Exercise: Line Planning in Public Transport

Exercise 1:

Use Zimpl to implement the variation of the line planning model of Grötschel, Borndörfer, and Pfetsch, which was introduced in the lecture.

$$
\begin{aligned}
\min \quad & \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in \mathcal{F}} c_\ell^f \, x_\ell^f + (1-\lambda) \sum_{(s,t) \in D} \sum_{p \in \mathcal{P}} \tau_p \, y_p \\
\text{s.t.} \quad & \sum_{p \in \mathcal{P}_{st}} y_p = d_{st} && \forall (s,t) \in D \\
& \sum_{p : a \in p} y_p \leq \sum_{\ell : e(a) \in \ell} \sum_{f \in F} \kappa_\ell^f \, x_\ell^f && \forall a \in A \\
& \sum_{f \in F} x_\ell^f \leq 1 && \forall \ell \in \mathcal{L} \\
& x_\ell^f \in \{0,1\} && \forall \ell \in \mathcal{L},\, f \in \mathcal{F} \\
& y_p \geq 0 && \forall p \in \mathcal{P}.
\end{aligned}
$$

Use an arc formulation instead of the path formulation for the passenger demand, i.e., introduce variables y_a^{st} that indicate the number of passengers from s to t on arc a and replace y_p. Use a flow formulation based on these variables to get a flow of d_{st} from s to t. This replaces $\sum_{p \in \mathcal{P}_{st}} y_p = d_{st}$.

The OD matrix is given for the afternoon traffic from 3 to 6 o'clock. Assume 100 € per edge as operating cost for each line. Note that lines are undirected. Furthermore, assume that one train can transport 600 passengers.

(a) In a first step use variables for all combinations of OD nodes, i.e., variables y_a^{st}, where $s, t \in V$. Solve the problem; find a set of lines with frequency of 9, i.e., each line operates every 20 minutes (cycle time). Set the weighting parameter $\lambda = 0.5$.

(b) Reduce the number of variables by using only combinations of OD-nodes with positive demand.

(c) Solve the problem by choosing a set of lines and a cycle time of 10 or 20 minutes.

(d) Change λ to 0 and to 1. What do the solutions show?

(e) Solve the model for $\lambda = 0.1$ and $\lambda = 0.97$. What are the values for costs and for travel time? (Stop the computations after some time, e.g. 1000 nodes).

(f) What are the differences (advantages/disadvantages) between an edge formulation and a path formulation for the passenger demand?

(g) Implement a new model where the passenger demand is fixed. Use the data given in load.dat which gives the number of passengers for each edge. Compute a cost minimal line plan using the same cost function as before.

Compare this solution with the solution of (f).

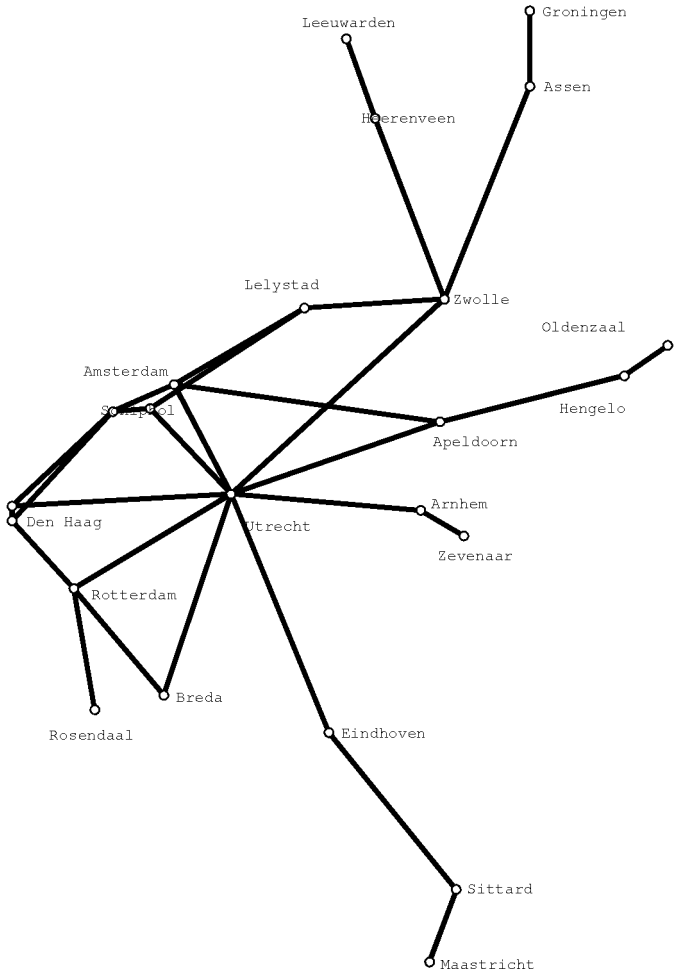
(h) The solution of (f) for $\lambda = 0.97$ contains too many lines. Compute a solution with less lines.

(i) The city of Apeldoorn ("Apd") requires approximately a 10-minute service at their station, i.e., every 10 minutes should be a stop of any line in Apeldoorn. Include this condition in your model.

Exercise 2:

Consider the variation of the model of Grötschel, Borndörfer, and Pfetsch. What are the consequences for the pricing problem of the line path variables?
(The capacity κ_ℓ^f depends only on the mode of the line and the frequency, i.e., $\kappa_\ell^f = \kappa^i \cdot f$ for line ℓ of mode i.
The cost of a line depends on the length of the line, the mode and the frequency, i.e., $c_\ell^f = \sum_{e \in \ell} c_e^i \cdot f$ for line ℓ of mode i.)

For several exercises we use data for the Dutch intercity network (taken from a GAMS model by Michael Bussieck). The network looks as follows:



The data is contained in several files:

- ▷ *nodes.dat* nodes of the graph.
- ▷ *edges.dat* edges of the graph. They are directed and the data contains forward and backward directions.
- ▷ *times.dat* travel times for each edge.
- ▷ *lines.dat* predefined lines. (Each row of the data contains "linename", "start node of edge" and "end node of edge". Use the command "proj" to get the linennames.)
- ▷ *od.dat* OD-matrix (number of passengers between two stations). Hint: the order is as in nodes.dat. Use command d[V*V] := read "od.dat" as "n+" comment '#';
- ▷ *times.dat* travel time on the edges.
- ▷ *load.dat* number of passengers between two OD nodes (stations).

We use the following abbreviations for station names:

Ah	Arnhem
Apd	Apeldoorn
Asd	Amsterdam CS
Asdz	Amsterdam Zuid WTC
Asn	Assen
Bd	Breda
Ehv	Eindhoven
Gn	Groningen
Gv	Den Haag HS
Gvc	Den Haag CS
Hgl	Hengelo
Hr	Heerenveen
Lls	Lelystad Centrum
Lw	Leeuwarden
Mt	Maastricht
Odzg	Oldenzaal Grens
Rsdg	Rosendaal Grens
Rtd	Rotterdam CS
Shl	Schiphol
Std	Sittard
Ut	Utrecht CS
Zl	Zwolle
Zvg	Zevenaar Grens