Planning and Optimizing Large Scale Telecommunication Networks

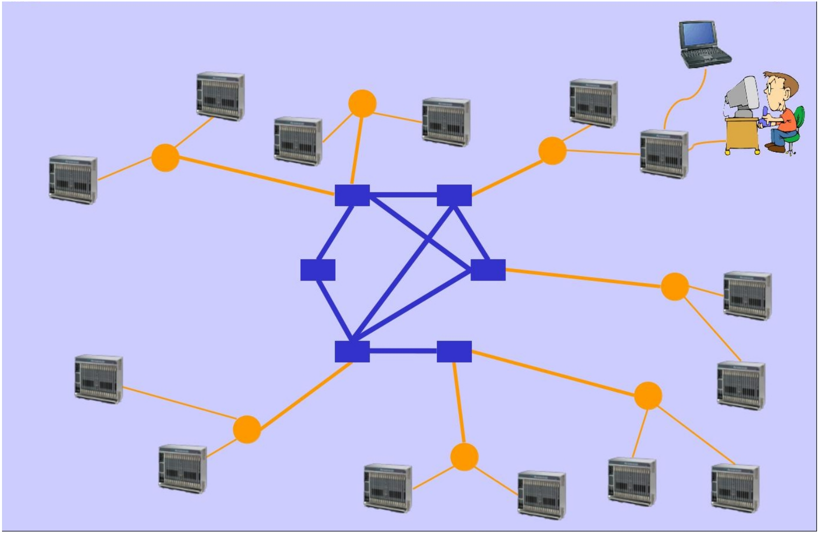
CO@Work

October 5, 2009

Maren Martens
Andreas Bley, Christian Raack

Contents

A Telecommunication Network



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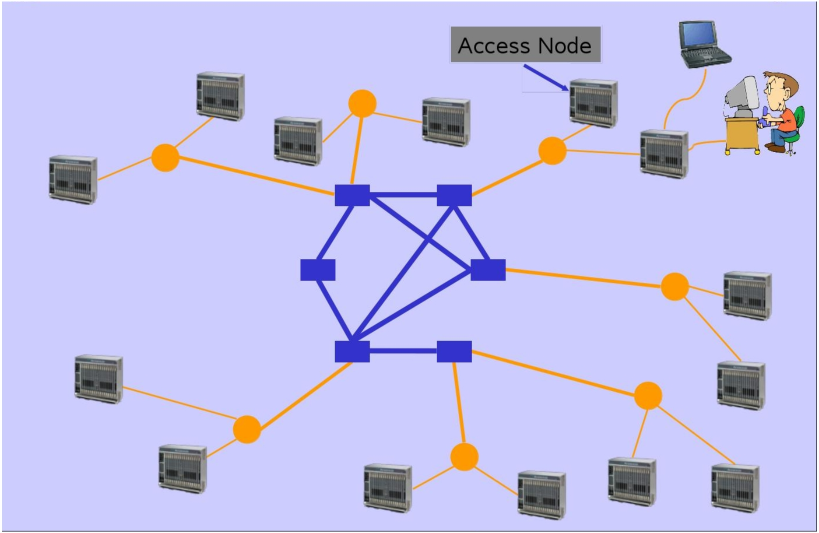

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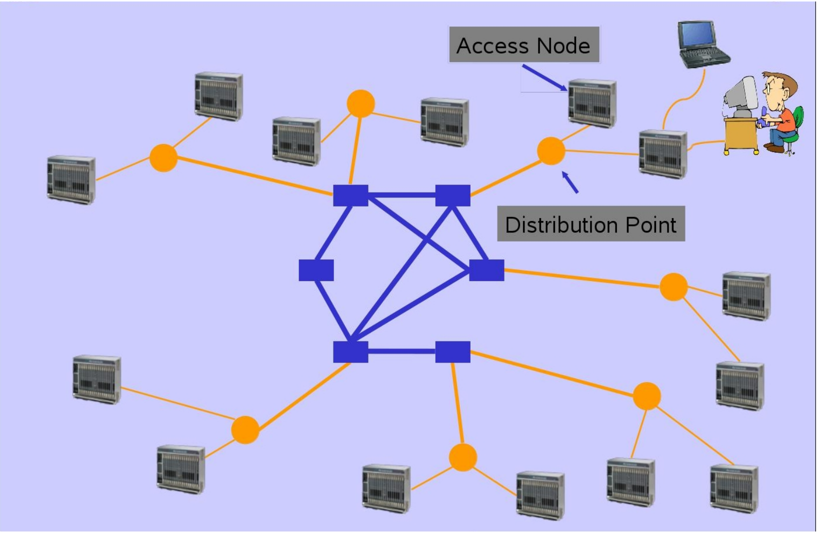

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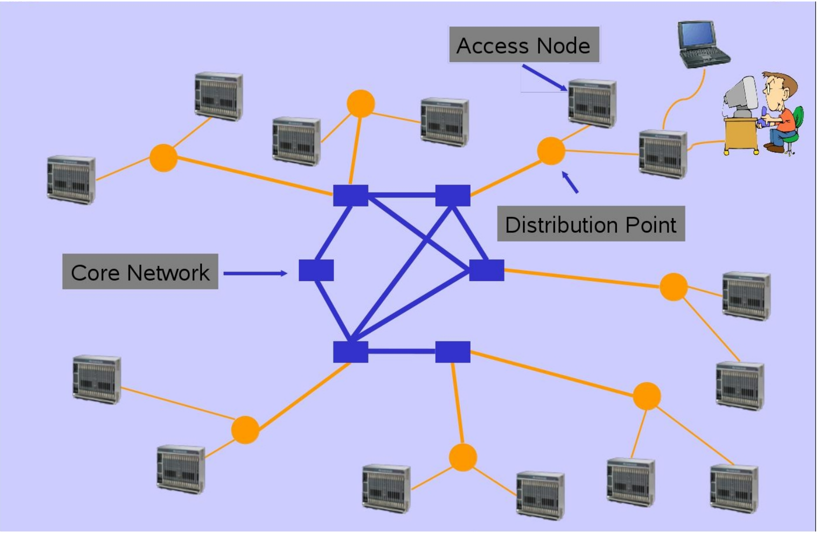

Planning Problems

Access Networks:

- How to connect customers to given routers?

Core Networks:

- Which links to use between routers?

Survivability:

- How to avoid interruptions?

Demands:

- Given customer demands, how to route them?

Access Networks

Access Networks

The Minimum Cost Flow Problem (MCFP)

Given: Digraph $D = (V, A)$, demands/supplies $b_v \in \mathbb{R}_0^+$ $(v \in V)$ with $\sum_{v \in V} b_v = 0$, arc cost c_a and capacities u_a $(a \in A)$

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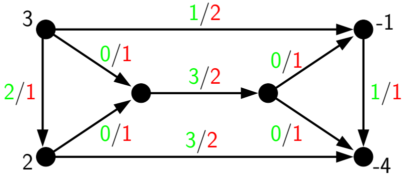

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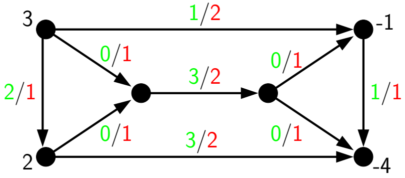

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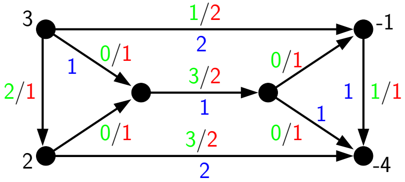

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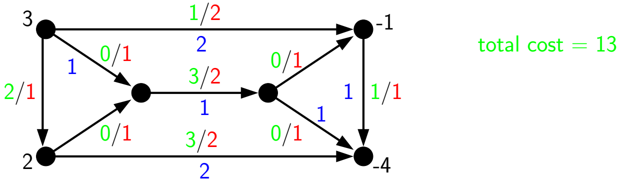

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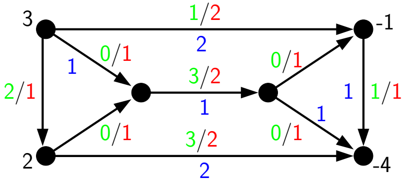

total cost = 13

Find: Flow $f : A \to \mathbb{R}_0^+$ of minimum cost, satisfying all demands

$$\min \sum_{a \in A} c_a f(a)$$

$$\text{s.t.} \quad \sum_{a \in \delta^+(v)} f(a) - \sum_{a \in \delta^-(v)} f(a) = b_v \qquad \forall\, v \in V$$

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Access Networks and the MCFP

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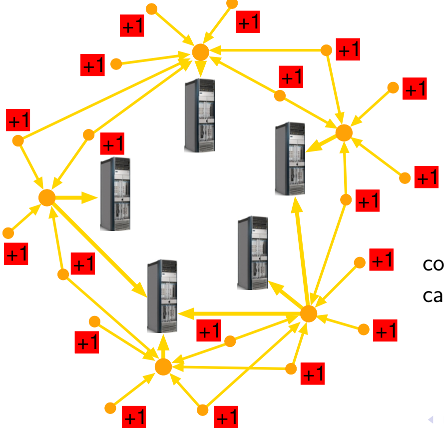

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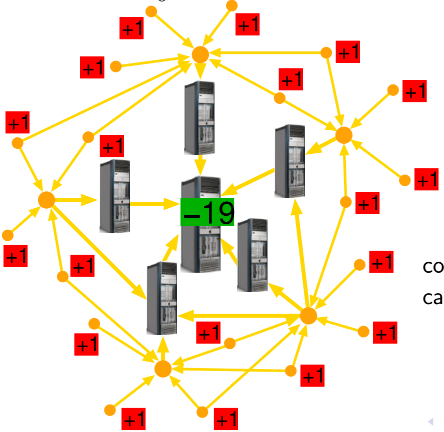

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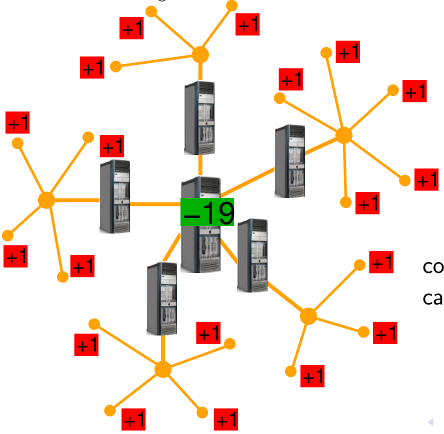

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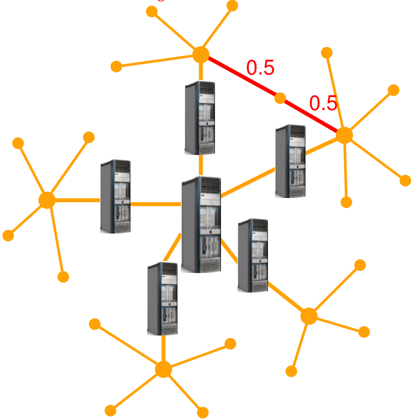

$\rightarrow f_e$ integral!

Algorithms for MCFP

$$\begin{aligned} \min \quad & \sum_{a \in A} c_a f(a) \\ \text{s.t.} \quad & \sum_{a \in \delta^+(v)} f(a) - \sum_{a \in \delta^-(v)} f(a) = b_v && \forall\ v \in V \\ & 0 \leq f(a) \leq u_a && \forall\ a \in A \end{aligned}$$

$\rightarrow$ Solvable in polynomial time for $f(a) \in \mathbb{R}_0^+$!

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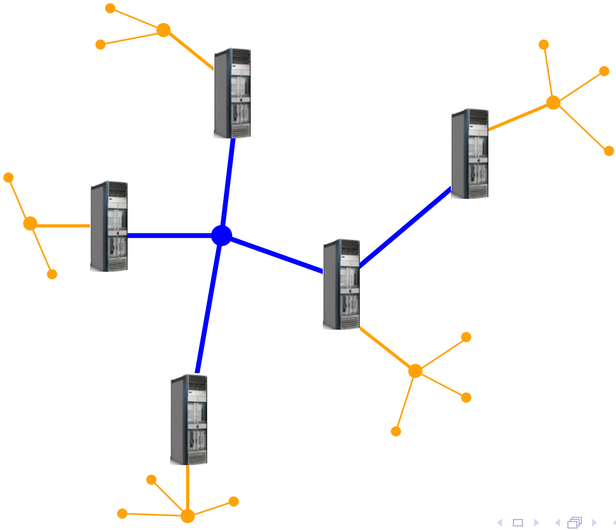
Both give integral optimal solution, if b and u are integral!

Core Networks

A Core Network



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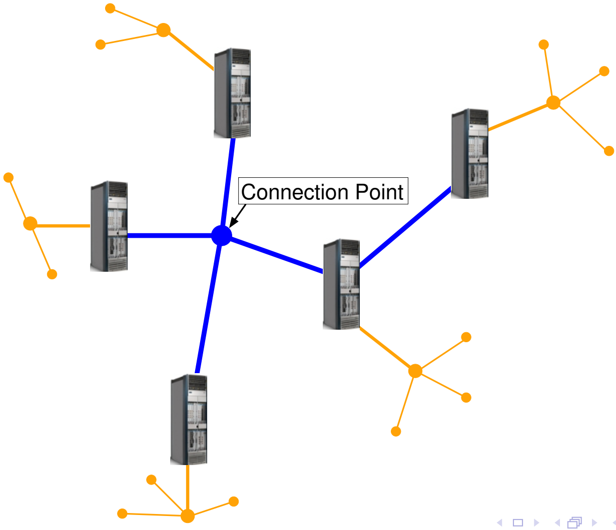

Steiner Trees

Definition

Given a graph $G = (V, E)$ with *terminals* $T \subseteq V$, a *Steiner tree* is a tree $S \subseteq E$ that connects all terminals in T.

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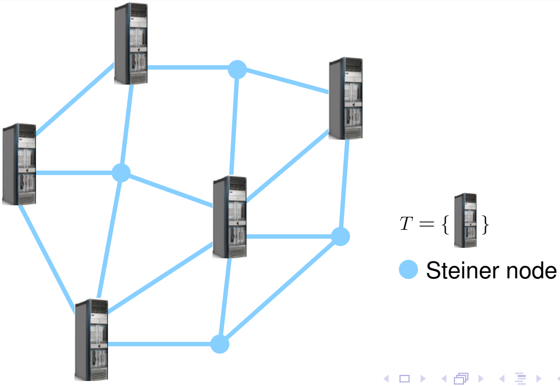

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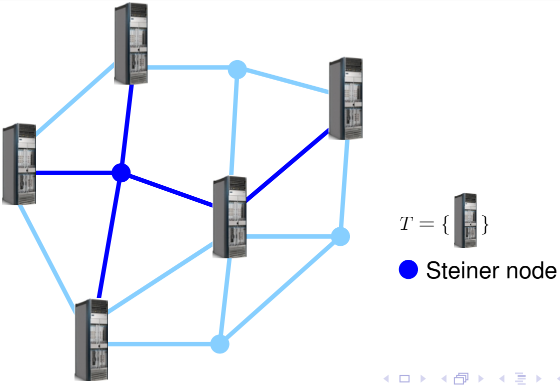

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Given: $G = (V, E)$, terminals $T \subseteq V$, edge weights c_e $(e \in E)$
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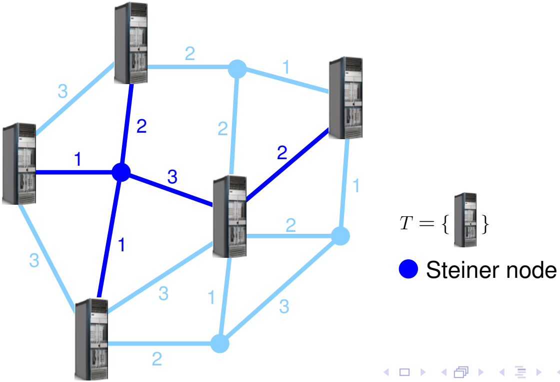

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Theorem (Garey & Johnson, 1979)

The Steiner Tree Problem is NP-complete.

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Simple special cases:

- $|T| = 2$: Shortest Path Problem
- $T = V$: Minimum Spanning Tree Problem

Formulation via Cuts (undirected)

Aneja 1980:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & x(\delta(U)) \geq 1 && \forall\, U \subset V : \emptyset \neq U \cap T \neq T \\ & x_e \in \mathbb{Z}_0^+ && \forall\, e \in E \end{aligned}$$

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→ The LP relaxation can also be solved in polynomial time. (separation time ≈ optimization time, [GLS1988])

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Directed vs. Undirected

- The directed cut formulation reflects all feasible Steiner trees.

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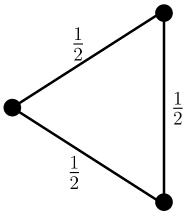

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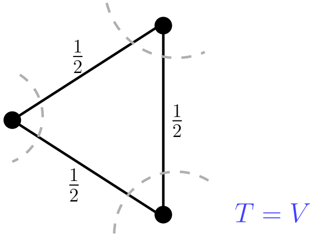

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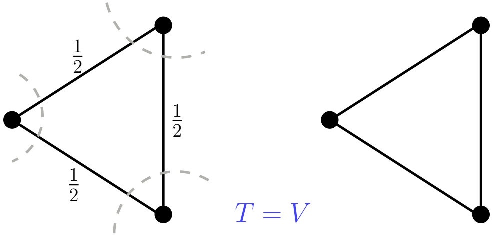

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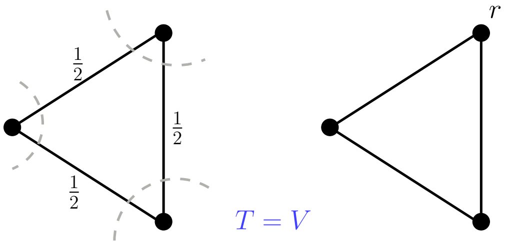

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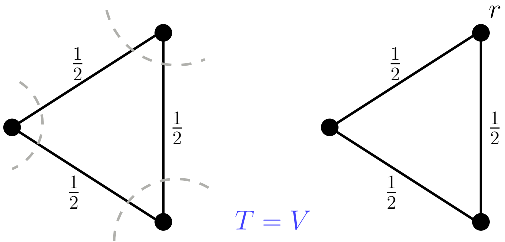

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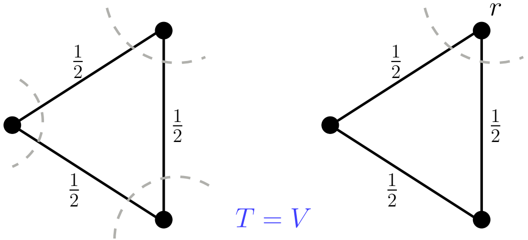

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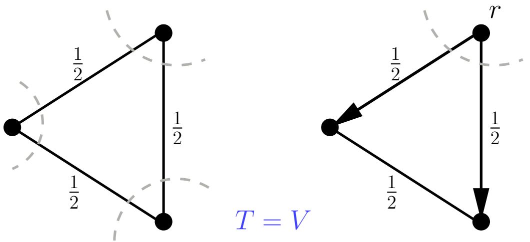

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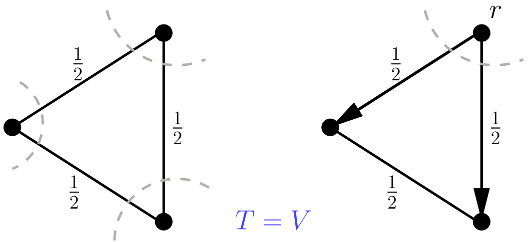

Theorem

The directed cut formulation is stronger than the undirected cut formulation.

Formulation via Flows

Problem with cuts: There are too many!

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On digraph $D = (V, A)$ with root $r \in T$ and $T' := T \setminus \{r\}$:

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$$f_t(a) \geq 0 \qquad \forall\, t \in T', a \in A$$

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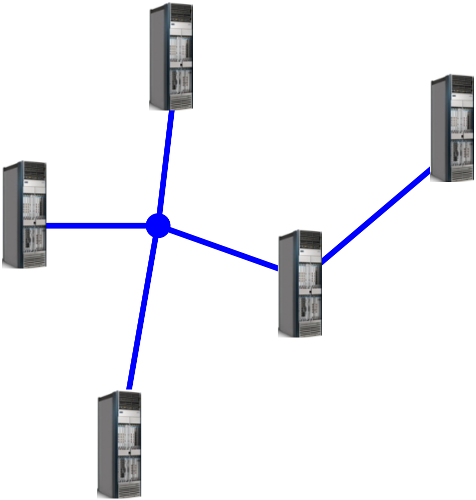
Survivability

Failures happen...



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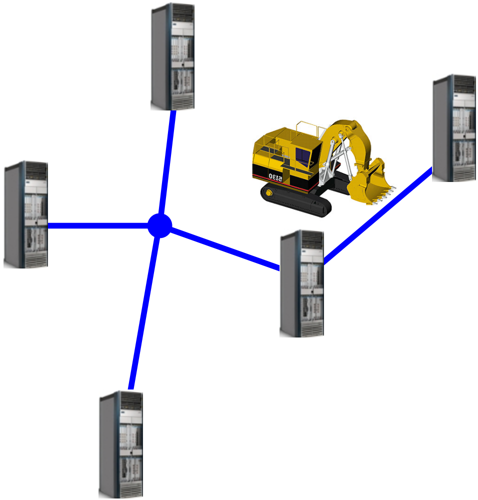

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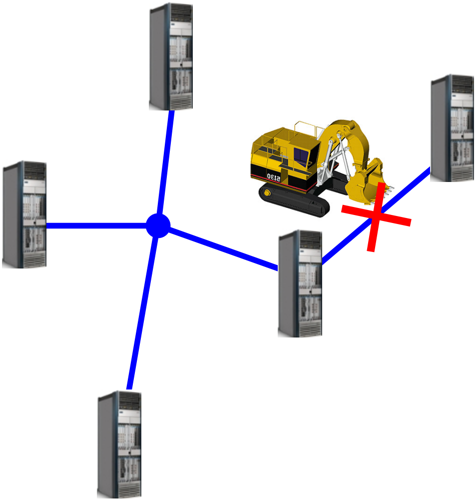

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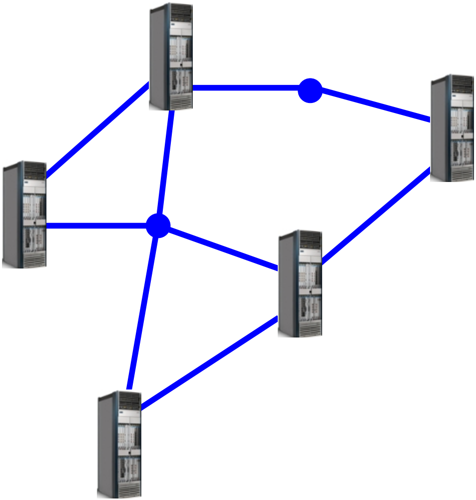

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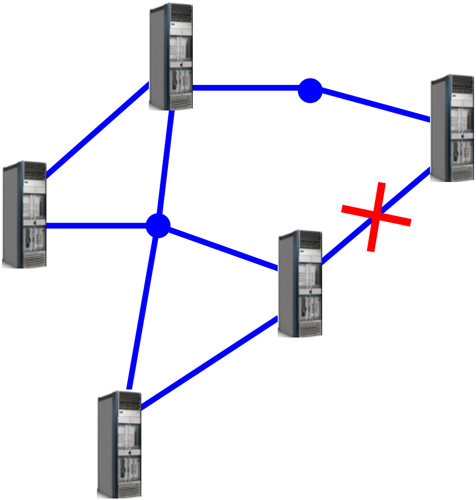

2-Connectivity

Given: $G = (V, E)$, edge cost c_e $(e \in E)$
Find: Minimum cost subgraph containing all $v \in V$ and having 2 node/edge-disjoint s-t-paths for all $s, t \in V$

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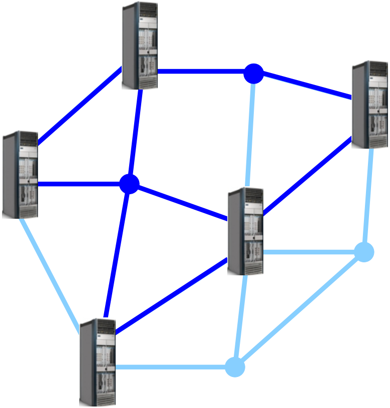

{0,1,2}-Survivable Network Design ({0,1,2}-SND)

Given: $G = (V, E)$, connectivity request $\rho_v \in \{0, 1, 2\}$ $(v \in V)$, edge cost c_e $(e \in E)$

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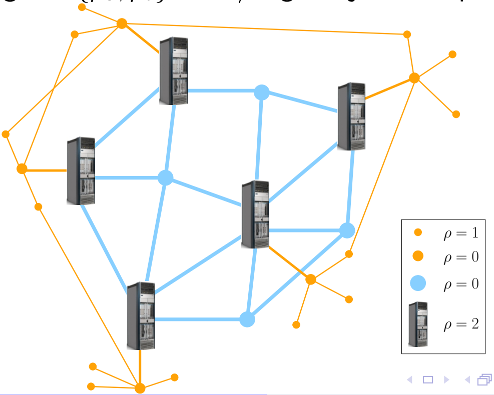

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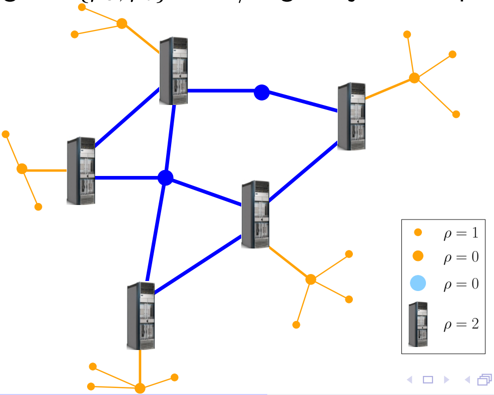

Algorithms for {0,1,2}-SND

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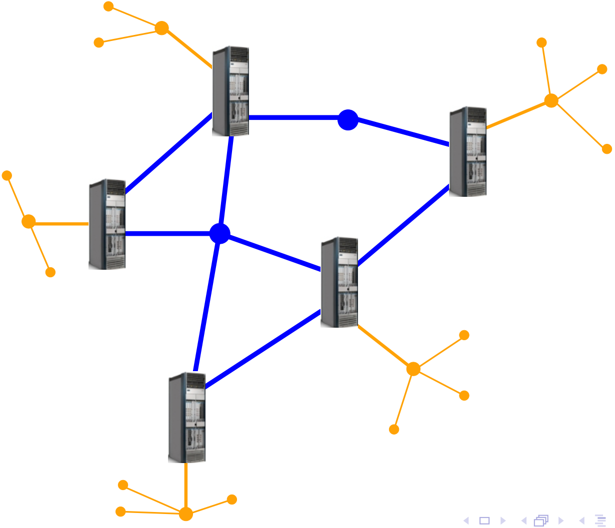
* 2-approximation for 2-node-connected subgraph (Khuller 1997)

Demands and Capacities

Are we done yet?



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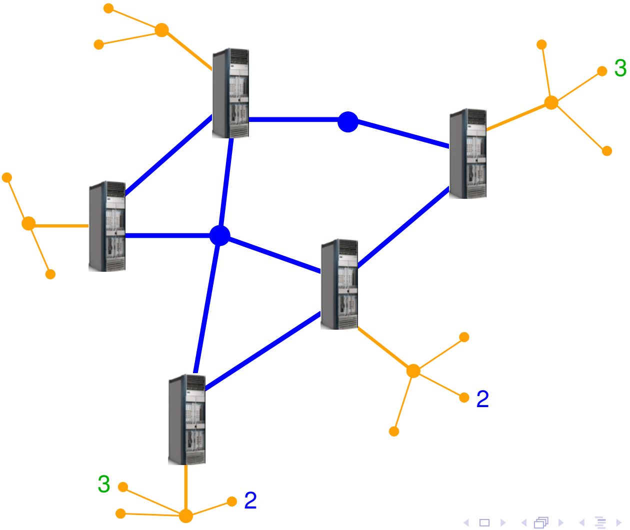

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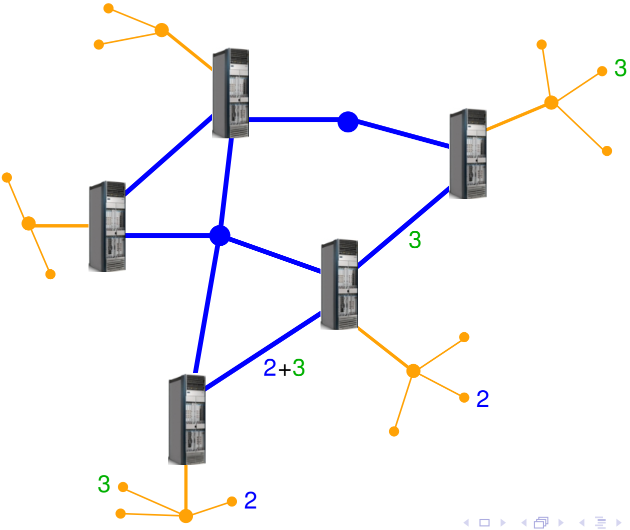

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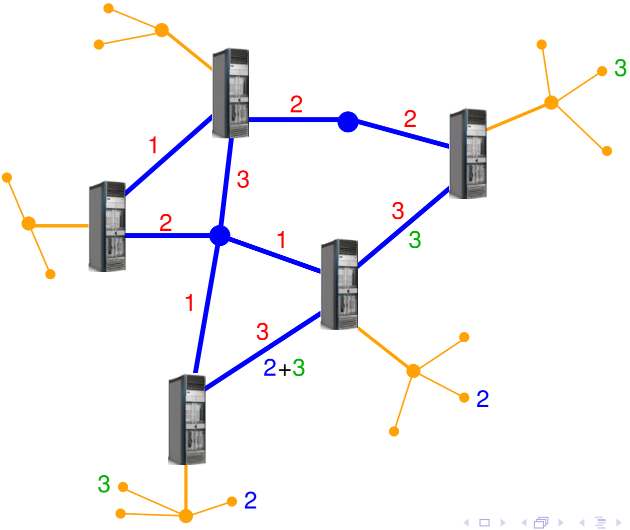

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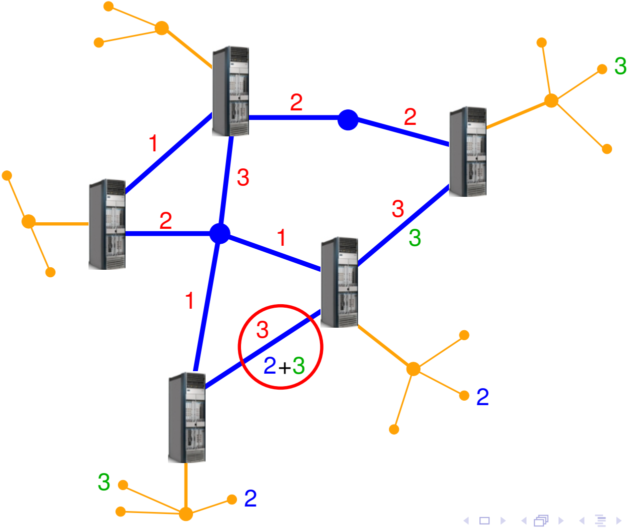

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Given: $G = (V, E)$, commodities $(s, t) \in V \times V$ with demands d_{st}, capacity modules $k \in K$ with capacities u_k, cost c_k

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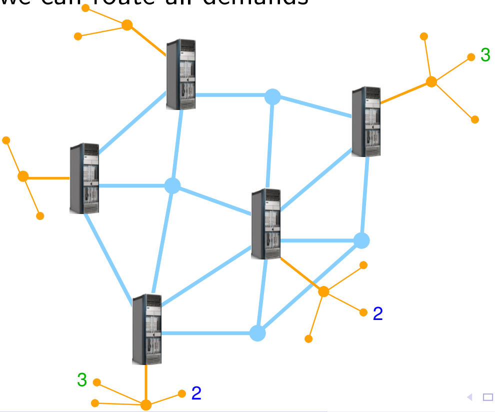

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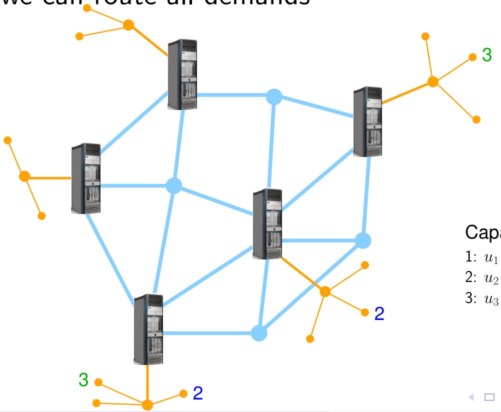

Capacity modules:

1: $u_1 = 2,\ c_1 = 20$
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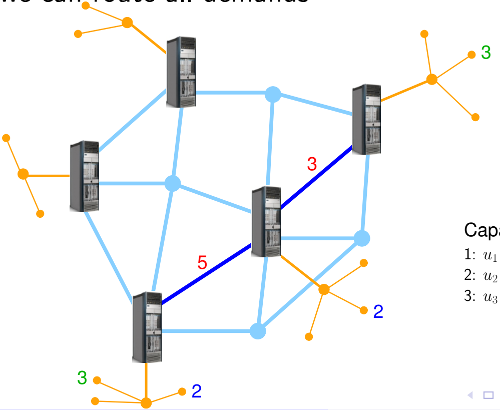

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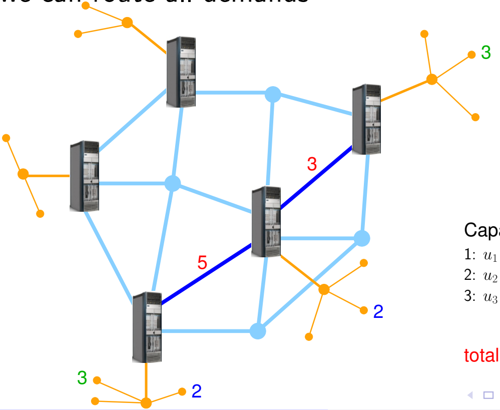

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total cost for core: 55

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[for $e = \{i, j\} \in E$: directed arcs $a_e = (i, j)$ and $\overleftarrow{a_e} = (j, i)$]

How to realize survivability?

(1+1)-protection:
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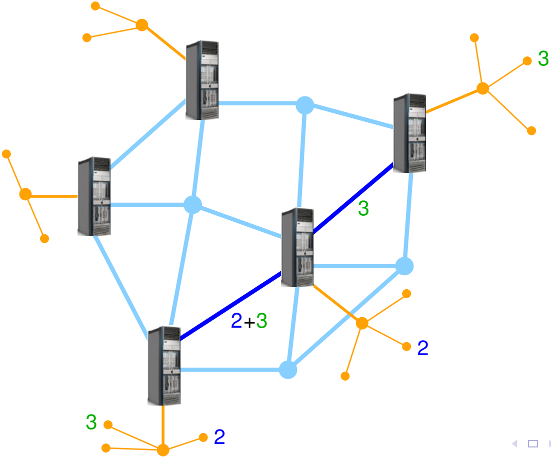

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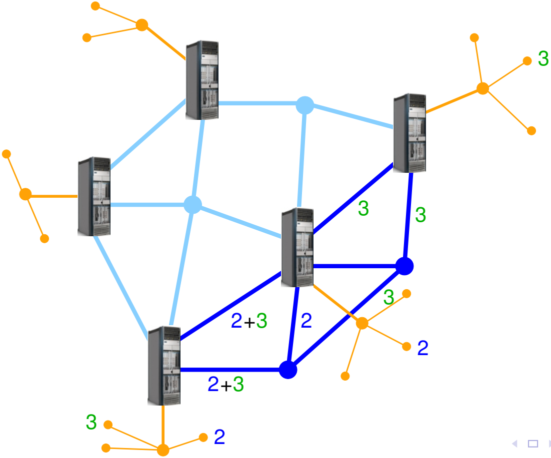

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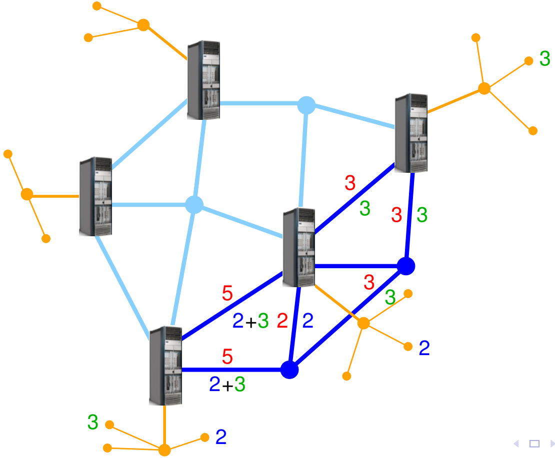

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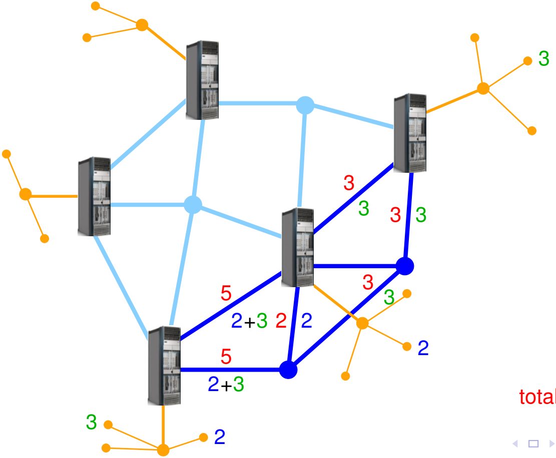

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Enjoy lunch!