Exercise 1

Show that the LP relaxation of the undirected cut formulation for the Steiner Tree Problem on a graph $G = (V, E)$,

$$\begin{array}{lll} \min & c^T x & \\ \text{s.t.} & x(\delta(U)) \geq 1 & \forall U \subset V, \emptyset \neq U \cap T \neq T \\ & x_e \in \mathbb{Z}_+ & \forall e \in E, \end{array} \tag{1}$$

is at most as strong as the LP relaxation of the directed cut formulation on the directed graph $D = (V, A)$ over G,

$$\begin{array}{lll} \min & c^T x & \\ \text{s.t.} & y(\delta^+(U)) \geq 1 & \forall U \subset V, r \in U, U \cap T' \neq T' \\ & y_{(i,j)} + y_{(j,i)} \leq x_{\{i,j\}} & \forall \{i, j\} \in E \\ & y_{(i,j)} \in \{0, 1\} & \forall (i, j) \in A \\ & x_{\{i,j\}} \in \mathbb{Z}_+ & \forall \{i, j\} \in E, \end{array} \tag{2}$$

i. e., show that for each fractional solution $(\mathbf{y}, \mathbf{x}) \in \mathbb{R}^A \times \mathbb{R}^E$ of (2), the projection $\mathbf{x} \in \mathbb{R}^E$ is feasible for (1).

Exercise 2

Implement a cut formulation for the Steiner Tree Problem as a ZIMPL model, extending the given ZIMPL file for reading a network instance and try it out for some instances of various sizes.

Now do the same for a flow model. What do you observe?

If you have time, implement a separation procedure for cuts using a max-flow/min-cut implementation...

Exercise 3

Setup an IP model for the *Node-Survivable Network Design Problem*: Given a graph $G = (V, E)$, terminals $T \subseteq V$, edge weights c_e for all $e \in E$, find a minimum cost subgraph containing all $v \in T$ that has at least two node-disjoint s-t-paths for all $s, t \in T$. (Two s-t-paths are *node-disjoint*, if they intersect only in s and t.)

Exercise 4

Replace the undirected cut constraints in the IP model for ConFL with other possibilities to enforce connectivity:

(i) directed cuts

(ii) flow variables and constraints

(iii) *customer cuts:* $x(\delta^-(U)) \geq 1 \; \forall U \subset V \cup C$ with $U \cap C \neq \emptyset$ (where the x-variables are defined both on the arcs of the directed graph over G and the assignment arcs)

Which model yields the strongest LP relaxations?

Exercise 5

Extend your favourite IP model for ConFL from Exercise 4 to one for the *k-Architecture Connected Facility Location Problem*. Here, the set of facilities is partitioned into k sets of facilities of different types (*architectures*): $F = F_1 \cup \cdots \cup F_k$; accordingly, an assignment arc (v, j) has the same architecture as the facility v in which it starts. Additionally, for each architecture ℓ there is a target coverage rate $p_k \in [0, 1]$, such that $0 \leq p_1 \leq \cdots \leq p_k \leq 1$. A feasible solution must ensure that for each architecture ℓ, the number of customers connected to a facility in $F_1 \cup \cdots \cup F_\ell$ is at least $p_k \cdot |C|$.

Exercise 6

Setup a version of the Capacitated Network Design IP, where traffic can be routed through multiple paths. For this, you just need to relax the variable definition of the flow variables to $f_{st,a} \geq 0$ and adapt the other constraints accordingly.

Which of the two models will give the better objective values for the same instance? Which one will solve faster? Verify your guess by implementing both models in ZIMPL and solving a few instances. Do the flow values in the solution of the multiple-path version look somehow strange?