Exercises: Equipment selection for surface mines

Christina Burt and Yao-ban Chan

September 28th, 2009

An important part of the process of modelling the equipment selection problem as a mixed-integer linear program lies in the discretization of time into time periods and equipment age into age brackets. We need to do this because cost functions are generally non-linear, and replacing them by step functions enables us to use a linear model.

Unfortunately, discretization does introduce some error in the process. In particular, the age of each piece of equipment is assessed at the beginning of each time period, and assumed to stay constant over that period. In these exercises, we modify the presented program to include corrections which do not require this assumption.

These exercises cover work published in: Burt, C. and Chan, Y., Accurate costing in mixed integer utilisation mining models. *MODSIM 2009 International Congress on Modelling and Simulation*, In R. Braddock et al. (eds) 18th IMACS World Congress - MODSIM09 International Congress on Modelling and Simulation, December 2009, pp. 74-80.

For reference, we reproduce the full model from the slides below. The variables are:

$\mathbf{x}_{e,j}^{k,l}$ 1 if we own a truck/loader of type e with ID j, in time period k, which is in age bracket l at the start of the period; 0 otherwise.

$\mathbf{f}_{e,j}^{k,l}$ the amount of time (as a proportion of total operating hours) that this truck works in period k.

$\mathbf{s}_{e,j}^{k,l}$ 1 if we sell a truck/loader of type e with ID j at the start of time period k, when it was in age bracket l; 0 otherwise.

The model is:

$$\begin{aligned}
\text{Minimise } & \sum_{e,j,k} \frac{F_e}{(1+I)^k} \left(\sum_l \mathbf{x}_{e,j}^{k,l} - \sum_l \mathbf{x}_{e,j}^{k,l} + \sum_l \mathbf{s}_{e,j}^{k,l} \right) + \sum_{e,j,k,l} \frac{V_e^{k,l}}{(1+I)^k} \mathbf{f}_{e,j}^{k,l} - \sum_{e,j,k,l} \frac{(1-D)^l F_e}{(1+I)^k} \mathbf{s}_{e,j}^{k,l} \\
\text{subject to } & \sum_{i',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \geq T^k \\
& \sum_{i \in X(A'),j,l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq \sum_{i' \in A',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \\
& \mathbf{x}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k+1,l} + \mathbf{x}_{e,j}^{k+1,l+1} + \mathbf{s}_{e,j}^{k+1,l} + \mathbf{s}_{e,j}^{k+1,l+1} \\
& \mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l} + \mathbf{x}_{e,j}^{k-1,l-1} \\
& \sum_l \mathbf{x}_{e,j}^{k,l} \leq 1 \\
& \sum_{k,l} \mathbf{s}_{e,j}^{k,l} \leq 1 \\
& \sum_{k'<k,l} \mathbf{s}_{e,j}^{k',l} + \sum_l \mathbf{x}_{e,j}^{k,l} \leq 1 \\
& \mathbf{f}_{e,j}^{k,l} \leq a_e^{k,l} \mathbf{x}_{e,j}^{k,l} \\
& \sum_{k'<k,l} \frac{O^{k'}}{B_0} \mathbf{f}_{e,j}^{k',l} \leq M + (l+1-\epsilon-M)\mathbf{x}_{e,j}^{k,l} \\
& \sum_{k'<k,l} \frac{O^{k'}}{B_0} \mathbf{f}_{e,j}^{k',l} \geq l + M(\mathbf{x}_{e,j}^{k,l} - 1) \\
& \mathbf{x}_{e,j}^{k,l}, \mathbf{s}_{e,j}^{k,l} \in \{0,1\}, \mathbf{f}_{e,j}^{k,l} \geq 0.
\end{aligned}$$

Consider a piece of equipment of type e and ID j. If $\mathbf{x}_{e,j}^{k,l} = 1$, then it starts time period k in age bracket l. Based on its utilisation in this period, it may advance to age bracket $l+1$ during this period, in which case we should account for that. Let $\boldsymbol{\beta}_{e,j}^{k,l}$ be the number of hours that this equipment works in age bracket $l+1$ in this period.

1. Write down the age of the equipment, in hours, at the start and at the end of time period k.

2. Using your expression from part 1, fill in the following equation:

$$\boldsymbol{\beta}_{e,j}^{k,l} = \begin{cases} \qquad\qquad\qquad\qquad & \text{if the equipment is owned and advances an age bracket} \\ & \text{otherwise.} \end{cases}$$

3. We need to convert the above equation into constraints without an 'if' clause. We do this by separately constraining $\boldsymbol{\beta}_{e,j}^{k,l}$ from above and below. Obviously $\boldsymbol{\beta}_{e,j}^{k,l} \geq 0$ — this is a strict lower bound when the equipment is not owned or does not advance an age bracket. First assume that the equipment is owned in this period, i.e. $\mathbf{x}_{e,j}^{k,l} = 1$.

   (a) Formulate a lower bound on $\boldsymbol{\beta}_{e,j}^{k,l}$ which is strict when the equipment advances an age bracket. This should be a non-strict, but valid, inequality when the equipment does not advance.

   (b) Assume that the equipment does not advance an age bracket in this period. Formulate a strict upper bound on $\boldsymbol{\beta}_{e,j}^{k,l}$. This should be invalid if the equipment advances.

   (c) Assume that the equipment advances an age bracket in this period. Formulate a strict upper bound on $\boldsymbol{\beta}_{e,j}^{k,l}$. This should be invalid if the equipment does not advance.

   (d) Formulate an expression, using $\mathbf{x}$ and $\mathbf{s}$ variables from period $k+1$, which is 1 if the equipment advances an age bracket in period k, and 0 otherwise.

   (e) Using your answer for part 3d, modify your constraint from part 3b so that it is still valid (but may be non-strict) if the equipment advances.

   (f) Using your answer for part 3d, modify your constraint from part 3c so that it is still valid (but may be non-strict) if the equipment does not advance.

   (g) If the equipment is not owned, then your constraint from 3a may be invalid. Modify it so that it is valid in this case.

   (h) If the equipment is not owned, then your constraint from 3f may be invalid. Modify it so that it is valid in this case.

(i) Using a big-M constraint, strictly constrain $\boldsymbol{\beta}_{e,j}^{k,l}$ from above when the equipment is not owned.

The constraints from parts 3e, 3g, 3h and 3i will be added to the model to bound $\boldsymbol{\beta}$.

4. Modify the operating cost so that hours spent in age bracket l are costed at $V_{e,j}^{k,l}$ per O^k hours and hours spent in age bracket $l+1$ are costed at $V_{e,j}^{k,l+1}$ per O^k hours.

5. (Bonus exercise) The availability constraint

$$\mathbf{f}_{e,j}^{k,l} \leq a_e^{k,l} \mathbf{x}_{e,j}^{k,l}$$

also calculates availability based on the age at the start of the time period. We will modify this to be more accurate. Firstly, we assume that the equipment is owned:

$$\mathbf{f}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k,l}$$

and devise a constraint which limits $\mathbf{f}$ based on availability.

(a) Given $\boldsymbol{\beta}_{e,j}^{k,l}$, formulate a lower bound on the total amount of time (not just utilised time) that the equipment spends in age bracket $l+1$ in period k.

(b) Using your answer from part 5a, formulate an upper bound on the total amount of time that the equipment spends in age bracket l in period k.

(c) Using your answer from part 5b, formulate a constraint which provides an upper bound on the amount of utilised time that the equipment spends in age bracket l in period k.

The constraint from part 5c will replace the availability constraint in the model.