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Life Cycle Optimization for Civil Engineering

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In Cooperation with Bilfinger SE
October 9th, 2015

Infrastructures

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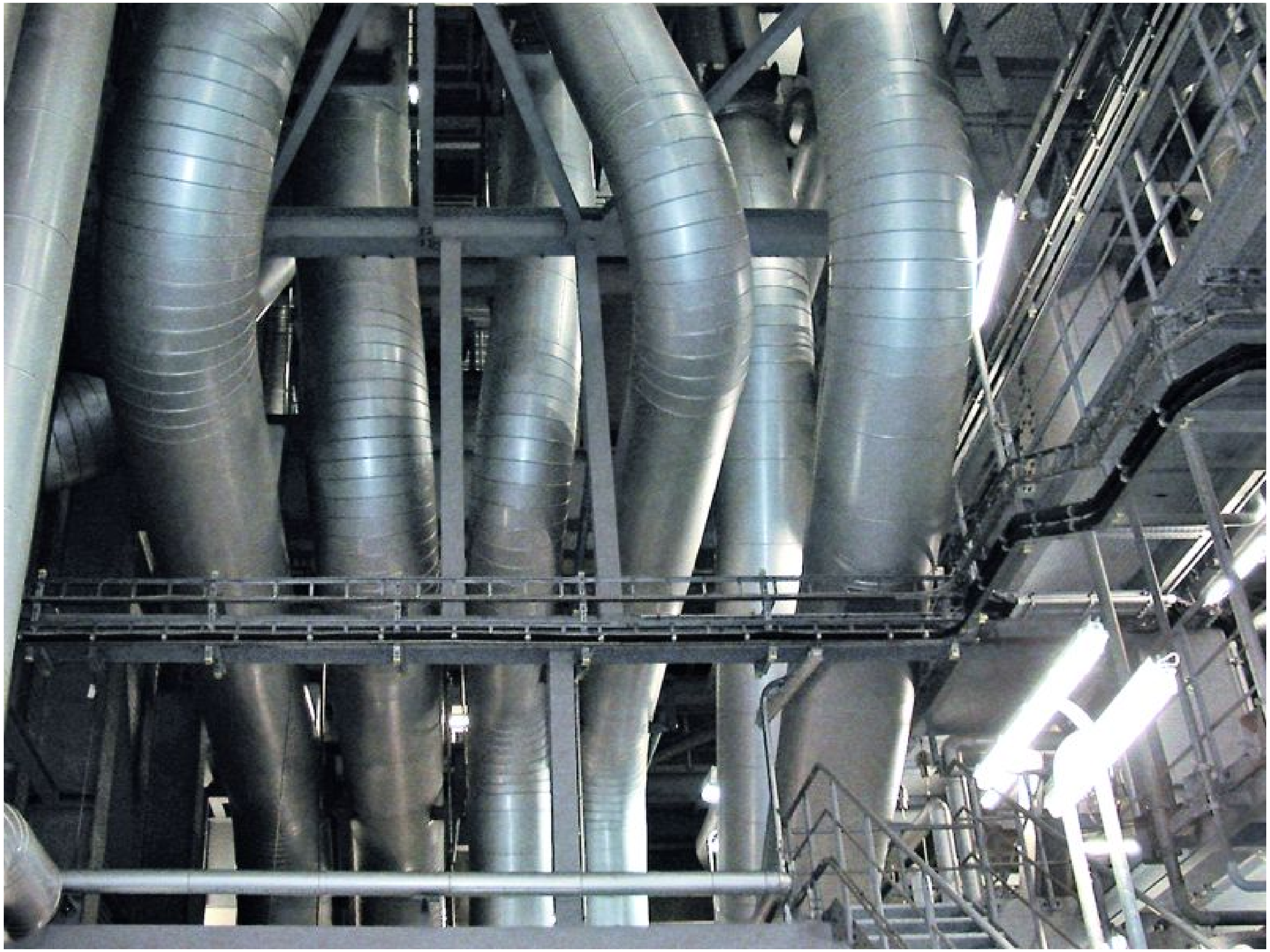
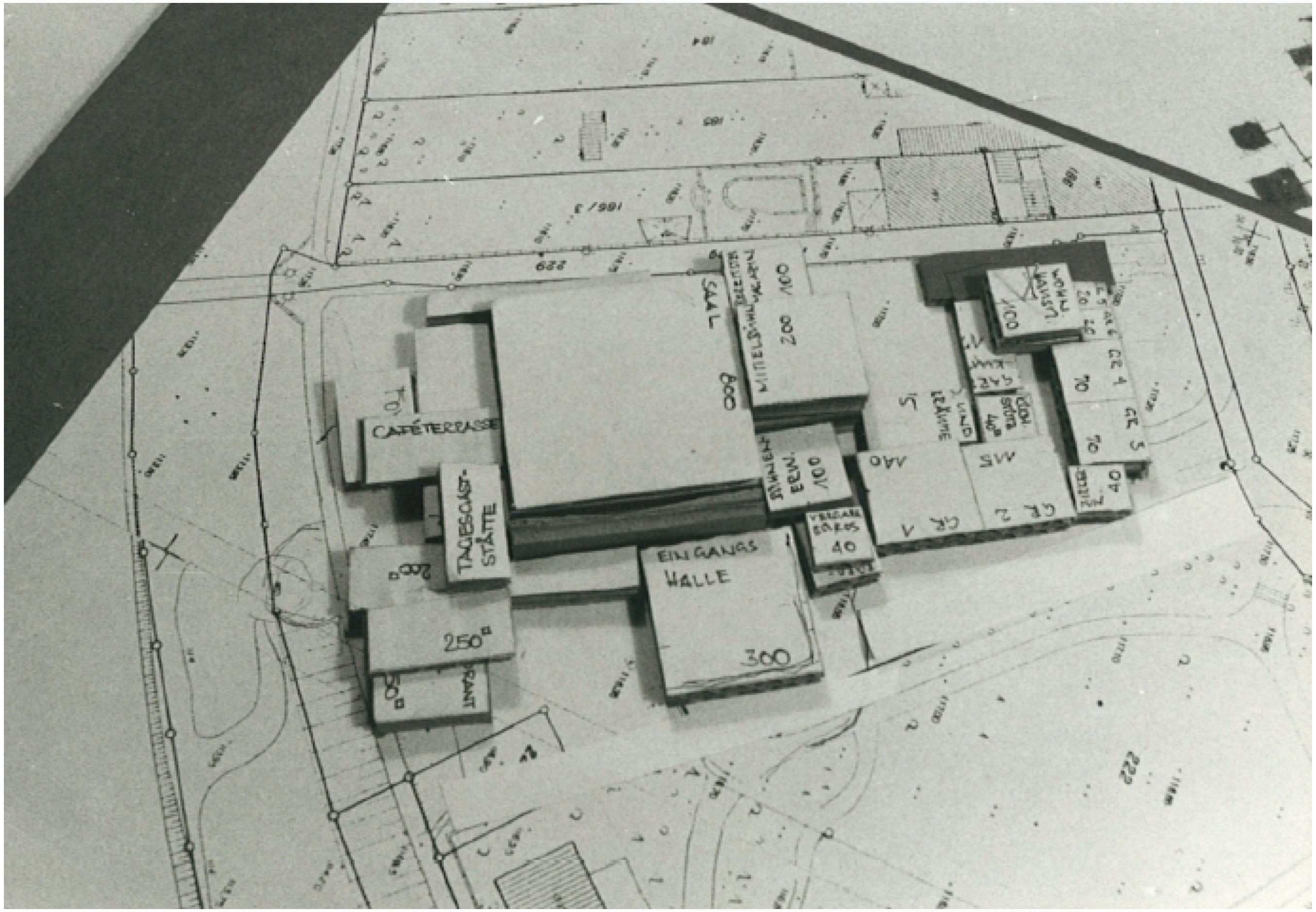
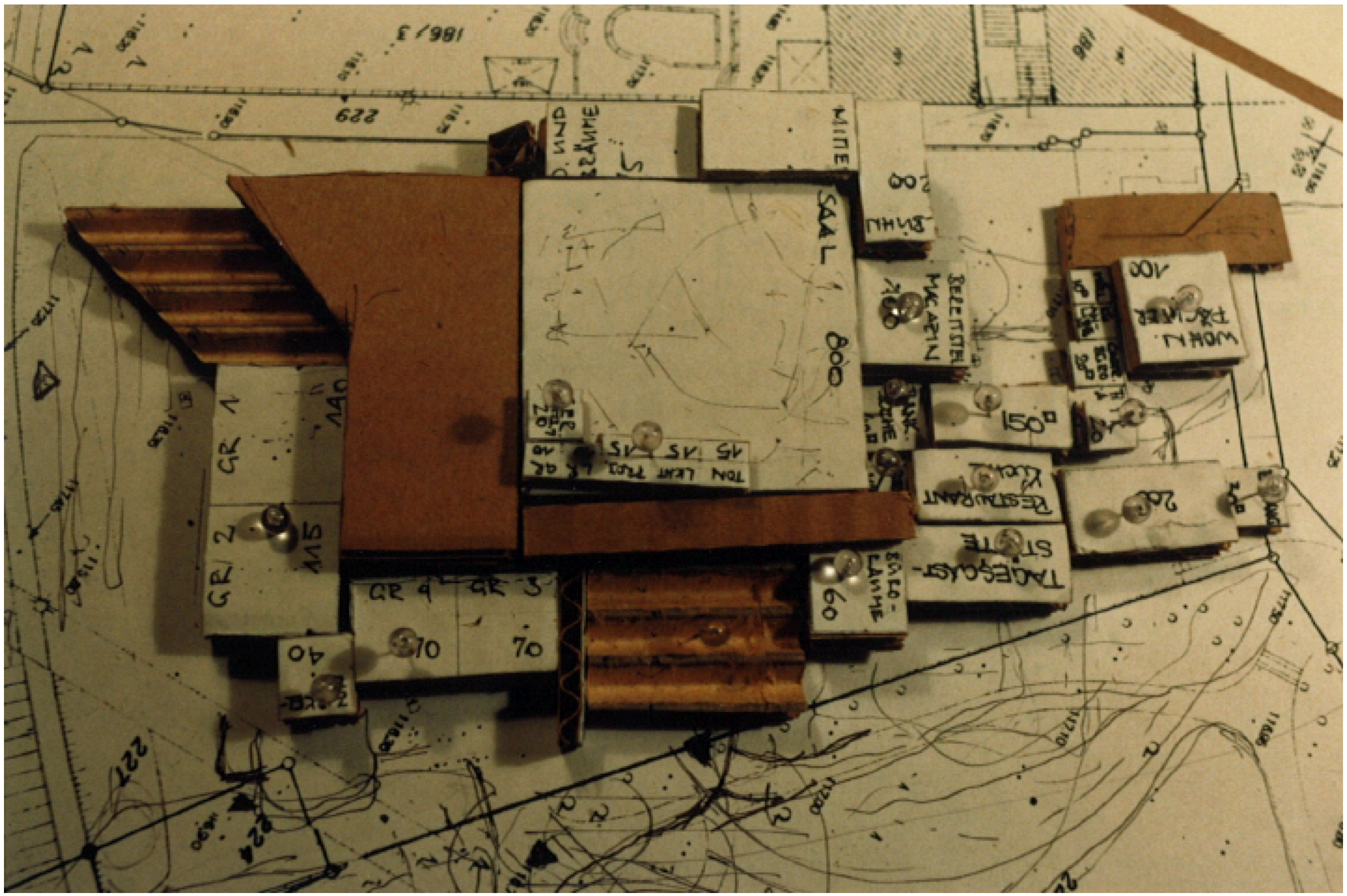


Infrastructures



Infrastructures



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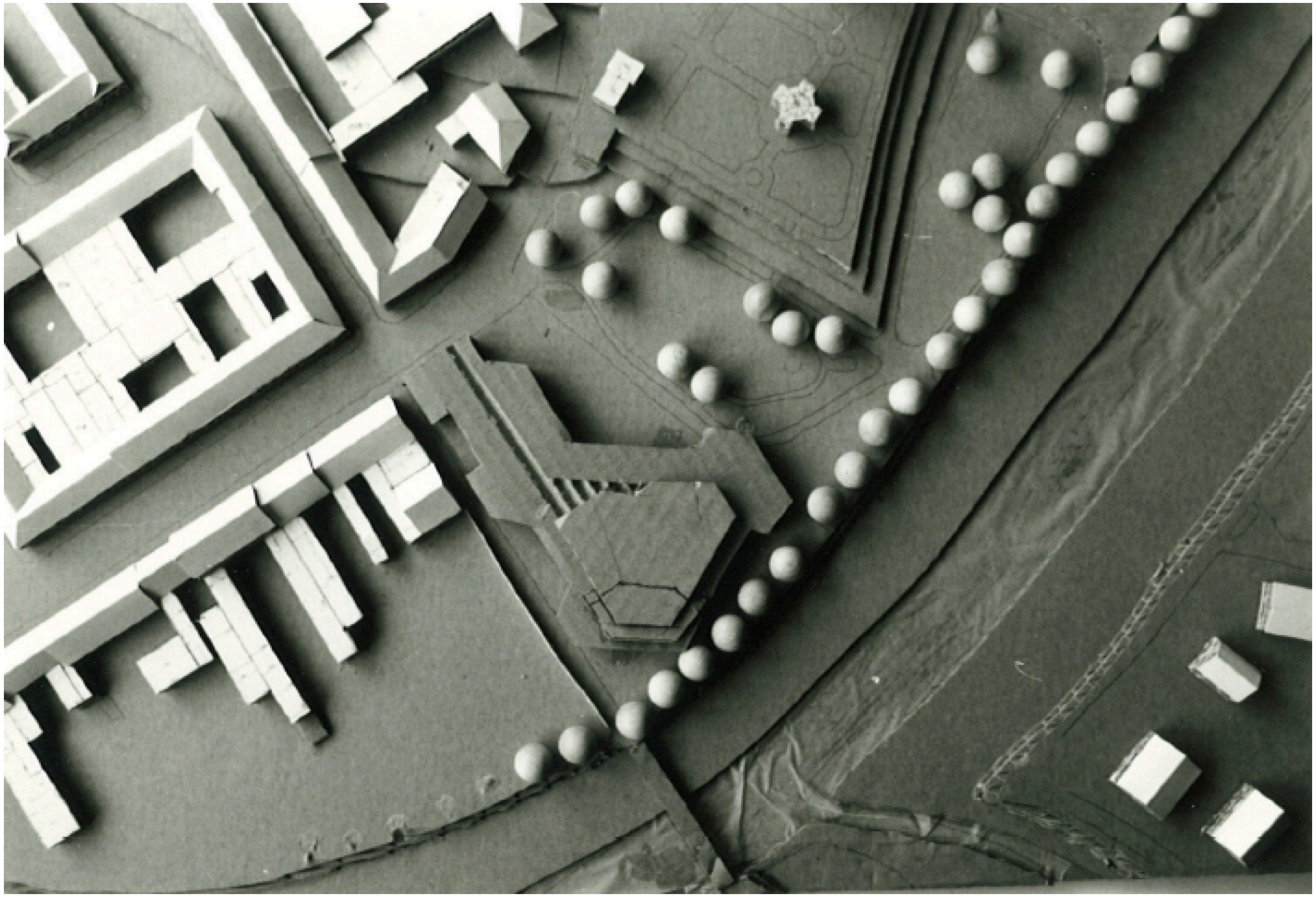
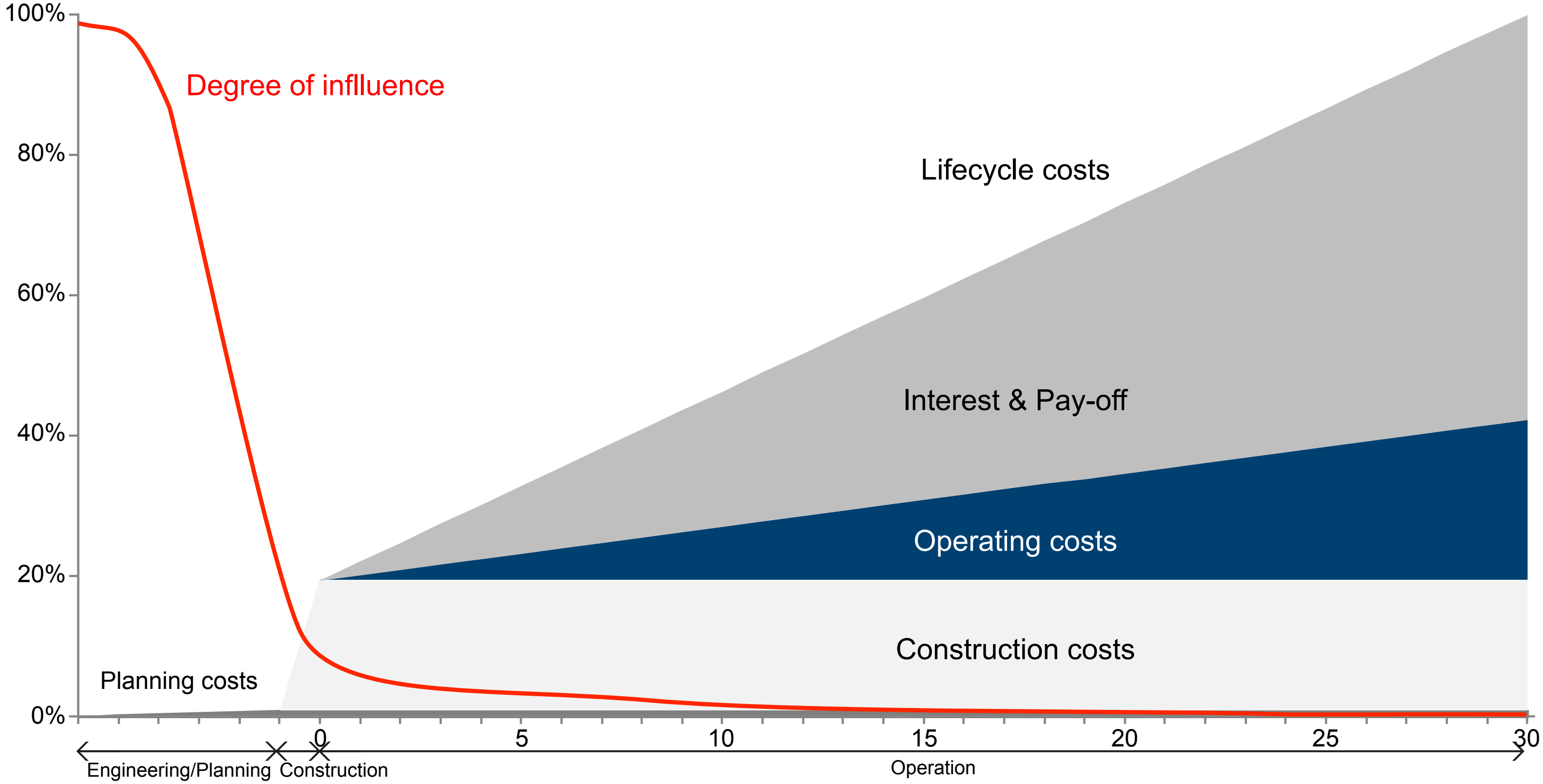


Importance of the planning phase



Complexity of the planning process

- Many degrees of freedom:
  - Geometry
  - Dimensions
  - Materials
  - ...
- Big leverage of the decisions
- Many side constraints and dependencies
- Current trends to extend the scope of planning:
  - Sustainability
  - Life cycle
  - ...

Challenges in the planning phase

Tasks

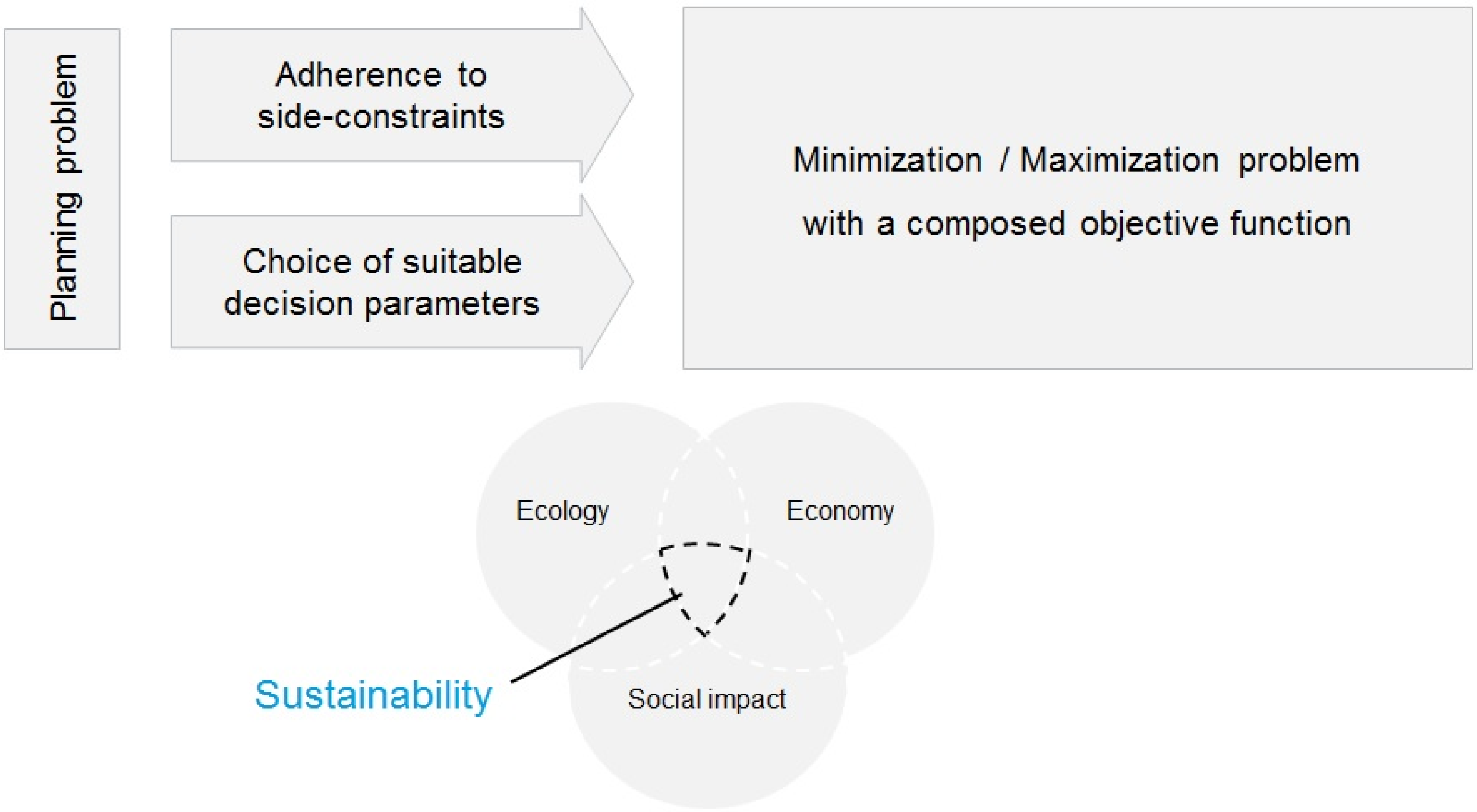
- Transformation of requirements into concrete planning results
- Optimization of
  - Design
  - Functionality
  - Equipment
  - Costs
  - Appointments

  over the life cycle
- Setting the course for optimal construction and operation

Difficulties

- Many specialized planners with different know-how and focus
- High complexity:
  - Limited amount of investigated variants
  - Successive decision-making
  - Missing feedback loops
  - Optimization of subtasks
- Limited time
- Changing side constraints

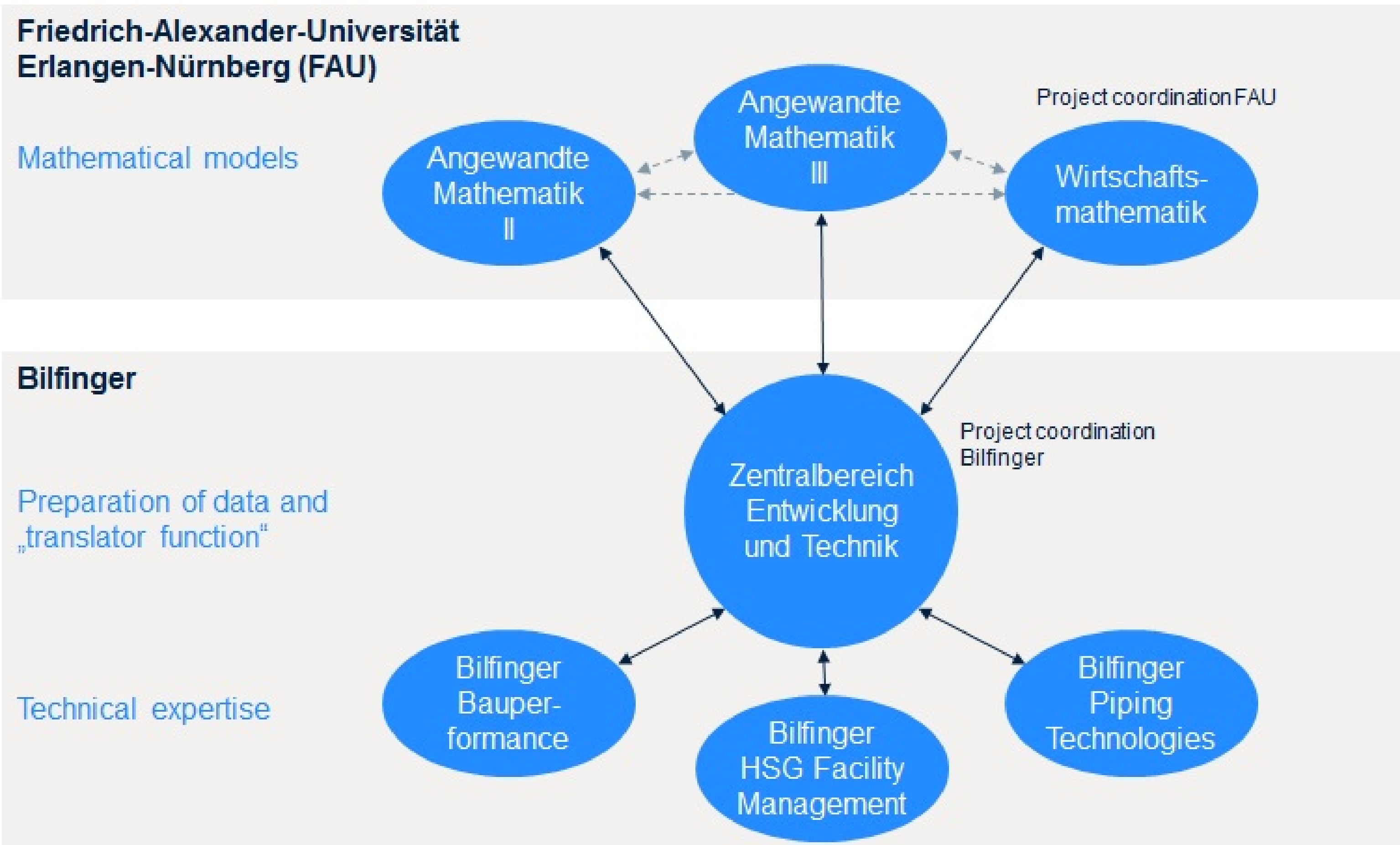
Optimization problem



Mathematical solution approach: LeOpIn

- LeOpIn = **L**ife-cycl**e** oriented **Op**timization for a resource- and energy-efficient **In**frastructure
- Supported under BMBF-program ”Mathematics for innovations in industry and services“
- Cooperation between
  - Friedrich-Alexander-Universität Erlangen-Nürnberg, Department of Mathematics
  - Bilfinger SE

Cooperation between Bilfinger and FAU



Goals and Vision of LeOpIn

Proof-of-concept study:
”Can planning problems be solved via mathematical optimization problems?“

- Explain fundamental dependencies and interconnections
- Extension of traditional solution approaches
- Methods for holistic optimization

Goals and Vision of LeOpIn

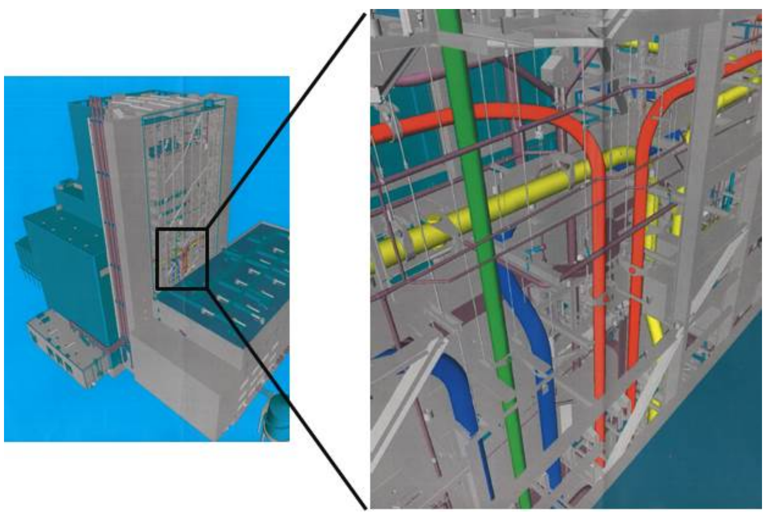
Advancement of the method to a tool mature for practical application:

- Holistic judgement of planning decisions
- Simple evidence for the impact of changes in the plan
- Integrated optimization of planning objects over the life cycle
  - Reduction of the investment costs
  - Considerable reduction of operation costs for the customer
- Acceleration of the planning process

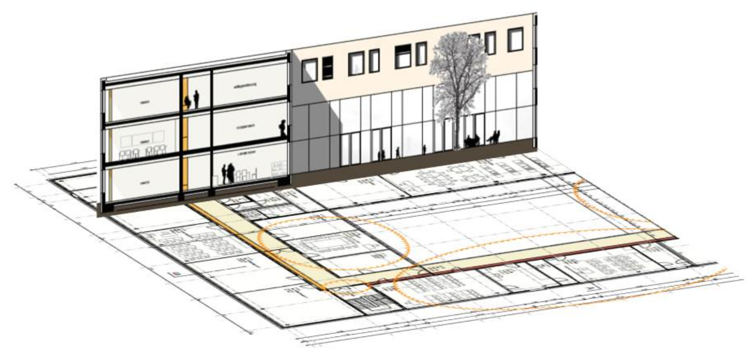
$\Longrightarrow$ The aim is not to replace the civil engineer
but to assist him in finding holistic solutions!

Strategy of LeOpIn

Development of the methodology based on specific application scenarios:



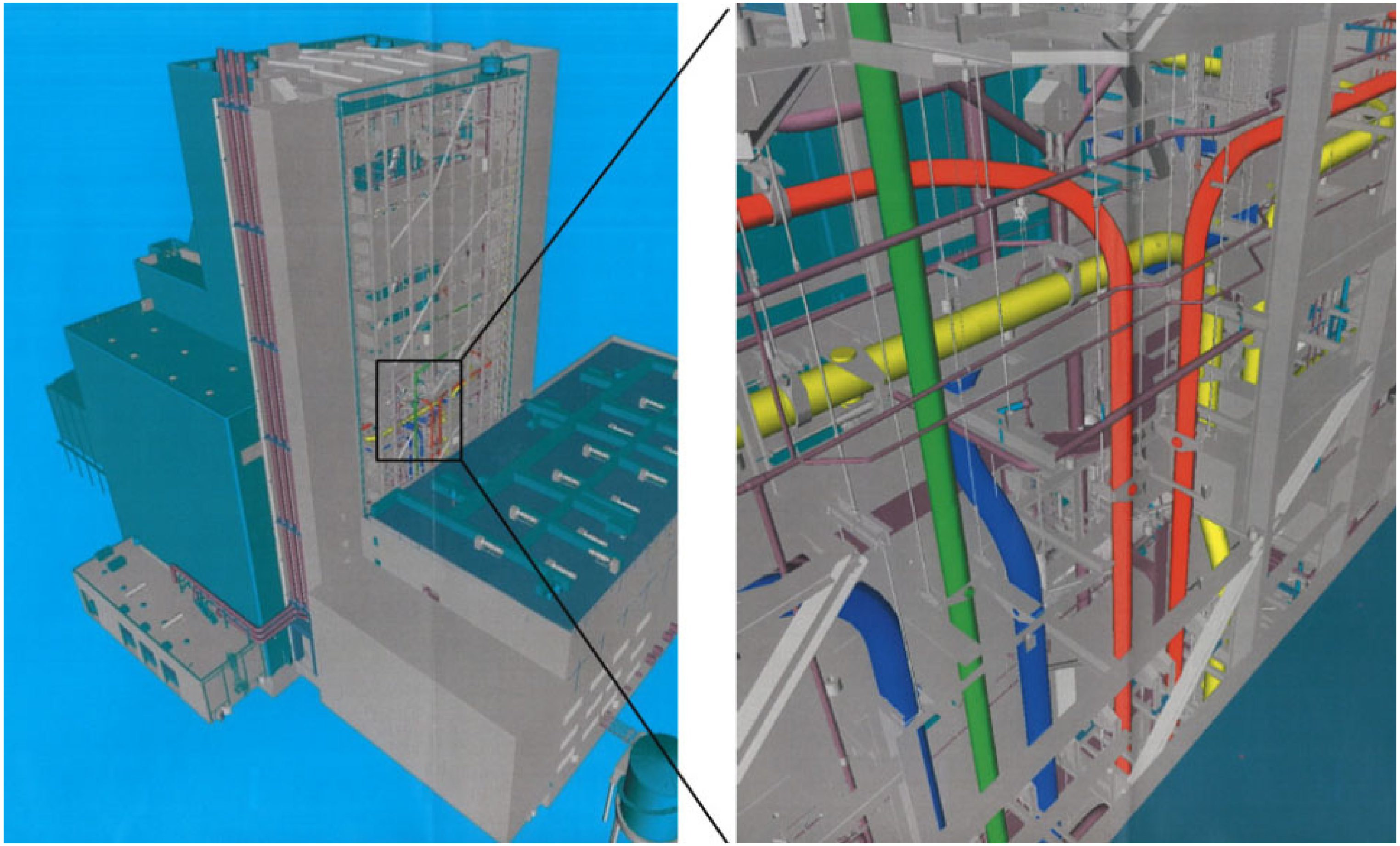
High-pressure pipe system



Building

Application to buildings

Application to power plants



Characteristics of the application scenarios

High-pressure pipe system

- Defined side constraints and parameters
- Defined physics
  - Structural mechanics
  - Fluid mechanics
  - Heat transmission
  - Material degradation
- Well-defined decision variables:
  - Line routing
  - Pipe thickness
  - Hanger location
  - Welding joints
  - Pipe bendings
  - ...
- Combination challenging

Building

- Side constraints and parameters still unclear
- Easy physics:
  - Heat transmission
  - Lighting
  - Fluid mechanics
  - Structural mechanics of minor importance
- Complex, coupled decision variables:
  - Shape of the building
  - Room, area, element location
  - Choice of product for the façade
  - TGA
  - ...
- Abstraction challenging

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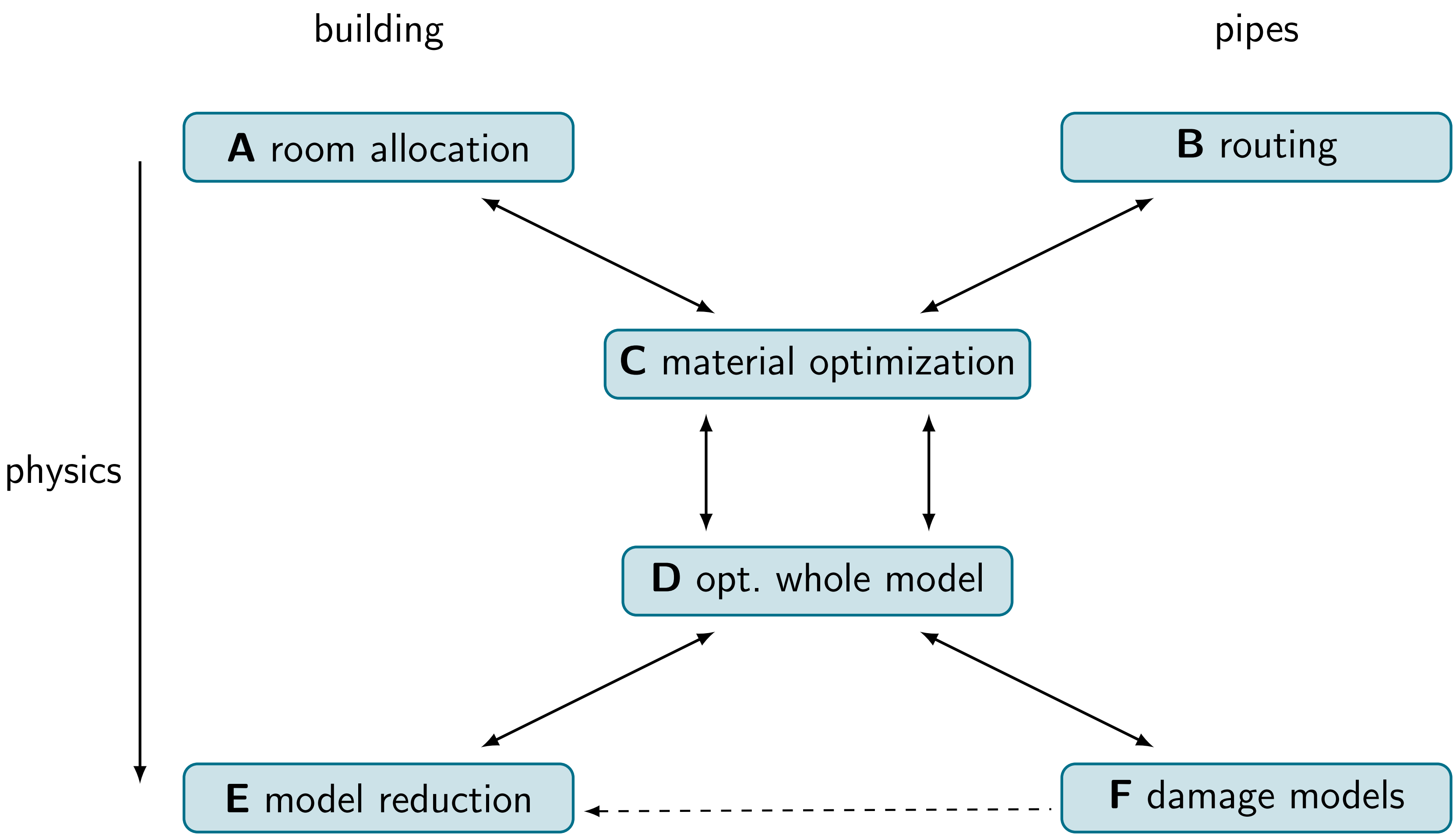
Challenges in LeOpIn

Scienctific research groups

- Complex, unstructured problem
- Hierarchical approach with feedback function from lower levels
- Connection of optimization methods and physical modelling

Bilfinger

- Reflection of the planning process
- Identification and weighting of side constraints, dependencies, influences
- Generalization to enable the application of mathematical optimization methods


building
pipes
A room allocation
B routing
C material optimization
physics
D opt. whole model
E model reduction
F damage models

Approach for the high-pressure pipe system

	B Pipe routing	C Structural optimization	D Efficient high-level model	F Material degradation
Input	▪ General side-constraints from geometry, material and transport	▪ Results from B ▪ Physical stress ▪ Specific side-constraints of the material	▪ Results from C ▪ Side-constraints from operation	▪ Results from D ▪ Specific material parameters
Output	▪ Possible paths ▪ Possible hanger locations ▪ Location of welding joints	▪ Pipe measures ▪ Hanger locations ▪ Local physical stress	▪ Optimal operation parameters ▪ Local life spans	▪ Detailed local prognosis of life spans

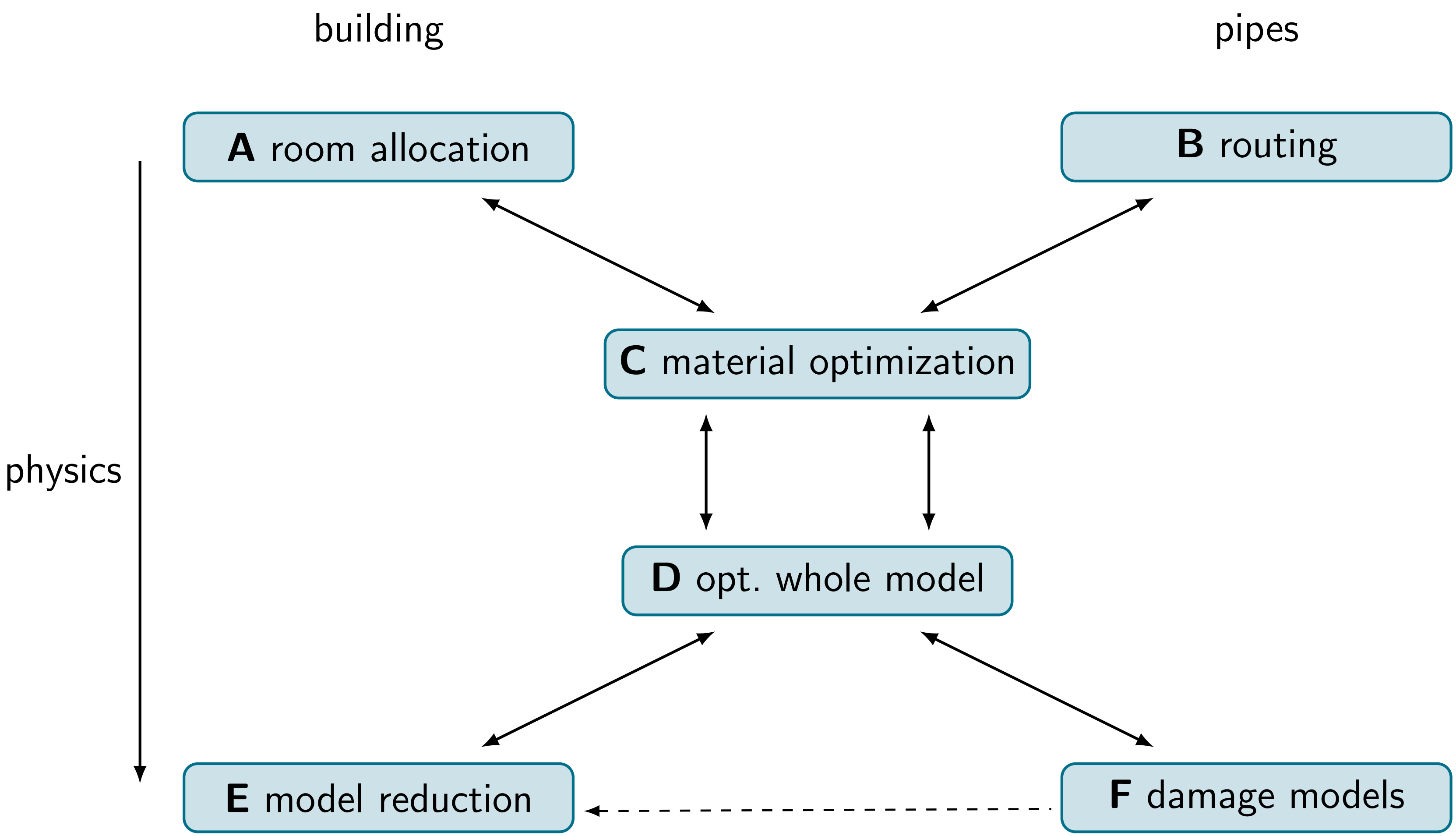
Optimization ⟷ Physical modelling

Approach for the building

	A Room location	C Structural optimization	D Efficient high-level model	E Fluid models
Input	▪ Side-constraints from estate, demand for land and room usage	▪ Results from A ▪ Side-constraints from TGA, room equipment and façade ▪ General climatic side-constraints	▪ Results from C ▪ Specific climatic side-constraints ▪ Distribution-, lighting- and shading systems	▪ Results from C and D ▪ Fluid models
Output	▪ Possible room configurations	▪ Choice of room equipment, TGA and façade	▪ Choice of systems ▪ Operation of TGA	▪ Local temperatures and flows

Optimization → Physical modelling

Overview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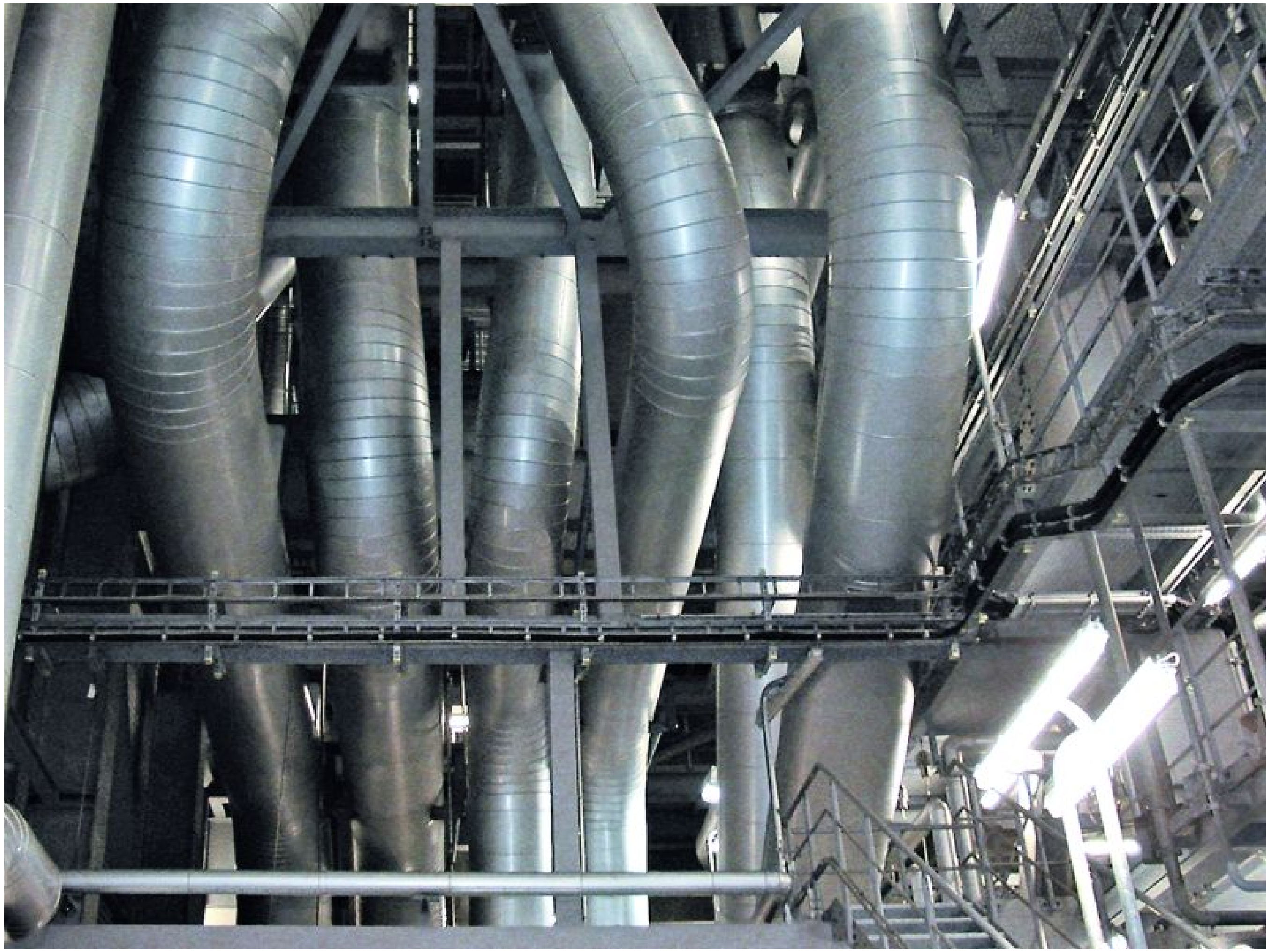
High Pressure Pipe System



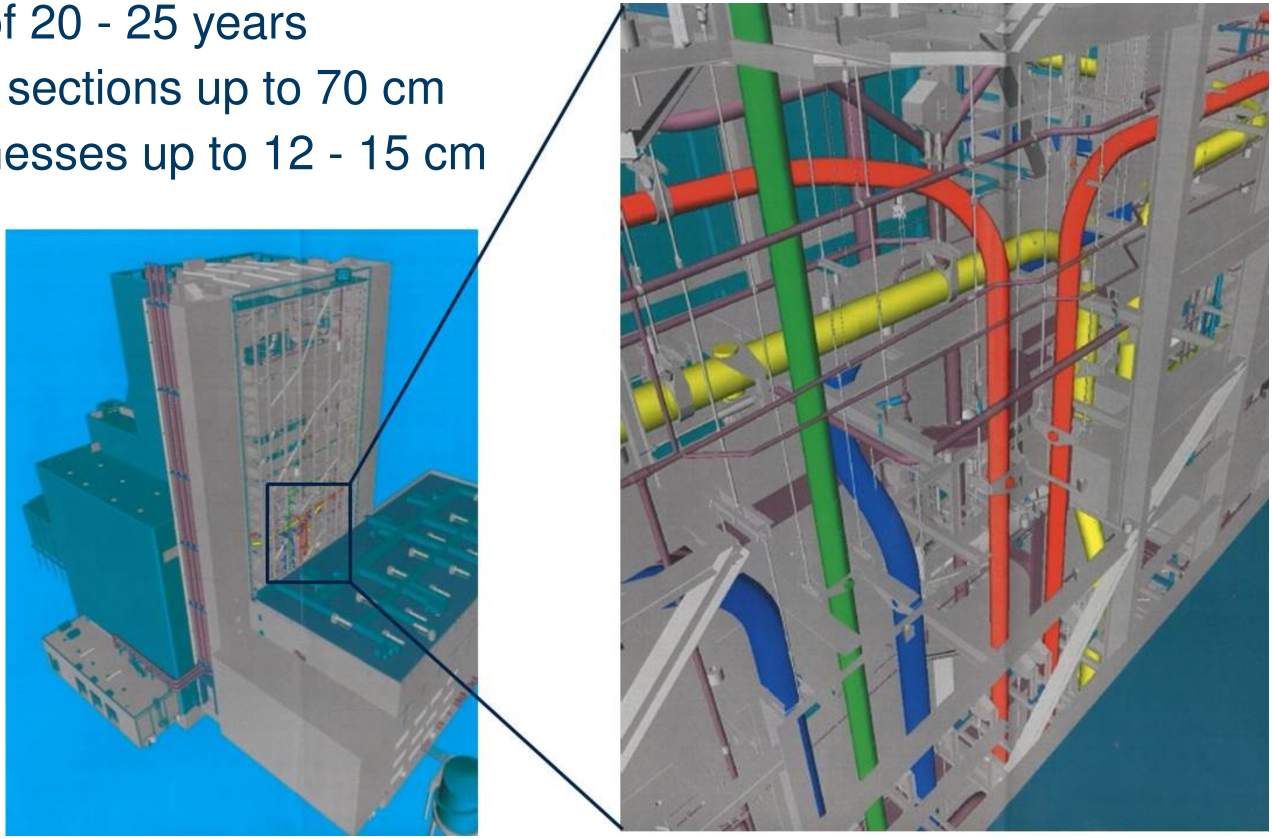
High Pressure Pipe System

High Pressure Pipe System

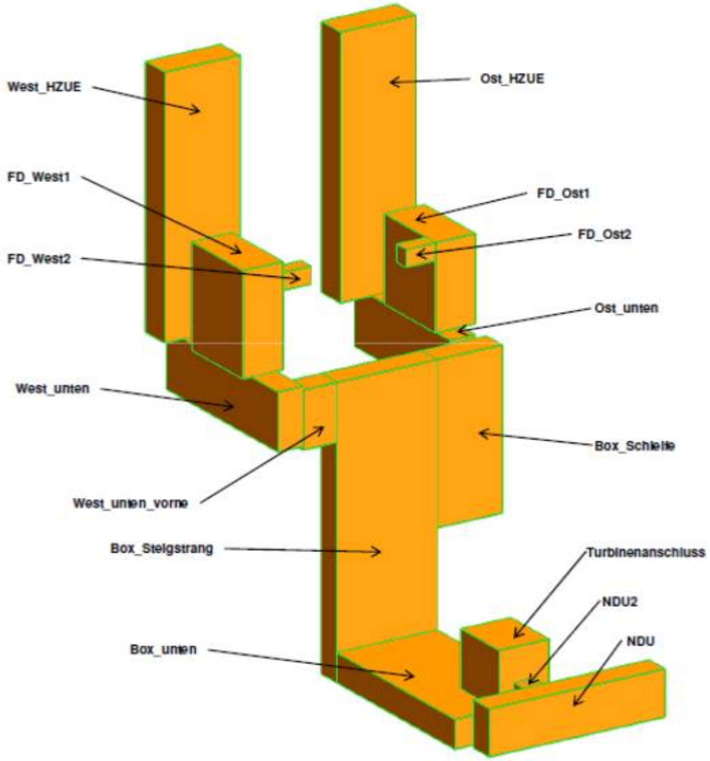
High Pressure Pipe System

Challenges in high-pressure pipe systems

- Temperatures above 600 degrees
- Pressures up to 300 bars
- Life span of 20 - 25 years
- Pipe cross sections up to 70 cm
- Pipe thicknesses up to 12 - 15 cm



The task

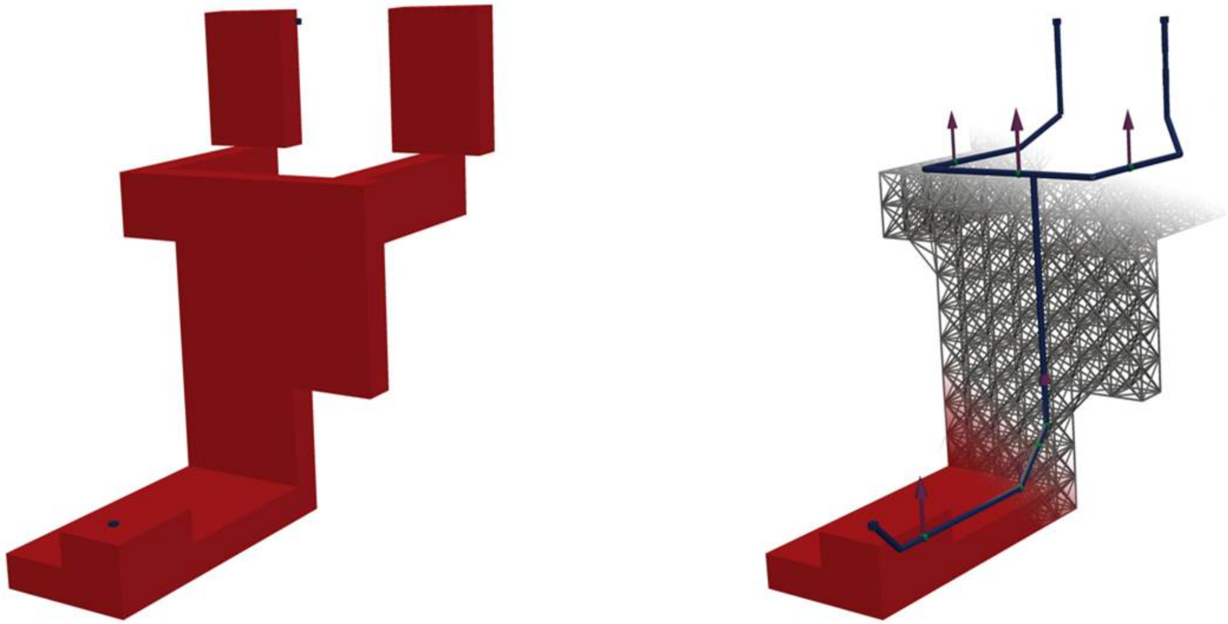


Goals:

- Optimal connection of the entry and exit points for a pipe system in a power plant
- Minimal cost over the life-cycle
- Minimal amount of CO_2-emissions
- Adherence to the side-constraints:
  - Geometry
  - Tensions
  - Transport restrictions
  - ...

1. Pipe routing (subproject B)

Goal: Coarse layout of the pipe routing and placement of hangers under simple physical side-constraints

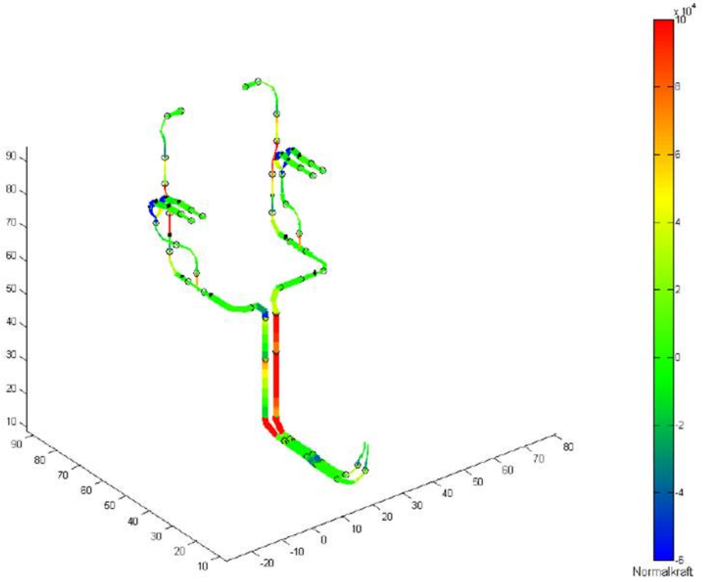


$\Longrightarrow$ Minimize the life-cycle costs

Objective function: $K_{\text{total}} = K_{\text{I}} + K_{\text{C}} + K_{\text{CO}_2}$

2. Topology optimization (subproject C)

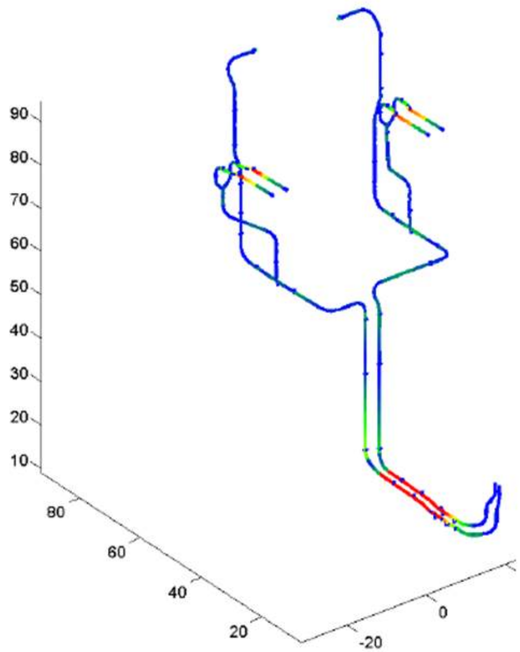
Goal: Optimal choice of material, pipe thicknesses, hanger positions, bending radii, etc., to optimize the pipe system while respecting all relevant side-constraints, e.g. tensions

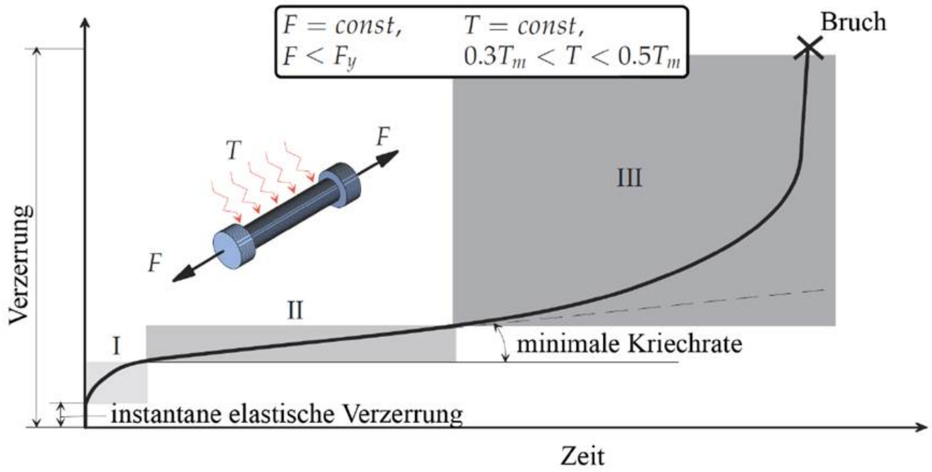


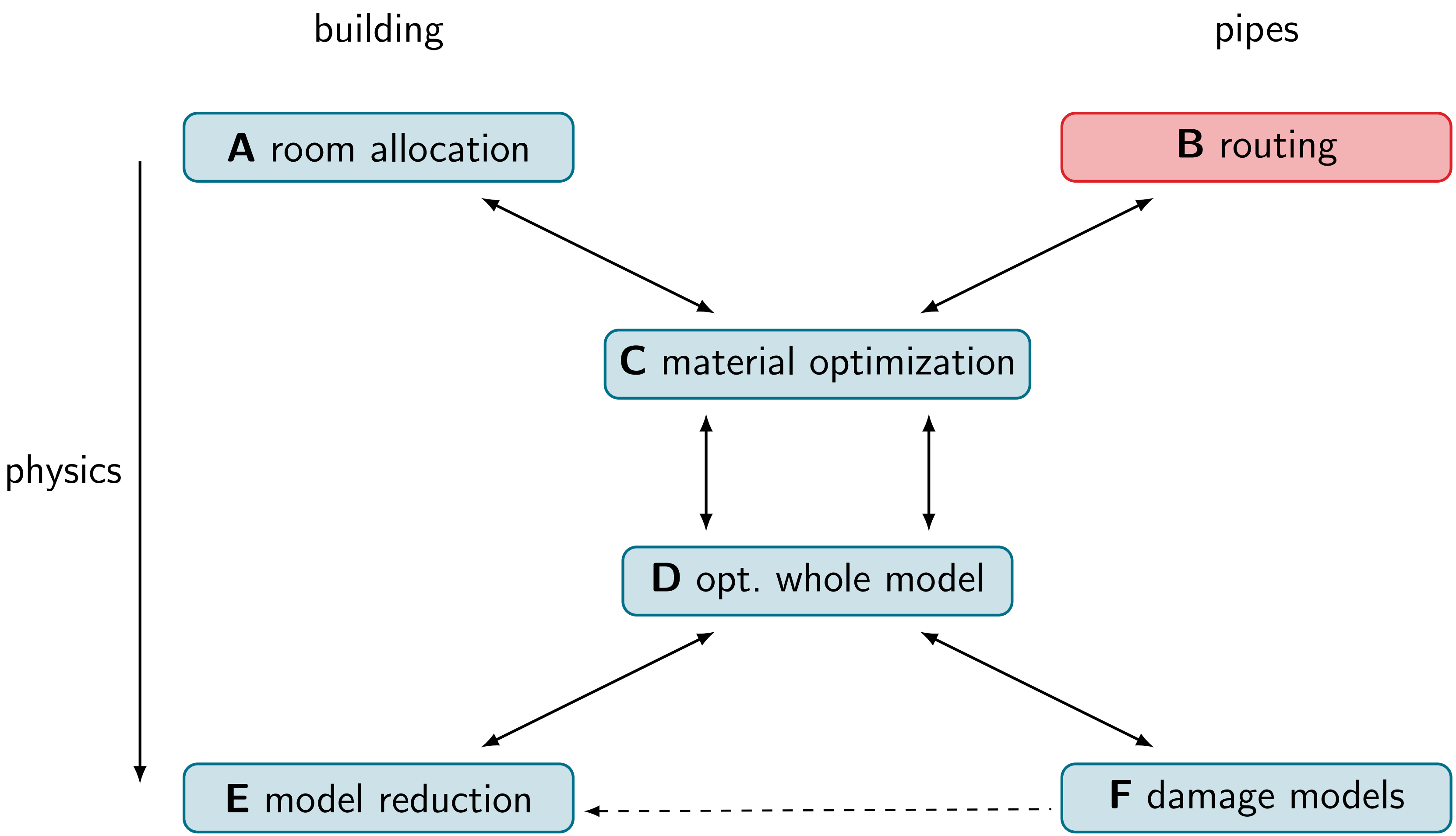
- Restrictions with respect to tensions: dimensioning for interior pressures $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ according to DIN EN 13480
- Geometrical restrictions

3. Models for material degradation (subproject F)

Goal: Development of a physically improved modelling of nonlinear effects and the material degradation of pipe systems



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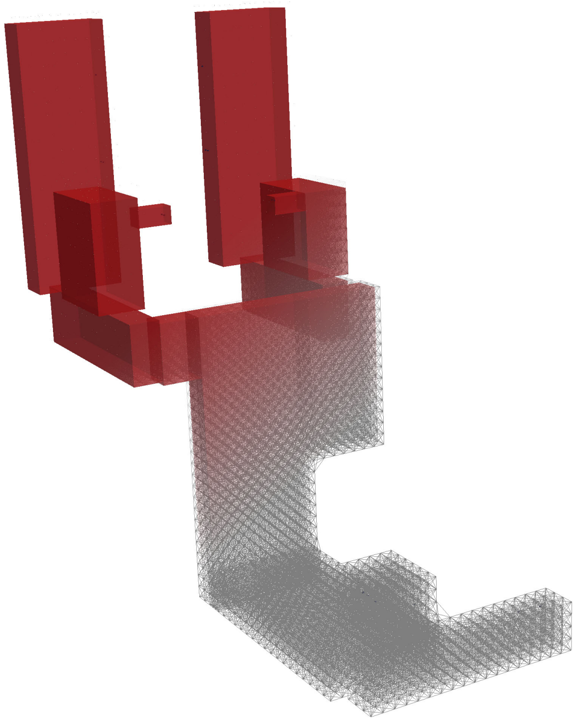
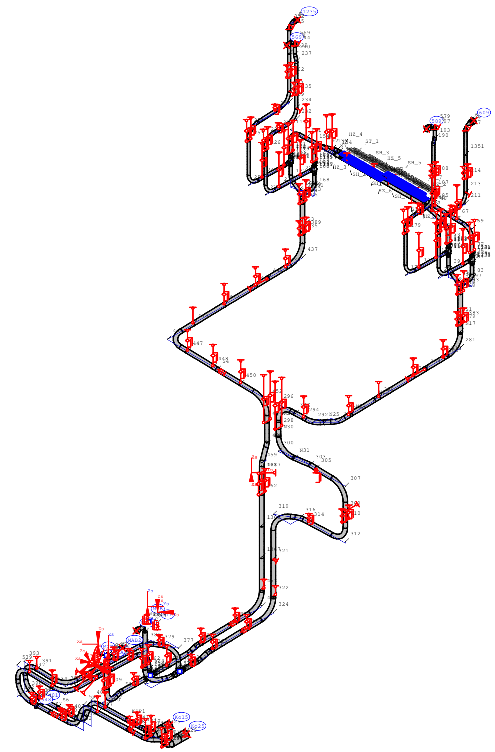




Problem

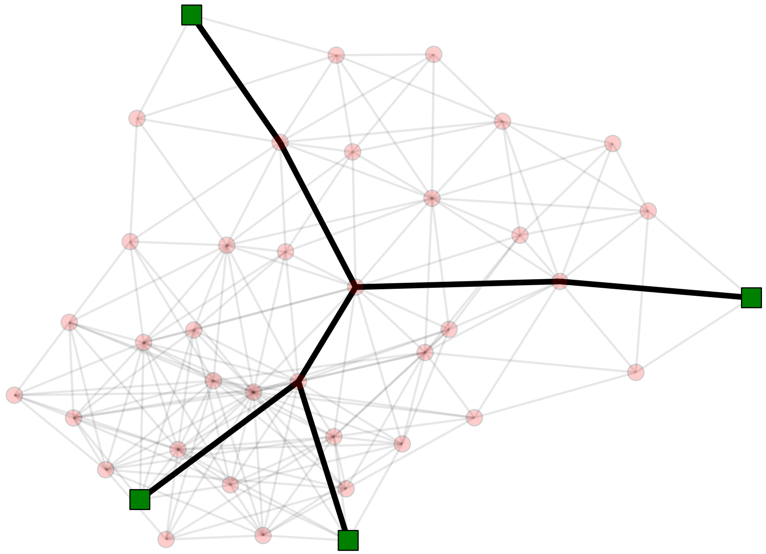
- Given a rough outline of a power plant
- Route a pipe through the plant considering physical constraints

Physical dimensions

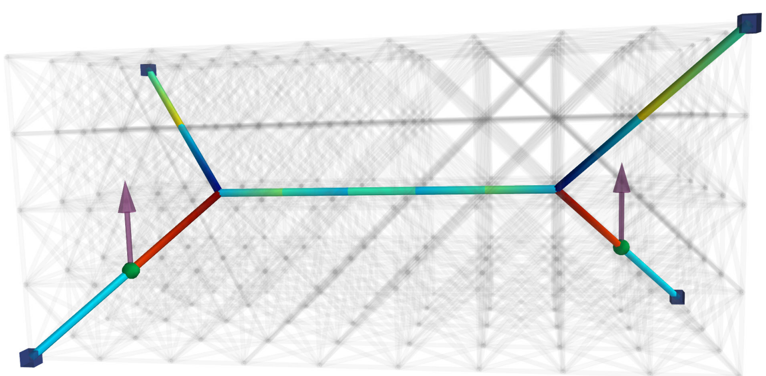
- Power plant: $50\,\text{m} \times 75\,\text{m} \times 100\,\text{m}$
- Pipe length: $\approx 450\,\text{m}$ per section
- Pipe diameter: $770\,\text{mm}$, wall thickness: $63\,\text{mm}$
- Working conditions: $631\,^\circ\text{C}$, $300\,\text{bar}$

Problem's ingredients

Combinatorics: Path/Steiner tree



Nonlinearities: Mechanics



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Basic problem statement

$$
\begin{aligned}
\min \; & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, \mathbf{u}(x, y)) \\
\text{s.t.} \; & \text{Steiner tree}(x) \\
& \text{pipe physics}(x, y, \mathbf{u}(x, y)) \\
& \text{hangers}(x, y, \mathbf{u}(x, y)) \\
& \text{industrial standards}(x, y, \mathbf{u}(x, y))
\end{aligned}
$$

Variables

- x Pipe variables
- y Hanger variables
- $\mathbf{u}$ Displacement variables (depend on x and y)

Cost function

Overall costs are given by

$$K_{\text{ges}} = K_I + K_B + K_{CO_2}.$$

With investment costs

$$K_I = K_{\text{Rohr}} + K_{\text{Biegung}} + K_{\text{Iso}} + K_{\text{Hänger}},$$

operation costs

$$K_B = K(\Delta H) + K(\Delta Q)$$

and CO_2 costs

$$K_{CO_2} = K_{CO_2,\Delta H} + K_{CO_2,\Delta Q} + K_{CO_2,\text{Stahl}} + K_{CO_2,\text{Iso}}$$

$$K_{\text{ges}} = c_L \cdot \text{length}(x) + c_B \cdot \text{bendings}(x) + c_{\text{hangers}}(\text{hangers}(y), \text{displacement}(u))$$

More than one inlet/outlet – Steiner tree

Definition: Steiner tree problem

Given graph $G = (V, E)$ with vertices V, edges E with weights $c : E \to \mathbb{R}^+$ and a set of terminal nodes $T \subseteq V$.
Find the cheapest tree S that includes all nodes in T.

- Huge catalogue of Steiner tree models available
- Usually few terminals in our application $\Rightarrow$ Use a flow formulation
- Computational study shows advantage over other models

More than one inlet/outlet – Steiner tree

Choose arbitrary $r \in T$ as root.

Constraints: Steiner tree(x)

$$\min \sum_{e \in E} c_e x_e$$

$$\text{s.t.}\ \forall t \in T \setminus \{r\}\ \forall v \in V: \sum_{a \in \delta^+(v)} f_a^t - \sum_{a \in \delta^-(v)} f_a^t = \begin{cases} 1 & \text{falls } v = r \\ 0 & \text{falls } v \neq r \wedge v \neq t \\ -1 & \text{falls } v = t \end{cases}$$

$$\forall t \in T\ \forall a \in A: \quad f_a^t \leqslant s_a$$

$$\forall e \in E: \quad s_{a_1} + s_{a_2} = x_e$$

$$\forall t \in T\ \forall a \in A: \quad f_a^t \geqslant 0$$

$$\forall a \in A: \quad x_e, s_a \in \{0, 1\}$$

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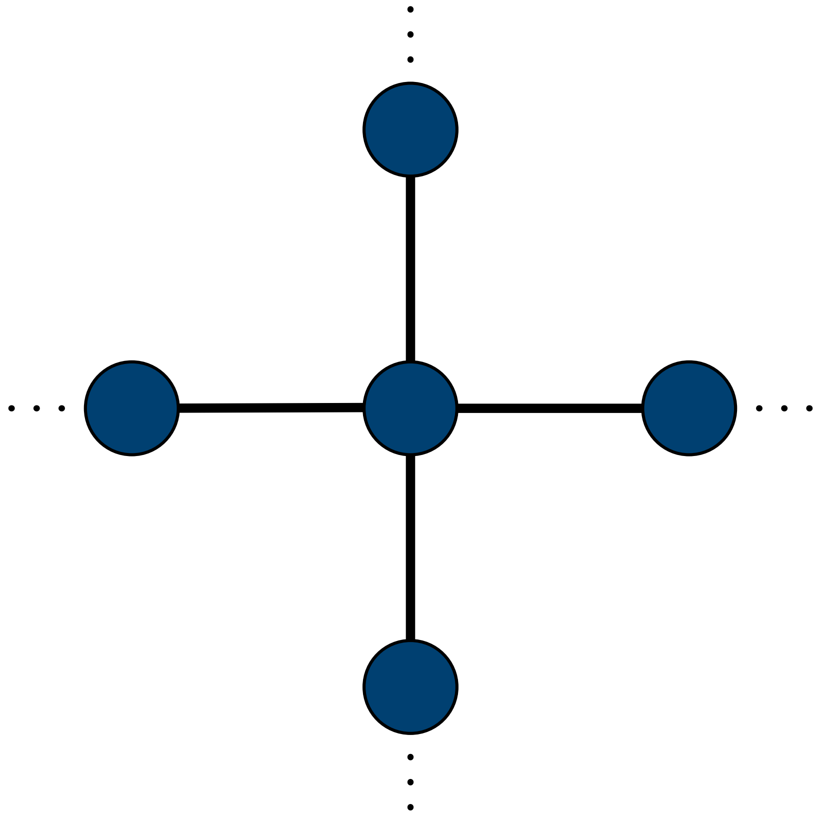
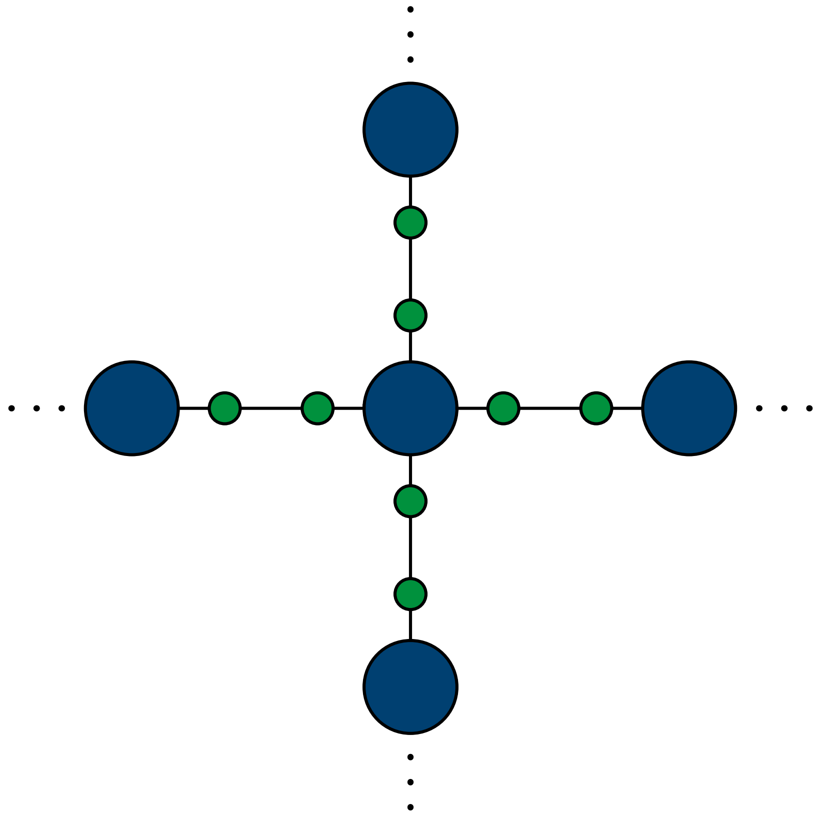
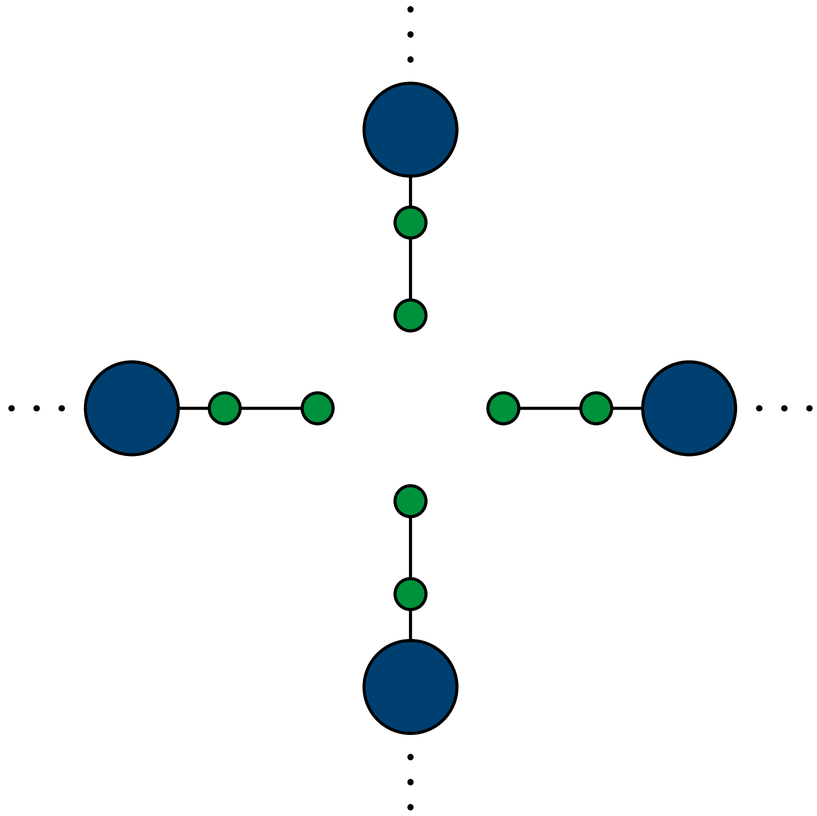
Extended graph for bending costs



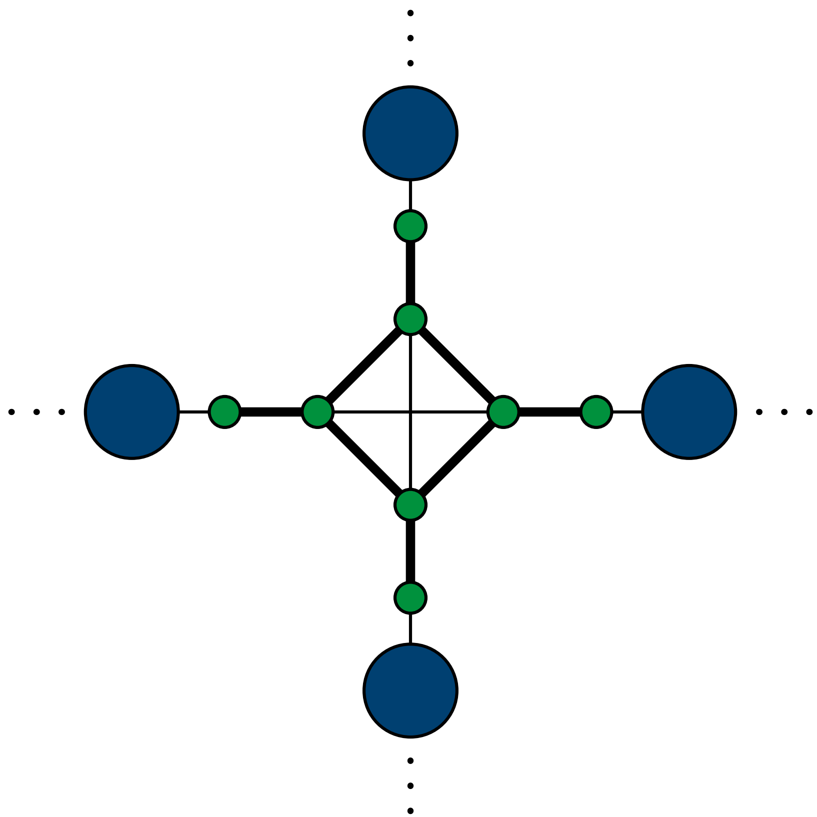
Extended graph for bending costs



Extended graph for bending costs



Extended graph for bending costs

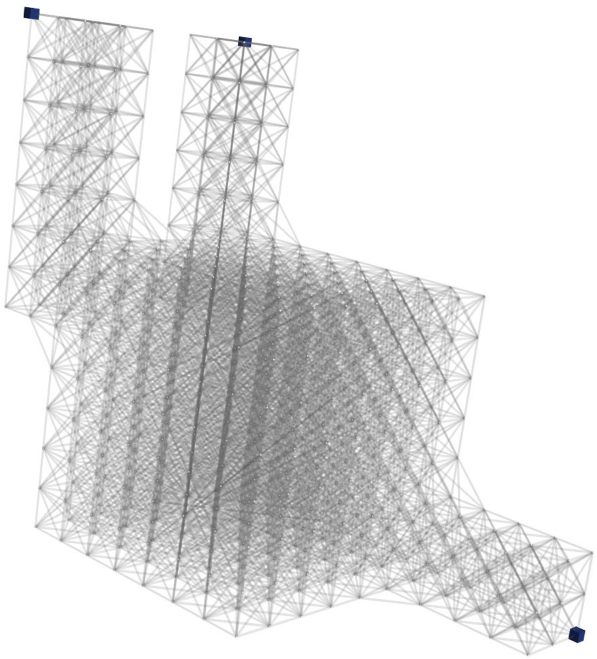


- Models bending costs for paths
- Gives lower bound for Steiner trees

Modeling the physics

Groundstructure approach

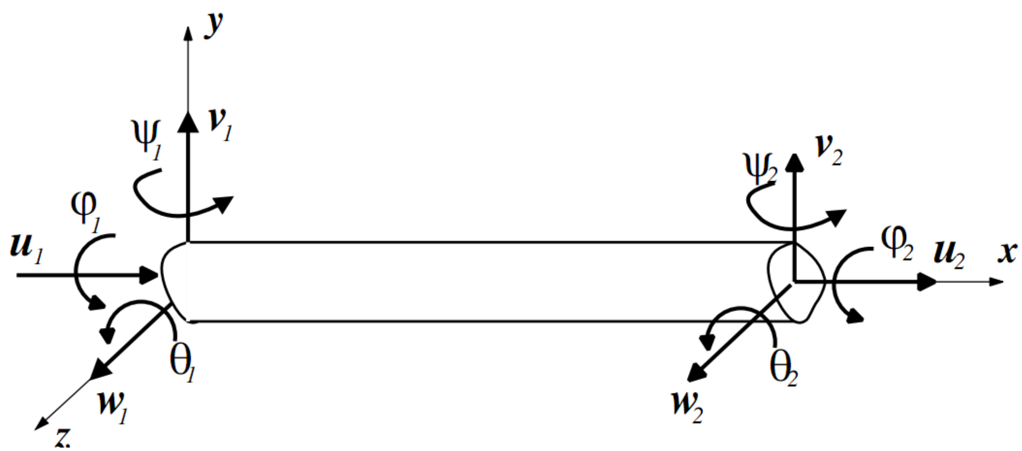
- Fixed and free nodes
- Potential elements between nodes
- Elements follow laws of linear elasticity



Linear Timoshenko beam

Assumptions

- Axial dimension is dominating length scale
- Cross-sections may rotate independently of the beam axis



- Variables for the displacement of the center line (u, v, w)
- Variables for the rotation of the cross-section (φ, θ, ψ)

Linear 3D Timoshenko beam equations

$$\begin{aligned} EAu'' + q_x &= 0 \\ kGA(v'' - \theta') + q_y &= 0 \\ kGA(w'' + \psi') + q_z &= 0 \\ GI_t\varphi'' + m_x &= 0 \\ EI_z\psi'' - kGA(w' + \psi) + m_y &= 0 \\ EI_y\theta'' + kGA(v' - \theta) + m_z &= 0 \end{aligned}$$

EA: extensional stiffness
kGA: shear stiffness with factor
GI_t: torsional stiffness
EI: bending stiffness

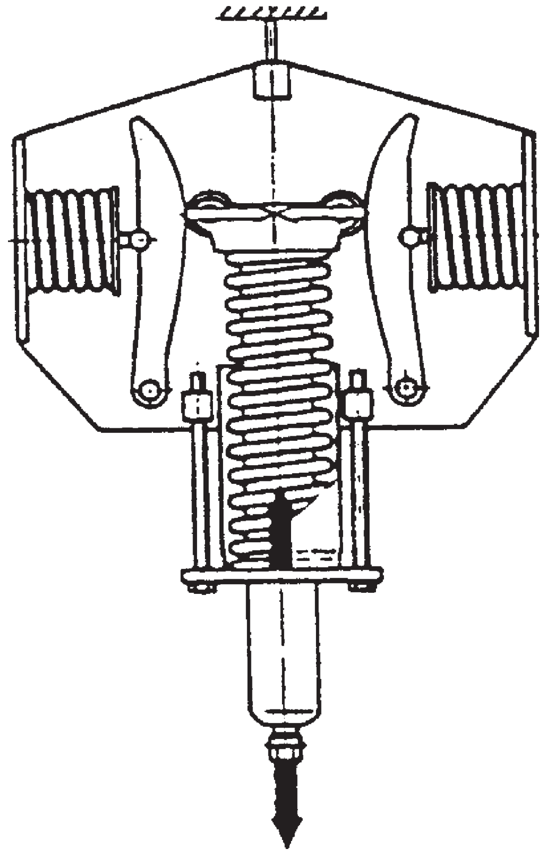
- Analytical solutions to homogeneous Timoshenko equations as ansatz functions give rise to proper stiffness matrices (Luo, 2008)
- Global stiffness matrix: $K(x) = \sum_{i=1}^{n} T_i^T K_i T_i x_i$

Constraints: pipe physics$(x, y, u(x, y))$

For hot an cold scenario:

$$K(x)\mathbf{u} = \sum_{e \in \mathcal{E}} g_i x_i + \sum_{n \in \mathcal{V}_{\text{free}}} l_j h_j$$

How to model the hangers?



Source: G. Wossog, Handbuch Rohrleitungsbau, p 594

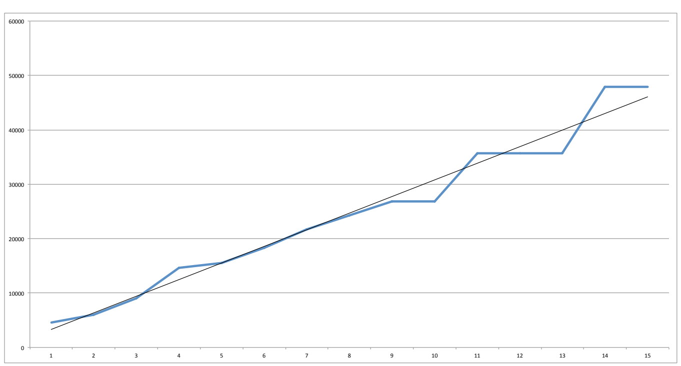
- Modeled as variable forces in opposite direction of gravity
- Variables: binary y_j and continuous h_j

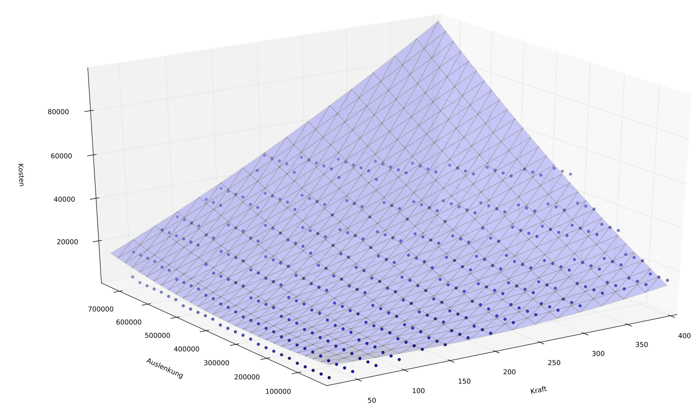
Constraints: hangers$(x, y, \mathbf{u}(x, y))$

$$h_j \leqslant h_{\max} y_j$$

$$y_j \leqslant \sum_{j \in i} x_i$$

$$c_{\text{fix}} y_j + c_{var} h_j \leqslant c_j \qquad \mathbf{u}_j(x, y)^T C h_j + c_u \mathbf{u}_j(x, y) + c_h h_j \leqslant c_j$$





Industrial standards (DIN EN 13480)

- Restrictions on the maximal supporting forces at fixed nodes
- Pipe must fulfill constraints based on moments
- Several constraints involve the difference of the moments from different loading scenarios

Constraints: industrial standards$(x, y, u(x, y))$

$$
\begin{aligned}
R(x)\mathbf{u} &\leqslant r(x)\\
M_e^h &= \hat{N}^h \mathbf{u}_e^h\\
M_e^c &= \hat{N}^c \mathbf{u}_e^c\\
M_e^D &= M_e^h - M_e^c\\
(M_{e1}^h)^2 + (M_{e2}^h)^2 + (M_{e3}^h)^2 &\leqslant (M_{\text{max}}^h)^2 \quad \text{if } x_e = 1\\
(M_{e1}^D)^2 + (M_{e2}^D)^2 + (M_{e3}^D)^2 &\leqslant (M_{\text{max}}^D)^2 \quad \text{if } x_e = 1
\end{aligned}
$$

Solution approaches

MINLP Model

$$\begin{aligned} \min \quad & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, u(x,y)) \\ \text{s.t.} \quad & \text{Steiner tree}(x) \\ & \text{pipe physics}(x, y, u(x,y)) \\ & \text{hangers}(x, y, u(x,y)) \\ & \text{industrial standards}(x, y, u(x,y)) \end{aligned}$$

MILP Model

- Linearize non-convex terms $\xi_{il} = x_i u_l$
- Can be done completely or adaptive

MISOCP Model

- Replace industrial standards with substitute constraint
- Problem becomes convex

Decomposition

- Decompose in master- and subproblem
- Both problems are convex

Linearization methods

Idea

Replace non-convex terms $x_i u_l = \xi_{il}$ with variable and use additional constraints.
Relies on ideas by Glover, first used by Stolpe in a structural/topology optimization context.

$$\begin{aligned} \underline{u} x_i \leqslant & \quad \xi_{li} & \leqslant \bar{u} x_i \\ \underline{u}(1 - x_i) \leqslant & \; u_l - \xi_{li} & \leqslant \bar{u}(1 - x_i) \end{aligned} \tag{Lin}$$

Two basic methods for linearization

- Linearize all terms from the start
- Adaptively linearize and add only needed constraints during optimization

Adaptive linearization

Definition

$$\mathcal{V}_{\text{affected}}(x) := \{n_i \in \mathcal{V} : \exists n_j \in \mathcal{V} : x_{(n_i,n_j)} = 1 \text{ or } x_{(n_j,n_i)} = 1\}$$

$$\mathcal{E}_{\text{affected}}(x) := \{(n_i, n_j) = e \in \mathcal{E} : n_i \in \mathcal{V}_{\text{affected}}(x) \text{ or } n_j \in \mathcal{V}_{\text{affected}}(x)\}$$

$$\text{prodrelax}(e) := \left\{ \begin{array}{rcl} \underline{u}x_e \leqslant & \xi_{le} & \leqslant \overline{u}x_e \\ \underline{u}(1-x_e) \leqslant & u_l - \xi_{le} & \leqslant \overline{u}(1-x_e) \end{array} : l \in I(e) \right\}$$

Changed constraints

$$\sum_{e\in\mathcal{E}} K_{c,e}\xi_{c,\cdot e} = \sum_{e\in\mathcal{E}} g_e x_e + \sum_{n\in\mathcal{V}_{\text{free}}} l_n h_n$$

$$\sum_{e\in\mathcal{E}} K_{h,e}\xi_{h,\cdot e} = \sum_{e\in\mathcal{E}} (g_e + f_e)\, x_e + \sum_{n\in\mathcal{V}_{\text{free}}} l_n h_n$$

Also technical standards constraints (linear displacement constraints, constraints involving moments).

$$
\begin{aligned}
\min\ & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, \mathbf{u}(x,y)) \\
\text{s.t.}\ & \text{Steiner tree}(x) \\
& \text{pipe physics}(x, y, \mathbf{u}(x,y), \xi) \\
& \text{hangers}(x, y, \mathbf{u}(x,y)) \\
& \text{industrial standards}(x, y, \mathbf{u}(x,y), \xi)
\end{aligned}
\qquad (\mathrm{P}_l)
$$

Algorithm 1: Adaptive linearization for the pipe optimization problem

solve (P_l)
if (P_l) is infeasible **then**
  return infeasible
else
  (P_l) gives solution $(x, \mathbf{u}, \xi)$
  while $L_{\text{cuts}} \longleftarrow$ Algorithm 2 is nonempty **do**
    add constraints in L_{cuts} to (P_l)
    solve (P_l) for solution $(x, \mathbf{u}, \xi)$
    if (P_l) is infeasible **then**
      return infeasible
  return solution $(x, \mathbf{u}, \xi)$

Algorithm 2: Check for violated product relaxation constraints

Input: Solution (x, u, ξ)

Output: List of violated constraints

$\mathcal{V}_{\text{affected}} = \{n \in \mathcal{V} : \; x_{(\cdot,n)} = 1 \text{ or } x_{(n,\cdot)} = 1\}$

$\mathcal{E}_{\text{affected}} = \{(n_i, n_j) = e \in \mathcal{E} : \; n_i \in \mathcal{V}_{\text{affected}} \text{ or } n_j \in \mathcal{V}_{\text{affected}}\}$

$L_{\text{cuts}} = \emptyset$

foreach $e \in \mathcal{E}_{\text{affected}}$ **do**

  if (x_e, u_e, ξ_e) violates some constraints (Lin) **then**

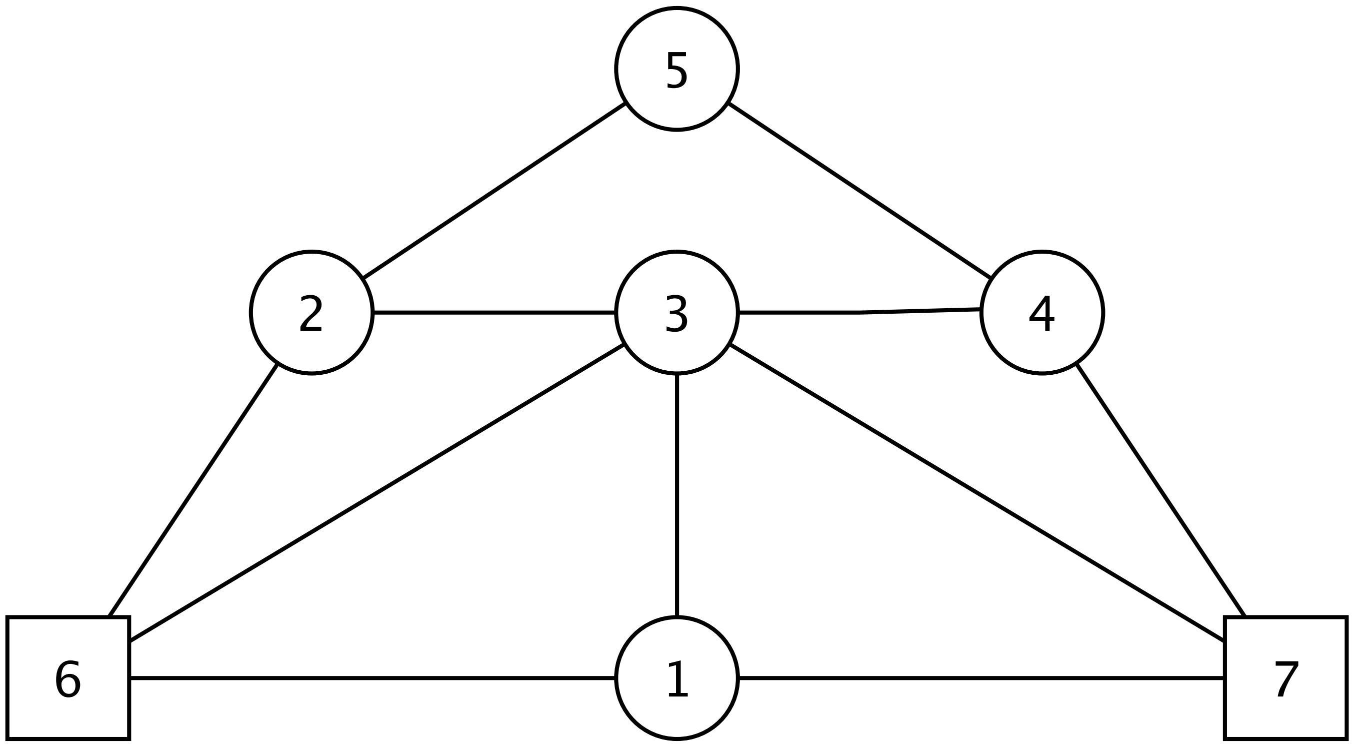
    $L_{\text{cuts}} = L_{\text{cuts}} \cup \text{prodrelax}(e)$

return L_{cuts}

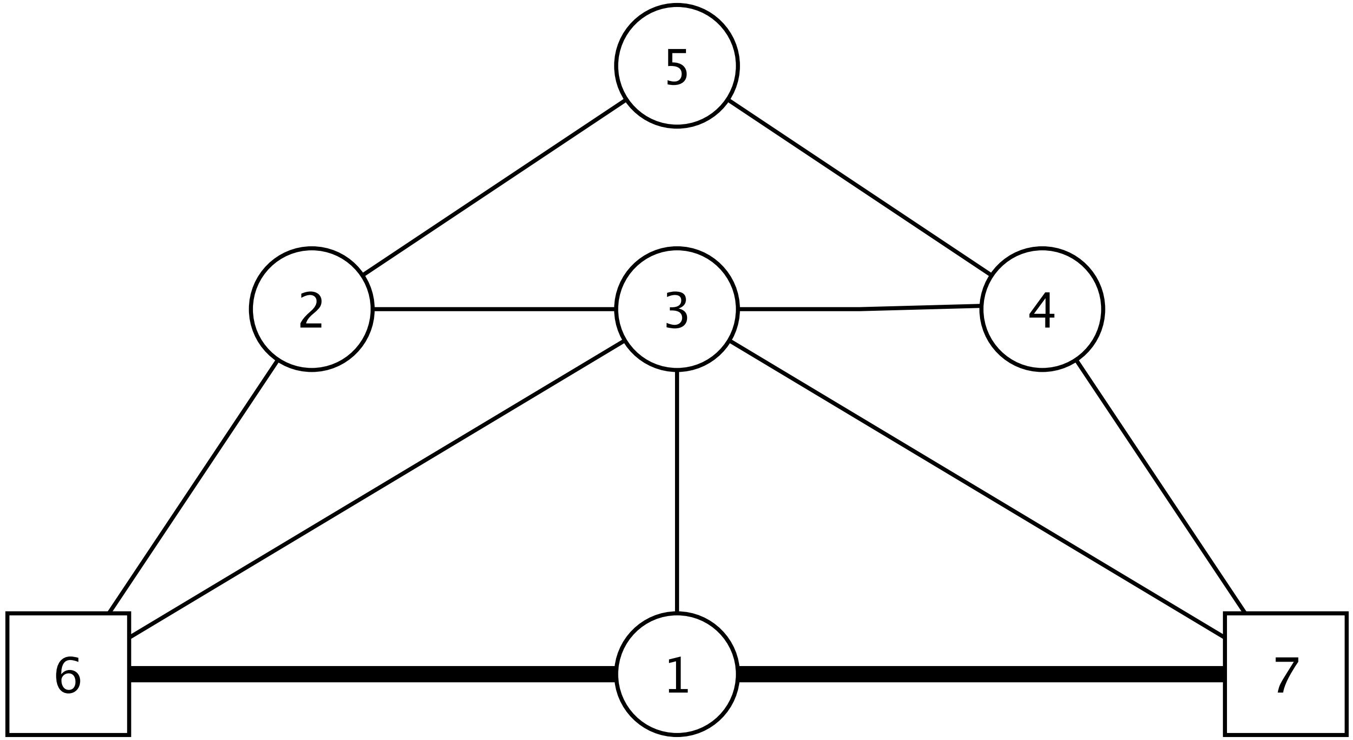
Obervation

Given a solution $(x, h, \tilde{u}, \xi)$ for the relaxed model (P_l). If constraints prodrelax(e) hold for all elements $e \in \mathcal{E}_{\text{affected}}(x)$ then a solution (x, h, u) for the original MINLP model can be reconstructed.

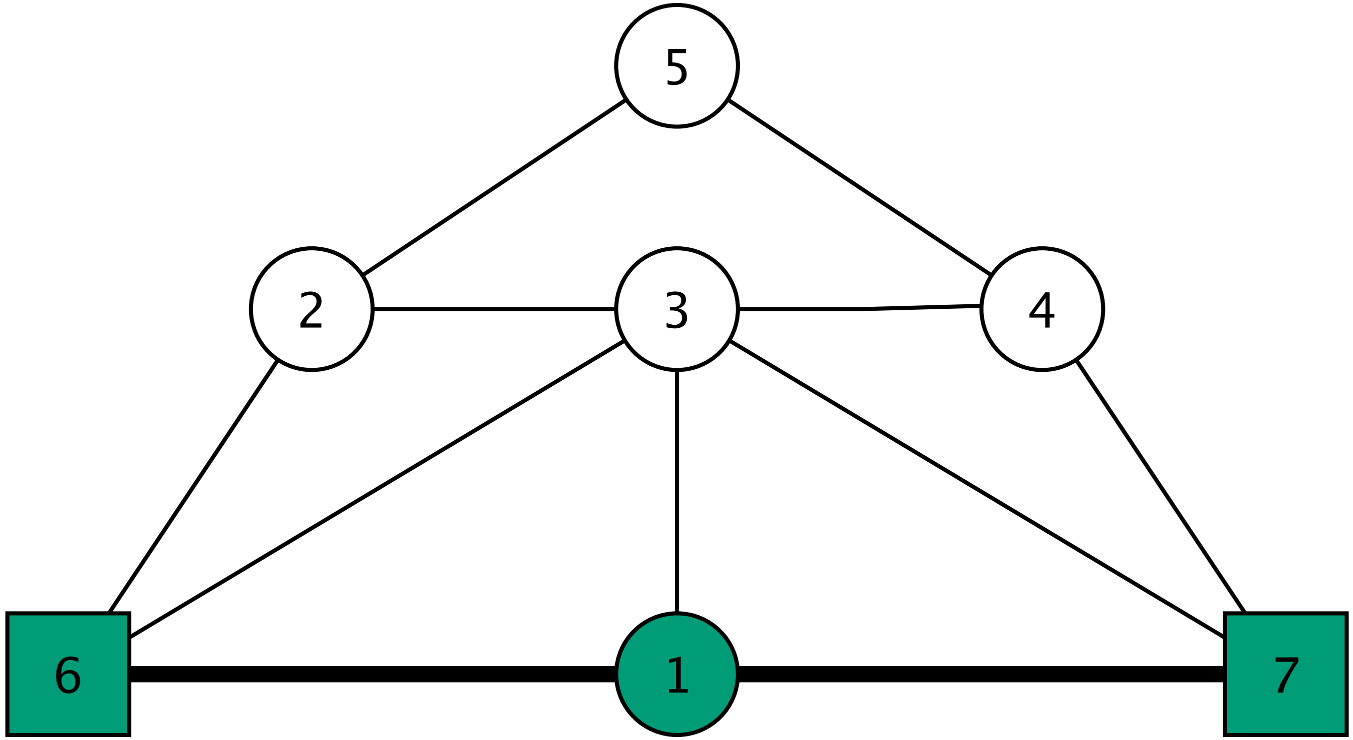
Adaptive linearization – Example



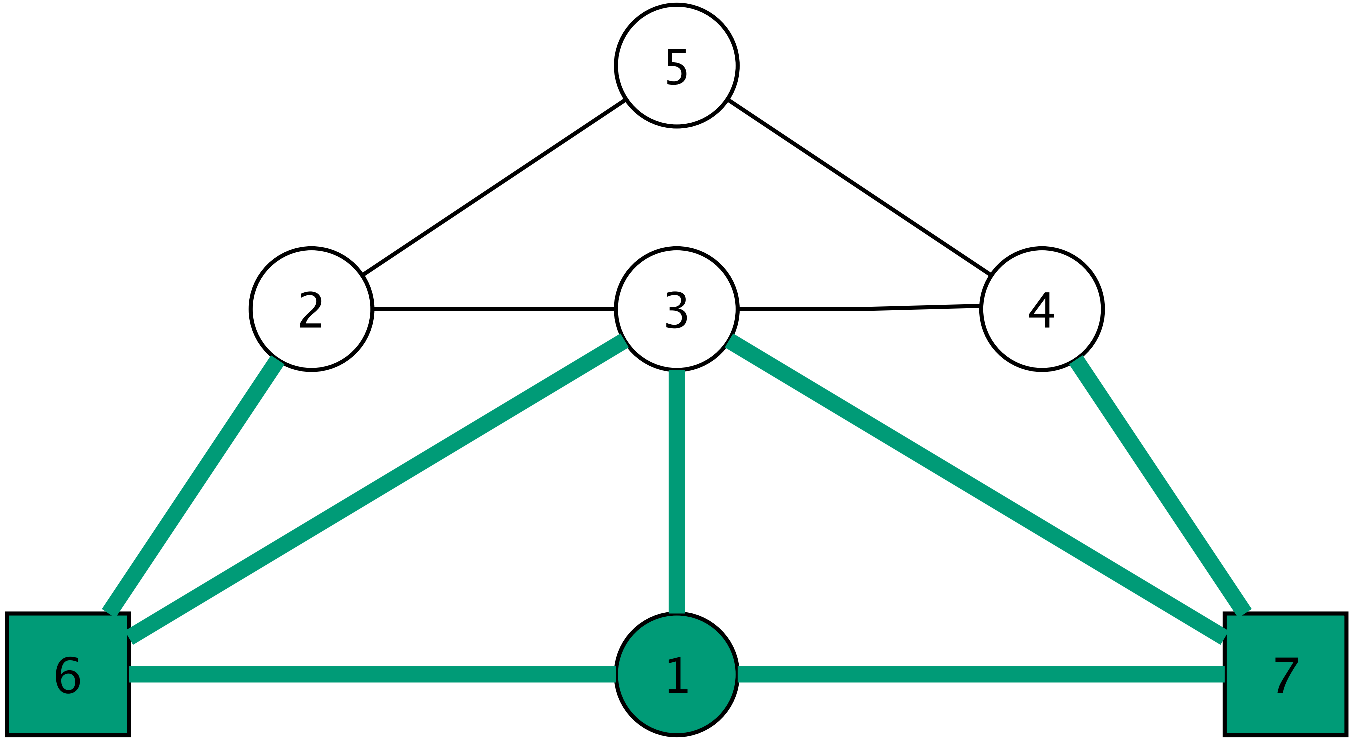
Adaptive linearization – Example



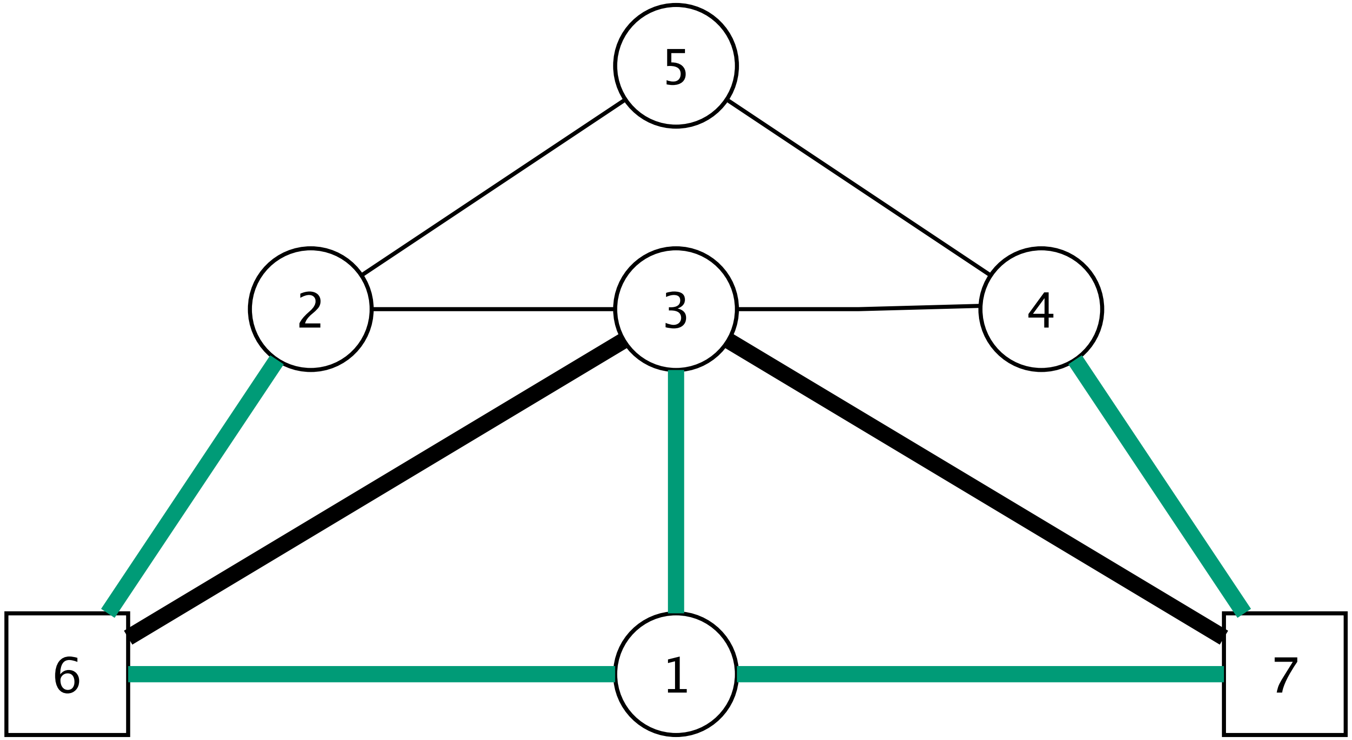
Adaptive linearization – Example



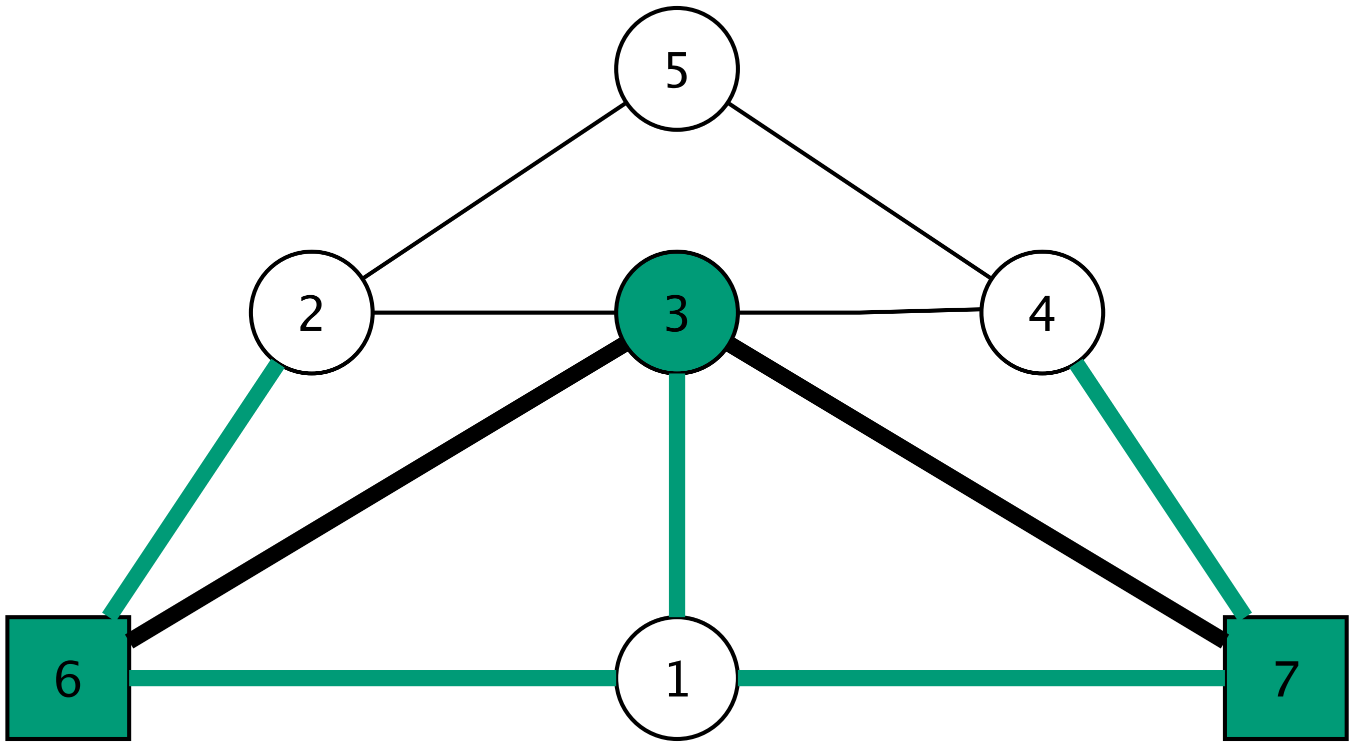
Adaptive linearization – Example



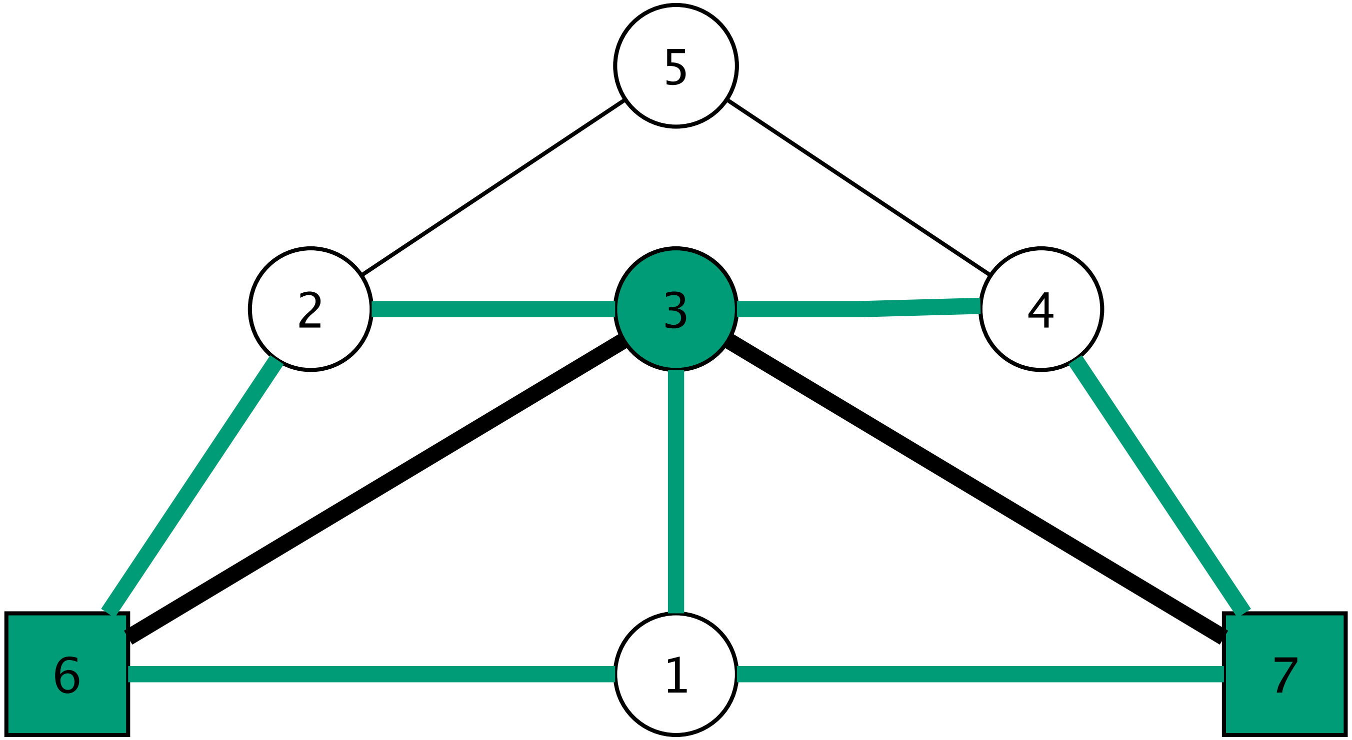
Adaptive linearization – Example



Adaptive linearization – Example



Adaptive linearization – Example



Decomposition approach

- General Idea: Benders [1962], Geoffrion [1972]
- Applied for truss problems: Muñoz and Stolpe [2011]

Masterproblem

$$\begin{aligned} \min\ & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, u(x, y)) \\ \text{s.t.}\ & \text{Steinertree}(x) \end{aligned}$$

$\Longleftrightarrow$

Subproblem for fixed $\hat{x}$

$$\begin{aligned} \min\ & c_{\text{hangers}}(y, u(\hat{x}, y)) \\ \text{s.t.}\ & \text{pipe physics}(\hat{x}, y, u(\hat{x}, y)) \\ & \text{hangers}(\hat{x}, y, u(\hat{x}, y)) \\ & \text{industrial standards}(\hat{x}, y, u(\hat{x}, y)) \end{aligned}$$

Decomposition algorithm

Set $k = 0$, *best* = None, $\phi = 0$, $\theta = \infty$
while $|\theta - \phi| > \varepsilon$ **do**
  Solve Masterproblem for a lower bound ϕ and solution x^k.
  if Check for infeasibility returns False **then**
    Solve Subproblem with fixed $\hat{x} = x^k$
  if Subproblem$(\hat{x})$ was feasible **then**
    Get solution y^k with costs γ (includes costs for hangers and pipe).
    if $\gamma < \theta$ **then**
      $\theta = \gamma$
      best $= (x^k, y^k)$
    Add Cost-Cut to Masterproblem
  else
    Add No-Good-Cut to Masterproblem
  $k = k + 1$

Masterproblem

$$\min \ \kappa$$

$$\sum_{a \in \mathcal{A}} c_a s_a \leqslant \kappa$$

$$\forall j \in F_k: \quad \sum_{i:\hat{x}_i^j = 0} x_i + \sum_{i:\hat{x}_i^j = 1} (1 - x_i) \geqslant 1$$

$$\forall j \in O_k: \quad \tilde{\gamma}_j - \tilde{\gamma}_j \left(\sum_{i:\hat{x}_i^j = 0} x_i + \sum_{i:\hat{x}_i^j = 1} (1 - x_i) \right) \leqslant \kappa$$

$$\forall t \in \mathcal{T} \setminus \{r\} \forall n \in \mathcal{V}: \quad \sum_{a \in \delta^+(n)} f_a^t - \sum_{a \in \delta^-(n)} f_a^t = \begin{cases} 1 & \text{if } n = r \\ -1 & \text{if } n = t \\ 0 & \text{else} \end{cases}$$

$$\forall a \in \mathcal{A}: \quad f_a^t \leqslant s_a$$

$$\forall e \in \mathcal{E}: \quad s_{(i,j)} + s_{(j,i)} = x_e$$

$$\forall a \in \mathcal{A}: \quad f_a \geqslant 0$$

$$x, s \in \{0, 1\}$$

Subproblem$(\hat{x})$

$$c_{\text{hangers}}(\hat{x}) := \min \sum_{n \in \mathcal{V}_{\text{free}}(\hat{x})} c_n$$

$$\forall \mathcal{V} \in \mathcal{V}_{\text{free}}(\hat{x}) : \gamma_1 h_n^2 + \gamma_2 h_n \mathbf{u}_n + \gamma_3 \mathbf{u}_n^2 + \gamma_4 h_n + \gamma_5 \mathbf{u}_n \leqslant c_n + M(1 - y_n)$$

$$K_c(\hat{x})\mathbf{u}_c = \sum_{e \in \mathcal{E}(\hat{x})} \hat{g}_e \hat{x}_e + \sum_{n \in \mathcal{V}} \tilde{h}_n$$

$$K_h(\hat{x})\mathbf{u}_h = \sum_{e \in \mathcal{E}(\hat{x})} g_e \hat{x}_e + \sum_{n \in \mathcal{V}} \tilde{h}_n$$

$$\forall n \in \mathcal{V}_{\text{free}}(\hat{x}) \quad h_n \leqslant h_{\max} y_n$$

$$R(\hat{x})\mathbf{u}_c \leqslant r(\hat{x})$$

$$R(\hat{x})\mathbf{u}_h \leqslant r(\hat{x})$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad N_e \mathbf{u}_{h,e} = \dot{\mathbf{u}}_{h,e}$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad N_e \mathbf{u}_{c,e} = \dot{\mathbf{u}}_{c,e}$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_1 (\dot{\mathbf{u}}_{h,e})_4^2 + \beta_2 (\dot{\mathbf{u}}_{h,e})_5^2 + \beta_3 (\dot{\mathbf{u}}_{h,e})_6^2 \leqslant \alpha_1^2$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_1 (\dot{\mathbf{u}}_{h,e})_{10}^2 + \beta_2 (\dot{\mathbf{u}}_{h,e})_{11}^2 + \beta_3 (\dot{\mathbf{u}}_{h,e})_{12}^2 \leqslant \alpha_1^2$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_1 (\dot{\mathbf{u}}_{h,e})_4 - \beta_4 (\dot{\mathbf{u}}_{c,e})_4 = (\bar{\mathbf{u}}_e)_4$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_2 (\dot{\mathbf{u}}_{h,e})_5 - \beta_5 (\dot{\mathbf{u}}_{c,e})_5 = (\bar{\mathbf{u}}_e)_5$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_3 (\dot{\mathbf{u}}_{h,e})_6 - \beta_6 (\dot{\mathbf{u}}_{c,e})_6 = (\bar{\mathbf{u}}_e)_6$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_1 (\dot{\mathbf{u}}_{h,e})_{10} - \beta_4 (\dot{\mathbf{u}}_{c,e})_{10} = (\bar{\mathbf{u}}_e)_{10}$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_2 (\dot{\mathbf{u}}_{h,e})_{11} - \beta_5 (\dot{\mathbf{u}}_{c,e})_{11} = (\bar{\mathbf{u}}_e)_{11}$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad \beta_3 (\dot{\mathbf{u}}_{h,e})_{12} - \beta_6 (\dot{\mathbf{u}}_{c,e})_{12} = (\bar{\mathbf{u}}_e)_{12}$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad (\bar{\mathbf{u}}_e)_4^2 + (\bar{\mathbf{u}}_e)_5^2 + (\bar{\mathbf{u}}_e)_6^2 \leqslant \alpha_2^2$$

$$\forall e \in \mathcal{E}(\hat{x}) : \quad (\bar{\mathbf{u}}_e)_{10}^2 + (\bar{\mathbf{u}}_e)_{11}^2 + (\bar{\mathbf{u}}_e)_{12}^2 \leqslant \alpha_2^2$$

Check of infeasibility

Idea

Fast check of infeasibility by solving a system of linear equations:

$$\begin{aligned} & K(\hat{x})u = b + \sum_{n \in \mathcal{V}} l_n h_n \\ \Leftrightarrow \quad & u = K^{-1}b + \sum_{n \in \mathcal{V}} K^{-1} l_n h_n \end{aligned}$$

Simple test whether $\underline{u} \leqslant u \leqslant \bar{u}$ is attainable

If

$$\sum_{n \in \mathcal{V}} \max \left\{ K^{-1} l_n h^{\max}, 0 \right\} \ngeqslant \underline{u} - K^{-1}b$$

or

$$\sum_{n \in \mathcal{V}} \min \left\{ K^{-1} l_n h^{\max}, 0 \right\} \nleqslant \bar{u} - K^{-1}b$$

holds, then the subproblem is infeasible.

Cutting planes for the subproblem

Subproblem

$$\begin{aligned} \min\ & \textstyle\sum_{n \in \mathcal{V}} c_n y_n \\ \text{s.t.}\ & Ku = b + \textstyle\sum_{n \in \mathcal{V}} l_n h_n \\ & 0 \leqslant h_n \leqslant h^{\max} y_n \\ & \underline{u} \leqslant u \leqslant \overline{u} \\ & y_n \in \{0, 1\} \end{aligned}$$

Cutting planes

$$\sum_{n:\gamma_n<0} \left(\lfloor \gamma_n h^{\max} \rfloor + \frac{(\{\gamma_n h^{\max}\} - \{\beta\})^+}{1 - \{\beta\}} \right) y_n +$$

$$\sum_{n:\gamma_n>0} \left(\lfloor \gamma_n h^{\max} \rfloor + \frac{(\{\gamma_n h^{\max}\} - \{\beta\})^+}{1 - \{\beta\}} - \frac{1}{1 - \{\beta\}} \gamma_n h^{\max} \right) y_n + \sum_{n:\gamma_n>0} \frac{1}{1 - \{\beta\}} \gamma_n h_n \leqslant \lfloor \beta \rfloor$$

with $\gamma_n := K^{-1} l_n$ and $\beta := \overline{u} - K^{-1} b$.

Derivation of the cutting planes

$$Ku = b + \sum_{n \in \mathcal{V}} l_n h_n \tag{1}$$

$$\Leftrightarrow \qquad u = K^{-1} b + \sum_{n \in \mathcal{V}} K^{-1} l_n h_n \tag{2}$$

- Use bounds $u \leqslant \bar{u}$ and (2) to get

$$\sum_{n \in \mathcal{V}} K^{-1} l_n h_n \leqslant \bar{u} - K^{-1} b \tag{3}$$

- Insert $h_n = h^{\text{max}} y_n - s_n$:

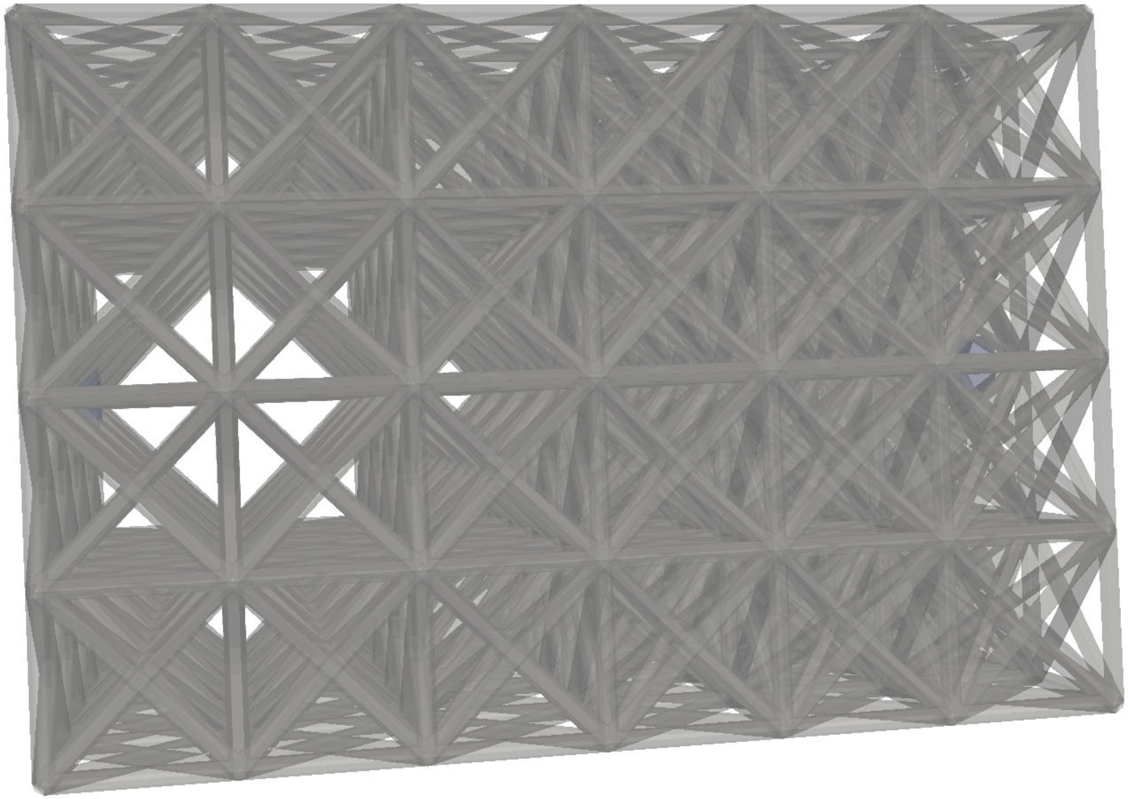
$$\sum_{n \in \mathcal{V}} K^{-1} l_n \left(h^{\text{max}} y_n - s_n \right) \leqslant \bar{u} - K^{-1} b \tag{4}$$

- Carefully aggregate slack variables and apply mixed integer rounding procedure.

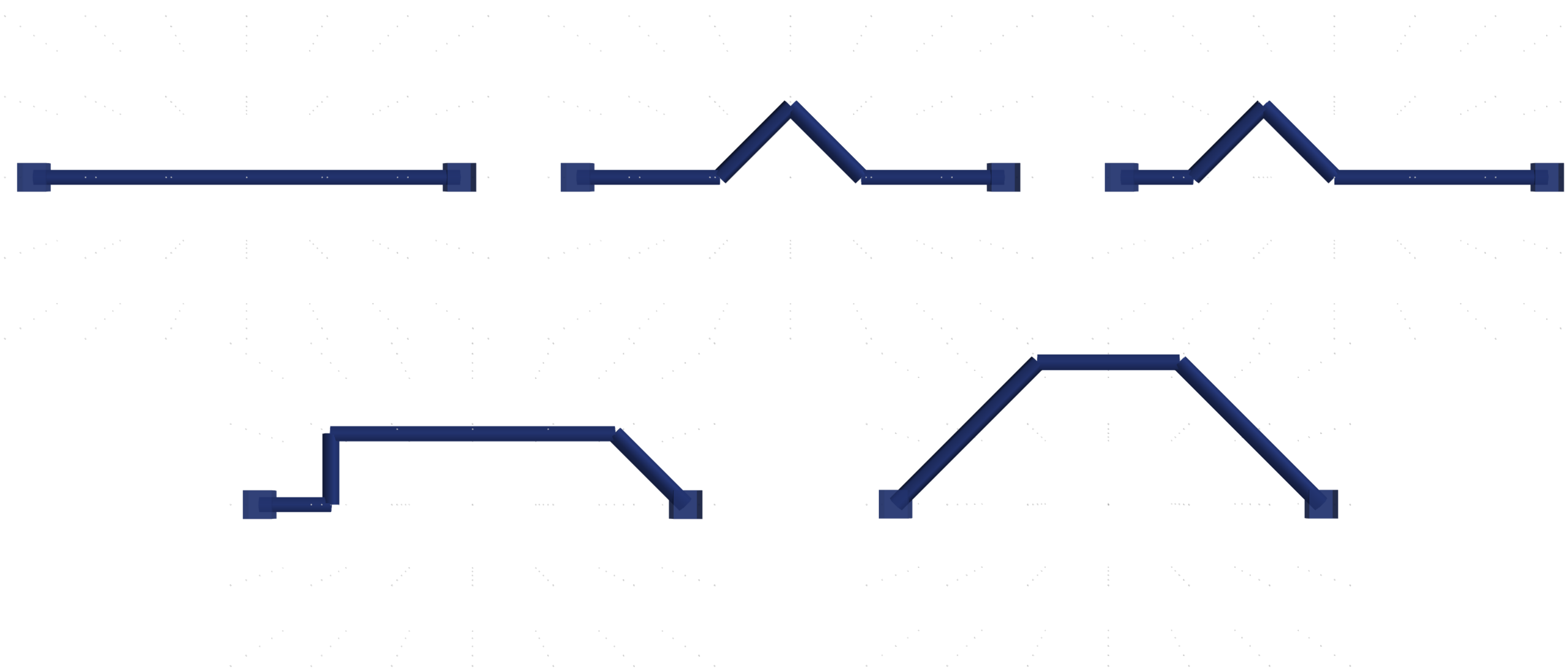
Some numerical tables

Name	$\|\mathcal{V}\|$	$\|\mathcal{E}\|$	$\|\mathcal{T}\|$	MIP (adaptiv) Time (s)	MIP (adaptiv) Gap (%)	Decomposition Time (s)	Decomposition $\#cb$	Decomposition Gap (%)
cube04	9	32	2	6.3	0	0.27	3	0
temptest	63	434	2	7200	41.7	515.6	2	0
BB-04	123	974	3	7200	27.8	7200	5074	0.07
BB-04_ff	141	964	3	7200	–	7200	5367	0.04
big-04	250	2241	4	7200	–	4249.8	10	0

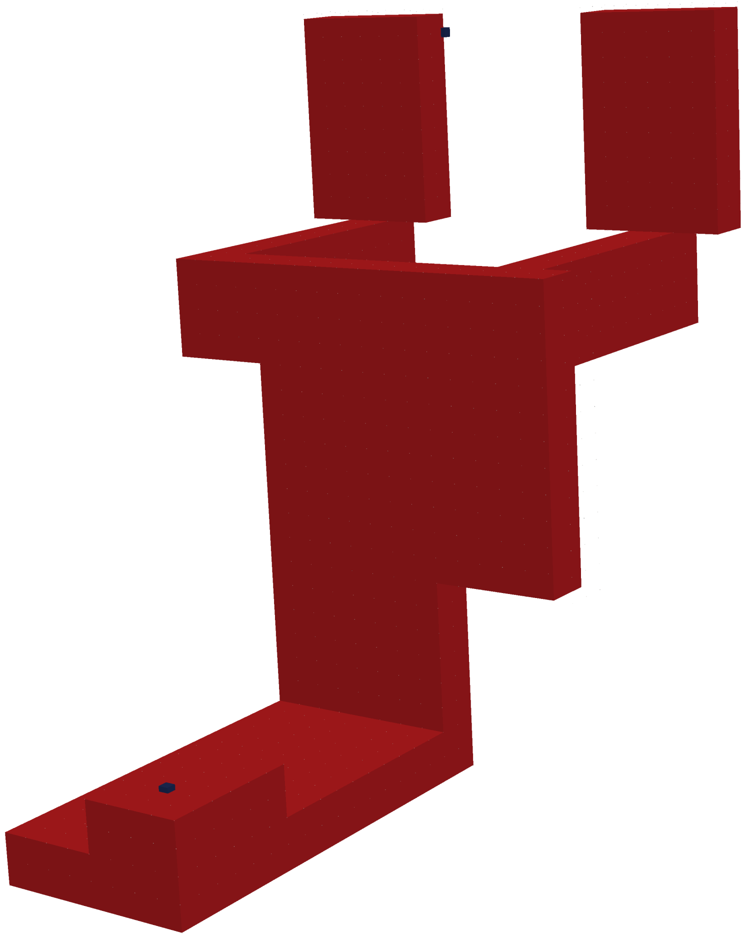
Examples – Physics matters!



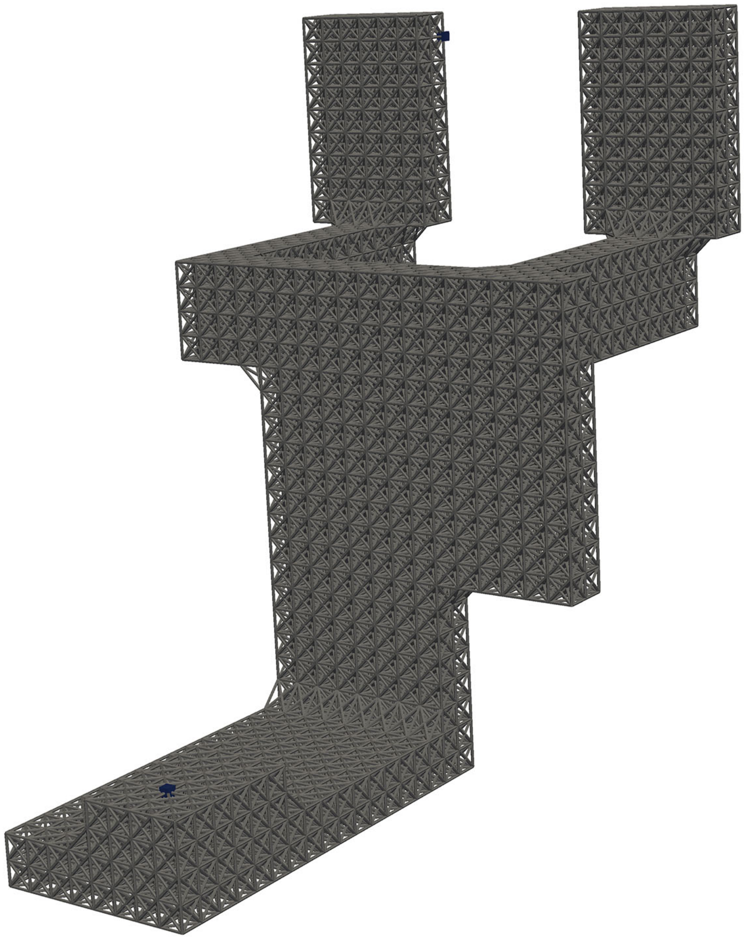
Examples – Physics matters!



Examples – Complete optimization

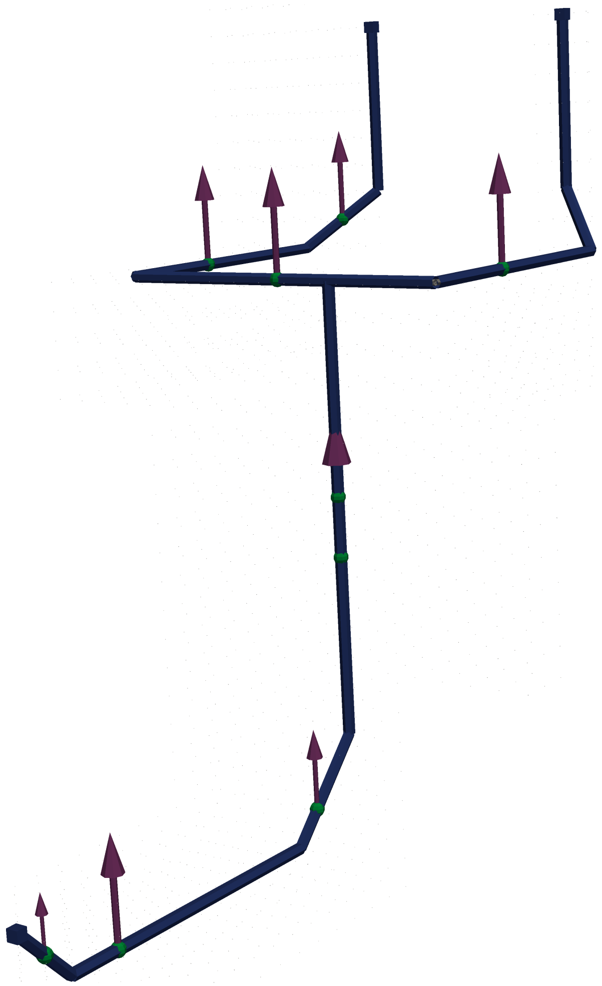


Examples – Complete optimization



Examples – Complete optimization

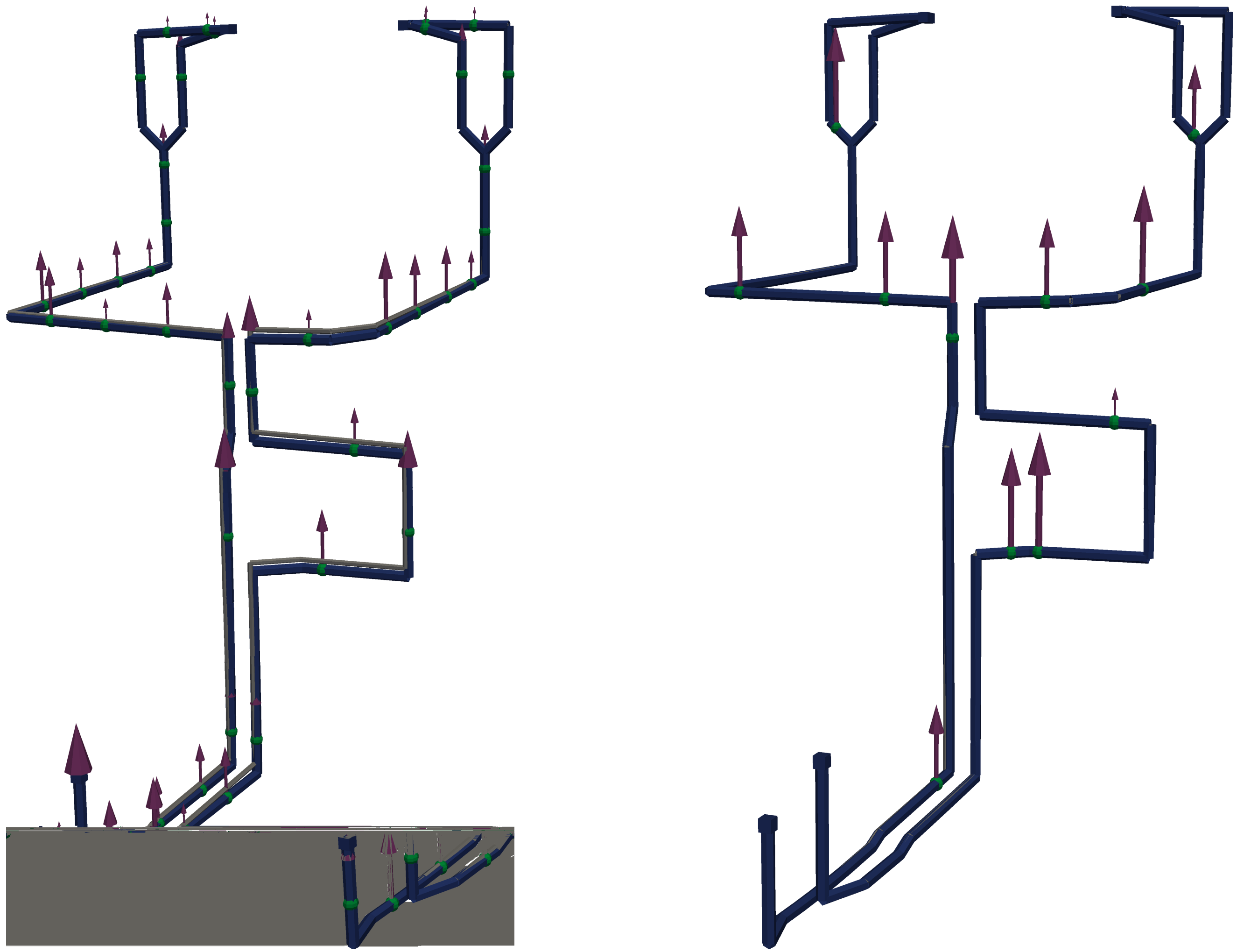
Examples – Complete optimization



Type	Amount[a]
Investment costs	2.369.084 €
Operating costs	7.671.916 €
CO_2 costs	1.500.286 €
Overall costs	11.212.866 €

[a]Assumptions based on data by Bilfinger

Examples – Subproblem: Choosing hangers



Examples – Subproblem: Choosing hangers

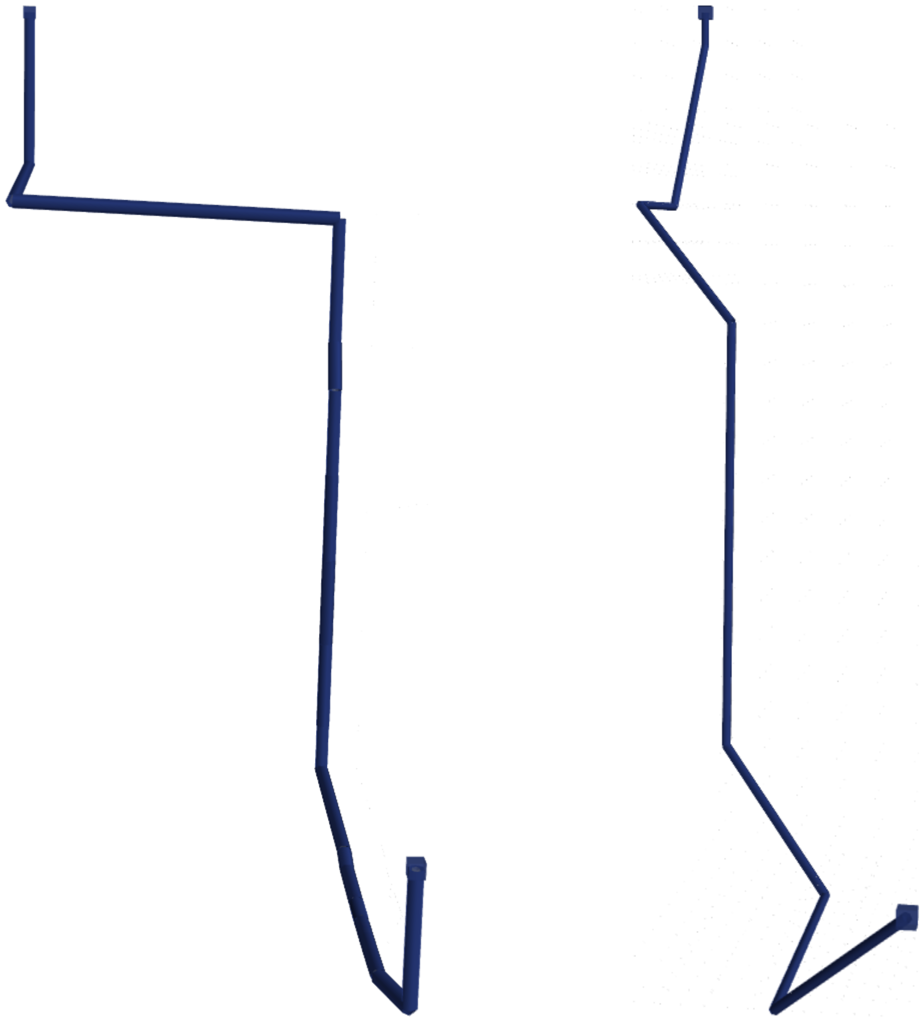
Original Pipe System

Type	Amount
Investment costs	4.820.084 €
Operating costs	24.413.594 €
CO_2 costs	4.772.725 €
Overall costs	34.006.403 €

Optimized Pipe System

Type	Amount
Investment costs	4.531.156 €
Operating costs	24.413.594 €
CO_2 costs	4.772.015 €
Overall costs	33.716.765 €

Examples – Masterproblem: Routing the pipe



Examples – Masterproblem: Routing the pipe

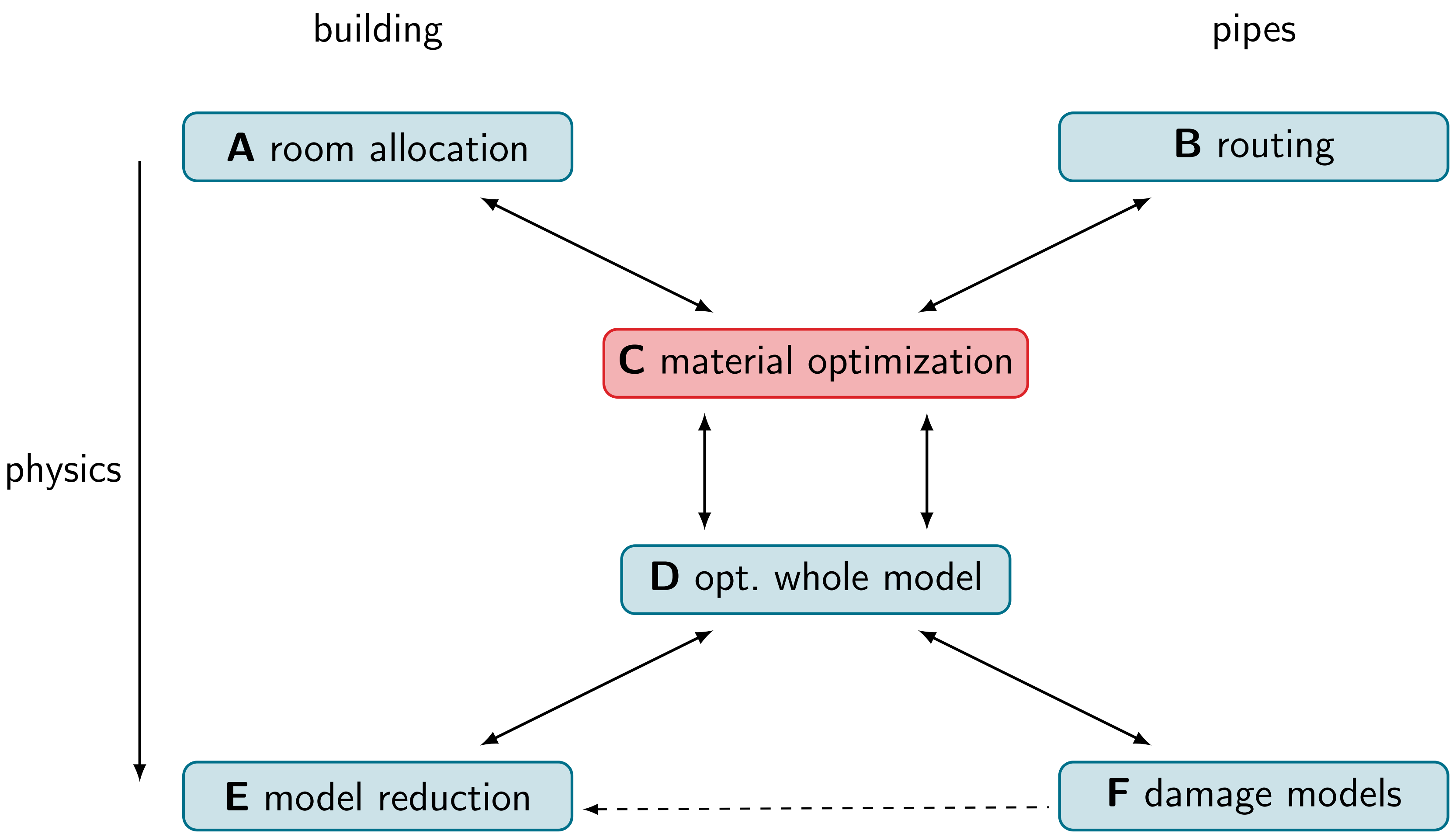
Original Pipe System

Type	Amount
Investment costs	1.504.890 €
Operating costs	8.510.002 €
CO_2 costs	1.662.596 €
Overall costs	11.677.488 €
Bendings	9 (528°)
Length	$154,9\mathrm{m}$

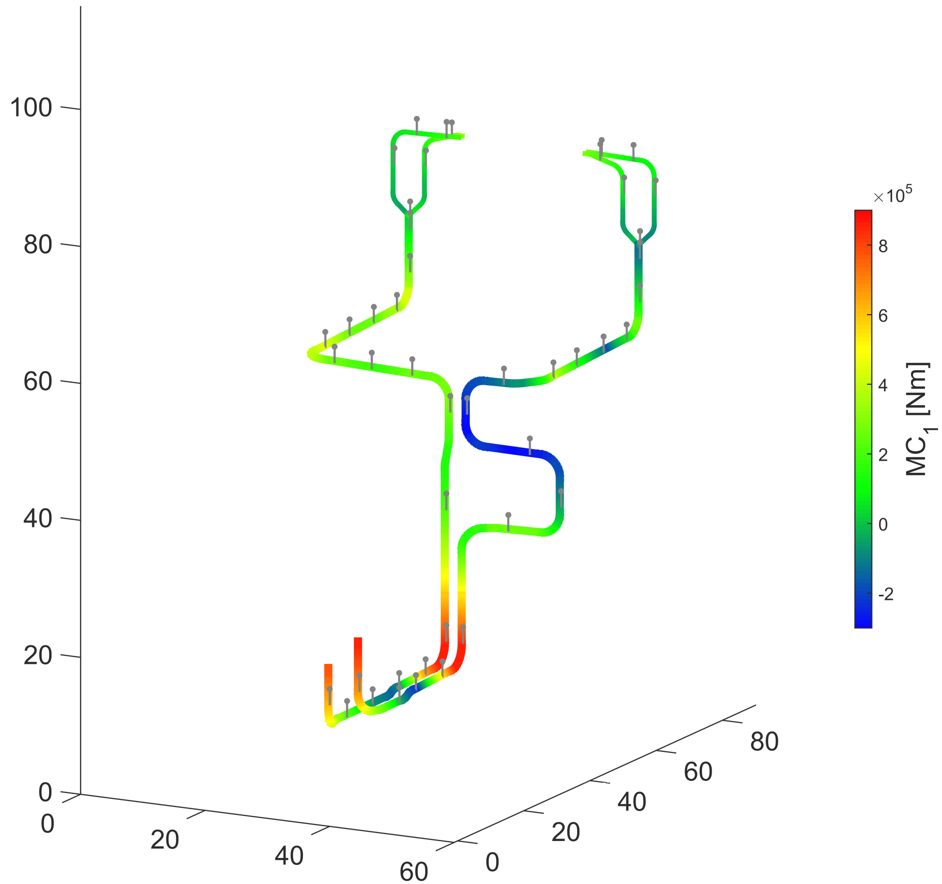
Optimized Pipe System

Type	Amount
Investment costs	1.172.310 €
Operating costs	6.459.276 €
CO_2 costs	1.262.028 €
Overall costs	8.893.615 €
Bendings	7 (363°)
Length	$121,6\mathrm{m}$

Improvement of 23%


building
pipes
A room allocation
B routing
C material optimization
physics
D opt. whole model
E model reduction
F damage models

Bilfinger piping system (initial)



Optimization of piping systems - Overview

Basic concept in subproject C

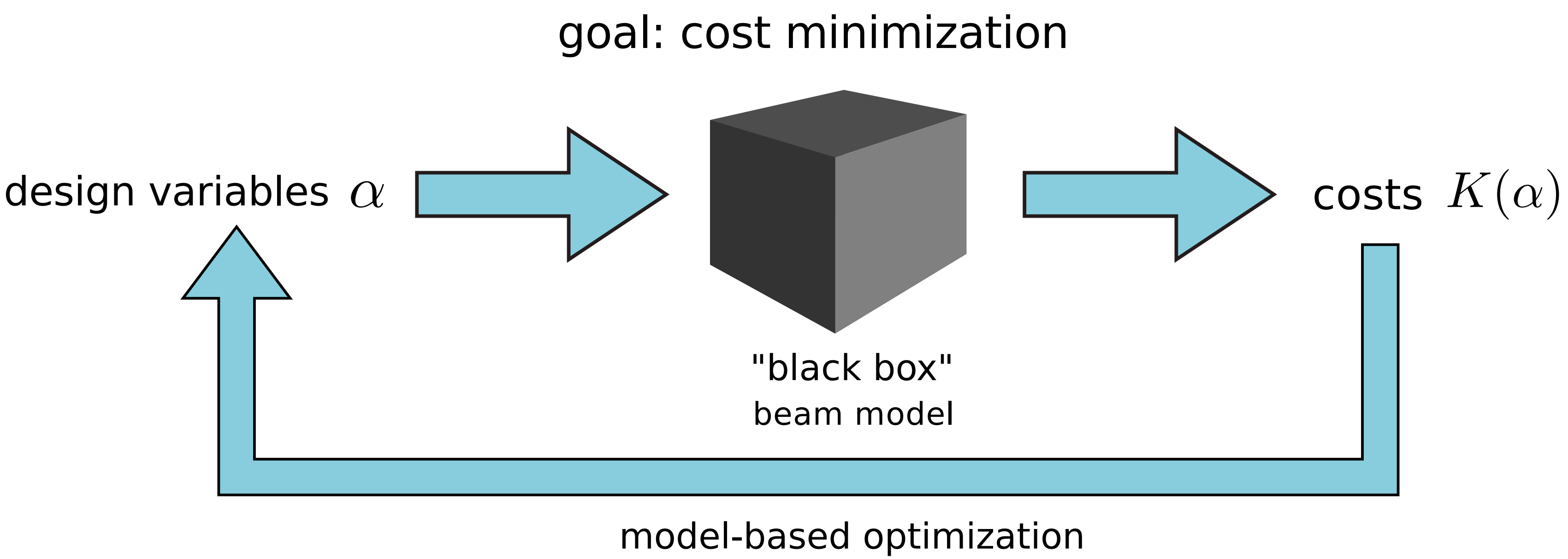
Input:

- Fixed **topology** of piping system
- Possible **diameters** of pipes, **hangers/supports** and **steel grades**
- Constraints on **stresses** and **geometry**

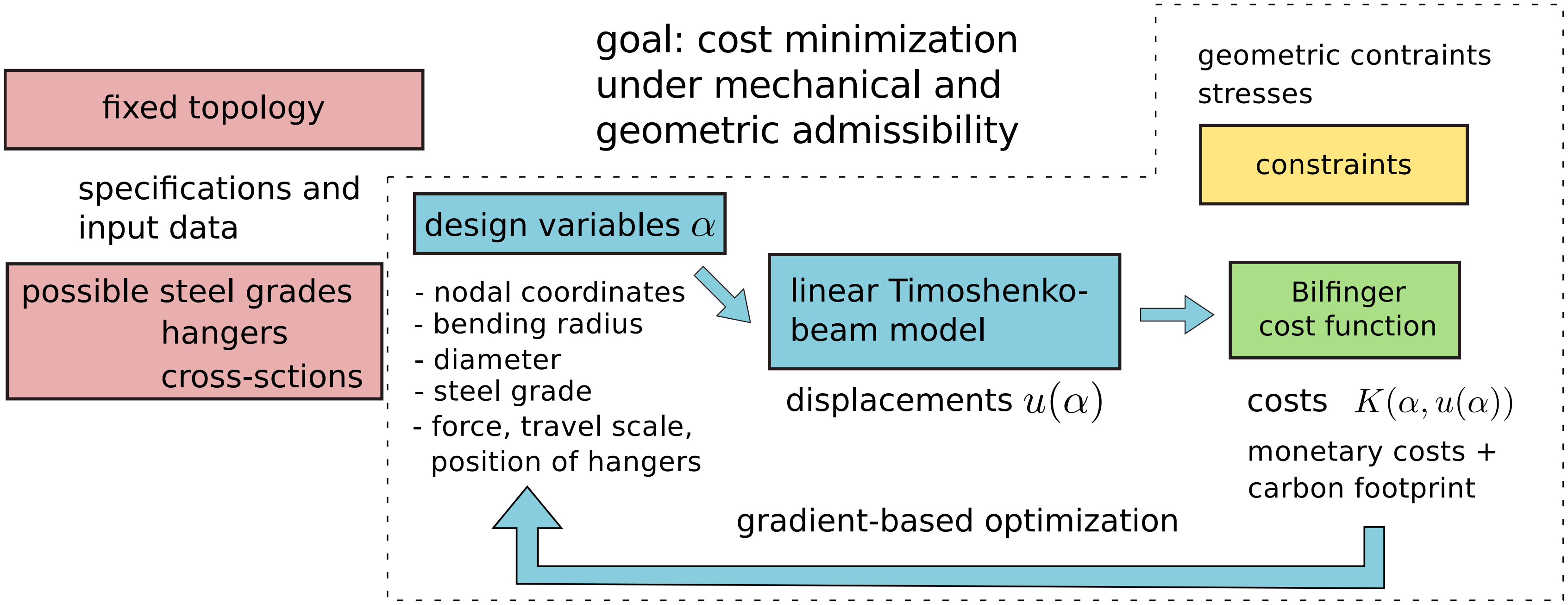
Output:

- Optimal **geometry of piping system**
- Optimal **steel grade** and **diameter**
- Force and **position** of **hangers/supports**

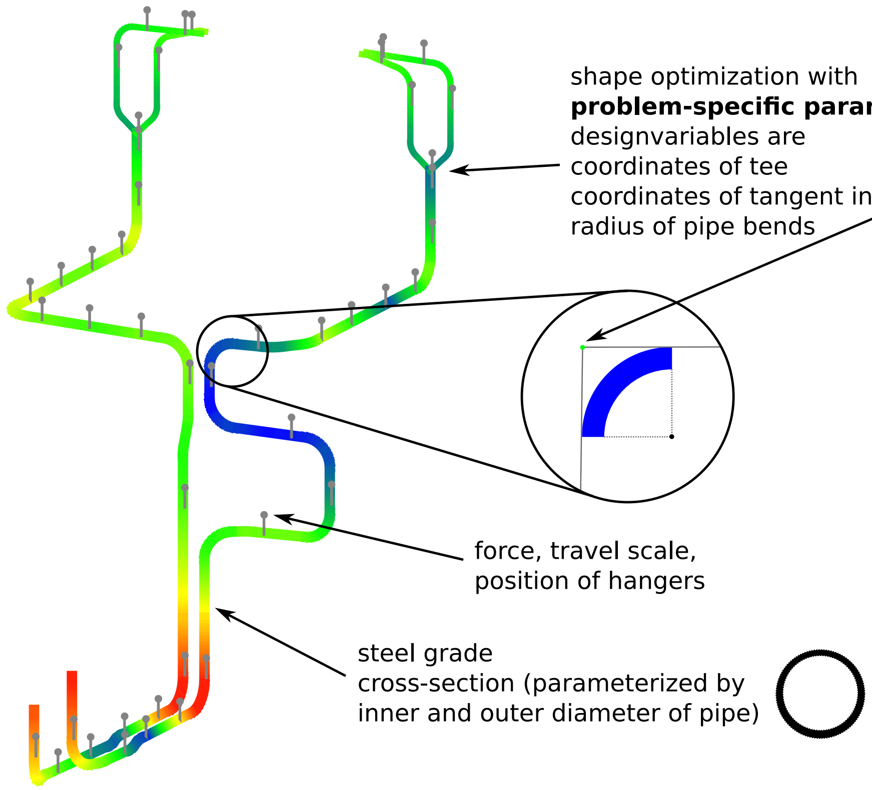
Optimization scheme (1)



Optimization scheme (2)



Design Variables



Entire model

Nonlinear optimization problem

$$\min_{\alpha \in \mathbb{R}^n} \sum \text{investment costs} + \sum \text{operating costs} + \sum CO_2 \text{ costs}$$

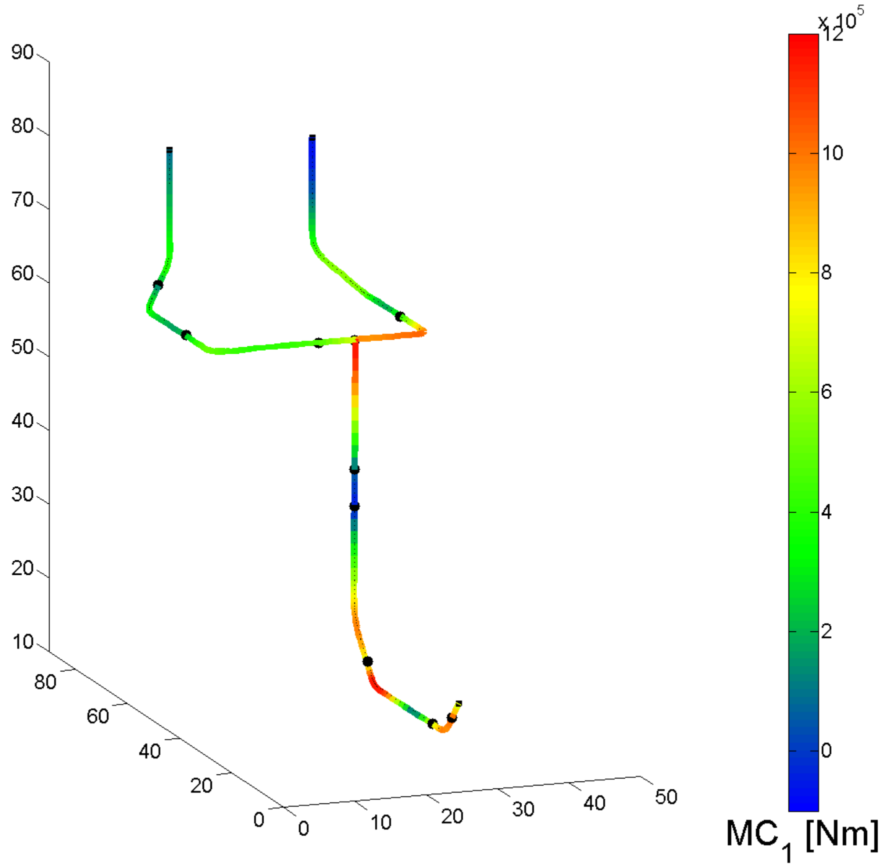
s.t. analysis model $K(\alpha)u = f(\alpha)$

admissible piping system (stress constraints, displacements, connection loads, geometry − boxes, resist against internal pressure, force of hangers)

geometrical constraints (max/min angle, radii, slope constraints)

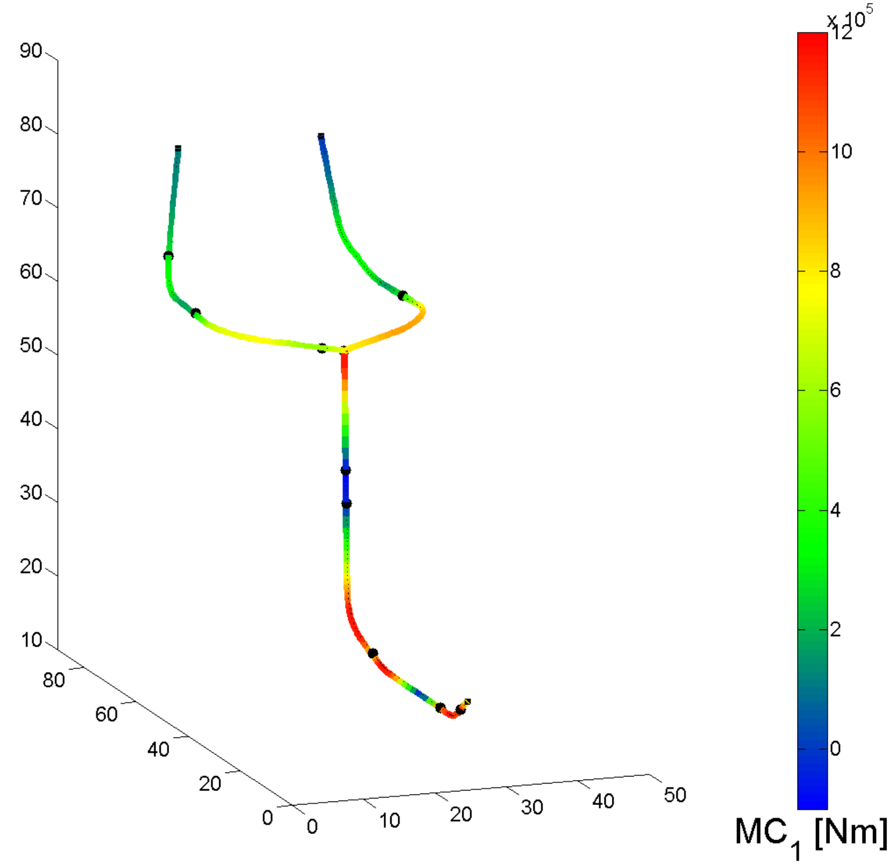
Results - academic example

Piping system (initial), costs: 11.352.112 €



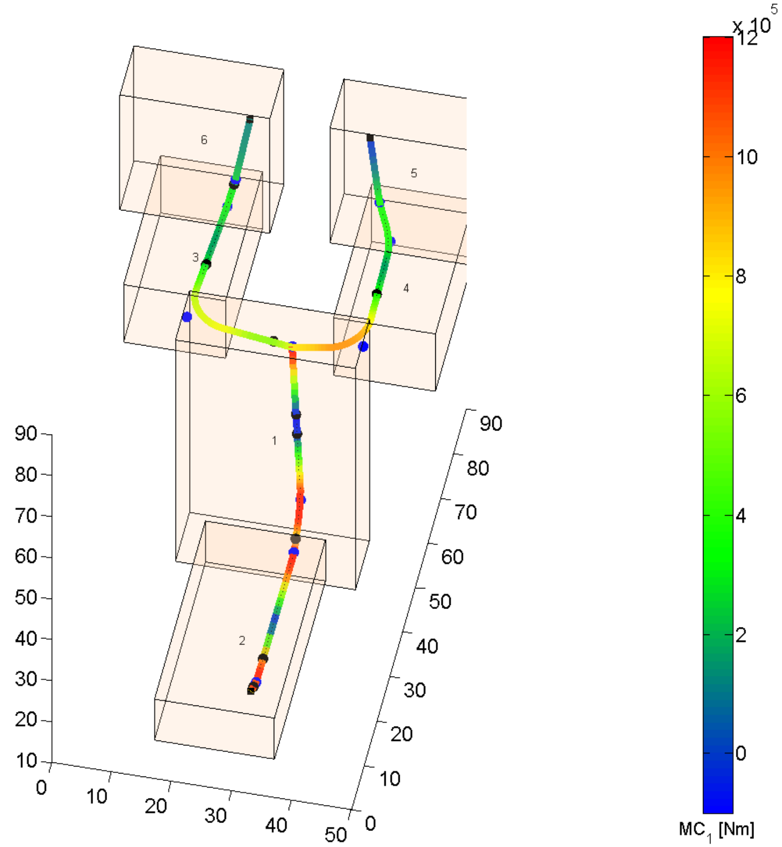
Results - academic example

After shape-optimization, costs: 8.266.301 €



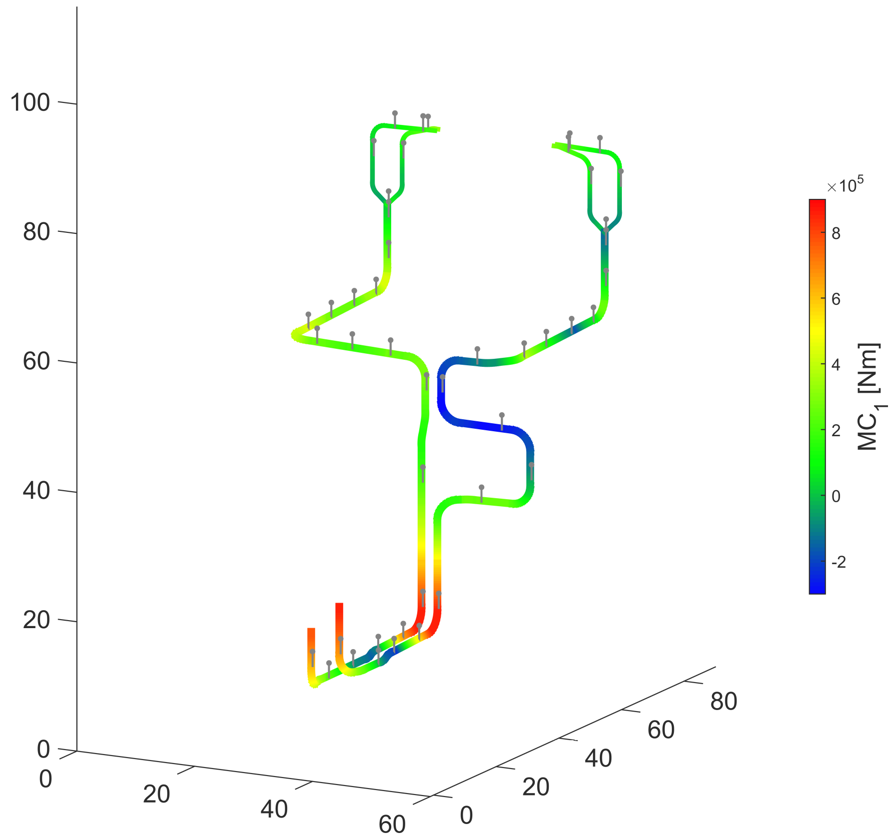
Results - academic example

Geometry-boxes



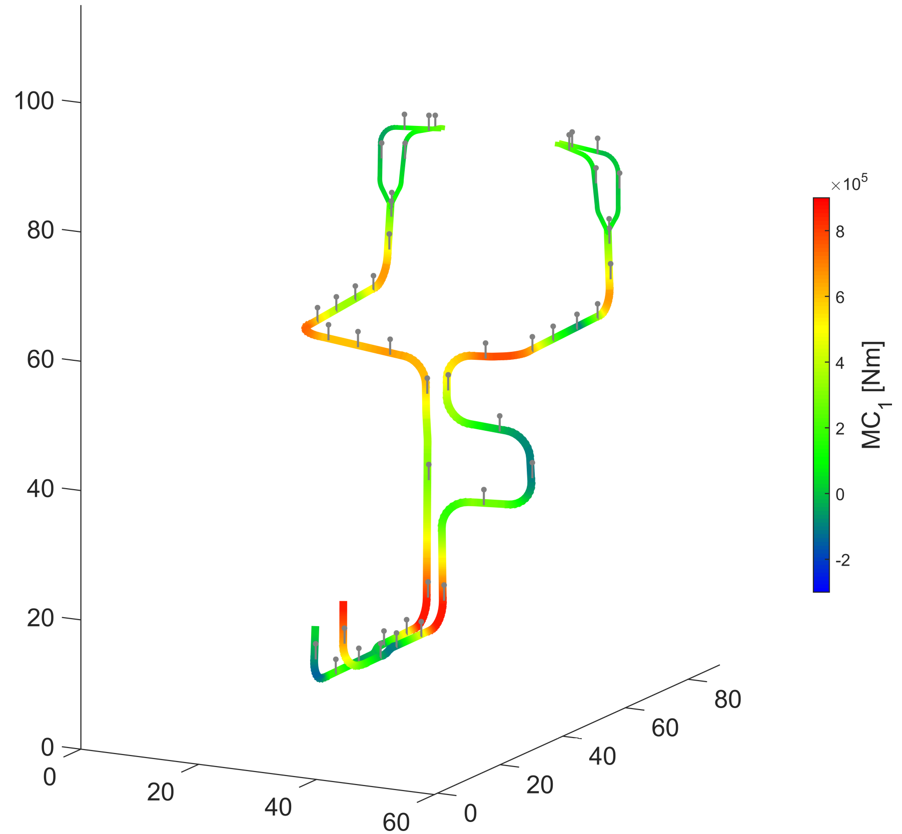
Results - Bilfinger piping system

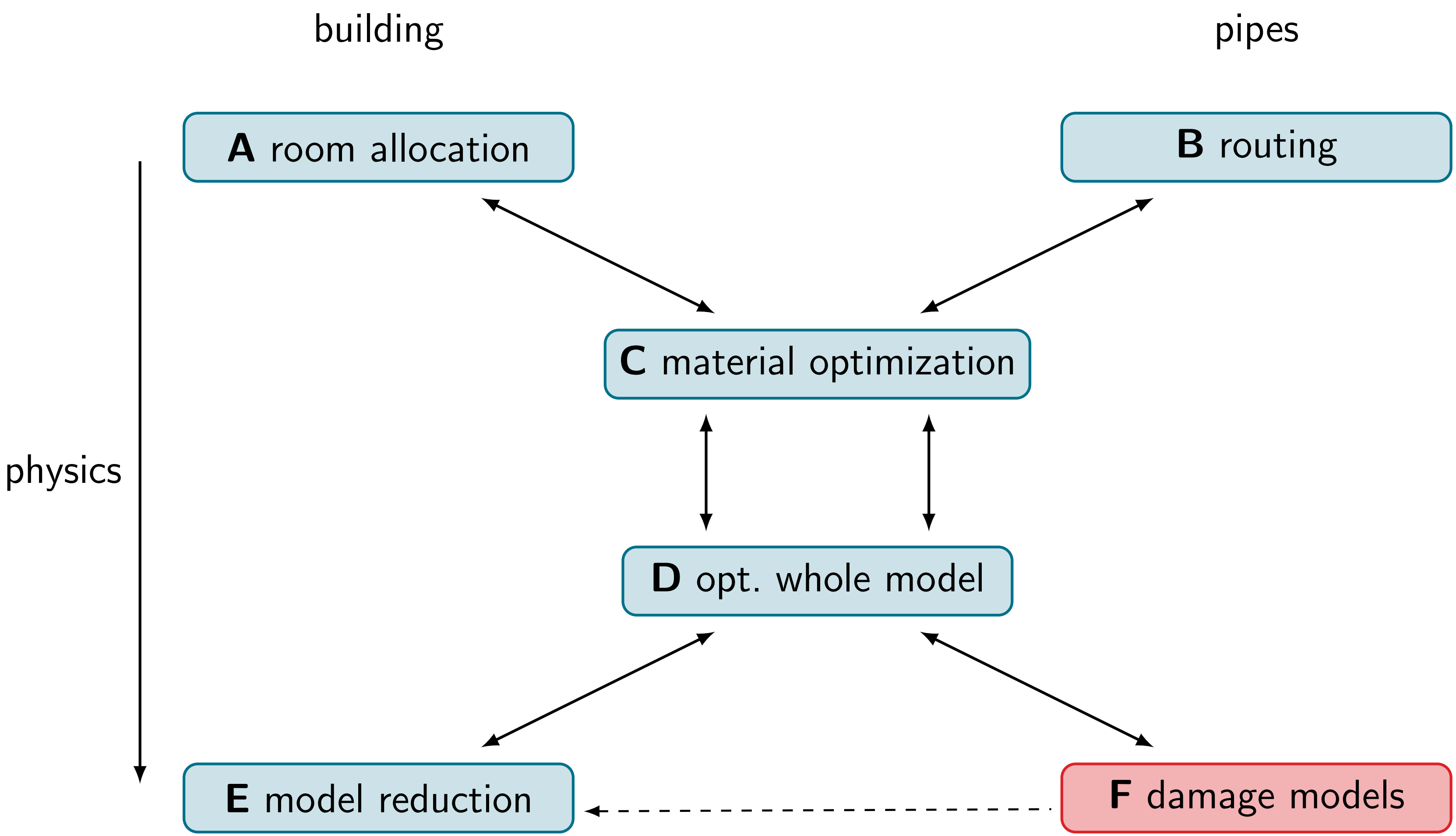
Bilfinger Piping System (initial), costs 26.557.306 €



Results - Bilfinger piping system

Bilfinger Piping System (full optimization), costs 22.064.639 €, 17% benefit




building
pipes
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physics

Overview sub project F

Simulation toolbox

- Linear and non-linear beam models:
  - straight / pre-twisted elements
  - simulation of arbitrary networks (dynamic / quasi-static / static)
- Different types of hangers (linear and non-linear):
  - spring hangers (modelled as truss)
  - rigid hangers (as stiff beams)
  - constant hangers (as dead load or special follower load)
- Different material laws:
  - linear visco-thermo-elasticity (Kelvin-Voigt damping)
  - plasticity (with linear hardening)
  - creep, creep damage

Modeling of a Pipe

Pipes are **thin three-dimensional** objects.

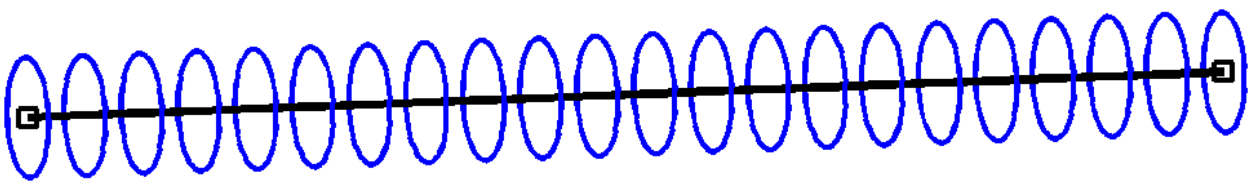
⇒ Reduction of the full 3D description seems reasonable

This is achieved in two steps:

- Assumptions on how the object can deform: **constrained motion**

$$\mathbf{u}^{\text{full}}(\mathbf{x}) = \mathbf{u}(x_1) + [\boldsymbol{\Lambda}(\boldsymbol{\phi}) - \mathbf{I}] \begin{pmatrix} 0 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= \text{displacement of centerline} + \text{rotation of cross sections}$$



- Insertion into 3D continuum equations and **averaging** over cross sections

⇒ Description of pipes by just one spatial variable: **beam theory**!

Two Beam Models

- Linear approximation $\Lambda v \approx v + \phi \times v$ and linear continuum equations
- $\Lambda \in SO(3)$ and non-linear continuum equations

linear Timoshenko beam (u, ϕ)	geometrically exact beam (u, Λ)
$\rho_0 A \ddot{u} - N' = N^{ext}$	$\rho_0 A \ddot{u} - (\Lambda N)' = N^{ext}$
$\rho_0 J \ddot{\phi} - M' - E_1 \times N = M^{ext}$	$\rho_0 I(J, \ddot{\Lambda}) - (\Lambda M)' - (u' + E_1) \times (\Lambda N) = M^{ext}$
$N = C_N \varepsilon^{lin} = C_N (u' - \phi \times E_1)$	$N = C_N \Gamma = C_N \Lambda^T (u' - (\Lambda - I) E_1)$
$M = C_M \kappa^{lin} = C_M \phi'$	$M = C_M \Upsilon = C_M \, \mathrm{vec}(\Lambda^T \Lambda' - \Lambda_0^T \Lambda_0')$

Matrices containing material and geometrical parameters

$$C_N = \begin{pmatrix} EA & 0 & 0 \\ 0 & GA_2 & 0 \\ 0 & 0 & GA_3 \end{pmatrix}, \quad C_M = \begin{pmatrix} GI_t & 0 & 0 \\ 0 & EI_2 & 0 \\ 0 & 0 & EI_3 \end{pmatrix}, \quad J = \begin{pmatrix} I_t & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$

Dynamics with nonlinear spring and constant hanger

Buckling of a pipe under thermal load

Quasi-static loading (gravity and heat) real piping system

Quasi-static loading (gravity and heat) real piping system

Dynamic setting with plasticity

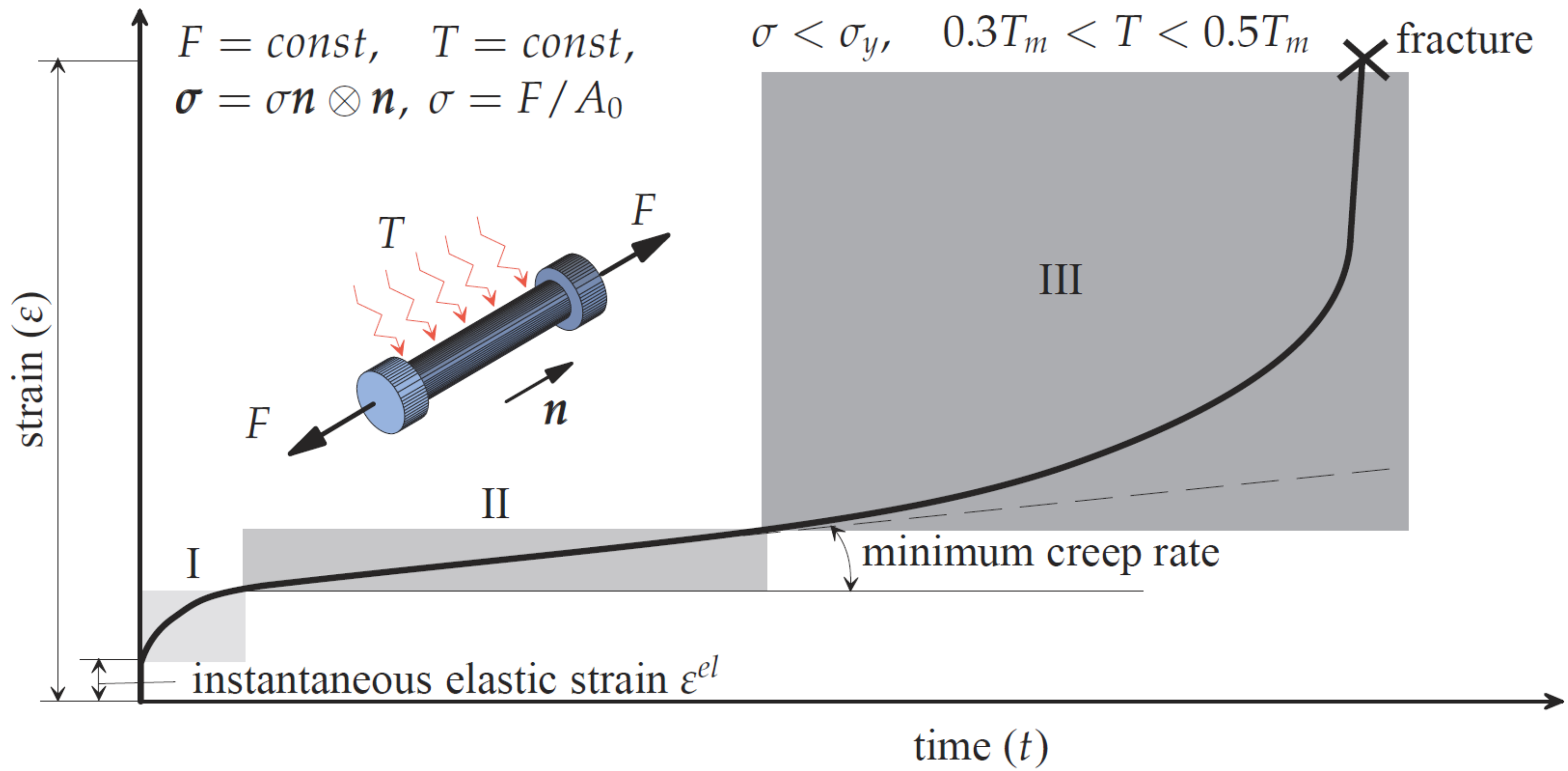
Dynamic setting with plasticity

Estimation of life cycle

- Multi-faceted problem:
  - Material history (plasticity, creep)
  - Material degradation (corrosion, erosion)
  - Material failure (crack initiation, fracture)
- Huge diversity of approaches and models available
- Occurring effects depend strongly on type of material and loads

$\Rightarrow$ Triple **material**, **process** and **environment** substantial for the choice of describing model

Creep Stages



(Figure: Modeling of High-Temperature Creep for Structural Analysis Applications, Konstantin Naumenko, 2006)

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Creep Model

Implemented model reproduces the three relevant creep stages.

Idea:

- Simple Bailey-Norton model (power creep law) describes second creep stage
- Third stage of accelerated creep until rupture is modeled by introducing a damage variable
- For the fist stage (of decelerating creep) hardening terms are added

Creep Model

Evolution equations for creep strain have to be coupled to beam equations

$$\dot{\boldsymbol{\Sigma}}_{cr} = d\,\dot{\varepsilon}_{cr}\frac{\mathbf{F}_{el}}{\|\mathbf{F}_{el}\|}$$

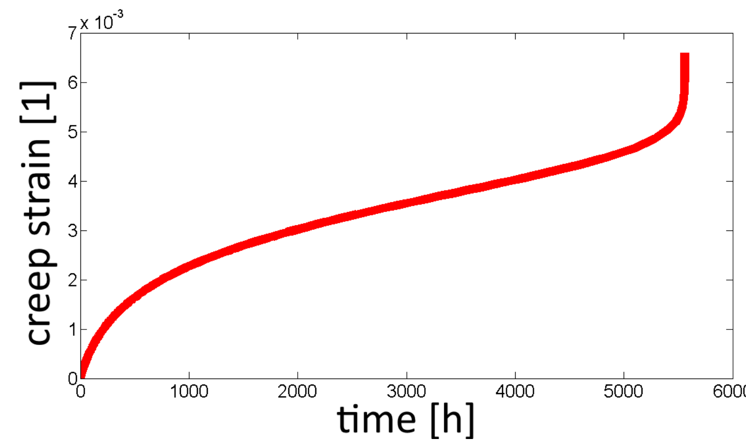
$$\dot{\varepsilon}_{cr} = A\left[1 + C\exp\left(-\frac{\varepsilon_{cr}}{k}\right)\right]\left(\frac{\|\mathbf{F}_{el}\|}{1-\omega}\right)^{m_1}$$

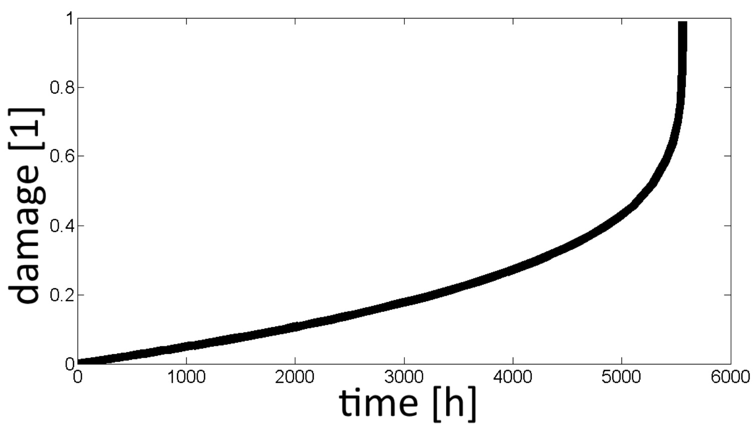
$$\dot{\omega} = B\,\frac{(\|\mathbf{F}_{el}\|)^{m_2}}{(1-\omega)^{m_3}}$$

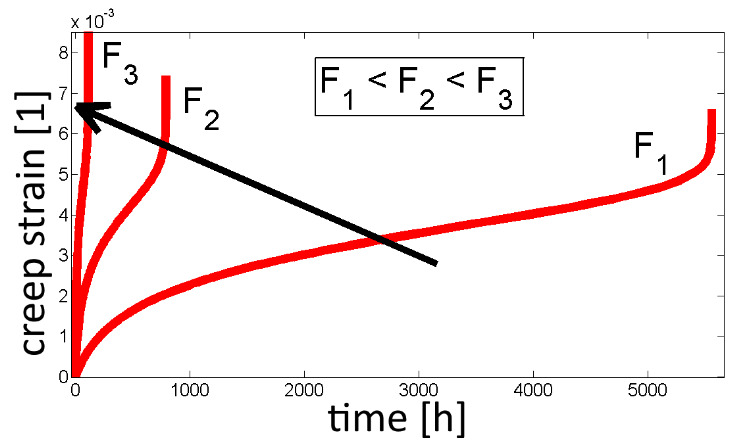
Material parameters

- Stage I: C, k
- Stage II: A and d, m_1
- Stage III: B and m_2, m_3

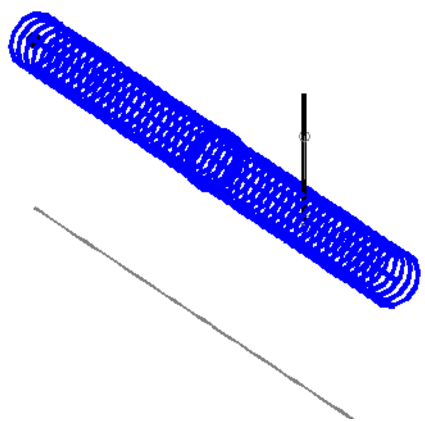
Creep Damage – Tension Test (qualitative validation)

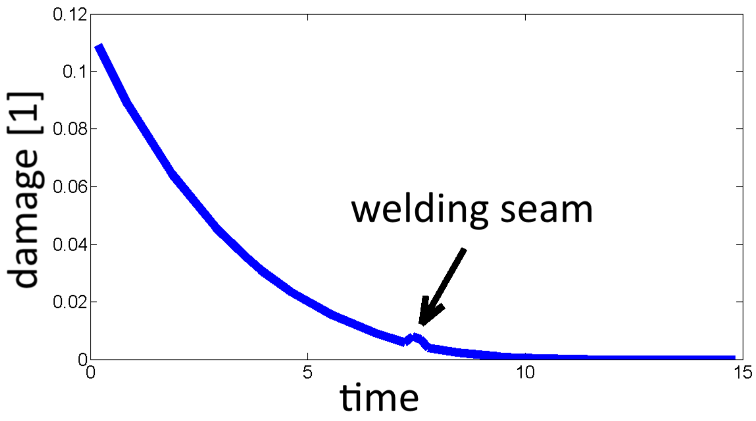






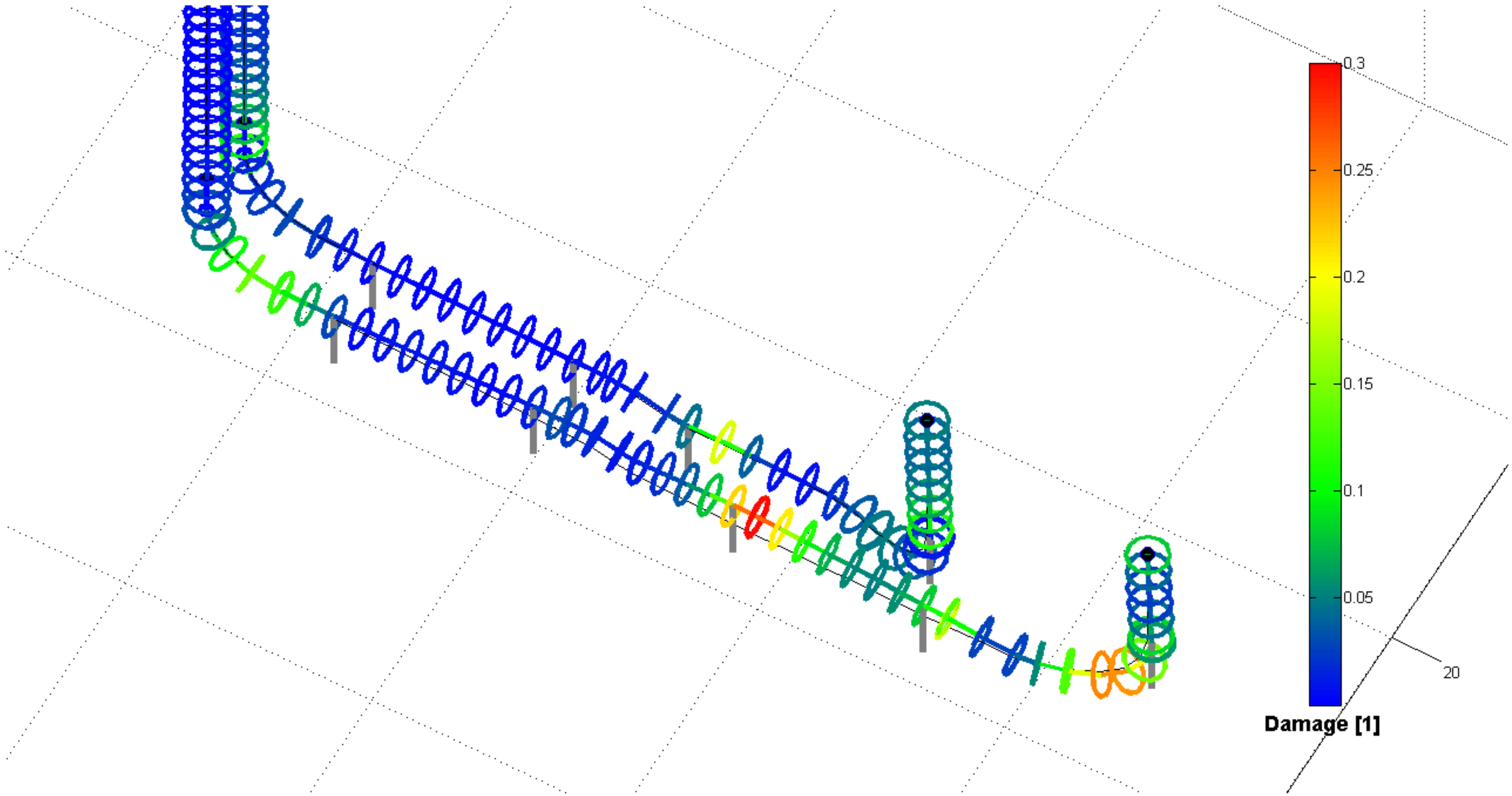
Creep Damage – Hanger and Welding Seam



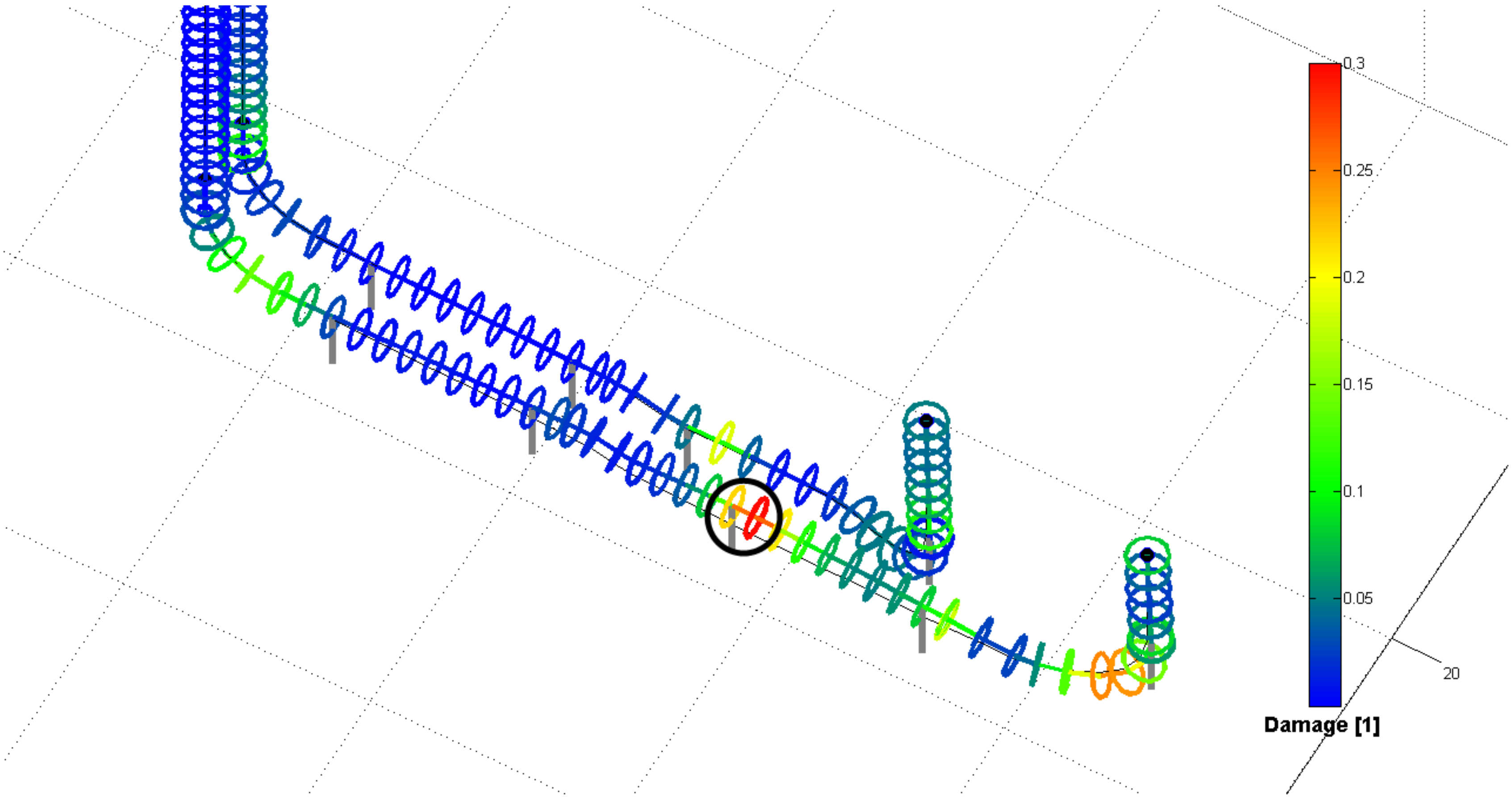


Creep Damage – Real System

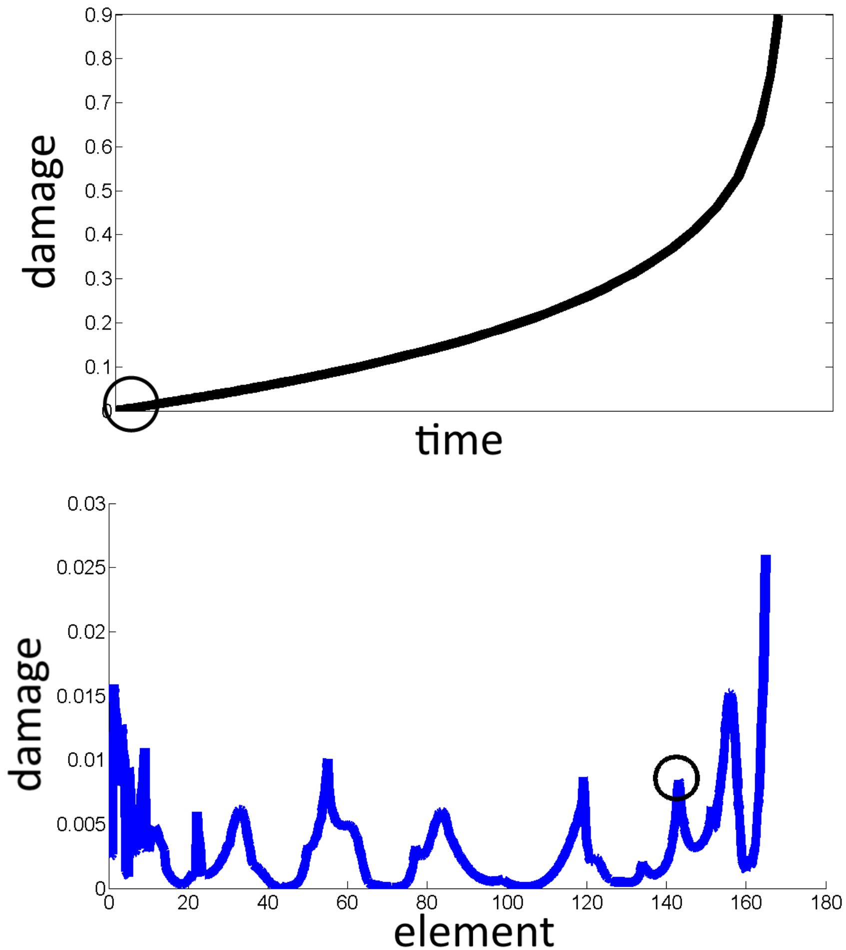
Creep Damage – Real System (detail analysis)



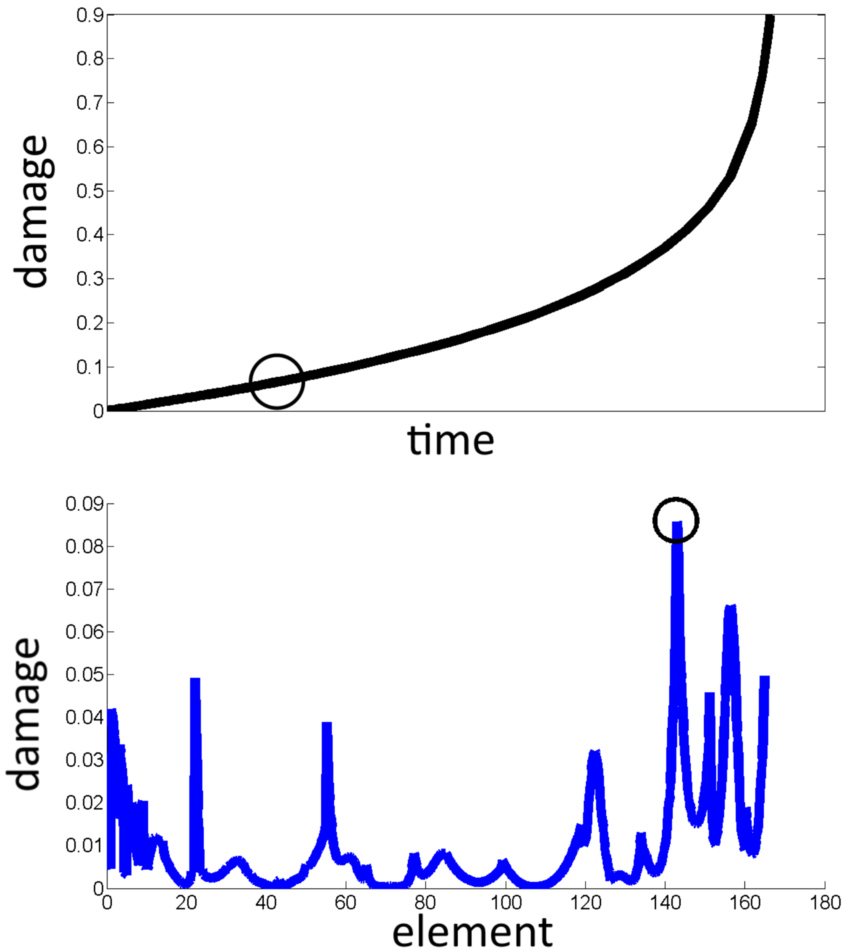
Creep Damage – Real System (detail analysis)



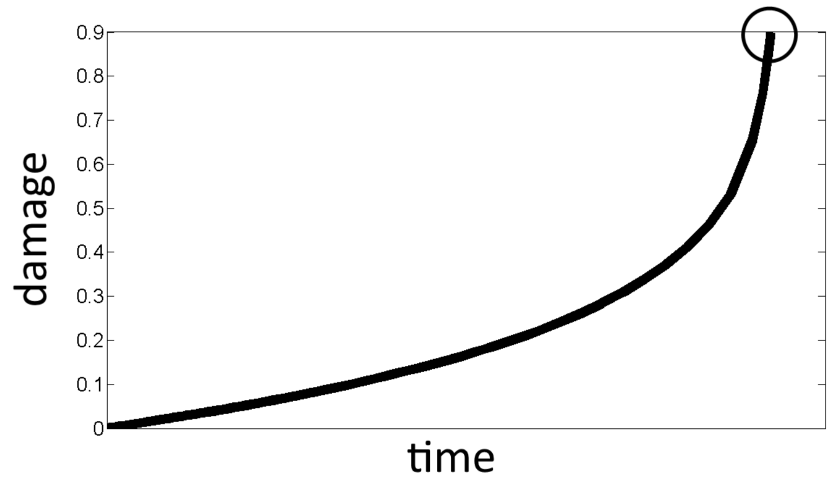
Creep Damage – Real System (detail analysis)

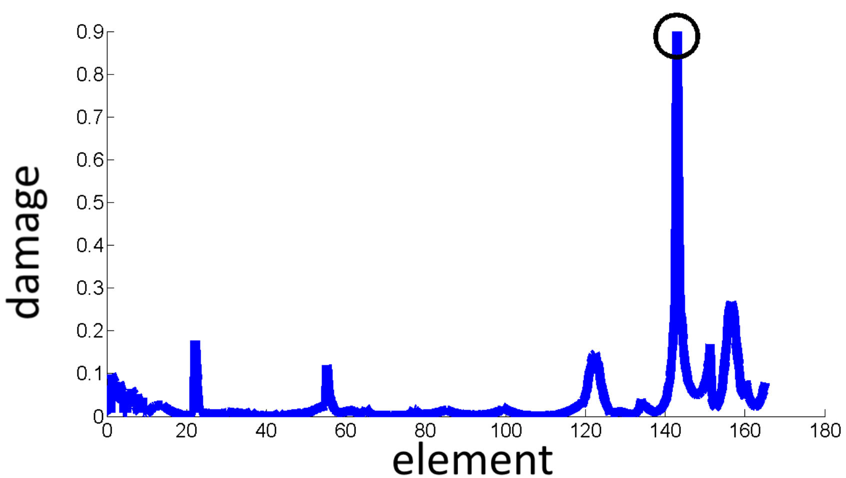


Creep Damage – Real System (detail analysis)



Creep Damage – Real System (detail analysis)





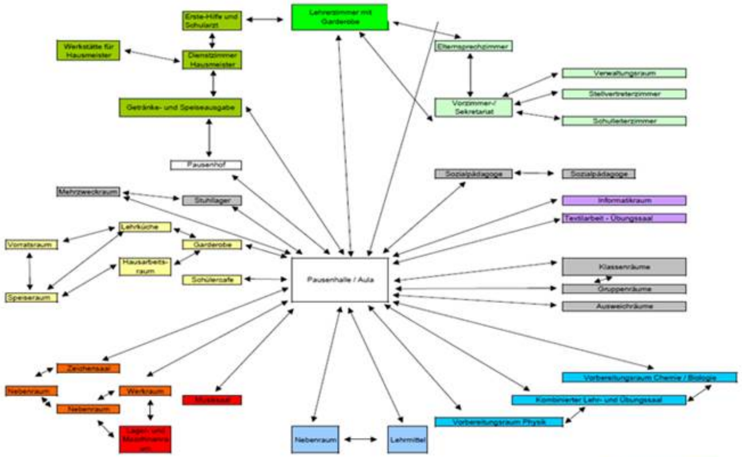
Building

Challenges in planning building

Raumprogramm

Art	Anzahl	jeweils qm
Allg. Unterrichtsraum für GS	08	66
Allg. Unterrichtsraum für HS (Regelklassen und spezielle Regeklassen)	08	66
Allg. Unterrichtsraum E-Zug	02	66
Gruppenraum	05	42
Mehrzweckraum	01	89
Lehrmittelraum	insg.	50
PCB-Saal	01	70
Vorbereitung Physik/Chemie	02	17
Musiksaal	01	70
Zeichensaal	01	70
Nebenraum Zeichensaal	01	20
Werkraum	01	66
Werkraum	01	75
Nebenraum Werken	01	20
Maschinenraum (bislang nicht vorhanden)	01	33
Textilarbeitsraum	01	60
Textilarbeitsraum	01	60

Funktionsschema



Aufriss

Side-constraints in the planning of buildings

- Estate with building line, height restrictions, etc.
- Specific side-constraints for different kinds of room utilization
- Escape routes
- Environmental factors (e.g. outside temperatures, solar imissions)
- Technical dependencies
- Costs (material costs, operating costs, CO_2-emisions)
- ...

Selection of the material

- Limited choice of materials and technical facilities
- Average values for material parameters and production costs

AUSSENWAND-Materialien – Tragende Schicht		MATERIALKENNWERTE			HERSTELLUNG		INSTAND-SETZUNG	WARTUNG + INSPEKTION
	Dicke	Wärmeleit-fähigkeit λ	Rohdichte ρ	Wärmespeicher-fähigkeit c	Herstellungs-kosten	CO_2-Emissionen	Lebensdauer	Kosten Wartung + Inspektion
	[cm]	[W/mK]			[€/m²]	[kg/m²]	[a]	[€/m²a]
Stahlbeton	variabel	2,3	2365	880	600 €/m²	309 kg/m²	50	0,10%
Porenbetonsteine	30	0,16	400	1000	61,09	57,6	50	0,10%
	36,5				74,36	70,08	50	0,10%
Kalksandstein	17,5	0,79	1900	880	52,8	43,5575	50	0,10%
	24				60,3	59,736	50	0,10%
	30				73,4	74,67	50	0,10%
	36,5				87,3	90,8485	50	0,10%
Hochlochziegel	24	0,1	740	1000	57,88	22,3776	50	0,10%
	30				69,65	27,972	50	0,10%
	36,5				76,07	34,0326	50	0,10%
Wärmedämmziegel (mit Perlit gefüllt)	30	0,08	806	1000	93,3	38,1	50	0,10%
	36,5				109,14	46,355	50	0,10%
Hüttenstein (Herstellungskosten und CO_2-Emissionen angenommen wie KS)	17,5	0,64	1600	1000	52,8	43,5575	50	0,10%
	24				60,3	59,736	50	0,10%
	30				73,4	74,67	50	0,10%
	36,5				87,3	90,8485	50	0,10%

Energy Balance

$\Longrightarrow$ Calculation based on DIN EN 18599

- Demand for heating:
  $Q_{h,b,i} = Q_{\text{sink}} - \eta \cdot Q_{\text{source}} - \Delta Q_{c,b}$
- Demand for cooling:
  $Q_{c,b,i} = (1 - \eta) \cdot Q_{\text{source}}$
- Consideration of the demand in heating/cooling for inlet air recycling
- Consideration of losses in heating/cooling energy at generation, distribution, delivery
- Consideration of the demand in electrical energy of the facilities

- Subdivision of the building into zones with different utilization requirements

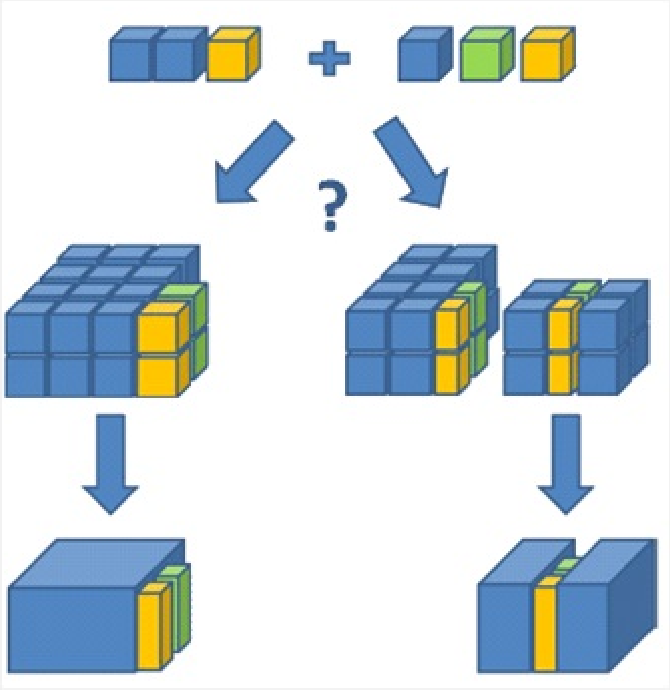
Nutzungsart	Sollinnen-temperatur	Zone
Einzelbüro	21°C	1
Büro (2-6 Arbeitsplätze)	21°C	2
Besprechungszimmer	21°C	3
Flur / Teeküche /Archiv	17°C	4
WC	21°C	5
Treppenhaus	unbeheizt	6

Calculation methods of DIN EN 18599 very complex with many case distinctions
$\Longrightarrow$ Simplifications necessary!

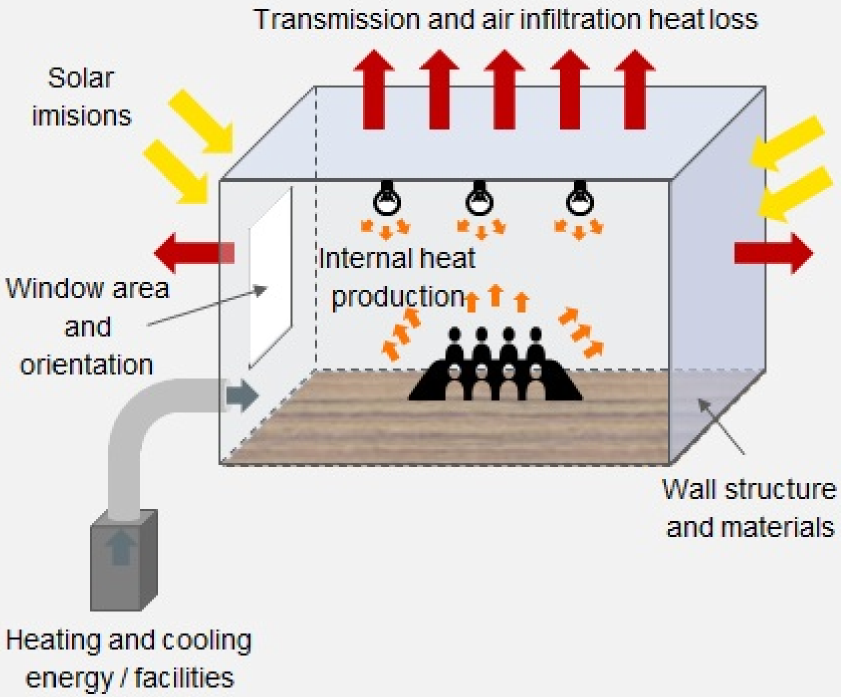
- Restriction to specific types of facilities, e.g. only variable-volume-flow-systems for the air transport selectable
- Generalizations, e.g. no special calculation of the pressure losses of the duct system

Minimization of costs and CO_2-emissions

- Room alignment and orientation



- Materials and energy system

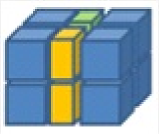


Task sharing and connections between subprojects

Room alignment

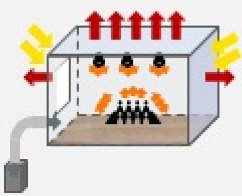
Building equipment

SP A
- Optimization of the room alignment
- Simplified calculation of energy demands
- General cost assumptions based on experience



↑ Material parameters

SP C
- Optimal choice of the material
- Optimization of the technical building equipment (Heating, cooling, air infiltration, lighting)
- Detailed calculation of energy demands
- Detailed calculation of investment and operating cost



Optimization

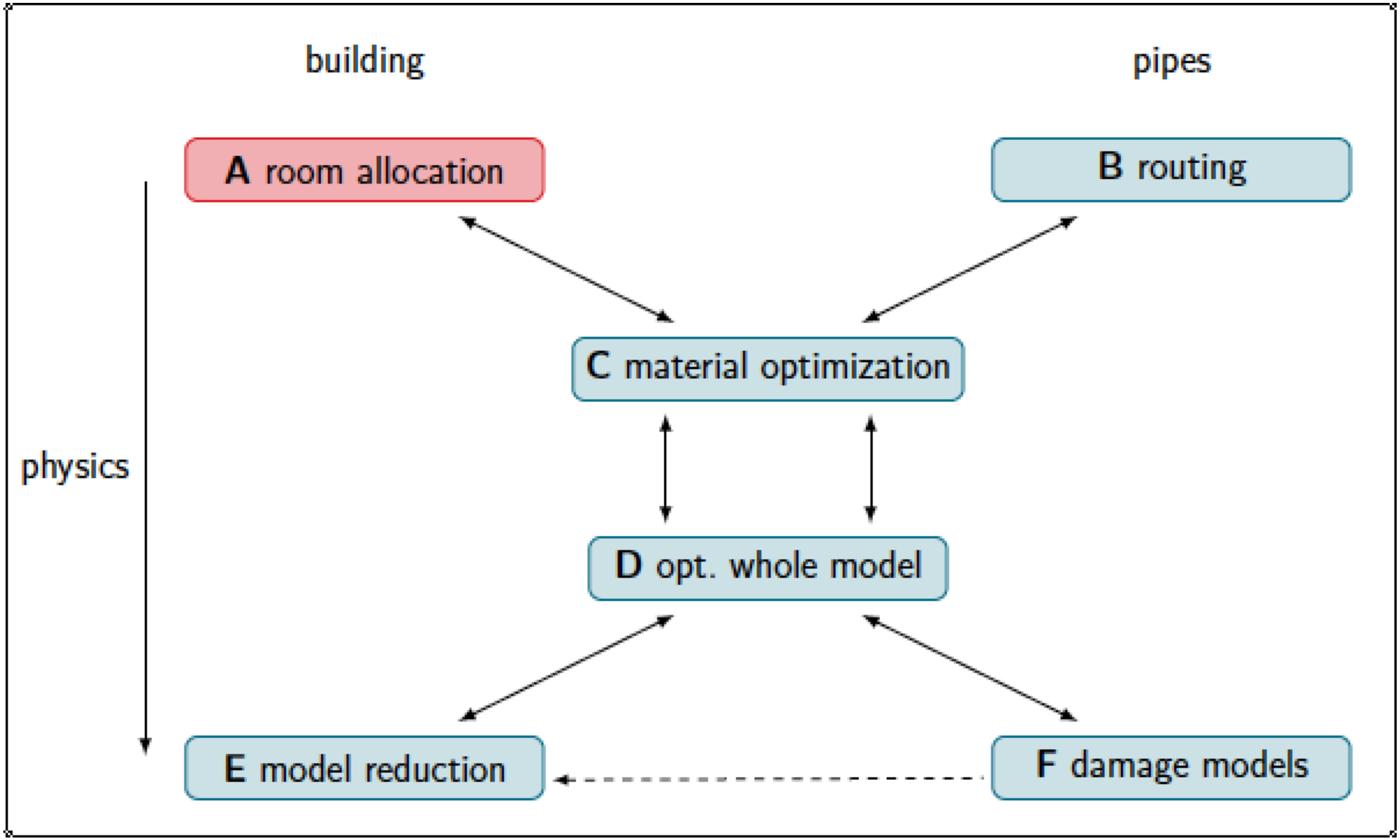
↑ Inside temperature

SP D
- Model reduction
- Statement of an integrated model by coupling the individual simulations

↑ Simulation results

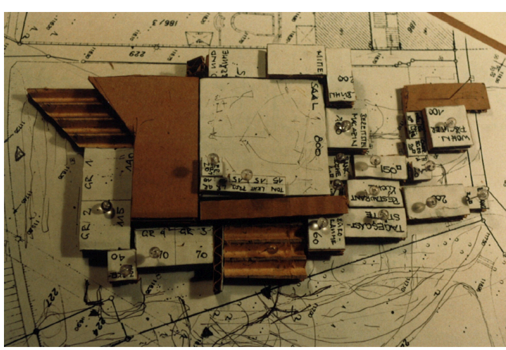
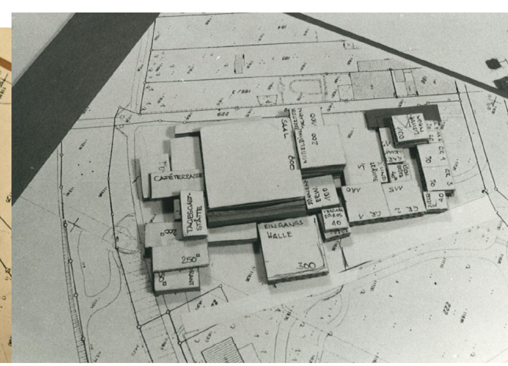
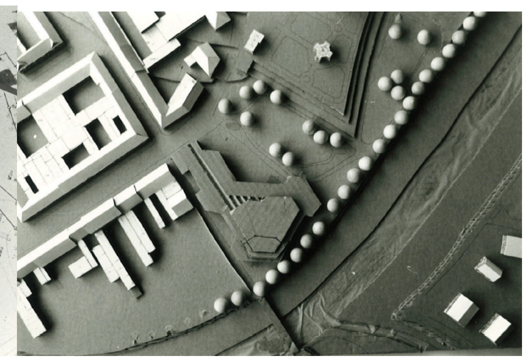
SP E
- Simulation of air circulation and heat transmission inside individual rooms

Complementary Simulation


building
pipes
A room allocation
B routing
C material optimization
physics
D opt. whole model
E model reduction
F damage models

Agenda for Room Allocation

- Problem definition
- Modeling as a MIP
  - Description of the feasible set
  - Description of the objective function
  - Difficulties in solving the MIP
- Alternative Procedure
  - Combination of a deterministic and random based approach
  - Random based local search

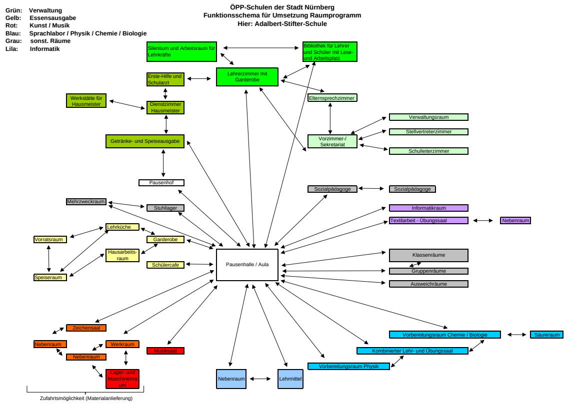
Problem Description

Given:

- Property specifications
- Space allocation plan
- Accounting model

Minimize:

- infrastructural costs
- operational costs
- energy costs



Problem Description

Given:

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- Space allocation plan
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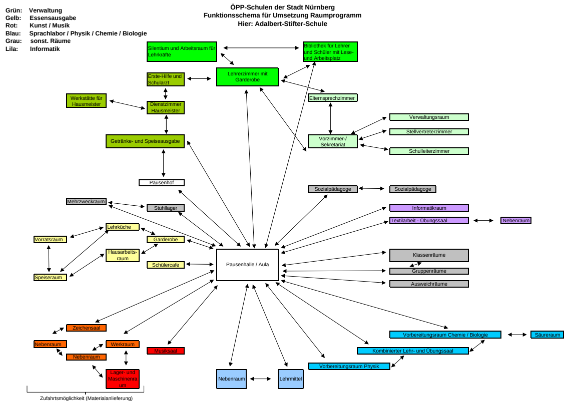
Minimize:

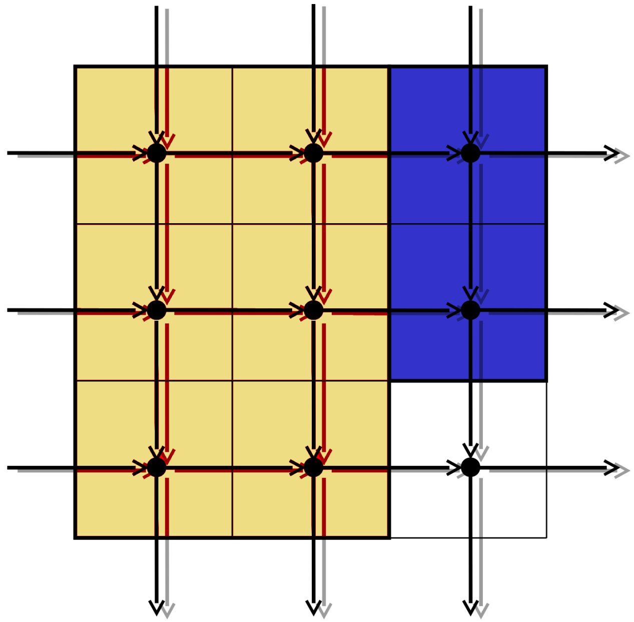
- infrastructural costs
- operational costs
- energy costs

along complete life cycle !



Mathematical modeling of the problem
Description of the solution space

- Reducing the problem to grid graphs



beheizter Raum
unbeheizter Raum

Mathematical modeling of the problem
Description of the solution space

- Constraints
  - Property specifications
  - Space allocation plan
  - Further constraints
    - static requirements
    - windows
    - stair cases and evaluators
    - guidelines for escape routes
    - . . .

$\Longrightarrow$ Modeling with continuous and integer variables

Mathematical modeling of the problem
Objective function

Accounting model:

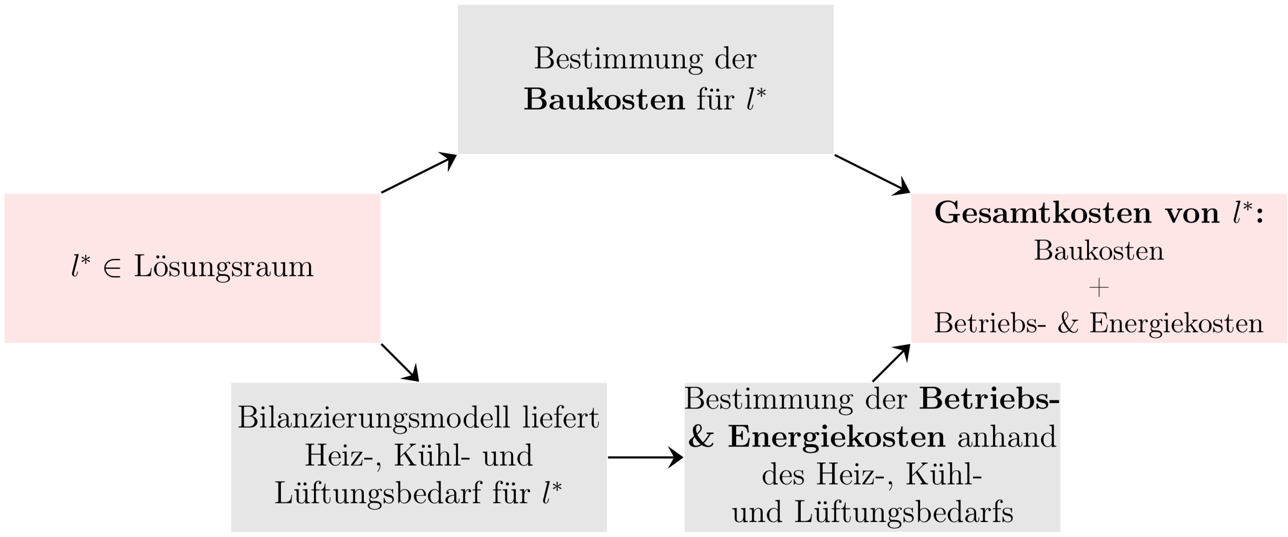
- heat transmission
- radiant heat
- air conditioning
- radiation of opaque components
- internal heat sources
- ...

$\Rightarrow$ Determining heat, cooling and air conditioning demand

Mathematical modeling of the problem
Objective function

Procedure to determine the objective function values

- investment costs
- operation and energy costs



Optimization of the planning of buildings

Objective function:

$K_{\text{total}} = K_{\text{investment}} + K_{\text{operation}}$

Investment cost function:

$K_{\text{I}} = K_{\text{insideT}}(V, \text{mat}, CO_2) + K_{\text{outsideT}}(V, \text{mat}, CO_2) + K_{\text{fundament}}(V, \text{mat}, CO_2) +$
$K_{\text{windows}}(A, \text{mat}, CO_2) + K_{\text{ceiling}}(V, \text{mat}, CO_2) + K_{\text{roof}}(V, \text{mat}, CO_2) +$
$K_{\text{floorCovering}}(A, \text{mat}, CO_2) + K_{\text{heating}}(\text{type}, \text{power}, CO_2) +$
$K_{\text{cooling}}(\text{type}, \text{power}, CO_2) + K_{\text{airInfiltration}}(\text{type}, \text{power}, CO_2) +$
$K_{\text{lighting}}(\text{type}, \text{power}, CO_2) + K_{\text{elevator}}(\text{type}, n_{\text{storeys}}, \text{mat}, CO_2) +$
$K_{\text{stairs}}(n_{\text{storeys}}, \text{mat}, CO_2) + K_{\text{riser}}(n_{\text{storeys}}, \text{mat}, CO_2)$

Operating cost function:

$K_{\text{O}} = K_{\text{energyConsumption}}(\text{EV}) + K_{\text{cleaning}}(\text{R}) + K_{\text{maintance+inspection}}(\text{WI})$

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Solving the mathematical model
Difficulties showing up

The whole model results in an MIP (MIXED INTEGER PROGRAM)

$$\min c^T x$$
$$Ax \leqslant b$$
$$x \in \mathbb{Z}^p \times \mathbb{R}^{n-p}$$

mit $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $p \in \{0, 1, \ldots, n\}$

Solving the mathematical model
Difficulties showing up

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$\Longrightarrow$ MIPs are inappropriate for floor planning problems (2D bin packing)

Solving the mathematical model
Difficulties showing up

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$$\begin{aligned}\min c^T x\\ Ax \leqslant b\\ x \in \mathbb{Z}^p \times \mathbb{R}^{n-p}\end{aligned}$$

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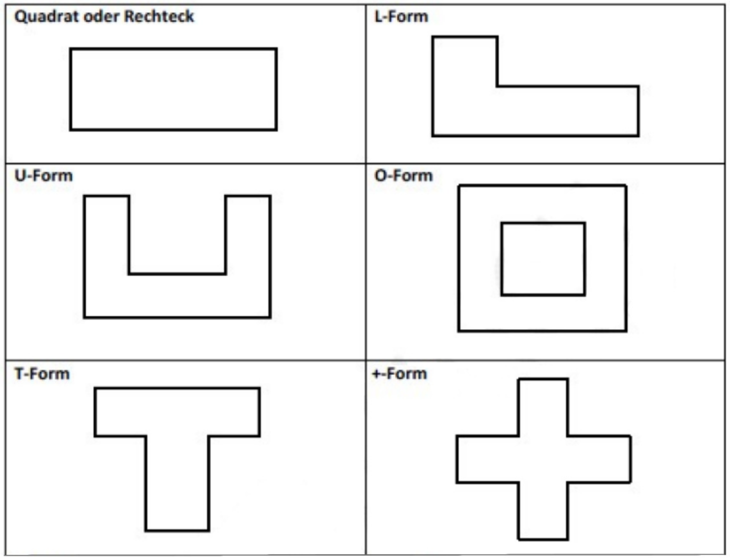
Alternative Procedure: Combination of deterministic and random approach

Alternative Procedure
Combination of deterministic and random approach

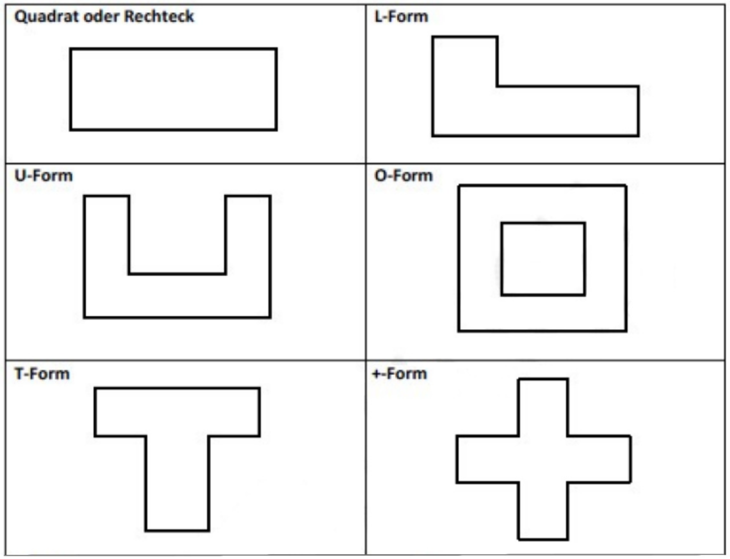
- Deterministic: shape of the building, fixed corridore/escape route and positions of elevators and stare cases



- randomized: room allocation

Alternative Procedure
Combination of deterministic and random approach

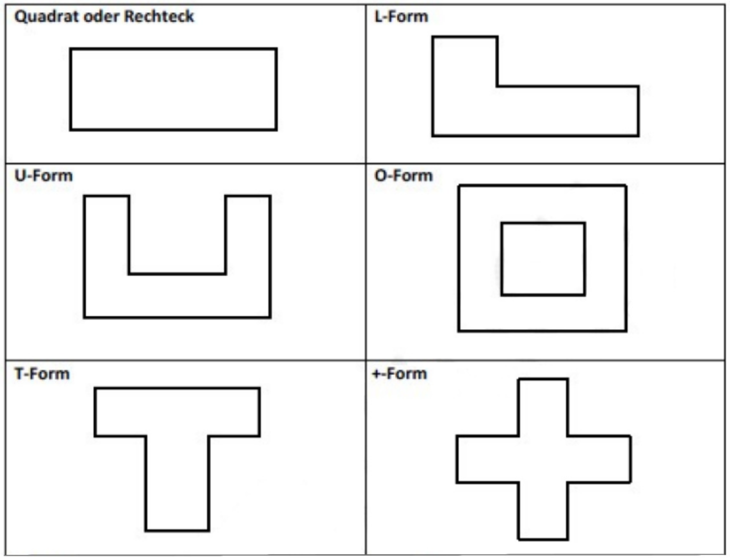
- Deterministic: shape of the building, fixed corridore/escape route and positions of elevators and stare cases



- randomized: room allocation by simulated annealing

Alternative Procedure
Combination of deterministic and random approach

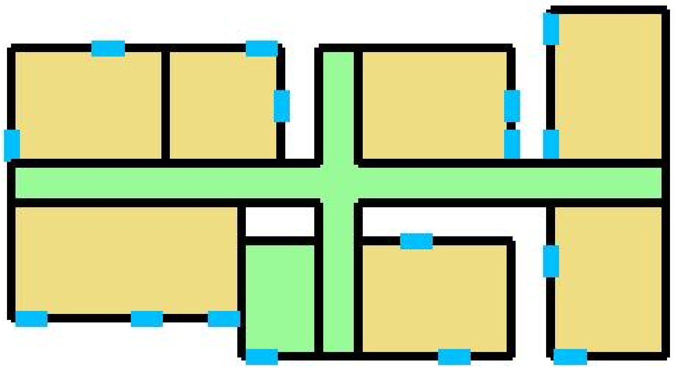
- Deterministic: shape of the building, fixed corridore/escape route and positions of elevators and stare cases



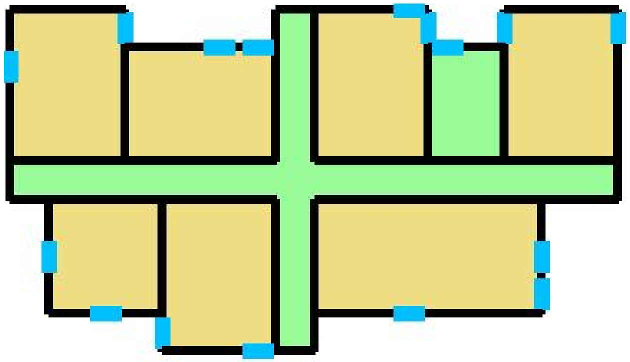
- randomized: room allocation by simulated annealing ☹

Alternative Procedure
Solutions generated by Simulated Annealing

Example with quadratic shape of the floor plan:



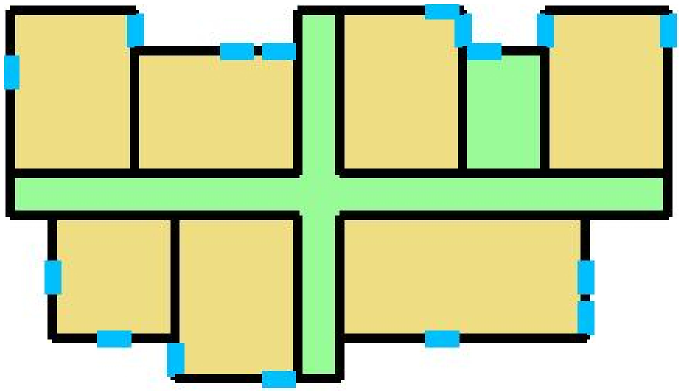
costs: 177.305 €



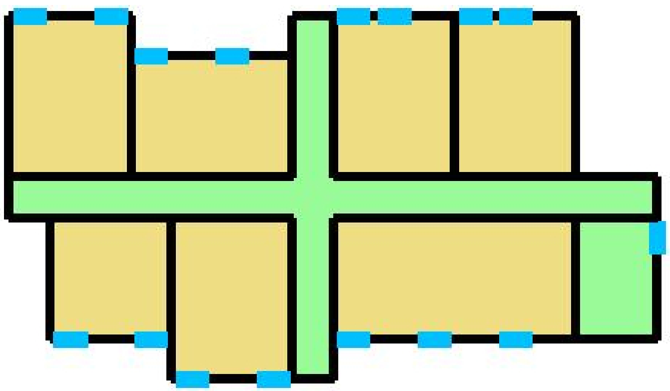
costs: 161.970 €

Alternative Procedure
Solutions generated by Simulated Annealing

Example with quadratic shape of the floor plan:



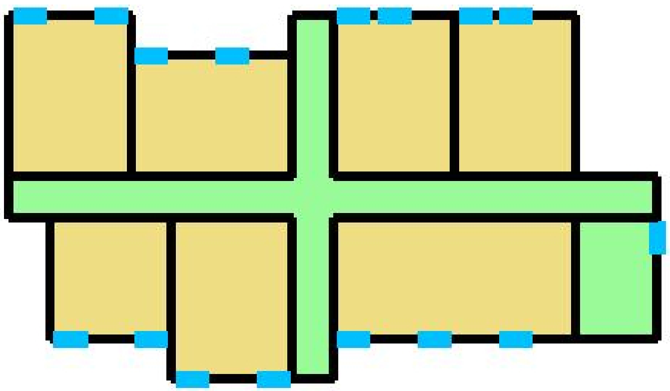
Kosten: 161.970 € $\Longrightarrow$



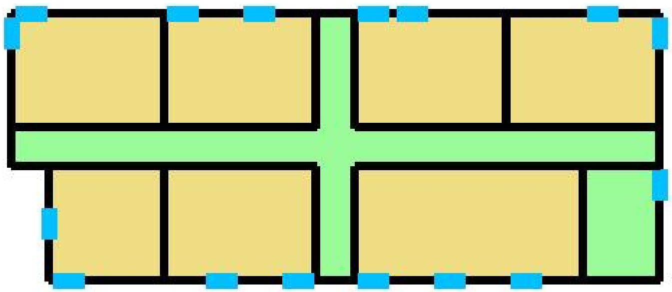
Kosten: 161.055 €

Alternative Procedure
Solutions generated by Simulated Annealing

Example with quadratic shape of the floor plan:

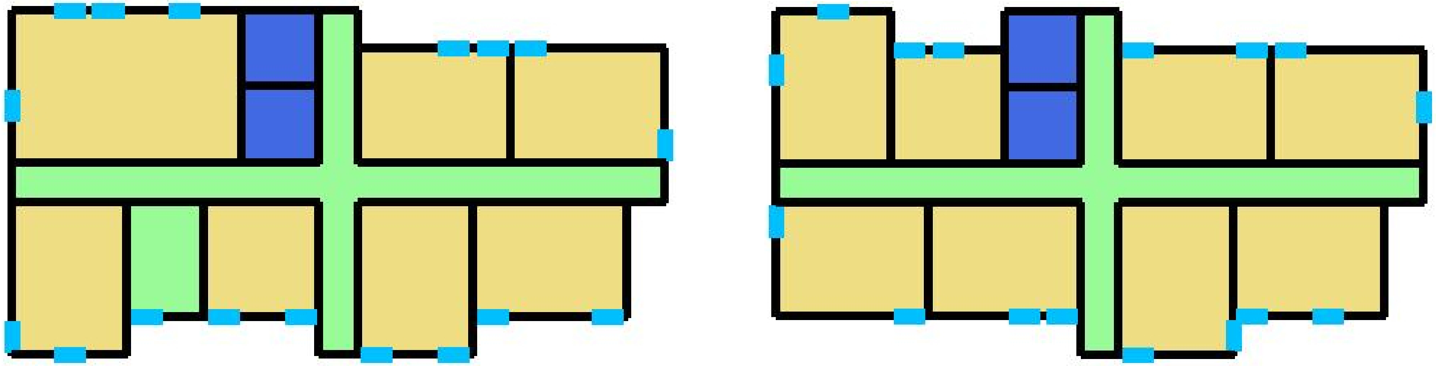


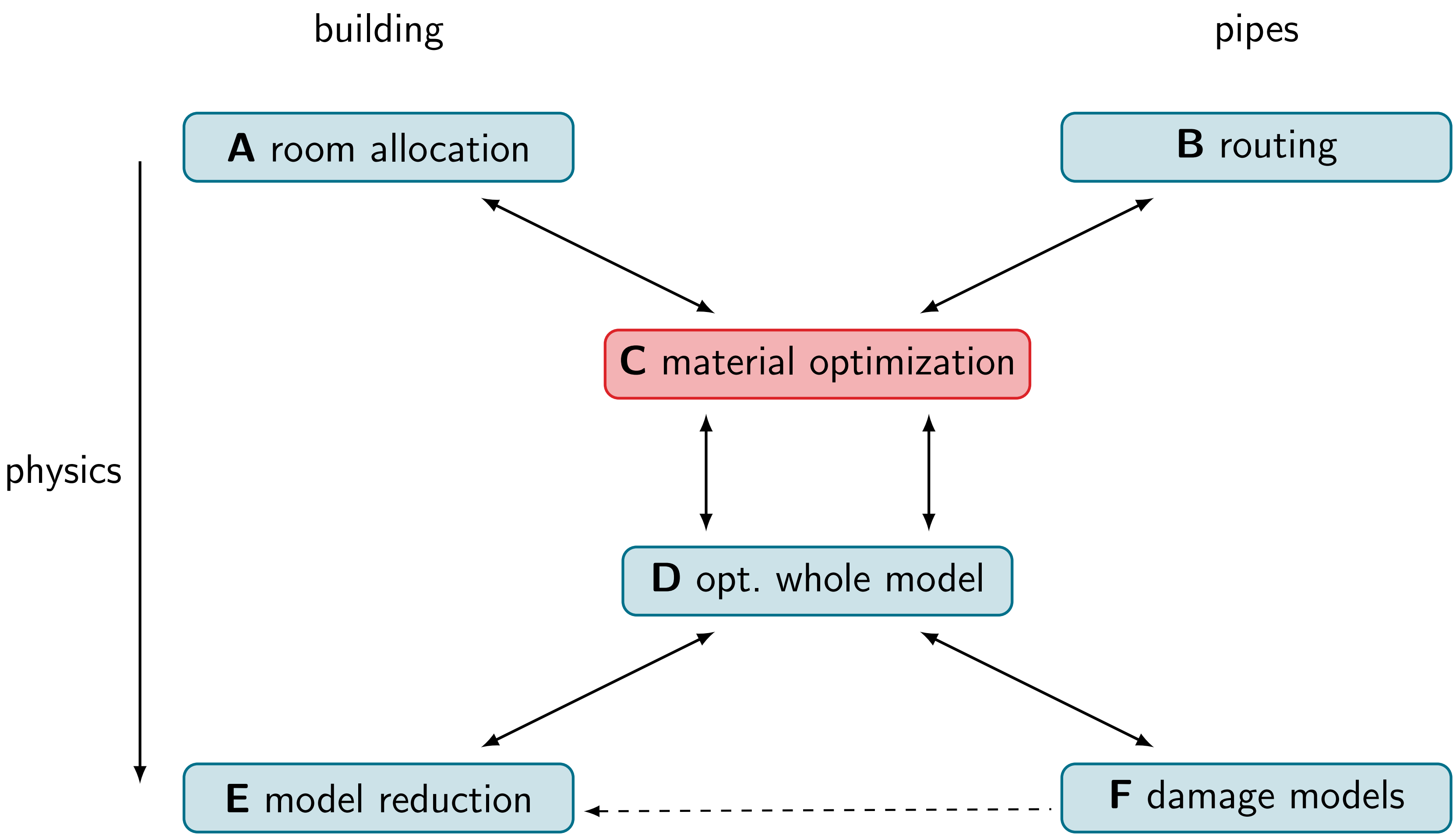
Kosten: 161.055 € $\Longrightarrow$ Kosten: 158.620 €



Alternative Procedure
Solutions generated by Simulated Annealing

Example with two floors with quadratic shape of the floor plan:




building
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F damage models

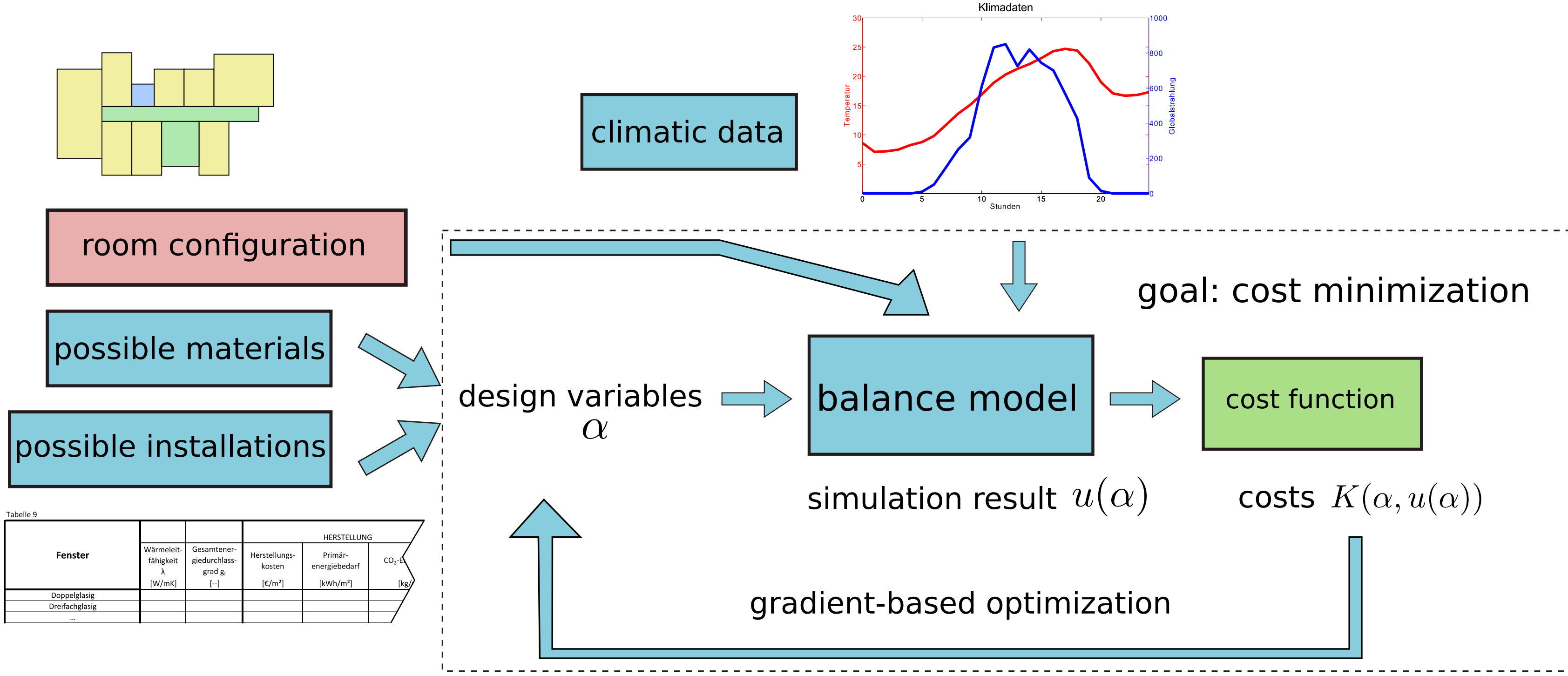
Optimization of buildings - Overview

Basic concept of building optimization in subproject C

Input: fixed **room configuration**, **catalogs** or **tables** of possible materials, climatic data

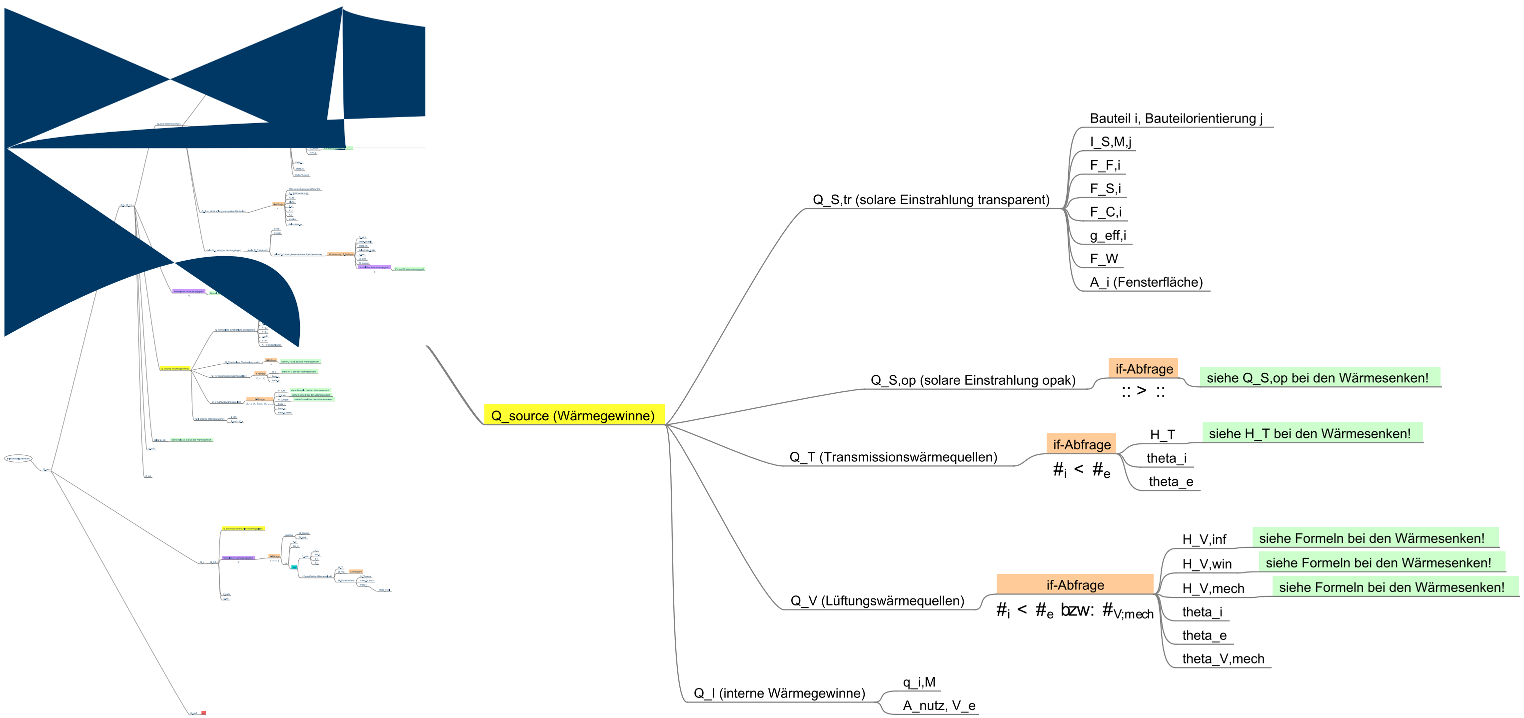
Output: optimal **materials** and **installations** (heating/cooling system, ...)

Optimization scheme



Balance model

Calculation of building energy demand by complex balance model based on DIN standards



Entire optimization model

Design variables

- Wall materials, windows, ...
- Variables for choice of installations (heating, cooling, ventilation, ...)

Objective function

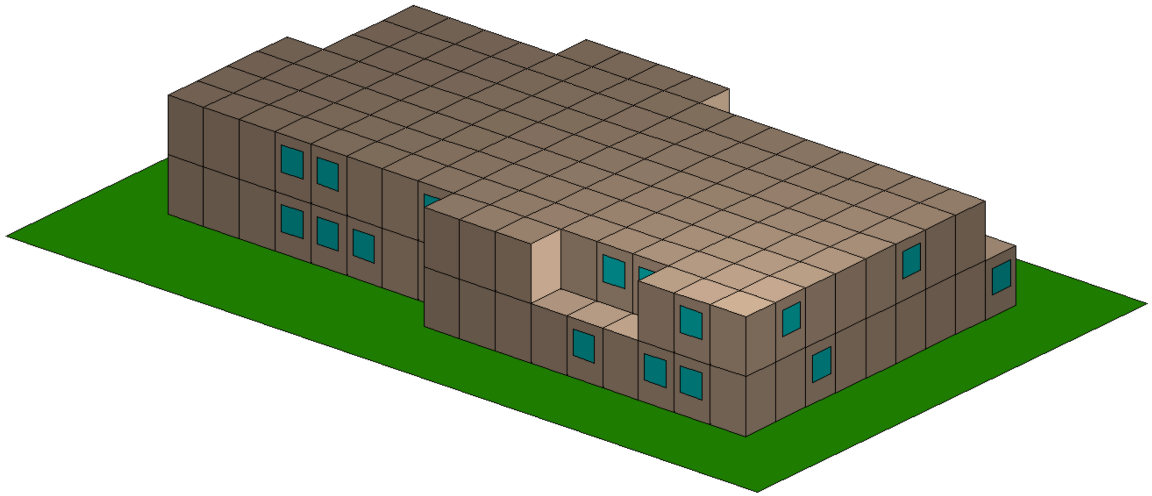
- Investment costs for installations
- Operation costs for installations
- Material costs

Simulation

- Simulation response by balance model
- Possible extension: simulation by 3D heat equation

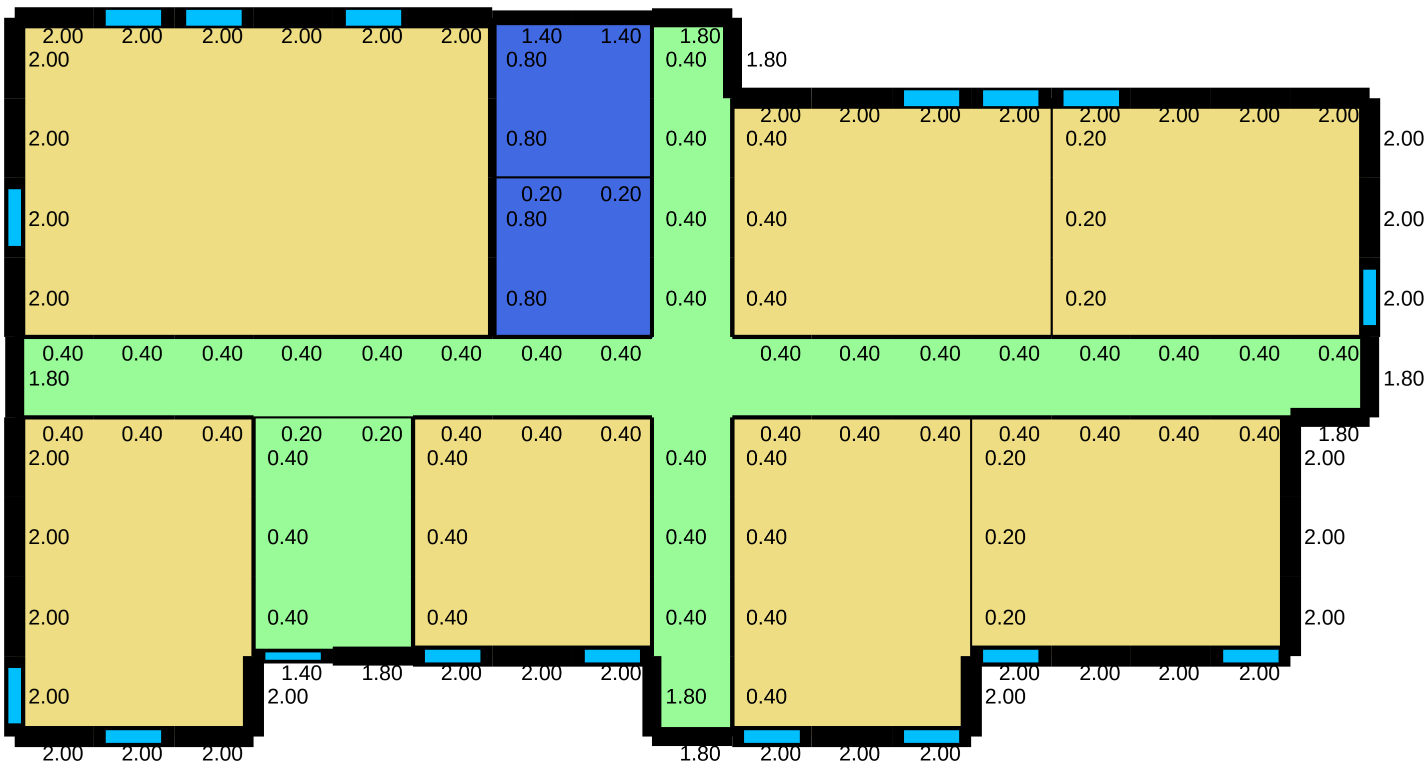
Academic example

Building (3D) with cube structure



Academic example

First floor with optimized wall materials and windows (thermal resistance)



Academic example

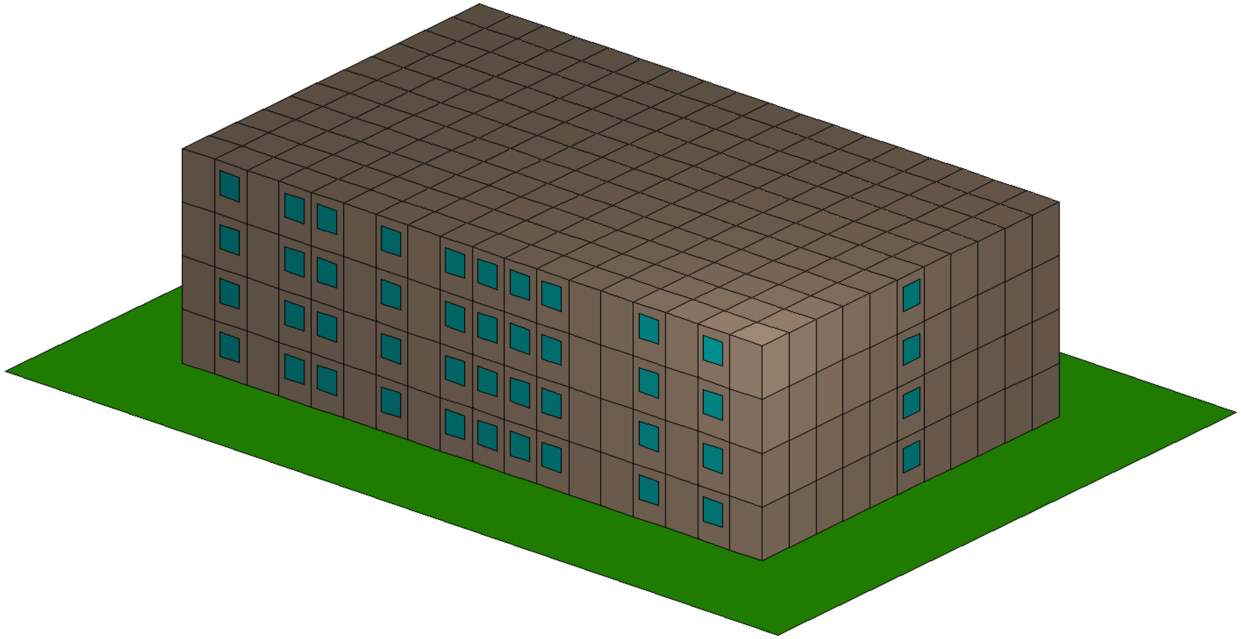
Breakdown of costs

Type of cost	Costs (init)	Costs (opti)	+/- (Absolute)
Material	847.202 €	890.100 €	-42.898 €
Inst. investment	576.987 €	178.992 €	397.995 €
Inst. operation	1.146.153 €	421.212 €	724.941 €
Total	2.570.342 €	1.490.304 €	1.080.038 €

Calculation time: < 1 s

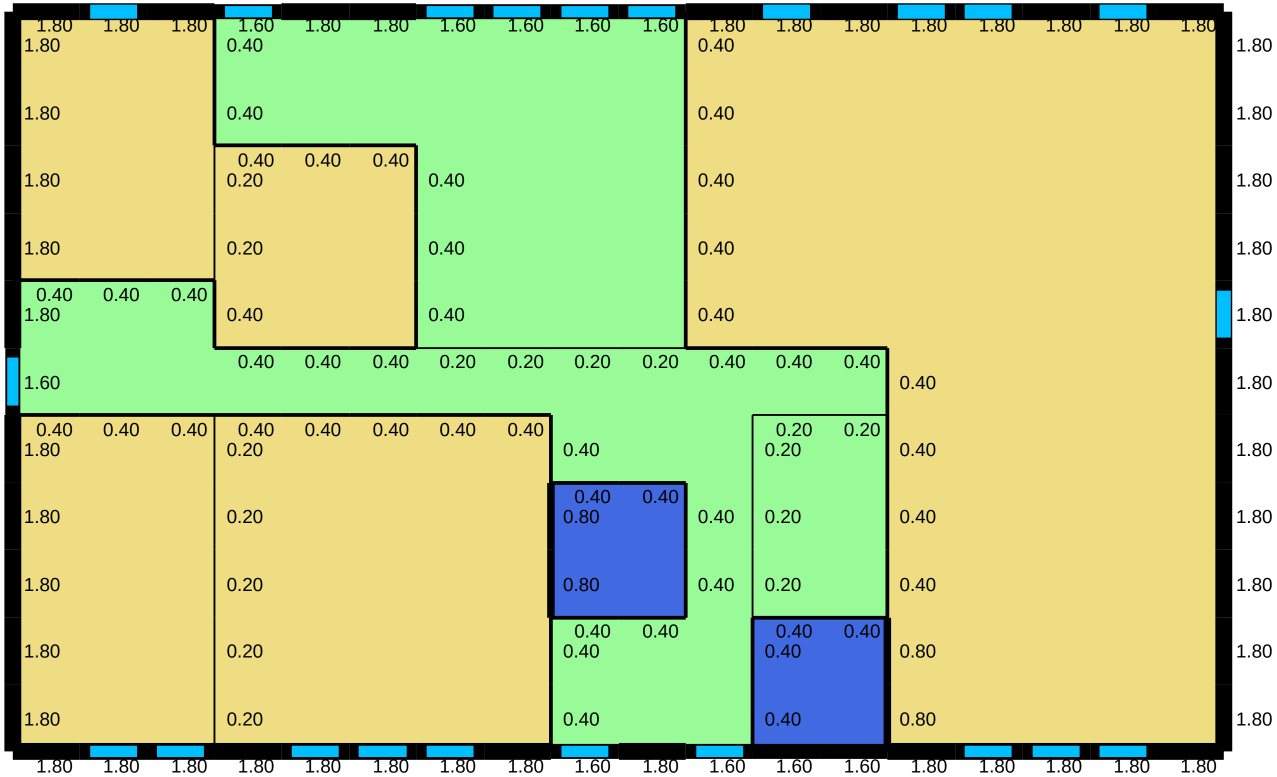
Real building (WTZ 1 Heilbronn)

Building (3D) with cube structure



Real building (WTZ 1 Heilbronn)

First floor with optimized wall materials and windows (thermal resistance)

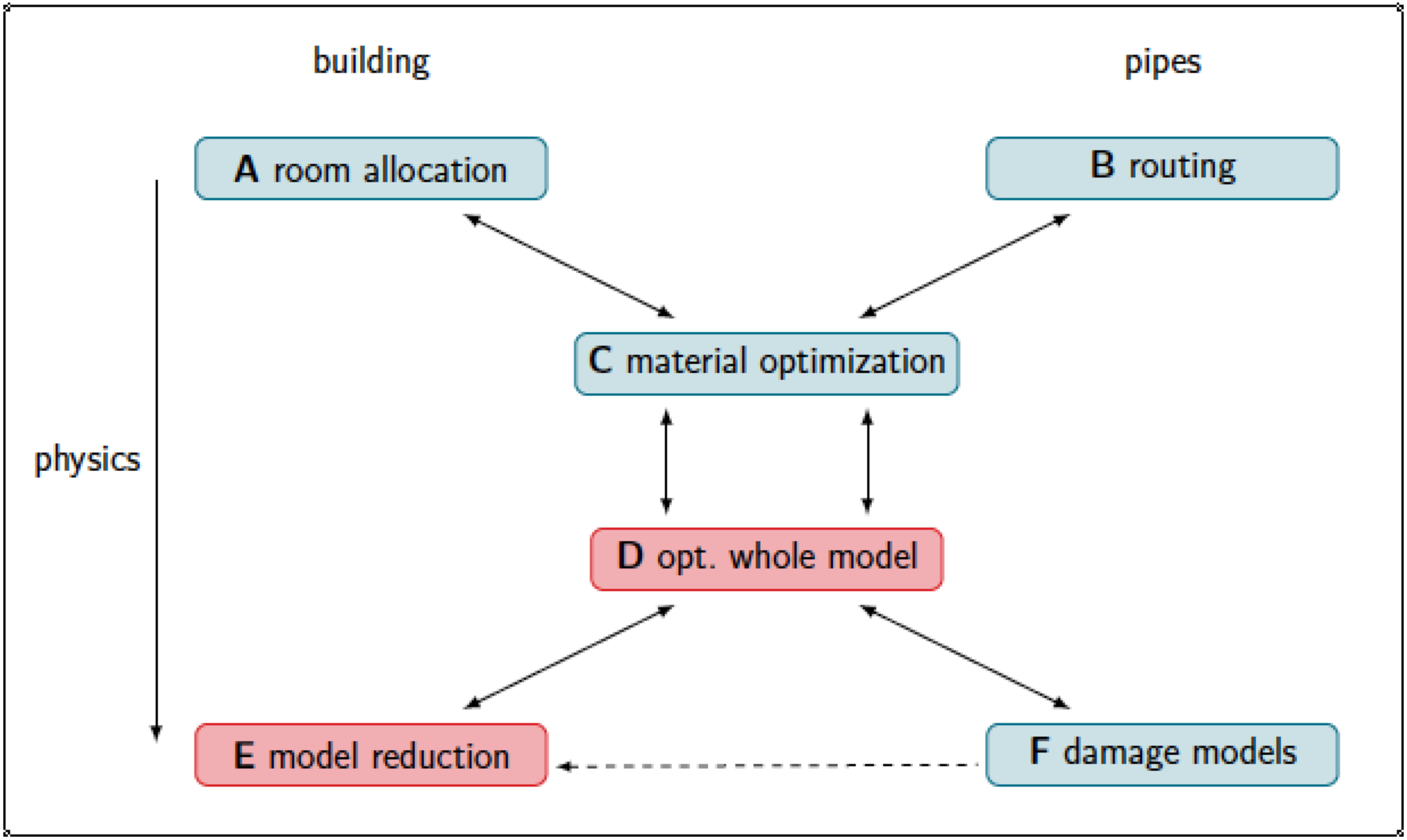


Real building (WTZ 1 Heilbronn)

Breakdown of costs

Type of cost	Costs (init)	Costs (opti)	+/- (Absolute)
Material	2.047.529 €	1.690.270 €	357.259 €
Inst. investment	1.026.904 €	328.894 €	698.010 €
Inst. operation	1.941.475 €	717.823 €	1.223.652 €
Total	5.015.909 €	2.736.987 €	2.278.922 €

Calculation time: 15 s


building
pipes
A room allocation
B routing
C material optimization
physics
D opt. whole model
E model reduction
F damage models

Motivation for Model Reduction

There are many applications in which the calculation time is a critical issue:

- Simulation of very large systems (Temperature in a building, damage of high pressure pipes, ...)
- Real time simulations of physical processes
- Optimization based on complex physical models

$\Rightarrow$ In all this applications we want to reduce the computation time

Use model reduction techniques to reduce the computation time significantly

The reduced basis method

Consider a discretized parabolic PDE with $u(t), f(t) \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$

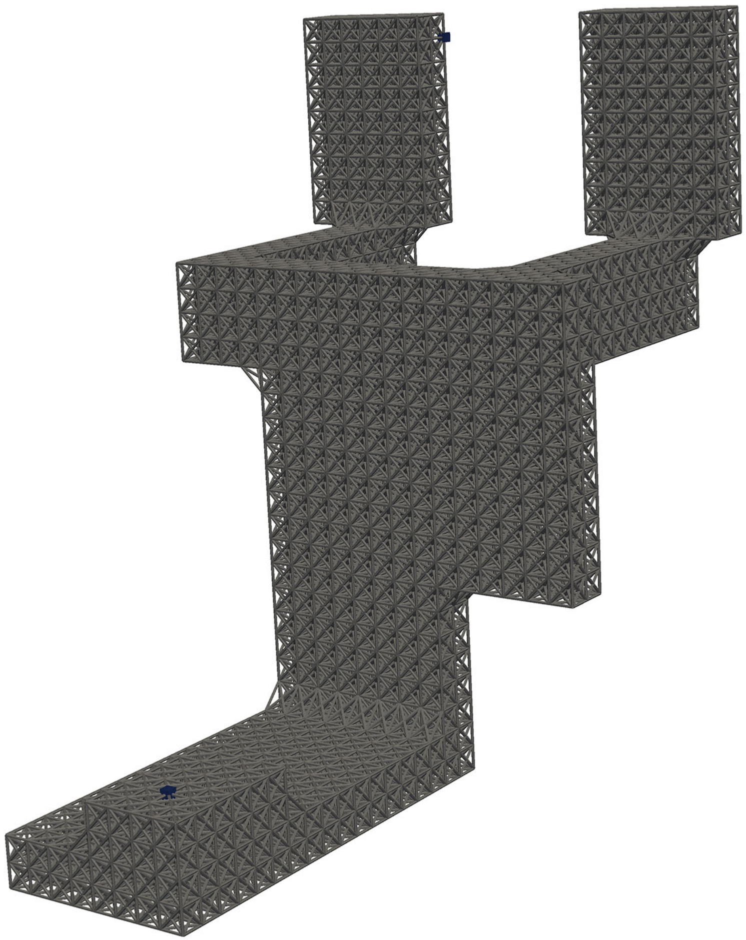
$$\frac{\partial}{\partial t} u(t) + A u(t) = f(t)$$

Assume we have a basis $\Phi_1, ..., \Phi_l, l << n$ with $\langle \Phi_i, \Phi_j \rangle = \delta_{ij}$. Additionally we define $\Lambda = (\Phi_1, ..., \Phi_l)$ as a matrix containing the basis functions columnwise.

Approximating the solution $u(t) \approx u^l(t) = \sum_{i=1}^{l} z_i(t)\Phi_i = \Lambda z(t),\ z(t) \in \mathbb{R}^l$ and projecting the system onto the subspace spanned by $\Phi_1, ..., \Phi_l$ we get:

$$\frac{\partial}{\partial t} z(t) + \underbrace{\Lambda^T A \Lambda}_{\tilde{A}} z(t) = \Lambda^T f(t)$$

Example of a large grid graph



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Offline/Online Decomposition

The reduced basis method can be decomposed into two phases:

Offline-phase

- Construction of basis functions
- Calculate all parts which are independent of input parameters ($\tilde{A}$, ...)
- Has to be calculated only once
- High computation time

Online-phase

- Calculate all parts which depend on input parameters (if necessary)
- Solve the reduced problem
- Low computation time

Proper orthogonal decomposition

Idea: Calculate the best approximating functions for the given problem

$$\min_{\Phi_1,\dots,\Phi_l \in \mathbb{R}^n} \int_0^T \|u(t) - \sum_{i=1}^{l} \langle u(t), \Phi_i \rangle \Phi_i\|^2 \, dt$$

$$\text{w.r.t. } \langle \Phi_i, \Phi_j \rangle = \delta_{ij}$$

The first order necessary optimality condition leads to the eigenvalue problem

$$\int_0^T \langle u(t), \Phi_i \rangle u(t) \, dt = \lambda_i \Phi_i, \quad \forall i = 1, \dots, l$$

Proper orthogonal decomposition

To calculate the eigenfunctions numerically we use the following procedure.

Calculate the solution u of the PDE for different time instances (snapshots) $t_0, t_1, ..., t_k$ and summarize them columnwise in a matrix

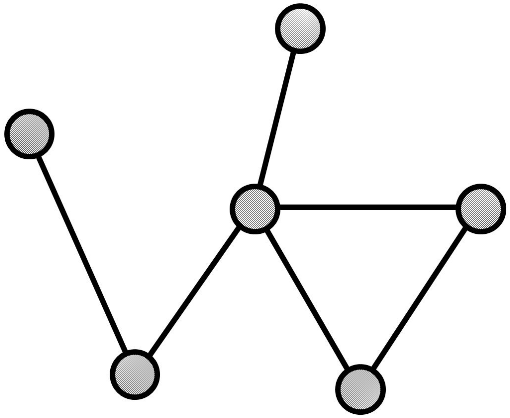
$$Y = (u(t_0), u(t_1), ...u(t_k)).$$

Solve by using singular value decomposition

$$Y^T Y \Phi_i = \lambda_i \Phi_i, \quad \forall i = 1, ..., l$$

Networks of PDEs

- Networks are represented by edges and nodes in the one dimensional case
- A PDE is defined on each edge of the network
- Different PDEs are coupled by coupling conditions on the nodes
- Boundary conditions can be defined on each node



Model reduction on networks

- **Reduce the whole network**
  - Easy to apply model reduction techniques
  - Reduced model can only be applied to one specific case
  - Extremely high offline computation times
- **Piecewise reduction of the network**
  Construct reduced model for subdomains which can be used multiple times in networks
  - Adapted algorithms are necessary to construct reduced models
  - Reduced models can be used to calculate different scenarios
  - Relatively small computation cost in the offline phase

$\Rightarrow$ **The second possibility is of great interest due to flexibility**

Basis Construction on a Subdomain

Problem formulation on Ω_i

$$
\begin{aligned}
\frac{\partial}{\partial t} u_i(x,t) - \kappa_i \Delta u_i(x,t) &= f_i(x,t) && \text{in } \Omega_i \\
u_i &= g_i && \text{on } \Gamma_i^D \\
u_i &= u|_{\Gamma_i^C} && \text{on } \Gamma_i^C \\
u_i(0) &= u_i^0 && \text{on } \Omega_i
\end{aligned}
$$

Problems:

- The boundary condition on Γ_i^C requires the solution of the network problem
- How can we ensure that the continuity coupling condition is also satisfied in the global reduced setting

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u_i(0) = u_i^0 \quad & \text{on } \Omega_i
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Problems:

- The boundary condition on Γ_i^C requires the solution of the network problem
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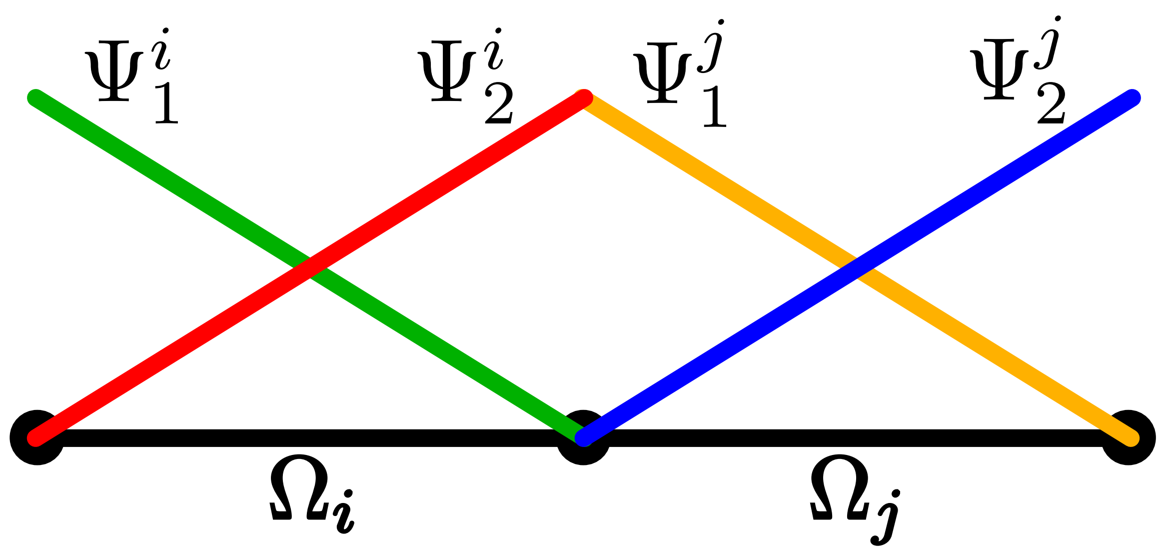
Assembling reduced basis

- Combine the stationary basis functions to semi-global linear ansatz function for every coupling and dirichlet boundary



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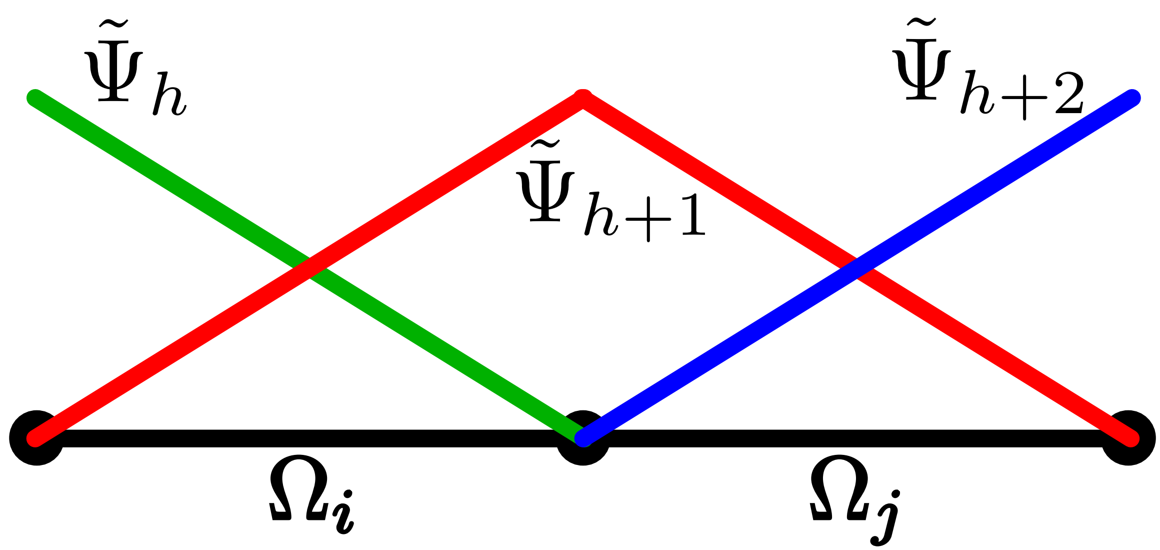

Assembling reduced basis

- Combine the stationary basis functions to semi-global linear ansatz function for every coupling and dirichlet boundary
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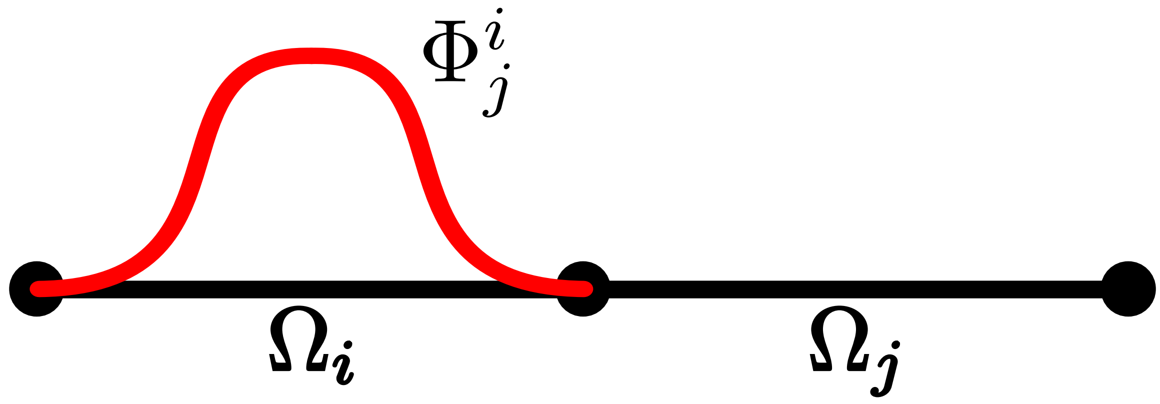

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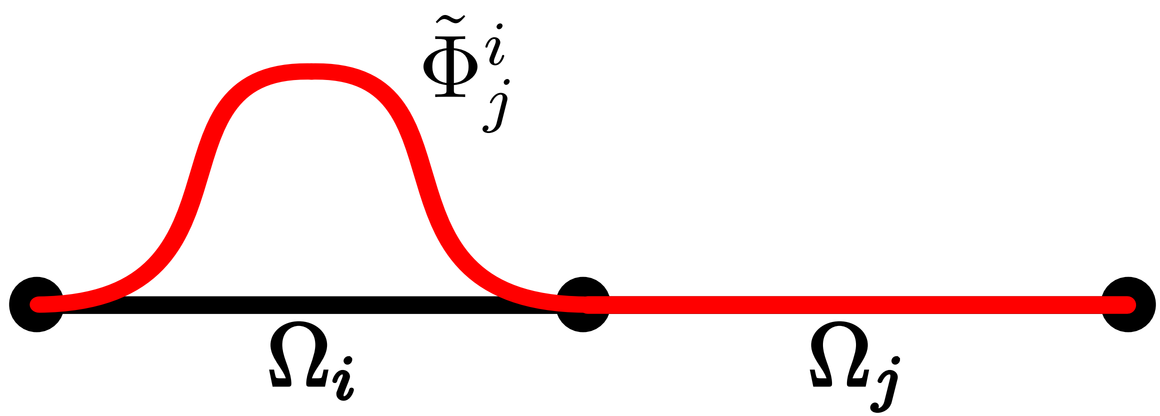
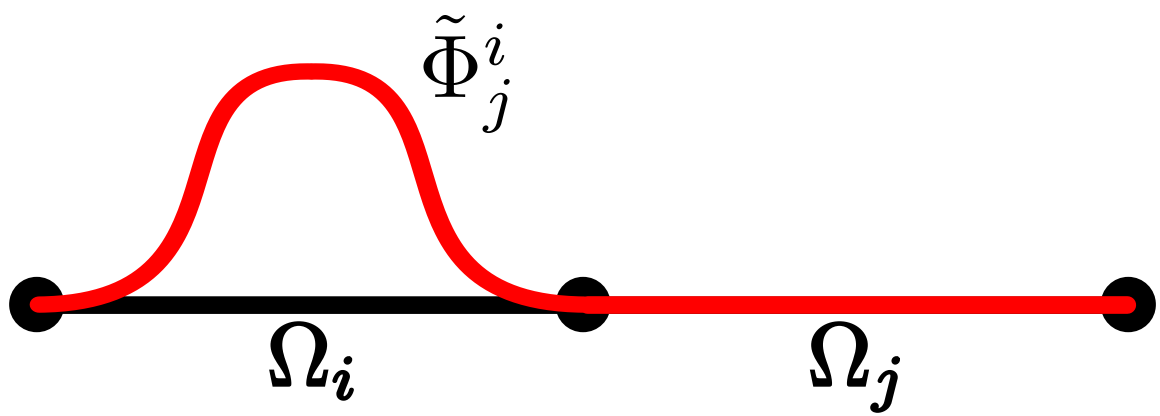


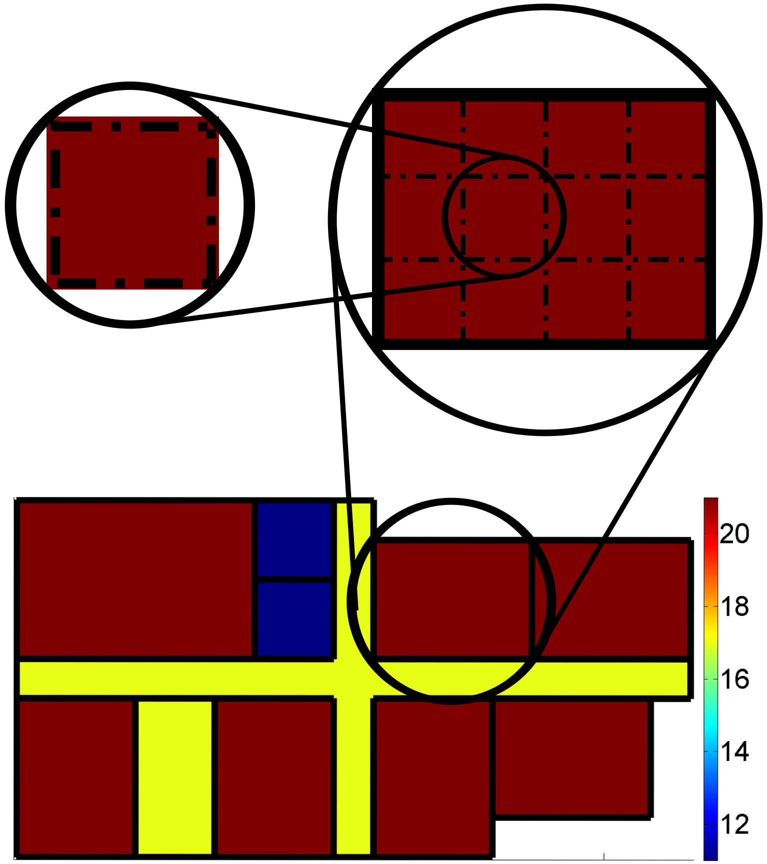
Assembling reduced basis

- Combine the stationary basis functions to semi-global linear ansatz function for every coupling and dirichlet boundary
- Extend every basis function Φ_j^i with zero on the whole network domain
- All those extended basis functions build the global reduced basis Λ



Example: 2D Heat equation - Office Building

- Divide domain in small blocks
- about 1.4 million pts, 140 blocks
- 16 types for every block (depending on the wall position)
- Parameter: only boundary parametrization
- POD basis: 50 per Element



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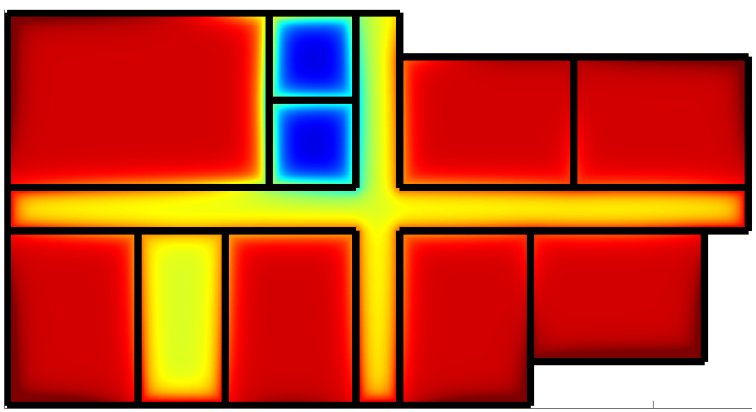
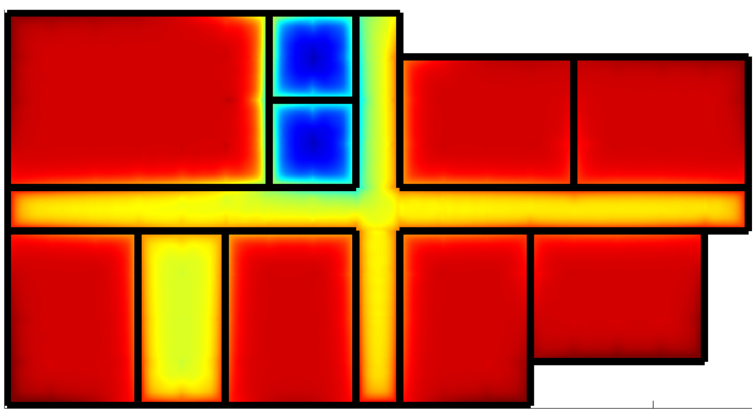
Comparison: full solution – reduced solution



full solution



reduced solution

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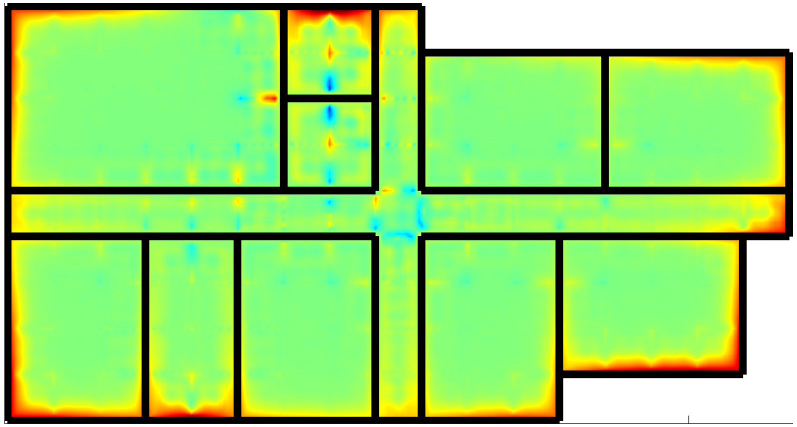

Relative error

Relative Error: $6.89e^{-3}$

computation times [s]

Offline phase:	1641
Online phase:	314
FEM solution:	1765
$\Rightarrow$ time saving:	83%

Summary

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Proof-of-concept study: ”Can planning problems be solved via mathematical optimization problems?“

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Answer

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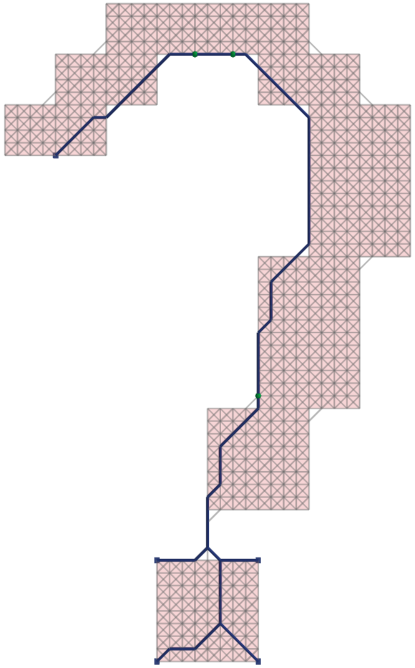
No, but ...

- We have proven that mathematics helps to push solutions for particular civil engineer problems to a new quality
- It is necessary to combine all fields of applied mathematics: numerics, continuous and integer optimization (and stochastics)
- a fascinating new appplication field for mathematicians with great challenges
- a topic that stays important for our society in the next decades



Any questions?

Thank you for your attention.