Exercise for Line Planning

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Mathematics for key technologies

Exercise 1 – Arc Formulation

$$\min \quad \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in \mathcal{F}} c_\ell^f \, x_\ell^f + (1-\lambda) \sum_{(s,t) \in D} \sum_{a \in A} \tau_a \, y_a^{st}$$

$$\text{s.t} \quad \sum_{a \in \delta^+(v)} y_a^{st} - \sum_{a \in \delta^-(v)} y_a^{st} = \begin{cases} d_{st} & v = s \\ 0 & v \neq s, t \end{cases} \qquad \forall\, v \in V, \forall\, (s,t) \in D$$

$$\sum_{(s,t) \in D} y_a^{st} \leq \sum_{\ell : e(a) \in \ell} \sum_{f \in F} \kappa_\ell^f \, x_\ell^f \qquad \forall\, a \in A$$

$$\sum_{f \in F} x_\ell^f \leq 1 \qquad \forall\, \ell \in \mathcal{L}$$

$$x_\ell^f \in \{0,1\} \qquad \forall\, \ell \in \mathcal{L},\ f \in \mathcal{F}$$

$$y_a^{st} \geq 0 \qquad \forall\, a \in A,\ (s,t) \in D.$$

Exercise 1

(a) objective value: $6.39892 \cdot 10^6$

(b) # demand pairs set $D := \{< s, t > \text{ in } O * O \text{ with } d[s, t] > 0\};$

(c) $\mathcal{F} = \{9, 18\}$; objective value: $6.39892 \cdot 10^6$

(d) $\lambda = 0$: objective value: $1.2724040 \cdot 10^7$
$\lambda = 1$: objective value: $66\,600$

(e) $\lambda = 0.1$: objective value: $1.1459016 \cdot 10^7$,
cost: $73\,800$, travel time: $1.2724040 \cdot 10^7$
$\lambda = 0.97$: objective value: $450\,625$,
cost: $68\,400$, travel time: $1.2809258 \cdot 10^7$

(g) cost: $73\,800$, travel time: $1.2724040 \cdot 10^7$

(h) introducing fixed cost of 1: solution with 14 lines (total operating cost: 68 400)

(i)

$$\sum_{\ell:\text{"}Apd\text{"}\in\ell} \sum_{f\in\mathcal{F}} x_\ell^f \cdot f \geq 18$$

Exercise 2 – Dual Program

$$
\begin{array}{llll}
\text{(DLP)} \quad \min & \displaystyle\sum_{(s,t)\in D} d_{st}\,\pi_{st} - \sum_{\ell\in\mathcal{L}} e\,\eta_e & & \\
\text{s.t.} & \displaystyle \pi_{st} - \sum_{a\in p} \mu_a & \leq \tau_p & \forall\, p \in \mathcal{P}_{st},\ (s,t)\in D \\
 & \displaystyle \kappa_\ell^f \sum_{e\in\ell} (\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_\ell & \leq \lambda\, c_\ell^f & \forall\, \ell\in\mathcal{L},\ f\in\mathcal{F} \\
 & \eta_\ell & \geq 0 & \forall\, \ell \in \mathcal{L} \\
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Reduced cost:

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\overline{\lambda c_\ell^f} = \lambda c_\ell^f - \kappa_\ell^f \sum_{e\in\ell}(\mu_{a(e)}+\mu_{\bar a(e)}) + \eta_\ell = \lambda \sum_{e\in\ell} c_e^i\, f - \kappa^i\, f \sum_{e\in\ell}(\mu_{a(e)}+\mu_{\bar a(e)}) + \eta_\ell
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\overline{\lambda c_\ell^f} < 0 \quad\Leftrightarrow\quad \sum_{e\in\ell}(\kappa^i(\mu_{a(e)}+\mu_{\bar{a}(e)}) - \lambda \sum_{e\in\ell} c_e^i) > \frac{\eta_\ell}{f}
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$\rightsquigarrow$ right hand side depends on line