Combinatorial Optimization at Work 2020

Traffic Optimization

Part I: Paths & Lagrange Relaxation

Part II: Vehicles & Crews

Part III: Pollsters & Vehicles

Zuse Institute Berlin, 22.09.2020

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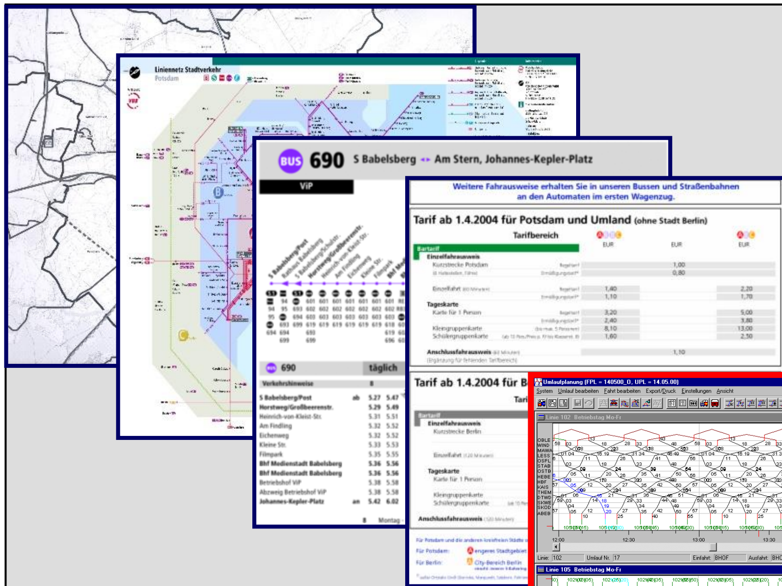
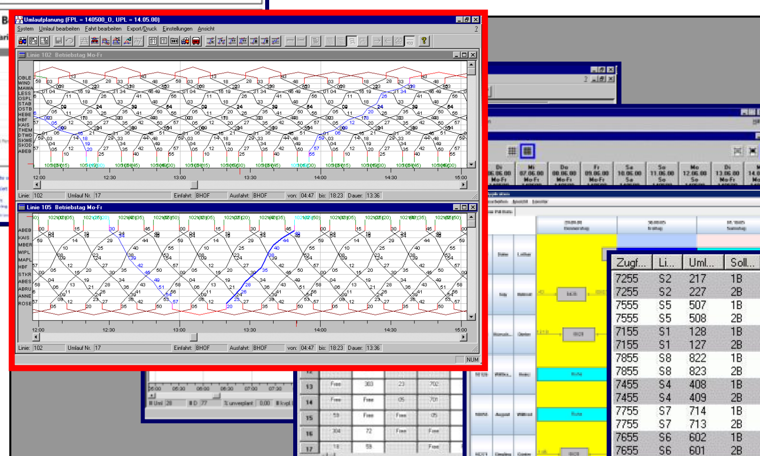
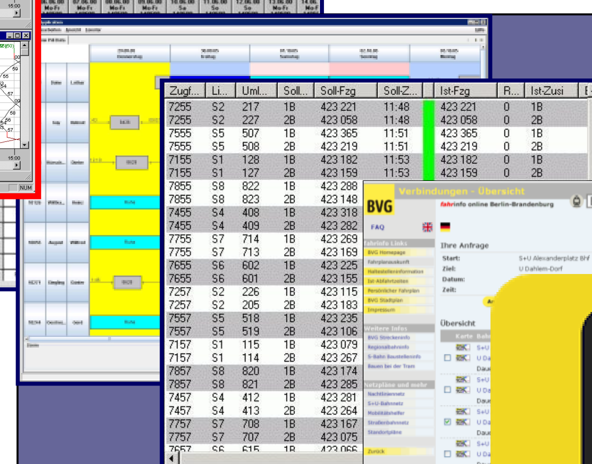
Planning Problems in Public Transit

Service Design



Operational Planning





Operations Control

Passenger Information

Vehicle Scheduling Problem

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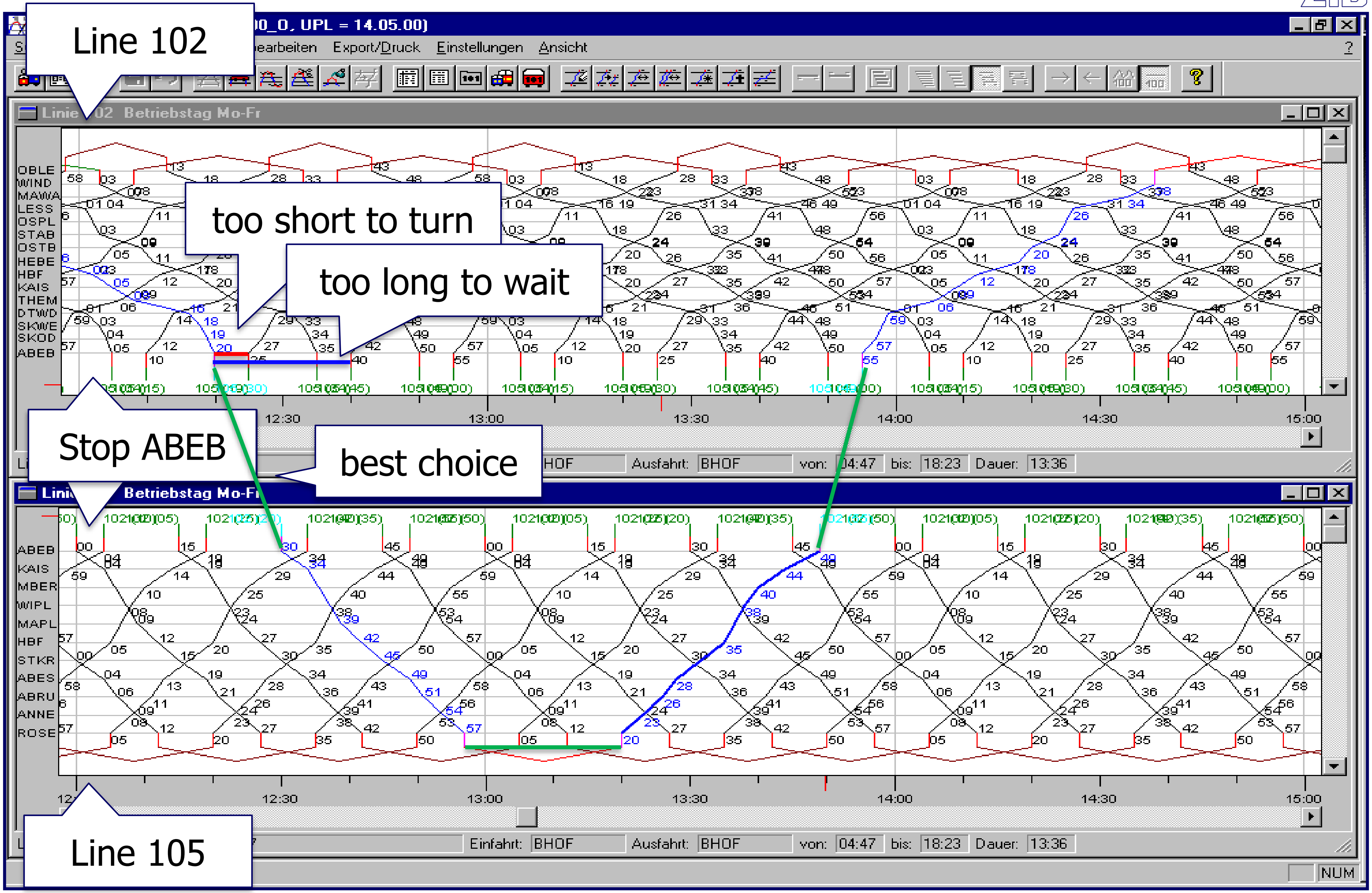

Max "Camel Curve" ≤ Min Fleet Size

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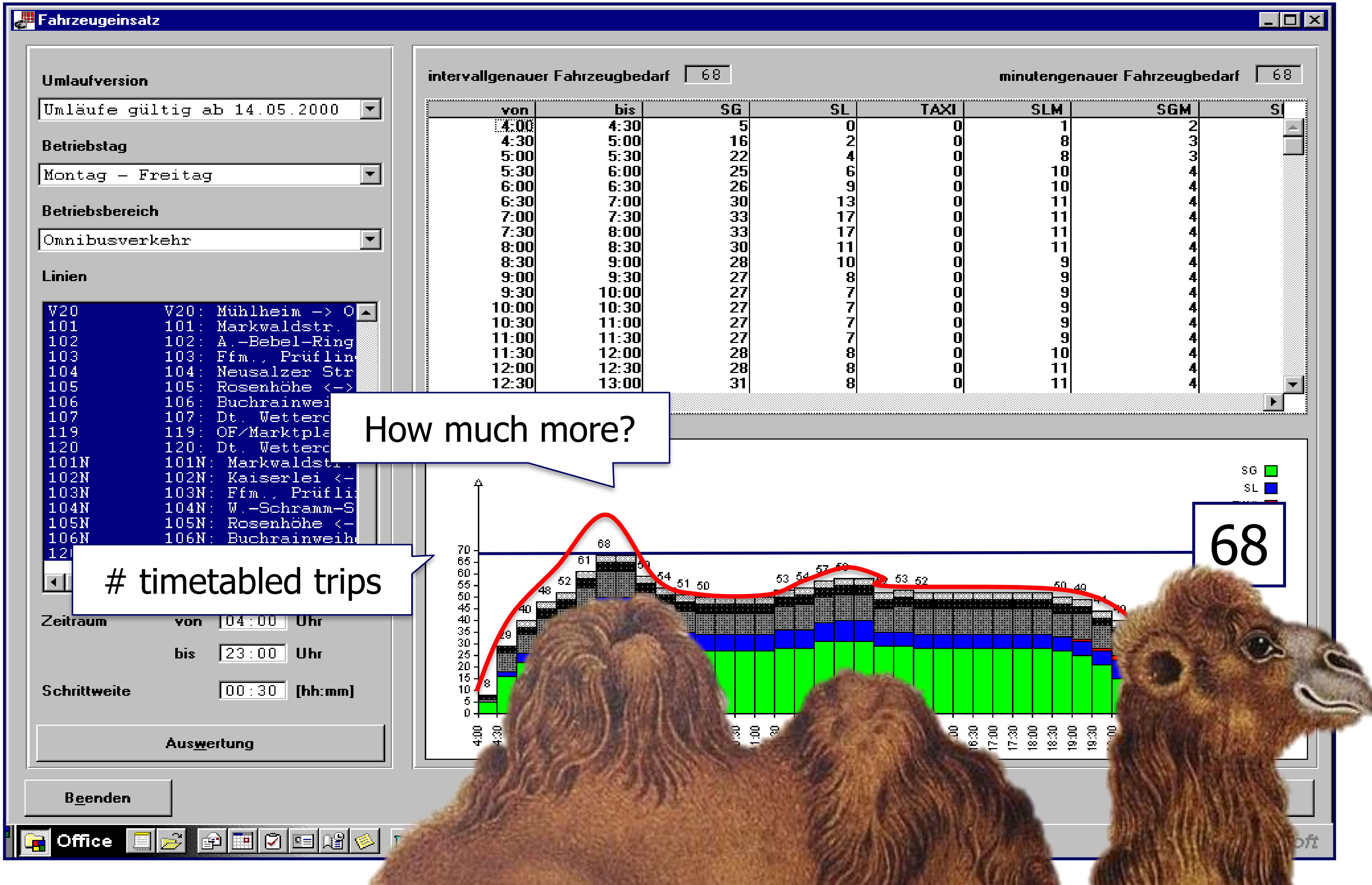
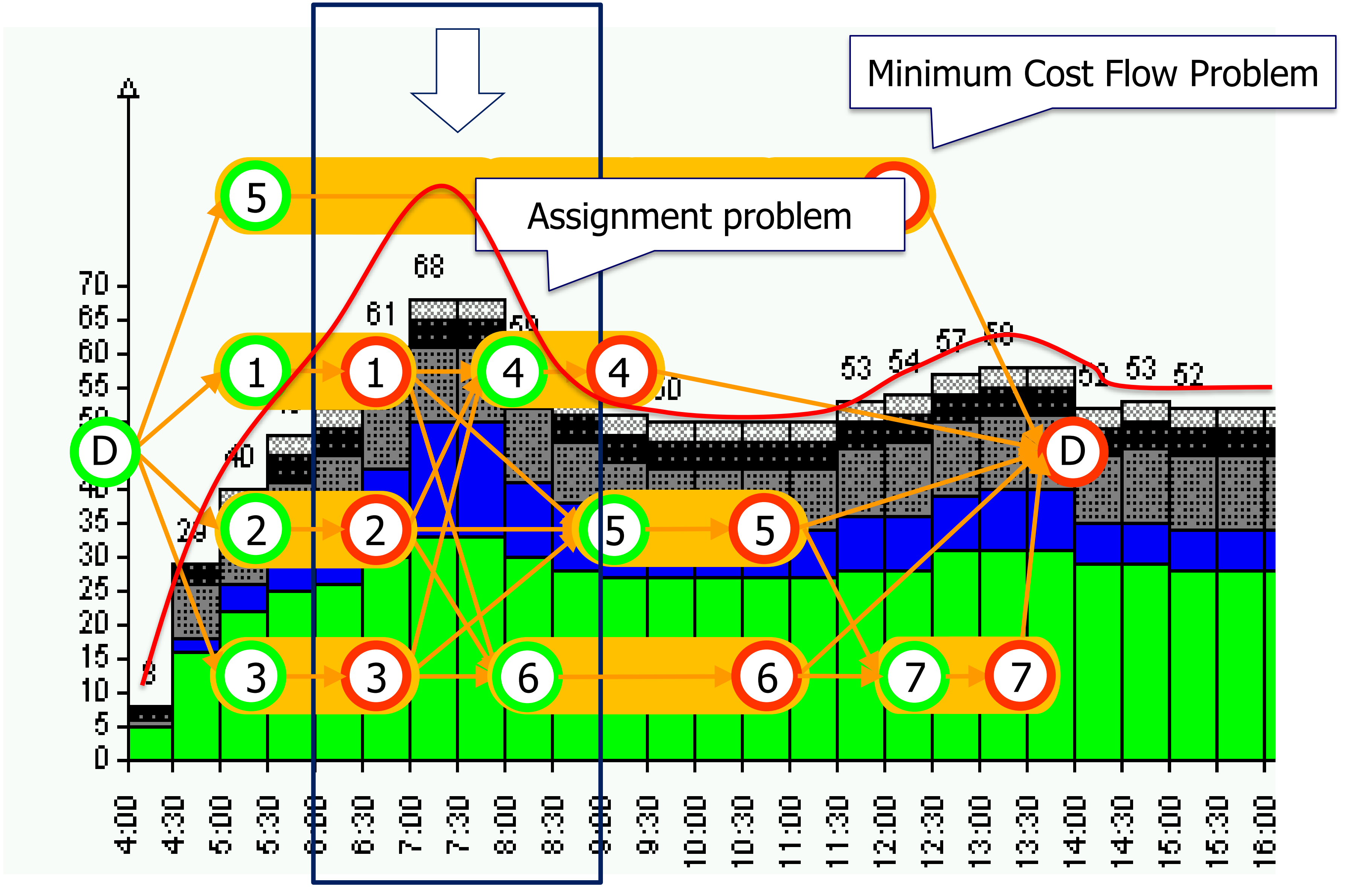


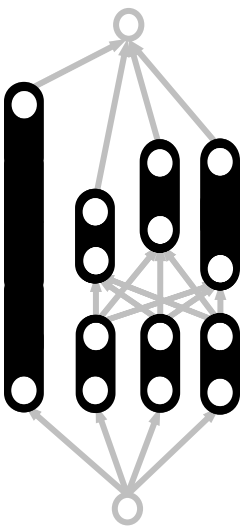
von	bis	SG	SL	TAXI	SLM	SGM	SI
4:00	4:30	5	0	0	1	2	
4:30	5:00	16	2	0	8	3	
5:00	5:30	22	4	0	8	3	
5:30	6:00	25	6	0	10	4	
6:00	6:30	26	9	0	10	4	
6:30	7:00	30	13	0	11	4	
7:00	7:30	33	17	0	11	4	
7:30	8:00	33	17	0	11	4	
8:00	8:30	30	11	0	11	4	
8:30	9:00	28	10	0	9	4	
9:00	9:30	27	8	0	9	4	
9:30	10:00	27	7	0	9	4	
10:00	10:30	27	7	0	9	4	
10:30	11:00	27	7	0	9	4	
11:00	11:30	27	7	0	9	4	
11:30	12:00	28	8	0	10	4	
12:00	12:30	28	8	0	11	4	
12:30	13:00	31	8	0	11	4	

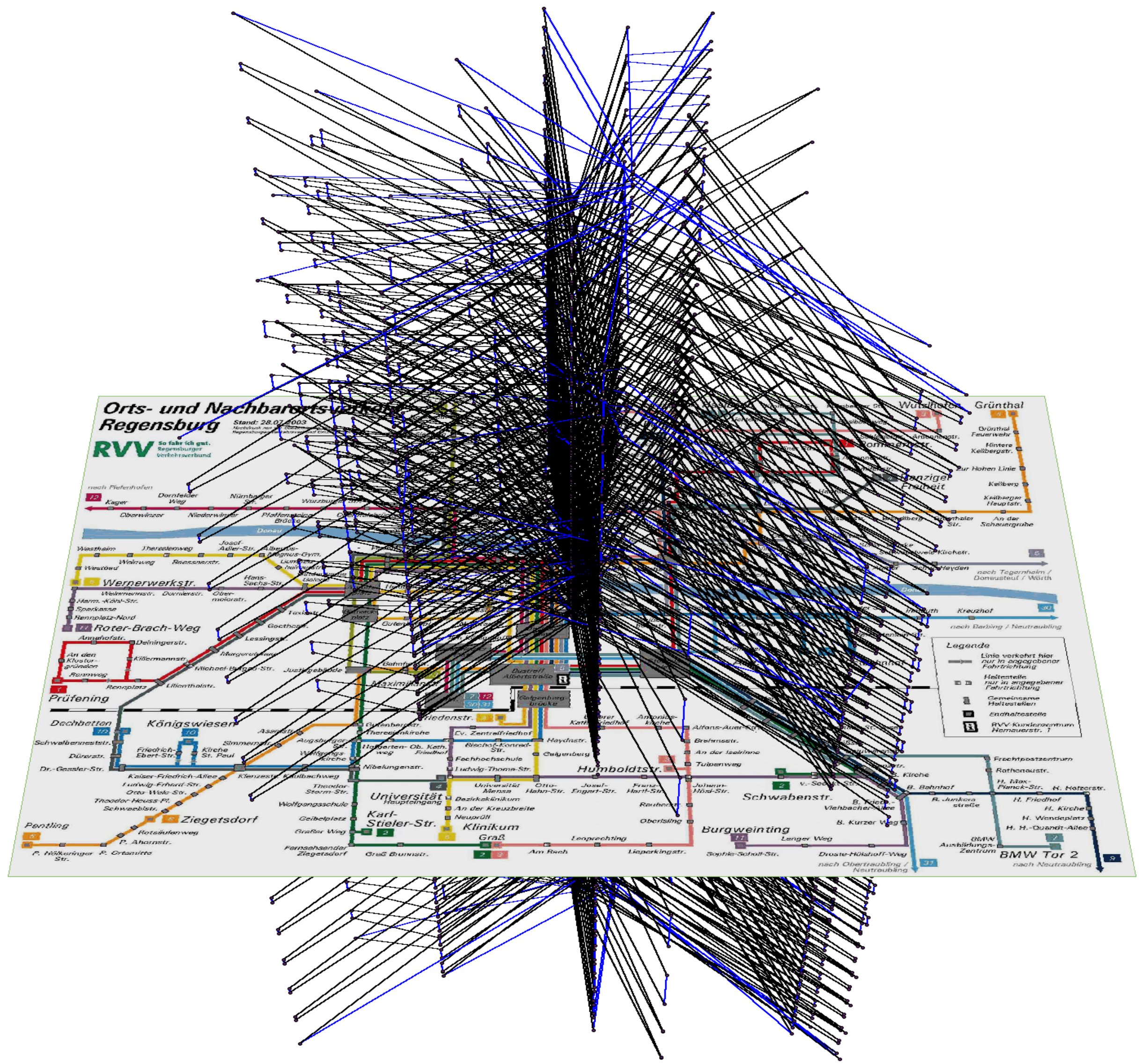
https://de.m.wikipedia.org/wiki/Datei:Das_Kamel.jpg

Flattening the Curve



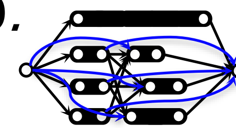
Vehicle Scheduling Graph (Only Timetabled Trips)





Single-Depot Vehicle Scheduling Problem

2.1 Def. (Single-Depot Vehicle Scheduling Problem): Let $D = (V, A, c)$ be a directed acyclic graph (DAG) with node set $V = T \mathbin{\dot\cup} \{s, t\}$ and arc weights $c \in \mathbb{R}^A_{\geq 0}$ s.t. $\delta^-(s) = \delta^+(t) = \emptyset$.



(SDVSP)	$\min c^T x$		objective
(i)	$x(\delta^+(v)) - x(\delta^-(v)) = 0$	$\forall v \neq s, t$	flow conservation
(ii)	$x(\delta^-(v)) = 1$	$\forall v \neq s, t$	flow constraints
(iii)	$0 \leq x \leq 1$		bounds
(iv)	x integer		integrality

Speech: T are the **timetabled trips**, s, t the **depot nodes**, A the **deadhead trips**.

a) $P^{SDVSP} := \text{conv}\,\{x \in \mathbb{R}^E : \text{(SDVSP) (i)} - \text{(iv)}\}$ **SDVSP polytope**

b) $P^{SDVSP}_{LP} := \text{conv}\,\{x \in \mathbb{R}^E : \text{(SDVSP) (i)} - \text{(iii)}\}$ **SD Flow Relax.**

2.2 Obs. (SDVSP): $P^{SDVSP} = P^{SDVSP}_{LP} \Longrightarrow$ SDVSP solvable in polytime.

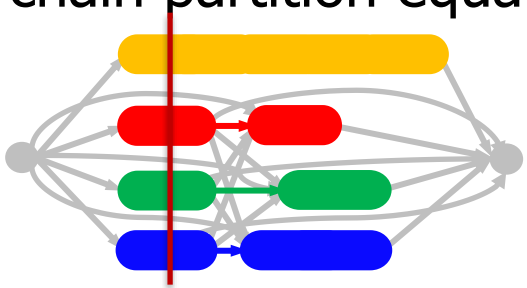
Proof: (SDVSP) (i) − (iii) is a minimum cost flow problem. □

2.1 Def. (Single-Depot Vehicle Scheduling Problem): Let $D = (V, A, c)$ be a directed acyclic graph (DAG) with node set $V = T \cup \{s, t\}$ and arc weights $c \in \mathbb{R}^A_{\geq 0}$ s.t. $\delta^+(s) = \delta^-(t) = T$.

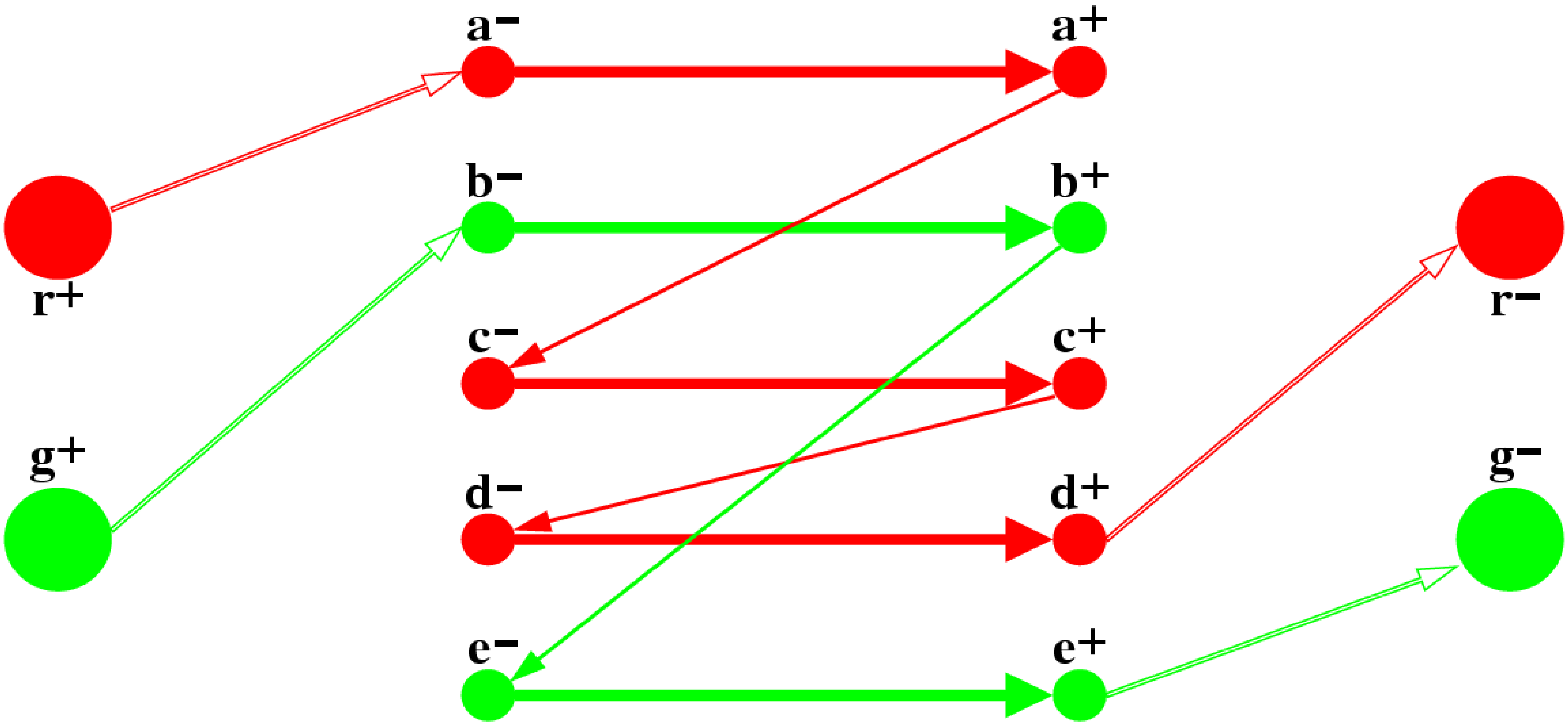
(SDVSP)	$\min c^T x$		objective
(i)	$x(\delta^+(v)) - x(\delta^-(v)) = 0$	$\forall v \neq s, t$	flow conservation
(ii)	$x_t = 1$	$\forall t \in T$	flow constraints
(iii)	$0 \leq x \leq 1$		bounds
(iv)	x integer		integrality

2.3 Obs. (Minimum Fleet Size): The min size of a homogenous fleet equals the max number of pairwise incompatible trips.

Proof: Define a partial ordered set $(T, \leq)$ via $u \leq v :\Leftrightarrow uv \in A$. By Dilworth's Theorem, the minimum size of a chain partition equals the maximum size of an antichain. Identify chains with vehicle rotations, antichains with pairwise incompatible trips. □



Multiple-Depot Vehicle Scheduling



Multiple-Depot Vehicle Scheduling Problem

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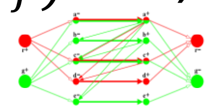
2.4 Def. (Multiple-Depot Vehicle Scheduling Problem): Let F be a set of **fleets** and $D = (V, A, c, \kappa)$ a directed acyclic multigraph with nodes $V = T \cup \{s_f, t_f : f \in F\}$, arcs $A = \cup_{f \in F} A_f$, weights $c \in \mathbb{R}^A_{\geq 0}$, and **capacities** $\kappa \in \mathbb{N}^F$; let $\delta^-(s_f) = \delta^+(t_f) = \emptyset$, $\delta^+(s_f), \delta^-(t_f) \subseteq A_f$.



(MDVSP)	$\min c^T x$		objective
(i)	$x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0$	$\forall v \neq s, t$ $\forall f \in F$	flow conservation per fleet
(ii)	$x\left(\delta^-(v)\right) = 1$	$\forall v \neq s, t$	flow constraints
(iii)	$s)) \leq \kappa_f$	$\forall f \in F$	fleet capacities
(iv)	$0 \leq x \leq 1$		bounds
(v)	x integer		integrality

all feasible fleets

Note: All fleets service the same timetabled trips, $\delta = \cup_{f \in F} \delta_f$.

a) $P^{MDVSP} := \text{conv}\{x \in \mathbb{R}^E : \text{(MDVSP) (i)} - \text{(v)}\}$ **MDVSP polytope**

b) $P_{LP}^{MDVSP} := \text{conv}\{x \in \mathbb{R}^E : \text{(MDVSP) (i)} - \text{(iv)}\}$ **MD Flow Relax.**

Multiple-Depot Vehicle Scheduling Problem

2.4 Def. (Multiple-Depot Vehicle Scheduling Problem): Let F be a set of **fleets** and $D = (V, A, c, \kappa)$ a directed acyclic multigraph with nodes $V = T \cup \{s_f, t_f : f \in F\}$, arcs $A = \cup_{f\in F} A_f$, weights $c \in \mathbb{R}^A_{\geq 0}$, and **capacities** $\kappa \in \mathbb{N}^F$; let $\delta^-(s_f) = \delta^+(t_f) = \emptyset$, $\delta^+(s_f), \delta^-(t_f) \subseteq A_f$.

(MDVSP)	$\min c^T x$		objective
(i)	$x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0$	$\forall v \neq s, t$ $\forall f \in F$	flow conservation per fleet
(ii)	$x\left(\delta^-(v)\right) = 1$	$\forall v \neq s, t$	flow constraints
(iii)	$x\left(\delta_f^+(s)\right) \leq \kappa_f$	$\forall f \in F$	fleet capacities
(iv)	$0 \leq x \leq 1$		bounds
(v)	x integer		integrality

2.5 Obs. (MDVSP): a) $P^{MDVSP} \subseteq P_{LP}^{MDVSP}$, in general $\subsetneq$. b) MDVSP is NP-hard.

Proof: a), b) Transformation from 1in3 3SAT with unneg. literals. □

Lagrange Relaxation I: Flow Constraints

2.6 Obs. (Multiple SDVSP Relaxation): The Lagrange relaxation of the (MDVSP) w.r.t. the flow constraints (ii) is …

(MDVSP)	$\min c^T x$		objective
(i)	$x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0$	$\forall v \neq s,t$ $\forall f \in F$	flow conservation per fleet
(ii)	$x\left(\delta^-(v)\right) = 1$	$\forall v \neq s,t$	flow constraints
(iii)	$x\left(\delta_f^+(s)\right) \leq \kappa_f$	$\forall f \in F$	fleet capacities
(iv)	$0 \leq x \leq 1$		bounds
(v)	x integer		integrality

2.6 Obs. (Multiple SDVSP Relaxation): The Lagrange relaxation of the (MDVSP) w.r.t. the flow constraints (ii) is

(LR(ii))	$\max_{\pi} \min c^T x - \sum_{v \neq s,t} \pi_v x(\delta^-(v))$	$+\pi^T 1$	objective
(i)	$x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0$	$\forall v \neq s,t$ $\forall f \in F$	flow cons. per fleet
(iii)	$x\left(\delta_f^+(s)\right) \leq \kappa_f$	$\forall f \in F$	capacities
(iv)	$0 \leq x \leq 1$		bounds
(v)	x integer		integrality

a) The subproblem (the inner minimization) decomposes into independent SDVSPs, one for each fleet.

b) For $c \geq 0$ and $\pi = 0$, the optimal objective of the subproblem is 0.

Aggregate Flow Conservation

2.7 Def. (Aggregate Flow Conservation): Consider an MDVSP $D = (V, A, c, \kappa)$.

(MDVSP′)	$\min c^T x$		objective
(i)	$x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0$	$\forall v \neq s, t$ $\forall f \in F$	flow conservation per fleet
(i′)	$x(\delta^+(v)) - x\big(\delta^-(v)\big) = 0$	$\forall v \neq s_f, t_f,$ $f \in F$	aggregate flow conservation
(ii)	$x\big(\delta^-(v)\big) = 1$	$\forall v \neq s, t$	flow constraints
(iii)	$x\left(\delta_f^+(s)\right) \leq \kappa_f$	$\forall f \in F$	fleet capacities
(iv)	$0 \leq x \leq 1$		bounds
(v)	x integer		integrality

2.8 Obs. (MDVSP’): (MDVSP′) $\Leftrightarrow$ (MDVSP), and this also holds for the LP relaxations.

Proof: $x\big(\delta^+(v)\big) - x\big(\delta^-(v)\big) = \sum_{f \in F} x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0.$ □

Lagrange Relaxation II: Flow Conservation

2.9 Obs. (Common SDVSP Relaxation): The Lagrange Relaxation of the (MDVSP’) w.r.t. the flow conservation constraints (i) is …

(MDVSP′)	$\min c^T x$		objective
(i)	$x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0$	$\forall v \neq s,t$ $\forall f \in F$	flow conservation per fleet
(i′)	$x(\delta^+(v)) - x\left(\delta^-(v)\right) = 0$	$\forall v \neq s_f, t_f,$ $f \in F$	aggregate flow conservation
(ii)	$x\left(\delta^-(v)\right) = 1$	$\forall v \neq s,t$	flow constraints
(iii)	$x\left(\delta_f^+(s)\right) \leq \kappa_f$	$\forall f \in F$	fleet capacities
(iv)	$0 \leq x \leq 1$		bounds
(v)	x integer		integrality

Lagrange Relaxation II: Flow Conservation

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2.9 Obs. (Common SDVSP Relaxation): The Lagrange relaxation of the (MDVSP') w.r.t. the flow conservation constraints (i) is …

$$\text{(LR(i))} \quad \max_{\pi} \min c^T x - \sum_{\substack{v \neq s,t \\ f \in F}} \pi_{vf} [x(\delta^-(v)) - x(\delta^-(v))]$$

(i')	$x(\delta^+(v)) - x(\delta^-(v)) = 0$	$\forall v \neq s_f, t_f, f \in F$
(ii)	$x(\delta^-(v)) = 1$	$\forall v \neq s, t$
(iii)	$x\left(\delta_f^+(s)\right) \leq \kappa_f$	$\forall f \in F$
(iv)	$0 \leq x \leq 1$	
(v)	x integer	

a) The subproblem is a common SDVSP (for a homogenized fleet).

b) For $c \geq 0$ and $\pi = 0$, the subproblem optimum can be > 0.

Solving the MDVSP

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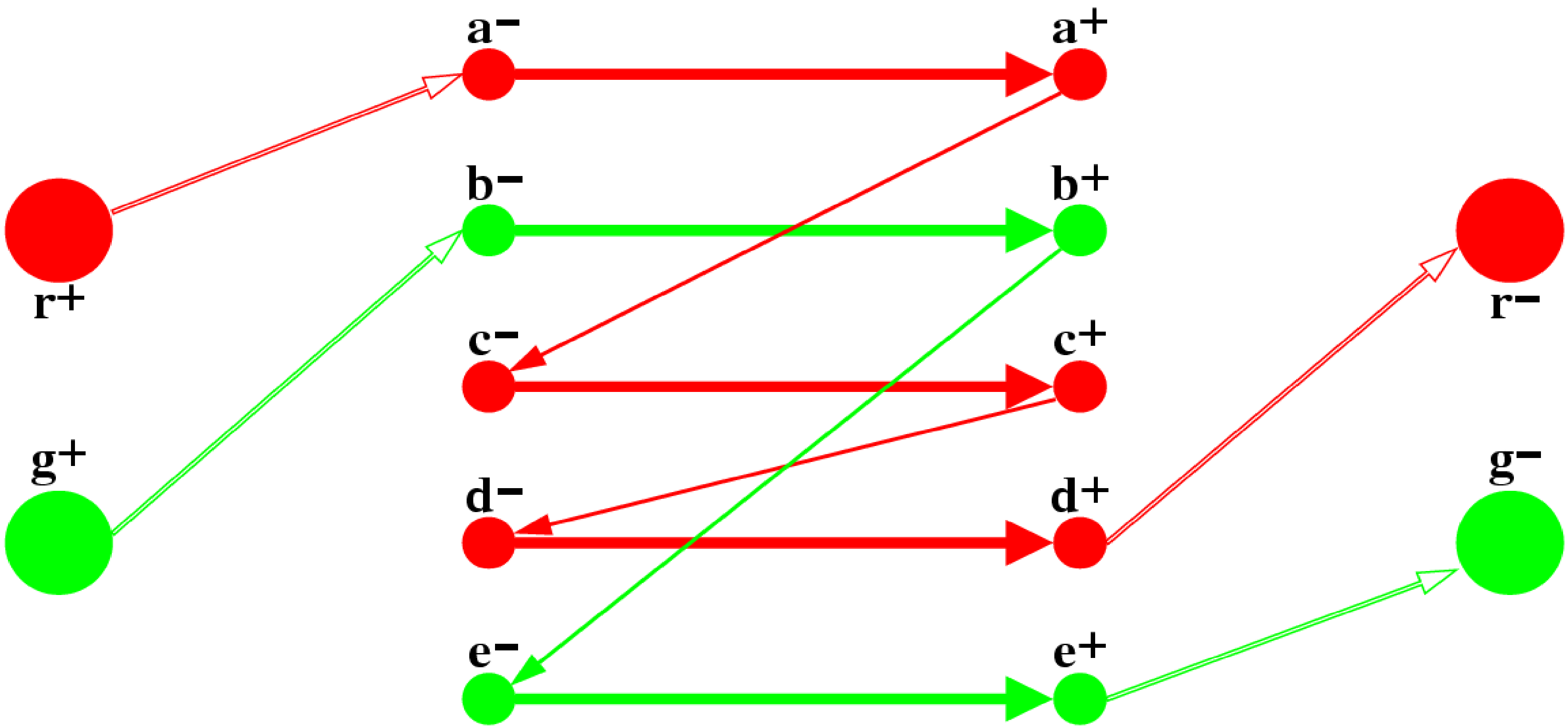
2.10 Alg. (Löbel [1997]):

Input: $D = (V, A, c, \kappa)$

Output: $x \approx \operatorname{argmin} \operatorname{MDVSP}(V, A, c, \kappa)$ (hopefully)

1. solve common SDVSP relaxation $\mathrm{LR(i)}$ // solve single fleet LR
2. forall $f \in F$ do
3. $\quad V_f \leftarrow \left\{v \in V : x\left(\delta_f^-(v)\right) = 1\right\} \cup \{s_f, t_f\}$ // trip2fleet assignment
4. endforall
5. forall $f \in F$ do
6. $\quad x_f \leftarrow \operatorname{argmin} \operatorname{SDVSP}\left(D\left[V_f\right]\right)$ // reoptimize each fleet
7. endforall
8. if satisfied then output $x = (x_f)$, stop endif
9. reassign some trips to other fleets by tabu search //reassign trips
10. goto 5

Solving the MDVSP



Solving Real World Urban Scenarios

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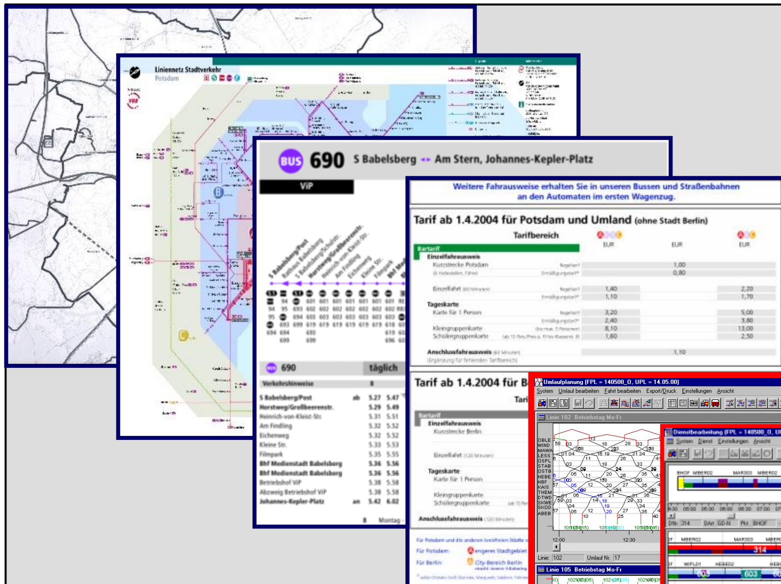
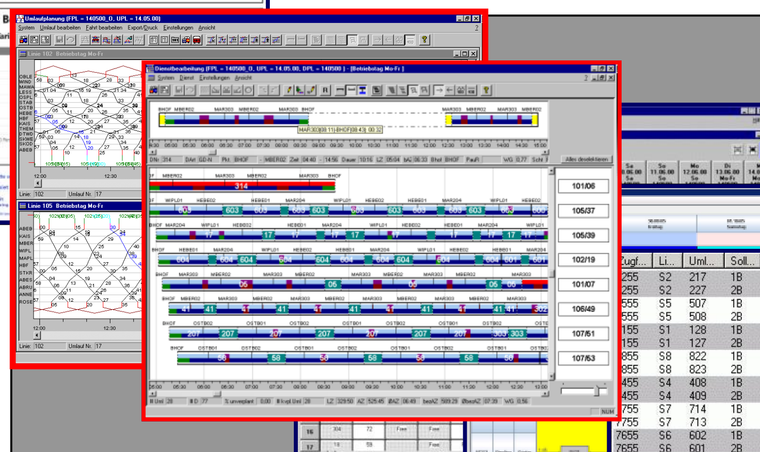
	BVG	*HHA*	*VHH*
depots	10	14	10
vehicle types	44	40	19
timetabled trips	25 000	16 000	5 500
deadheads	70 000 000	15 100 000	10 000 000
cpu mins	200	50	28

Planning Problems in Public Transit

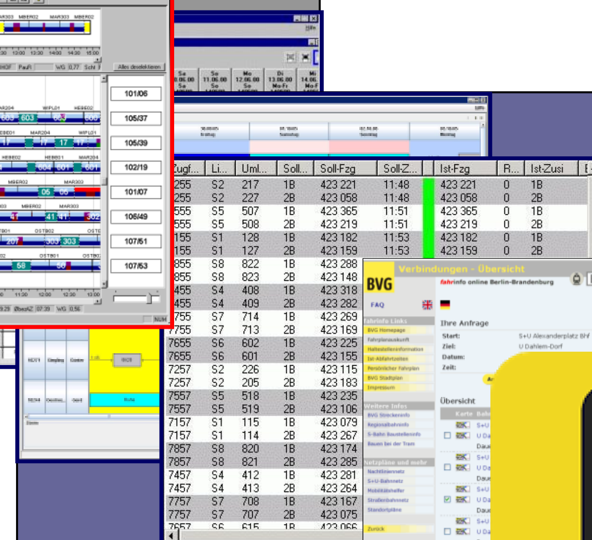
Service Design



Operational Planning

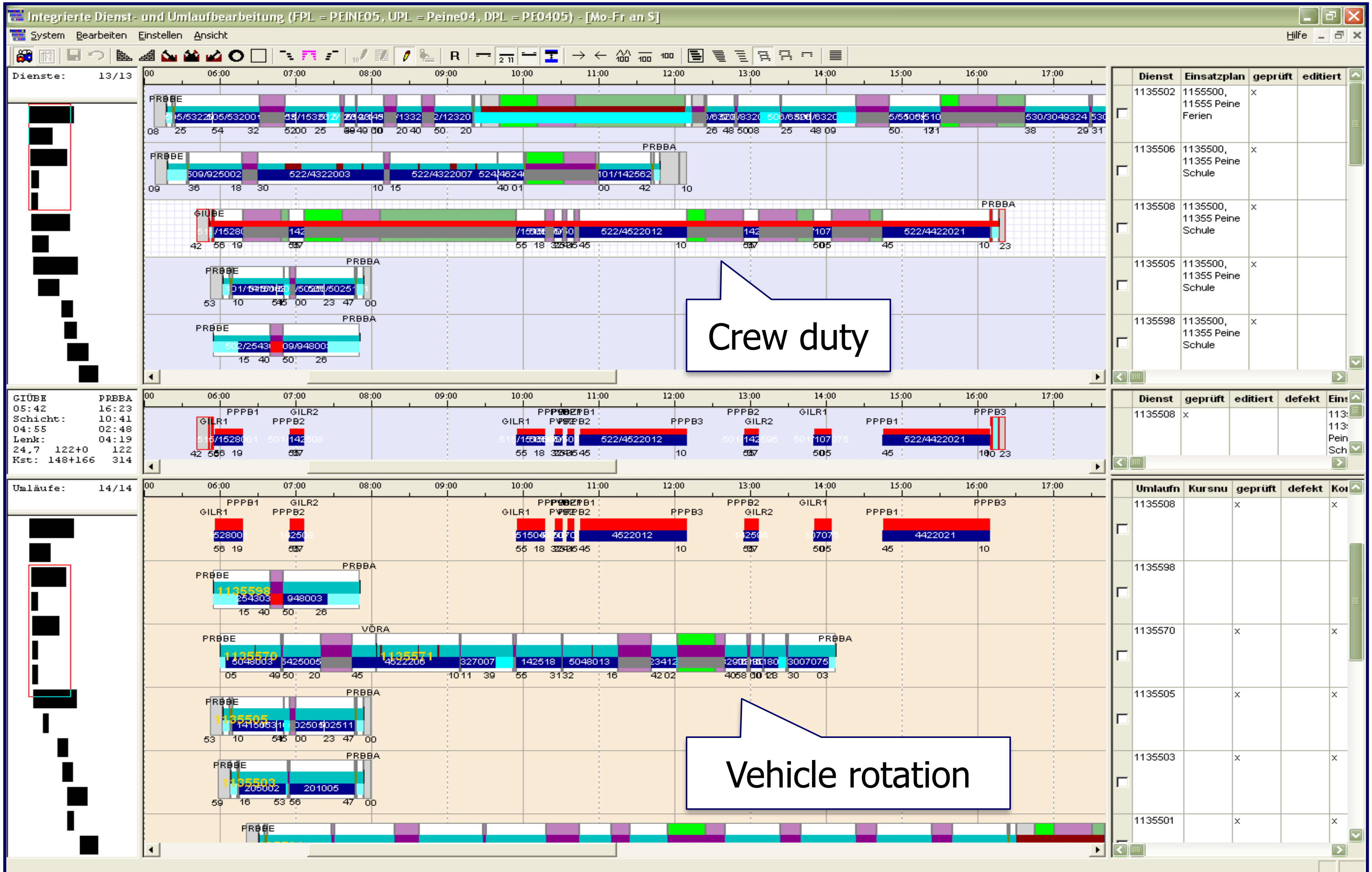


Operations Control

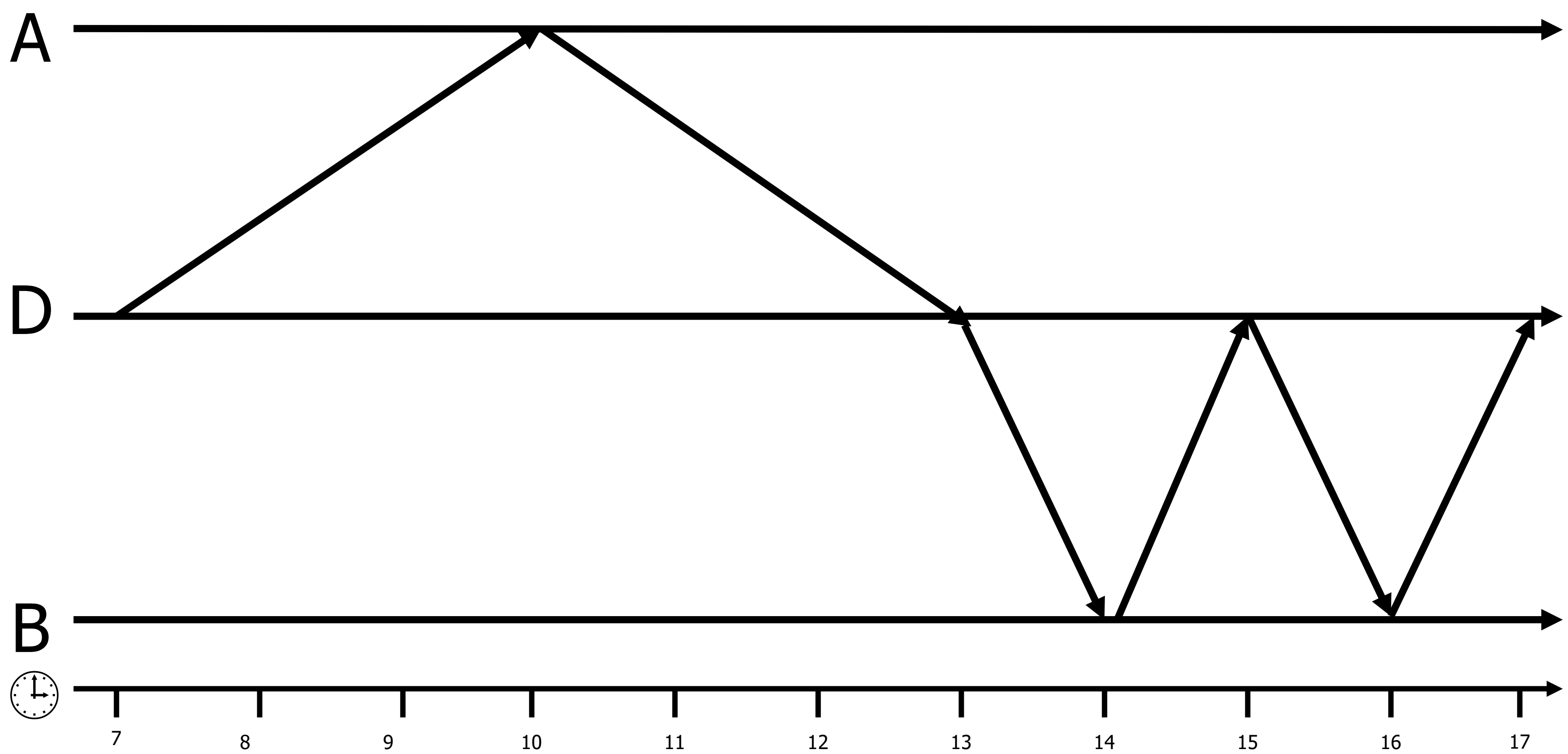


Passenger Information

Crew Scheduling

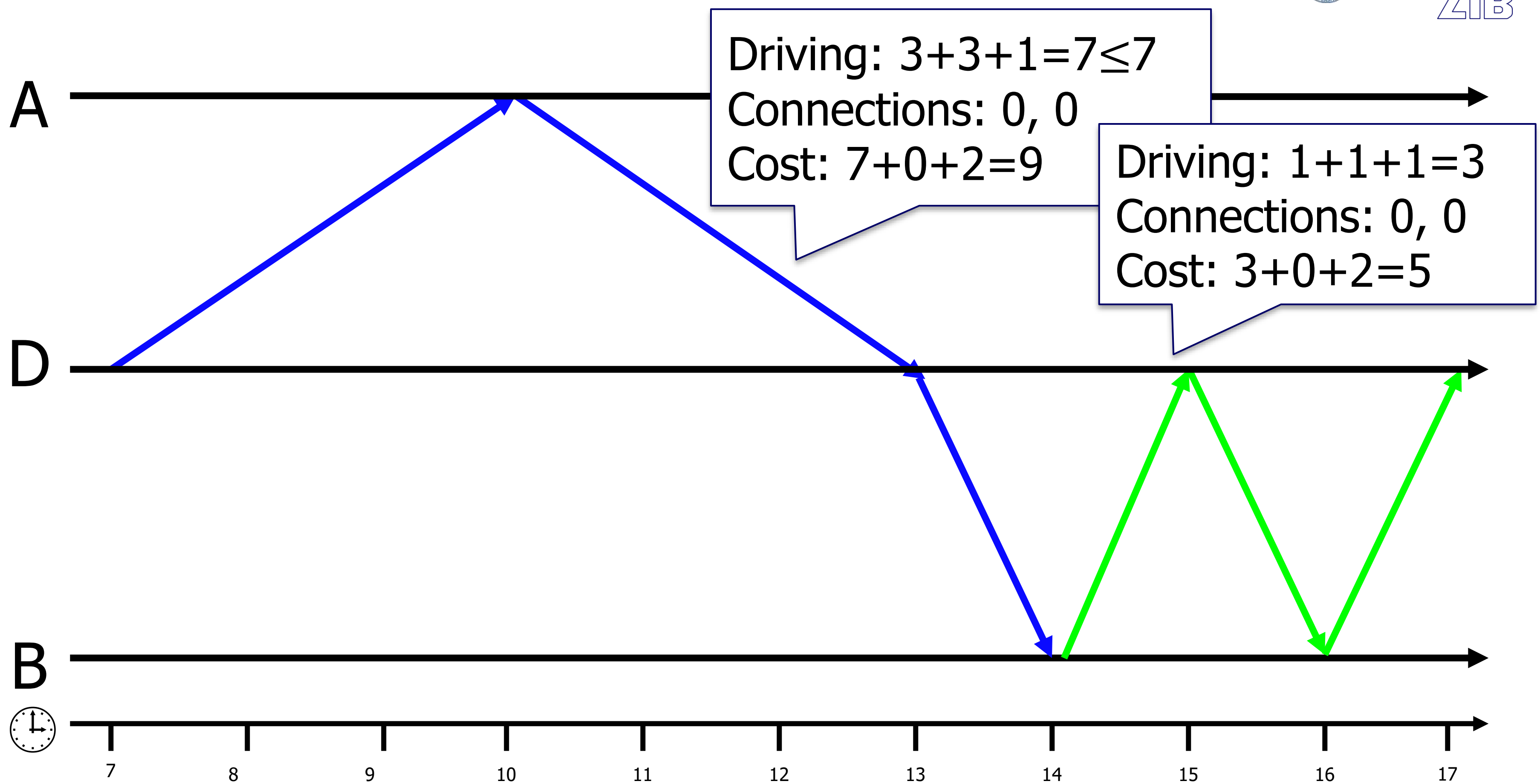


Crew Scheduling Example



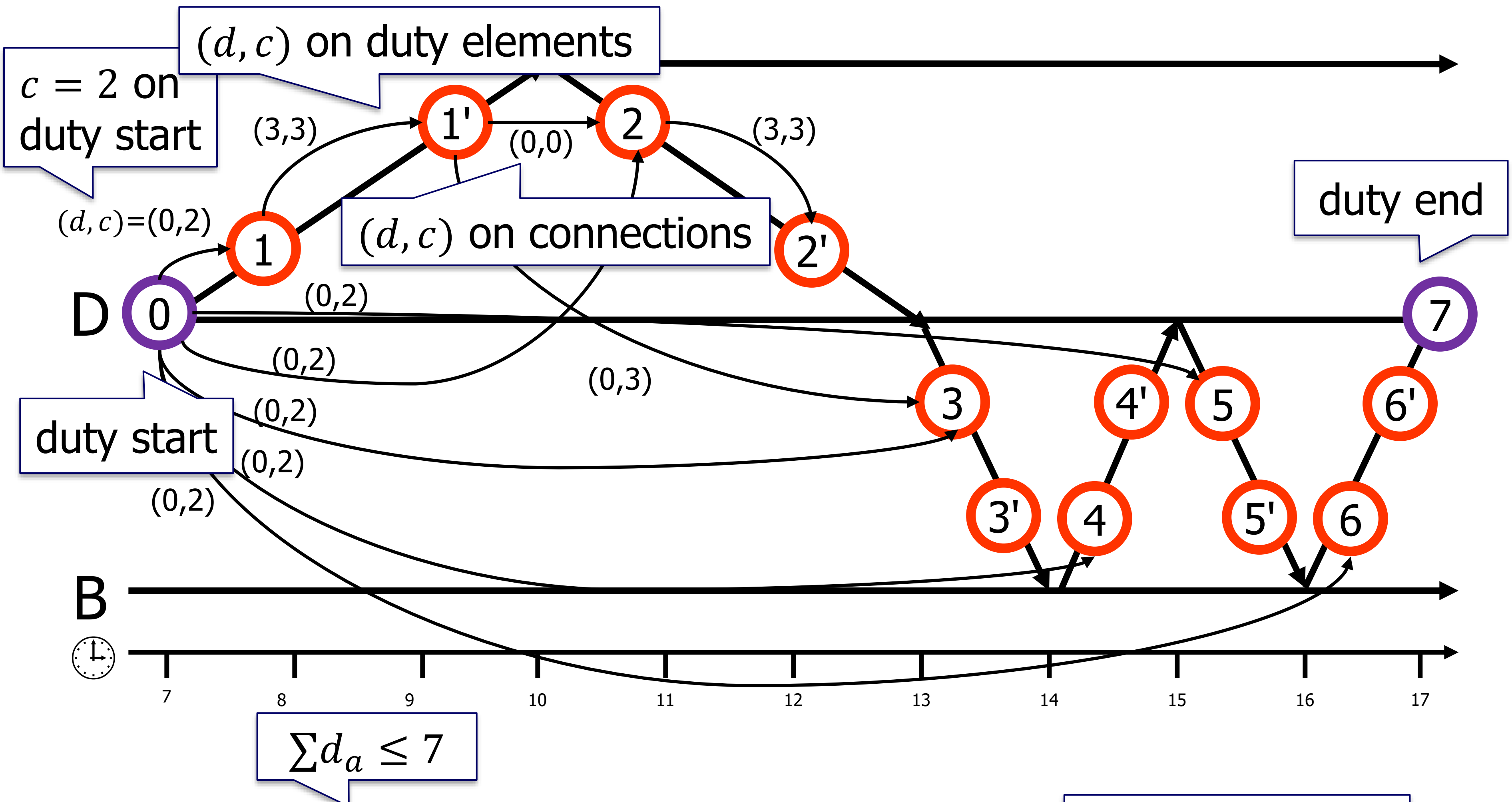
- Rules: Driving time ≤ 7 h, connections ≤ 3 h
- Costs: 2 + duty time

Crew Scheduling Example



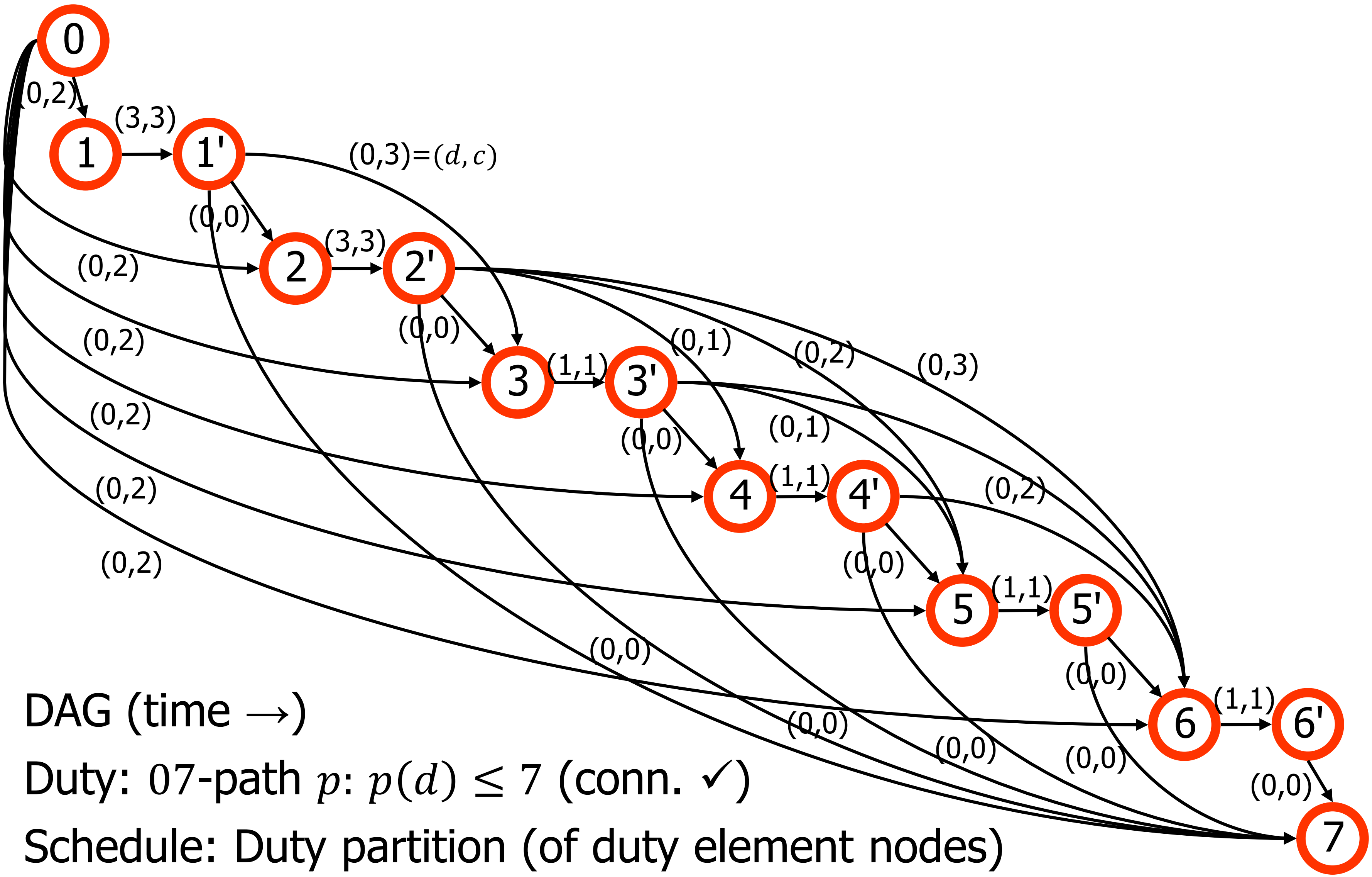
- Rules: Driving time ≤ 7 h, connections ≤ 3 h
- Costs: 2 + duty time

Graph Theoretic Model



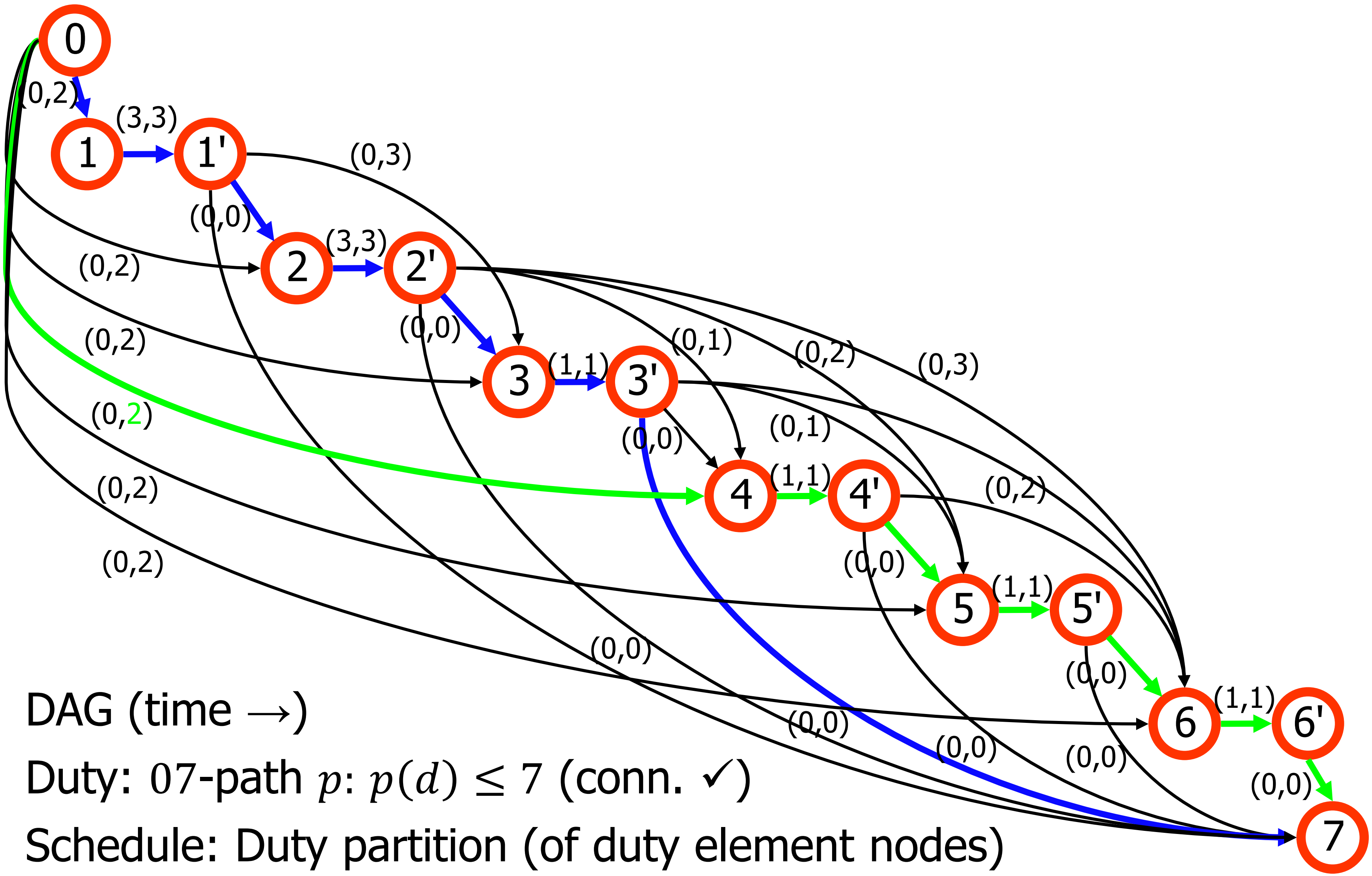
- Rules: Driving time ≤ 7 h, connections ≤ 3 h — arc construction
- Costs: 2 + duty time — $\sum c_a$

Graph Theoretic Model



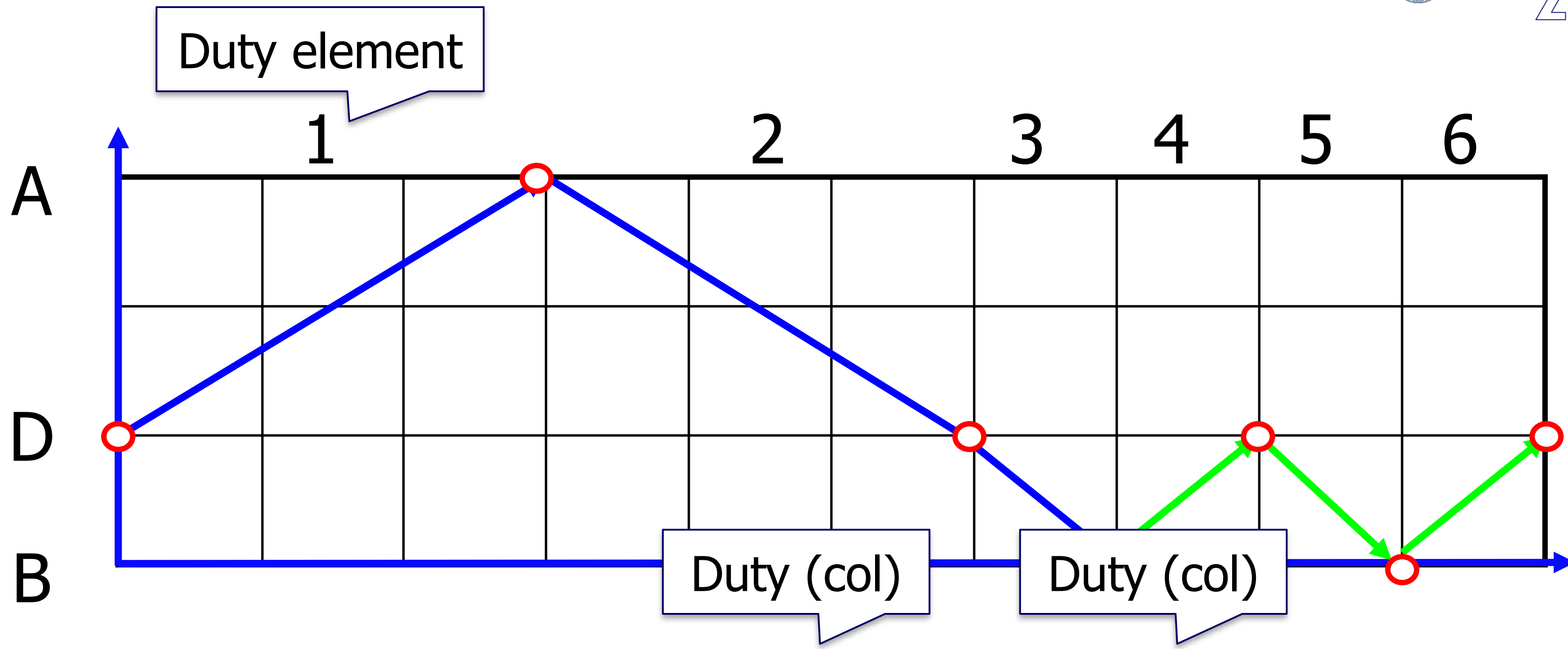
- DAG (time $\rightarrow$)
- Duty: 07-path p: $p(d) \leq 7$ (conn. ✓)
- Schedule: Duty partition (of duty element nodes)

Graph Theoretic Model



- DAG (time →)
- Duty: 07-path p: $p(d) \leq 7$ (conn. ✓)
- Schedule: Duty partition (of duty element nodes)

Duty Table



no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4
1	1						1	1										
2		1					1		1	1	1	1						
[illegible]	[illegible]	[illegible]	1					1	1				1	1	1			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]					1			1			1	1	
5					1						1			1		1		1
6						1						1			1		1	1

no	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
c	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9
1	1	1	1	1							1	1	1					1	
2	1	1	1	1	1	1	1							1	1	1			1
[illegible]	1				1	1	1	1	1		1	1		1	1		1	1	1
[illegible]		1			1			1	1	1	1	1	1	1	1	1	1	1	1
5			1			1		1		1	1		1	1		1	1	1	1
6				1			1		1	1		1	1		1	1	1	1	1

Duty element

IP Model

no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	
1	1						1	1											1	1	1	1							1	1	1					1		1
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	1
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	1
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	1
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	1
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	1
	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31	x32	x33	x34	x35	x36	x37	

01 duty variables

$$\min \quad 5x_1+5x_2+\ldots+12x_{36}+9x_{37}$$

$$x_1+x_7+x_8+x_{19}+x_{20}+x_{21}+x_{22}+x_{29}+x_{30}+x_{31}+x_{36}=1$$

$$x_2+x_7+x_9+x_{10}+x_{11}+x_{12}+x_{19}+x_{20}+x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{32}+x_{33}+x_{34}+x_{37}=1$$

$$x_3+x_8+x_9+x_{13}+x_{14}+x_{15}+x_{19}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27}+x_{29}+x_{30}+x_{32}+x_{33}+x_{35}+x_{36}+x_{37}=1$$

$$x_4+x_{10}+x_{13}+x_{16}+x_{17}+x_{20}+x_{23}+x_{26}+x_{27}+x_{28}+x_{29}+x_{30}+x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}=1$$

$$x_5+x_{11}+x_{14}+x_{16}+x_{18}+x_{21}+x_{24}+x_{26}+x_{28}+x_{29}+x_{31}+x_{32}+x_{34}+x_{35}+x_{36}+x_{37}=1$$

$$x_6+x_{12}+x_{15}+x_{17}+x_{18}+x_{22}+x_{25}+x_{27}+x_{28}+x_{30}+x_{31}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}=1$$

$$x_1,\ldots,x_{37} \geq 0$$

$$x_1,\ldots,x_{37} \text{ integer}$$

IP Model

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$\min 5x_1+5x_2+\ldots+12x_{36}+9x_{37}$

$x_1+x_7+x_8+x_{19}+x_{20}+x_{21}+x_{22}+x_{29}+x_{30}+x_{31}+x_{36}=1$

$x_2+x_7+x_9+x_{10}+x_{11}+x_{12}+x_{19}+x_{20}+x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{32}+x_{33}+x_{34}+x_{37}=1$

$x_3+x_8+x_9+x_{13}+x_{14}+x_{15}+x_{19}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27}+x_{29}+x_{30}+x_{32}+x_{33}+x_{35}+x_{36}+x_{37}=1$

$x_4+x_{10}+x_{13}+x_{16}+x_{17}+x_{20}+x_{23}+x_{26}+x_{27}+x_{28}+x_{29}+x_{30}+x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}=1$

$x_5+x_{11}+x_{14}+x_{16}+x_{18}+x_{21}+x_{24}+x_{26}+x_{28}+x_{29}+x_{31}+x_{32}+x_{34}+x_{35}+x_{36}+x_{37}=1$

$x_6+x_{12}+x_{15}+x_{17}+x_{18}+x_{22}+x_{25}+x_{27}+x_{28}+x_{30}+x_{31}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}=1$

$0 \le x_1,\ldots,x_{37} \le 1$

$x_1,\ldots,x_{37}$ integer

	(SPP)	$\min c^T x$		objective		$\min c^T x$
$\Leftrightarrow$	(i)	$\sum_{i \in j} x_j = 1$	$\forall$ duty elements i	partitioning	$\Leftrightarrow$	$Ax = 1$
	(ii)	$x \ge 0$		bounds		$x \ge 0$
	(iii)	x integer		integrality		x integer

2.11 Def. (Set Partitioning Problem): An IP with all 01-equations (and nonneg. constraints) is called a **set partitioning problem**.

Note: We partition the duty elements into duties.

IP Model

(SPP)	$\min c^T x$		objective		$\min c^T x$
(i)	$\Sigma_{i \in j} x_j = 1$	$\forall$ duty elements i	partitioning	$\Leftrightarrow$	$Ax = 1$
(ii)	$x \geq 0$		bounds		$x \geq 0$
(iii)	x integer		integrality		x integer

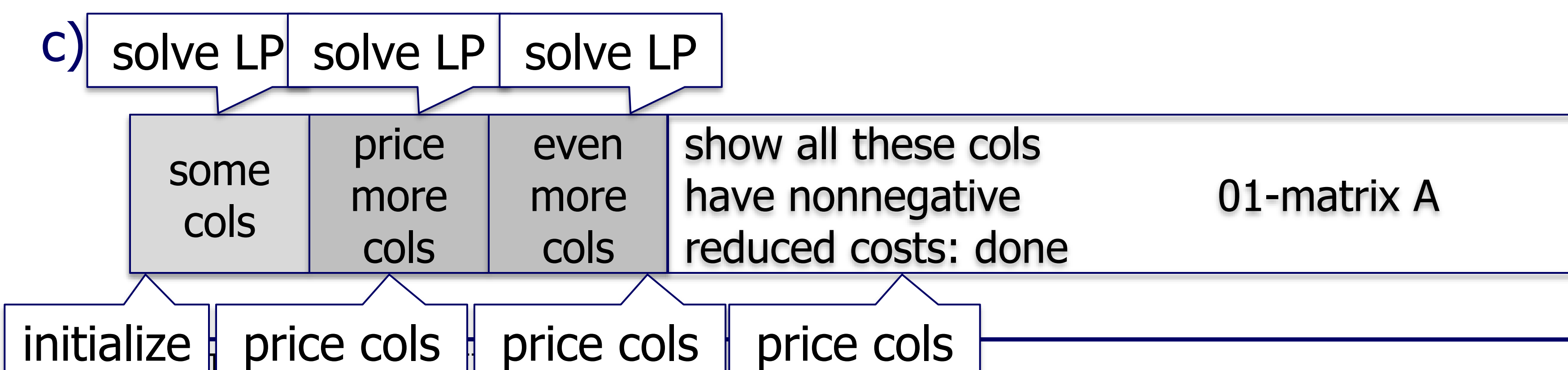
2.11 Def. (Set Partitioning Problem): An IP with all 01-equations (and nonneg. constraints) is called a **set partitioning problem**.

2.12 Obs. (Crew Scheduling): In crew scheduling applications

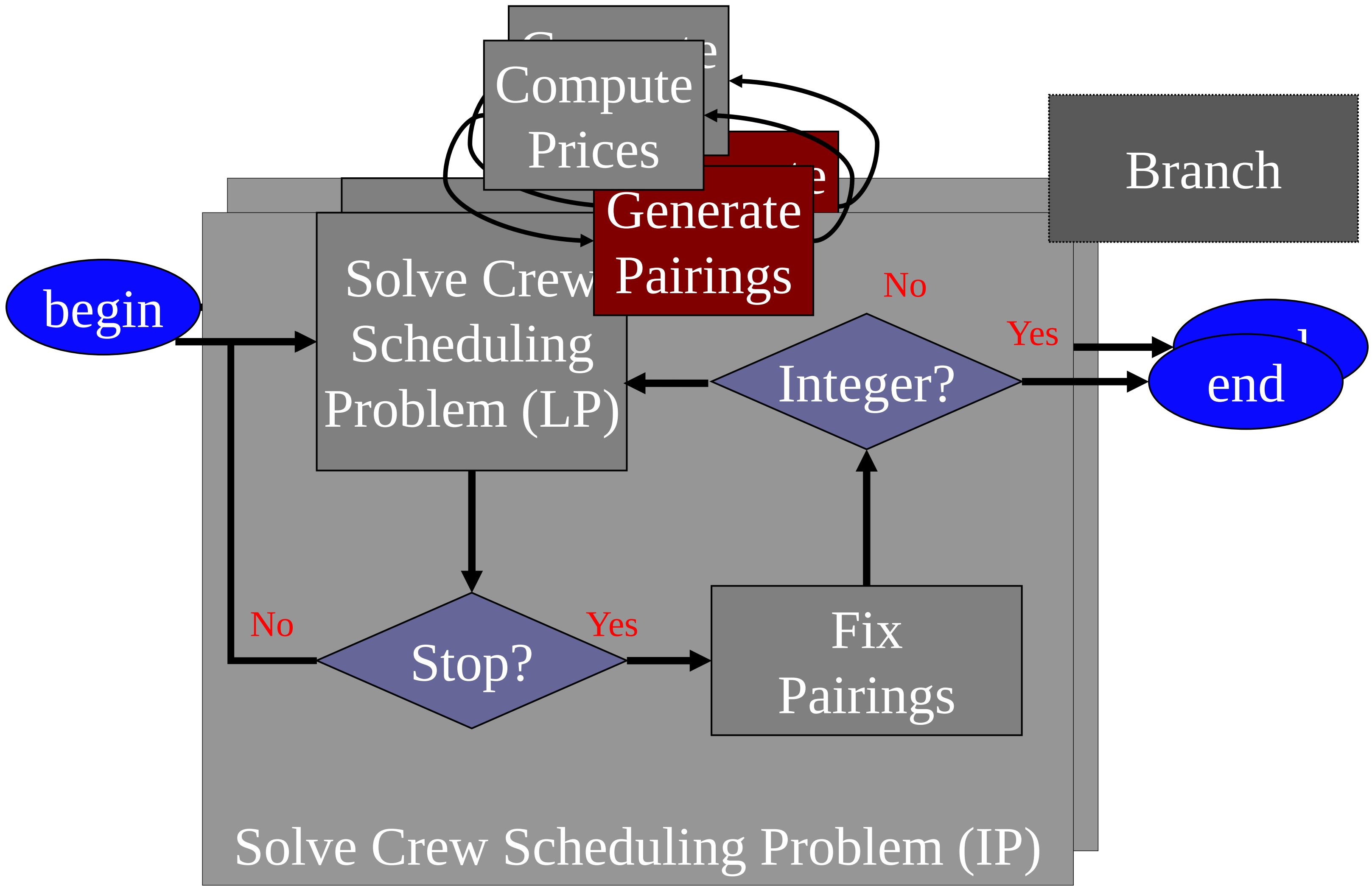
a) $m = \#\text{rows} = \#\text{duty elements}$ is small

b) $n = \#\text{cols} = \#\text{duties} = \#\text{duty paths}$ is large (exponential in $\langle D \rangle$)

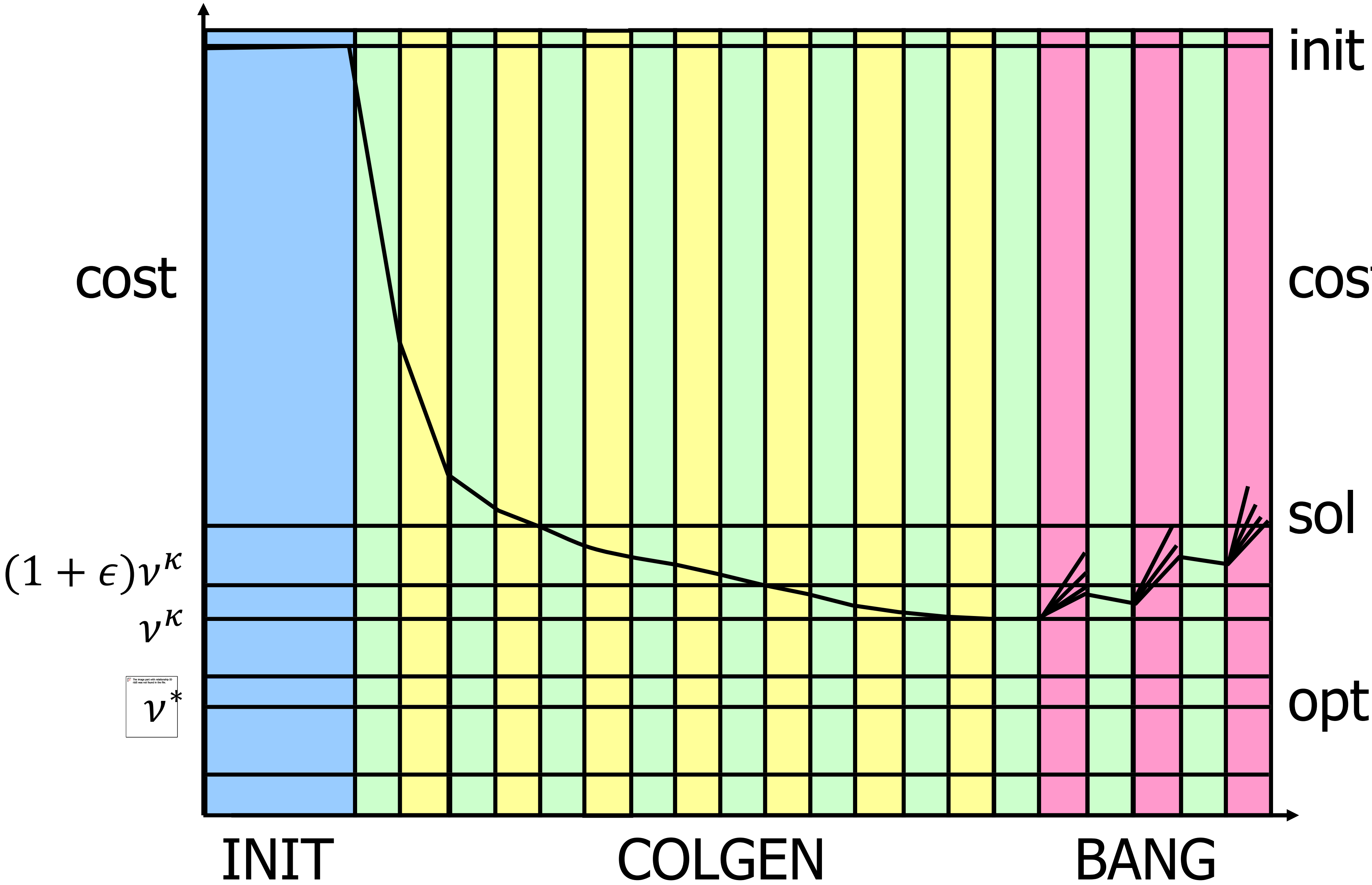
c)



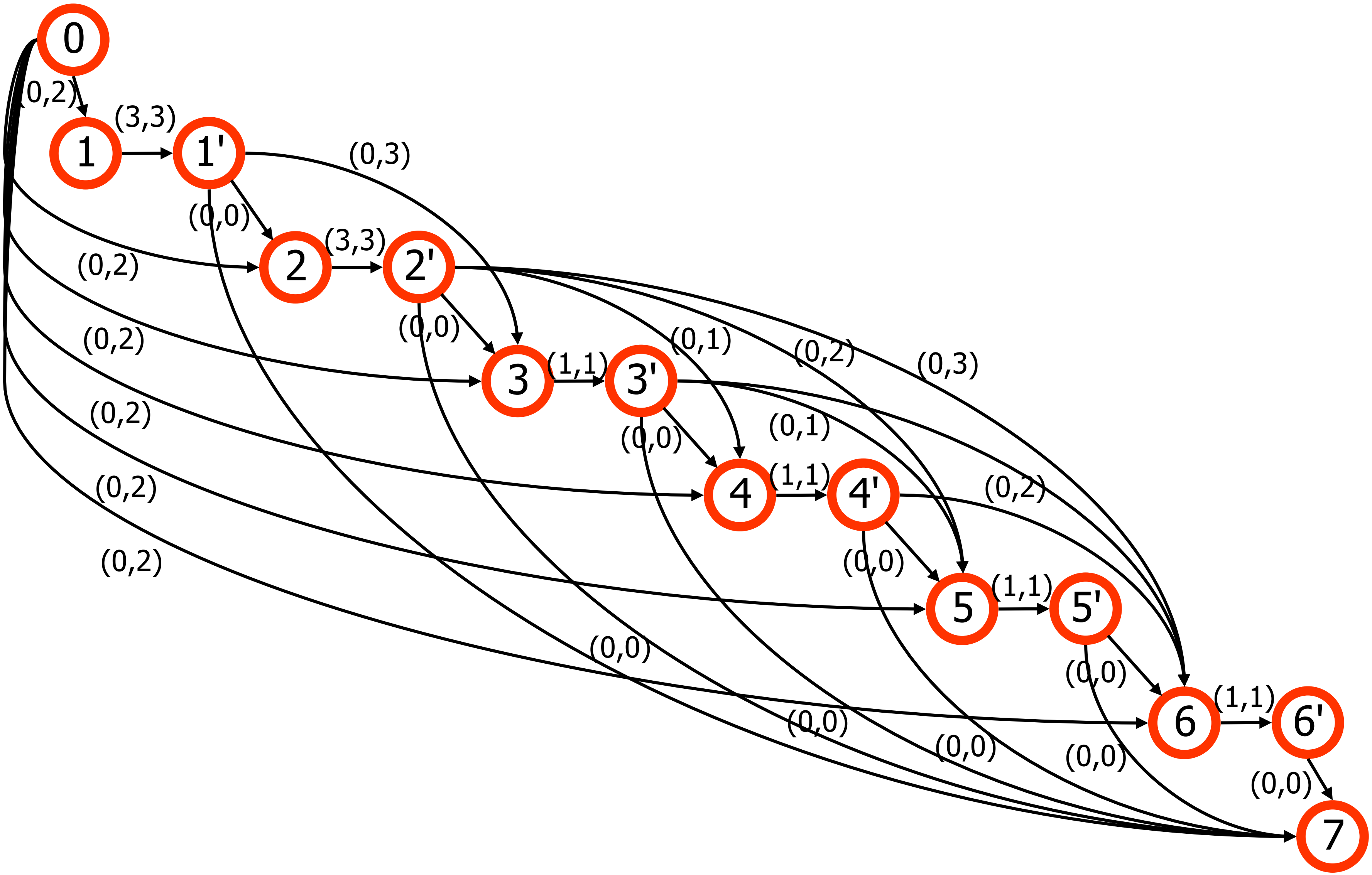
Column Generation Method



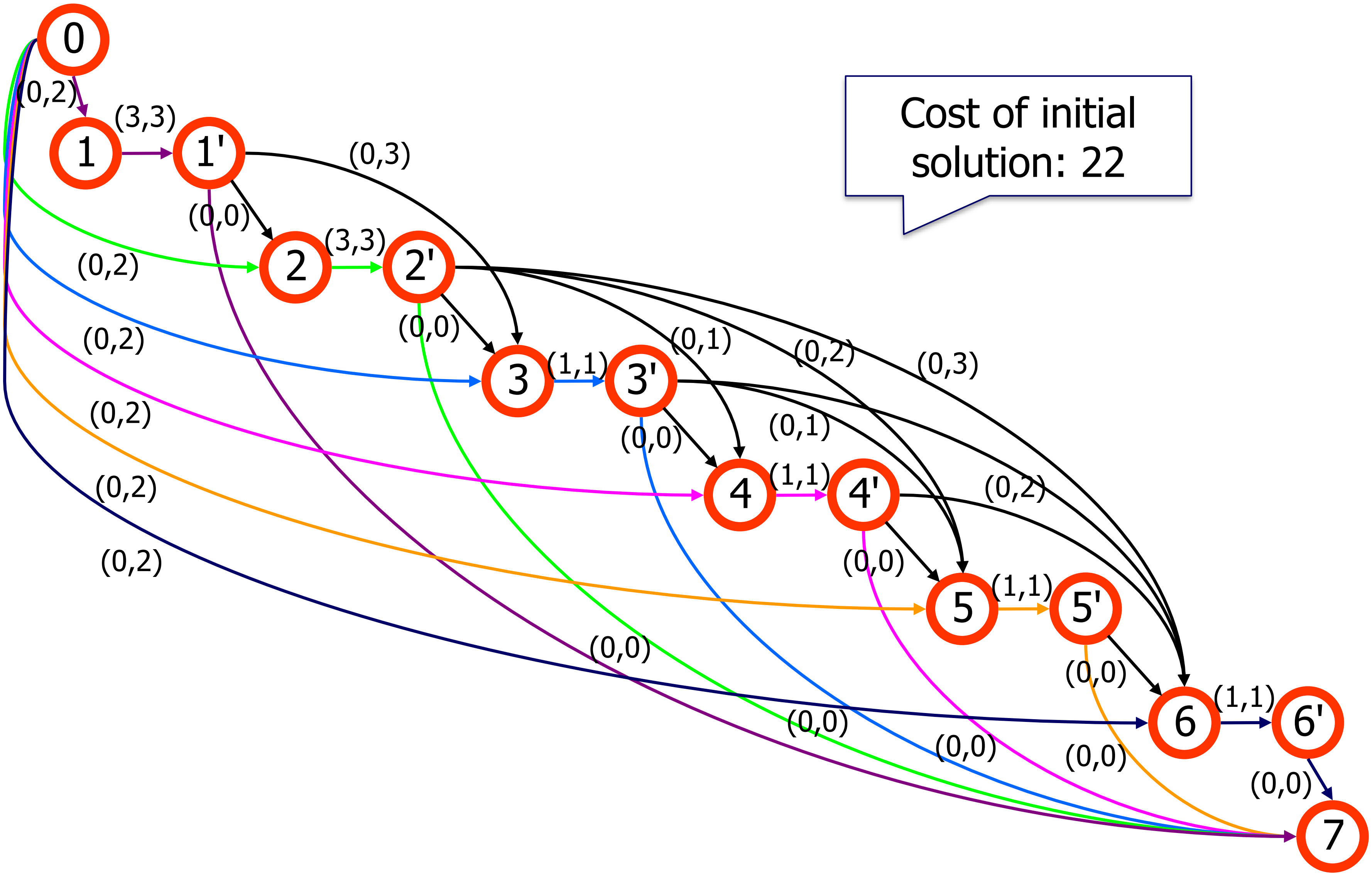
Branch-and-Generate



Crew Scheduling Graph



Initialization



Cost of initial solution: 22

Column Generation: 1st LP

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ZIB

no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		5
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	5
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	3
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	3
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	3
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	3
x	1	1	1	1	1	1																																

primal LP

$$\min\ 5x_1 + 5x_2 + 3x_3 + 3x_4 + 3x_5 + 3x_6$$
$$x_1 = 1,\quad x_2 = 1,\quad x_3 = 1,\quad x_4 = 1,\quad x_5 = 1,\quad x_6 = 1$$
$$x_1, \dots, x_6 \geq 0$$

$$x_1^* = \cdots = x_6^* = 1$$

$\Leftrightarrow$

dual LP

$$\max\ y_1 + y_2 + y_3 + y_4 + y_5 + y_6$$
$$y_1 \leq 5,\quad y_2 \leq 5,\quad y_3 \leq 3,\quad y_4 \leq 3,\quad y_5 \leq 3,\quad y_6 \leq 3$$
$$y_1, \dots, y_6 \text{ free}$$

$$y^* = (5,5,3,3,3,3)^T$$

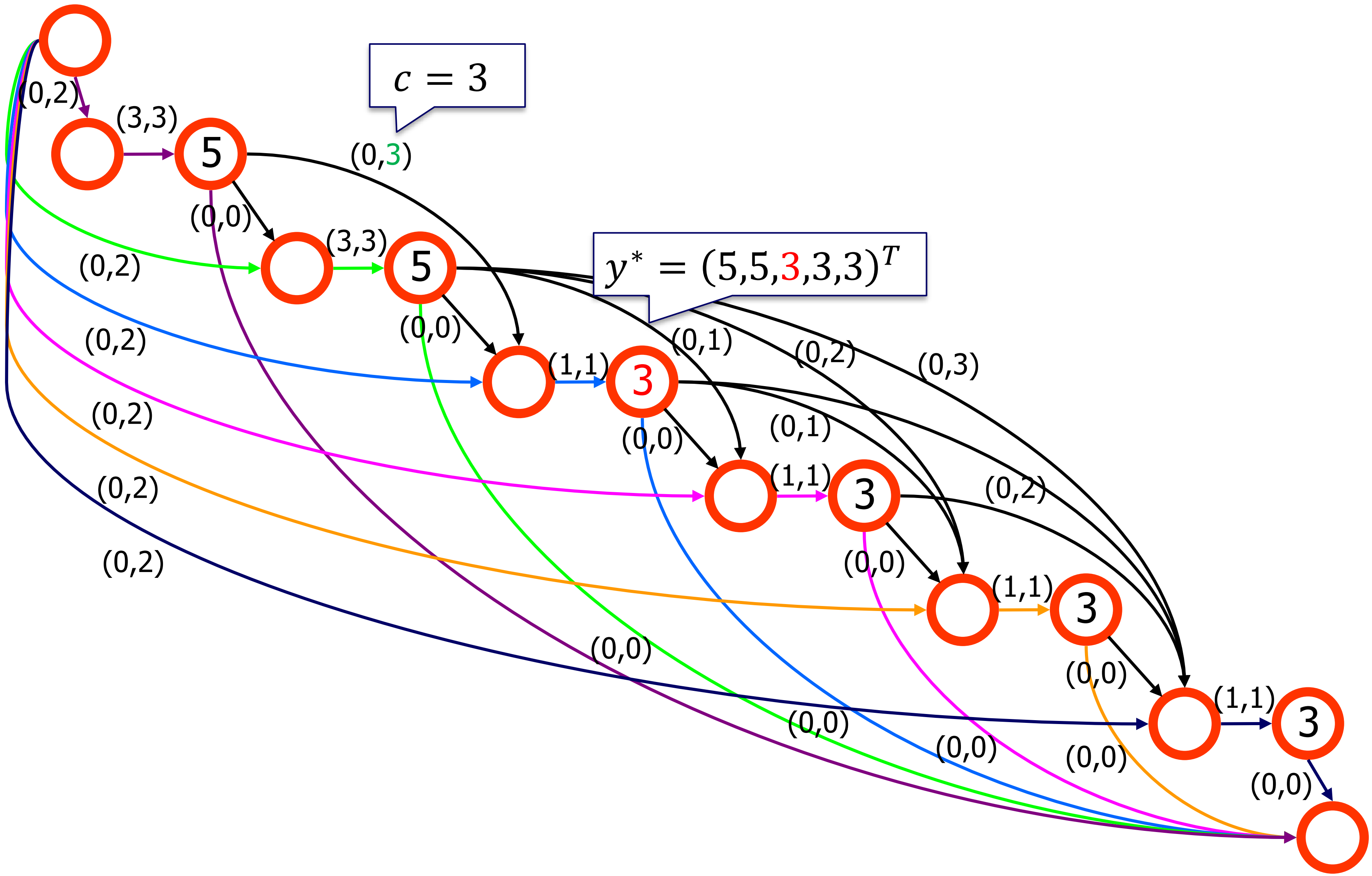
2.13 Obs: (Pricing Problem): The pricing problem is to find a duty path p s.t.

$$0 > \bar{c}_p = c(p) - y^T A_{\cdot p} = \sum_{a \in p} c_a - \sum_{v \in p} y_v$$

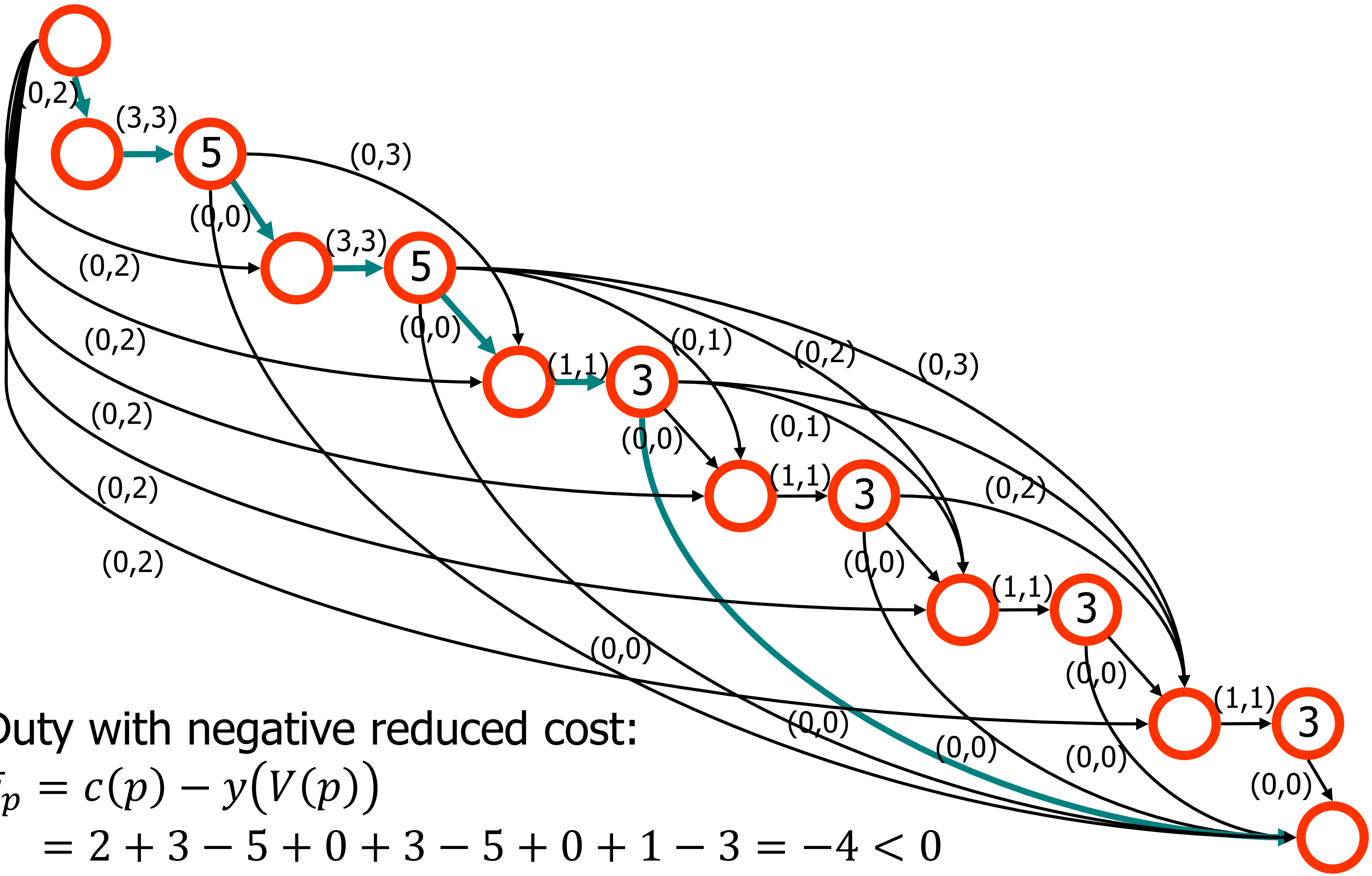
or to prove none exists. This is a constrained shortest path problem in an acyclic digraph.

Cf. Lecture 1

Column Generation: 1st Pricing Problem



Column Generation: 1st Pricing Problem

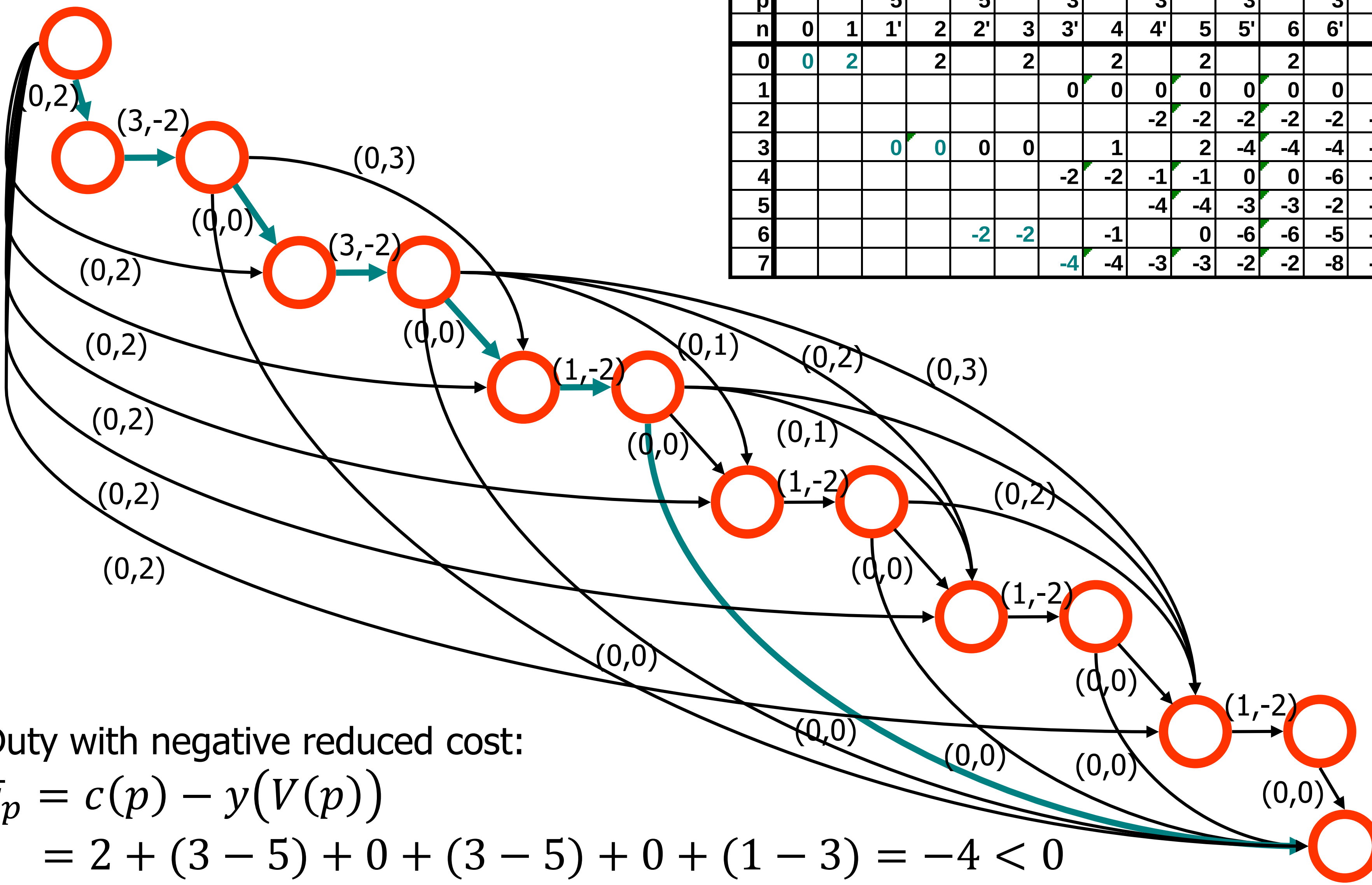


Duty with negative reduced cost:

$$\bar{c}_p = c(p) - y\big(V(p)\big)$$
$$= 2 + 3 - 5 + 0 + 3 - 5 + 0 + 1 - 3 = -4 < 0$$

Column Generation: 1st Pricing Problem

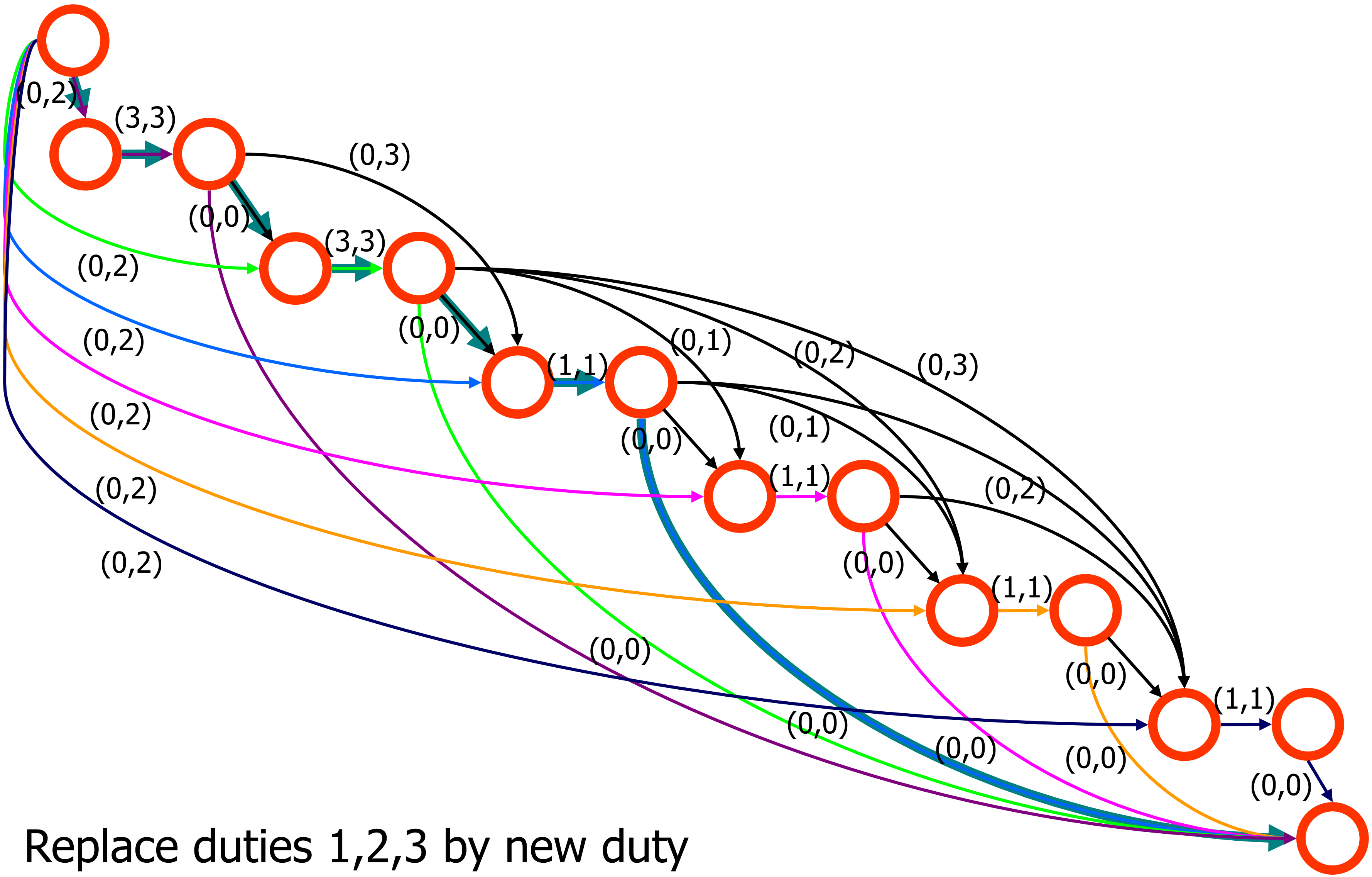
p			5		5		3		3		3		3	
n	**0**	**1**	**1'**	**2**	**2'**	**3**	**3'**	**4**	**4'**	**5**	**5'**	**6**	**6'**	**7**
0	0	2		2		2		2		2		2		
1							0	0	0	0	0	0	0	0
2									-2	-2	-2	-2	-2	-2
3			0	0	0	0		1		2	-4	-4	-4	-4
4							-2	-2	-1	-1	0	0	-6	-6
5									-4	-4	-3	-3	-2	-4
6					-2	-2		-1		0	-6	-6	-5	-6
7							-4	-4	-3	-3	-2	-2	-8	-8



Duty with negative reduced cost:

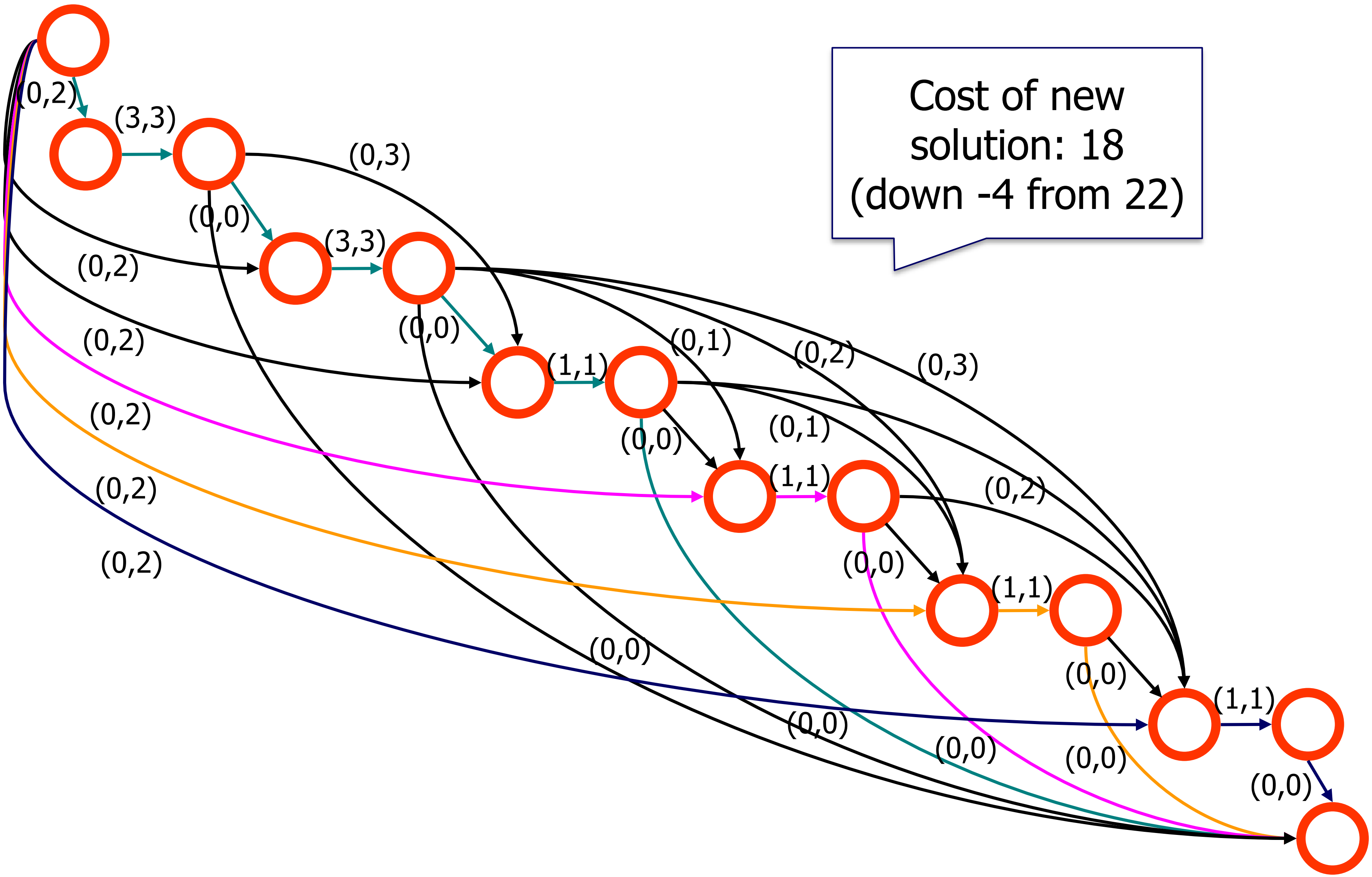
$$\bar{c}_p = c(p) - y\big(V(p)\big)$$
$$= 2 + (3-5) + 0 + (3-5) + 0 + (1-3) = -4 < 0$$

Column Generation: 1st Col Addition



- Replace duties 1,2,3 by new duty

Column Generation: 1st Col Addition



Cost of new solution: 18 (down -4 from 22)

Column Generation: 2^{nd} LP

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no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		1
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	5
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	3
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	3
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	3
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	3
x	1	1	1																1																			

primal LP

$$
\begin{aligned}
\min \quad & 5x_1 + 5x_2 + 3x_3 + 3x_4 + 3x_5 + 3x_6 + 9x_{19} \\
& x_1 + x_{19} = 1 \\
& x_2 + x_{19} = 1 \\
& x_3 + x_{19} = 1 \\
& x_4 = 1 \\
& x_5 = 1 \\
& x_6 = 1 \\
& x_1, \dots, \boldsymbol{x_6}, \boldsymbol{x_{19}} \geq 0
\end{aligned}
$$

$\Leftrightarrow$

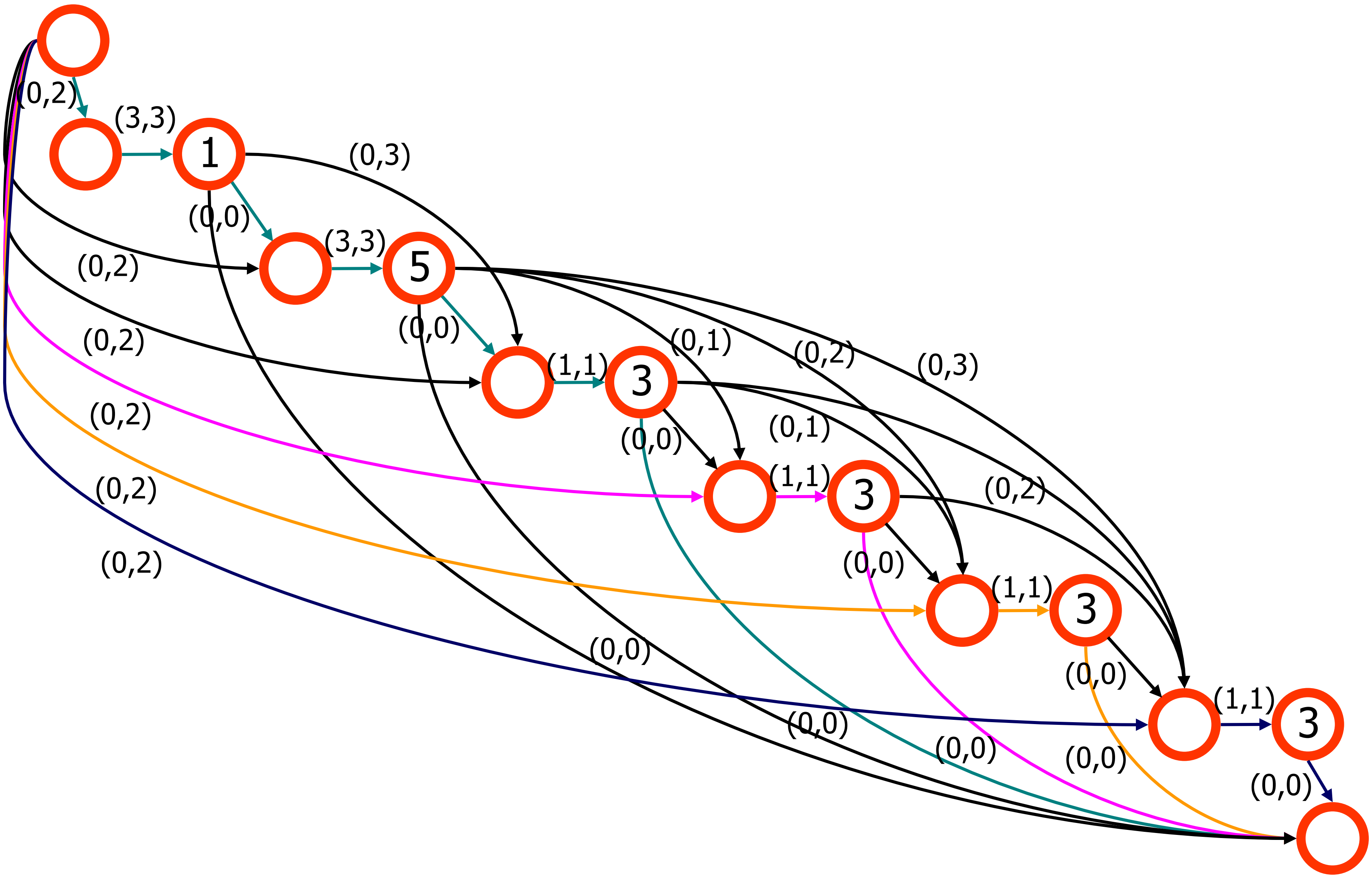
dual LP

$$
\begin{aligned}
\max \quad & y_1 + y_2 + y_3 + y_4 + y_5 + y_6 \\
& y_1 \leq 5 \\
& y_2 \leq 5 \\
& y_3 \leq 3 \\
& y_4 \leq 3 \\
& y_5 \leq 3 \\
& y_6 \leq 3 \\
& y_1 + y_2 + y_3 \leq 9 \\
& y_1, \dots, y_6 \text{ free}
\end{aligned}
$$

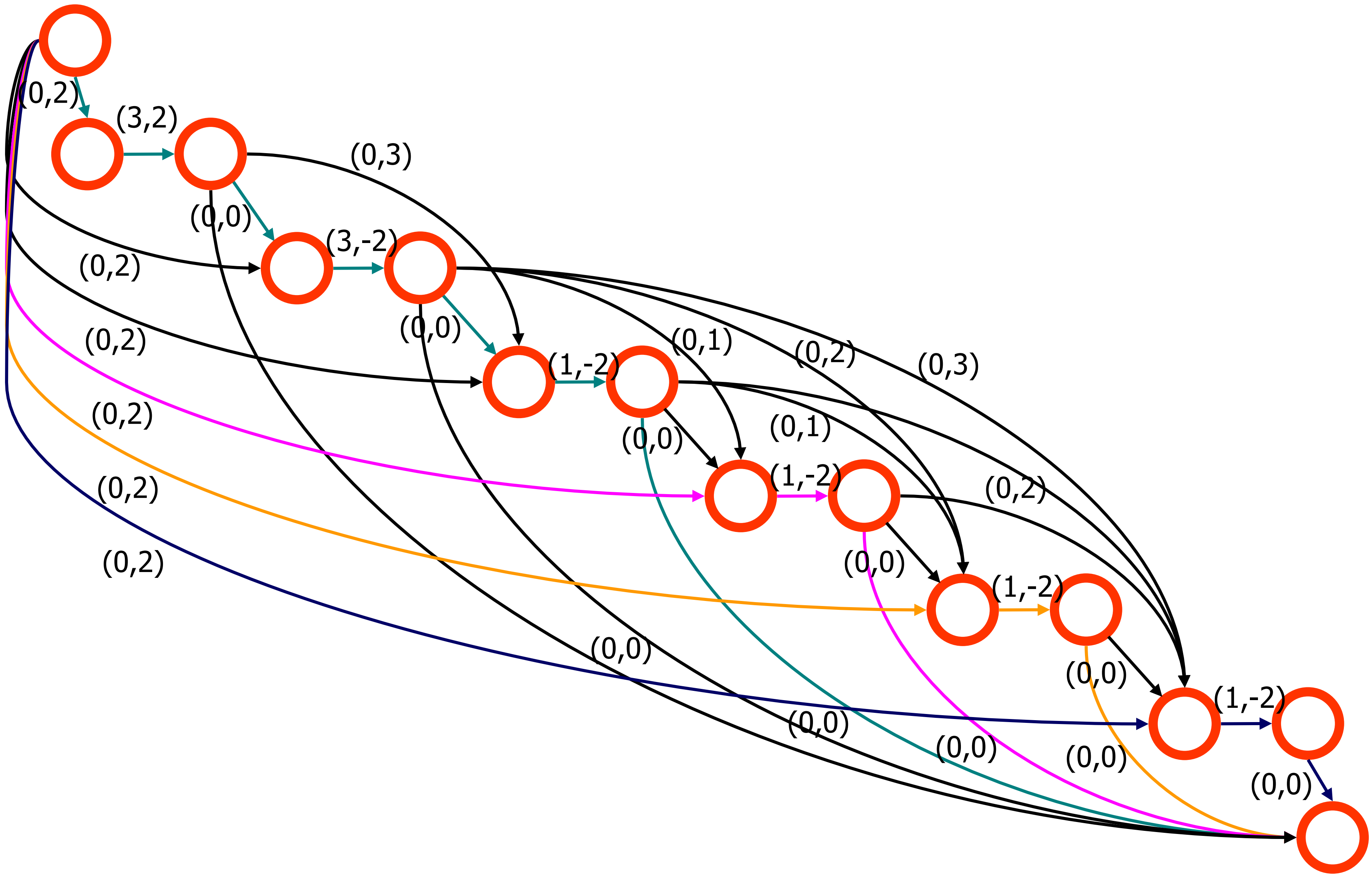
$x_4^* = x_5^* = x_6^* = x_{19}^* = 1$

$y^* = (1,5,3,3,3,3)^T$
(or other optimum)

Column Generation: 2nd Pricing Problem

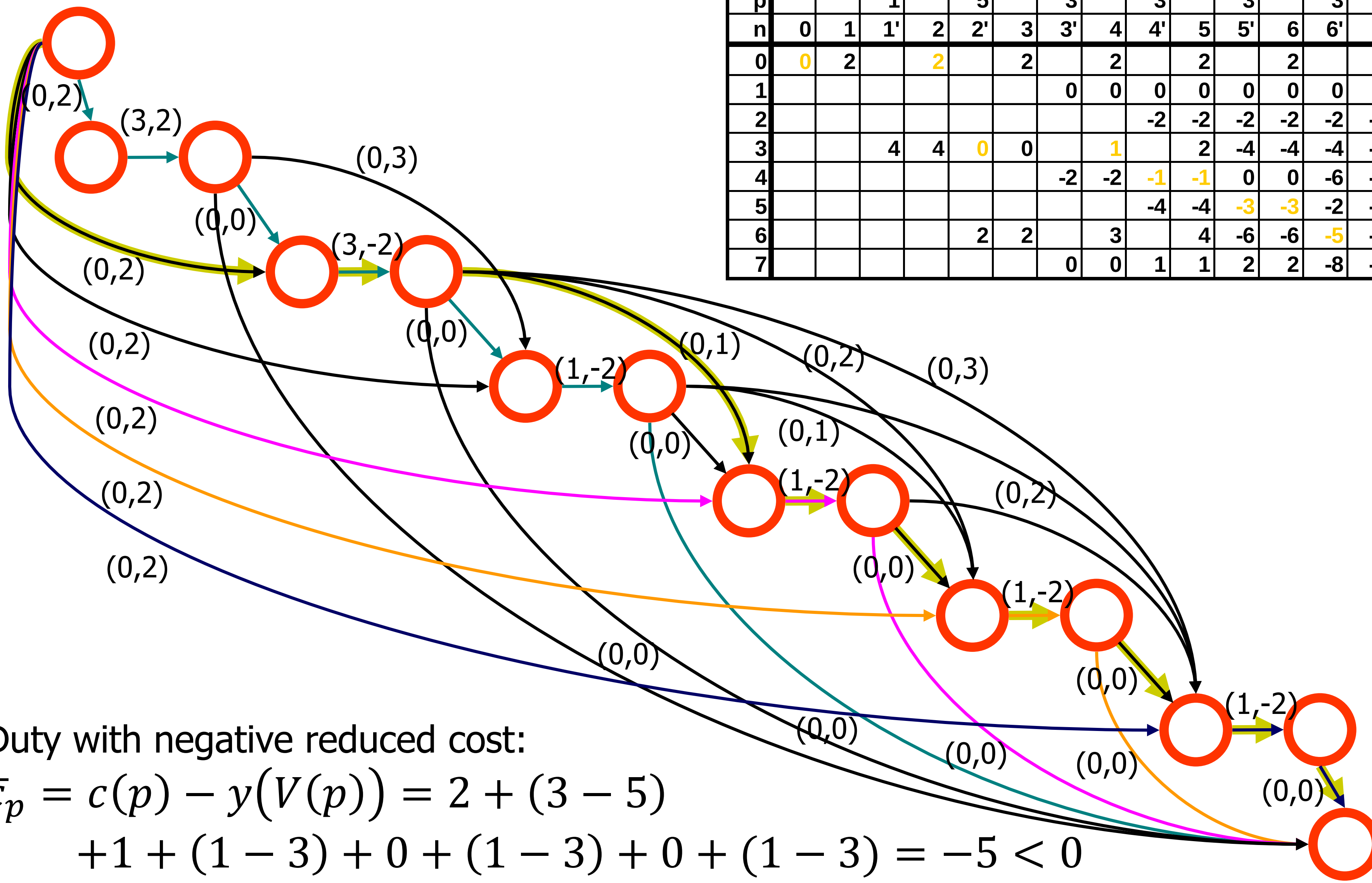


Column Generation: 2^{nd} Pricing Problem



Column Generation: 2nd Pricing Problem

p			1		5		3		3		3		3	
n	**0**	**1**	**1'**	**2**	**2'**	**3**	**3'**	**4**	**4'**	**5**	**5'**	**6**	**6'**	**7**
0	0	2		2		2		2		2		2		
1							0	0	0	0	0	0	0	0
2									-2	-2	-2	-2	-2	-2
3			4	4	0	0		1		2	-4	-4	-4	-4
4							-2	-2	-1	-1	0	0	-6	-6
5									-4	-4	-3	-3	-2	-4
6					2	2		3		4	-6	-6	-5	-6
7							0	0	1	1	2	2	-8	-8

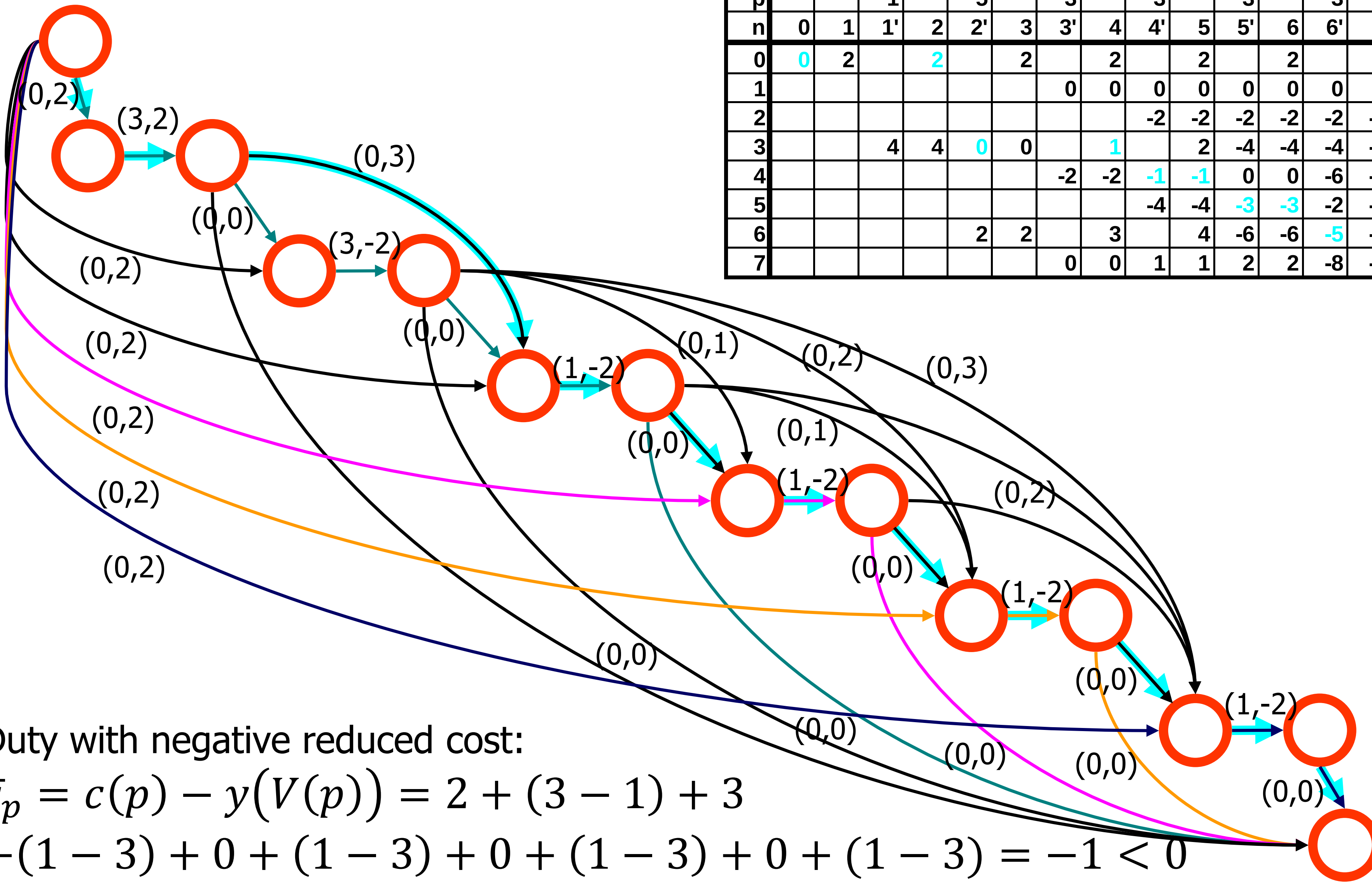


Duty with negative reduced cost:

$$\bar{c}_p = c(p) - y\big(V(p)\big) = 2 + (3 - 5)$$
$$+1 + (1 - 3) + 0 + (1 - 3) + 0 + (1 - 3) = -5 < 0$$

Column Generation: 2nd Pricing Problem

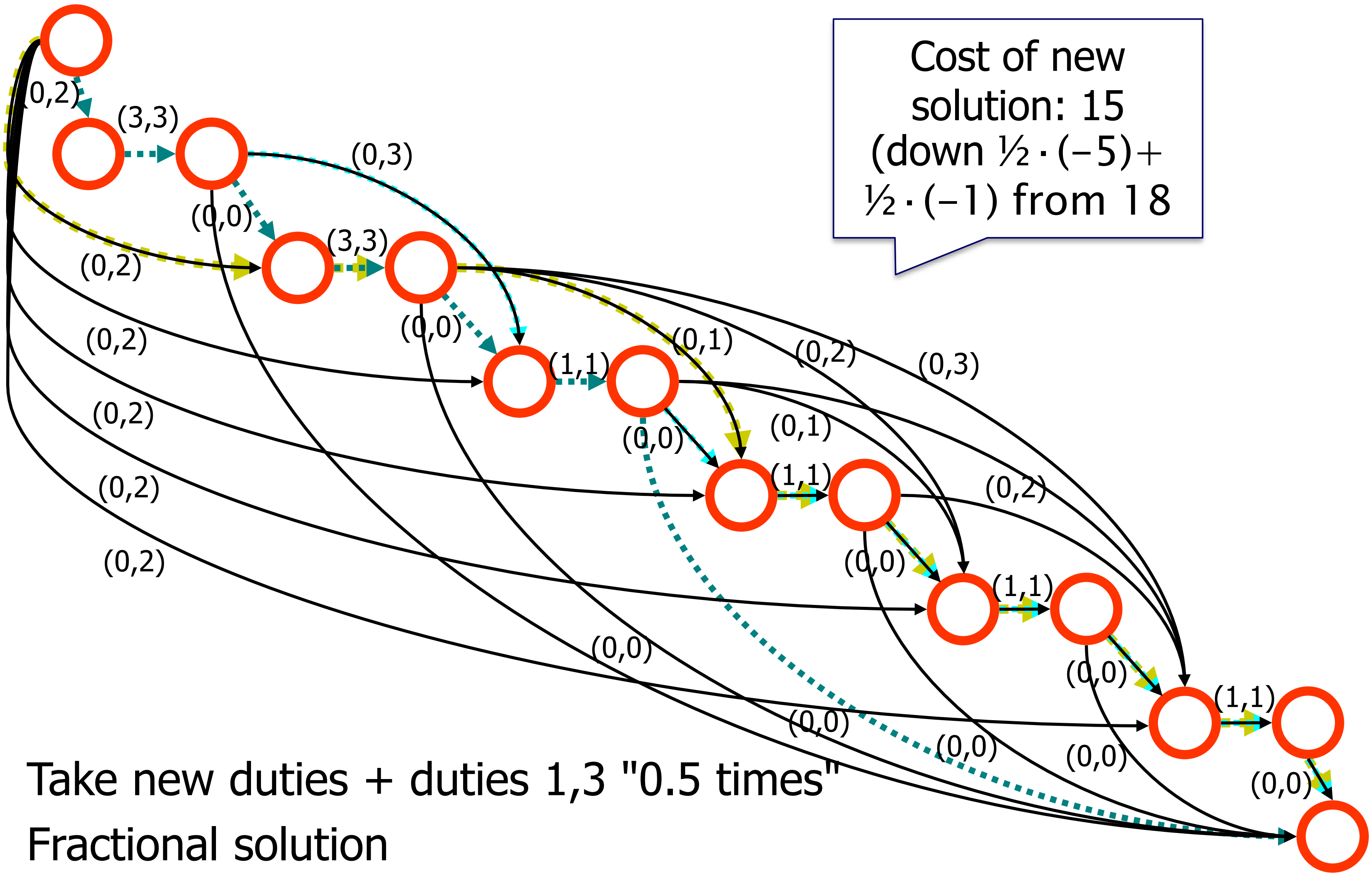
p			1		5		3		3		3		3	
n	**0**	**1**	**1'**	**2**	**2'**	**3**	**3'**	**4**	**4'**	**5**	**5'**	**6**	**6'**	**7**
0	0	2		2		2		2		2		2		
1							0	0	0	0	0	0	0	0
2									-2	-2	-2	-2	-2	-2
3			4	4	0	0		1		2	-4	-4	-4	-4
4							-2	-2	-1	-1	0	0	-6	-6
5									-4	-4	-3	-3	-2	-4
6					2	2		3		4	-6	-6	-5	-6
7							0	0	1	1	2	2	-8	-8



Duty with negative reduced cost:

$$\bar{c}_p = c(p) - y\big(V(p)\big) = 2 + (3-1) + 3$$
$$+(1-3) + 0 + (1-3) + 0 + (1-3) + 0 + (1-3) = -1 < 0$$

Column Generation: 2nd Col Addition



Cost of new solution: 15 (down ½·(−5)+ ½·(−1) from 18

- Take new duties + duties 1,3 "0.5 times"
- Fractional solution

Column Generation: 4th LP

no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		5
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	3
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	1
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	2
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	2
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	1
x																			1									1										

primal LP

$$
\begin{array}{lllllllllll}
\min & 5x_1 + 5x_2 + 3x_3 + 3x_4 + 3x_5 + 3x_6 + 9x_{19} + 9x_{34} + 12x_{36} \\
& x_1 & & & & & + x_{19} & & + x_{36} & = 1 \\
& & x_2 & & & & + x_{19} & + x_{34} & & = 1 \\
& & & x_3 & & & + x_{19} & & + x_{36} & = 1 \\
& & & & x_4 & & & + x_{34} & + x_{36} & = 1 \\
& & & & & x_5 & & + x_{34} & + x_{36} & = 1 \\
& & & & & x_6 & & + x_{34} & + x_{36} & = 1 \\
& x_1, \dots, x_6, x_{19}, x_{34}, x_{36} \geq 0
\end{array}
$$

$\Leftrightarrow$

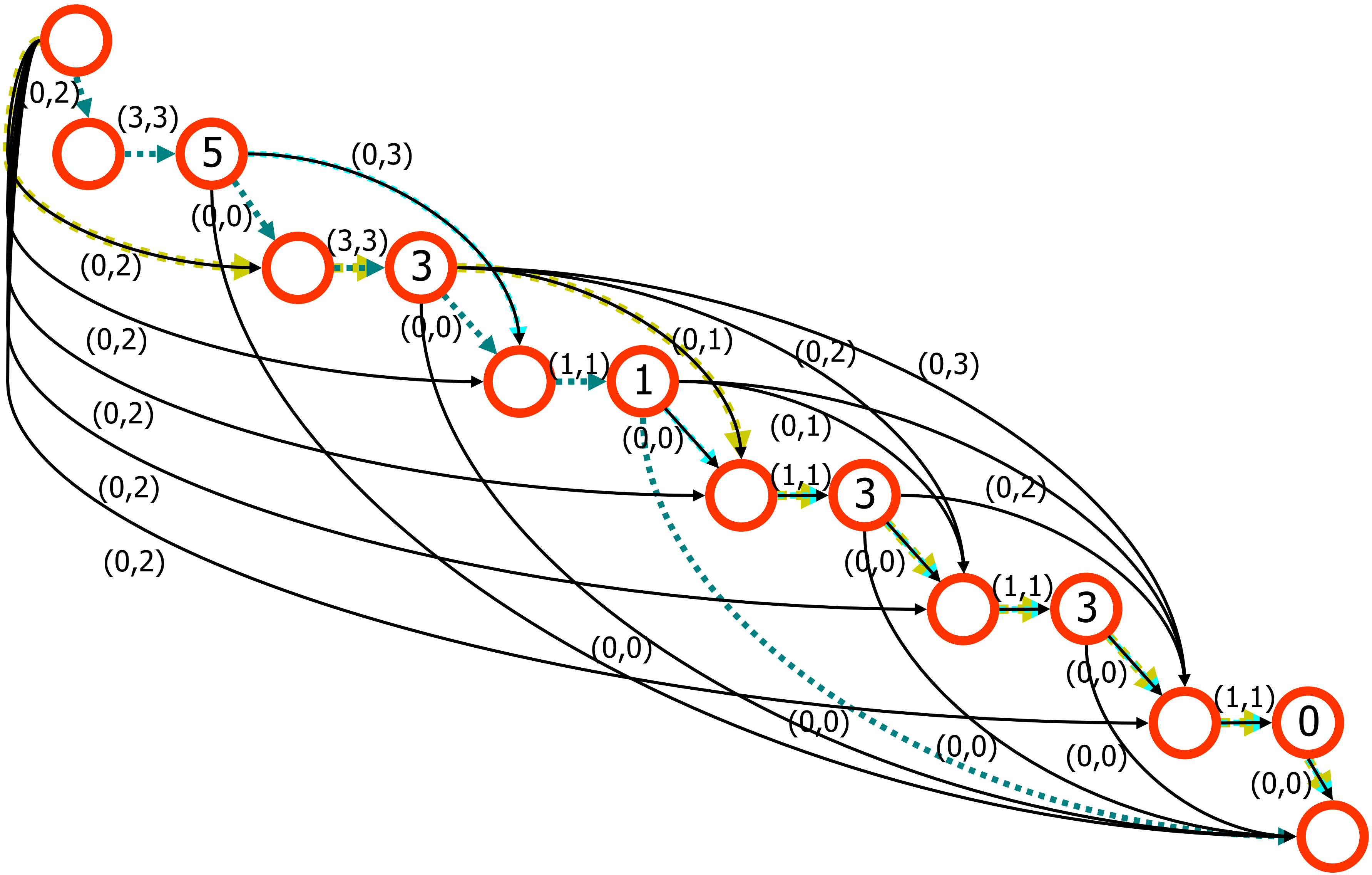
dual LP

$$
\begin{array}{ll}
\max & y_1 + y_2 + y_3 + y_4 + y_5 + y_6 \\
& y_1 \leq 5 \\
& y_2 \leq 5 \\
& y_3 \leq 3 \\
& y_4 \leq 3 \\
& y_5 \leq 3 \\
& y_6 \leq 3 \\
& y_1 + y_2 + y_3 \leq 9 \\
& y_4 + y_5 + y_6 \leq 5 \\
& y_2 + y_4 + y_5 + y_6 \leq 9 \\
& y_1 + y_3 + y_4 + y_5 + y_6 \leq 12 \\
& y_1, \dots, y_6 \text{ free}
\end{array}
$$

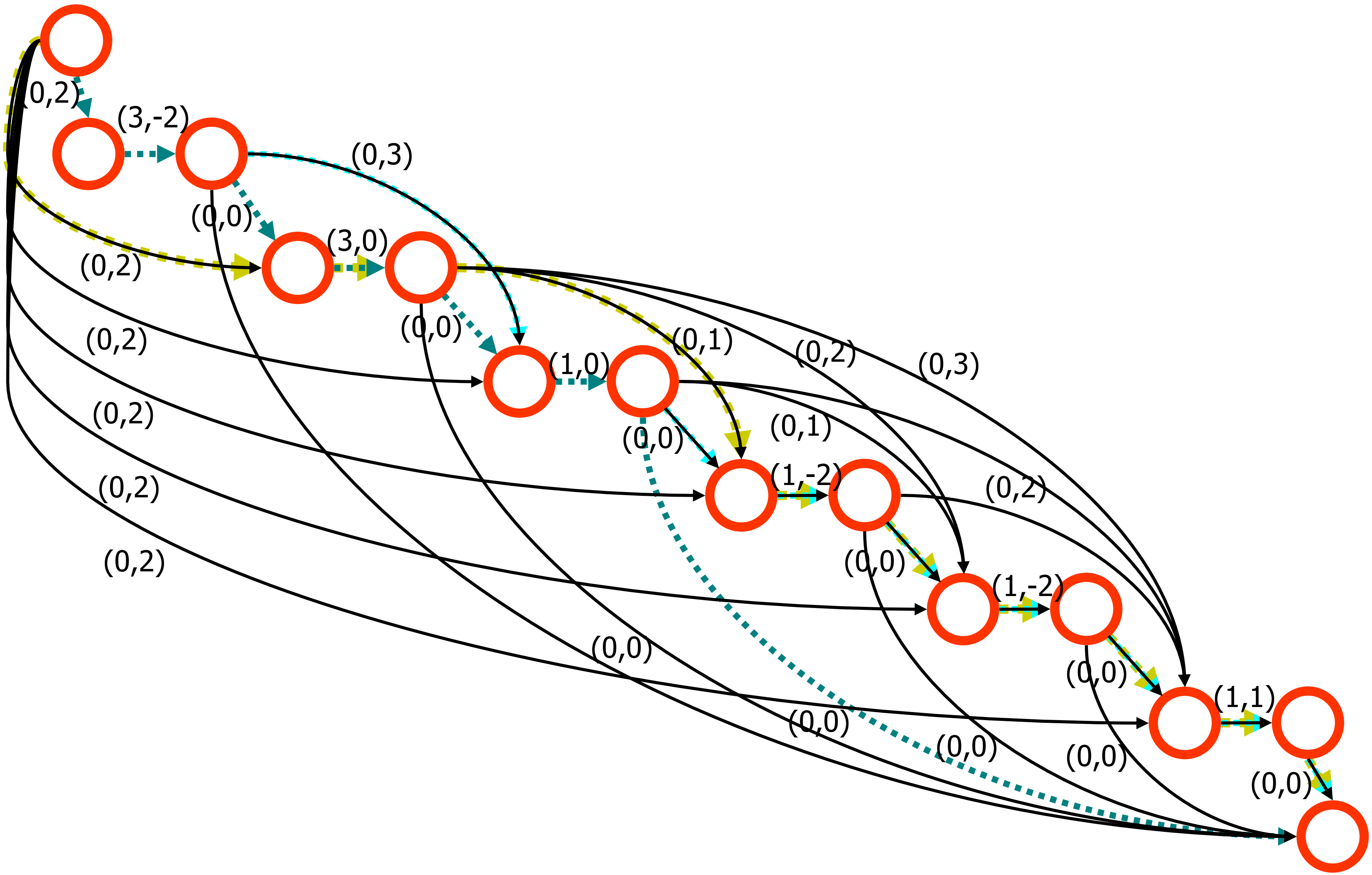
$x_{19}^* = x_{34}^* = x_{35}^* = 0.5$

$y^* = (5,3,1,2,2,1)^T$

Column Generation: 3rd Pricing Problem



Column Generation: 3rd Pricing Problem



Column Generation: 3rd Pricing Problem

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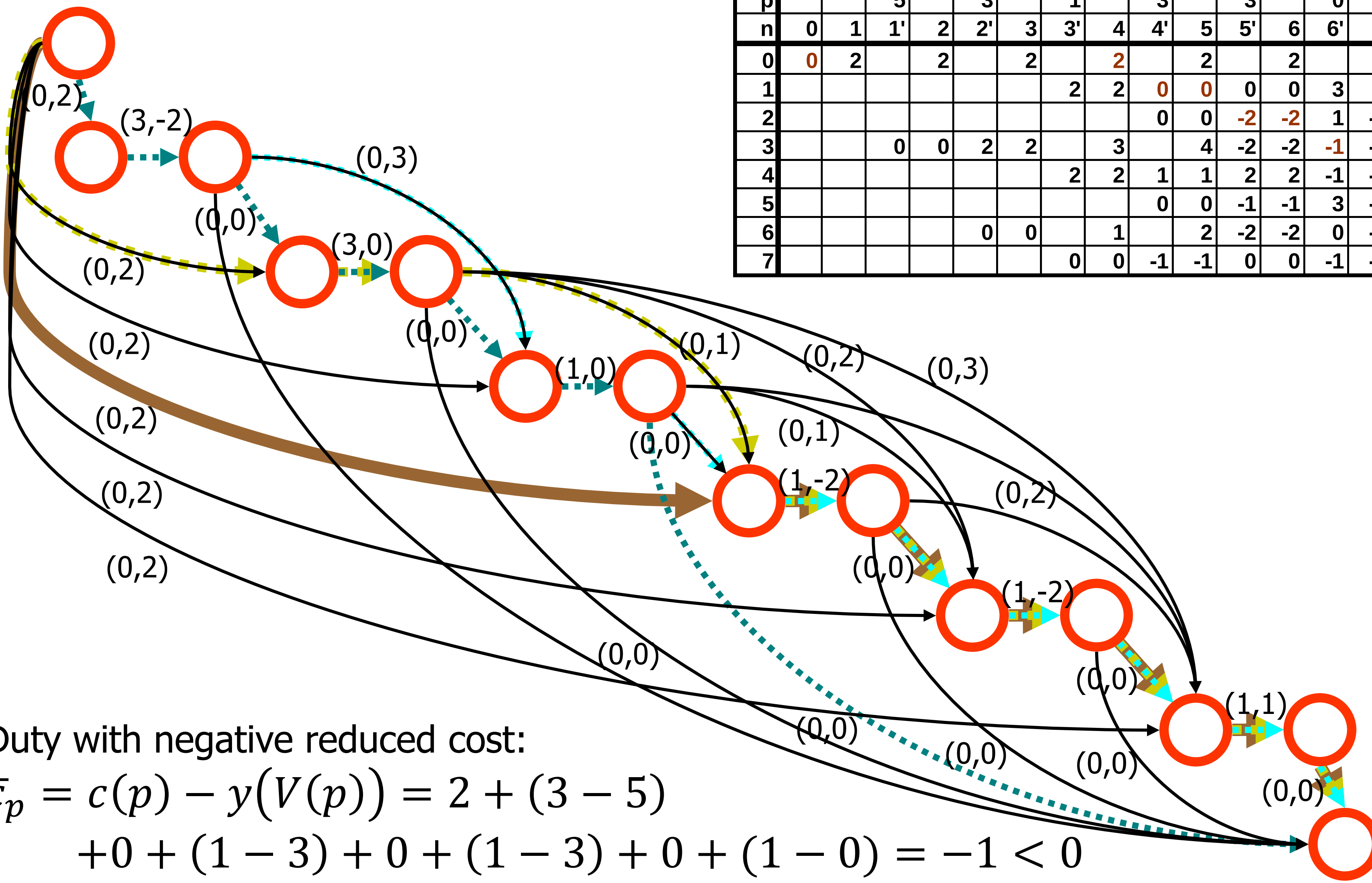
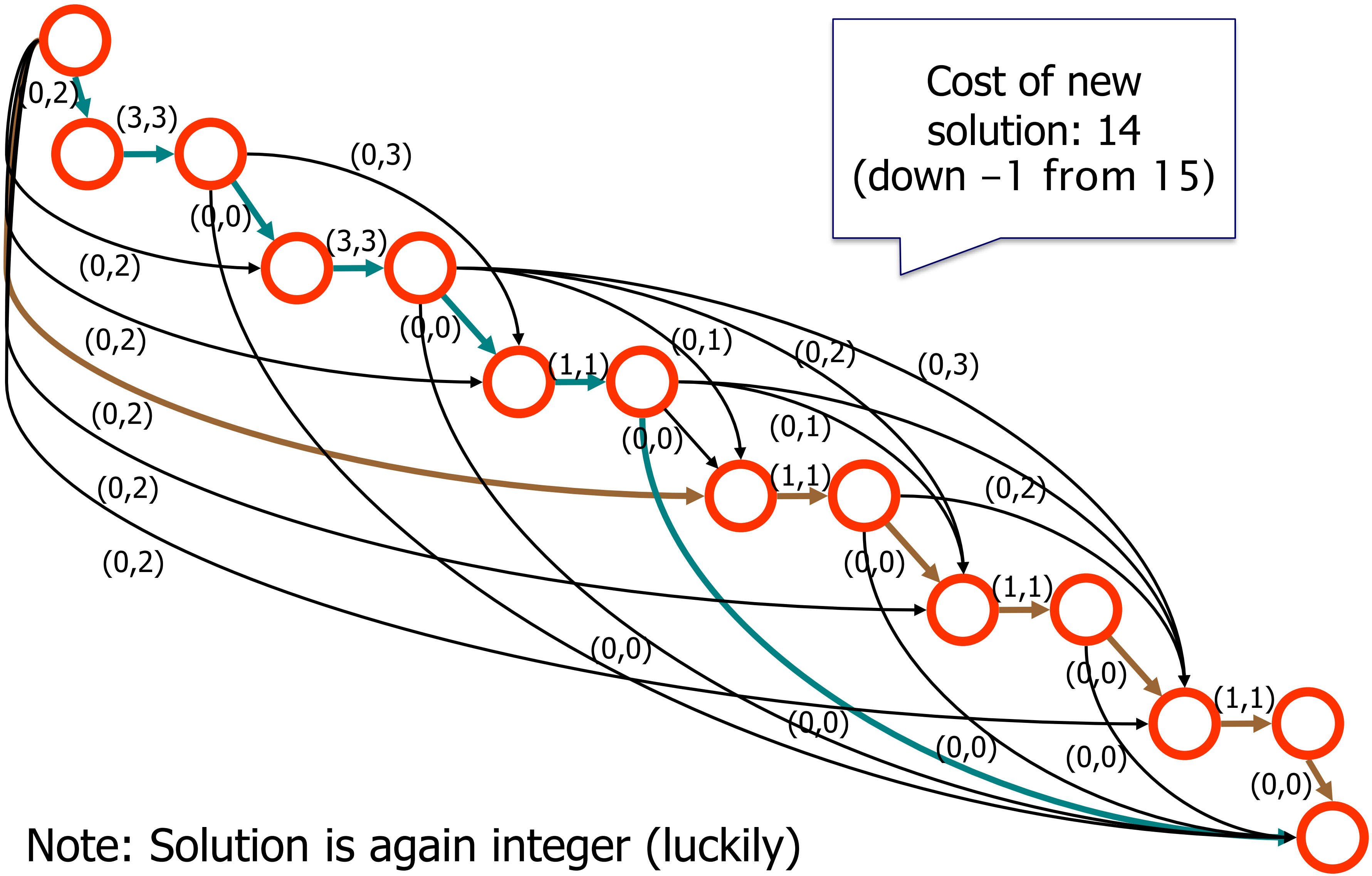
p			5		3		1		3		3		0	
n	0	1	1'	2	2'	3	3'	4	4'	5	5'	6	6'	7
0	0	2		2		2		2		2		2		
1							2	2	0	0	0	0	3	0
2									0	0	-2	-2	1	-2
3			0	0	2	2		3		4	-2	-2	-1	-2
4							2	2	1	1	2	2	-1	-1
5									0	0	-1	-1	3	-1
6					0	0		1		2	-2	-2	0	-2
7							0	0	-1	-1	0	0	-1	-1



Duty with negative reduced cost:

$$\bar{c}_p = c(p) - y\big(V(p)\big) = 2 + (3-5)$$
$$+0 + (1-3) + 0 + (1-3) + 0 + (1-0) = -1 < 0$$

Column Generation: 3rd Col Addition



Cost of new solution: 14 (down −1 from 15)

- Note: Solution is again integer (luckily)

Column Generation: 4th LP

no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		5
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	3
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	1
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	2
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	2
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	1
x																			1									1										

primal LP

$$
\begin{array}{lrcl}
\min & 5x_1 + 5x_2 + 3x_3 + 3x_4 + 3x_5 + 3x_6 + 9x_{19} + 5x_{28} + 9x_{34} + 12x_{36} & & \\
 & x_1 + x_{19} + x_{36} & = & 1 \\
 & x_2 + x_{19} + x_{34} & = & 1 \\
 & x_3 + x_{19} + x_{36} & = & 1 \\
 & x_4 + x_{28} + x_{34} + x_{36} & = & 1 \\
 & x_5 + x_{28} + x_{34} + x_{36} & = & 1 \\
 & x_6 + x_{28} + x_{34} + x_{36} & = & 1 \\
 & x_1, \dots, x_6, x_{19}, x_{28}, x_{34}, x_{36} & \geq & 0
\end{array}
$$

$\Leftrightarrow$

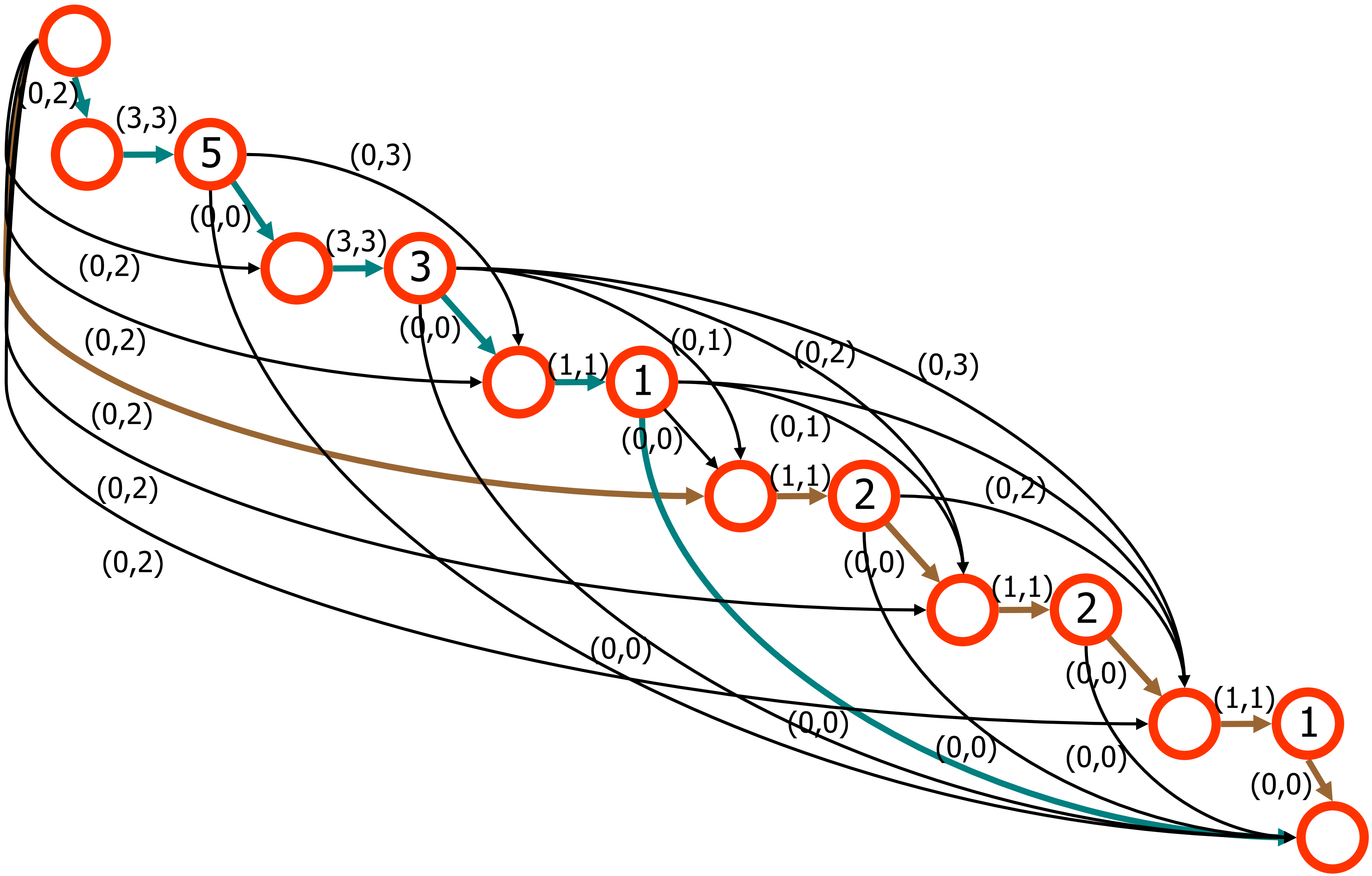
dual LP

$$
\begin{array}{lrcl}
\max & y_1 + y_2 + y_3 + y_4 + y_5 + y_6 & & \\
 & y_1 & \leq & 5 \\
 & y_2 & \leq & 5 \\
 & y_3 & \leq & 3 \\
 & y_4 & \leq & 3 \\
 & y_5 & \leq & 3 \\
 & y_6 & \leq & 3 \\
 & y_1 + y_2 + y_3 & \leq & 9 \\
 & y_4 + y_5 + y_6 & \leq & 5 \\
 & y_2 + y_4 + y_5 + y_6 & \leq & 9 \\
 & y_1 + y_3 + y_4 + y_5 + y_6 & \leq & 12 \\
 & y_1, \dots, y_6 \text{ free} & &
\end{array}
$$

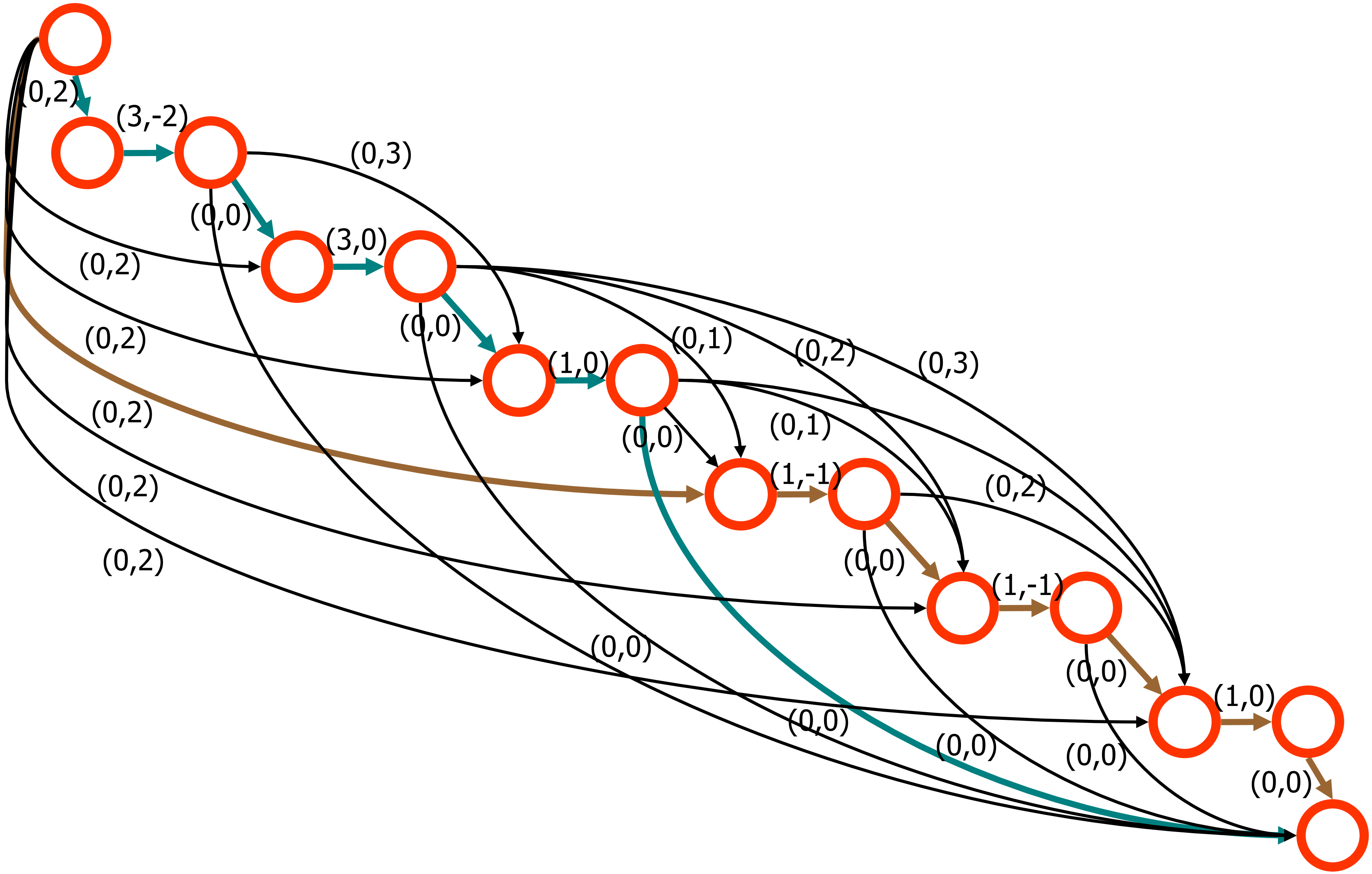
$x_{19}^* = x_{28}^* = 1$

$y^* = (5,3,1,2,2,1)^T$

Column Generation: 4th Pricing Problem

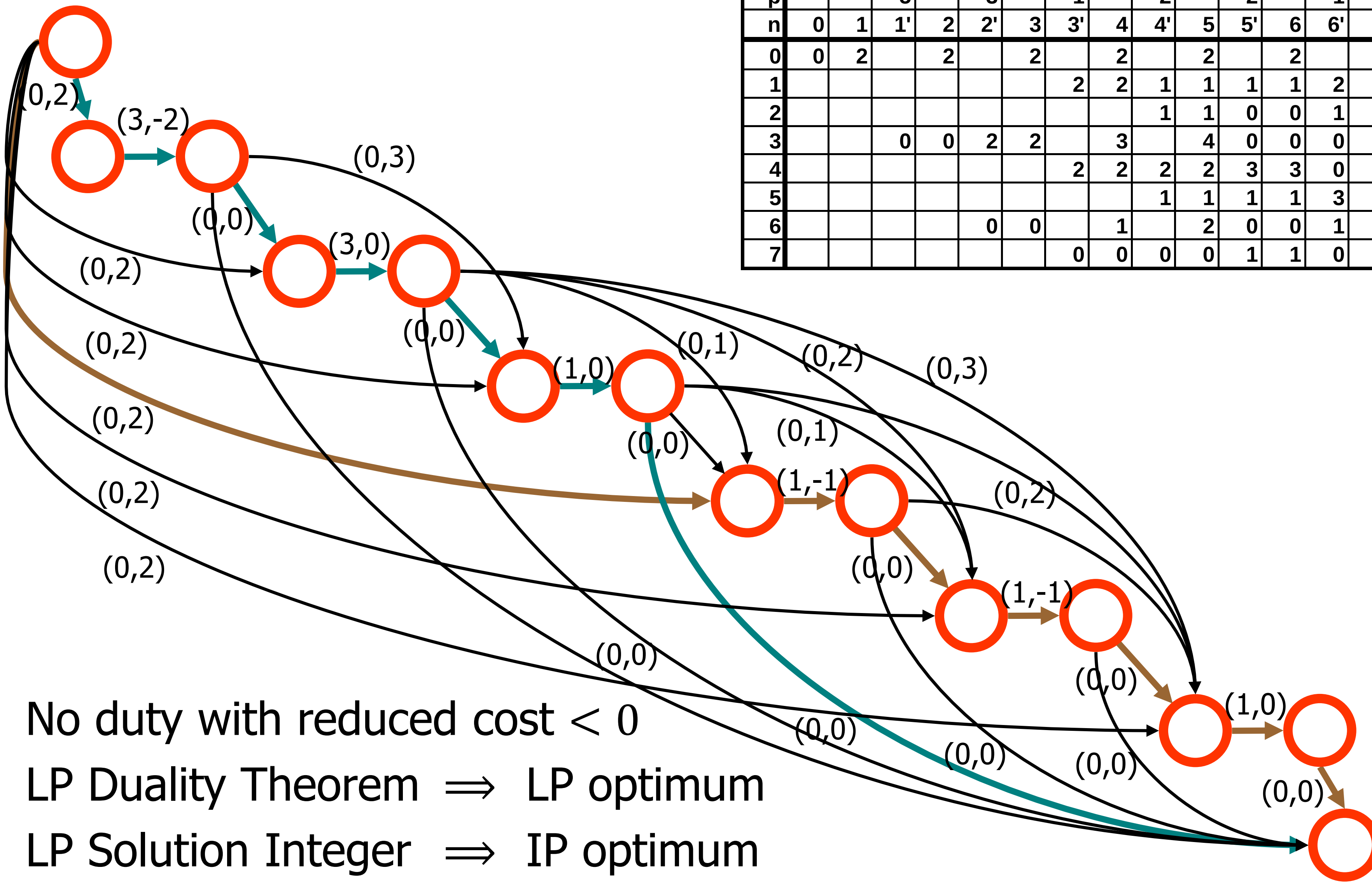


Column Generation: 4th Pricing Problem



Column Generation: 4th Pricing Problem

p			5		3		1		2		2		1	
n	0	1	1'	2	2'	3	3'	4	4'	5	5'	6	6'	7
0	0	2		2		2		2		2		2		
1							2	2	1	1	1	1	2	1
2									1	1	0	0	1	0
3			0	0	2	2		3		4	0	0	0	0
4							2	2	2	2	3	3	0	0
5									1	1	1	1	3	1
6					0	0		1		2	0	0	1	0
7							0	0	0	0	1	1	0	0



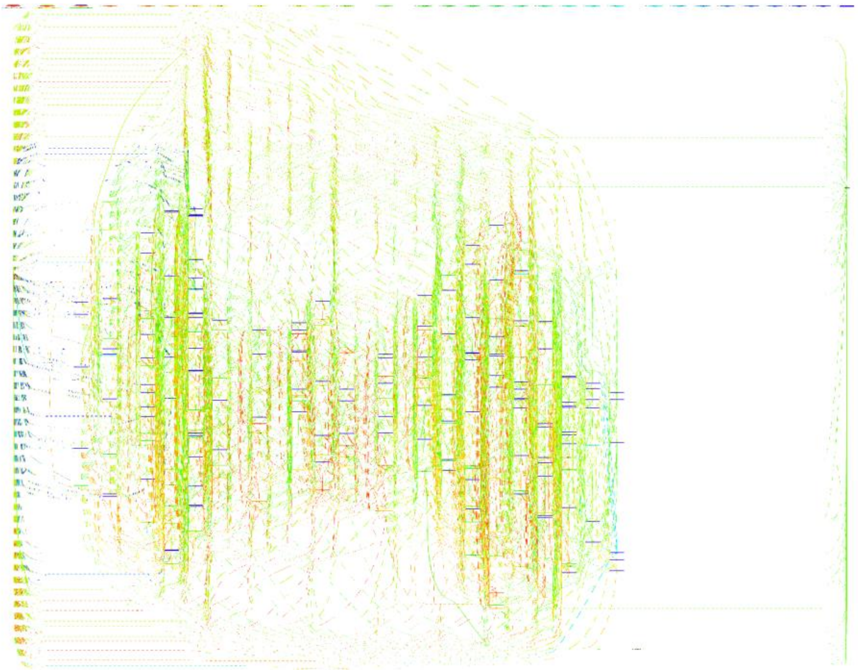
- No duty with reduced cost < 0
- LP Duality Theorem ⟹ LP optimum
- LP Solution Integer ⟹ IP optimum

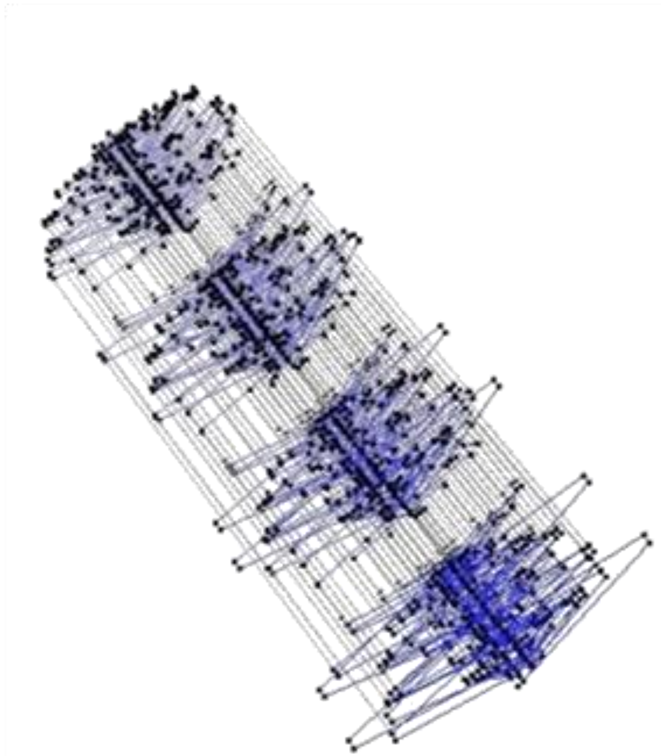
Solving Real World Crew Scheduling Problems

Article	*Constraints*	*Variables*	*Time*
Charnes & Miller [1956]	6	17	by hand
Hoffman & Padberg [1993]	145	1 053 137	5 min
Bixby, Gregory, Lustig, Marsten, Shanno [1992]	837	12 753 313	249 sec
Barnhart, Johnson, Nemhauser, Savelsbergh, Vance [1998]	>10 000	nobody knows	several days

- Exploit problem structure to price
- Solve large-scale LPs
- Use specialized branching strategies (not on individual variables)

Exploiting Structure





Public Transit

- Short, but wide; peaks
- Short paths
- Need to handle complex rules

Airline Industry

- Long, but thin; day structure
- Enumerate duty periods
- Can use k-shortest path alg.

Solving Large Set Partitioning LPs

(SPP)	$\min c^T x$		objective		$\min c^T x$
(i)	$\Sigma_{j\in d} x_j = 1$	$\forall$ duties d	partitioning	$\Leftrightarrow$	$Ax = 1$
(ii)	$x \geq 0$		bounds		$x \geq 0$
(iii)	x integer		integrality		x integer

2.11 Def. (Set Partitioning Problem): An IP with all 01-equations (and nonneg. constraints) is called a **set partitioning problem**.

2.14 Obs. (Box Lagrange Relaxation): The LPP relaxation of an SPP can be solved by Lagrangean relaxation as follows:

reduced costs

$$\begin{array}{ccccc} \min c^T x & = & \min c^T x & = & \max_\lambda \min (c^T - \lambda^T A)x + \lambda^T 1 \\ Ax = 1 & & Ax = 1 & & \\ x \geq 0 & & 0 \leq x \leq 1 & & 0 \leq x \leq 1 \end{array}$$

Rapid Branching

- Sort candidate variables by reduced costs

$$B^* = \{j_1, \dots, j_m\}, \quad \bar{c}_{j_1} \leq \cdots \leq \bar{c}_{j_m}$$

- Branch on set Q^k at branch j, $k = m, \dots, 0$

$$Q^k := \{x_{j_1} = \cdots = x_{j_k} = 1\}$$

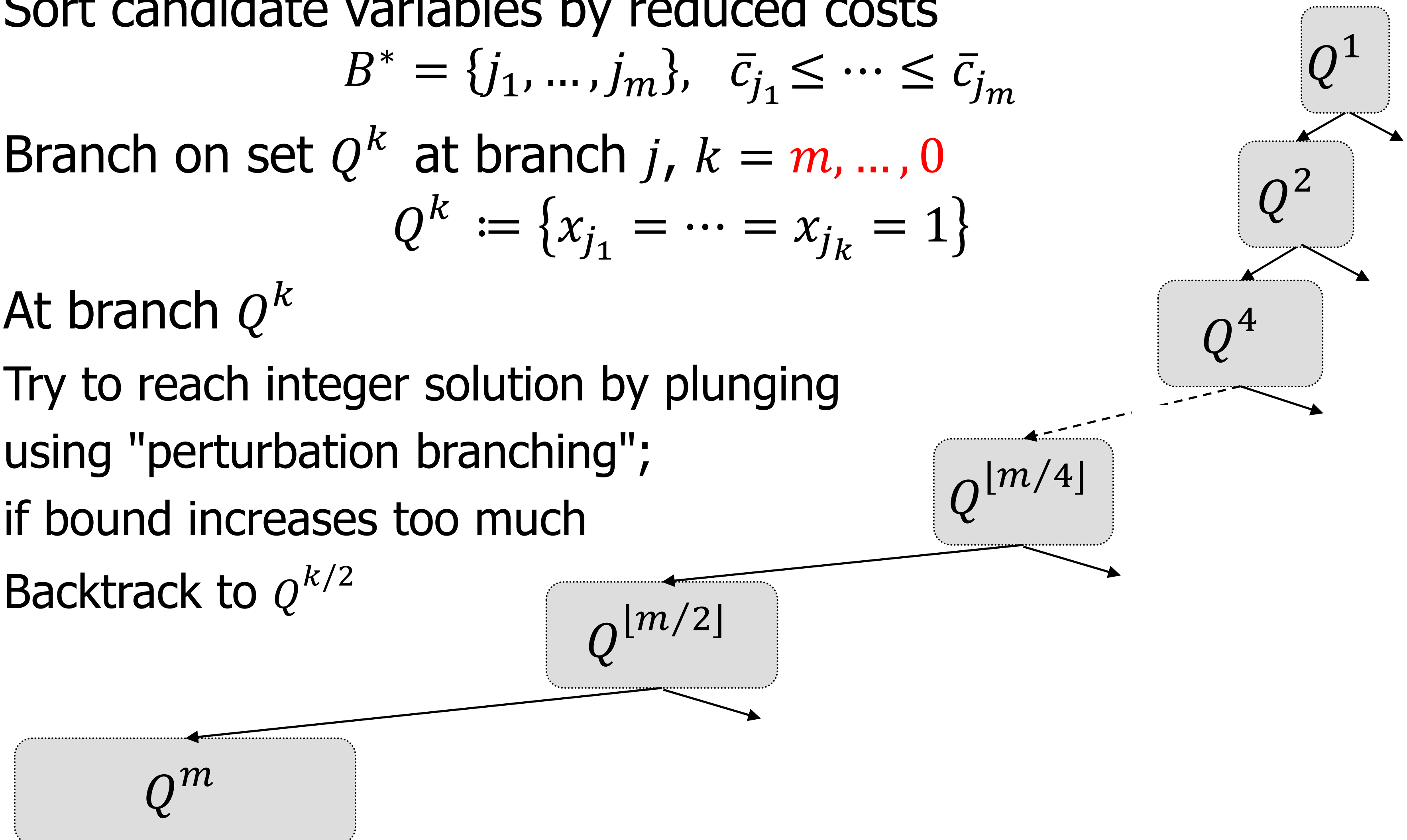
- At branch Q^k

  Try to reach integer solution by plunging

  using "perturbation branching";

  if bound increases too much

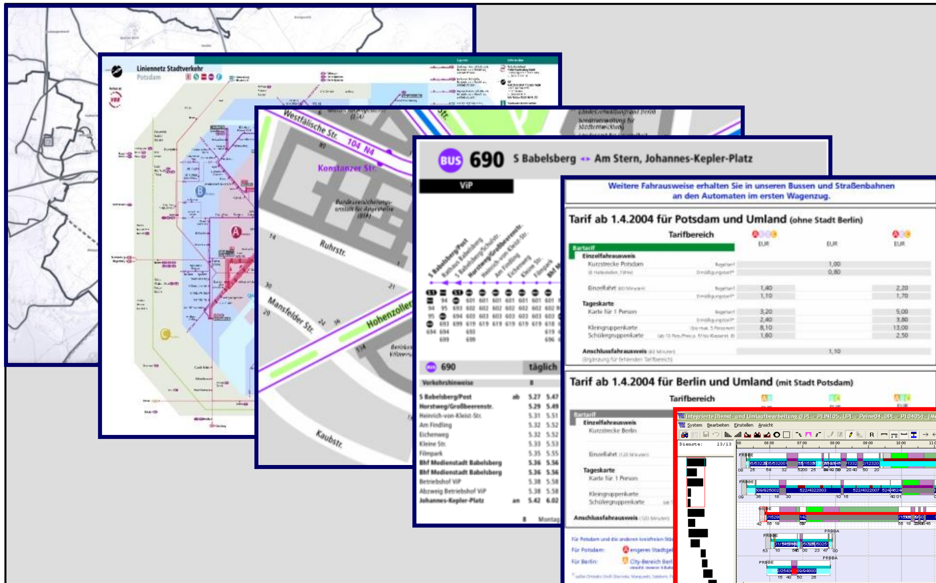
  Backtrack to $Q^{k/2}$



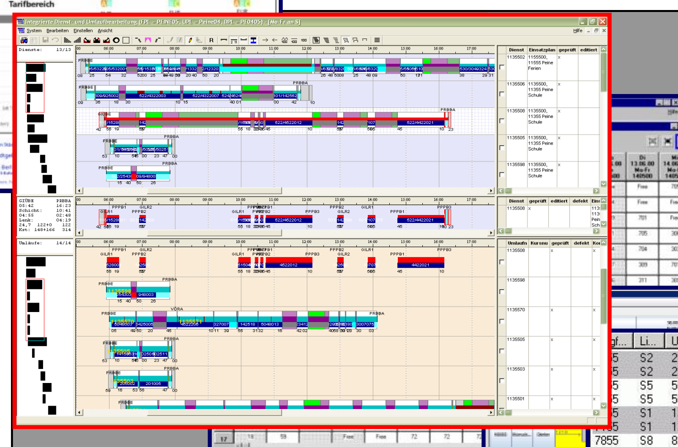
Planning Problems in Public Transit

Freie Universität Berlin
ZIB

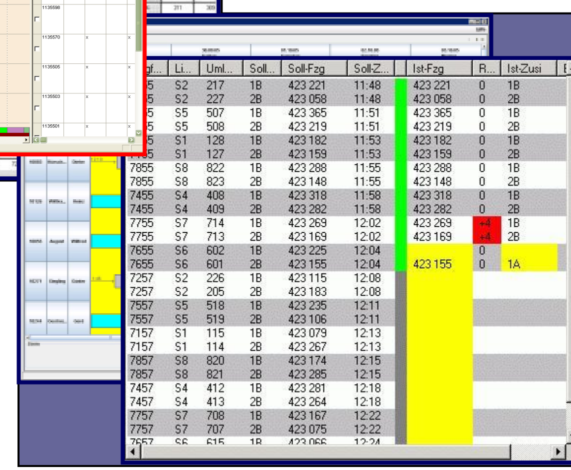
Service Design



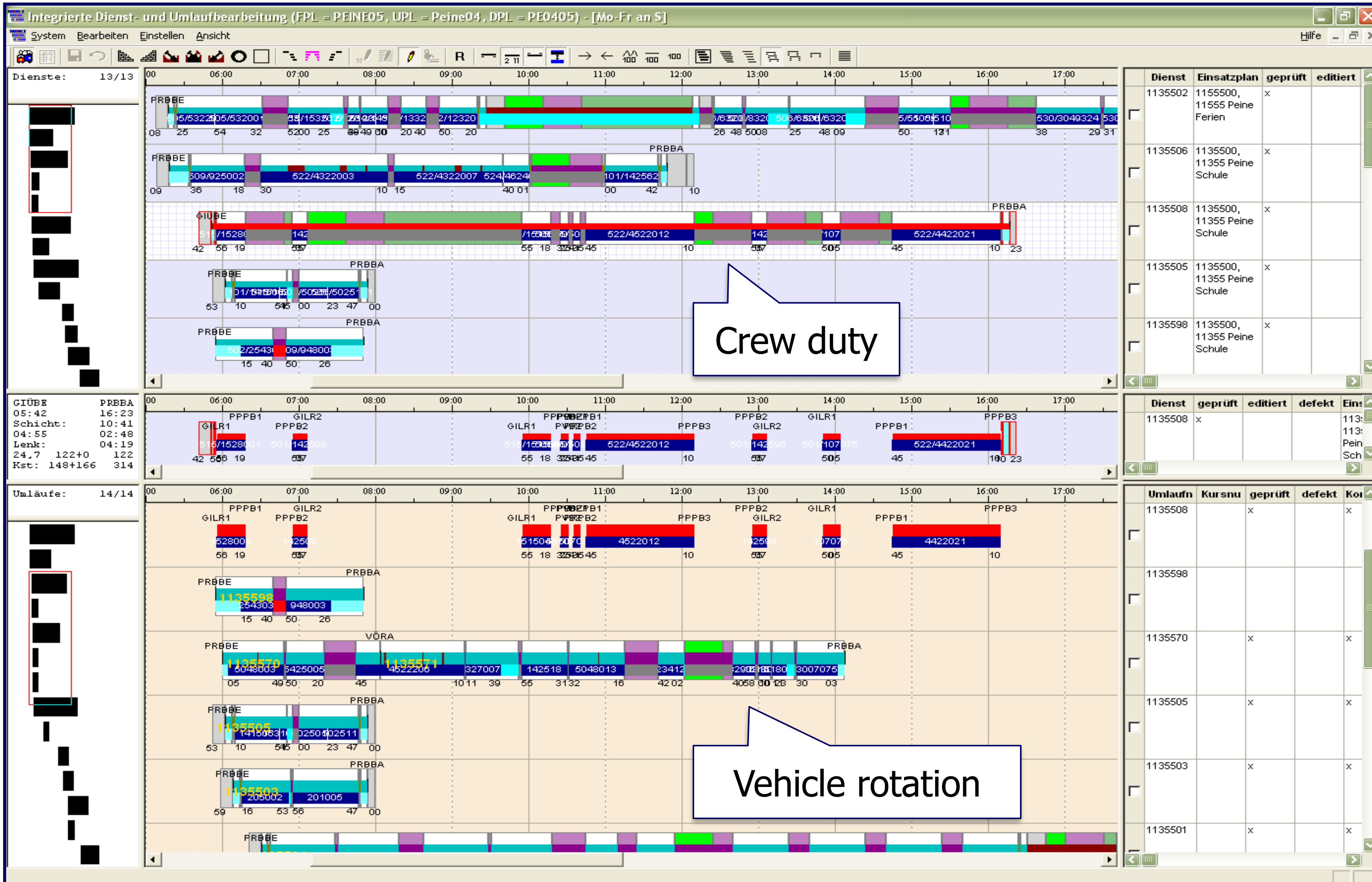
Operational Planning



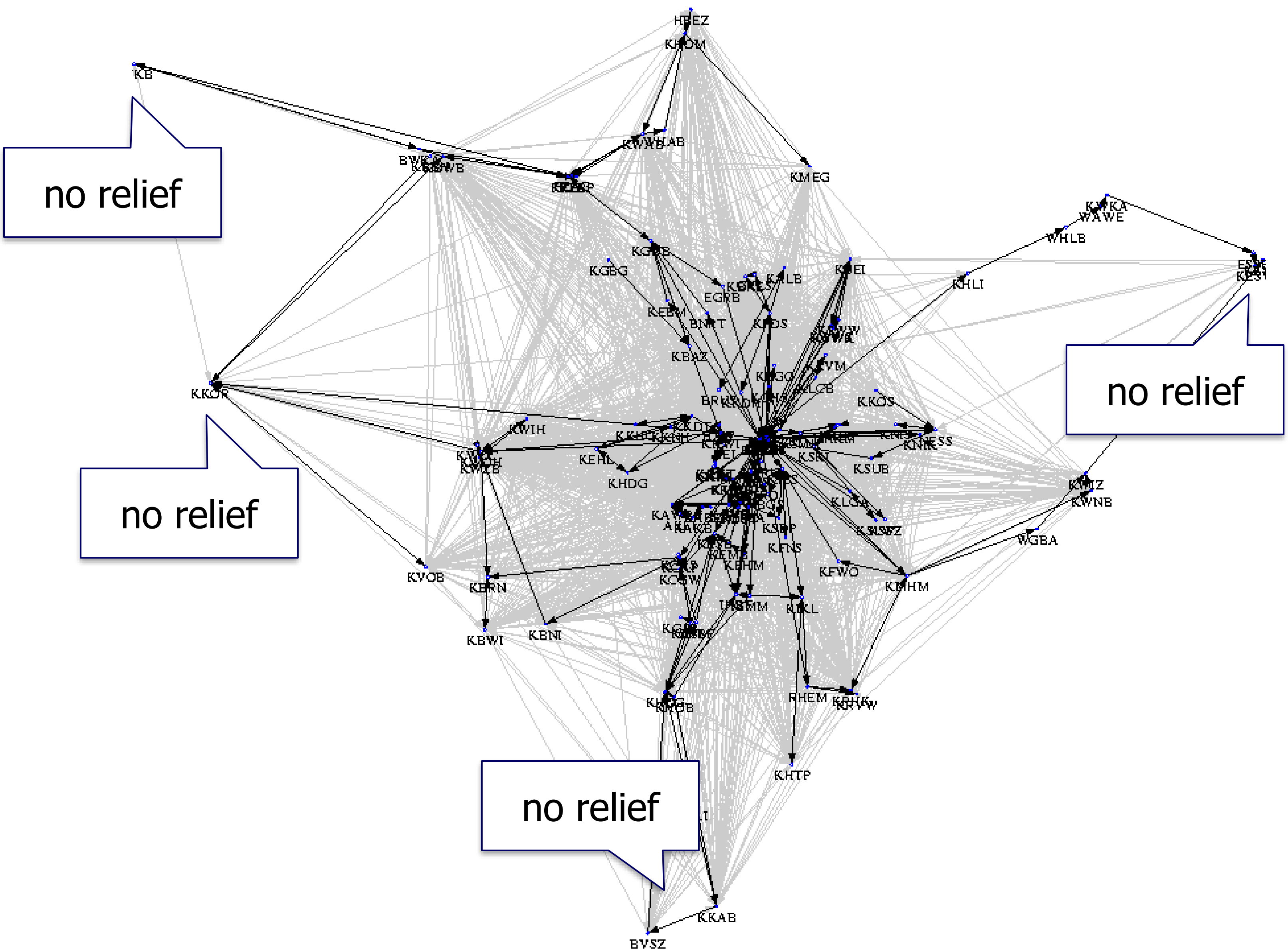
Operations Control



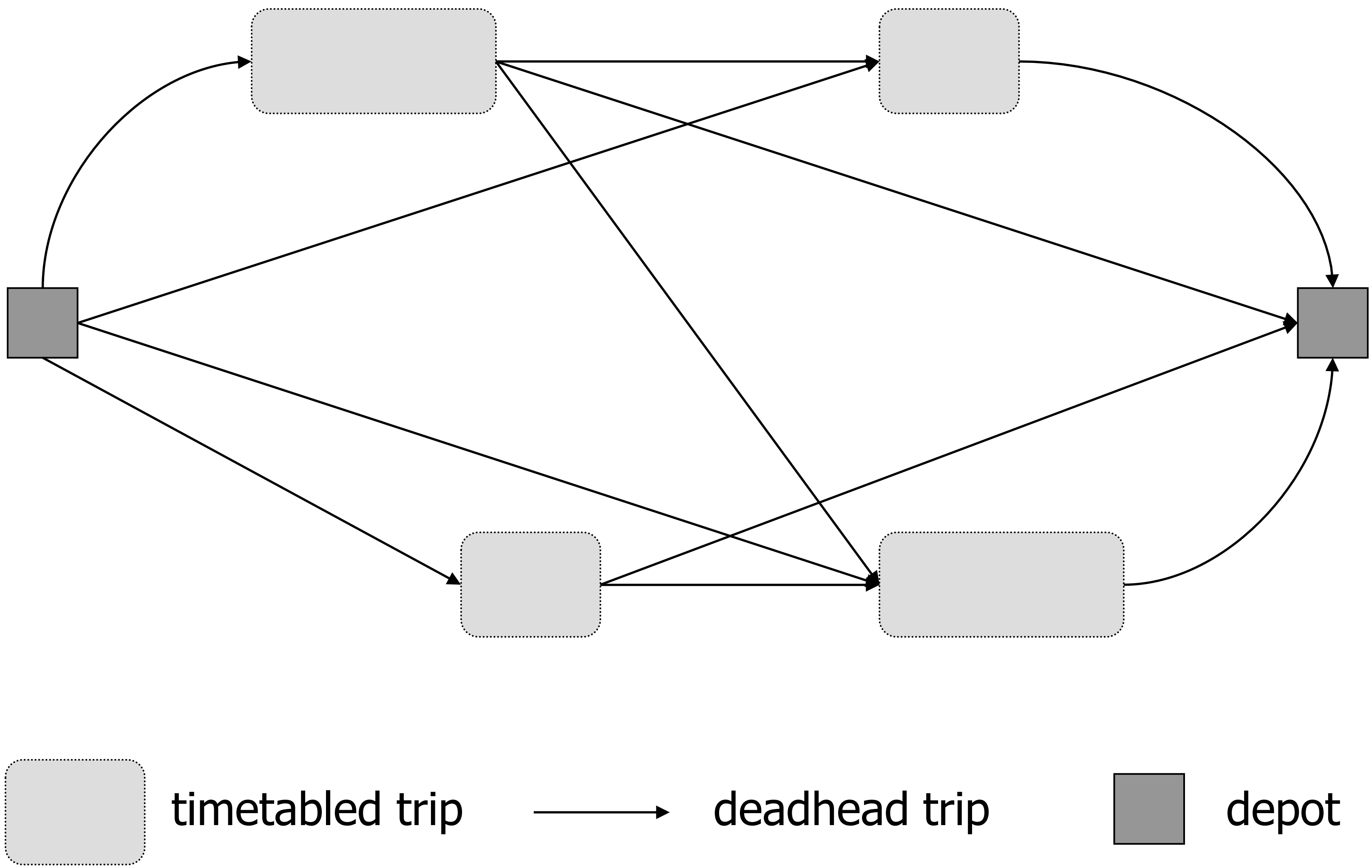
Integrated Vehicle and Duty Scheduling



Regional Scenarios

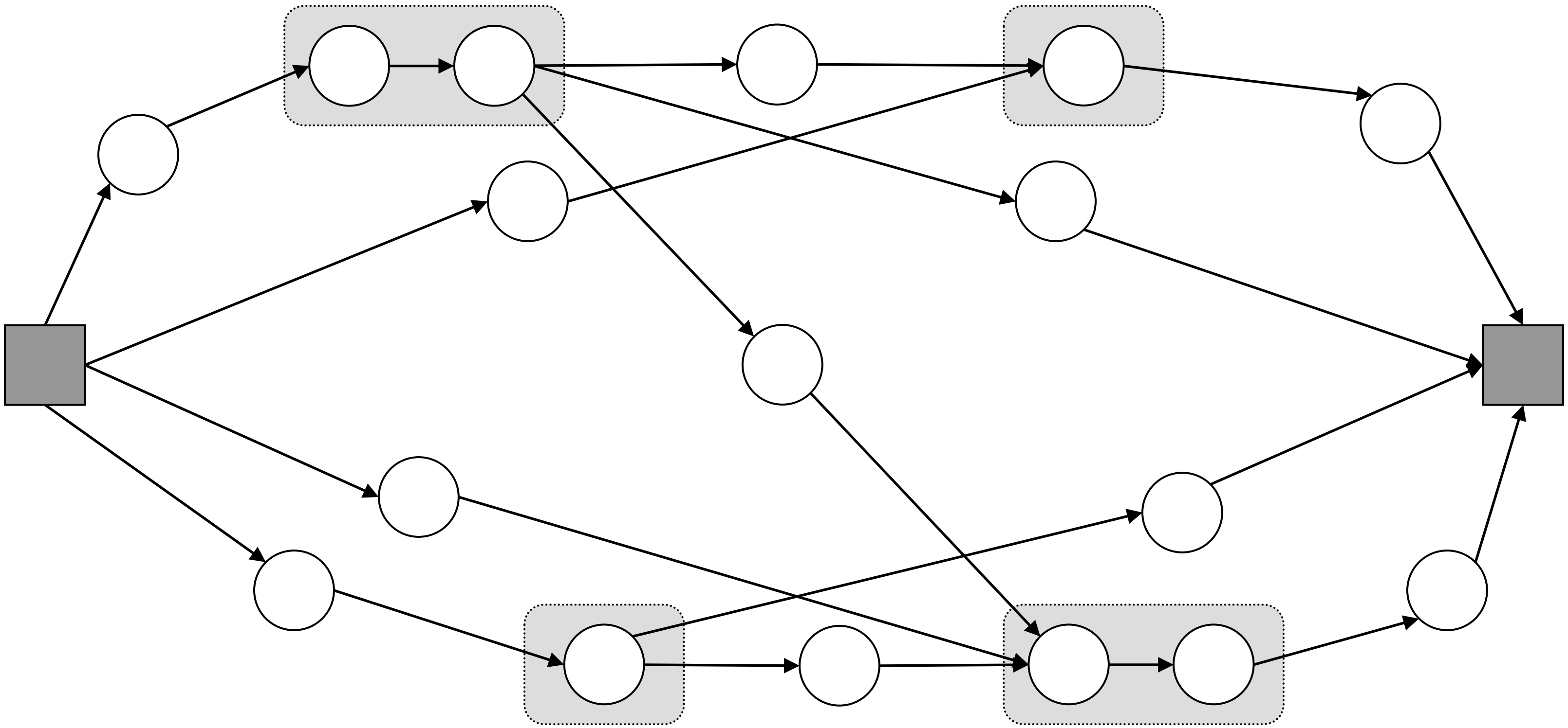


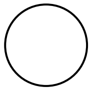
Vehicle Scheduling Graph



timetabled trip deadhead trip depot

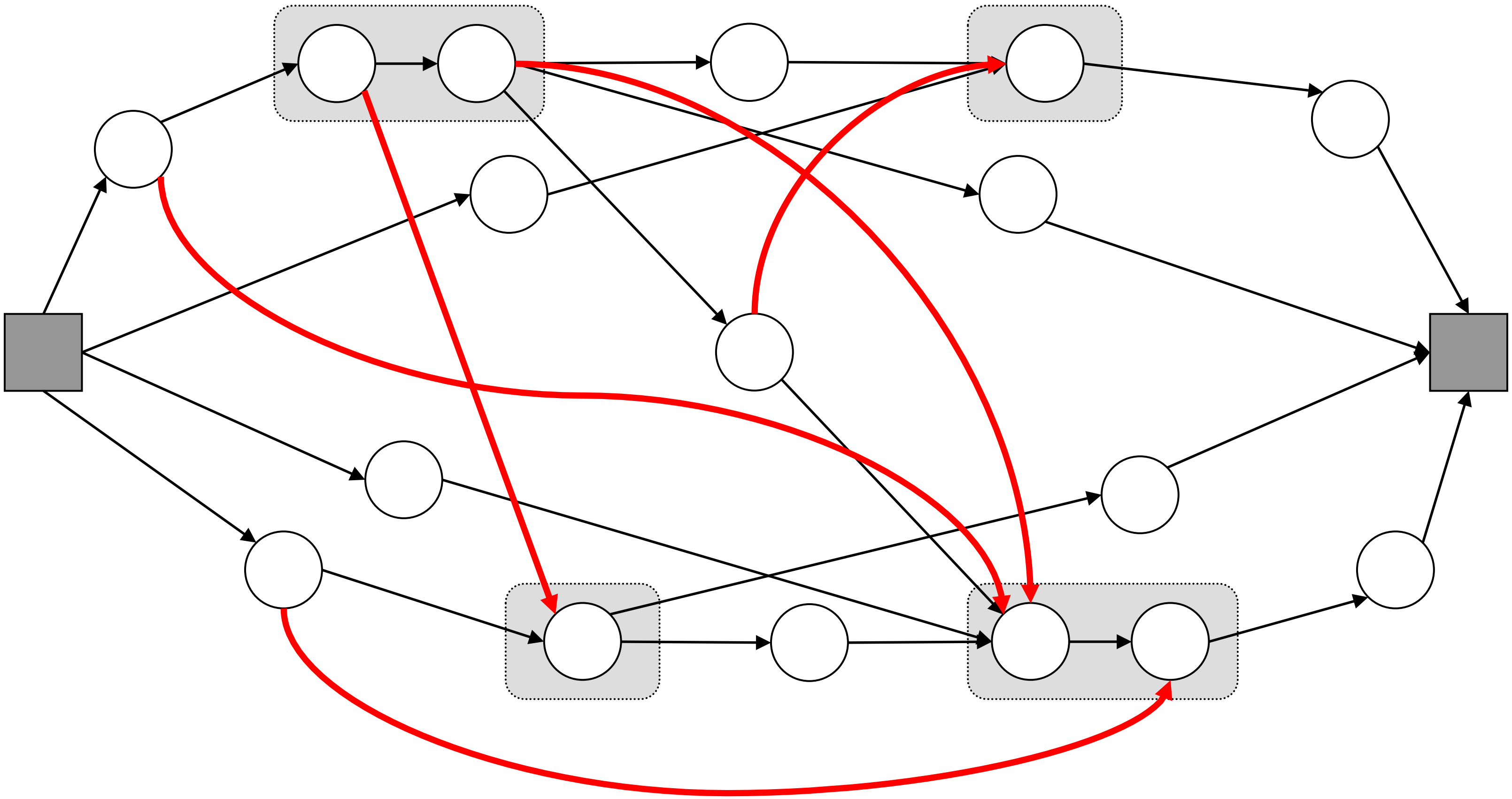
Duty Elements





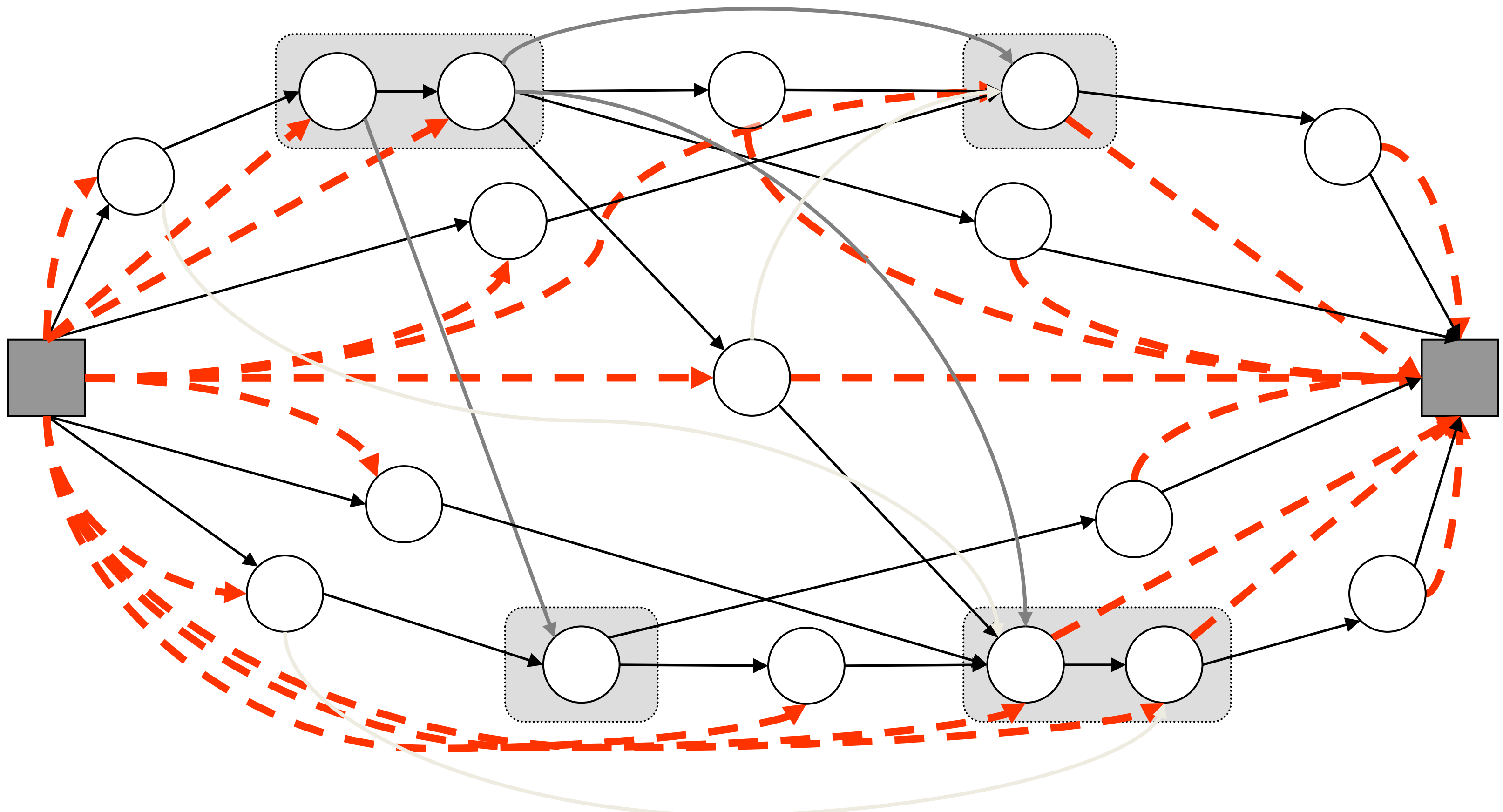
duty element

Change-Overs



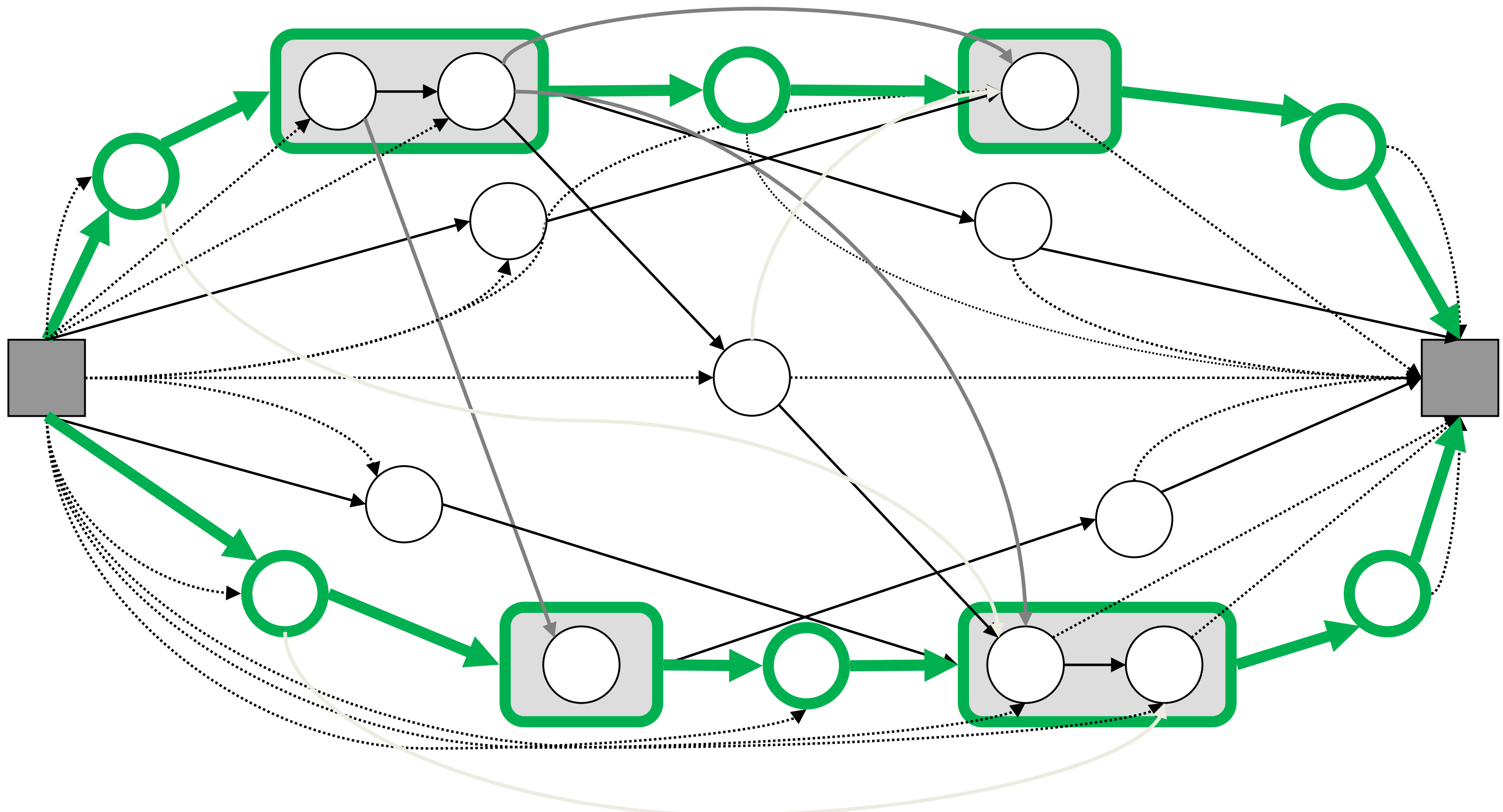
change-overs for drivers

Start and End of Duties

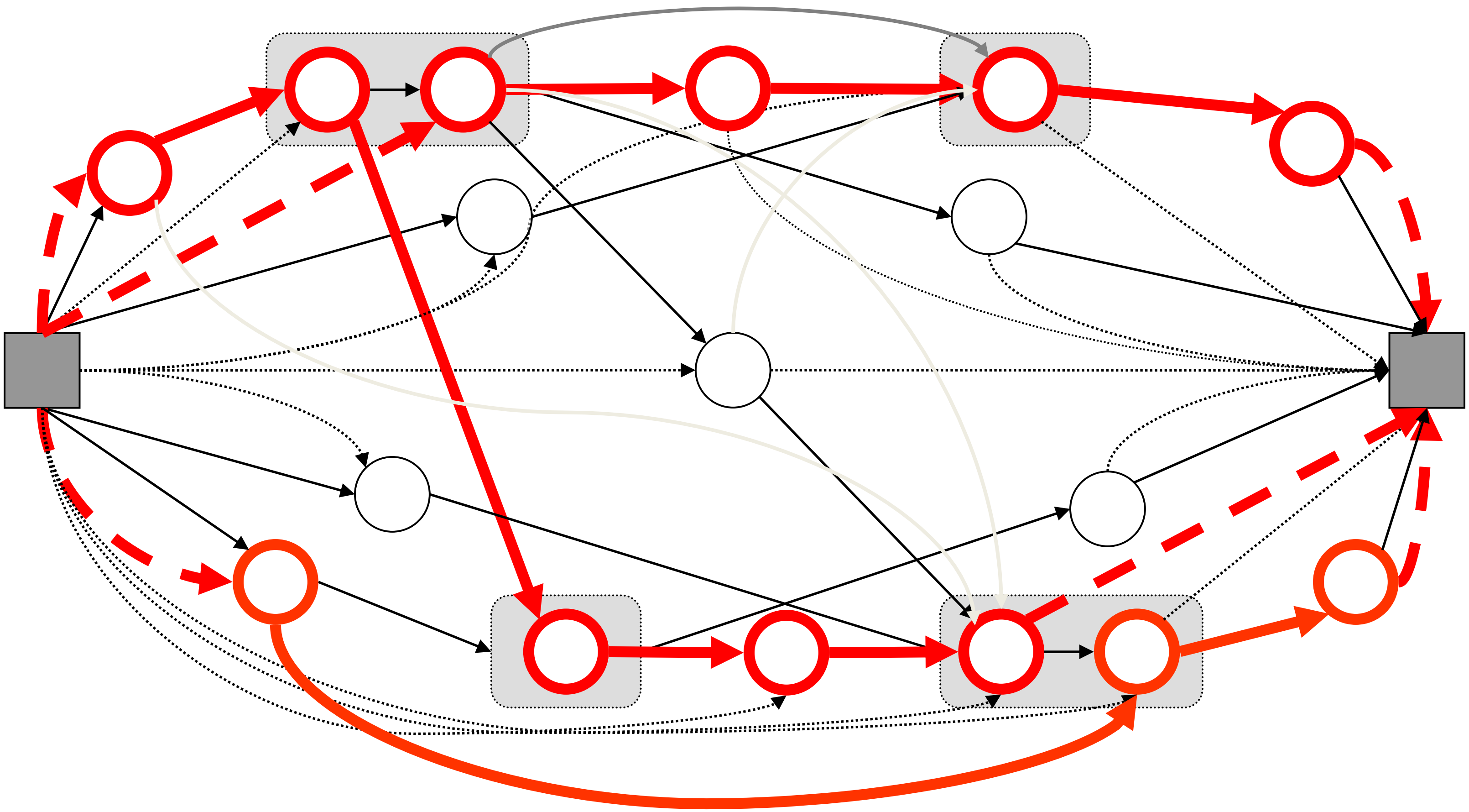


start or end of a duty

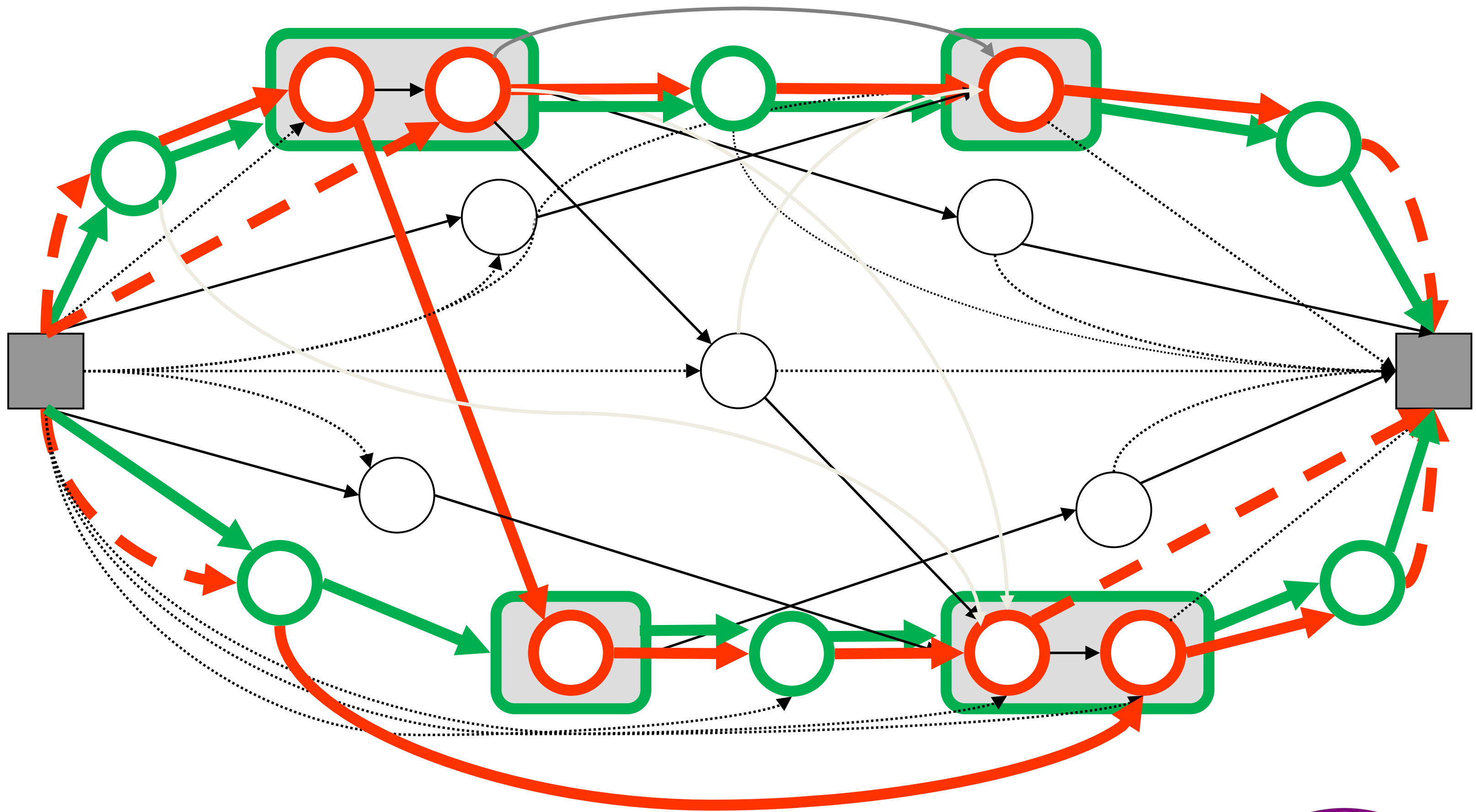
Vehicle Rotations

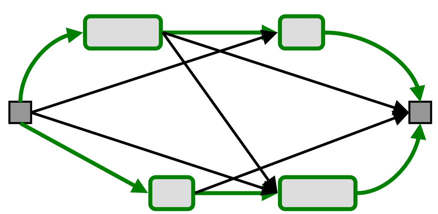


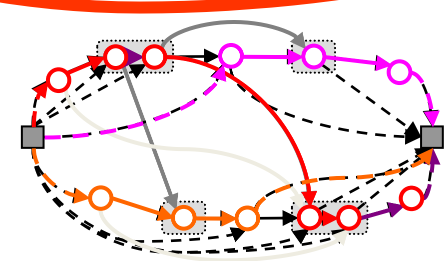
Crew Duties

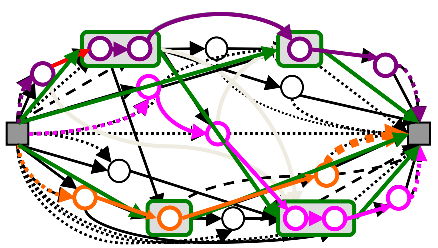


Compatible Schedules









2.15 Def. (Integrated Vehicle and Crew Scheduling Problem):
Let E be the set of timetabled and deadhead trips that require a driver. Then the **Integrated Vehicle and Crew Scheduling Problem** can be formulated as

$$
\begin{array}{llcll}
\text{(ISP)} & \min c^T x & + & d^T y & \\
(1)(\text{i}) & x\left(\delta_f^+(v)\right) - x\left(\delta_f^-(v)\right) = 0 & & & \forall v \neq s,t,\ f \in F \\
(1)(\text{ii}) & x\left(\delta^-(v)\right) = 1 & & & \forall v \neq s,t \\
(1)(\text{iii}) & x\left(\delta_f^+(s)\right) \leq \kappa_f & & & \forall f \in F \\
(2) & & & Ay = 1 & \\
(3) & x_e & = & y(e) & \forall e \in E \\
(4) & 0 \leq x \leq 1 & & 0 \leq y \leq 1 & \\
(5) & x \text{ integer} & & y \text{ integer} &
\end{array}
$$

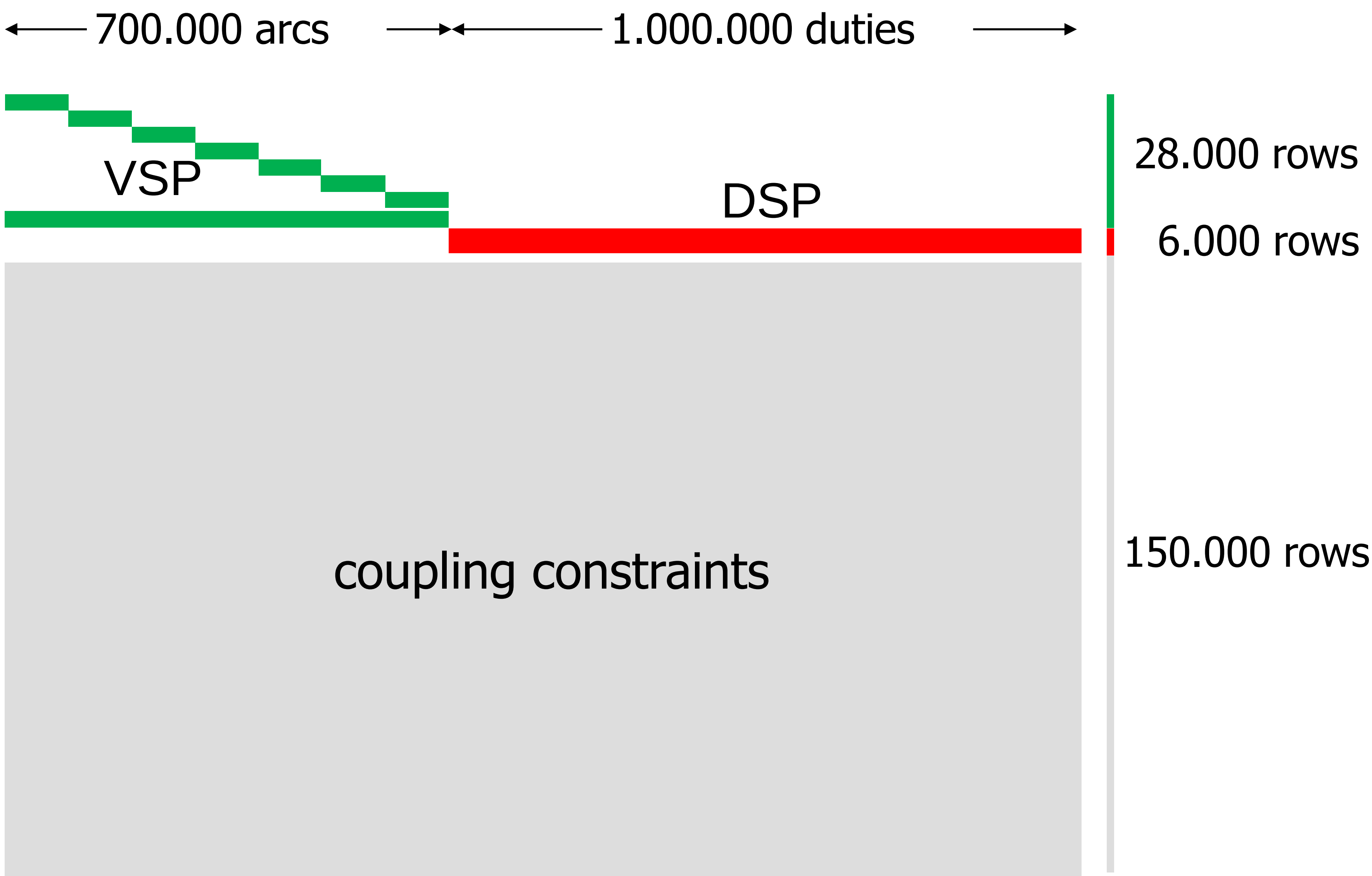
The constraints (3) are the **coupling constraints**.

2.15 Def. (Integrated Vehicle and Crew Scheduling Problem): Let E be the set of timetabled and deadhead trips that require a driver. Then the **Integrated Vehicle and Crew Scheduling Problem** can be formulated as

$$
\begin{array}{llcl}
\text{(ISP)} & \min c^T x & + & d^T y \\
(1) & Bx =/\leq b & & \\
(2) & & & Ay = 1 \\
(3) & Cx & = & Dy \\
(4) & 0 \leq x \leq 1 & & 0 \leq y \leq 1 \\
(5) & x \text{ integer} & & y \text{ integer}
\end{array}
$$

The constraints (3) are the **coupling constraints**.

Integrated Vehicle and Crew Scheduling Problem



2.16 Obs. (Lagrange Relaxation of the ISP): A Lagrange relaxation of the ISP with respect to the coupling constraints decomposes the ISP into an MDVSP and an SPP:

$$
\begin{array}{llcl}
\min & c^T x & + & d^T y \\
(1) & Bx =/\leq b & & \\
(2) & & & Ay = 1 \\
(3) & Cx & = & Dy \\
 & x \text{ binary} & & y \text{ binary}
\end{array}
$$

reduced costs

reduced costs

$$
\geq \max_{\lambda} \overbrace{\underbrace{\min_{\substack{x \text{ fulfills (1) and} \\ x \in \{0,1\}^m}} (c^T - \lambda^T C)x}_{=: f_V(\lambda)} + \underbrace{\max_{\substack{y \text{ fulfills (2) and} \\ y \in [0,1]^n}} (d^T - \lambda^T D)y}_{=: f_D(\lambda)}}^{f(\lambda) :=}
$$

Solving Integrated V&C Scheduling Problems

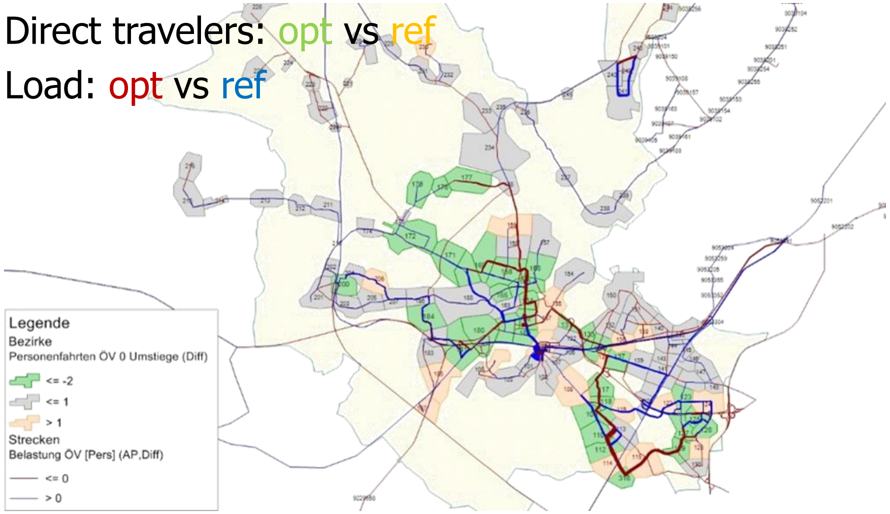
Article	*depots*	*trips*	*veh.*	*dut.*	*Problem*
Ball et al. [1983]	1	1 000	--	133	sequential planning
Scott [1985]	1	456	54	--	VSP + duty cost estimate
Tosini & Vercellis [1988]	17	300	--	--	VSP + additional constraints
Falkner & Ryan [1992]	1	182	--	41	DSP + additional constraints
Patrikalakis et al. [1992]	--	111	20	45	DSP + min cost flow
Gaffi & Nonato [1997]	28	257	44	65	ISP without driver releases
Freling [1997]	1	296	38	90	ISP
Friberg & Haase [1997]	1	30	--	--	DSP + SPP to optimality
Freling et al. [2000]	1	476	9	23	ISP
Huisman [2004]	--	653	67	117	ISP
Weider [2007]	7	3 698	209	260	ISP + caps + resource cons

Integrated Scheduling Problems

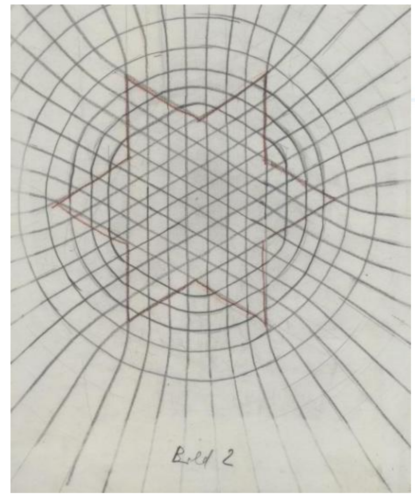
Line Planning & Pax Routing

Direct travelers: opt vs ref

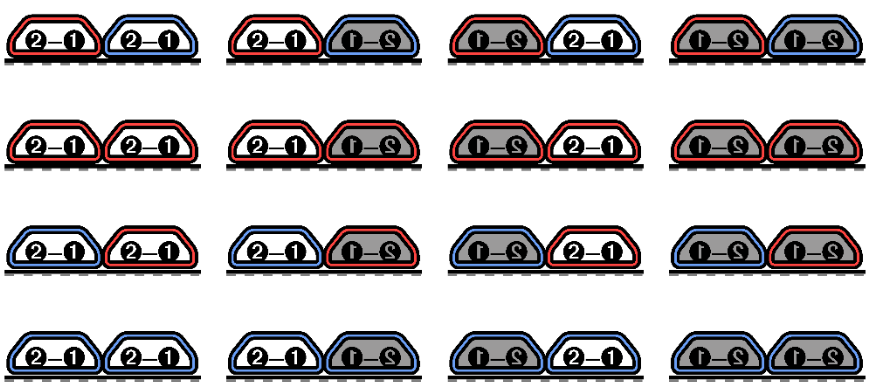
Load: opt vs ref



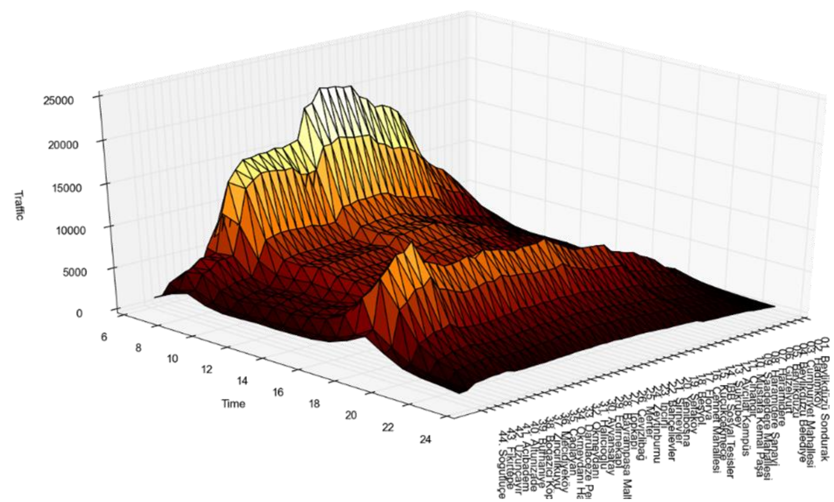
Network Design



Train Scheduling and Composition

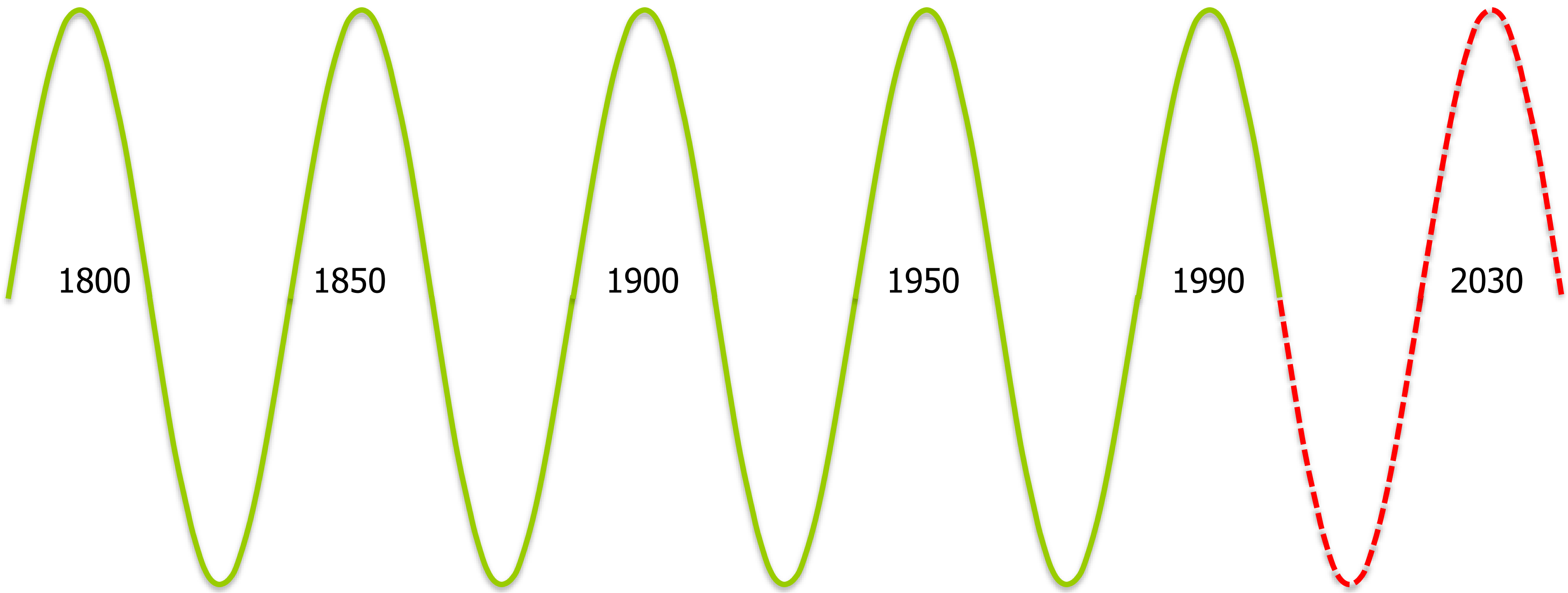


Static and Over Time



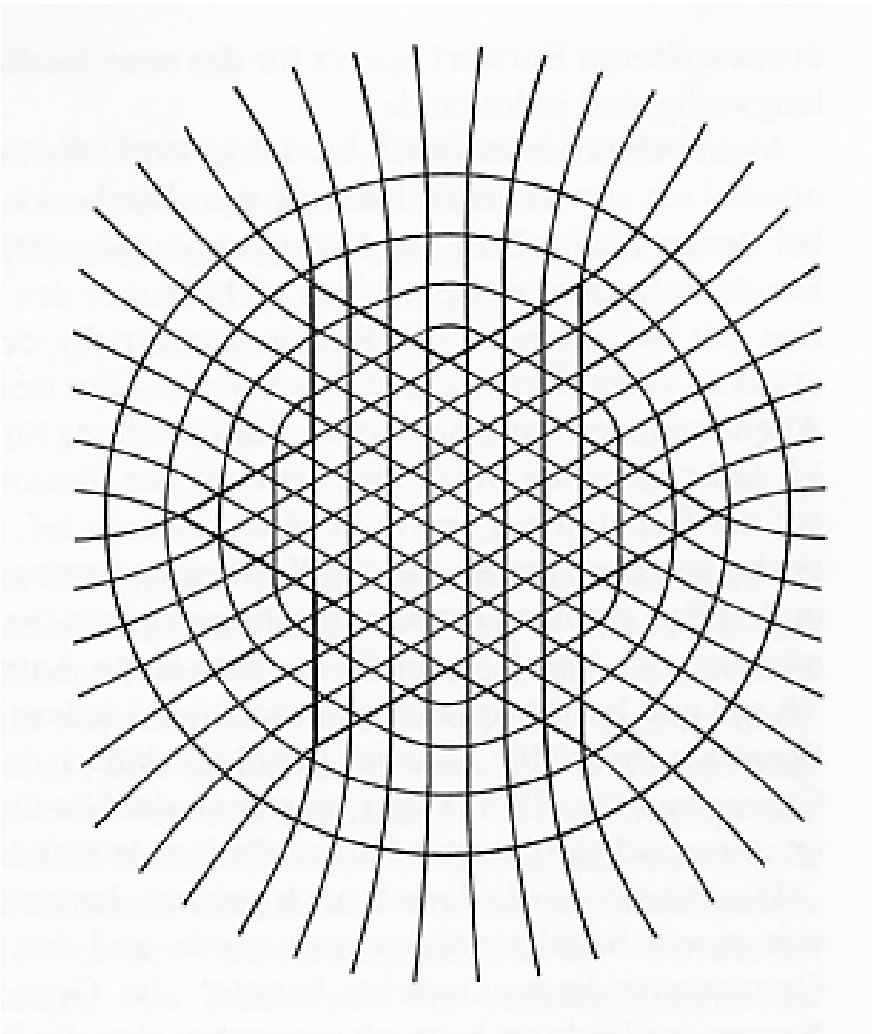
The Future of Traffic

Steam Engine	Railways	Electric Power	Automobile	Information Techn	Traffic Optimization
Textile Industry	Steel Industry	Chemical Industry	Petro Chemistry	Structured Inform.	Unstruct. Inform.



1. Kondratieff 2. Kondratieff 3. Kondratieff 4. Kondratieff 5. Kondratieff 6. Kondratieff

Thank you for your attention



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