Duty Scheduling Exercise

Ralf Borndörfer

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Exercise 1) Use Zimpl to implement a set covering model for a vehicle and crew scheduling problem. There is data for three days: Saturday (`dsp1-sa`), Friday (`dsp1-fr`), and Thursday (`dsp1-th`). The objective coefficients and the indices of the nonzero elements of the columns are listed in the data files in the format `c obj col` and `r row col`, where row and column are indices, and `obj` is a floating point number.

a) Use `zimpl` to implement a set covering model. (Hint: use the template files `dsp1-skeleton.zpl`; it is used with a command line such as

```
zimpl -DFILE=<FILENAME> dsp1-skeleton.zpl
```

that defines a file to read.)
b) Solve these models using SCIP.
c) Compare the solutions of LP-relaxations to those of the IPs.
d) Solve the set partitioning variant of the models.

Exercise 2) Consider the digraph in the file `dsp2-dutch.dat`. This digraph contains a number of simple directed paths (without node repetitions) of cost at most 20 000. Let us assigne to each path a disutility that is equal to the travel time plus some constant offset. The task in this exercise is to partition the nodes in the graph into cost constrained paths of minimum total disutility. We assume that a path must contain at least one arc, i.e., a node alone is not a path.

a) Implement a depth first serach algorithm that enumerates all simple paths in the network given in the file dsp2-dutch.dat with a cost of a most 20 000. How many such paths are there? (Hint: use the template file `dsp2-skeleton.cpp`.)
b) Use `zimpl` to implement a set partitioning model for the path partitioning problem. (Hint: use the template files `dsp2a-skeleton.zpl` and `dsp2b-skeleton.zpl`; they are used with a command line such as

```
zimpl -DOFFSET=<VALUE> dsp2a-skeleton.zpl
```

that defines the constant `OFFEST`. If you didn't solve a), use the data in file `dsp2.dat`.)
c) Use your model to compute the optimal IP and LP values of the path partitioning problems associated with the offsets 0, 100, 500, and 1 000. Are the LP solutions integral?

Exercise 3) Consider again the digraph of the dutch network. The task in this exercise is to cover the nodes of this digraph by directed cost constrained paths of minimum disutility using a column generation method. In this exercise, we want to generate these columns using the dynamic program that was implemented in the constrained shortest path exercise. Applied to a digraph that contains directed cycles, the dynamic program produces non-simple paths, i.e., paths with cycles, such that the columns in a corresponding set covering model have coefficients larger than one, namely, equal to the number of times that a path visits a node. An optimum integer solution of such a model never uses a non-simple q-path if the triangle inequality holds for the objective function; the LP relaxation will, however, in general be weaker. For these reasons, this formulation is called a *q-path relaxation*.

a) Implement a column generator for cost contrained q-paths based on the dynamic program from the constrained shortest path exercise. (Hint: use the template `pricer_csp.c`.)
b) Implement a column generation algorithm to solve the LP relaxation of the q-path relaxation using SCIP. (Hints: use the template in the tar-ball `dsp.tgz`. For this exercise use the data file of the dutch network contained in the directory `dsp3/data/`. This network has an additional source node to handle the pricing problem. An cost offset of 100 for each path is already in the data of the network.)
c) Solve the LP relaxation of the q-path relaxation. Compare the value to the optimal LP value for simple paths computed in the previous exercise. Which one is smaller?
d) Solve the LP relaxation of the q-path relaxation. Compare the value to the optimal LP value for simple paths computed in the previous exercise. Which one is smaller?