Domenico Salvagnin – CPLEX Optimization – IBM Italy
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Recent MILP advances
CO@W2015

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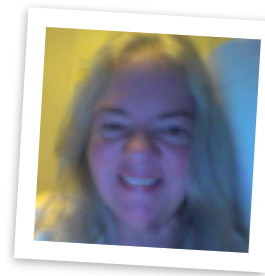

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CPLEX Solver

- Problem types that can be solved:
  - LP: linear programs
  - QP: quadratic programs (convex objective)
  - QCP: quadratically constrained programs (convex constraints)
  - MIP: mixed integer linear programs
  - MIQP: mixed integer quadratic programs
  - MIQCP: mixed integer quadratically constrained programs
  - MINLP: mixed integer nonconvex quadratic objective programs
- Continuous algorithms (LP, QP, QCP):
  - primal simplex
  - dual simplex
  - barrier

CPLEX Features

- Tools to improve solving MIP models:
  - parameter tuning tool
    - find settings that work best for your problem class
  - polishing
    - improve primal bound
  - solution pool
    - get multiple solutions out of a solve
  - populate
    - get more and diverse solutions
  - rich callback API
    - try your research ideas inside state-of-the-art environment

- Tools to analyze MIP model issues:
  - FeasOpt (infeasible problems)
    - find minimal relaxation of problem to make it feasible
  - Conflict Refiner (infeasible problems)
    - extract a minimal infeasible subproblem to show the issue
  - Condition Number and MIP kappa (numerical issues)
    - estimate the ill-conditioning and numerics of an LP/MIP solve

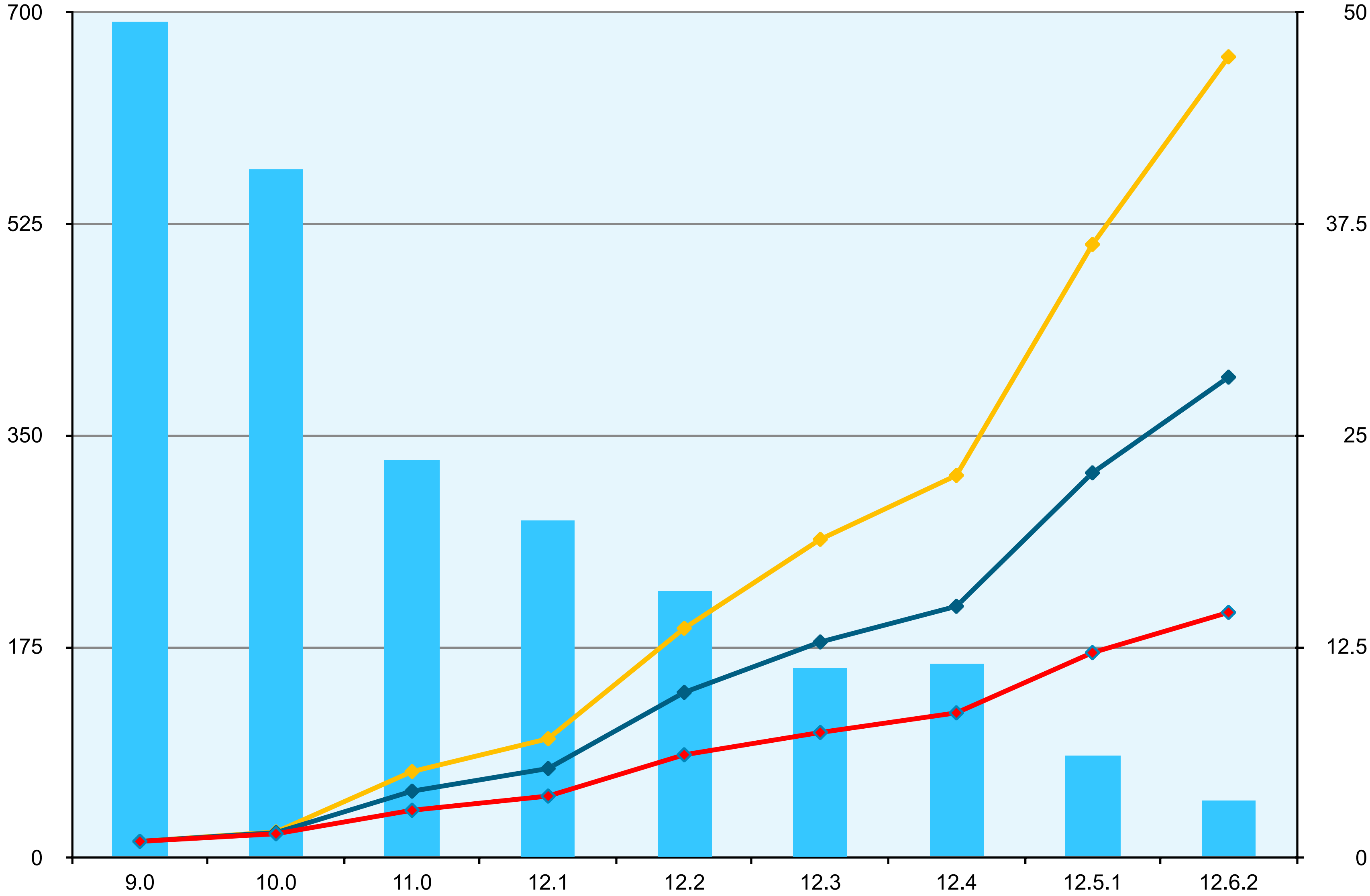
CPLEX Everywhere

- Available on many platforms:
  - Windows
  - Linux
  - Mac
  - zOS/AIX/Solaris/Sparc/HPux

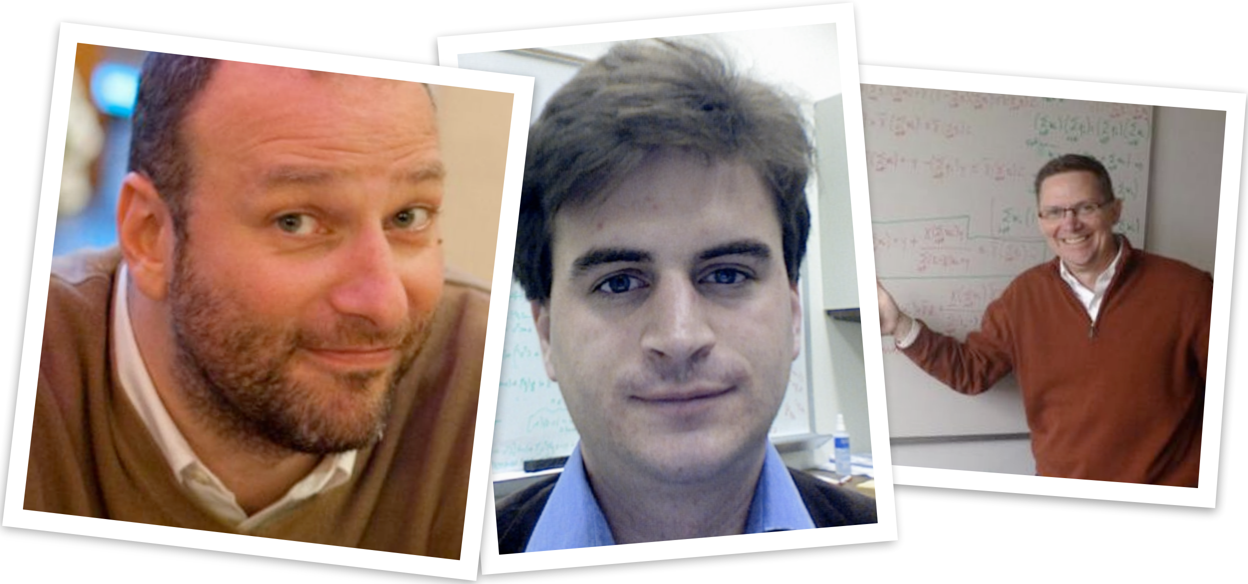
- Scalable to many scenarios:
  - Sequential
  - Shared-memory parallel
  - Distributed-memory parallel
  - Remote Object API
  - Cloud

see Daniel's talk

CPLEX keeps getting better everyday ☺



All the cool kids do MINLP these days…



…but this is supposed to be a MILP session!

(don't worry, let's just redefine "recent"…)

Recent advances in MILP

1) Parallel cut loop

2) Local implied bound cuts

3) Lift-and-project cuts

PARALLEL CUT LOOP

exploiting performance variability at root node

Performance Variability

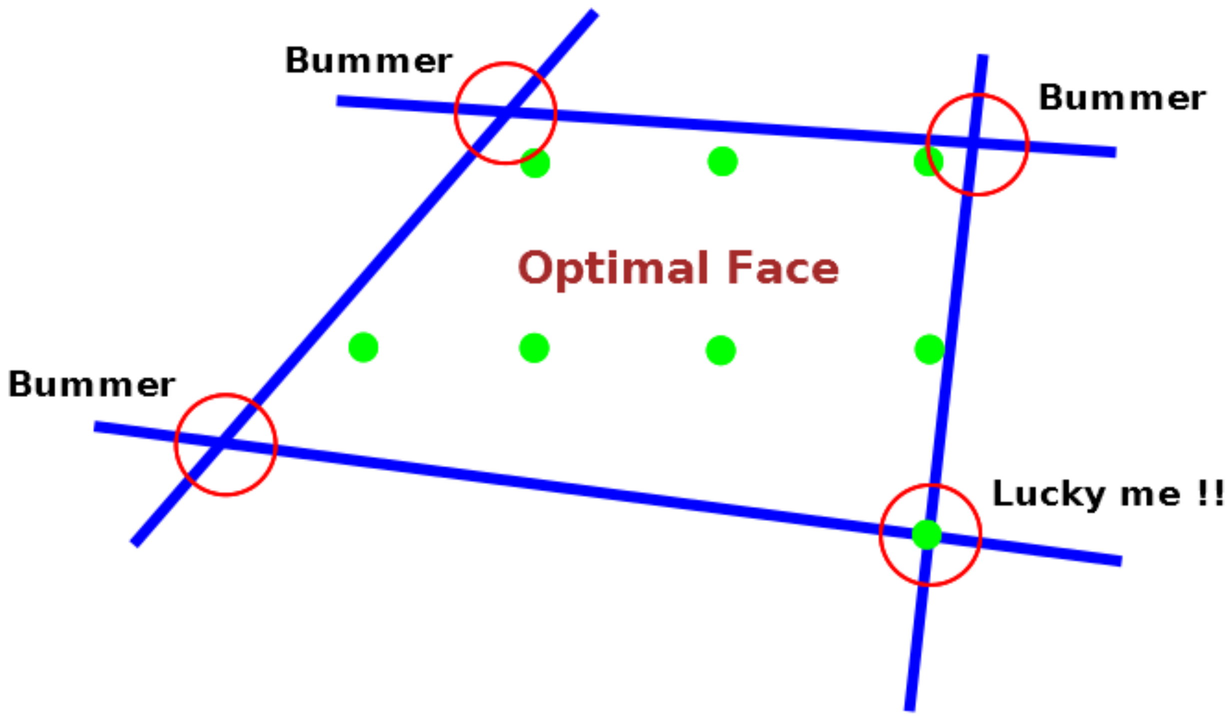
- Performance variability is a well known phenomenon **intrinsic** to MIP:
  - Danna (MIP Workshop, 2008)
  - Koch et al. (Math. Prog. Comp., 2011)
  - Achterberg and Wunderling (Facets of Comb. Optimization, 2013)
  - Lodi and Tramontani (INFORMS 2013 Tutorial)
  - Fischetti and Monaci (Op. Res., 2014)

- MIP Solvers contain various ingredients:
  - Heuristics
  - Cutting planes
  - Criteria for branching variable selection
  - ...

- Seemingly performance neutral changes (in the platform, in the code, or in the parameter setting) may have a big impact
  - on all those ingredients (separated cuts, heuristic solution found, picked variable to branch on)
  - and therefore on the whole solution process!

- A natural source of performance variability is the choice of the **initial LP basis**

- Because of **dual degeneracy**, there may be tons of equivalent (but **different**!) optimal solution/bases of the first LP relaxation



- Equivalent (but different) optimal solution/bases may lead to different:
  - cutting planes
  - heuristic solutions
  - ...
- Reoptimizing with different cutting planes amplifies the diversification
- The final root node could be very different in terms of:
  - Fractional solution
  - Dual and primal bounds
  - Cuts active in the LP
- Branching rules will take different decisions
- Branch-and-cut will then amplify the diversification again

Simulating Performance Variability: CPLEX random seed

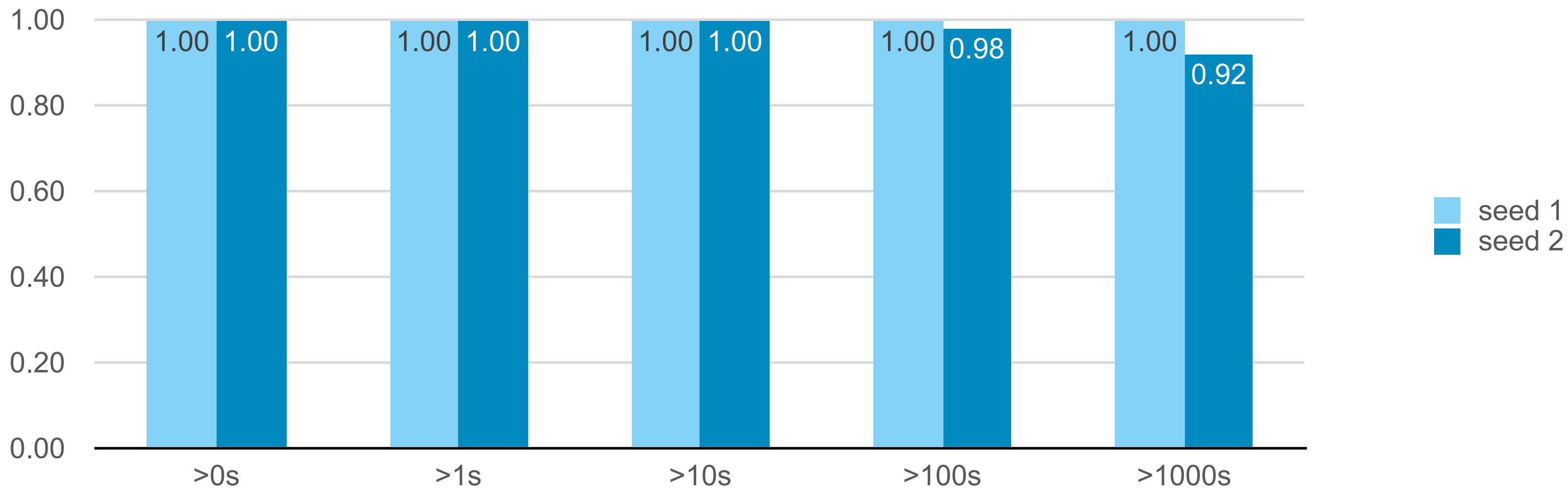
- Use CPLEX random seed parameter `CPXPARAM_RandomSeed` to simulate a performance neutral change in the environment (very convenient!)

- Compare:
  - Solver A: CPLEX 12.6.0 with random seed 1
  - Solver B: CPLEX 12.6.0 with random seed 2

- Test set: 3224 models classified according to their difficulty:
  - [0, 10k]: all models but the ones for which the solvers provide an inconsistent answer
  - [*T*, 10k]: all models for which one of the solver takes at least *T* seconds

- The comparison is fair because the difficulty of a given model is defined by all the compared solver: there is no bias against any of the compared solvers

- Comparison: (shifted) geometric mean of computing times



3084 models (75% affected)

1877 models (91% affected)

1112 models (95% affected)

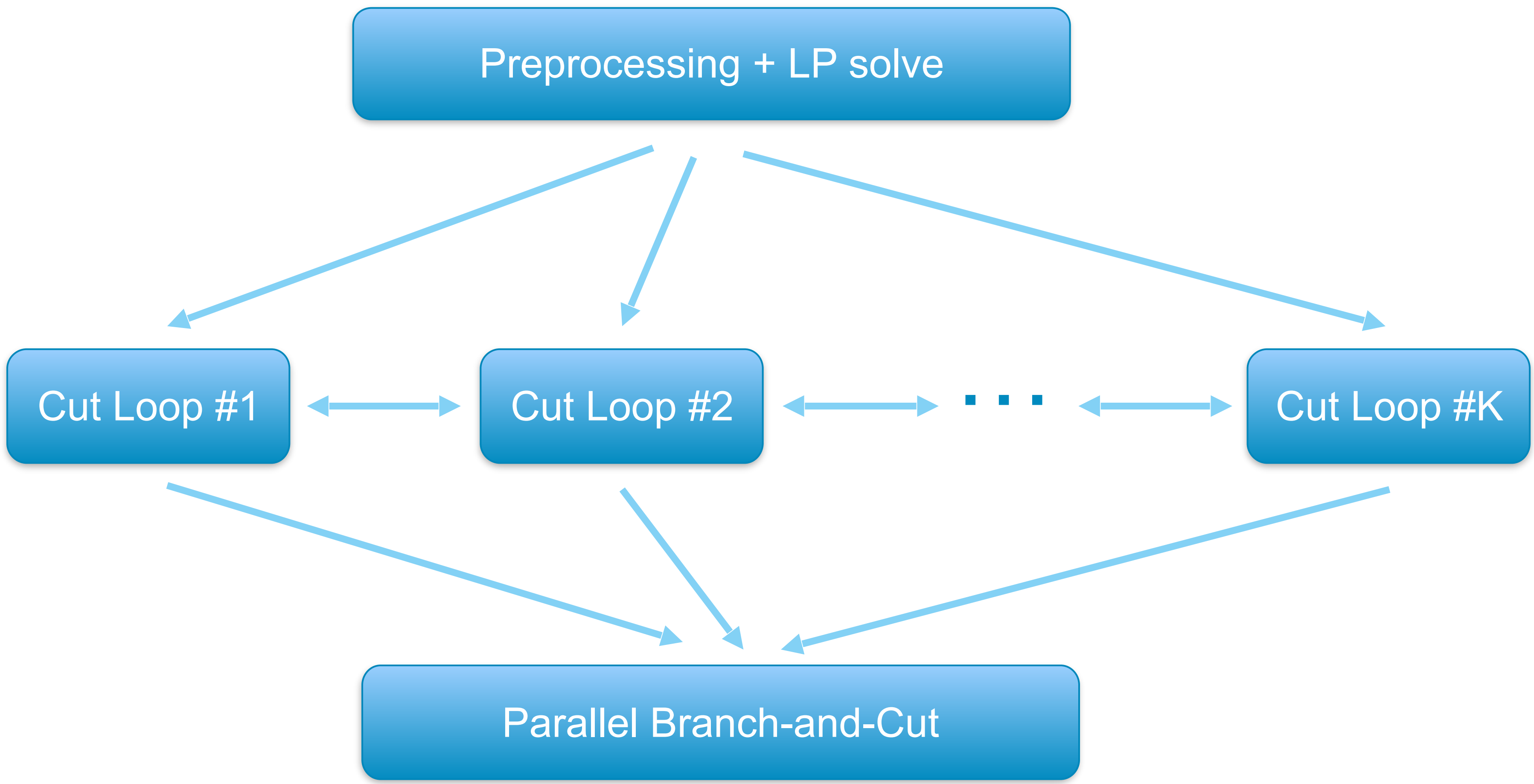
572 models (98% affected)

218 models (100% affected)

- 91% of the models in [1,10k] affected:
  - the random seed is effective to simulate performance variability.
- ~200 models in [1k,10k] **cannot** measure performance difference of less than 10%:
  - Small test sets are less robust to outliers,
  - Harder models are typically the ones exhibiting the larger variability.

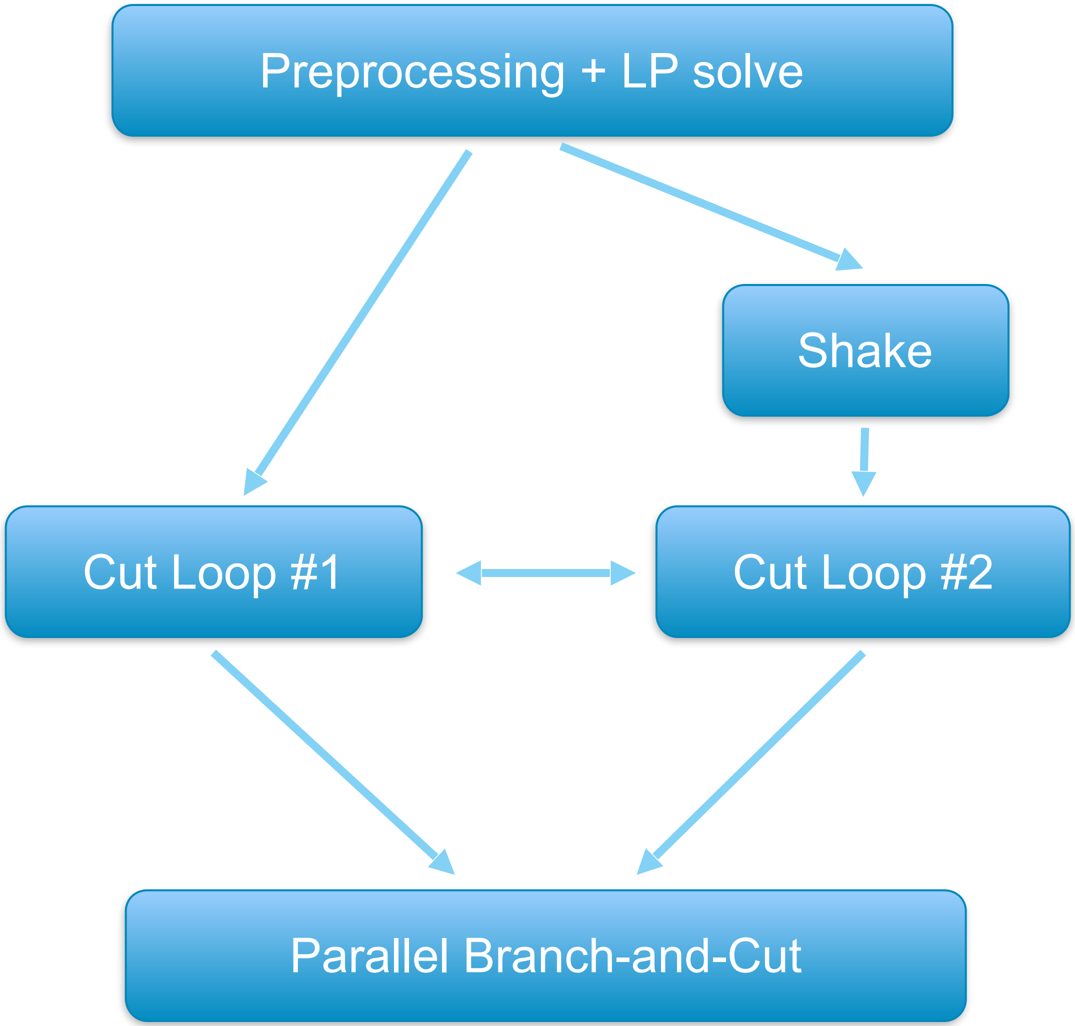
Can we take advantage of that?

Main Concept

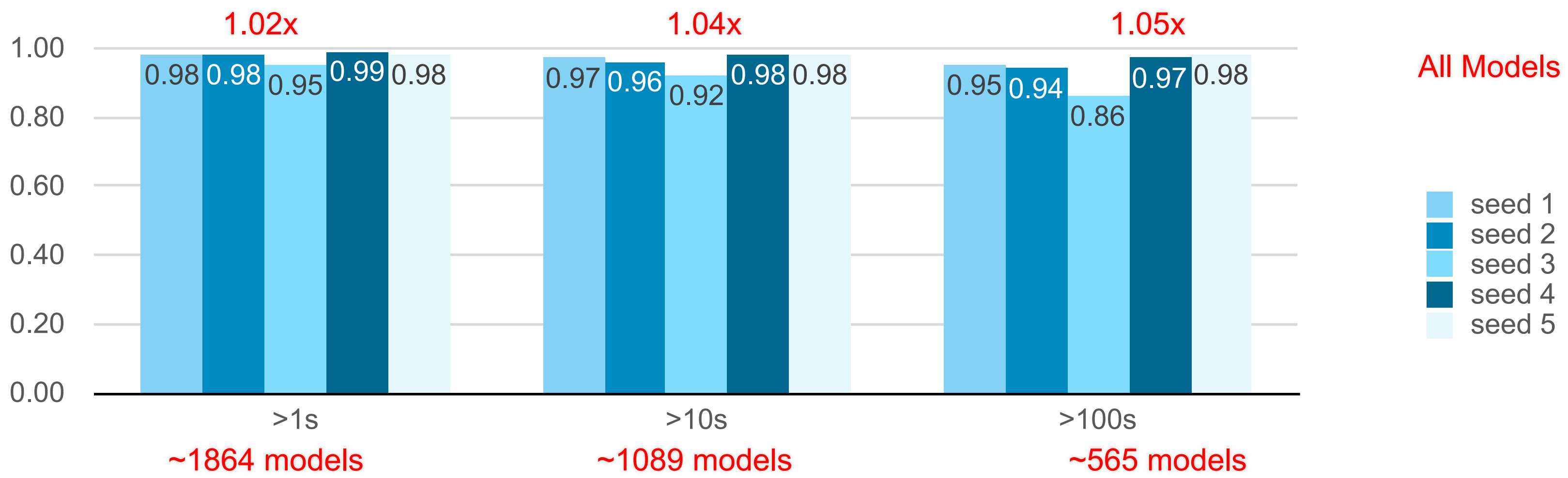


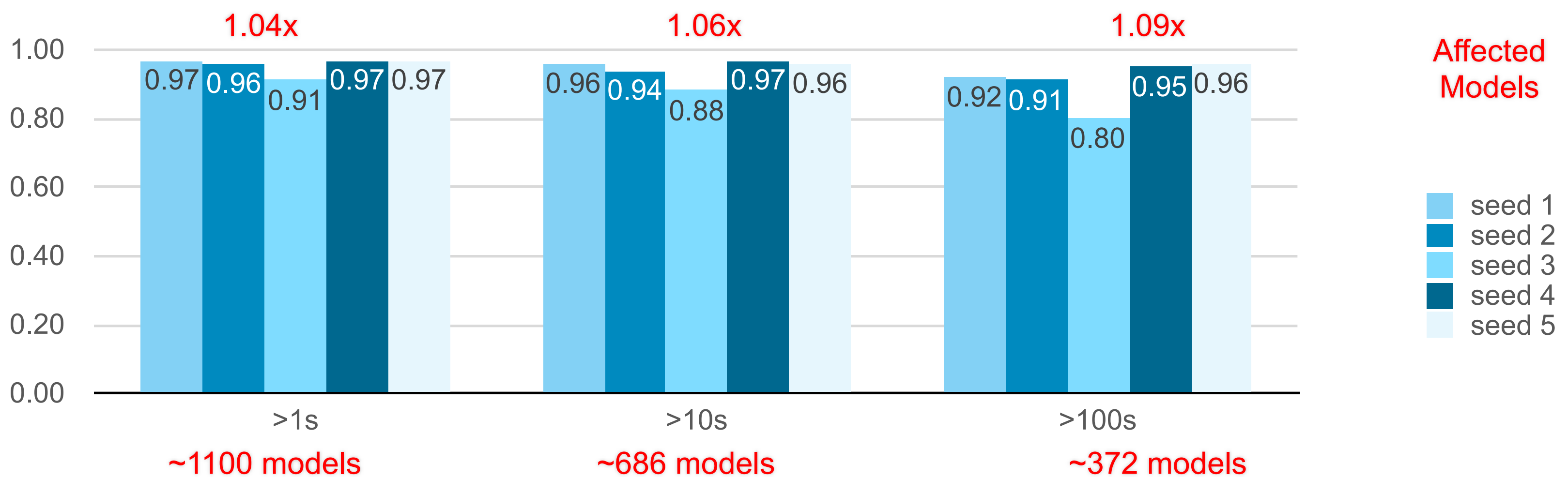
Implementation within CPLEX 12.5.1

- Forget about large K (we just use K = 2)
  - multi-threading is not for free
  - we also want to apply other methods at the root node
  - more cuts typically means better dual bound, but better dual bound does not mean less time to solve...
- Create diversification in the parallel cut loop by changing
  - the basis
  - the random seed
  - other parameters
- Be conservative with the cuts:
  - share cuts between the two cut loops
  - but be clever when selecting the cuts to be shared!


Preprocessing + LP solve
Shake
Cut Loop #1
Cut Loop #2
Parallel Branch-and-Cut

Parallel cut loop results (CPLEX 12.5.1)





LOCAL IMPLIED BOUND CUTS

Implications and implied bound cuts

$$z = 1 \rightarrow y \leq D$$

$$z \in \{0, 1\}, L \leq y \leq U, D < U$$

$$\Downarrow$$

$$y \leq U + (D - U)z$$

Discovered during presolve and probing

- **Global implied bound cuts**:
  - Globally valid implications
  - **Globally valid bounds** on implied non-binary variables
  - Separated at the root node and at branch-and-bound nodes

- **Local implied bound cuts (new in CPLEX 12.6.1):**
  - Globally valid implications
  - **Locally valid bounds** on implied non-binary variables
  - Separated at branch-and-bound **nodes only**

- **New parameter** `CPXPARAM_MIP_Cuts_LocalImplied` to control local implied bound cuts:
  - -1 = disabled
  - 0 = automatic (let CPLEX choose, currently equivalent to -1)
  - 1 = moderate: separated at starts of new dives, under conservative restrictions
  - 2 = aggressive: separated at starts of new dives, more often
  - 3 = very aggressive: potentially separated at every node

- Enabling moderate separation of local implied bound cuts:
  - 1-2% degradation (neutral?) on our test bed

- **Key points to be investigated**:
  - Cut strength versus cut applicability
    - Local cuts are stronger, but global cuts applies to the whole tree
  - Interaction with other cut types
  - Cut filtering and cut selection
    - Local cuts are violated more often than global cuts: special filtering rules?

- **Disabled by default, but fundamental in some cases** (e.g., to handle “big-M constraints”)

MIQP models from classification problems (Brooks, OR 2011):

$$
\begin{cases}
\min \frac{1}{2} w^T Q w + c^T y + 2c^T z \\
z_i = 0 \rightarrow K_i(w^T a_i + \alpha) + y_i \geq 1 \\
z_i \in \{0, 1\} \\
0 \leq y_i \leq 2 \\
-M \leq w_j \leq M \\
-M \leq \alpha \leq M
\end{cases}
$$

Key points:

- Implications are the **core of the problem**
- Very **large bounds** on α and w make any global cut completely ineffective.

Did you notice something odd in the previous model?

$$z_i = 0 \rightarrow K_i(w^T a_i + \alpha) + y_i \geq 1$$

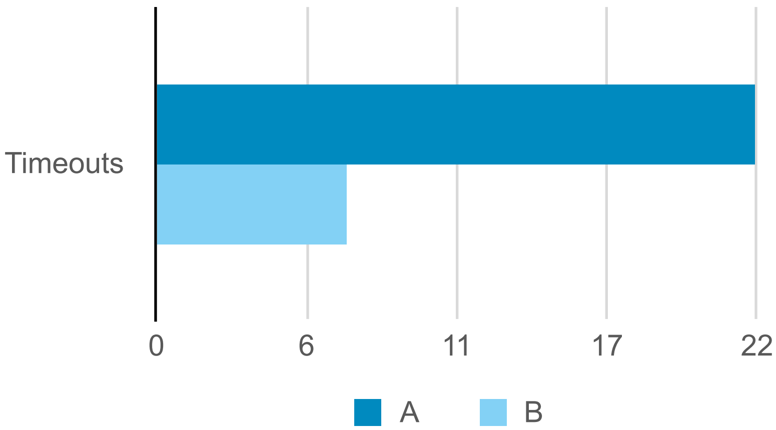
- Are these linear constraints?
- No, they are indicator constraints:
  - Advanced modeling feature natively supported by the CPLEX solver
  - Not just a modeling convenience
  - More numerically robust than their big-M counterpart and possibly more efficient as well

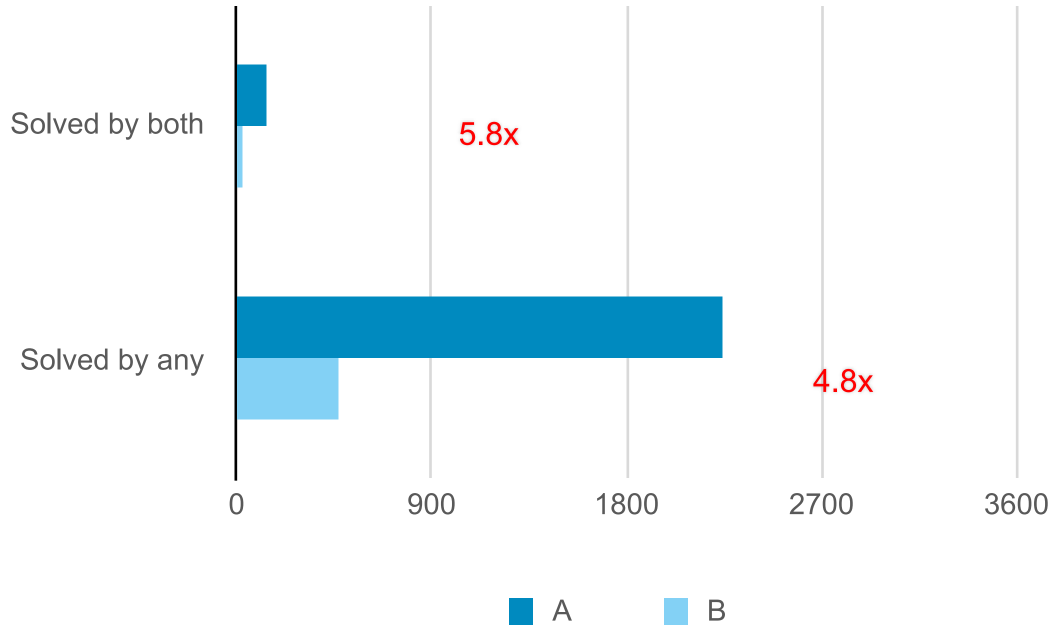
$$z_i = 0 \rightarrow K_i(w^T a_i + \alpha) + y_i \geq 1$$

$$\begin{cases} K_i(w^T a_i + \alpha) + y_i - s_i = 1 \\ s_i \geq -L_i z_i \end{cases}$$

- Bounds L_i are **just too big** and global implied bound cuts are totally ineffective
- Local implied bound cuts:
  - Two levels of branching immediately yield **reasonable local bounds** on α and ω_j variables
  - Local bounds L_i on s_i variables become tight
  - **Local implied bound cuts are effective**

Results on classification models





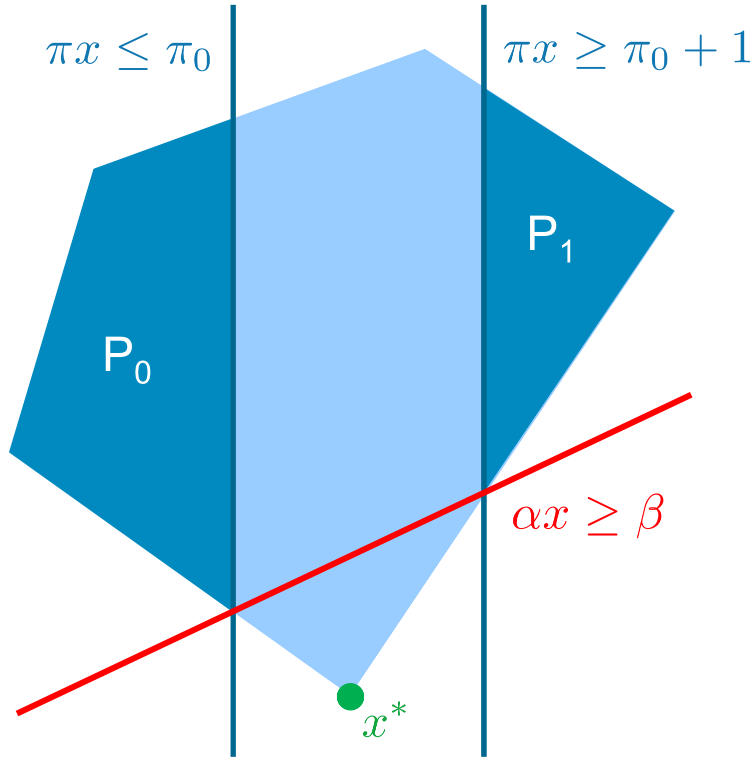
A: CPLEX 12.6.1 default
B: CPLEX 12.6.1 + aggressive local implied bound cuts

LIFT & PROJECT CUTS

CGLP magic

CGLP basics

$$(u) Ax = b$$
$$(u_0) - \pi x \geq -\pi_0$$
$$x \geq 0$$



$$(v) Ax = b$$
$$(v_0) \pi x \geq \pi_0 + 1$$
$$x \geq 0$$

$$\begin{cases} \min \boxed{\alpha x^* - \beta} \\ \alpha \geq uA - u_0\pi \\ \beta \leq ub - u_0\pi_0 \\ \alpha \geq vA + v_0\pi \\ \beta \leq vb + v_0(\pi_0 + 1) \\ u_0, v_0 \geq 0 \end{cases}$$

maximize violation

cut valid for P_0

cut valid for P_1

- All violated inequalities make the CGLP unbounded.
- By construction can only distinguish between violated and not-violated inequalities…
- Normalization is crucial!!!
- Twice the size of the original LP…

Lift and Project Cuts à la Pierre (Math. Prog. Comp., 2012)

$$\begin{cases} \min \alpha x^* - \beta \\ \alpha \geq uA - u_0\pi \\ \beta \leq ub - u_0\pi_0 \\ \alpha \geq vA + v_0\pi \\ \beta \leq vb + v_0(\pi_0 + 1) \\ u_0, v_0 \geq 0 \end{cases}$$

$$u_0 + v_0 = 1$$

$$d^* = \pi x^* - \pi_0$$

duality

$$\begin{cases} \max \pi y - (\pi_0 + 1) \\ \boxed{Ay = b} \\ 0 \leq \boxed{y \leq x^*/d^*} \end{cases}$$

Same size and structure of original problem

*Automatically works on the support of x**

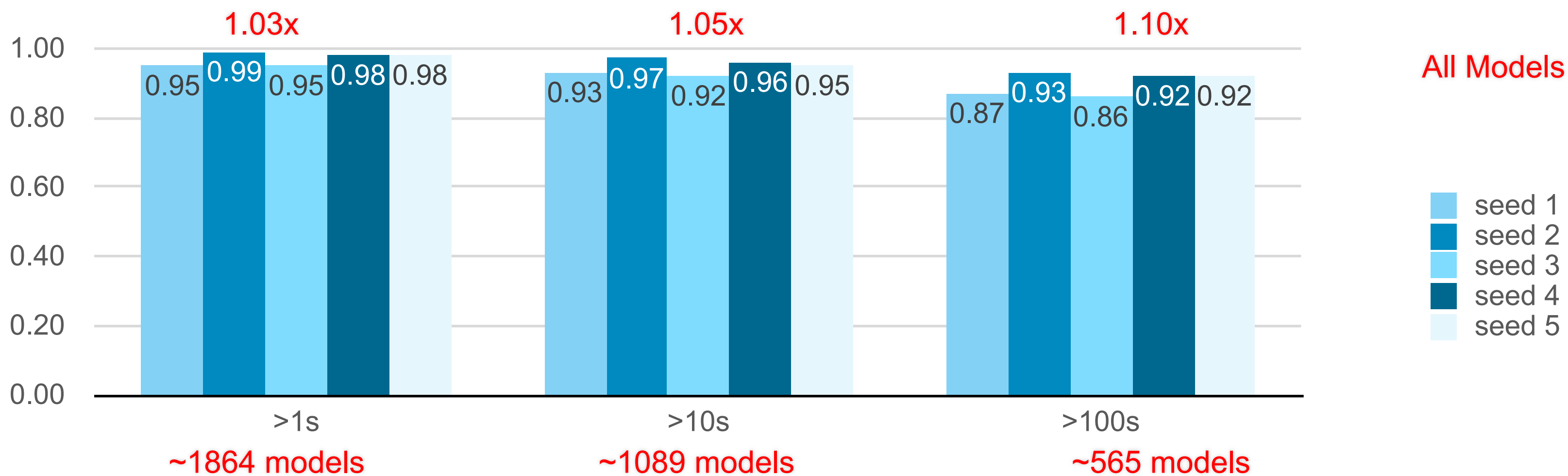
Lift and Project Cuts in CPLEX 12.5.1

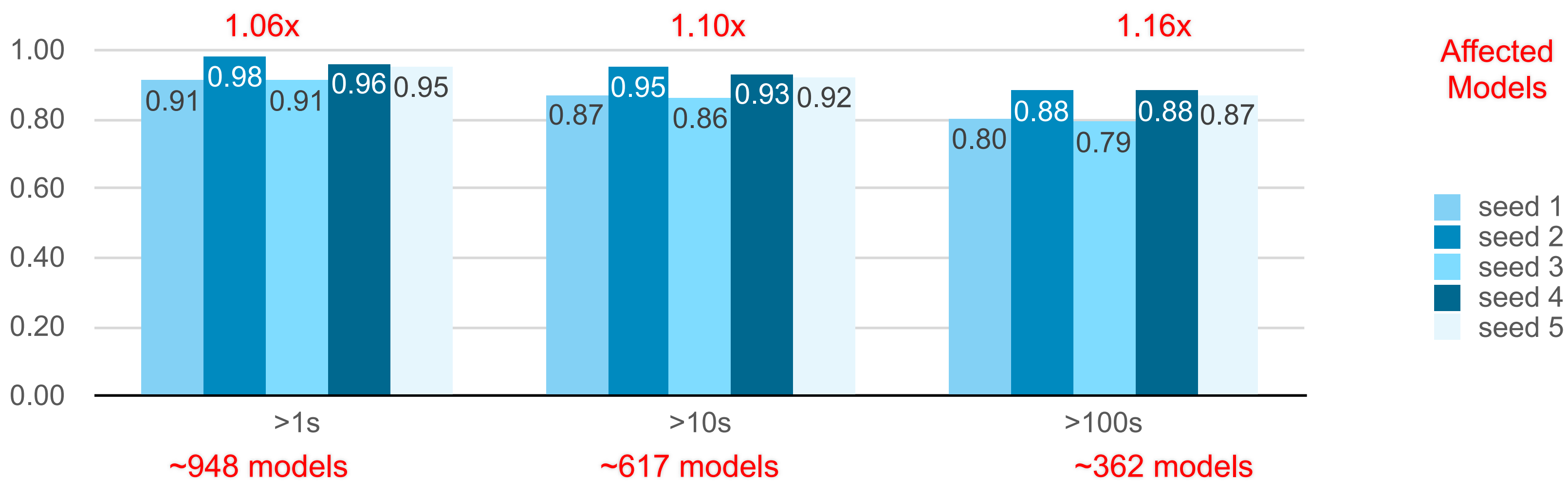
- Pierre's CGLP to separate **rank-1 cuts** only. Advantages:
  - Tends to produce cleaner and sparser cuts
  - Enforces the separation of low-rank inequalities (not a big surprise)
  - Breaks the correlation between optimization and separation

- Plus **some tricks**:
  - Heuristic reduction of CGLP to
    - Speed up the separation
    - Get rid of constraints you don't want to use to produce the cut
  - Clever selection of the disjunctions to use

- Not explored but worthy to mention:
  - Rank-1 heuristics: Dash and Goycoolea (Math. Prog. Comp., 2010)
  - Relax-and-cut: Fischetti and S. (Math. Prog. Comp., 2011)

L&P results (CPLEX 12.5.1)





Questions?

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