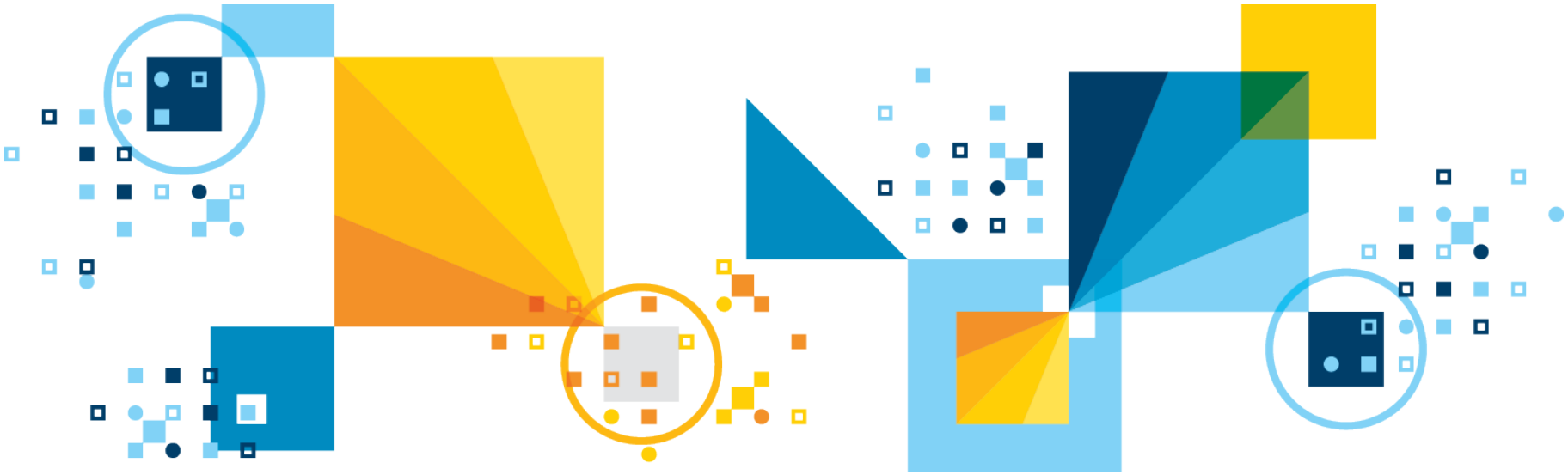
CPLEX Optimization – IBM Germany

2015-10-06

Using computing resources with IBM ILOG CPLEX

CO@W2015



- Hardware resources
  - Multiple cores/threads
  - Multiple machines
  - No machines
- Software resources
  - Interfacing with CPLEX
  - Interacting with Python

Using more than one core/thread to solve a problem

Sequential branch and bound

Search tree

T

$\min c^T x$
$Ax = b$
x_j in {0, 1}

Pseudo code:

```
while !T.is_empty
  n = T.get_next()
  n.solve()
  if not (n.is_integer() ||
          n.is_infeasible())
    j = n.fractional_index()
    v = n.fractional_value(j)
    T.create(n + “x[j] <= floor(v)”)
    T.create(n + “x[j] >= ceil(v)”)
```

Sequential branch and bound

1. 

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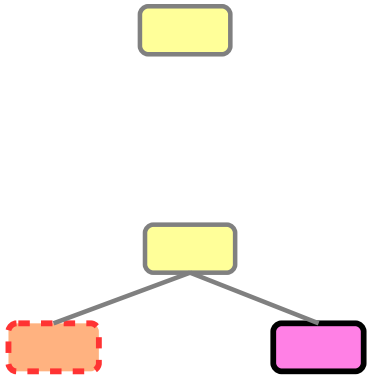

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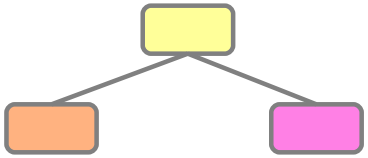

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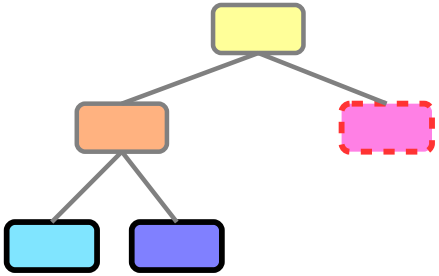

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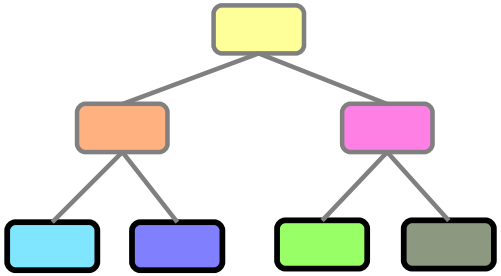

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```

Parallel branch and bound

```
while !T.is_empty
  T.lock()
  n = T.get_next()
  T.unlock()
  n.solve()
  if not (n.is_integer() ||
          n.is_infeasible())
    j = n.fractional_index()
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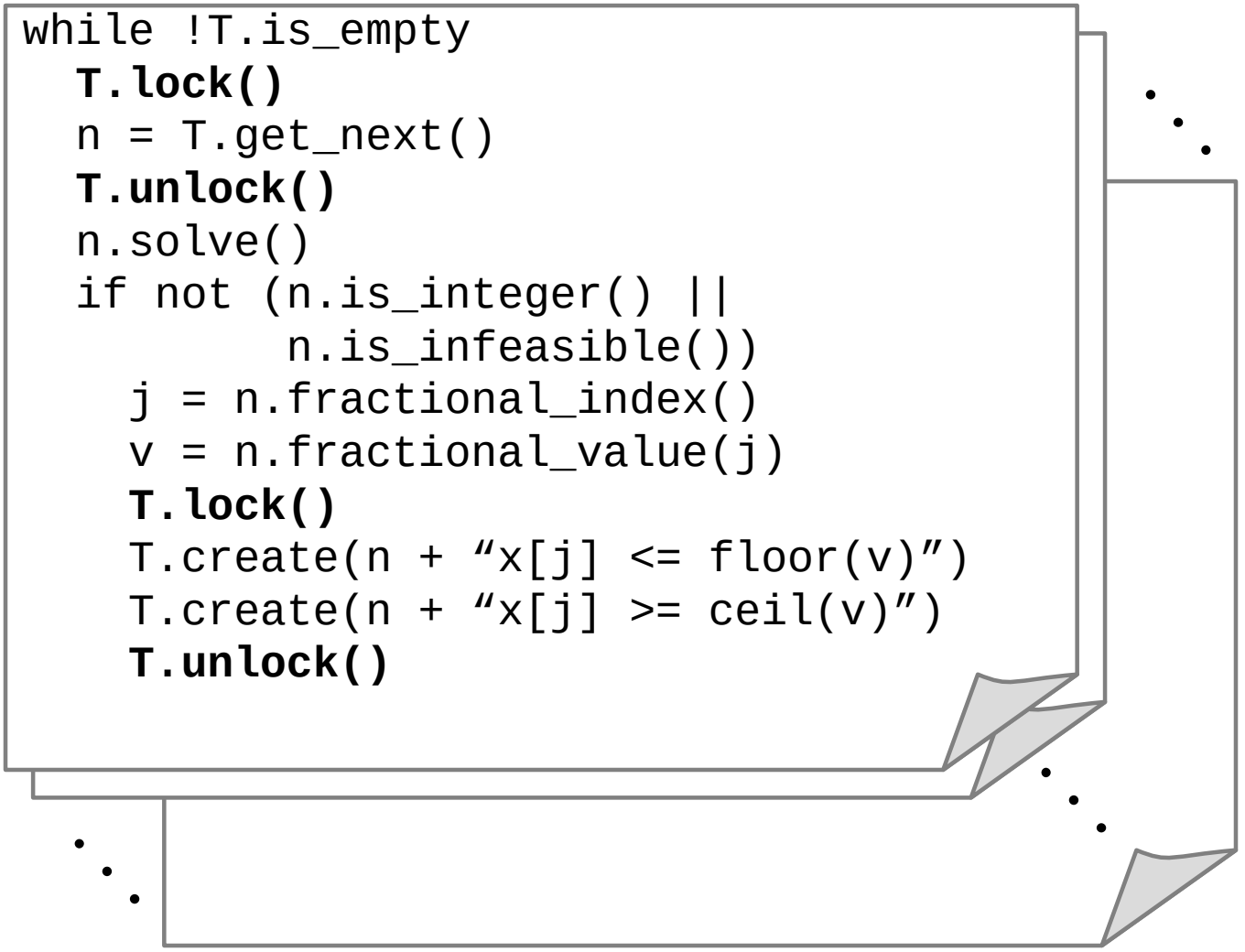
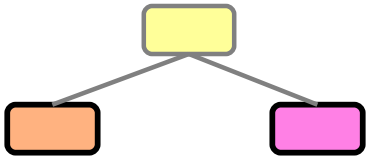


Parallel branch and bound

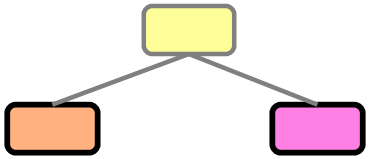
Thread 1



Thread 2

Parallel branch and bound

Thread 1

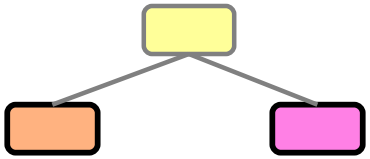


Thread 2

T.lock()

T.lock()

Parallel branch and bound



Thread 1

T.lock()

T.get_next()

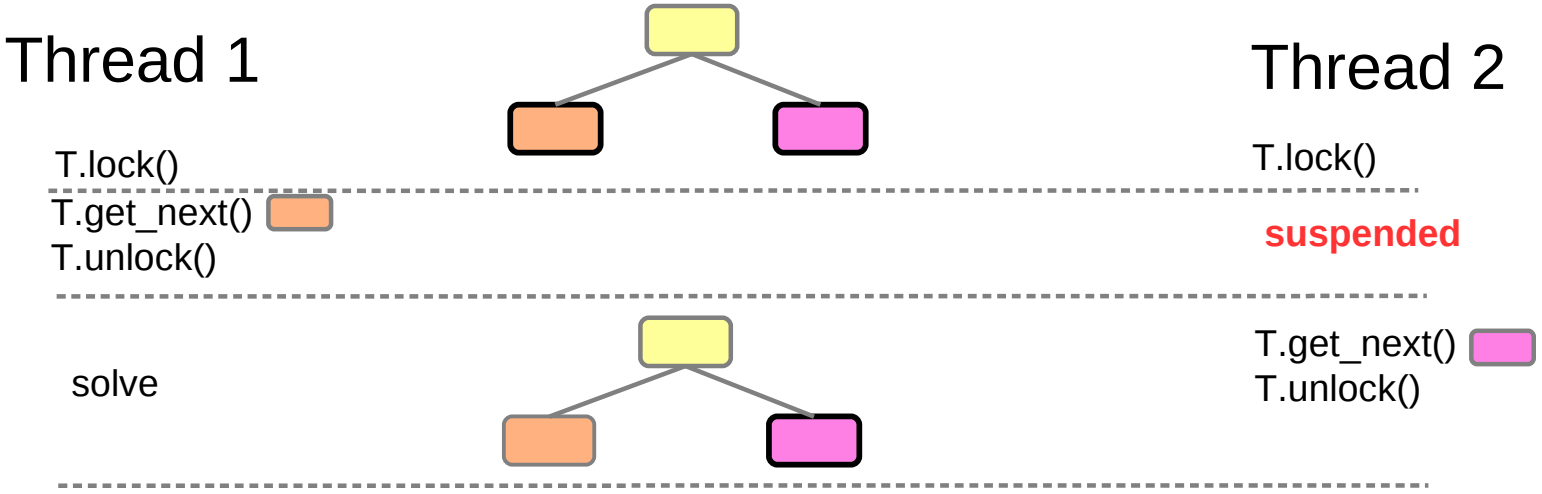
T.unlock()

Thread 2

T.lock()

suspended

Parallel branch and bound



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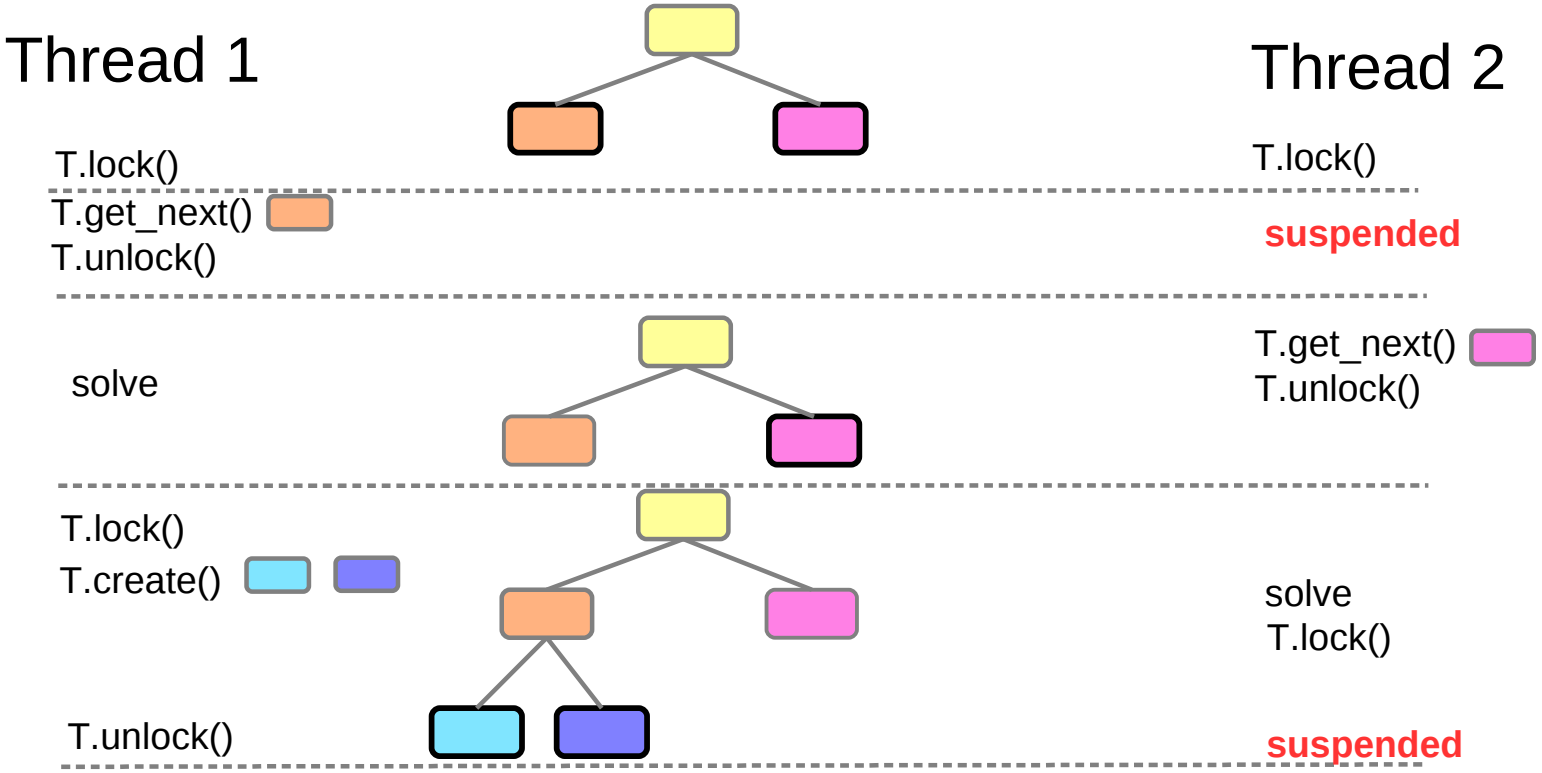

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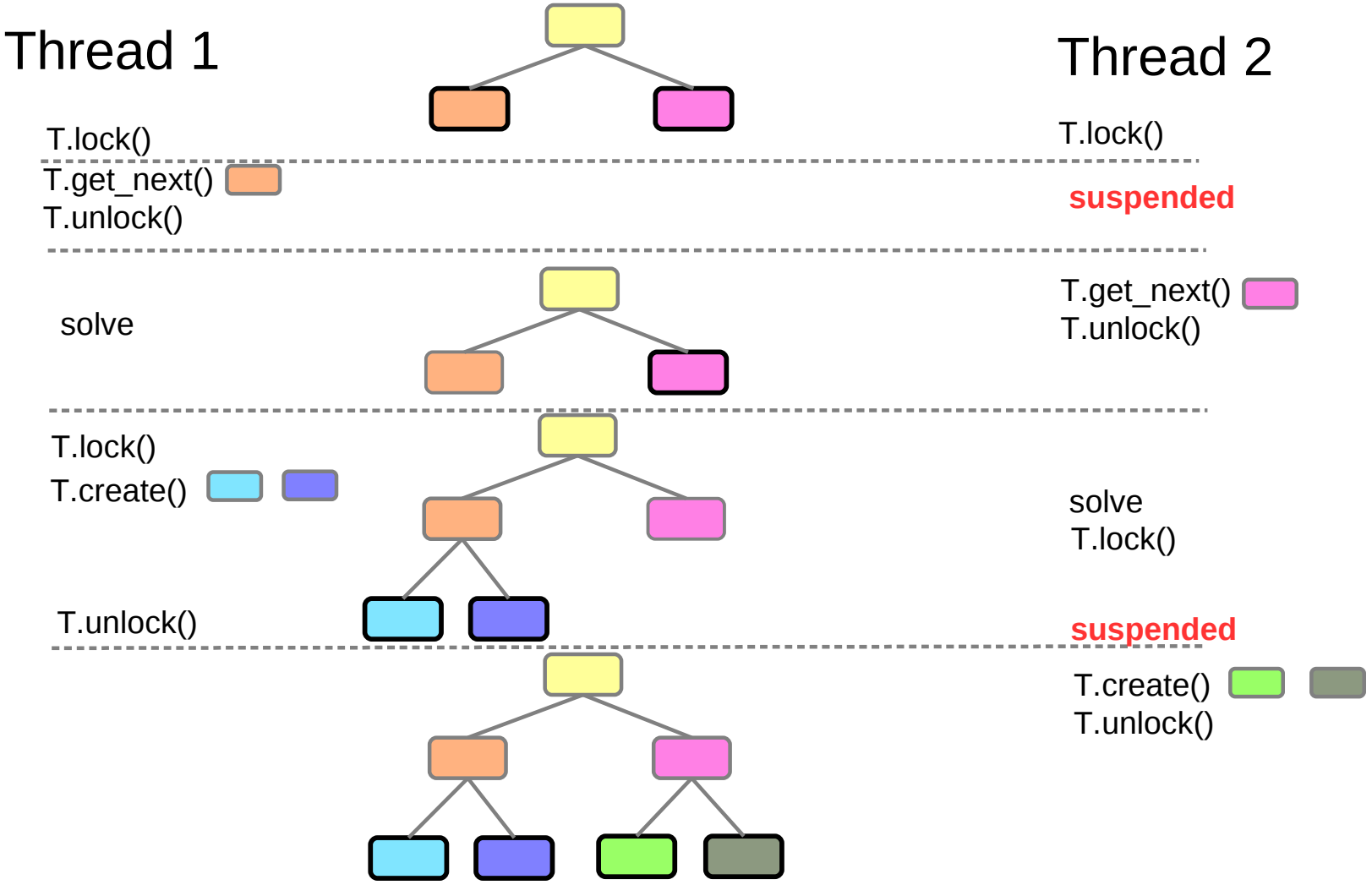
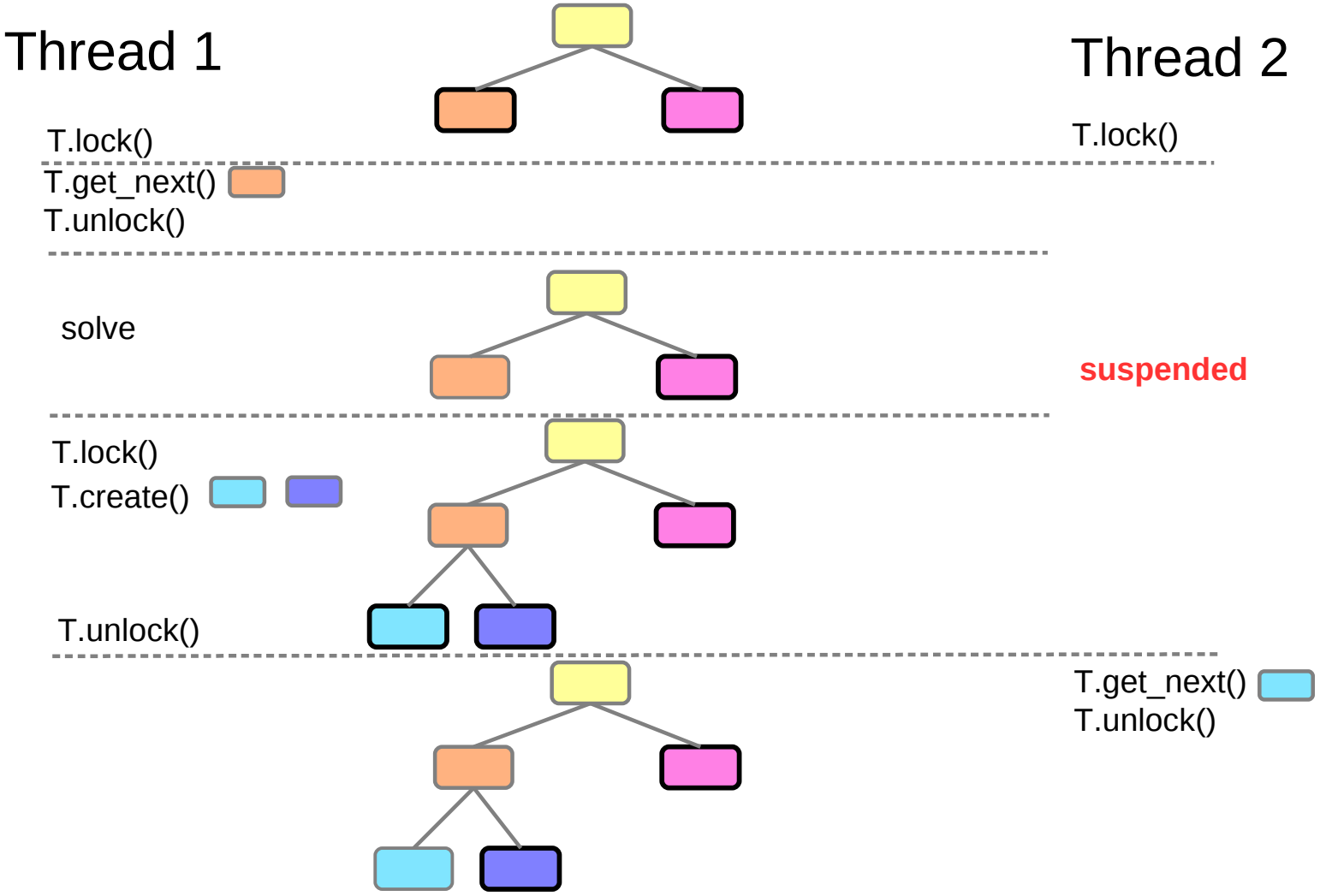


Not deterministic!



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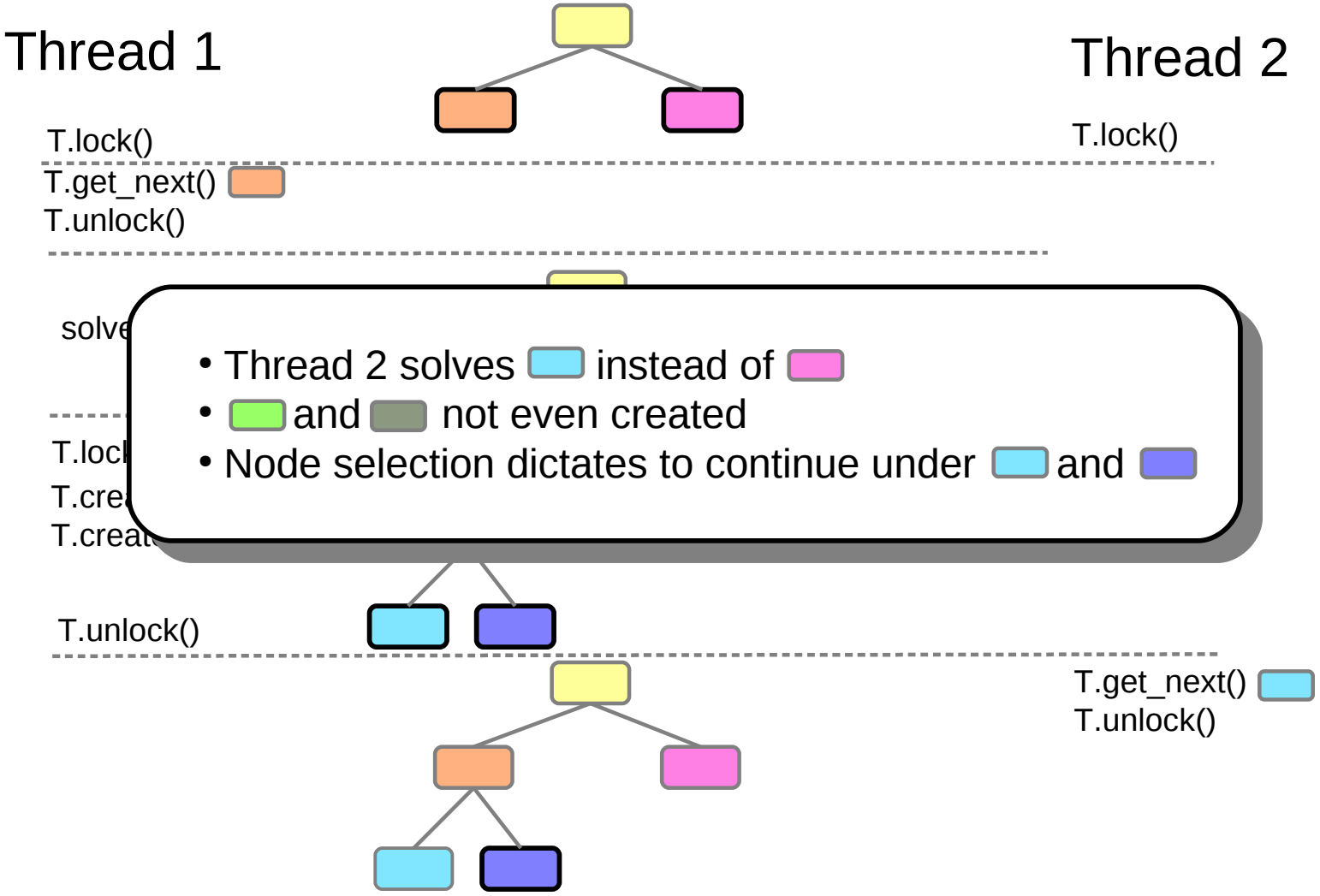

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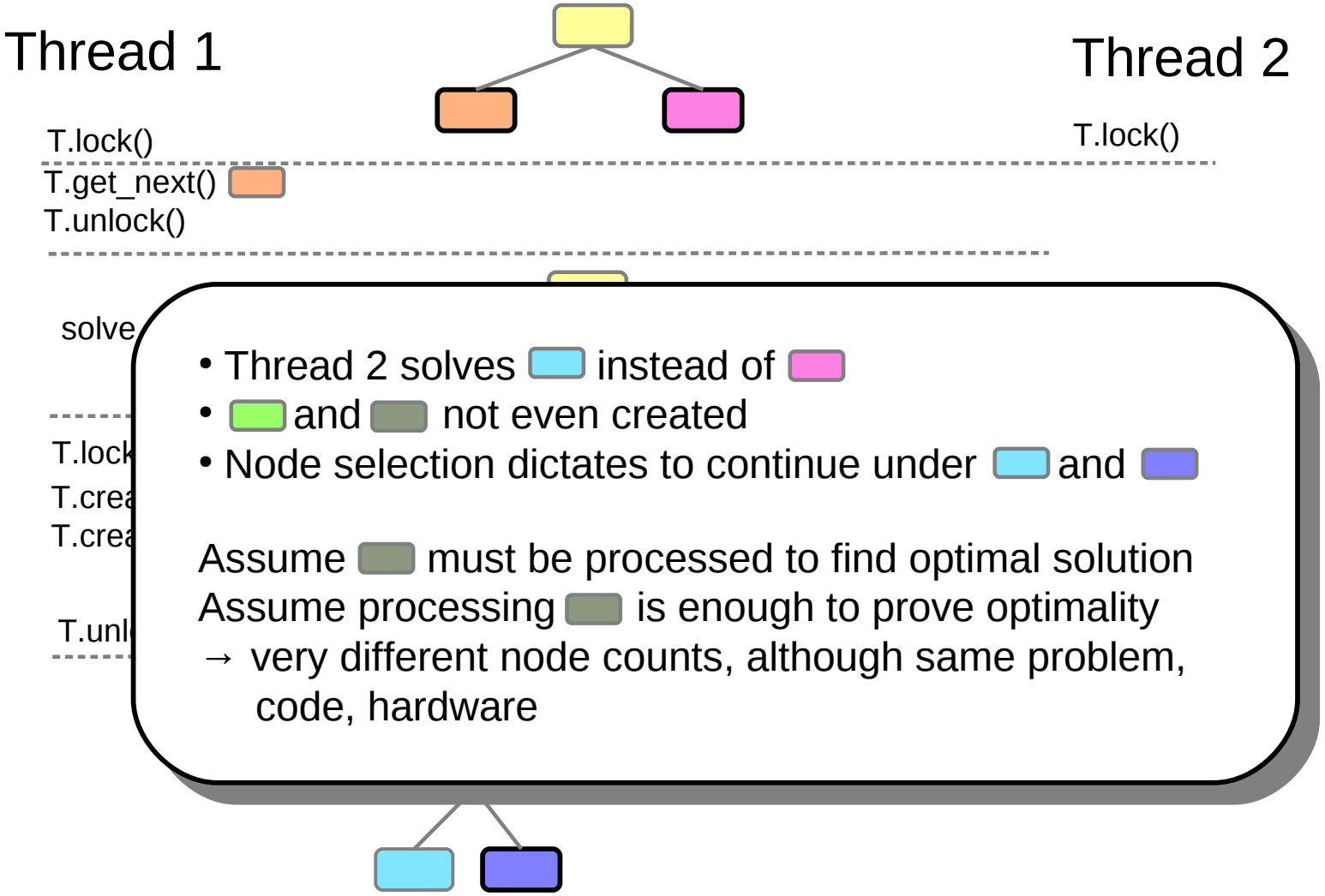

Issues with non-determinism

- opportunistic behavior of OS scheduler
- very minor side effects (cache misses, page swaps, …) can change order of threads

This is a problem in practice:

gmu-35-40		
Time	Iterations	Nodes
21.31	2236199	444838
18.46	2060789	322282
4.62	**253871**	**51778**
31.55	3300767	509641
11.89	1238162	177236
79.50	**7212334**	**1307340**
21.10	2955104	595939
16.27	1953450	401507
13.61	1410170	261508
17.10	1948228	410949

iis-pima-cov		
Time	Iterations	Nodes
62.92	**1678290**	**17744**
139.72	5475373	47428
84.39	2075243	26460
80.37	2766163	22016
71.25	1753092	17513
69.18	1615592	21636
117.43	3002680	40299
72.41	1704471	21301
124.01	4863607	42298
191.31	**7898071**	**71043**

glass4		
Time	Iterations	Nodes
41.60	6577338	340192
70.57	11800428	637471
29.30	**4075955**	**304913**
34.31	6026804	208466
160.12	29207354	1082067
244.62	**35189917**	**1893985**
54.40	8700758	512053
74.64	11307243	698508
47.19	8600157	389436
54.42	8275014	544172

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- → results are not repeatable
- → debugging becomes painful
- → benchmarking is complicated
- → assessment of algorithmic changes is difficult
  "Is my new cut really helpful or is this just a random change?"

→ We need a solver that operates in a deterministic way!

Deterministic multi-threading

opportunistic — OS scheduler, cache misses, swaps, ...
→ order in which lock is granted to threads is not deterministic

deterministic — make this order deterministic:

- use **deterministic time** → same in any run
- grant lock to first thread to arrive in **deterministic time**

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Kendo	“Kendo: Efficient Deterministic Multithreading in Software” http://people.csail.mit.edu/mareko/asplos073-olszewski.pdf → use number of retired store instructions as time (x86)

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CPLEX

deterministic time = number of array accesses
- ➔ works the same on all hardware
- ➔ explicit instrumentation of source code

```
for (int i = 0; i < n; ++i)
   x[i] = y[i] * 0.5;
DETCLOCK_INDEX_ARRAY (2 * i);
```

Deterministic multi-threading

- Deterministic threading is not for free
  - settle for a particular path through the search tree
  - increases wait/idle times in locks

gmu-35-40		
Time	Iterations	Nodes
4.62	253871	51778
79.50	7212334	1307340
38.48	**4124364**	**737994**

iis-pima-cov		
Time	Iterations	Nodes
62.92	1678290	17744
191.31	7898071	71043
72.84	**1382605**	**20449**

glass4		
Time	Iterations	Nodes
29.30	4075955	304913
244.62	35189917	1893985
91.68	**11737409**	**528160**

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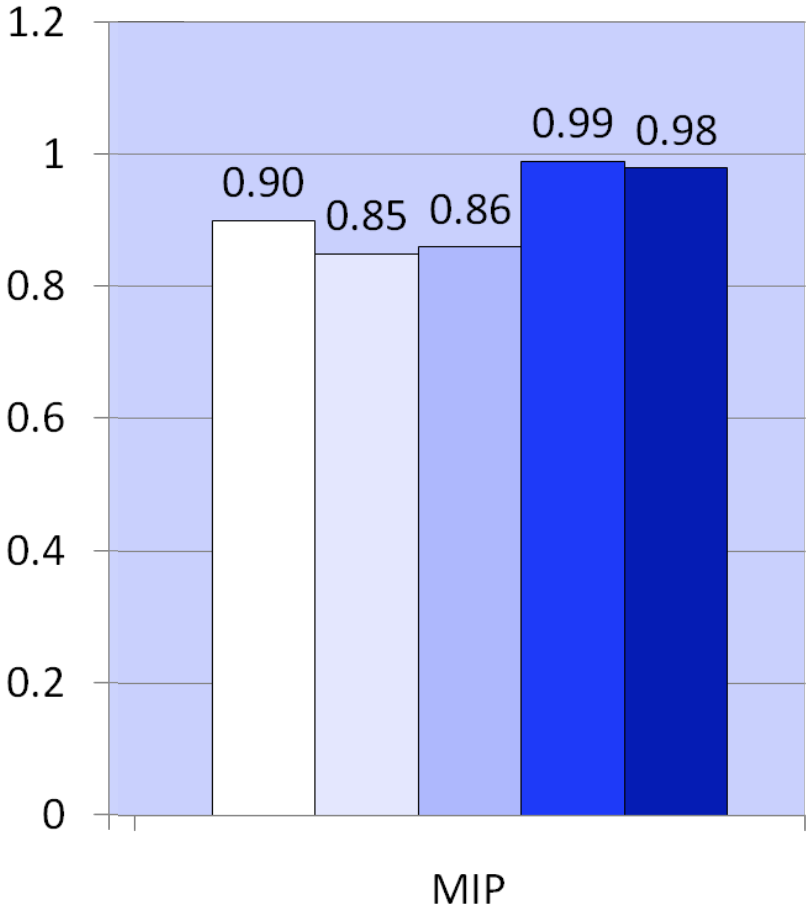

deterministic vs. opportunistic threading

- All
- [1,10k]
- [10,10k]
- [100,10k]
- [1k,10k]

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Deterministic multi-threading

- Deterministic threading is not for free
  - settle for a particular path through the search tree
  - increases wait/idle times in locks
- Deterministic threading is unrelated to performance variability
  - parallel cut loop is still meaningful
- CPLEX extensions of Kendo framework allow efficient parallel cutloop
  - → opportunistic code, but results are deterministic
- All parallel algorithms in CPLEX are available as opportunistc/deterministic
  - → select at runtime with a parameter

Using more than one machine to solve a problem

Distributed parallel MIP

CPLEX implements distributed parallel MIP solving

- one master process to coordinate the search
- several worker processes to perform the heavy lifting
- usually one master/worker per machine (can configure otherwise)
- communication only between master and workers

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- 3 different ways of communication:

`ssh`	→ master starts workers via ssh
	→ communication via pipes
`TCP/IP`	→ workers run a server-like process to which master connects
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`MPI`	→ all processes run within an MPI communicator
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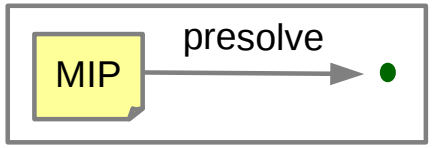
- 2 phases:
  1. “rampup” → create an initial set of open nodes
  2. “tree search” → perform distributed parallel b&b

Phase I: rampup

1. master runs presolve to create a root node



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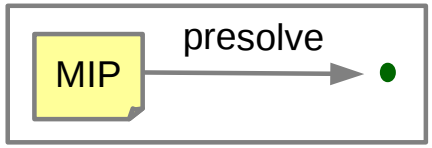
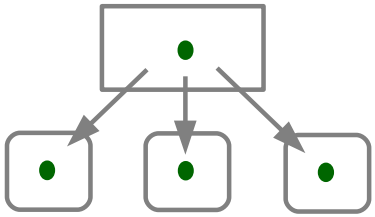


2. master sends root to workers



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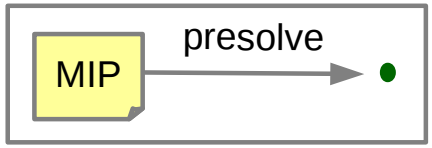

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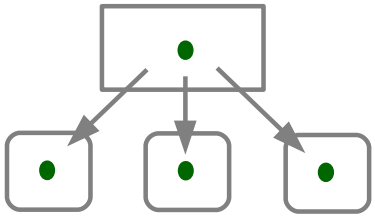
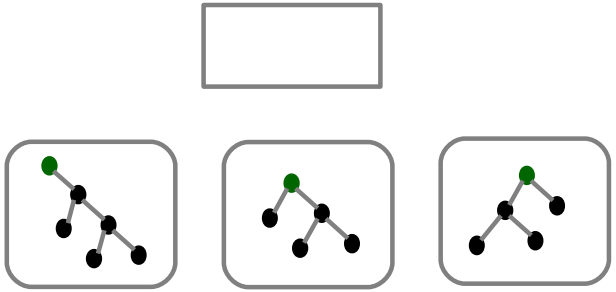


3. each worker starts b&b with different parameters

→ each worker produces a different tree



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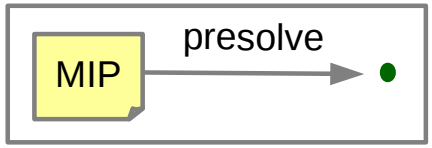

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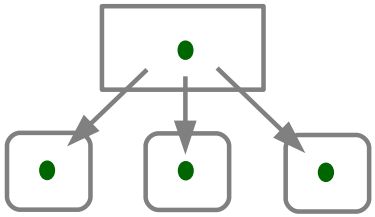

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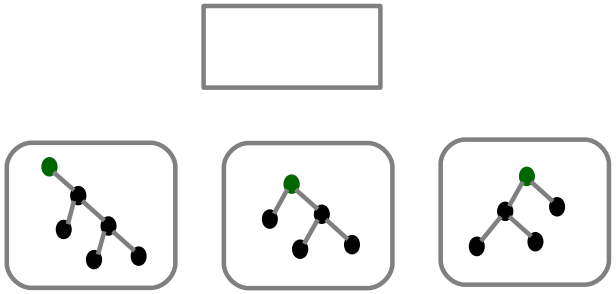
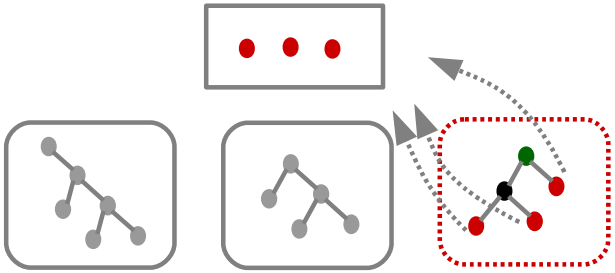
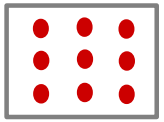


4. master eventually
   - stops workers
   - selects a winner
   - collects open nodes from winner
   → list of “supernodes”



Phase II: tree search

5. master starts with list of supernodes



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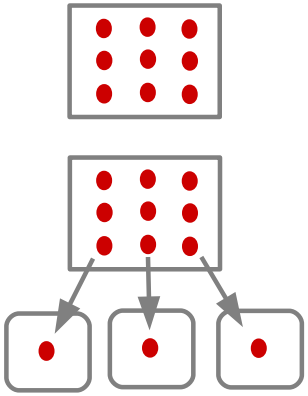

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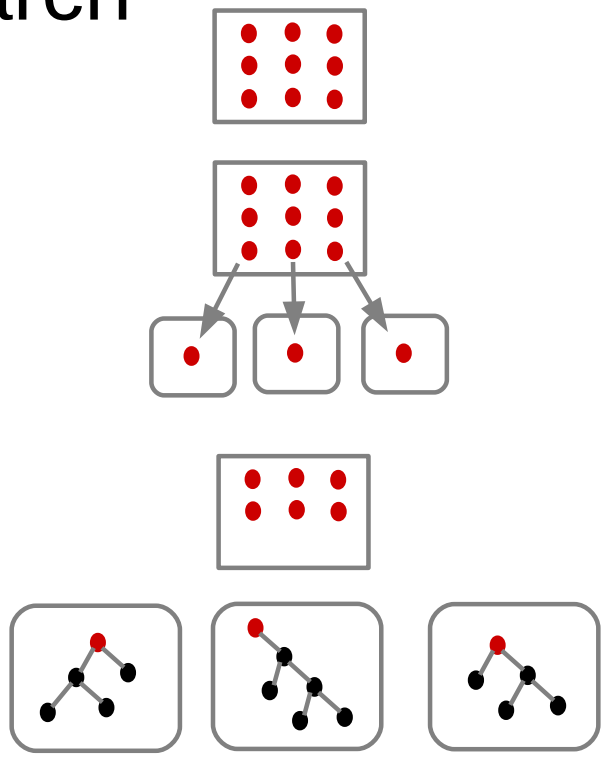

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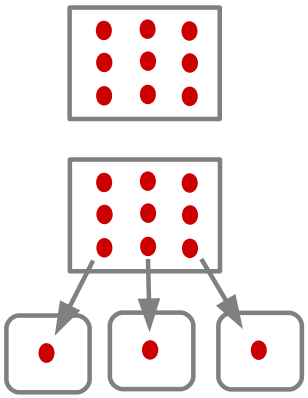

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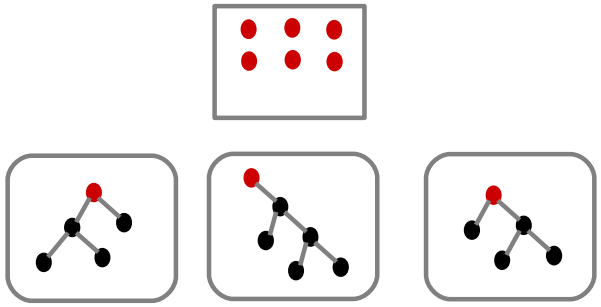
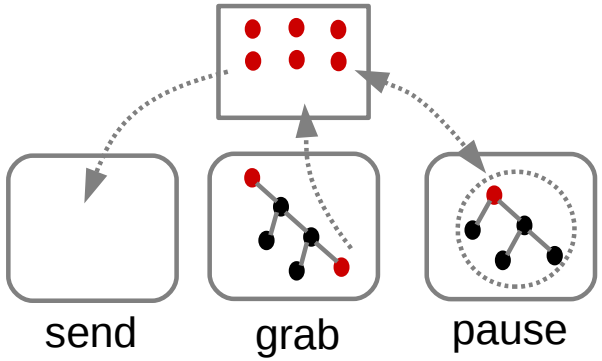


8. Master can
   - send new supernodes (if idle)
   - grab nodes to produce new supernodes
   - pause supernode (exchange)



Distributed parallel MIP

Variants/improvements

1. exchange information (incumbents, bound tightenings, ...)

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2. in rampup, start some workers with special settings
   - aggressive heuristics → quickly find good solutions
   - aggressive cuts → quickly improve dual bound
   - ...

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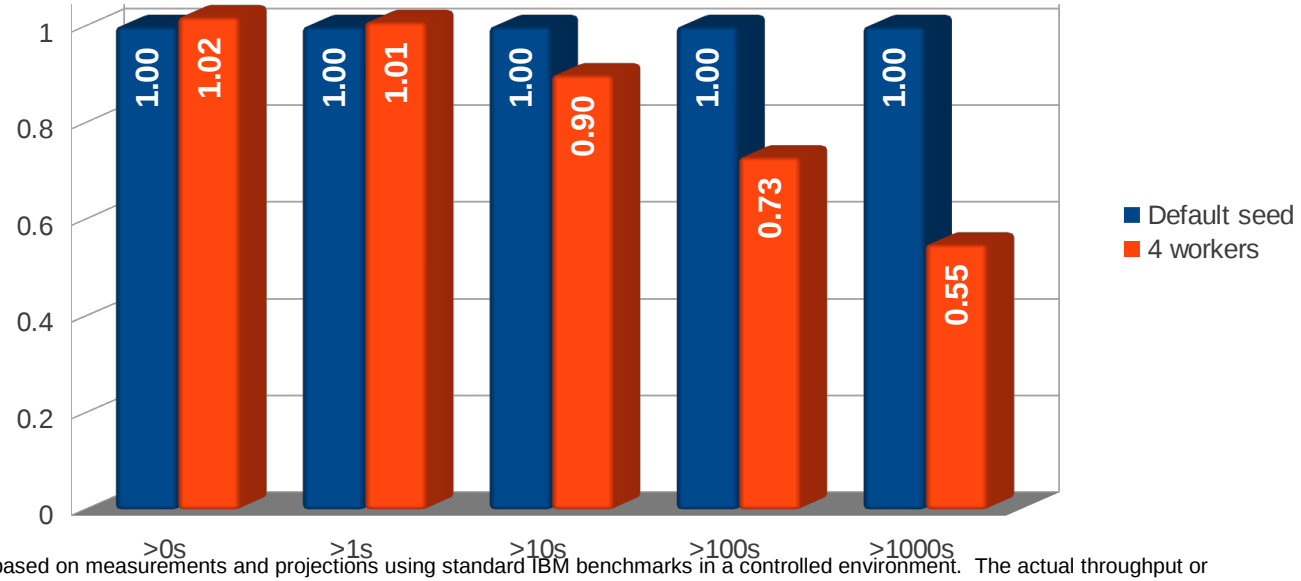
1. exchange information (incumbents, bound tightenings, ...)
2. in rampup, start some workers with special settings
   - aggressive heuristics → quickly find good solutions
   - aggressive cuts → quickly improve dual bound
   - ...
3. never stop the rampup phase → exploit performance variability



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Solving a problem without a machine

CPLEX in the cloud

What if no local resources to solve model?

- model too hard
- only need to solve once in a while
- ...

→ solve in the cloud

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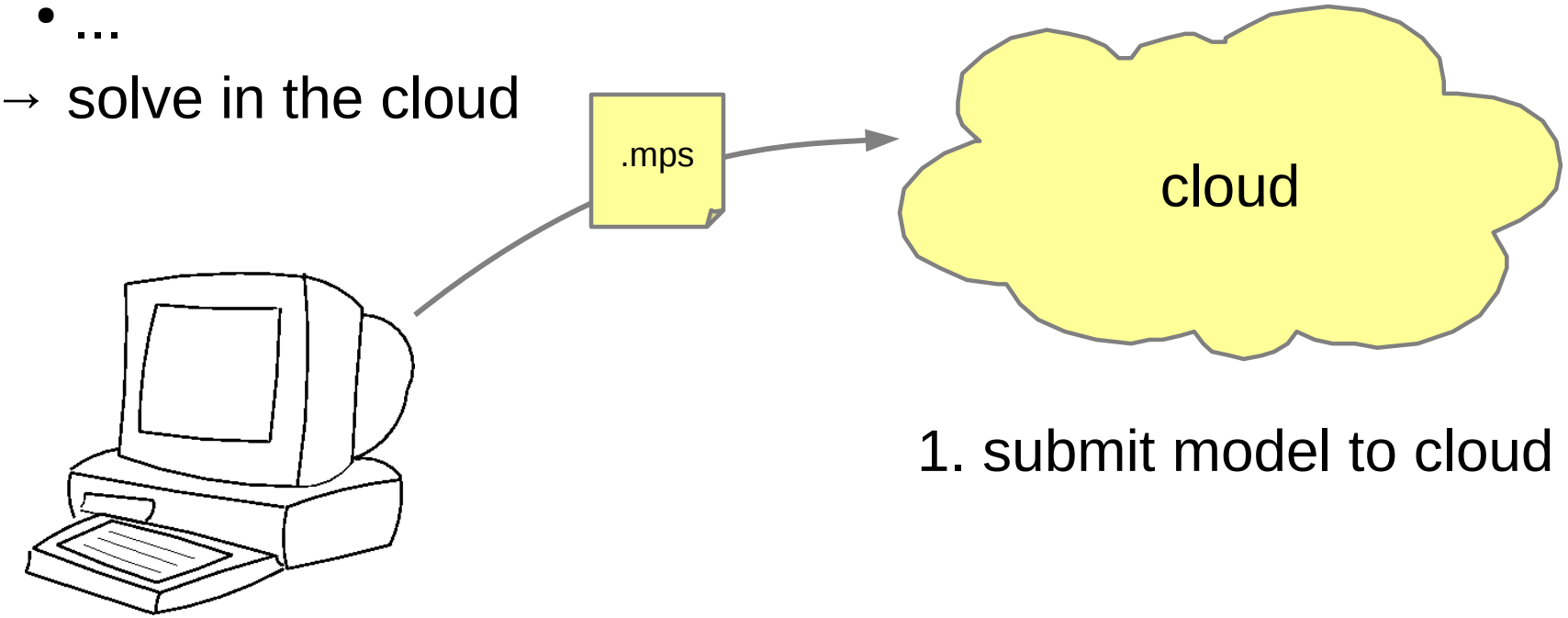


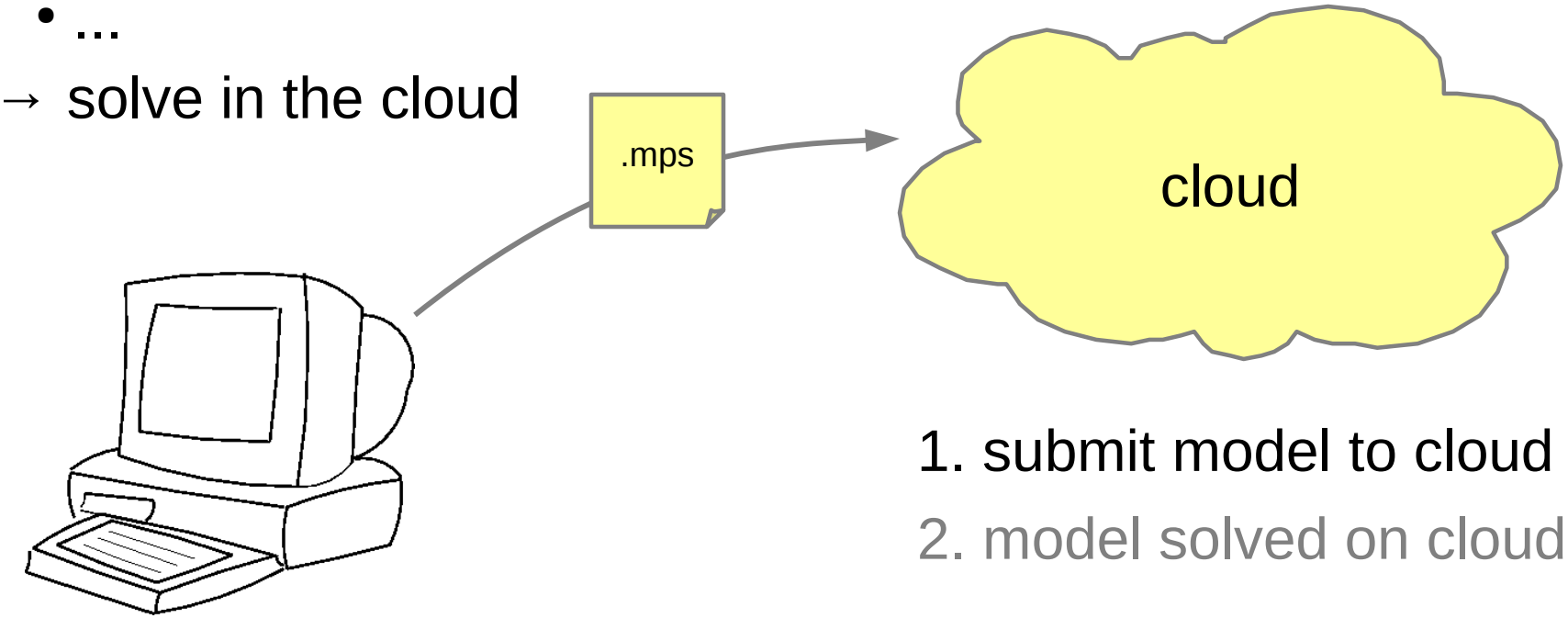
1. submit model to cloud

CPLEX in the cloud

What if no local resources to solve model?

- model too hard
- only need to solve once in a while
- ...

→ solve in the cloud



1. submit model to cloud
2. model solved on cloud

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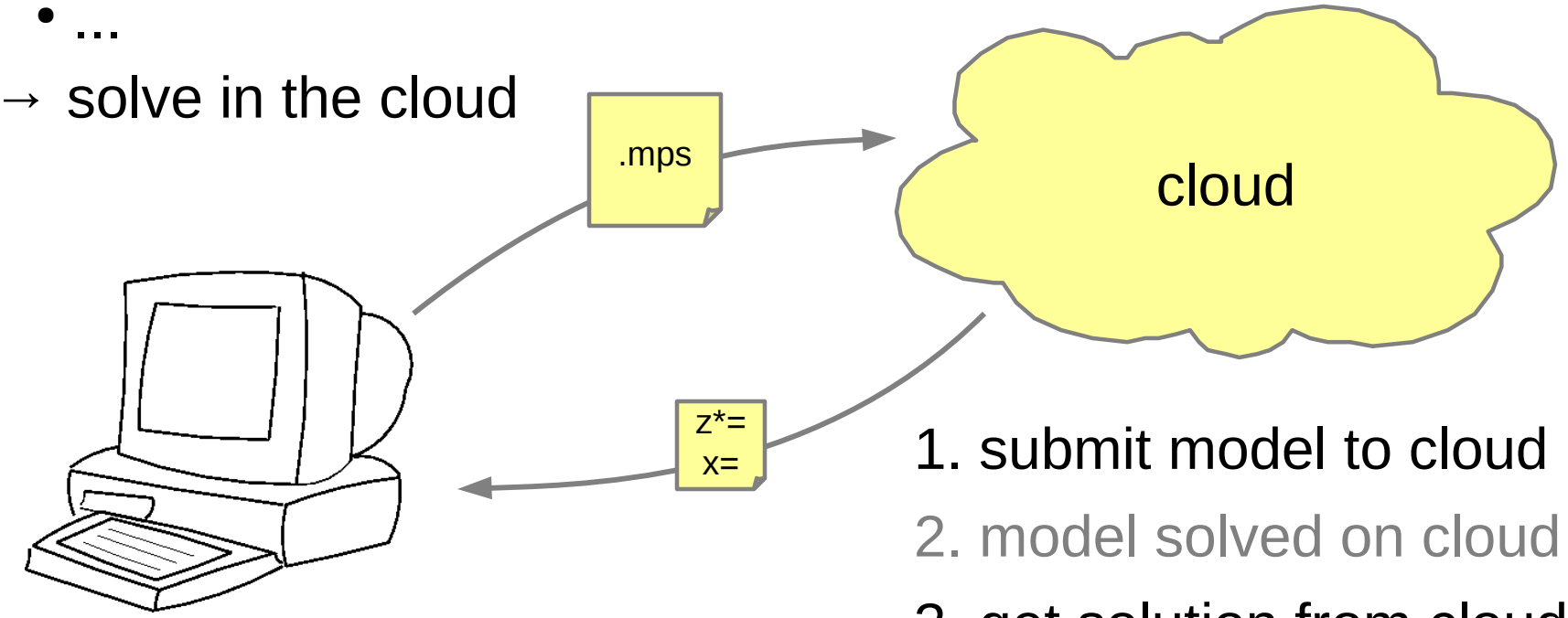

1. submit model to cloud
2. model solved on cloud
3. get solution from cloud

CPLEX in the cloud

Two ways to access CPLEX in the cloud

1. Dropsolve

www.ibm.com/software/analytics/docloud
dropsolve-oaas.docloud.ibmcloud.com/software/analytics/docloud

Drag and drop your model file from disk to web browser

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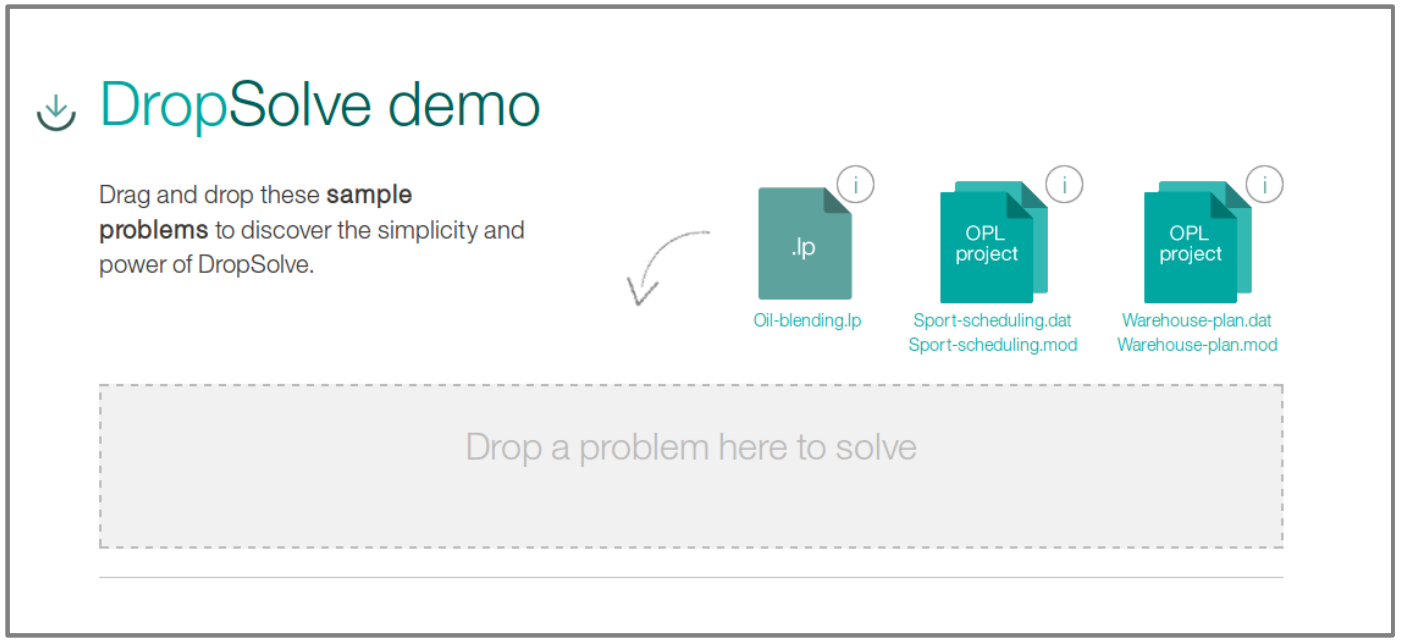

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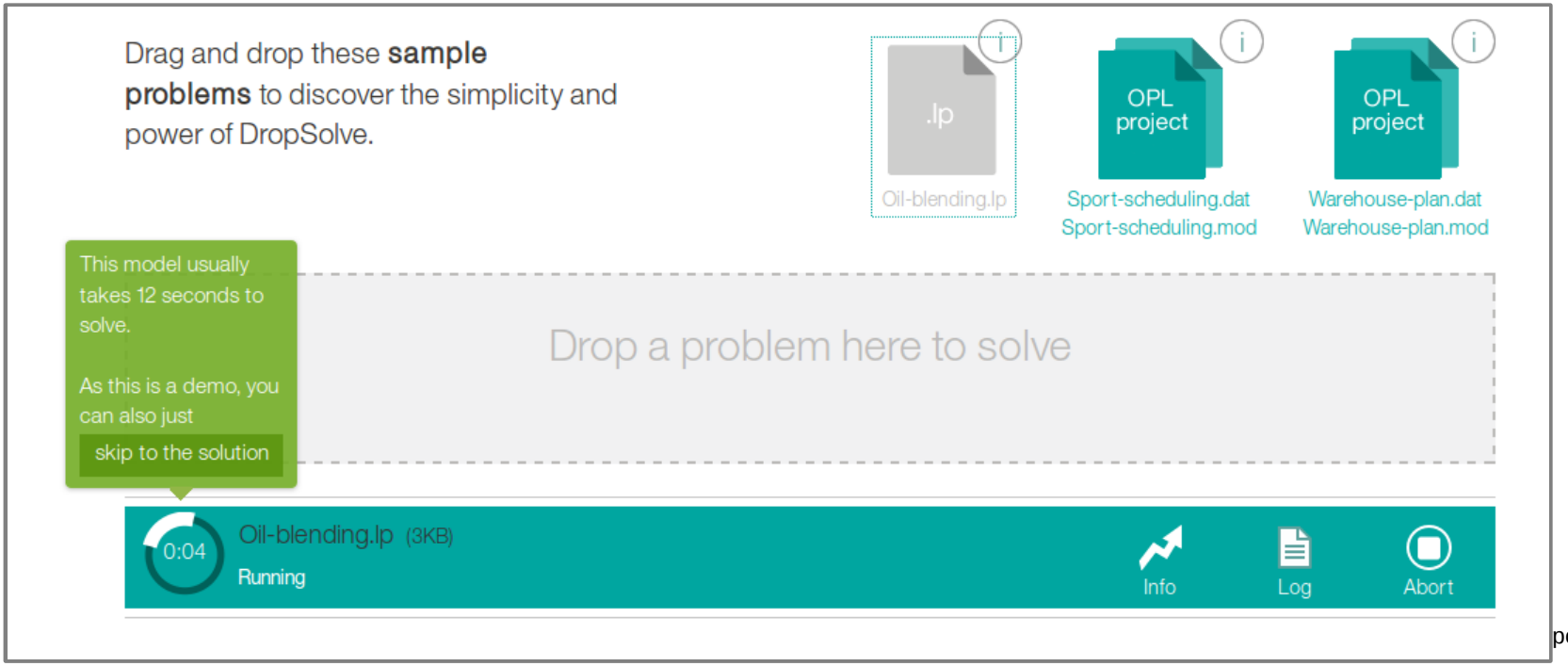

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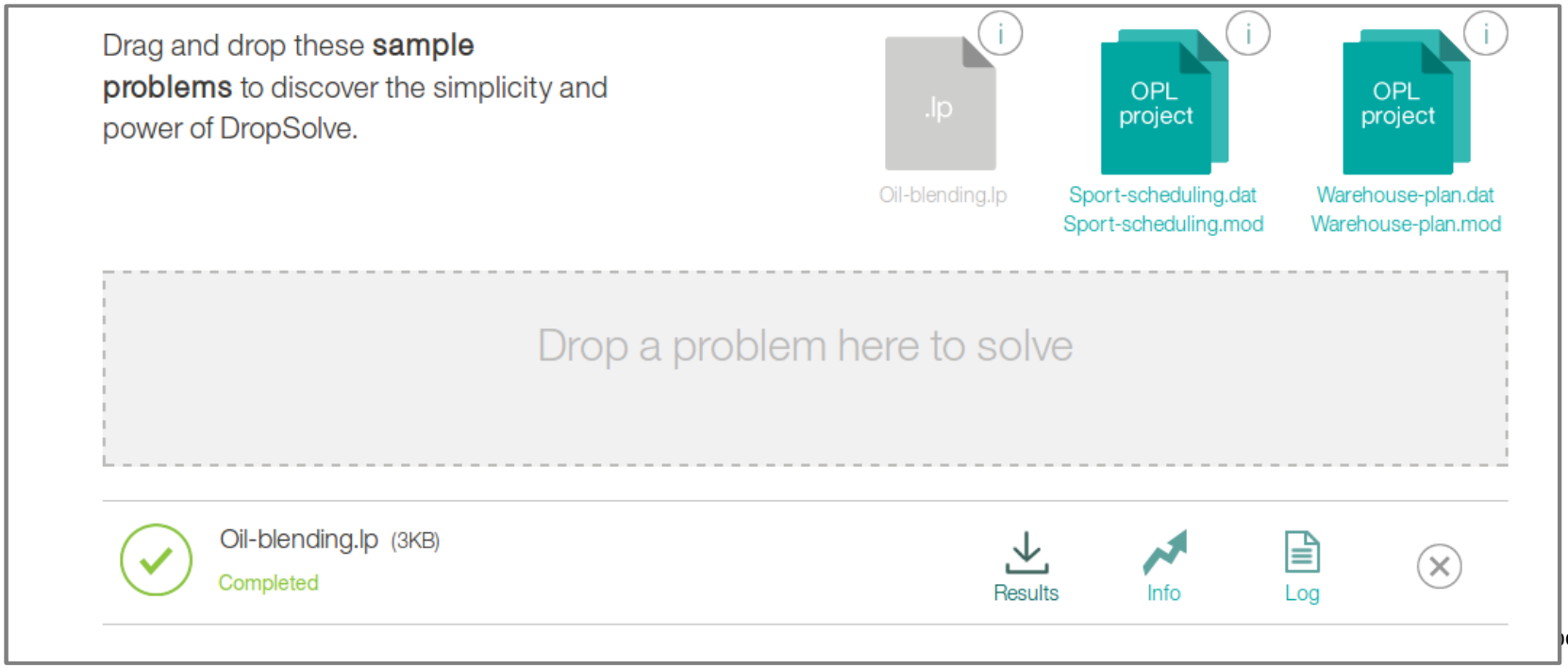


CPLEX in the cloud

Two ways to access CPLEX in the cloud

2. REST API

https://developer.ibm.com/docloud/
https://developer.ibm.com/docloud/docs/welcome/

Access the solve service via its REST API

CPLEX in the cloud

Two ways to access CPLEX in the cloud

2. REST API

https://developer.ibm.com/docloud/
https://developer.ibm.com/docloud/docs/welcome/

Access the solve service via its REST API

- ready-to-use clients provided for Java™ and Python, e.g.

```
JobExecutor executor = JobExecutorFactory.createDefault();
JobClient jobclient = JobClientFactory.createDefault(BASE_URL,
                                                     APIKEY_CLIENTID);
jobclient.newRequest().input(new File("model.mps"))
                      .output(new File("x.sol"))
                      .execute(executor).get();
```

- With any HTTP client (cURL, ...)

CPLEX in the cloud

Sign up for a free trial

www.ibm.com/software/analytics/docloud

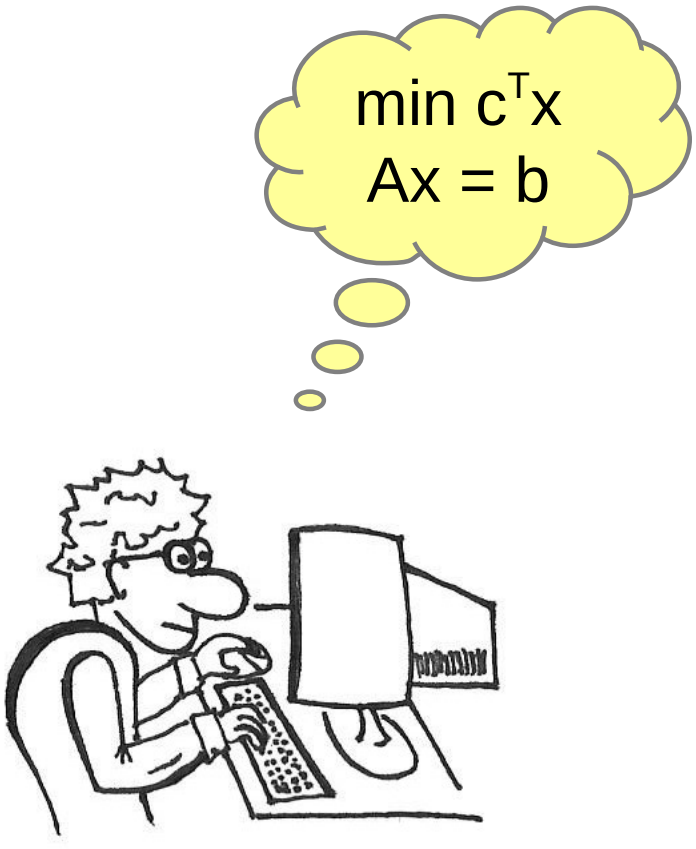
https://developer.ibm.com/docloud/

free CPLEX community edition

www.cplex.com

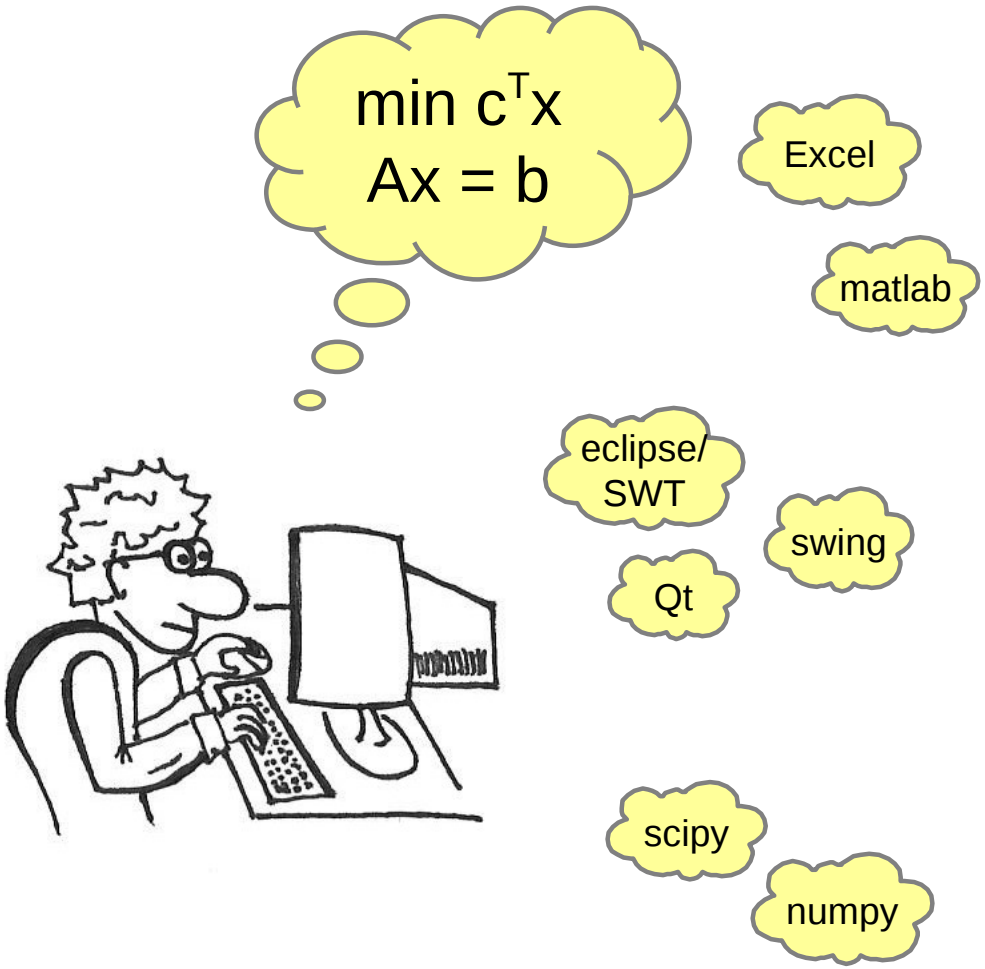
Exploiting existing software resources

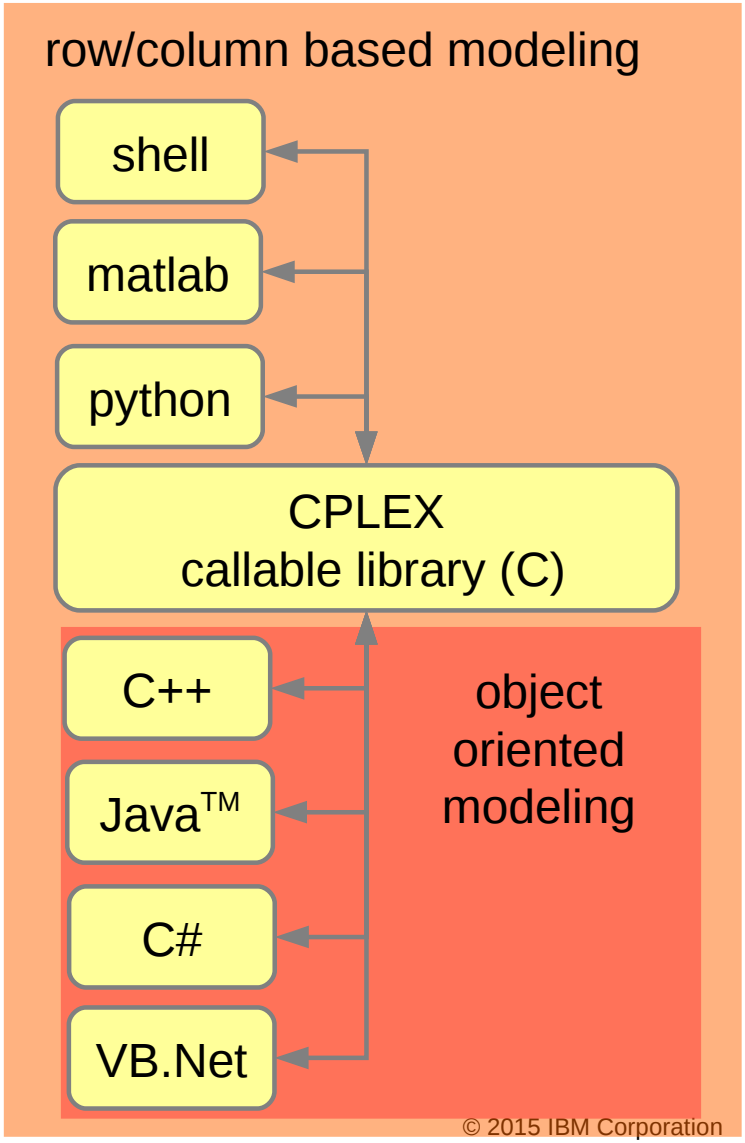
Interfacing with CPLEX



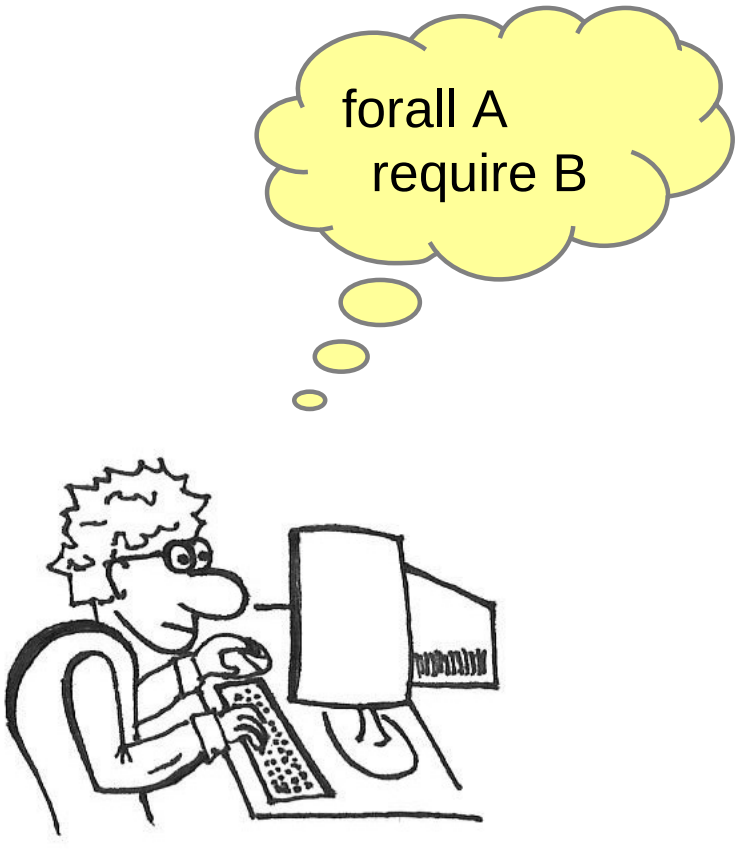
CPLEX
callable library (C)

Interfacing with CPLEX





Interfacing with CPLEX



Optimization **P**rogramming **L**anguage (OPL)

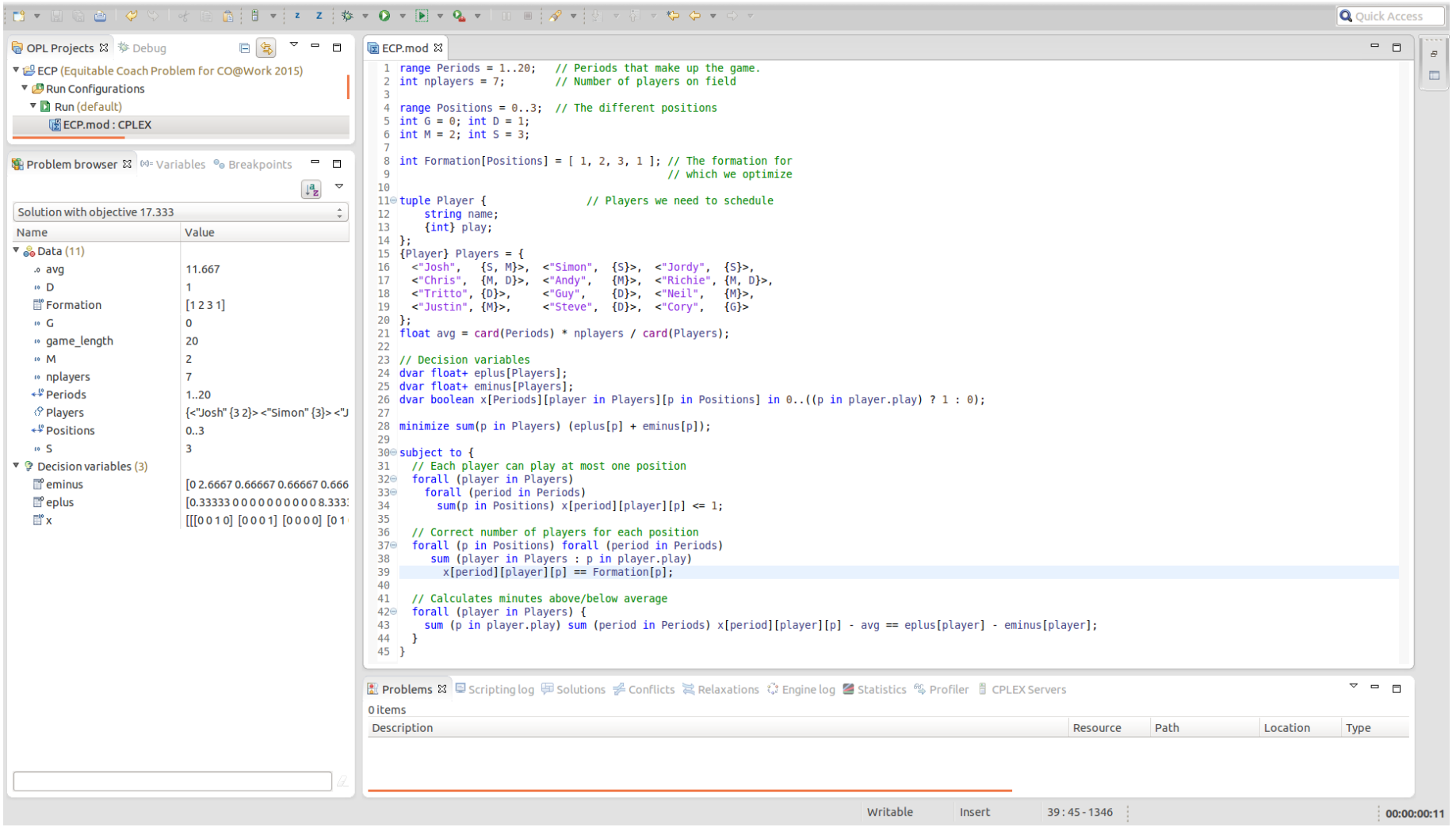
- write models in a more descriptive form
- write models in a more compact form
- faster prototyping, easier maintenance
- easier access to data (Excel, database, ...)

$\sum\sum x_{ijk} <= 1 \quad \text{for i in I}$

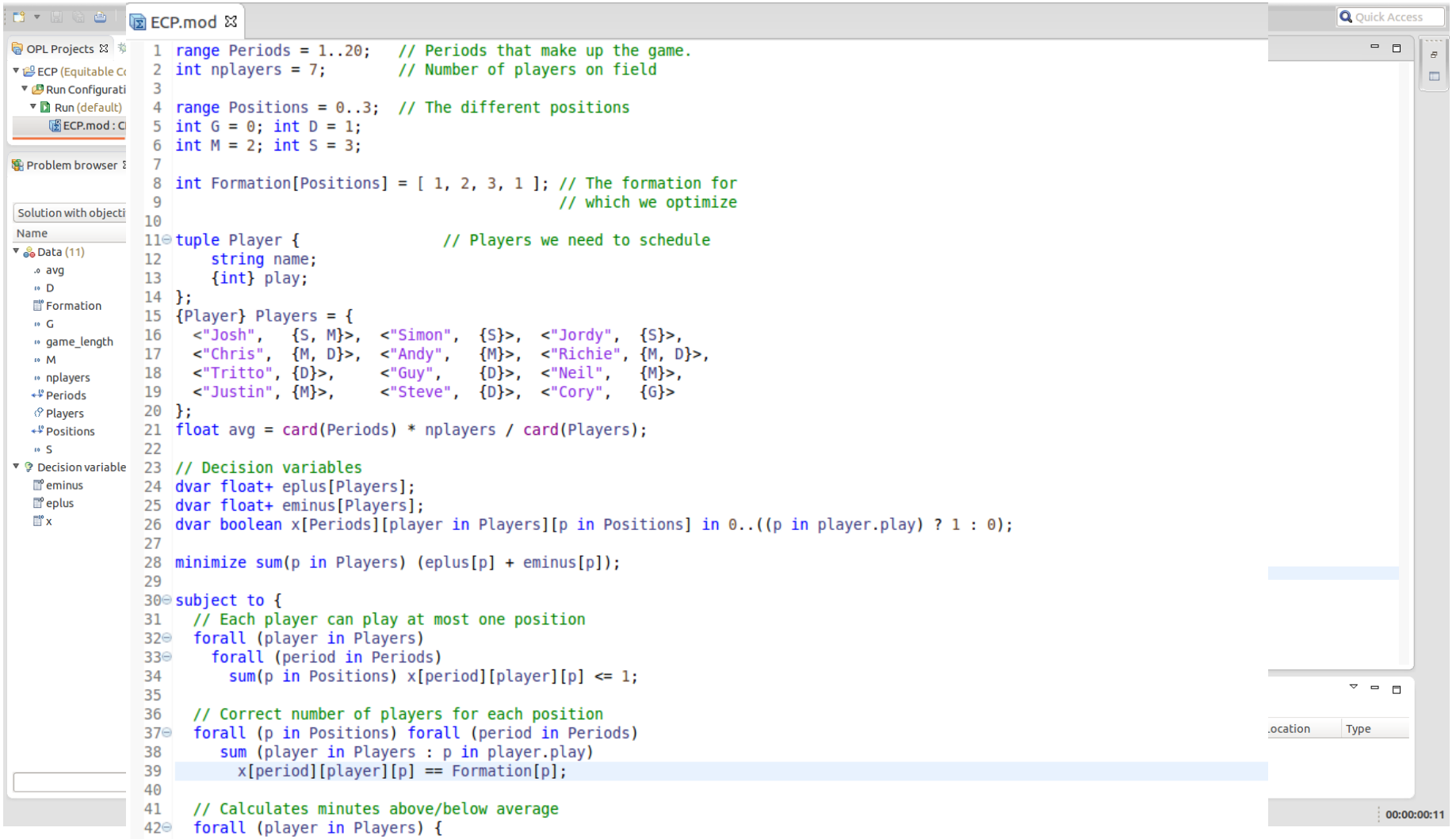
```
forall (i in I)
  sum (j in J)
    sum (k in K)
      x[i][j][k] <= 1;
```

- scriptable
- model editor
- IDE support (eclipse based)

Interfacing with CPLEX



Interfacing with CPLEX




```
1  range Periods = 1..20;   // Periods that make up the game.
2  int nplayers = 7;        // Number of players on field
3
4  range Positions = 0..3;  // The different positions
5  int G = 0; int D = 1;
6  int M = 2; int S = 3;
7
8  int Formation[Positions] = [ 1, 2, 3, 1 ]; // The formation for
9                                             // which we optimize
10
11 tuple Player {               // Players we need to schedule
12     string name;
13     {int} play;
14 };
15 {Player} Players = {
16   <"Josh",   {S, M}>,  <"Simon",  {S}>,  <"Jordy",  {S}>,
17   <"Chris",  {M, D}>,  <"Andy",   {M}>,  <"Richie", {M, D}>,
18   <"Tritto", {D}>,     <"Guy",    {D}>,  <"Neil",   {M}>,
19   <"Justin", {M}>,     <"Steve",  {D}>,  <"Cory",   {G}>
20 };
21 float avg = card(Periods) * nplayers / card(Players);
22
23 // Decision variables
24 dvar float+ eplus[Players];
25 dvar float+ eminus[Players];
26 dvar boolean x[Periods][player in Players][p in Positions] in 0..((p in player.play) ? 1 : 0);
27
28 minimize sum(p in Players) (eplus[p] + eminus[p]);
29
30 subject to {
31   // Each player can play at most one position
32   forall (player in Players)
33     forall (period in Periods)
34       sum(p in Positions) x[period][player][p] <= 1;
35
36   // Correct number of players for each position
37   forall (p in Positions) forall (period in Periods)
38      sum (player in Players : p in player.play)
39        x[period][player][p] == Formation[p];
40
41   // Calculates minutes above/below average
42   forall (player in Players) {
43     sum (p in player.play) sum (period in Periods) x[period][player][p] - avg == eplus[player] - eminus[player];
44   }
45 }
```

docplex

Most recent addition

- https://pypi.python.org/pypi/docplex
- pure Python modeling API (no native code)
- open source (`pypi`, `github`)
- prepared to connect to local or cloud CPLEX

→ write your model in Python
→ hook up with the whole Python software ecosystem

docplex

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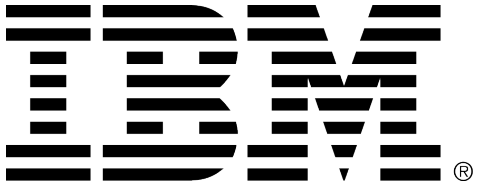
- prepared to connect to local or cloud CPLEX

→ write your model in Python
→ hook up with the whole Python software ecosystem

For example

- Equitable Coach Problem
- list of players from the internet (web service)
- graphical display of solution

→ use `iPython`/`Jupyter` notebook and Python libraries



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