

Exercises: Equipment selection for surface mines

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An important part of the process of modelling the equipment selection problem as a mixed-integer linear program lies in the discretization of time into time periods and equipment age into age brackets.

We need to do this because cost functions are generally non-linear, and replacing them by step functions enables us to use a linear model.

Unfortunately, discretization does introduce some error in the process.

In particular, the age of each piece of equipment is assessed at the beginning of each time period, and assumed to stay constant over that period.

In these exercises, we modify the presented program to include corrections which do not require this assumption.

Consider a piece of equipment of type e and ID j . If $\mathbf{x}_{e,j}^{k,l} = 1$, then it starts time period k in age bracket l . Based on its utilisation in this period, it may advance to age bracket $l + 1$ during this period, in which case we should account for that.

Let $\beta_{e,j}^{k,l}$ be the number of hours that this equipment works in age bracket $l + 1$ in this period.

1. Write down the age of the equipment, in hours, at the start and at the end of time period k .

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Solution: At the start of the period, the age is

$$\sum_{k' < k, l} \mathbf{f}_{e,j}^{k',l} O^{k'}.$$

At the end of the period, the age is

$$\sum_{k' \leq k, l} \mathbf{f}_{e,j}^{k',l} O^{k'}.$$

2. Using your expression from part 1, fill in the following equation:

$$\beta_{e,j}^{k,l} = \left\{ \right.$$

if the equipment is owned
and advances an age bracket
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Solution:

$$\beta_{e,j}^{k,l} = \left\{ \begin{array}{l} \sum_{k' \leq k, l} \mathbf{f}_{e,j}^{k',l} O^{k'} - (l+1)B_0 \text{ if the equipment is owned} \\ \text{and advances an age bracket} \\ 0 \text{ otherwise.} \end{array} \right.$$

3. We need to convert the above equation into constraints without an 'if' clause. We do this by separately constraining $\beta_{e,j}^{k,l}$ from above and below. Obviously $\beta_{e,j}^{k,l} \geq 0$ — this is a strict lower bound when the equipment is not owned or does not advance an age bracket. First assume that the equipment is owned in this period, i.e. $x_{e,j}^{k,l} = 1$.
- a Formulate a lower bound on $\beta_{e,j}^{k,l}$ which is strict when the equipment advances an age bracket. This should be a non-strict, but valid, inequality when the equipment does not advance.

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Solution:

$$\beta_{e,j}^{k,l} \geq \sum_{k' \leq k, l} f_{e,j}^{k',l} O^{k'} - (l+1)B_0.$$

3.

- b Assume that the equipment does not advance an age bracket in this period. Formulate a strict upper bound on $\beta_{e,j}^{k,l}$. This should be invalid if the equipment advances.

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Solution:

$$\beta_{e,j}^{k,l} \leq 0.$$

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Solution:

$$\beta_{e,j}^{k,l} \leq \sum_{k' \leq k,l} \mathbf{f}_{e,j}^{k',l} O^{k'} - (l+1)B_0.$$

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- d Formulate an expression, using \mathbf{x} and \mathbf{s} variables from period $k + 1$, which is 1 if the equipment advances an age bracket in period k , and 0 otherwise.

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Solution: One such expression is $\mathbf{x}_{e,j}^{k+1,l+1} + \mathbf{s}_{e,j}^{k+1,l+1}$.

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- e Using your answer for part 3d, modify your constraint from part 3b so that it is still valid (but may be non-strict) if the equipment advances.

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Solution:

$$\beta_{e,j}^{k,l} \leq M(\mathbf{x}_{e,j}^{k+1,l+1} + \mathbf{s}_{e,j}^{k+1,l+1}).$$

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- f Using your answer for part 3d, modify your constraint from part 3c so that it is still valid (but may be non-strict) if the equipment does not advance.

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Solution:

$$\beta_{e,j}^{k,l} \leq \sum_{k' \leq k,l} \mathbf{f}_{e,j}^{k',l} O^{k'} - (l+1)B_0 + M(1 - \mathbf{x}_{e,j}^{k+1,l+1} - \mathbf{s}_{e,j}^{k+1,l+1}).$$

3. We now no longer assume that the equipment is owned.
 - g If the equipment is not owned, then your constraint from 3a may be invalid. Modify it so that it is valid in this case.

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- g If the equipment is not owned, then your constraint from 3a may be invalid. Modify it so that it is valid in this case.

Solution:

$$\beta_{e,j}^{k,l} \geq \sum_{k' \leq k,l} \mathbf{f}_{e,j}^{k',l} O^{k'} - (l+1)B_0 - M(1 - \mathbf{x}_{e,j}^{k,l}).$$

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- h If the equipment is not owned, then your constraint from 3f may be invalid. Modify it so that it is valid in this case.

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Solution:

$$\beta_{e,j}^{k,l} \leq \sum_{k' \leq k,l} \mathbf{f}_{e,j}^{k',l} O^{k'} - (l+1)B_0 + M(1 - \mathbf{x}_{e,j}^{k,l} - \mathbf{x}_{e,j}^{k+1,l+1} - \mathbf{s}_{e,j}^{k+1,l+1}).$$

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- i Using a big- M constraint, strictly constrain $\beta_{e,j}^{k,l}$ from above when the equipment is not owned.

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Solution:

$$\beta_{e,j}^{k,l} \leq Mx_{e,j}^{k,l}.$$

The constraints from parts 3e, 3g, 3h and 3i will be added to the model to bound β .

4. Modify the operating cost so that hours spent in age bracket l are costed at $V_{e,j}^{k,l}$ per O^k hours and hours spent in age bracket $l + 1$ are costed at $V_{e,j}^{k,l+1}$ per O^k hours.

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Solution:

$$\text{Operating cost} = \sum_{k,l} \frac{V_e^{k,l}}{(1+I)^k} (\mathbf{f}_{e,j}^{k,l} - \frac{1}{O^k} \beta_{e,j}^{k,l}) + \sum_{k,l} \frac{V_e^{k,l+1}}{(1+I)^k O^k} \beta_{e,j}^{k,l}.$$

Bonus exercise on your problem sheet!