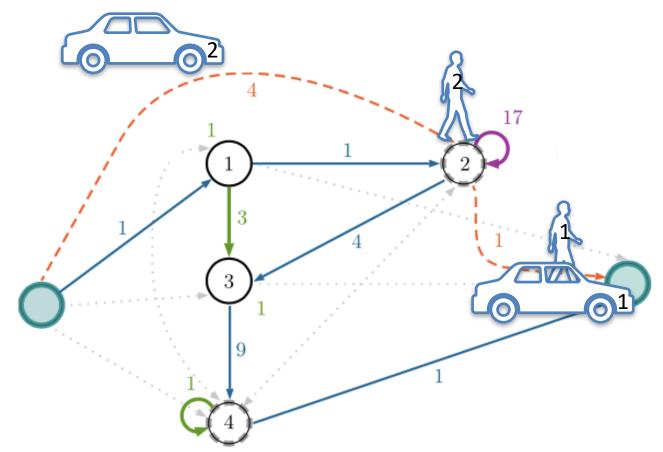
A Mixed Linear and Integer Programming Problem





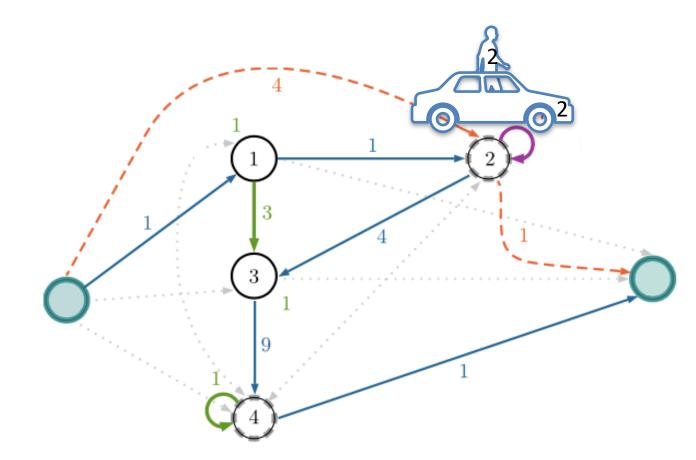
A Mixed Linear and Integer Programming Problem





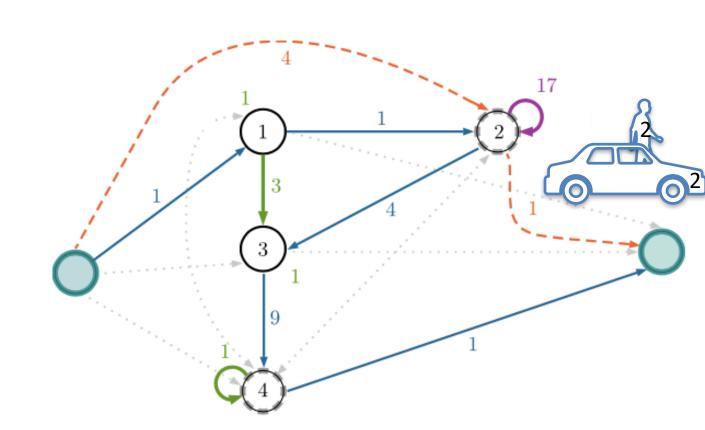
A Mixed Linear and Integer Programming Problem





A Mixed Linear and Integer Programming Problem



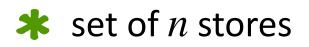


Definitions: sets



***** *E*: pollsters

***** *K*: vehicles



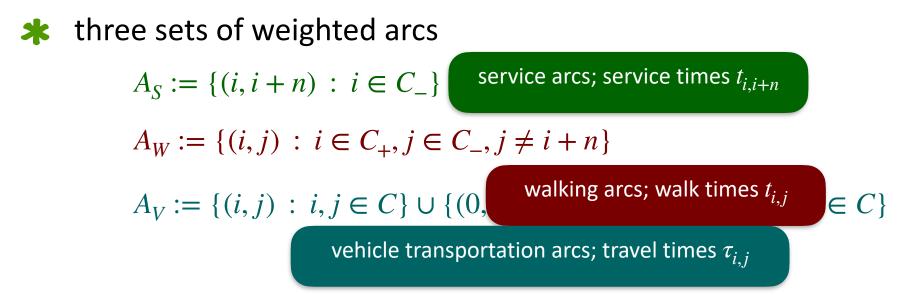
***** S: days within time horizon for planning

Definitions: network



- two nodes for each store (customer)
 - $C_{-} := \{1, ..., n\}$ (arrival at store)
 - $C_+ := \{n + 1, \dots, 2n\}$ (departure from store)
- ***** two nodes 0, 2n + 1 for depot

*
$$C := C_{-} \cup C_{+}, \quad V := C \cup \{0, 2n + 1\}$$



Definitions: paths and routes



- ★ A walking path for a pollster is a simple path from $i \in C_{-}$ to $j \in C_{+}$ with alternating arcs from the sets A_{S} and A_{W} .
- * A vehicle path is a directed path between two nodes in V using only arcs from A_V .
- * A service route for a pollster is a dipath from 0 to 2n + 1 consisting of an alternating sequence of vehicle and walking subpaths.
- ***** A vehicle route is a vehicle path from 0 to 2n + 1.
- ***** The duration of a route is the sum of its arc weights.
- * A service route is feasible if it does not exceed a maximum allowed duration, including time for a lunch break.

Definitions: schedules



- Vehicles pick-up and deliver pollsters to certain stores; pollsters can share vehicles.
- ***** Vehicle fleet is homogeneous, with vehicle capacity Q.
- Pollster "fleet" is homogeneous: any pollster can visit any store.
- ★ A daily schedule consists of a set of feasible pollster routes and "compatible" vehicle routes, i.e., for any $(i, j) \in A_V$:
 - if (i, j) is contained in some service route, then it is contained in a vehicle route
 - \triangleright (i, j) is not contained in more than Q service routes

The IPVRP



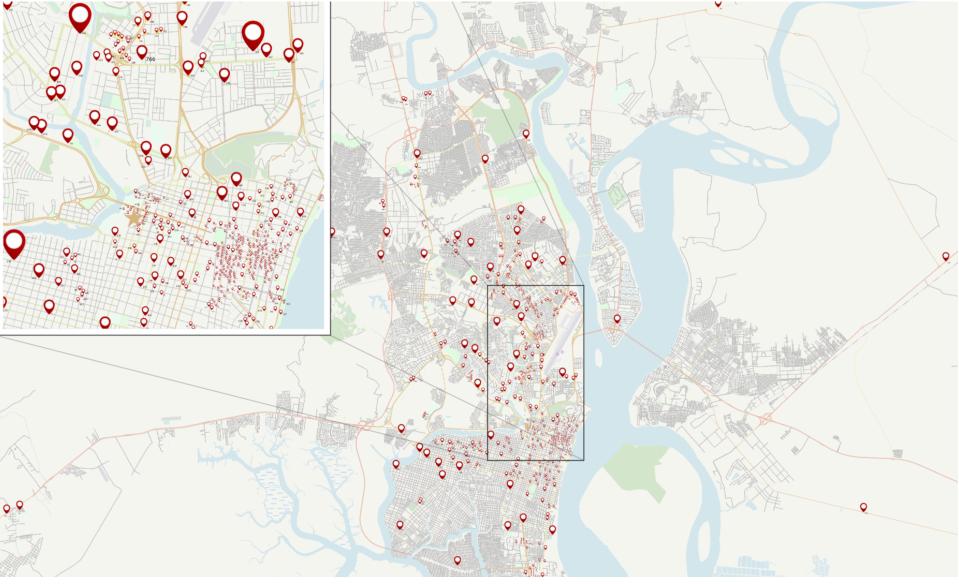
Task:

Find a set of daily schedules, at most one for each day in a given time horizon, such that:

- ***** Each store is visited once.
- * Number of working days is minimized.
- * Number of service routes is minimized.
- * Number of vehicle routes is minimized.

The INEC instance





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Pollster routing: variables



Binary variables:

 $x_{i,i+n}^{e,s}, \forall (i,i+n) \in A_S, e \in E, s \in S : x_{i,i+n}^{e,s} = 1 \Leftrightarrow e \text{ visits store } i \text{ on day } s$

 $x_{i,j}^{e,s}, \forall (i,j) \in A_W, e \in E, s \in S: x_{i,j}^{e,s} = 1 \Leftrightarrow e \text{ walks from } i \text{ to } j \text{ on day } s$

 $z_{i,j}^{e,s}, \forall (i,j) \in A_V, e \in E, s \in S: z_{i,j}^{e,s} = 1 \Leftrightarrow e \text{ is transported from } i \text{ to } j \text{ on day } s$

 $b_i^{e,s}, \forall i \in C_-, e \in E, s \in S: \quad b_i^{e,s} = 1 \Leftrightarrow i \text{ is start of a walking path for } e \text{ on day } s$

 $f_i^{e,s}, \forall i \in C_+, e \in E, s \in S: f_i^{e,s} = 1 \Leftrightarrow i \text{ is end of a walking path of } e \text{ on day } s$

 $u_s, s \in S: u_s = 1 \Leftrightarrow s$ has a daily schedule assigned to it



$$\sum_{s \in S} \sum_{e \in E} x_{i,i+n}^{e,s} = 1, \quad \forall i \in C_{-},$$

$$\sum_{(j,i) \in A_W} x_{j,i}^{e,s} - x_{i,i+n}^{e,s} = -b_i^{e,s}, \quad \forall i \in C_{-}, e \in E, s \in S,$$

$$x_{i-n,i}^{e,s} - \sum_{(i,j) \in A_W} x_{i,j}^{e,s} = f_i^{e,s}, \quad \forall i \in C_{+}, e \in E, s \in S,$$

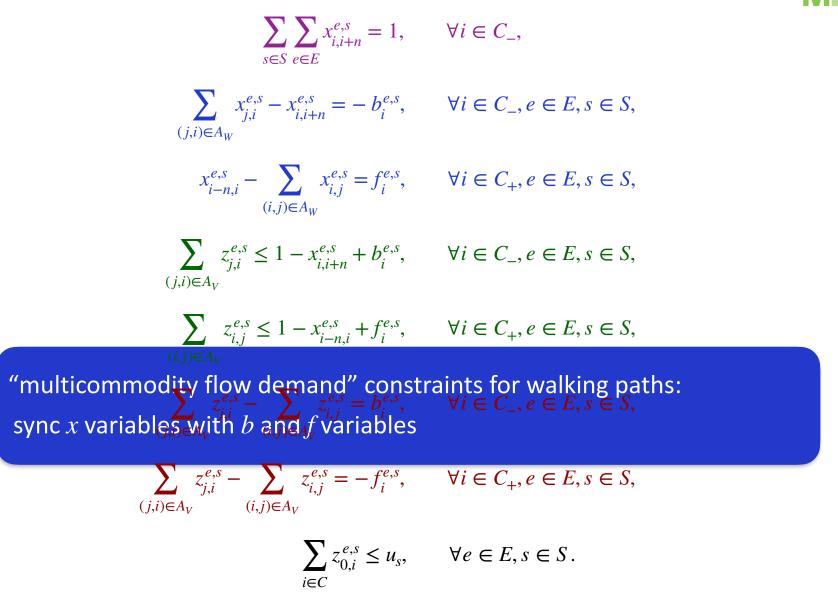
$$\sum_{(j,i) \in A_V} z_{j,i}^{e,s} \le 1 - x_{i,i+n}^{e,s} + b_i^{e,s}, \quad \forall i \in C_{-}, e \in E, s \in S,$$

$$\sum_{(j,i) \in A_V} z_{i,j}^{e,s} \le 1 - x_{i-n,i}^{e,s} + f_i^{e,s}, \quad \forall i \in C_{+}, e \in E, s \in S,$$
each store is visited by one pollster on one day

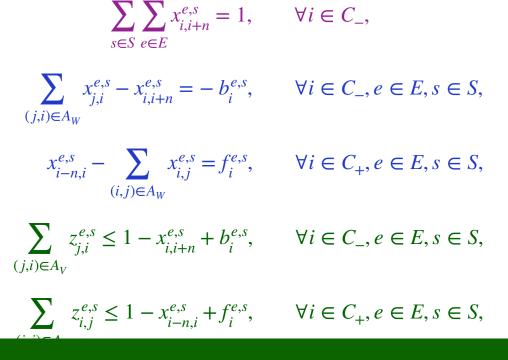
 $\sum_{(j,i)\in A_V} z_{j,i}^{e,s} - \sum_{(i,j)\in A_V} z_{i,j}^{e,s} = -f_i^{e,s}, \qquad \forall i\in C_+, e\in E, s\in S,$

$$\sum_{i \in C} z_{0,i}^{e,s} \le u_s, \qquad \forall e \in E, s \in S.$$







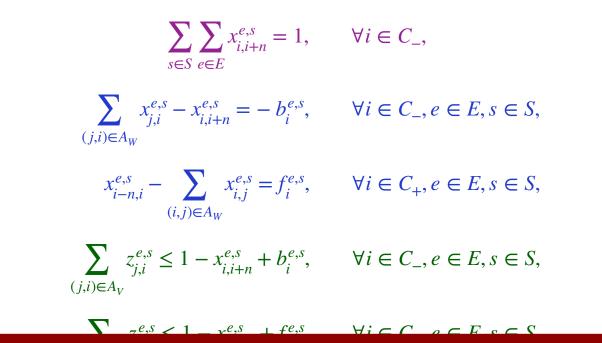


degree constraints for vehicle transportation: forbid arcs that cannot connect properly to walking paths

$$\sum_{(j,i)\in A_V} z_{j,i}^{e,s} - \sum_{(i,j)\in A_V} z_{i,j}^{e,s} = -f_i^{e,s}, \qquad \forall i \in C_+, e \in E, s \in S,$$

$$\sum_{i \in C} z_{0,i}^{e,s} \le u_s, \qquad \forall e \in E, s \in S.$$



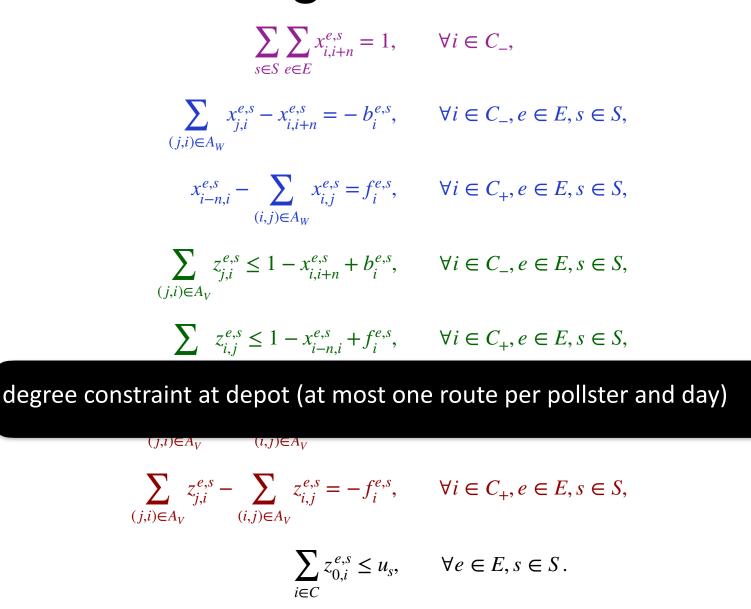


"multicommodity flow demand" constraints for vehicle transportation (pickup and delivery of pollsters): sync z variables with b and f variables

$$\sum_{(j,i)\in A_V} z_{j,i}^{e,s} - \sum_{(i,j)\in A_V} z_{i,j}^{e,s} = -f_i^{e,s}, \qquad \forall i \in C_+, e \in E, s \in S,$$

$$\sum_{i \in C} z_{0,i}^{e,s} \le u_s, \qquad \forall e \in E, s \in S.$$





Vehicle routing: variables



Binary variables:

 $y_{i,j}^{k,s}, \forall (i,j) \in A_V, k \in K, s \in S: y_{i,j}^{k,s} = 1 \Leftrightarrow k \text{ travels through } (i,j) \text{ on day } s$

Remind that...

 $z_{i,j}^{e,s}, \forall (i,j) \in A_V, e \in E, s \in S : \quad z_{i,j}^{e,s} = 1 \Leftrightarrow e \text{ is transported from } i \text{ to } j \text{ on day } s$ $b_i^{e,s}, \forall i \in C_-, e \in E, s \in S : \quad b_i^{e,s} = 1 \Leftrightarrow i \text{ is start of a walking path for } e \text{ on day } s$ $f_i^{e,s}, \forall i \in C_+, e \in E, s \in S : \quad f_i^{e,s} = 1 \Leftrightarrow i \text{ is end of a walking path of } e \text{ on day } s$ $u_s, s \in S : \quad u_s = 1 \Leftrightarrow i \text{ if day } s \text{ has a daily schedule assigned to it}$

k.s



$$\sum_{k \in K} \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = \sum_{e \in E} b_i^{e,s}, \qquad \forall i \in C_-, \forall s \in S,$$
$$\sum_{k \in K} \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = \sum_{e \in E} f_i^{e,s}, \qquad \forall i \in C_+, \forall s \in S,$$
$$\sum_{j,i) \in A_V} y_{j,i}^{k,s} - \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = 0, \qquad \forall i \in C, k \in K, s \in S,$$

degree constraints for vehicle routes: out-degree is required to be 1 if node is starting or ending node of some walking path, 0 otherwise



$$\sum_{k \in K} \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = \sum_{e \in E} b_i^{e,s}, \qquad \forall i \in C_-, \forall s \in S,$$

 $\sum \sum y_{i,j}^{k,s} = \sum f_i^{e,s}, \qquad \forall i \in C_+, \forall s \in S,$ $k \in K(i, j) \in A_{V}$ $e \in E$

$$\sum_{(j,i)\in A_V} y_{j,i}^{k,s} - \sum_{(i,j)\in A_V} y_{i,j}^{k,s} = 0, \qquad \forall i \in C, k \in K, s \in S,$$

multicommodity flow conservation constraints at stores





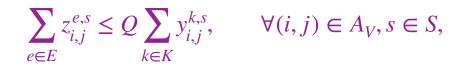
$$\sum_{k \in K} \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = \sum_{e \in E} b_i^{e,s}, \qquad \forall i \in C_-, \forall s \in S,$$

 $\sum_{k \in K} \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = \sum_{e \in E} f_i^{e,s}, \qquad \forall i \in C_+, \forall s \in S,$

$$\sum_{(i,i)\in A_{V}} y_{j,i}^{k,s} - \sum_{(i,i)\in A_{V}} y_{i,j}^{k,s} = 0, \qquad \forall i \in C, k \in K, s \in S,$$

degree constraint at depot:

at most one route allowed per vehicle, and only on days used in the schedule

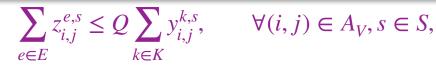




$$\sum_{k \in K} \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = \sum_{e \in E} b_i^{e,s}, \quad \forall i \in C_-, \forall s \in S,$$
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$$\sum_{(j,i) \in A_V} y_{j,i}^{k,s} - \sum_{(i,j) \in A_V} y_{i,j}^{k,s} = 0, \quad \forall i \in C, k \in K, s \in S,$$

each arc of the vehicle network with positive transportation demand must be covered by a vehicle route;

account for vehicle capacity Q



Shift-length: Variables



Variables

 B_i , $\forall i \in V$:arrival time at store i, if $i \in C_-$ departure time from store i - n, if $i \in C_+$ duration of longest service route, if i = 2n + 1equals to 0 for consistency, if i = 0

 $w_i^{e,s}, \forall i \in C_-, e \in E, s \in S: w_i^{e,s} = 1 \Leftrightarrow e \text{ takes break after visiting } i \text{ on day } s$ (in this case, B_{i+n} is departure time after break)

Remind that...

$$x_{i,j}^{e,s}, \forall (i,j) \in A_W, e \in E, s \in S : \quad x_{i,j}^{e,s} = 1 \Leftrightarrow e \text{ walks from } i \text{ to } j \text{ on day } s$$

 $y_{i,j}^{k,s}, \forall (i,j) \in A_V, k \in K, s \in S : \quad y_{i,j}^{k,s} = 1 \Leftrightarrow k \text{ travels through } (i,j) \text{ on day } s$



$$B_{i+n} \ge B_i + t_{i,i+n} + P \sum_{e \in E} \sum_{s \in S} w_i^{e,s}, \qquad \forall (i, i+n) \in A_S,$$

$$B_j \ge B_i + t_{i,j} - M\left(1 - \sum_{e \in E} \sum_{s \in S} x_{i,j}^{e,s}\right), \qquad \forall (i,j) \in A_W,$$

$$B_j \geq B_i + \tau_{i,j} - M\left(1 - \sum_{k \in K} \sum_{s \in S} y_{i,j}^{k,s}\right), \qquad \forall (i,j) \in A_V,$$

account for service times and duration of lunch breaks (when present) Parameters:

- $t_{i,i+n}$: service time at store *i*
 - *P* : duration of lunch break



$$B_{i+n} \ge B_i + t_{i,i+n} + P \sum_{e \in E} \sum_{s \in S} w_i^{e,s}, \qquad \forall (i, i+n) \in A_S,$$

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account for walking times between consecutive stores Parameters: $B_{2n+1} \leq B_{\max}$, $t_{i,j}$: walking time from store *i* to store *j*

M: sufficiently large constant

 $B_0 = 0,$



$$B_{i+n} \ge B_i + t_{i,i+n} + P \sum_{e \in E} \sum_{s \in S} w_i^{e,s}, \qquad \forall (i, i+n) \in A_S,$$

$$B_j \ge B_i + t_{i,j} - M\left(1 - \sum_{e \in E} \sum_{s \in S} x_{i,j}^{e,s}\right), \qquad \forall (i,j) \in A_W,$$

$$B_j \ge B_i + \tau_{i,j} - M\left(1 - \sum \sum y_{i,j}^{k,s}\right), \qquad \forall (i,j) \in A_V,$$

account for vehicle transportation times

when j=2n+1, maximum duration of a service route is computed Parameters:

 $\boldsymbol{D}_{()}$

- $\tau_{i,j}$: driving time from node *i* to node *j*
- *M* : sufficiently large constant

 $\mathbf{v},$



$$B_{i+n} \ge B_i + t_{i,i+n} + P \sum_{e \in E} \sum_{s \in S} w_i^{e,s}, \qquad \forall (i, i+n) \in A_S,$$

$$B_j \ge B_i + t_{i,j} - M\left(1 - \sum_{e \in E} \sum_{s \in S} x_{i,j}^{e,s}\right), \qquad \forall (i,j) \in A_W,$$

$$B_j \geq B_i + \tau_{i,j} - M\left(1 - \sum_{k \in K} \sum_{s \in S} y_{i,j}^{k,s}\right), \qquad \forall (i,j) \in A_V,$$

specify upper bound for route length Parameters:

 B_{max} : maximum allowed duration for a route

 $B_0 = 0,$

Pollster breaks



$$T_0 \sum_{e \in E} \sum_{s \in S} w_i^{e,s} \le B_i + t_{i,i+n} \le T_1 + M \left(1 - \sum_{e \in E} \sum_{s \in S} w_i^{e,s} \right), \qquad \forall i \in C_-,$$

$$w_i^{e,s} \le x_{i,i+n}^{e,s}, \quad \forall i \in C_-, e \in E, s \in S,$$

each break must start within the prescribed time window Parameters:

- [T_0, T_1] : time window for starting of break
 - $t_{i,i+n}$: service time at store *i*
 - *M* : sufficiently large constant

Pollster breaks



$$T_0 \sum_{e \in E} \sum_{s \in S} w_i^{e,s} \le B_i + t_{i,i+n} \le T_1 + M \left(1 - \sum_{e \in E} \sum_{s \in S} w_i^{e,s} \right), \qquad \forall i \in C_-,$$

$$w_i^{e,s} \le x_{i,i+n}^{e,s}, \quad \forall i \in C_-, e \in E, s \in S,$$

sync variables $w_i^{e,s}$ and $x_{i,i+n}^{e,s}$: pollster e can have a break after visiting store i on day s only if eserves i on that day

Pollster breaks



$$T_0 \sum_{e \in E} \sum_{s \in S} w_i^{e,s} \le B_i + t_{i,i+n} \le T_1 + M \left(1 - \sum_{e \in E} \sum_{s \in S} w_i^{e,s} \right), \qquad \forall i \in C_-,$$

 $w_i^{e,s} \le x_{i,i+n}^{e,s}, \quad \forall i \in C_-, e \in E, s \in S,$

each pollster must have exactly one break on each service route

Objective function



 $\min \kappa_0 \sum u_s + \kappa_1 \sum \sum \sum y_{0,i}^{k,s} + \kappa_2 \sum \sum \sum z_{0,i}^{e,s}$ $s \in S \ k \in K \ i \in C$ $s \in S \ k \in K \ i \in C$ $s \in S$

Weighted sum of three components:

- number of working days
- total number of required vehicle routes
- total number of required service routes

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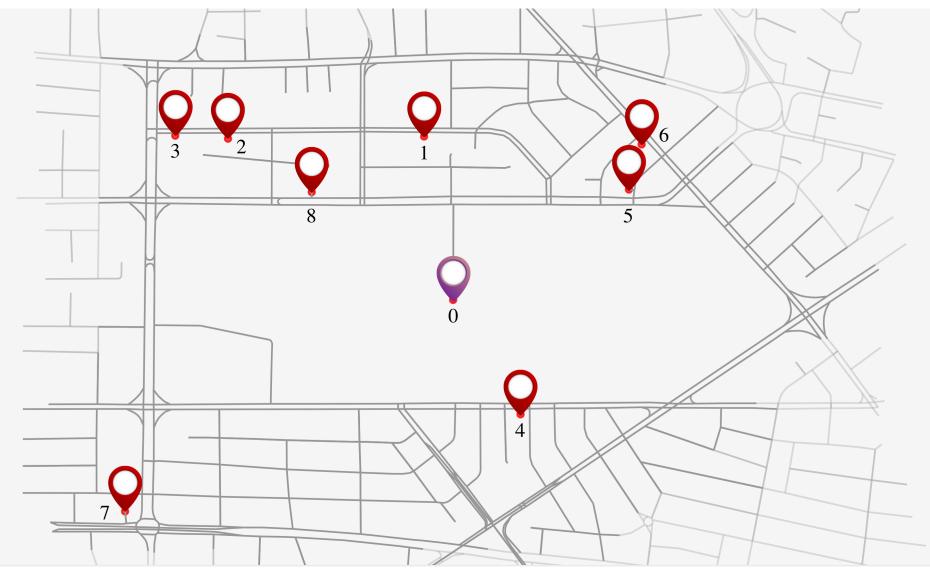
Modeling via mixed integer programming

Computational results



A "toy" example





A "toy" example



- * 8 stores, 2 pollsters, 2 vehicles, 3 days in time horizon
- * Service times in [2; 34]
- * Walking times in [2; 28]
- * Vehicle transportation times in [0.5; 7]
- ***** Shift-duration $B_{\text{max}} = 120$.
- * Pollster break duration P = 20, and must be taken in time window [50; 90]
- ***** Objective coefficients: $\kappa_0 = 200$, $\kappa_1 = 100$, $\kappa_2 = 40$.

A "toy" example



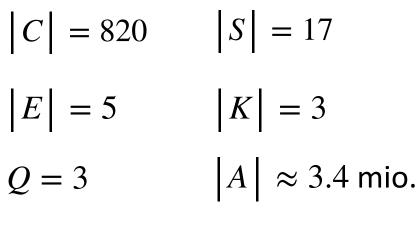


Day 1

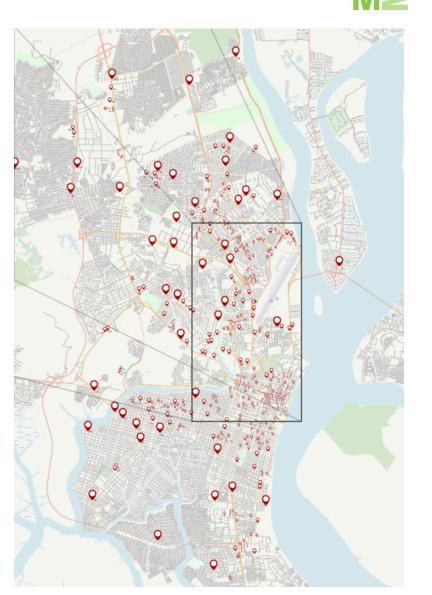
Day 2

- MIP model with 2923 variables and 1683 rows
- Solved on a MacBook Pro i7 2.6GHz with 16 GB RAM, OSX Catalina, using Gurobi
 9.0.2. as MIP solver, TimeLimit= 3600 (best solution and LB found within 2 minutes)
- ***** Feasible solution with Gap= 52.8%, uses 1 vehicle, 2 pollsters, 2 days.

The INEC complete instance



- P = 8 $B_{\text{max}} = 84$
- > 420 mio. binary variables!!!
- > 49 mio. constraints!!!

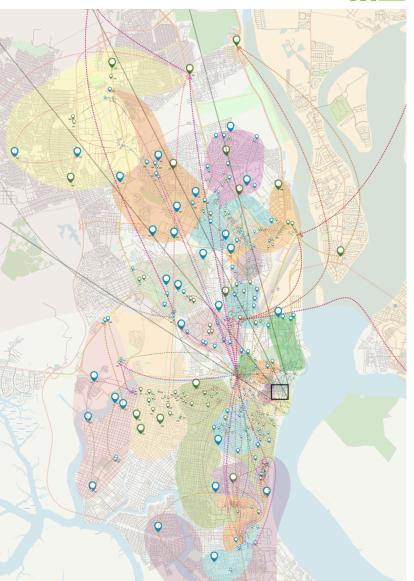


Mode

Mat

Solution approach

- Partition of the instance into "half-daily snapshots" (Graph partitioning techniques balancing total node load).
- Route pollsters and vehicles in each partition without considering lunch break.
- Link partial solutions into a complete schedule (matching problem).





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Results



30 partitions were constructed, with aggregated service times varying in [90; 210]

- Each partition could be solved using 2 pollsters and 1 vehicle.
- Global schedule with 15 days, 2 pollsters, 1 vehicle (previous: 17d, 5p, 3v)



Outline



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Conclusions & Outlook



- Presented a mixed integer programming model for scheduling the visits of stores and for the integrated routing of pollsters and vehicles.
- * Model is hard to solve even for small "toy" instances.
- Three-phase solution approach: partitioning of stores into half-day instances, partial (vehicle + pollster) routing, matching of routes into daily schedules.

Future work

- ***** Tighten linear relaxation (cutting-planes)
- Alternative formulations? Column generation?



Thank you for your attention!!!





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