Vehicle Scheduling Exercise

Ralf Borndörfer

October 03, 2009

Exercise 1) Consider the vehicle scheduling problem in the file `arcs.dat`:

```
#  dh  tail  head  depot    cost
a   0     0     1      2    1800
a   1     0     2      2    1800
a   2     0     3      0    1320
a   3     0     4      2    1800
...
```

File `arcs.dat`.

The file has the following format. A line starting with `#` contains comments. A line starting with `a` defines an arc in terms of five numbers, namely, a deadhead trip index `dh`, a start node `tail`, an end node `head`, a depot or vehicle type `depot`, for which this arc is feasible, and a `cost`. There can be several arcs with the same deadhead index, tail, and head, but different depots; these arcs model the feasibility of the same deadhead trip for serveral types of vehicles. `dh`, `tail`, `head`, and `type` are integer numbers starting with 0, costs are nonnegative integer numbers. The artificial node 0 denotes the beginning and the end of the vehicle rotations, the other nodes represent timetabled trips. The problem in file `arcs.dat` involves 11 depots (= vehicle types), 7 130 timetabled trips, 7 131 nodes, 75 137 deadhead trips, and 126 992 arcs.

a) Fomulate and solve the vehicle scheduling problem using Zimpl and `Scip`.
b) What is the optimal objective value of the IP and the LP relaxation?
c) How many vehicles are used in each case?
d) Is this the minimum fleet size (= number of vehicles)?

Exercise 2) This exercise continues Exercise 1. Files `tripduals.dat` and `flowduals.dat` contain multipliers for the flow and the flow conservation constraints of a multi-commodity flow formulation of the vehicle scheduling problem:

```
# node    dual
v    1    1740.000000
v    2    2280.000000
v    3    1200.000000
v    4    2040.000000
...
```

File `tripduals.dat`.

```
# node depot     dual
v    0     0      420.000000
v    0     1        0.000000
v    0     2      240.000000
v    0     3     3330.000000
...
```

File `flowduals.dat`.

Each noncomment line of file `tripduals.dat`, starting with `v`, contains a node index (= index of a timetabled trip, different from 0) and an associated multiplier. Each noncomment line of file `flowduals.dat` contains a node index, a depot (= vehicle type), and an associated multiplier. The multipliers can be arbitrary floating point numbers.

a) Construct an m-depot relaxation of the vehicle scheduling problem by ignoring the flow constraints. Show that this m-depot relaxation is a minimum cost flow problem. What is the optimal objective value?
b) Improve the m-depot relaxation by adding a Lagrange relaxation of the flow constraints, using the multipliers from file `tripduals.dat`. Show that this Lagrangean relaxation with fixed multipliers is a min-cost flow problem. What is the optimal objective value?
c) Construct a 1-depot relaxation of the vehicle scheduling problem by pretending that all arcs are feasible for vehicle type 1. Show that the 1-depot relaxation is a min-cost flow problem. What is the optimal objective value?
d) Improve the 1-depot relaxation by adding a Lagrange relaxation for the flow conservation constraints associated with the individual vehicle types, using the mulotipliers in file `flowduals.dat`. Show that this Lagrangean relaxation with fixed multipliers is also a min-cost flow problem. What is the optimal objective value?
e) Which relaxation is better?

Exercise 3) A public transit company services the lines of the lengths and with the hourly frequencies and transport capacity demands listed in file `lines.dat`. In addition, the company must provide 5 buses per hour at each stop. There are two bus types: small buses with 80 seats, and large buses with 150 seats. These operate linewise, i.e., a bus never changes a line. The average speed is 18 km/h, and a turn takes 10 minutes. How many vehicles are needed to operate all lines?

a) Formulate an integer programming model for this problem.
b) Implement and solve the model using Zimpl and Scip. What is the minimum number of buses?
c) Suppose there are only 145 large buses. How many small buses are needed?
d) Suppose there are only 145 large buses and 150 small buses. What now? Reduce turn times? Drive faster? Leave some passengers behind? Use your model to come up with some "solution".