

CPLEX Optimization – IBM Germany 2015-10-06

### Using computing resources with IBM ILOG CPLEX CO@W2015



- Hardware resources
  - -Multiple cores/threads
  - -Multiple machines
  - -No machines
- Software resources
  - -Interfacing with CPLEX
  - -Interacting with Python





# Using more than one core/thread to solve a problem



#### Sequential branch and bound

Search tree



Pseudo code:

1.



#### Sequential branch and bound



#### Sequential branch and bound



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#### Sequential branch and bound



while !T.is_empty
n = T.get_next()
n.solve()
if not (n.is_integer()
n.is_infeasible())
<pre>j = n.fractional_index()</pre>
<pre>v = n.fractional_value(j)</pre>
T.create(n + "x[j] <= floor(v)")
T.create(n + "x[j] >= ceil(v)")



```
while !T.is_empty
  T.lock()
  n = T.get_next()
  T.unlock()
  n.solve()
  if not (n.is_integer() ||
          n.is infeasible())
    j = n.fractional_index()
    v = n.fractional_value(j)
    T.lock()
    T.create(n + "x[j] <= floor(v)")</pre>
    T.create(n + "x[j] >= ceil(v)")
    T.unlock()
```



Thread 1



#### Thread 2





















#### Not deterministic!



#### Not deterministic! Thread 1 Thread 2 T.lock() T.lock() T.get\_next() T.unlock() solve Thread 2 solves — instead of and **m** not even created T.loc Node selection dictates to continue under and T.cre T.crea T.unlock() T.get\_next() T.unlock()





#### Issues with non-determinism

- opportunistic behavior of OS scheduler
- very minor side effects (cache misses, page swaps, ...) can change order of threads

	gmu-35-40	
Time	Iterations	Nodes
21.31	2236199	444838
18.46	2060789	322282
4.62	253871	51778
31.55	3300767	509641
11.89	1238162	177236
79.50	7212334	1307340
21.10	2955104	595939
16.27	1953450	401507
13.61	1410170	261508
17.10	1948228	410949

#### This is a problem in practice:

	iis-pima-cov	
Time	Iterations	Nodes
62.92	1678290	17744
139.72	5475373	47428
84.39	2075243	26460
80.37	2766163	22016
71.25	1753092	17513
69.18	1615592	21636
117.43	3002680	40299
72.41	1704471	21301
124.01	4863607	42298
191.31	7898071	71043

	glass4	
Time	Iterations	Nodes
41.60	6577338	340192
70.57	11800428	637471
29.30	4075955	304913
34.31	6026804	208466
160.12	29207354	1082067
244.62	35189917	1893985
54.40	8700758	512053
74.64	11307243	698508
47.19	8600157	389436
54.42	8275014	544172



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- → results are not repeatable
- debugging becomes painful
- benchmarking is complicated
- assessment of algorithmic changes is difficult
   "Is my new cut really helpful or is this just a random change?"

#### → We need a solver that operates in a <u>deterministic</u> way!



opportunistic OS scheduler, cache misses, swaps, ...

 $\rightarrow$  order in which lock is granted to threads is not deterministic

deterministic make this order deterministic:

- use deterministic time → same in any run
- grant lock to first thread to arrive in **deterministic time**



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Kendo

"Kendo: Efficient Deterministic Multithreading in Software" http://people.csail.mit.edu/mareko/asplos073-olszewski.pdf → use number of retired store instructions as time (x86)



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CPLEX

deterministic time = number of array accesses

- works the same on all hardware
- explicit instrumentation of source code

for (int i = 0; i < n; ++i)
 x[i] = y[i] \* 0.5;
DETCLOCK\_INDEX\_ARRAY (2 \* i);</pre>



- Deterministic threading is not for free
  - settle for a particular path through the search tree
  - increases wait/idle times in locks

	gmu-35-40	
Time	Iterations	Nodes
4.62	253871	51778
79.50	7212334	1307340
38.48	4124364	737994

	iis-pima-cov	
Time	Iterations	Nodes
62.92	1678290	17744
191.31	7898071	71043
72.84	1382605	20449

	glass4	
Time	Iterations	Nodes
29.30	4075955	304913
244.62	35189917	1893985
91.68	11737409	528160



- > Deterministic threading is not for free
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- > Deterministic threading is not for free
  - settle for a particular path through the search tree
  - increases wait/idle times in locks
- > Deterministic threading is unrelated to performance variability
  - parallel cut loop is still meaningful
- > CPLEX extensions of Kendo framework allow efficient parallel cutloop
  - opportunistic code, but results are deterministic
- > All parallel algorithms in CPLEX are available as opportunistc/deterministic
  - select at runtime with a parameter



## Using more than one machine to solve a problem



CPLEX implements distributed parallel MIP solving

- one master process to coordinate the search
- several worker processes to perform the heavy lifting
- usually one master/worker per machine (can configure otherwise)
- communication only between master and workers



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- 3 different ways of communication:
  - $ssh \rightarrow master starts workers via ssh$ 
    - $\rightarrow$  communication via pipes
  - $TCP/IP \rightarrow$  workers run a server-like process to which master connects
    - $\rightarrow$  communication via sockets
  - MPI  $\rightarrow$  all processes run within an MPI communicator
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- 2 phases:
  - 1. "rampup"  $\rightarrow$  create an initial set of open nodes
  - 2. "tree search"  $\rightarrow$  perform distributed parallel b&b



#### Phase I: rampup

1. master runs presolve to create a root node





#### Phase I: rampup

- 1. master runs presolve to create a root node
- 2. master sends root to workers





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- 1. master runs presolve to create a root node
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- 3. each worker starts b&b with different parameters
- $\rightarrow\,$  each worker produces a different tree









- 1. master runs presolve to create a root node
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- 3. each worker starts b&b with different parameters
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- 4. master eventually
  - stops workers

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- selects a winner
- collects open nodes from winner
- $\rightarrow$  list of "supernodes"



presolve

MIP







#### Phase II: tree search

5. master starts with list of supernodes

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- 5. master starts with list of supernodes
- 6. master sends a supernode to each worker





#### Phase II: tree search

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7. workers solve supernode as MIP





#### Phase II: tree search

- 5. master starts with list of supernodes
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7. workers solve supernode as MIP

- 8. Master can
  - send new supernodes (if idle)
  - grab nodes to produce new supernodes
  - pause supernode (exchange)







#### Variants/improvements

1. exchange information (incumbents, bound tightenings, ...)



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2. in rampup, start some workers with special settings

- > aggressive heuristics  $\rightarrow$  quickly find good solutions
- $\rightarrow$  aggressive cuts  $\rightarrow$  quickly improve dual bound
- ≻ ...



Variants/improvements

≻ ...

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- 1. exchange information (incumbents, bound tightenings, ...)
- 2. in rampup, start some workers with special settings
  - > aggressive heuristics  $\rightarrow$  quickly find good solutions
  - > aggressive cuts  $\rightarrow$  quickly improve dual bound
- 3. never stop the rampup phase  $\rightarrow$  exploit performance variability



performance that any user will experience will vary depending upon many factors, including considerations such as the amount of multiprogramming in the user's job stream, the I/O configuration, the storage configuration, and the workload processed. Therefore, no assurance can be given that an individual user will achieve results similar to those stated here.



### Solving a problem without a machine



- model too hard
- only need to solve once in a while
- ...
- $\rightarrow$  solve in the cloud



- model too hard
- only need to solve once in a while





- model too hard
- only need to solve once in a while





- model too hard
- only need to solve once in a while





#### Two ways to access CPLEX in the cloud

#### 1. Dropsolve

www.ibm.com/software/analytics/docloud dropsolve-oaas.docloud.ibmcloud.com/software/analytics/docloud



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Oil-blending.lp Sport-scheduling.dat Warehouse-plan.da Sport-scheduling.mod Warehouse-plan.mo	<b>problems</b> to discover the simplicity and power of DropSolve.	.lp	OPL project	OPL project
Drop a problem here to solve		Oil-blending.lp	Sport-scheduling.dat Sport-scheduling.mod	Warehouse-plan.dat Warehouse-plan.mo
	Drop	a problem here to sol	ve	



## Two ways to access CPLEX in the cloud 2. REST API

https://developer.ibm.com/docloud/
https://developer.ibm.com/docloud/docs/welcome/

Access the solve service via its REST API



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Access the solve service via its REST API

• ready-to-use clients provided for Java<sup>™</sup> and Python, e.g.

• With any HTTP client (cURL, ...)

**IBM Analytics** 



#### CPLEX in the cloud

#### Sign up for a free trial

www.ibm.com/software/analytics/docloud

https://developer.ibm.com/docloud/

## free CPLEX community edition



## Exploiting existing software resources









**IBM Analytics** 



#### Interfacing with CPLEX



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#### Optimization Programming Language (OPL)

- write models in a more descriptive form
- write models in a more compact form
- faster prototyping, easier maintenance
- easier access to data (Excel, database, ...)

 $\sum x_{ijk} \le 1$  for i in I

- scriptable
- model editor
- IDE support (eclipse based)



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🔻 🚺 Run (defauli	t) 4	range Po	Positions = 03; // The different positions			
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	6	int M =	2; int S = 3;			
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#### docplex

#### Most recent addition

- https://pypi.python.org/pypi/docplex
- <u>pure</u> Python modeling API (<u>no</u> native code)
- open source (pypi, github)
- prepared to connect to local or cloud CPLEX
- $\rightarrow$  write your model in Python
- $\rightarrow$  hook up with the whole Python software ecosystem



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For example

- Equitable Coach Problem
- list of players from the internet (web service)
- graphical display of solution

 $\rightarrow$  use iPython/Jupyter notebook and Python libraries

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