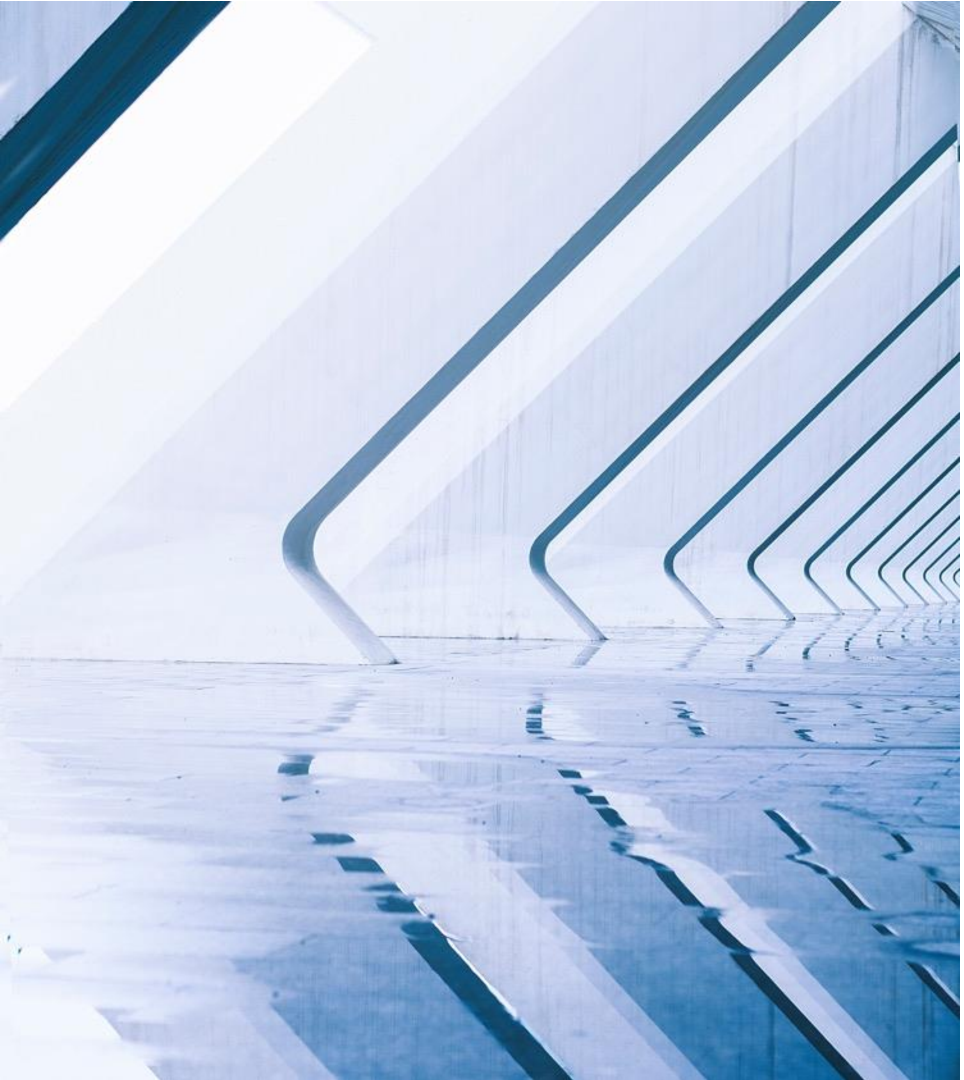


FICO

Cutting planes

Strengthening model formulations on the fly

Timo Berthold

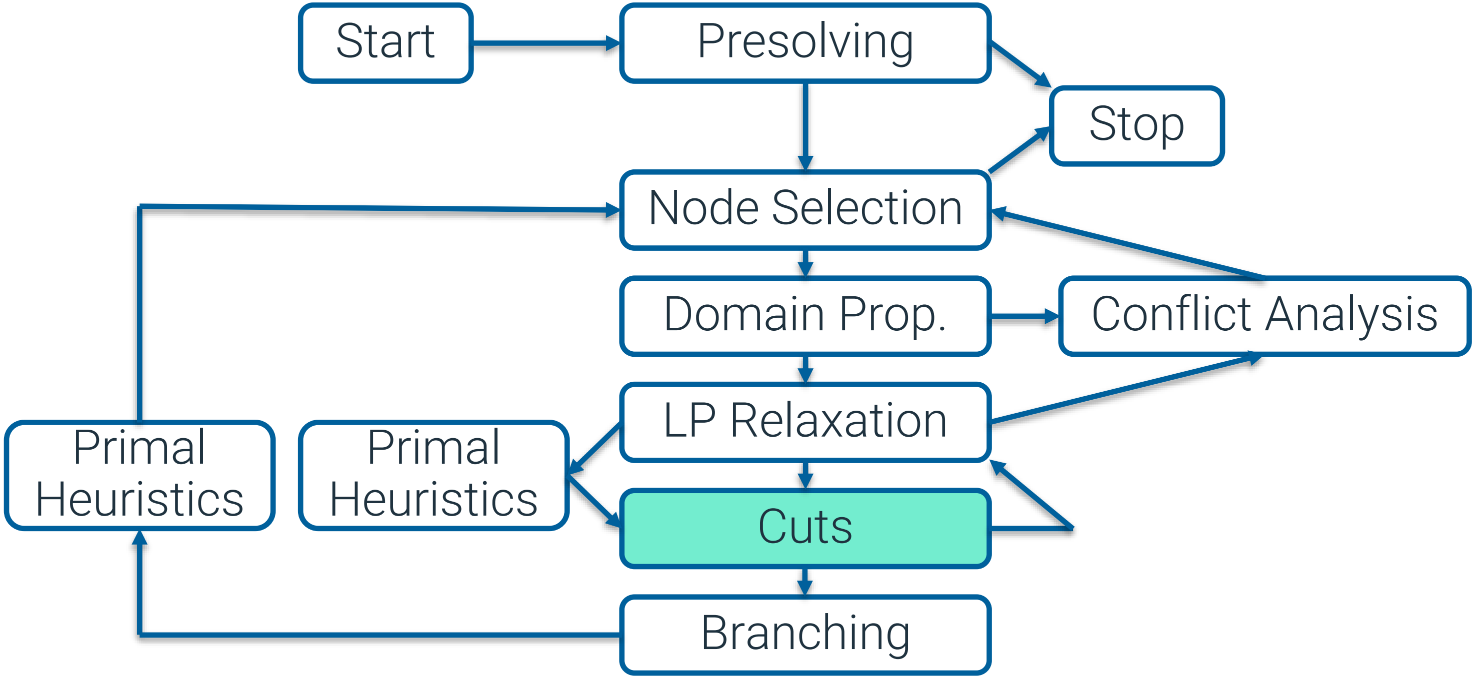
Agenda

- Cutting
- Generic (matrix-based) cuts
- Problem specific cuts
- Cut selection

Motivation

- Original MIP formulation can almost always be improved
  - Fewer constraints and variables
  - Less data to process
  - Smaller difference between space of feasible continuous and feasible integer solutions
- Two techniques:
  - Presolving: Logic reductions of the model before the main search starts
  - Cutting planes: Generating additional constraints that tighten the formulation
- Three principles occur at many places in cutting and presolving:
  - Rounding: Integer multiples of integer variables take integer values
  - Lifting: Fixing a variable at a bound can make constraints infeasible or redundant
  - Disjunction: Binary variable must take one of two values

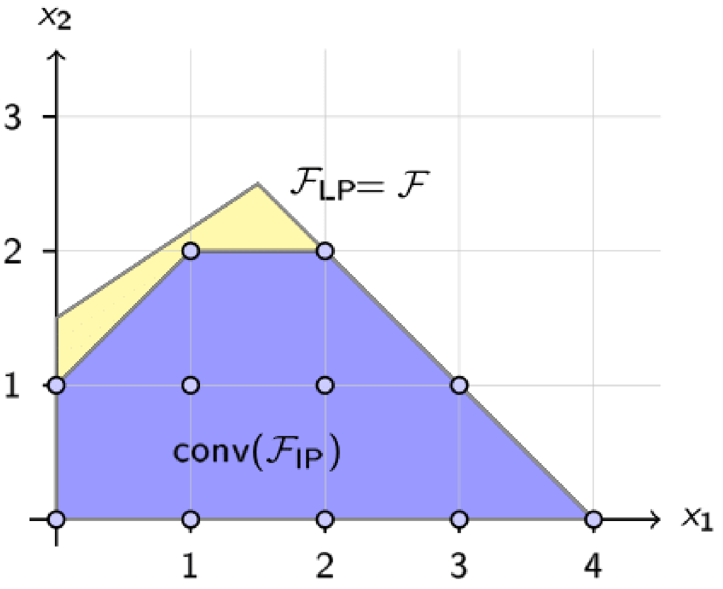
MIP Solver Flowchart



Cutting

General cutting plane method: Colorful picture

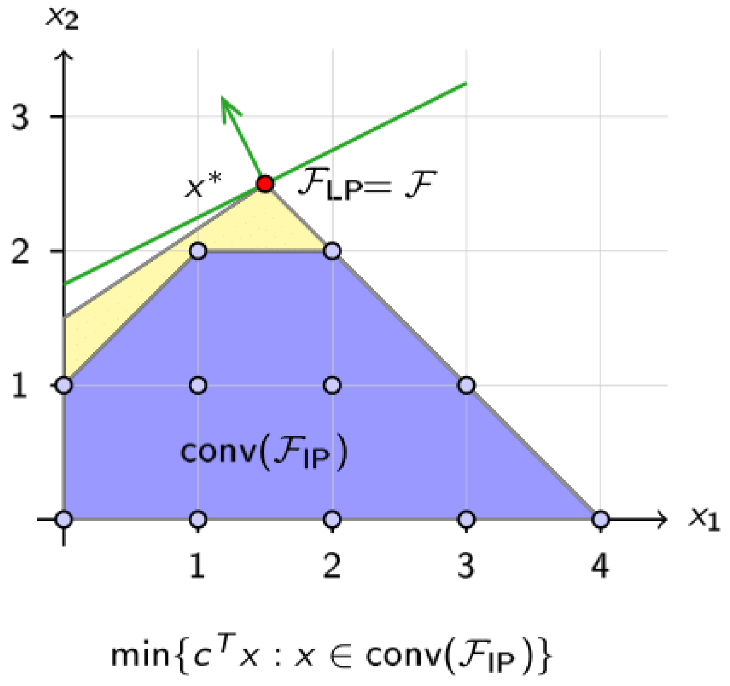
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

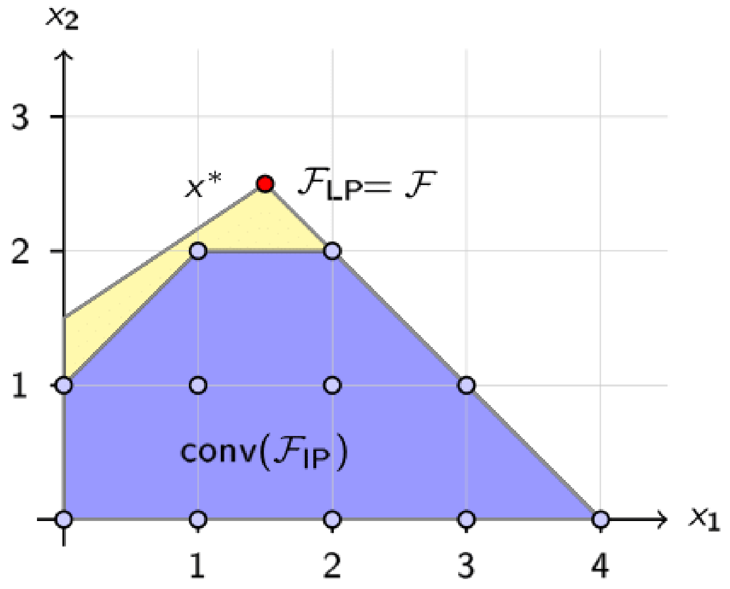
General cutting plane method: Colorful picture

1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



General cutting plane method: Colorful picture

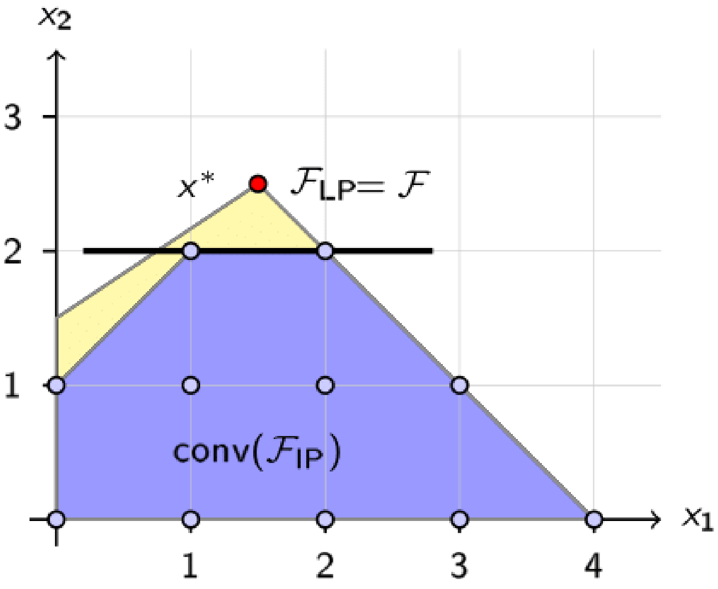
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

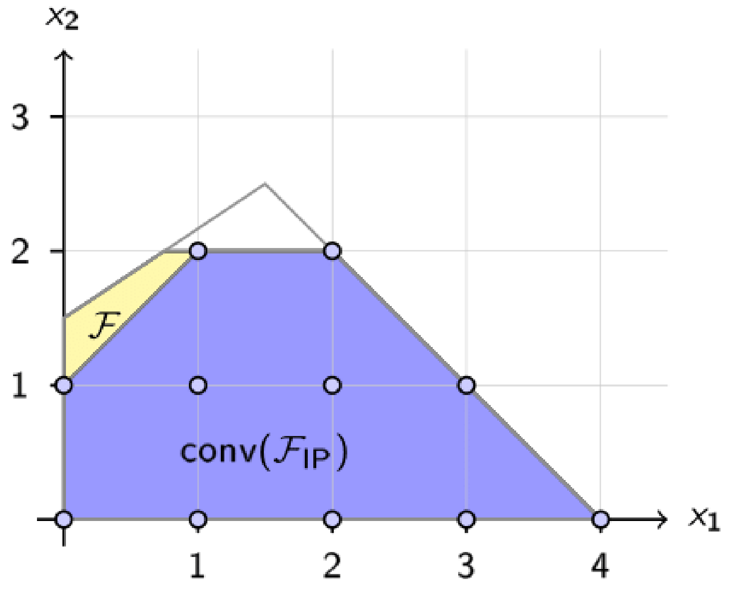
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

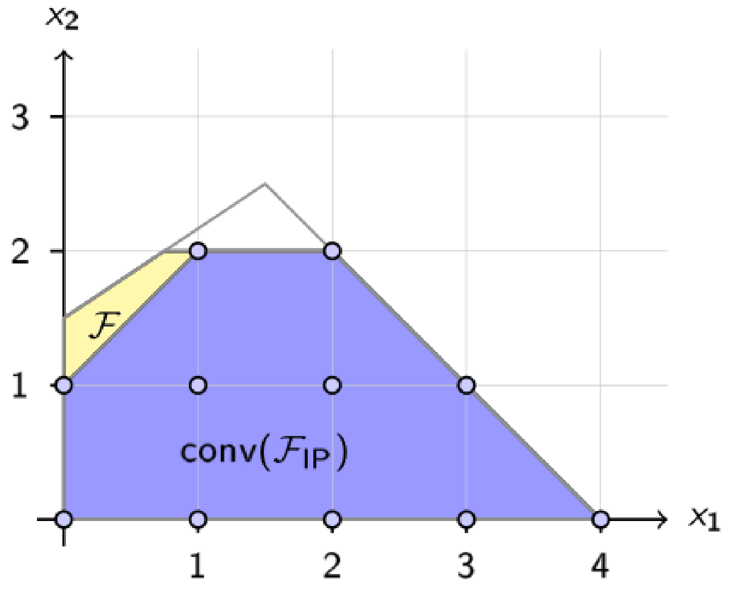
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

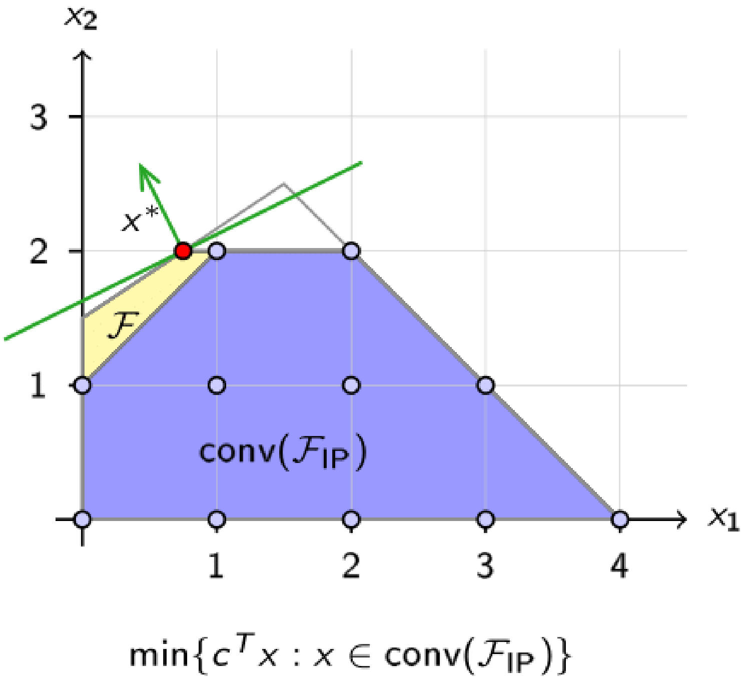
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

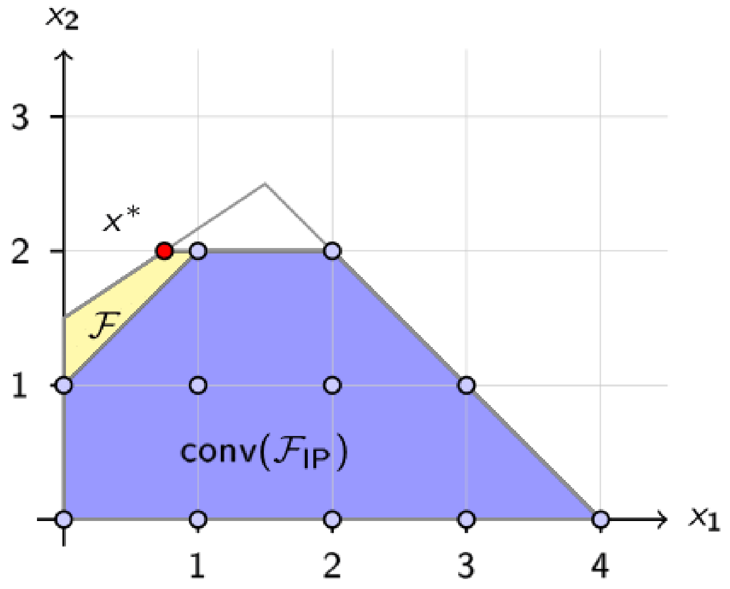
General cutting plane method: Colorful picture

1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



General cutting plane method: Colorful picture

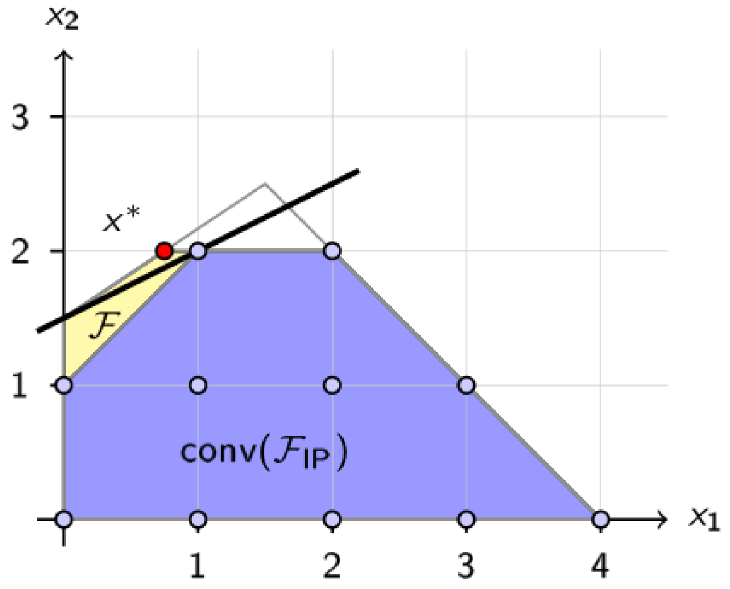
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

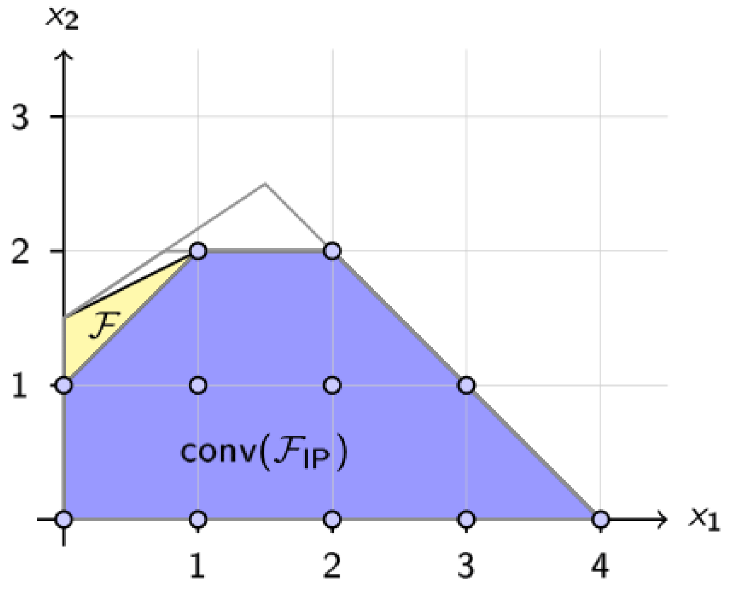
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

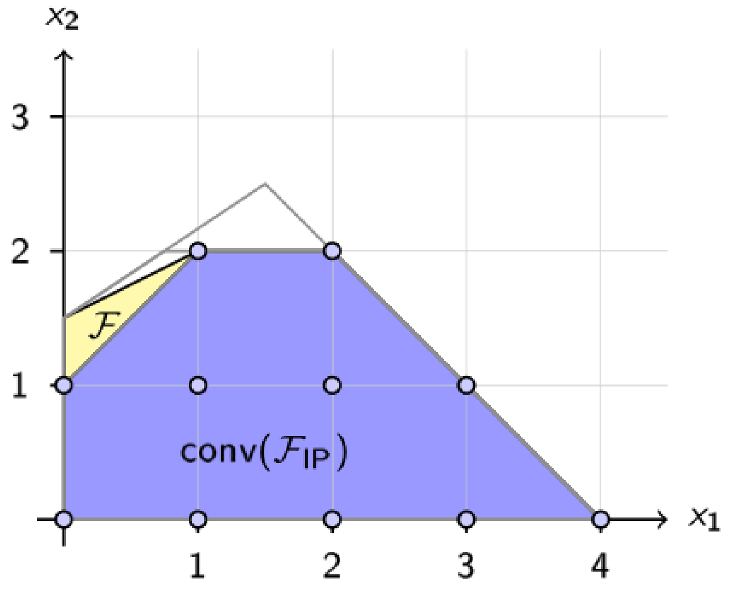
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

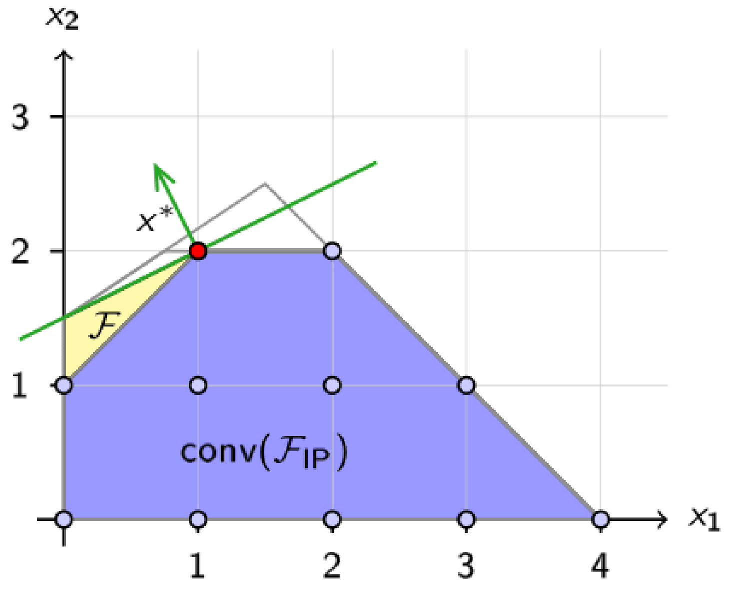
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

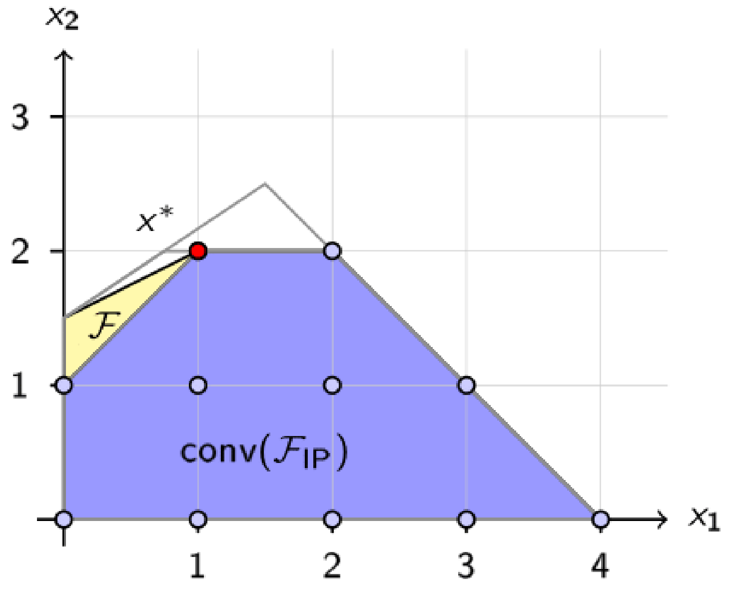
1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

General cutting plane method: Colorful picture

1. Initialize: $F \leftarrow F_{LP}$
2. Solve $x^* \leftarrow \min\{c^T x \mid x \in F\}$
3. If $x^* \in F_{IP}$:
   Stop!
4. Add inequality to F that is:
   - Valid for $conv(F_{IP})$ and
   - Violated by x^*
5. Goto 2.



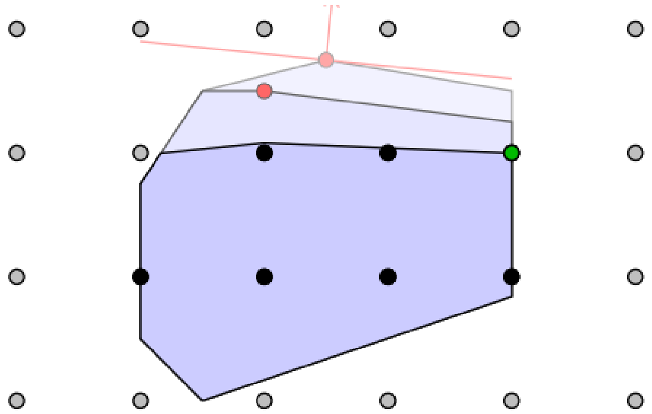
$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{IP})\}$

Classes of cuts

- General,“matrix-based“ cuts:
  - Gomory cuts
  - complemented MIR cuts
  - Gomory mixed integer cuts
  - strong Chvátal-Gomory cuts
  - {0, ½}-cuts
  - implied bound cuts .
  - Split cuts
  - Lift-and-project cuts
  - Mod-k cuts
  - ...
- Combinatorial, „problem-specific cuts“:
  - 0-1 knapsack problem
  - stable set problem
  - 0-1 single node flow problem
  - multi-commodity-flow problem
  - ...

Global and local cuts

- Cutting plane generation works in rounds:
  - Solve LP, remove cuts, generate cuts, filter cuts, select cuts, add cuts, repeat
- Heavily at the root node
  - Often around 20 rounds of cuts, sometimes more than 100
- Less heavy in the tree
  - Not at every node
  - Much less rounds and fewer cuts per round
  - Should we generate local cuts in the tree?



Matrix-based cuts: Gomory & friends

Gomory cuts (1958)

- Consider LP in basic representation, i.e., all rows look like $x_i + \sum \bar{a}_{ij} x_j = \bar{b}_i$
- Split up and write integer parts to the left side and fractional parts to the right side
  - Right hand side less than 1, left hand side integer, hence right side less than 0 for any feasible integer solution
- This is the Gomory cut: $\bar{b}_i - \lfloor \bar{b}_i \rfloor - \sum (\lfloor \bar{a}_{ij} \rfloor - \bar{a}_{ij}) x_j \leq 0$
  - Does not hold for the current LP solution (since $x_j = 0$)
- Add slack variable, add Gomory cut to $Ax = b$, iterate
- Similar idea works for mixed-integer programming (Gomory 1960)

Chvátal-Gomory (Chvátal 1973)

- Works on original matrix. Only for pure integer constraints.
- Let A_j be the j-th column of A and $\lambda \in \mathbb{R}^m_{\geq 0}$
- Aggregate: $\sum \lambda A_j x_j \leq \lambda b$
- Rounding, step 1: $\sum \lfloor \lambda A_j \rfloor x_j \leq \lambda b$
  - Valid, since $\sum \lfloor \lambda A_j \rfloor x_j \leq \sum \lambda A_j x_j$ and $x_j \geq 0$
- Rounding, step 2: $\sum \lfloor \lambda A_j \rfloor x_j \leq \lfloor \lambda b \rfloor$
  - Valid, since $x \in \mathbb{Z}^n$

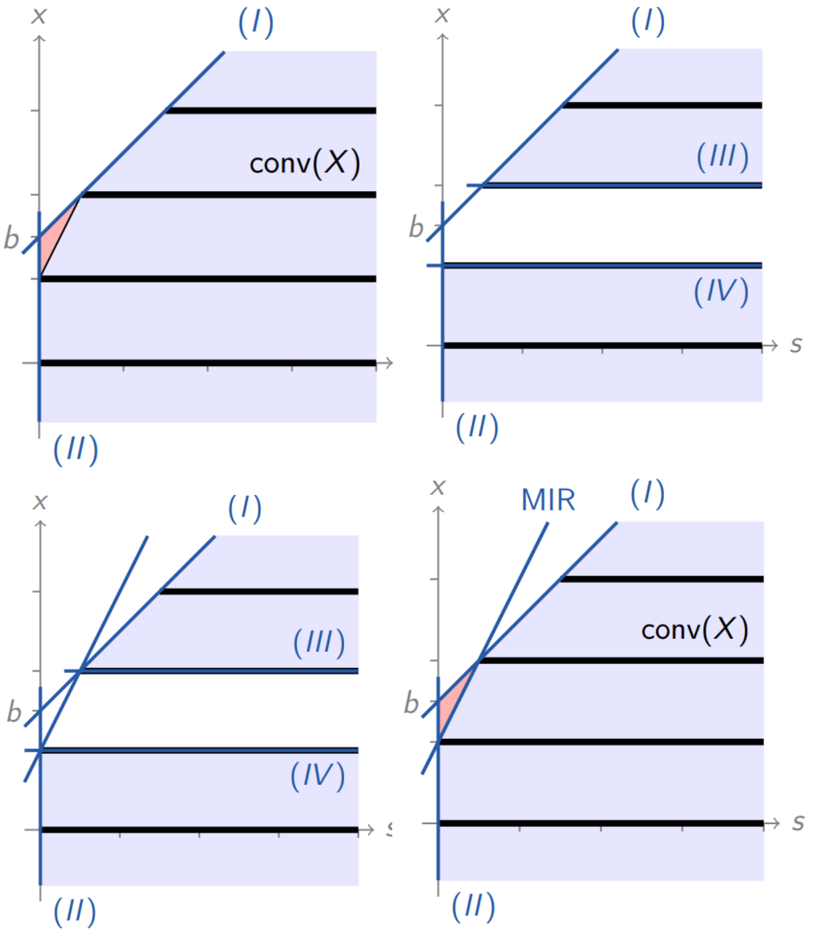
{0, ½} and mod-k (Caprara&Fischetti 1996, Caprara et al 2000)

- How to choose λ for Chvátal-Gomory cuts?
  - Many heuristics exist...
  - λ can be replaced by λ- $\lfloor\lambda\rfloor \in [0,1)^m$
- Important special case: $\lambda \epsilon \{0, \frac{1}{2}\}^m$
  - For subclass of {0,½}-cuts, there are efficient algorithms to compute strongest cut
- Many important sets of facet-defining inequalities can be expressed as {0,½}-cuts
  - Odd cycle inequalities for stable set
  - Comb inequalities for TSP
  - Blossom inequalities for b-matching
- Generalization: mod-k cuts with $\lambda \epsilon \{0, \frac{1}{k}, \dots, \frac{k-1}{k}\}^m$

FICO

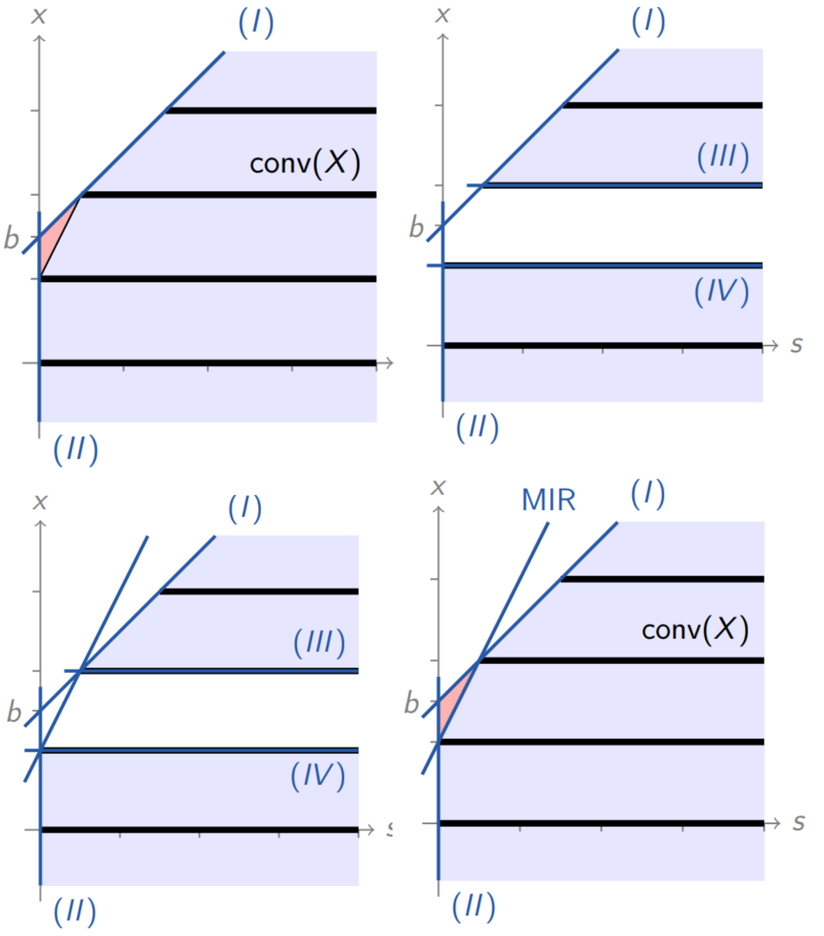
Mixed-Integer Rounding (MIR)

- Mixed-Integer set:
  - $X \;:= \{(x, s) \in \mathbb{Z} \times \mathbb{R}: x \leq b + s(I), s \geq 0(II)\}$
  - Inequalities do not suffice to describe conv(X)
- Disjunctive Argument:
  - If an inequality is valid for X_1 and for X_2, it is also valid for $X_1 \cup X_2$.
  - Here: $x \geq \lceil b \rceil$ (III) and $x \leq \lfloor b \rfloor$ (IV)
- MIR inequality: $x \leq \lfloor b \rfloor + \frac{s}{1-(b-\lfloor b \rfloor)}$
  - This is (I) + $(b - \lfloor b \rfloor)$(III)
  - This is (II) + $(1 - (b - \lfloor b \rfloor))$(IV)



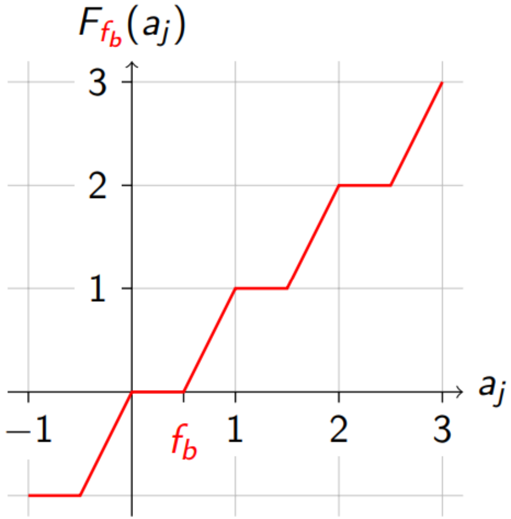
Mixed-Integer Rounding (MIR)

- Mixed-Integer set:
  - $X := \{(x, s) \in \mathbb{Z} \times \mathbb{R}: x \leq b + s \text{(I)}, s \geq 0 \text{(II)}\}$
  - Inequalities do not suffice to describe conv(X)
- Disjunctive Argument:
  - If an inequality is valid for X_1 and for X_2, it is also valid for $X_1 \cup X_2$.
  - Here: $x \geq \lceil b \rceil$ (III) and $x \leq \lfloor b \rfloor$ (IV)
- MIR inequality: $x \leq \lfloor b \rfloor + \frac{s}{1-(b-\lfloor b \rfloor)}$
  - This is (I) $+ (b - \lfloor b \rfloor)$(III)
  - This is (II) $+ (1 - (b - \lfloor b \rfloor))$(IV)



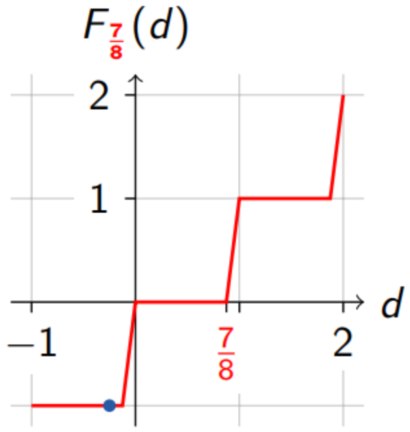
Complemented MIR

- Mixed knapsack set
  - $X := \{(x, s) \in \mathbb{Z}_{+}^{n} \times \mathbb{R}_{+}: \sum a_j x_j \leq b + s, x_j \leq u_j\}$
- General MIR inequality:
  - $\sum a_j x_j \leq \lfloor b \rfloor + \frac{s}{1-f_b}$ with $f_b = (b - \lfloor b \rfloor)$
- C-Mir inequality:
  - Divide by positive δ (typically integer multiple of some a_j)
  - Complement some of the integers ($x_j = u_j - \bar{x}_j$)
  - $\sum F_f(a_j) x_j \leq \lfloor b \rfloor - \frac{s}{1-f_b}$



Example: Complemented MIR

- Mixed knapsack, two general integers, one continuous:
  - $x_1 + 4x_2 \leq \frac{11}{2} + s$, bounds: $x_1,\ x_2 \leq 2$
- General MIR inequality ($\delta = 1$, no complements):
  - $x_1 + 4x_2 \leq 5 + 2s$
- Example of a c-MIR inequality:
  - Use $\delta = 4,\ x_1 = 2 - \bar{x}_1$
  - $-\frac{1}{4}\bar{x}_1 + x_2 \leq \frac{7}{8} + \frac{1}{4}s$
    - Reformulation of original knapsack
    - Substituted the complement, scaled, brought constant to the right-hand side
  - Apply MIR procedure to this reformulation
  - $-1\bar{x}_1 + x_2 \leq 0 + 2s \rightarrow$ Substitute back to get $x_1 + x_2 \leq 2 + 2s$



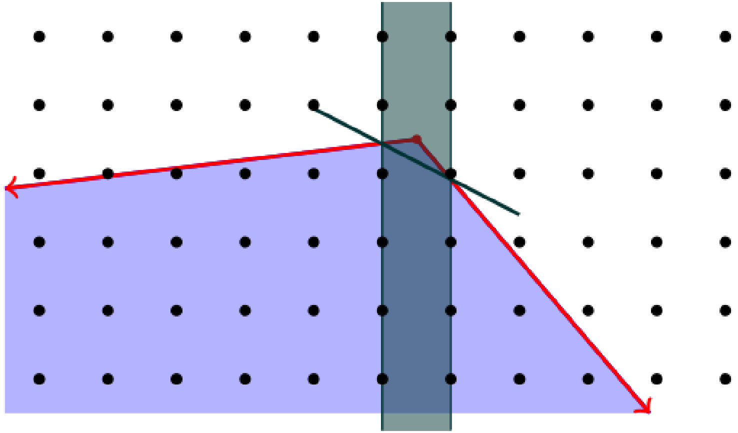
C-MIR in practice

C-MIR separation procedure of Marchand and Wolsey (1998, 2001):

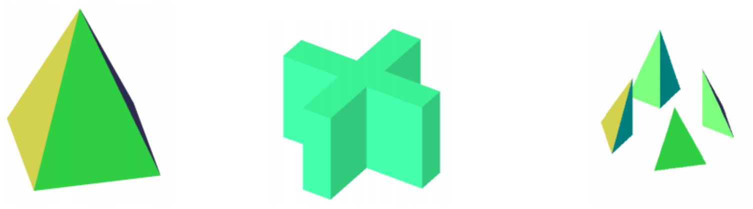
1. For each constraint of the problem:
2. Apply MIR procedure to constraint:
   - Complement variables whose LP value is closer to upper bound
   - For each coefficient a_j and each $\gamma \in \{1,2,4,8\}$ divide constraint by $\delta = \gamma|a_j|$
   - Apply MIR formula
   - Choose most violated cut from this set of MIR cuts
   - Check if complementing one more (or one less) variable yields larger violation
3. If no violated cut was found (and no limit reached):
   - Add other constraint to the current constraint s.t. a continuous variable is canceled
   - Go to 2

Intersection cuts (Balas 1971)

- Given fractional solution $\tilde{x}$ and the simplex tableau.
  - Apply arbitrary disjunction $\pi^T x \leq \pi_0 \vee \pi^T x \geq \pi_0+1$ that defines lattice-free split set
  - Intersect extreme rays of cone defined by optimal basis with boundary of split set
  - Hyperplane through these points is a valid cut
  - $\sum_{j\in N} \max\left(\frac{-\pi_j - \sum_{i\in B}\pi_i a_{ij}}{\pi^T\tilde{x}-\pi_0}, \frac{\pi_j+\sum_{i\in B}\pi_i a_{ij}}{1+\pi_0-\pi^T\tilde{x}}\right)x_j \geq 1$



- Various generalizations possible:
  - cross cuts, sphere cuts,….



P^{LP} the cross set $P^{LP} \setminus$ cross set

image source: Oktay Günlük

Lift-and-project (Balas et al 1993)

- Intersection cut from an infeasible LP solution

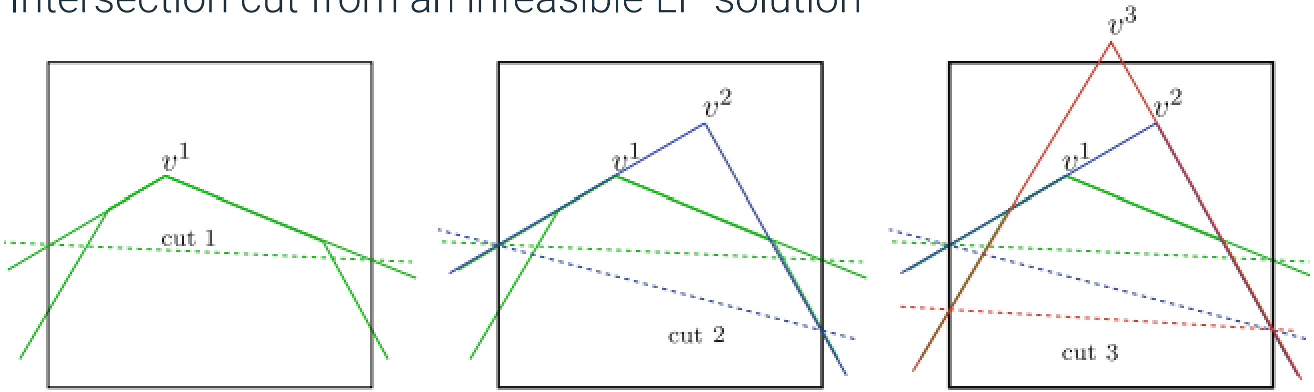


image source: Balas & Margot

- Can be computed by auxiliary „cut-generating“ LP
  - Can compute deepest lift-and-project cut
  - Can be computationally expensive
  - Cheaper: perform sequence of infeasible pivots on original LP (Balas & Perregaard 2002)

Quiz time

- Mixed-Integer Rounding Cuts
  a) Rely on a disjunctive argument
  b) Are less powerful than Mixed-Integer Gomory Cuts
  c) Require the solution of an Auxiliary LP
- $\{0, {}^{1}/_{2}\}$-cuts work
  a) On a graph structure
  b) On the original constraint matrix
  c) On the Simplex tableau
- In MIP solvers, cut generation is typically applied
  a) Only at the root node
  b) Aggressively at the root and moderately at some tree nodes
  c) The same way at all nodes

Quiz time

- Mixed-Integer Rounding Cuts
  a) Rely on a disjunctive argument
  b) Are less powerful than Mixed-Integer Gomory Cuts
  c) Require the solution of an Auxiliary LP
- $\{0, {}^1\!/_2\}$-cuts work
  a) On a graph structure
  b) On the original constraint matrix
  c) On the Simplex tableau
- In MIP solvers, cut generation is typically applied
  a) Only at the root node
  b) Aggressively at the root and moderately at some tree nodes
  c) The same way at all nodes

Combinatorial Cuts

Knapsack Cover cuts (Balas & Zemel 1978)

- Feasible set of knapsack problem: $X^K := \{x \in \{0,1\}^n : \sum a_j x \leq b\}$ with $(b \in \mathbb{Z}_+, a_j \in \mathbb{Z}_+)$
- Minimal Cover: subset C of the variables s.t.
  - $\sum_{j \in C} a_j > b$
  - $\sum_{j \in C \setminus \{i\}} a_j \leq b$ for all $i \in C$
- Minimal Cover Inequality
  - $\sum_{j \in C} x_j \leq |C| - 1$
- Example: $5x_1 + 6x_2 + 2x_3 + 2x_4 \leq 8$
  - Minimal cover: $C = \{2,3,4\}$
  - Minimal cover inequality: $x_2 + x_3 + x_4 \leq 2$

Lifting cover inequalities

- Can we incorporate variable x_1 in our cover cut $x_2 + x_3 + x_4 \leq 2$?
  - Is there an α_i s.t. $\alpha_i x_i + \sum_{j \in C} x_j \leq |C| - 1$, $i \notin C$ is a valid inequality for X^K ?
- Disjunctive argument:
- $\alpha_i x_i + \sum_{j \in C} x_j \leq |C| - 1$ is valid for $X^K \cap \{x \in \{0,1\}^n : x_i = 0\}$ for all α_i
- $\alpha_i x_i + \sum_{j \in C} x_j \leq |C| - 1$ is valid for $X^K \cap \{x \in \{0,1\}^n : x_i = 1\}$
  $\Leftrightarrow \alpha_i \cdot 1 + \max\left\{\sum_{j \in C} x_j : \sum a_j x \leq b, x \in \{0,1\}^n, x_i = 1\right\} \leq |C| - 1$
  $\Leftrightarrow \alpha_i \leq |C| - 1 - \max\left\{\sum_{j \in C} x_j : \sum a_j x \leq b, x \in \{0,1\}^n, x_i = 1\right\}$
- $\alpha_1 x_1 + x_2 + x_3 + x_4 \leq 2$ is valid for $\{x \in \{0,1\}^4 : 5x_1 + 6x_2 + 2x_3 + 2x_4 \leq 8\}$
  - $\Leftrightarrow \alpha_1 \leq 2 - \max\{x_2 + x_3 + x_4 : 6x_2 + 2x_3 + 2x_4 \leq 3, x \in \{0,1\}^4\} \Leftrightarrow \alpha_1 \leq 1$
- $x_1 + x_2 + x_3 + x_4 \leq 2$ is a valid inequality!

Knapsack cover cuts: Little Exercise

- Consider the knapsack problem $\max\{\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 \mid 3x_1 + 4x_2 + 7x_3 + 9x_4 \leq 11, x \in \{0,1\}^4\}$
  - What is the optimal solution vector?
  - Find a knapsack cover cut that cuts this solution off
  - Can we lift another variable into that cut?

Knapsack cover cuts: Little Exercise

- Consider the knapsack problem $\max\{x_1 + x_2 + x_3 \mid 3x_1 + 4x_2 + 7x_3 + 9x_4 \leq 11, x \in \{0,1\}^4\}$
  - What is the optimal solution vector?
  - Find a knapsack cover cut that cuts this solution off
  - Can we lift another variable into that cut?

  - The optimal LP solution is $x^* = (1, 1, \frac{4}{7}, 0)$ with a solution value of $2\frac{4}{7}$.

Knapsack cover cuts: Little Exercise

- Consider the knapsack problem $\max\{\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 \mid 3x_1 + 4x_2 + 7x_3 + 9x_4 \leq 11, x \in \{0,1\}^4\}$
  - What is the optimal solution vector?
  - Find a knapsack cover cut that cuts this solution off
  - Can we lift another variable into that cut?

  - The optimal LP solution is $\mathrm{x}^* = (1, 1, \frac{4}{7}, 0)$ with a solution value of $2\frac{4}{7}$.
  - $\{1,2,3\}$ is a cover, since $3 + 4 + 7 = 14 > 11$, but $3 + 4$ and $3 + 7$ and $4 + 7$ are all ≤ 11.
    - $x_1 + x_2 + x_3 \leq 2$ is a cover cut
    - Since $x_1^* + x_2^* + x_3^* = 2\frac{4}{7} > 2$, it cuts off the LP optimum

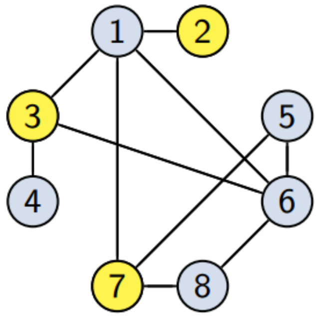
Knapsack cover cuts: Little Exercise

- Consider the knapsack problem $\max\{x_1 + x_2 + x_3 \mid 3x_1 + 4x_2 + 7x_3 + 9x_4 \leq 11, x \in \{0,1\}^4\}$
  - What is the optimal solution vector?
  - Find a knapsack cover cut that cuts this solution off
  - Can we lift another variable into that cut?

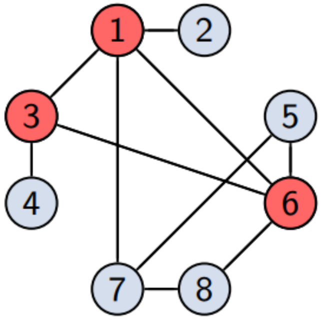
- The optimal LP solution is $x^* = (1, 1, \frac{4}{7}, 0)$ with a solution value of $2\frac{4}{7}$.
- $\{1,2,3\}$ is a cover, since $3 + 4 + 7 = 14 > 11$, but $3 + 4$ and $3 + 7$ and $4 + 7$ are all ≤ 11.
  - $x_1 + x_2 + x_3 \leq 2$ is a cover cut
  - Since $x_1^* + x_2^* + x_3^* = 2\frac{4}{7} > 2$, it cuts off the LP optimum
- Only x_4 remains. Lift it with any $\alpha_4 \leq 2 - \max\{x_1 + x_2 + x_3 | 3x_1 + 4x_2 + 7x_3 \leq 2\} = 2$
- $x_1 + x_2 + x_3 + 2x_4 \leq 2$ is a valid inequality

The stable set problem (Chvátal 1975)

- Given a graph $G = (V, E)$. A stable set is a set of non-adjacent vertices.
  - Stable Set: $S \subseteq V$, for all $u, v \in S : (u, v) \notin E$
- Stable set polytope for graph $G = (V, E)$:
  - $conv(\{x \in \{0,1\}^{|V|}: x_u + x_v \leq 1 \; for \; all \; (u, v) \in E\})$
- Given a graph $G = (V, E)$. A clique is a set of pairwise adjacent vertices.
  - Clique: $S \subseteq V$, for all $u, v \in S : (u, v) \in E$
- Clique inequalities: $\sum_{j \in C} x_j \leq 1$
  - Valid for stable set polytope



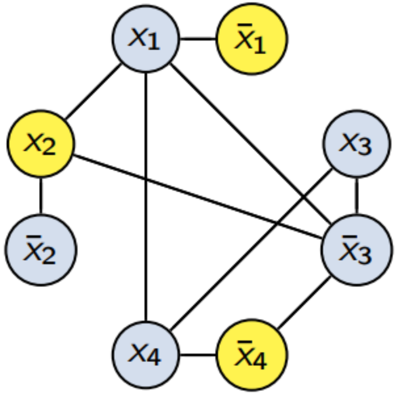
Stable set



Clique

Cutting from the clique table/graph

- Clique Graph: A graph $G = (V, E)$.
  - A node for every binary variable x_j and for its complement $\bar{x}_j := 1 - x_j$
  - Add an edge $(x_i, x_j) \in E$ whenever we find that in any feasible MIP solution x_j and x_j cannot be one at the same time.
    - Can come from assigment constraints $\sum x_i = 1$, but also from 2-elementary knapsack covers, or from constraints such as $x_1 + x_2 + x_3 \geq 2$, from probing (later),...
- Feasible MIP solution corresponds to stable set in clique graph
- Stable set polytope on conflict graph is relaxation of MIP's feasible region
- Separation algorithm: Find maximal violated cliques in clique graph
  - Heuristic / greedy DFS tree search



Implied bound cuts

- Given a continuous variable with an upper bound:
  - $y \leq u$
- and a binary variable x that implies a new upper bound on y,
  - E.g., $x = 0 \rightarrow y \leq \bar{u} < u$
- We can lift x into $y \leq u$
  - Implied bound cut: $y \leq u + (\bar{u} - u)(1 - x)$
  - Important special case: $\bar{u} = 0$. The implied bound cut is $y \leq ux$
- Example $\max\{y_1 - y_2 - x : y_1 + y_2 \leq 10x; y_1 \leq 5\}$. LP optimum: $y_1 = 5, y_2 = 0, x = \frac{1}{2}$
  - Implied bound cut: $y_1 \leq 5x$ cuts off the LP optimum
- Implications are detected and stored in presolving, but also detected locally

Cut Selection

Luxury problem: Which cuts should we use?

- How many cuts should be generated for a relaxation solution?
  - One?
    - Will provide a new relaxation solution
    - Expensive to re-solve relaxation for each cut
  - As many as possible?
    - Relaxation solution only needs to be cut off once
    - Cuts increase the size of the model
    - Cutting plane separators might be expensive
- Balancing is important:
  - Multiple rounds, limited number of cuts per round, replace old with new ones
  - Carefully choose which cuts complement each other nicely

Cut selection: What does a good cut look like?

- Numerically stable:
  - Coefficient range not too large, neither the absolute values
  - Hard criterion, throw cuts away that fail this
- Efficient:
  - Distance of hyperplane to the LP solution, cut as deeps as possible into the polyhedron
  - Soft criterion, minimum efficacy should be met
- Orthogonal w.r.t. other cuts
  - Ideally, pairwise almost orthogonal, each cut „cuts off a different part of the polyhedron“
- Almost parallel to the objective:
  - Exactly parallel is bad (degeneracy!), throw cut away (and only use as dual bound)
  - Almost parallel should trigger progress in dual bound

Cut selection: How does a good cut look like?

- Sparse: Only a few (integer) variables
- Recent result: Cutting towards primal solution (use directed distance between LP optimum and incumbent)

Selection process:

- Aggregate different measures and compute a single score
- Greedily select cut with highest score, remove similar cuts, iterate until no cut left or maximum number of cuts / cut elements hit

Quiz time

- Which of the following is true?
  a) There‘s a unique minimal cover for each knapsack constraint
  b) A knapsack constraint might have multiple minimal covers

- Clique cuts were originally introduced for
  a) The knapsack problem
  b) The stable set problem
  c) The travelling salesman problem

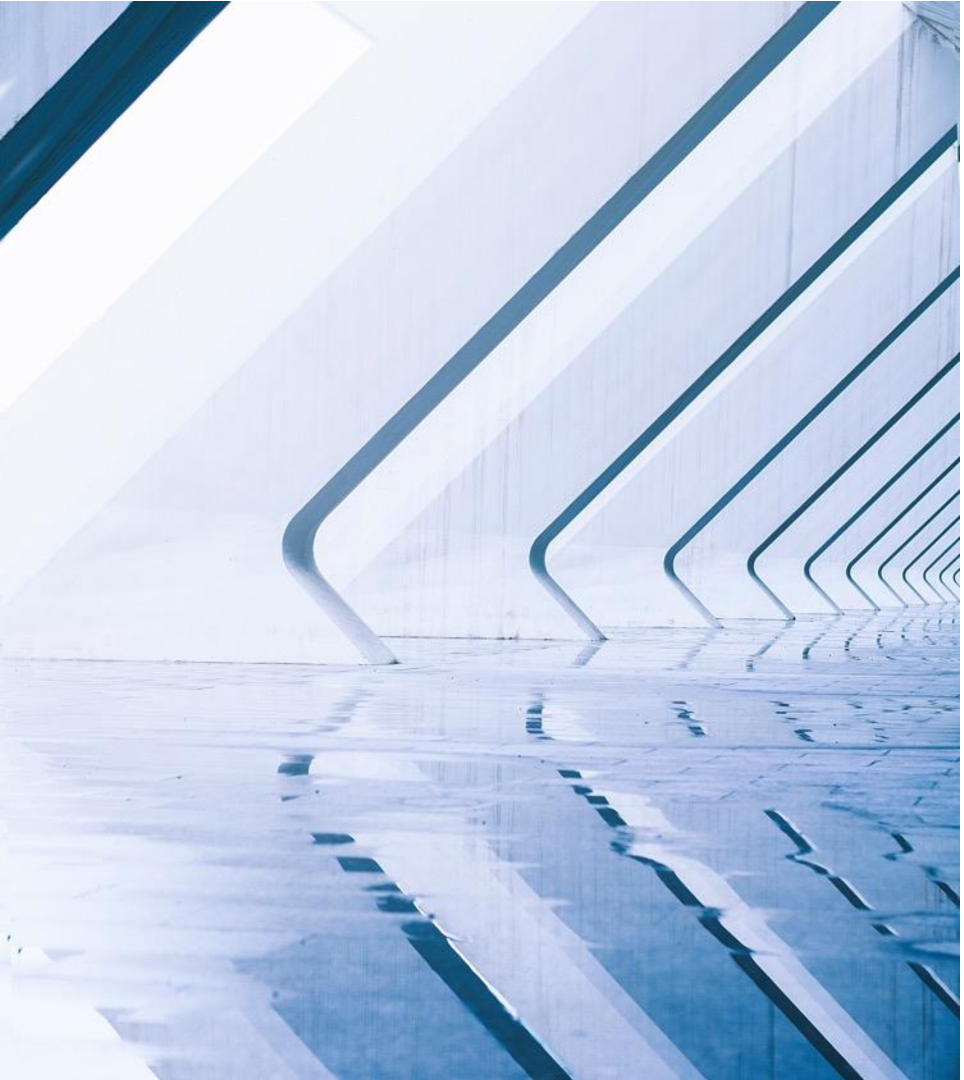
- Cut selection adresses the problem
  a) When to call cutting plane separators
  b) Which cutting plane separators to call
  c) Which of the generated cuts to add to the LP

Quiz time

- Which of the following is true?
  a) There's a unique minimal cover for each knapsack constraint
  b) A knapsack constraint might have multiple minimal covers

- Clique cuts were originally introduced for
  a) The knapsack problem
  b) The stable set problem
  c) The travelling salesman problem

- Cut selection adresses the problem
  a) When to call cutting plane separators
  b) Which cutting plane separators to call
  c) Which of the generated cuts to add to the LP



FICO®

Thank You!

Timo Berthold