Basic Concepts of Constraint Integer Programming

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MODAL
Mathematical Optimization and Data Analysis Laboratories

Outline

SCIP – Solving Constraint Integer Programs

Outline

SCIP – Solving Constraint Integer Programs

4 methodologies in optimization

An integrated method

SCIP: Solving Constraint Integer Programs

The Solving Process of SCIP

Mixed-Integer Linear Programming: MIP

Problem class

$$\begin{aligned} \min \quad & c^T x \\ s.t. \quad & Ax \leq b \\ & (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C \end{aligned}$$

- continuous and integer variables
- linear objective function
- linear constraints

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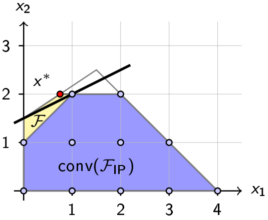
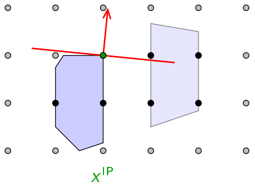


$\min\{c^T x : x \in \text{conv}(\mathcal{F}_{\text{IP}})\}$

Methods

- LP relaxation
- cutting planes
- branch-and-bound



More details: Tobias Achterberg and Bob Bixby, Oct. 5

Satisfiability Solving: SAT

Problem class

$$
\begin{aligned}
&\exists x \in \{0,1\}^n \\
s.t.\ &\bigvee_{i \in \mathcal{Y}_k} x_i \vee \bigvee_{i \in \mathcal{N}_k} \neg x_i \text{ for } k = 1, \dots, m, \\
&\qquad\qquad\qquad \mathcal{Y}_k, \mathcal{N}_k \subseteq \{1, \dots, n\}
\end{aligned}
$$

- binary variables and their negation
- (linear) clauses
- feasible assignment?

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Methods

- unit propagation

$$x_1 \vee x_3$$
$$x_1 \vee x_2 \vee x_4$$
$$\neg x_3$$

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$$\begin{array}{r} x_1 \vee x_3 \\ x_1 \vee x_2 \vee x_4 \\ \neg x_3 \end{array} \Rightarrow \begin{array}{r} x_1 \\ x_1 \vee x_2 \vee x_4 \\ \neg x_3 \end{array}$$

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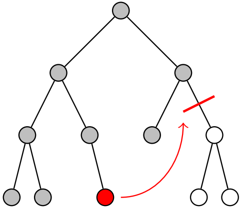

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- tree search
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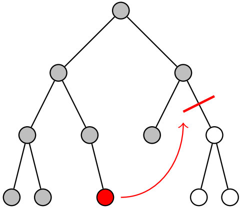

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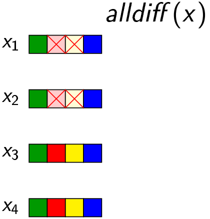
Constraint Programming: CP

Problem class

$$\begin{aligned} \min \quad & c(x) \\ s.t. \quad & x \in F_k \text{ for } k = 1, \dots, m \\ & (x_I, x_N) \in \mathbb{Z}^I \times X \end{aligned}$$

- arbitrary variable domains (usually finite: FD)
- arbitrary constraints
- arbitrary objective function



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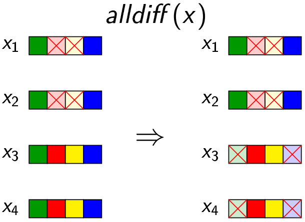

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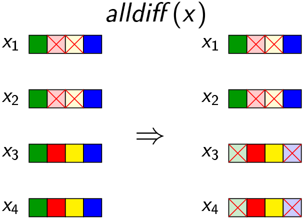
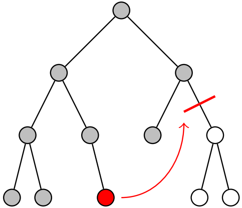


Methods

- constraint propagation
- tree search
- conflict analysis/no-good learning



Mixed-Integer Nonlinear Programming: MINLP

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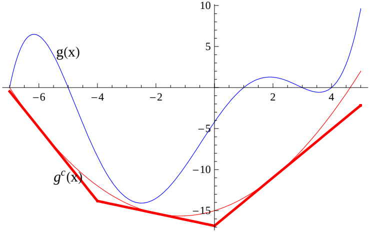

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- nonlinear objective function
- nonlinear constraint functions

More details: Ralf Lenz, Jesco Humpola, Pierre Bonami, Oct. 1 & 8

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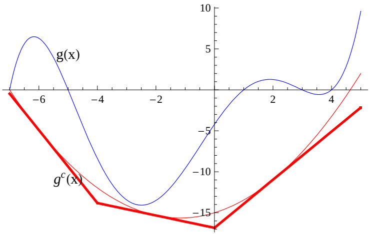

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- outer approximation
- convex relaxation
- bound tightening

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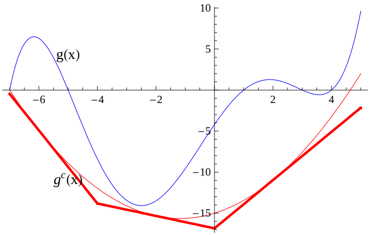

Methods

- outer approximation
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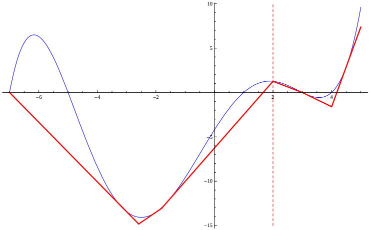
- bound tightening
- spatial branch-and-bound



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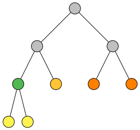
SCIP – Solving Constraint Integer Programs

Integrating CP, MIP, SAT, and MINLP

Search: in MIP



in SAT

in CP

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Integrating CP, MIP, SAT, and MINLP

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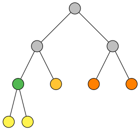

in MIP

\+ LP relaxation

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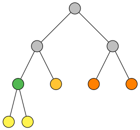

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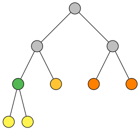

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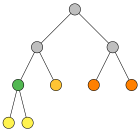

in MIP

+ LP relaxation
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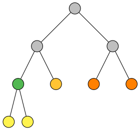

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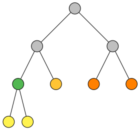

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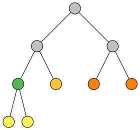

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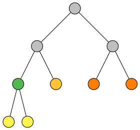

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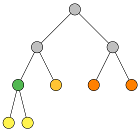

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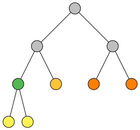

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High-level and low-level integration

▷ interaction of different algorithms

▷ combination of algorithmic techniques

e.g., Althaus, Bockmayr, Elf, Jünger, Kasper, Mehlhorn 2002;
Hooker 2007;
Achterberg 2007;
Berthold, Heinz, Vigerske 2010;
Vigerske 2013;
...

Constraint Integer Programming: CIP

Low-level integration of solving techniques into one algorithm

- CP+SAT+MIP [Achterberg 2007, 2009]
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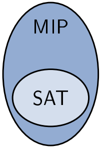

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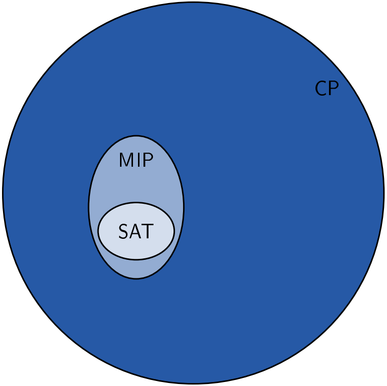

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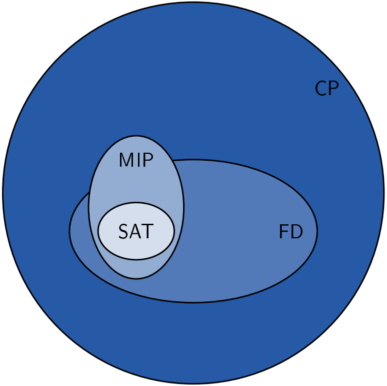

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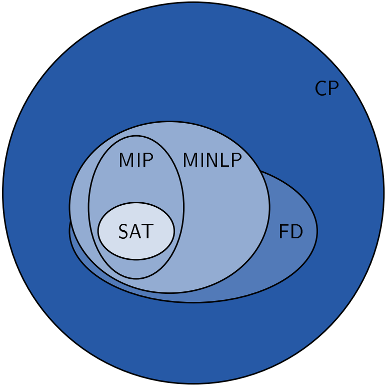

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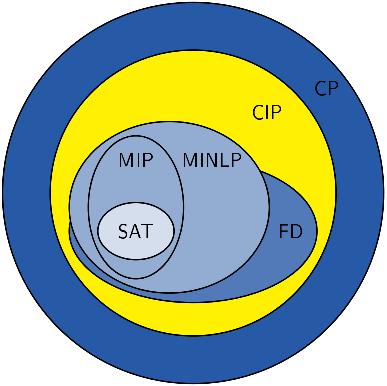
- condition: after fixing all integers, CIP can be solved as an LP or NLP



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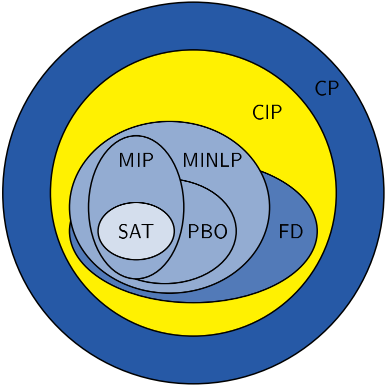

Example: Pseudoboolean Optimization

The AND constraint

- $y = \prod_{i \in J} x_i$
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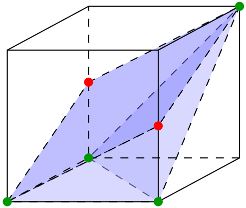
- $y = 1 \Rightarrow x_i = 1 \;\forall i \in J$
- $y = 0 \wedge \exists k : x_i = 1 \;\forall i \in J \setminus \{k\} \Rightarrow x_k = 0$

Linearization of $y = x_1 \wedge \cdots \wedge x_n$

$$\sum_{i=1}^{n} x_i - y \leq n - 1$$

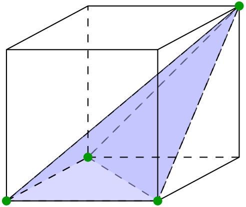
$$\sum_{i=1}^{n} x_i - n\,y \geq 0$$



- 2 constraints
- contains fractional vertices

$$\sum_{i=1}^{n} x_i - y \leq n - 1$$

$$x_i - y \geq 0 \quad \text{for } i = 1, \ldots, n$$



- n + 1 constraints
- only integer vertices

Algorithms

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- good: fast subproblem processing
- bad: LP relaxation has no knowledge about the nonlinear structure

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SCIP can implement all strategies

default: propagation + weak linearization + separation

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An integrated method

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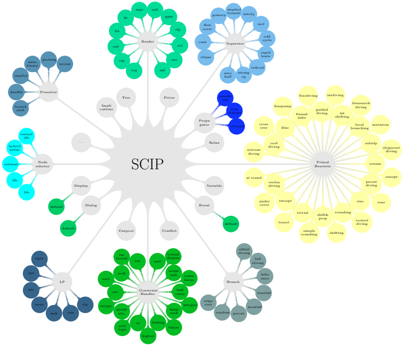
The Solving Process of SCIP

Why use SCIP?

SCIP: Solving Constraint Integer Programs . . .

- provides a full-scale MIP and MINLP solver



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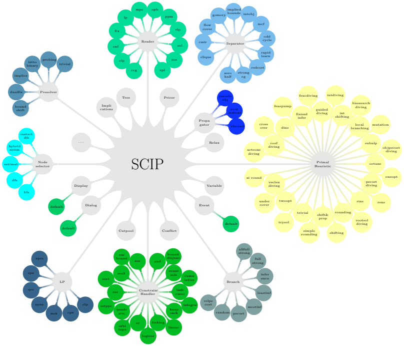

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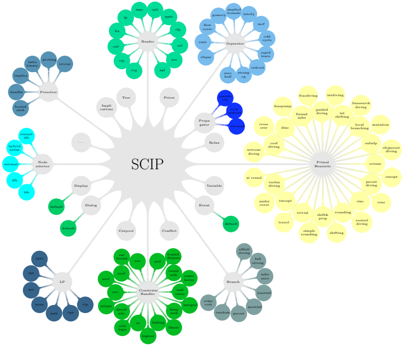

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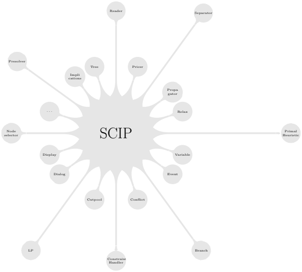

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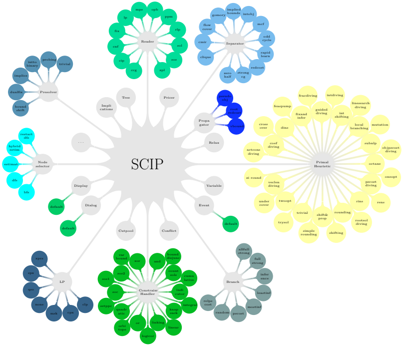

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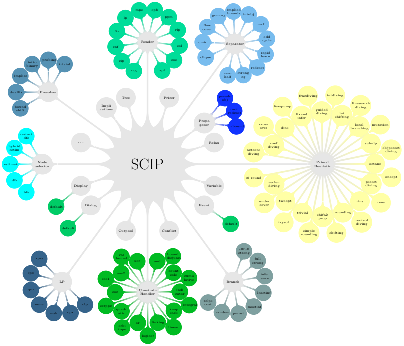

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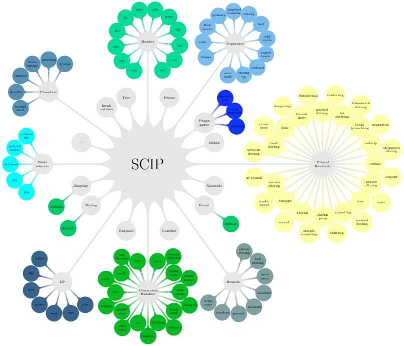

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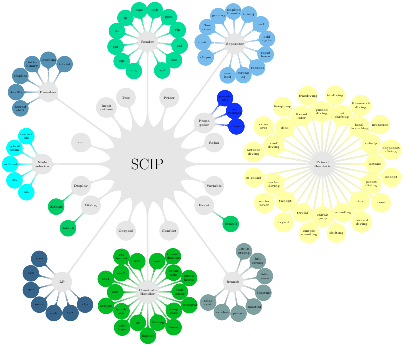
- ▷ is open: available in source code



Plugin-based design

SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- ...

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Plugins

- external callback objects
- interact with the framework through a very detailed interface

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- SCIP knows for each plugin type:
  - the number of available plugins
  - priority defining the calling order (usually)
- SCIP does not know any structure behind a plugin

$\Rightarrow$ plugins are black boxes for the SCIP core

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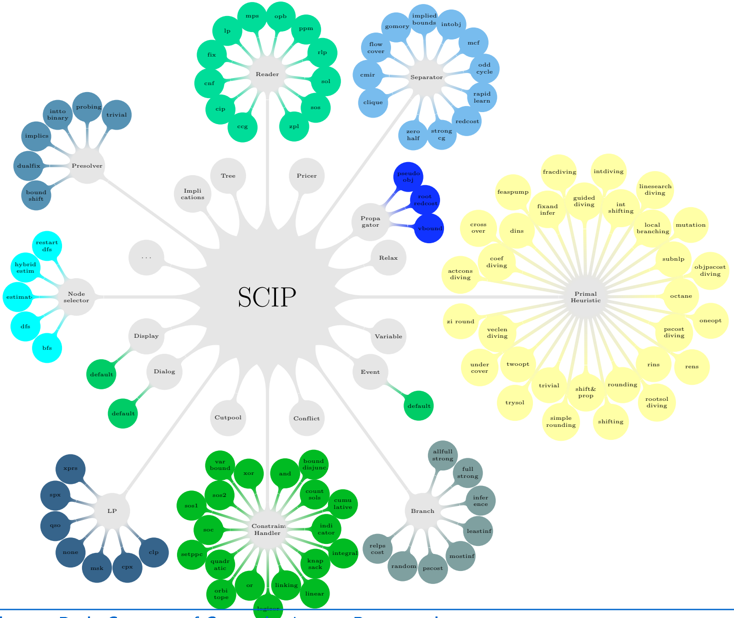
$\Rightarrow$ plugins are black boxes for the SCIP core

$\Rightarrow$ Very flexible branch-and-bound based search algorithm

Types of Plugins

- Constraint handler: assures feasibility, strengthens formulation
- Separator: adds cuts, improves dual bound
- Pricer: allows dynamic generation of variables
- Heuristic: searches solutions, improves primal bound
- Branching rule: how to divide the problem?
- Node selection: which subproblem should be regarded next?
- Presolver: simplifies the problem in advance, strengthens structure
- Propagator: simplifies problem, improves dual bound locally
- Reader: reads problems from different formats
- Event handler: catches events (e.g., bound changes, new solutions)
- Display: allows modification of output
- ...

A closer look: branching rules



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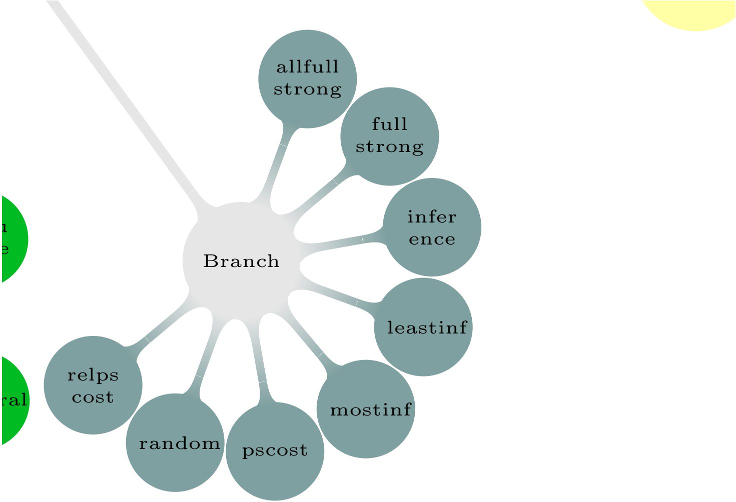


What does SCIP know about branching rules?

- SCIP knows the number of available branching rules
- each branching rule has a priority
- SCIP calls the branching rule in decreasing order of priority
- the interface defines the possible results of a call:
  - branched
  - reduced domains
  - added constraints
  - detected cutoff
  - did not run

Outline

SCIP – Solving Constraint Integer Programs

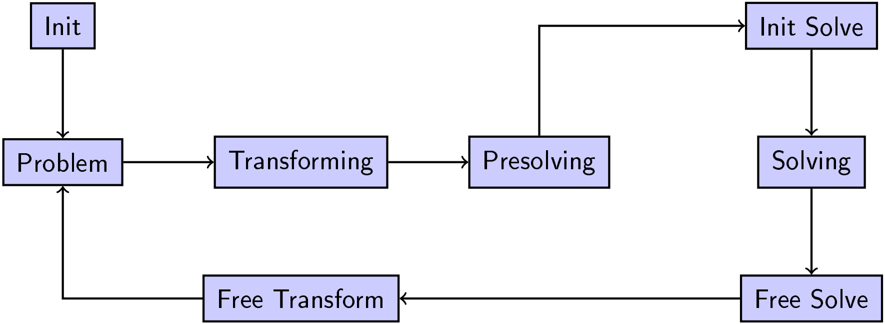
4 methodologies in optimization

An integrated method

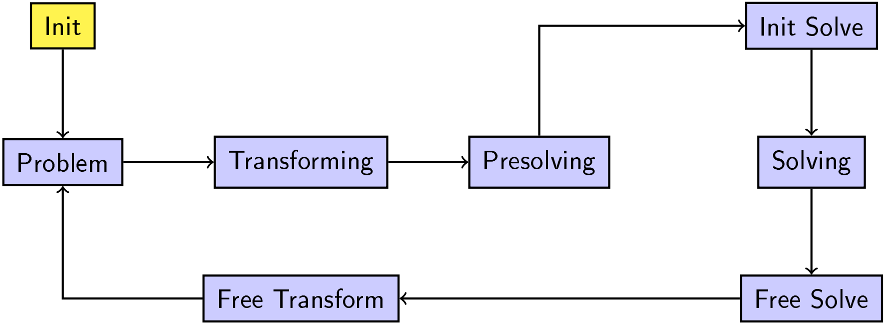
SCIP: Solving Constraint Integer Programs

The Solving Process of SCIP

Operational Stages

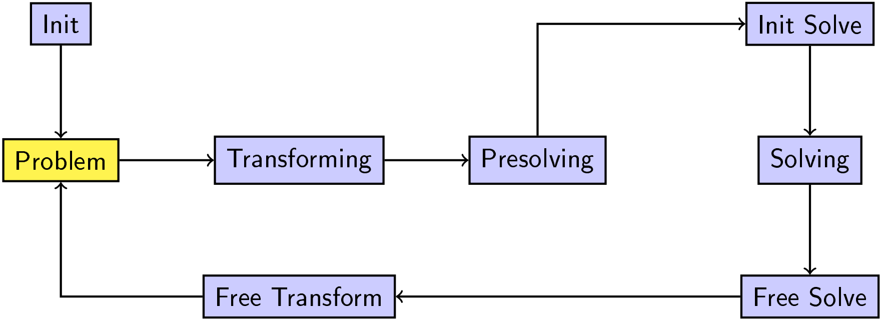


Operational Stages



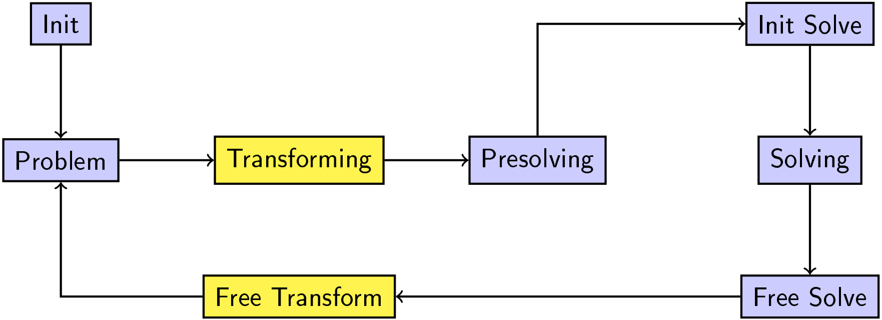
- Basic data structures are allocated and initialized.
- User includes required plugins (or just takes default plugins).

Problem Specification



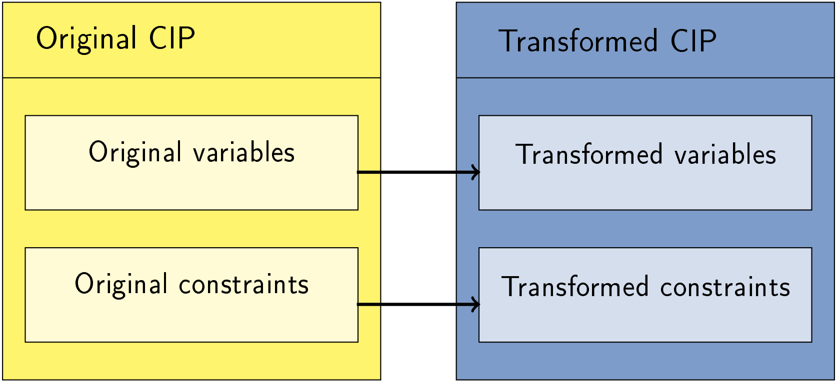
- User creates and modifies the original problem instance.
- Problem creation is usually done in file readers.

Transformation



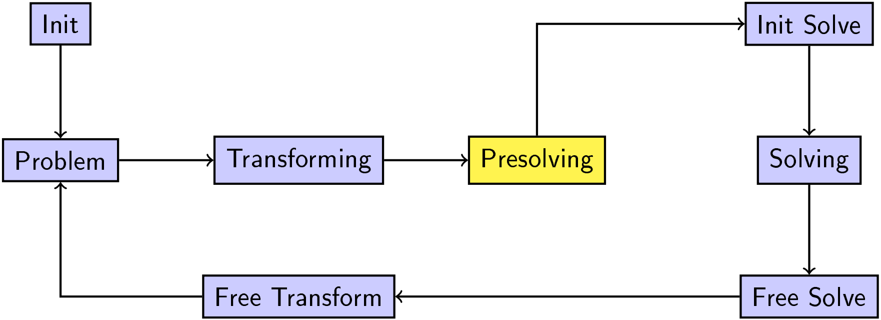
- Creates a working copy of the original problem.

Original and Transformed Problem

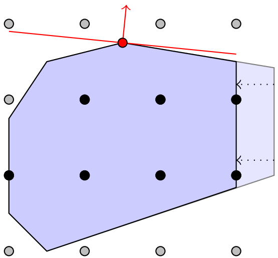


- data is copied into separate memory area
- presolving and solving operate on transformed problem
- original data can only be modified in problem modification stage

Presolving


Init
Problem
Transforming
Presolving
Init Solve
Solving
Free Solve
Free Transform

Presolving



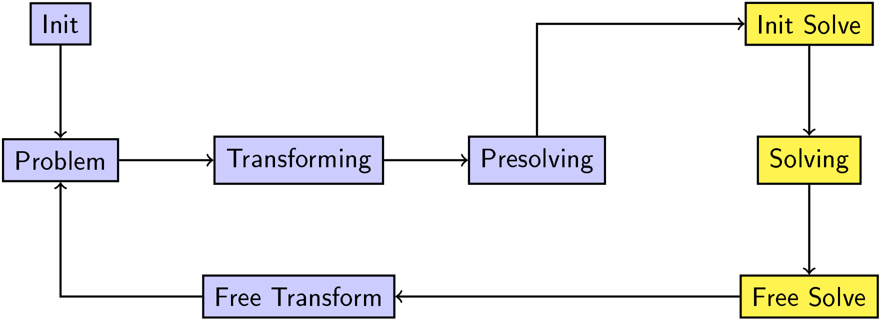
Task

- reduce size of model by removing irrelevant information
- strengthen LP relaxation by exploiting integrality information
- make the LP relaxation numerically more stable
- extract useful information

Primal Reductions:
- based on feasibility reasoning
- no feasible solution is cut off

Dual Reductions:
- consider objective function
- at least one optimal solution remains

Solving


Init
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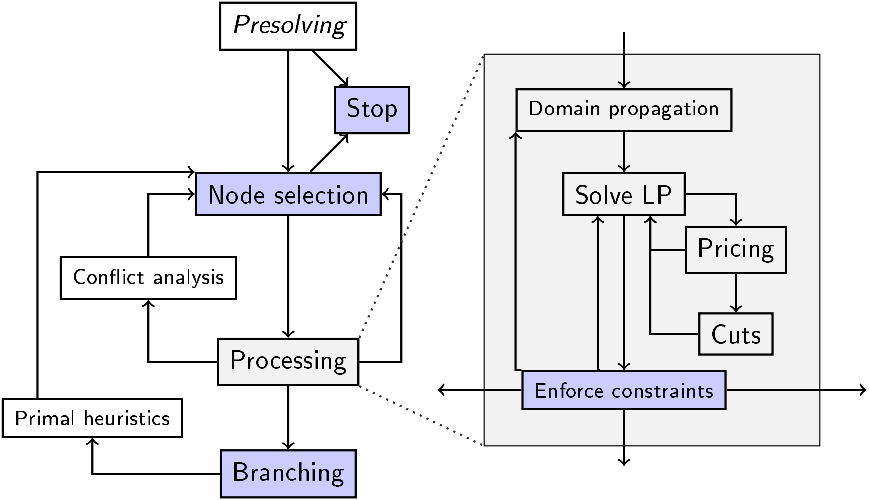
Flow Chart SCIP



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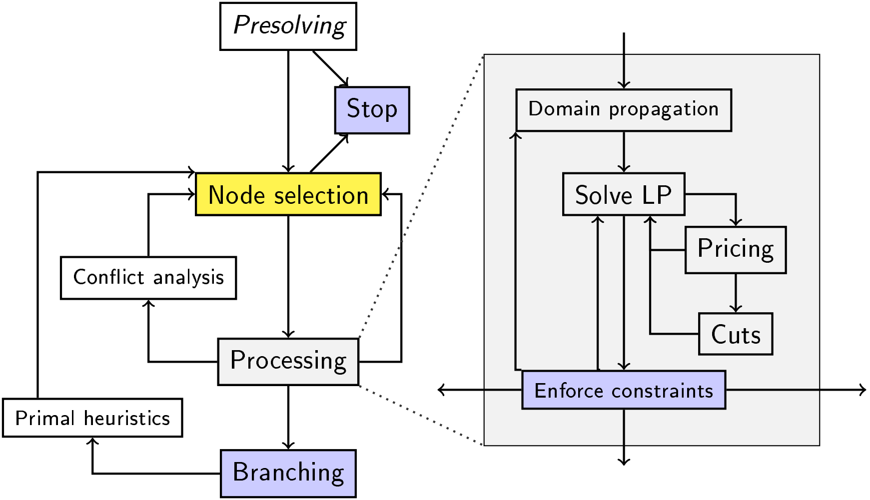
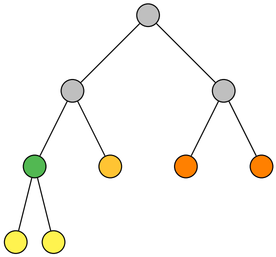


Node Selection



Task

- improve primal bound
- keep comp. effort small
- improve global dual bound

Techniques

- basic rules
  - depth first search (DFS)
    $\rightarrow$ early feasible solutions
  - best bound search (BBS)
    $\rightarrow$ improve dual bound
  - best estimate search (BES)
    $\rightarrow$ improve primal bound
- combinations
  - BBS or BES with plunging
  - hybrid BES/BBS
  - interleaved BES/BBS

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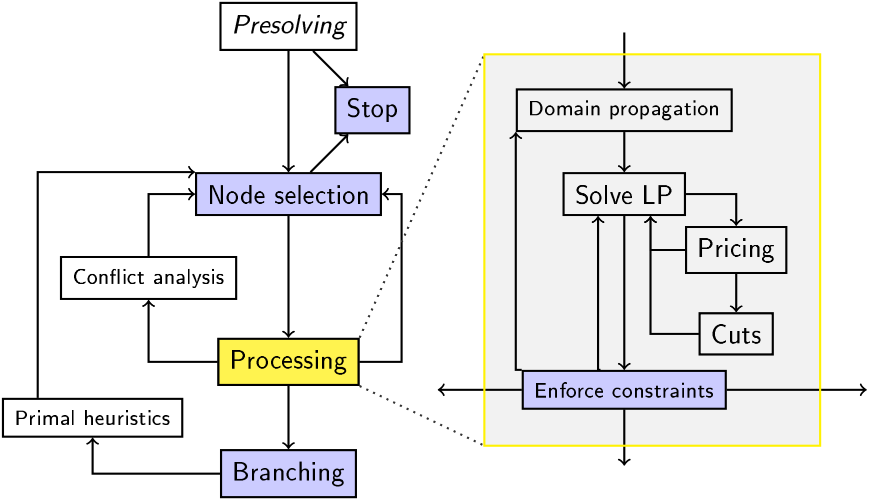

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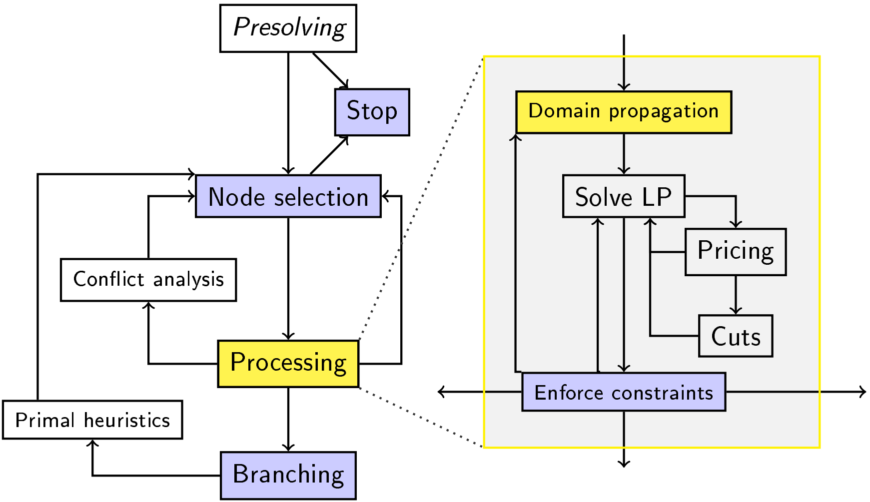

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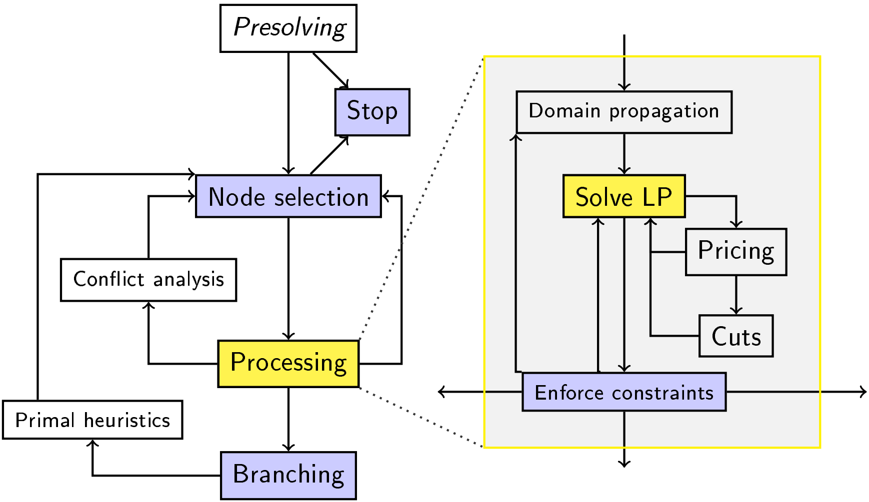
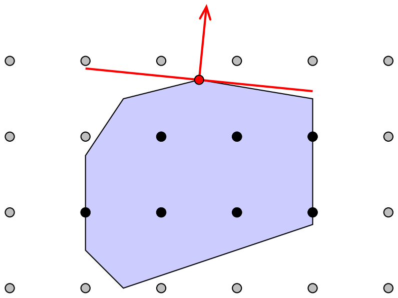


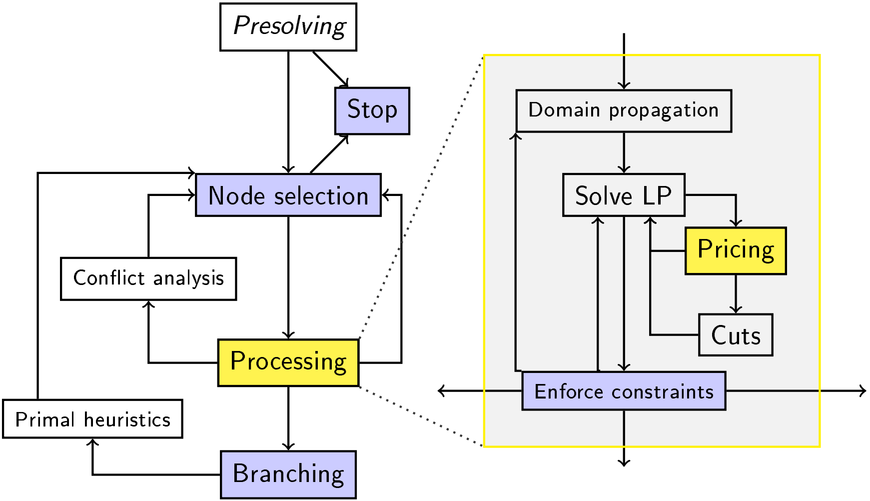
LP Solving

- LP solver is a black box
- interface to different LP solvers: SoPlex, CPLEX, XPress, Gurobi, CLP, ...
- primal/dual simplex
- barrier with/without crossover



- double-check feasibility
- check condition number
- address numerical troubles by changing parameters: scaling, tolerances, solving from scratch, other simplex

Flow Chart SCIP



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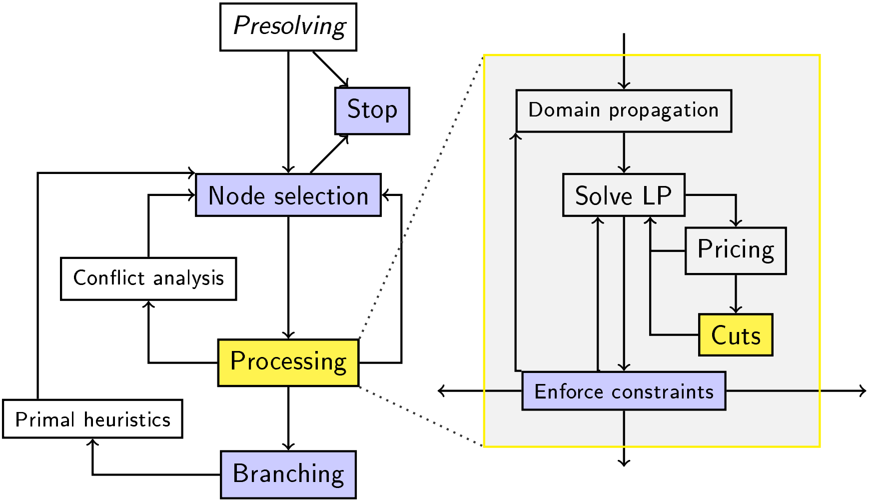

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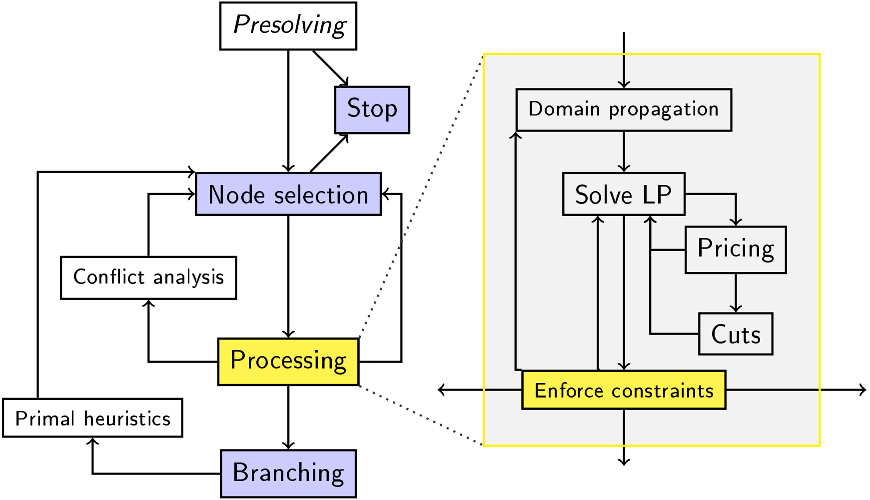

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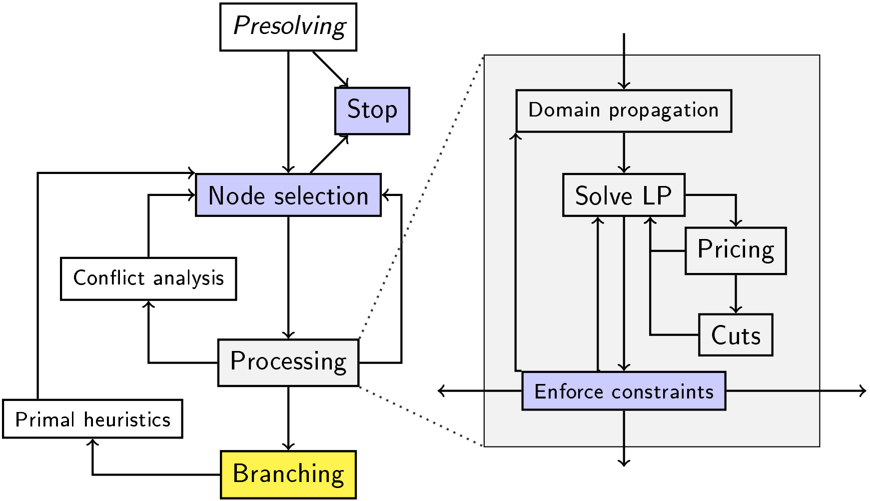

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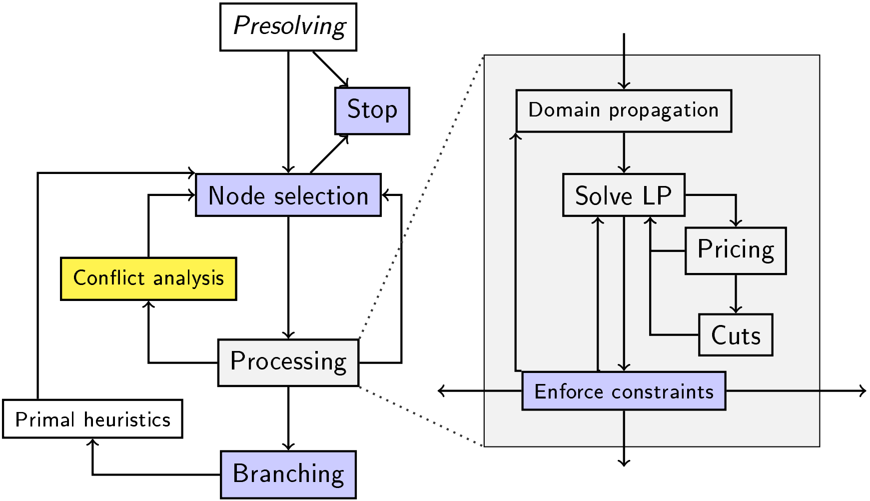
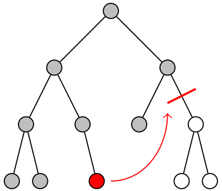


Conflict Analysis



Task

- Analyze infeasibility
- Derive valid constraints
- Help to prune other nodes

Techniques

- Analyze:
  - Propagation conflicts
  - Infeasible LPs
  - Bound-exceeding LPs
  - Strong branching conflicts
- Detection:
  - Cut in conflict graph
  - LP: Dual ray heuristic
- Use conflicts:
  - Only for propagation
  - As cutting planes

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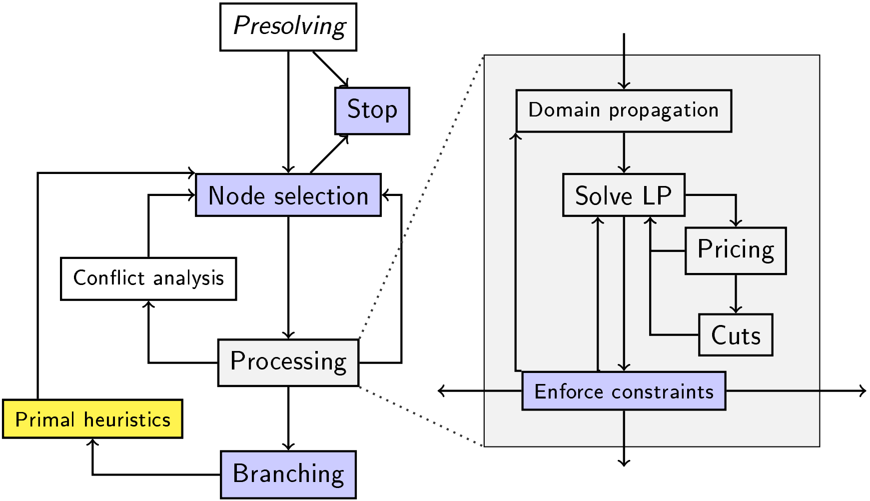

Summary

Take-away messages

- optimization paradigms: CP, SAT, MIP, MINLP
- CIP: an algorithmic, solution-driven integration
- SCIP: a flexible tool for computational research in optimization

Next

The most powerful plugin: constraint handlers