

Overview on Applications: Is (advanced) mathematics useful? CO@W Berlin

Martin Grötschel

2.10.2015



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- Institut für Mathematik, Technische Universität Berlin (TUB)
- Forschungszentrum MATHEON
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

<http://www.zib.de/groetschel>

Remark

- For some of the photos and pictures shown in the lecture the copy right is unclear. That is why these pictures have been removed from the slides.



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 - a) MATHEON
 - b) Traffic
 - c) Telecom



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About mathematics

- The most challenging intellectual endeavor of mankind
- Beauty, esthetics
- Eternal truths
- But mathematics is more than that: an indispensable tool for almost all other scientific and economical activities



The role of mathematics in this century

- It is my firm belief – not only an advertisement pitch – that mathematics is THE scientific endeavor of the 21st century.
- Advanced sciences and industries have learned that.
- Unfortunately this is hardly known by the general public.
- But also many mathematicians are not aware of this role.



The role of mathematics in this century

Moreover

- If a country wants to grow, it needs to improve the skills of its population, and in particular, its mathematical education. And the mathematical education has to focus on both, the elementary and the top level.
(Eric Hanushek, KunMo Chung at MENAO)
- If a company wants to stay competitive, it needs mathematics.
- To understand the world (by scientific means), mathematics is indispensable.

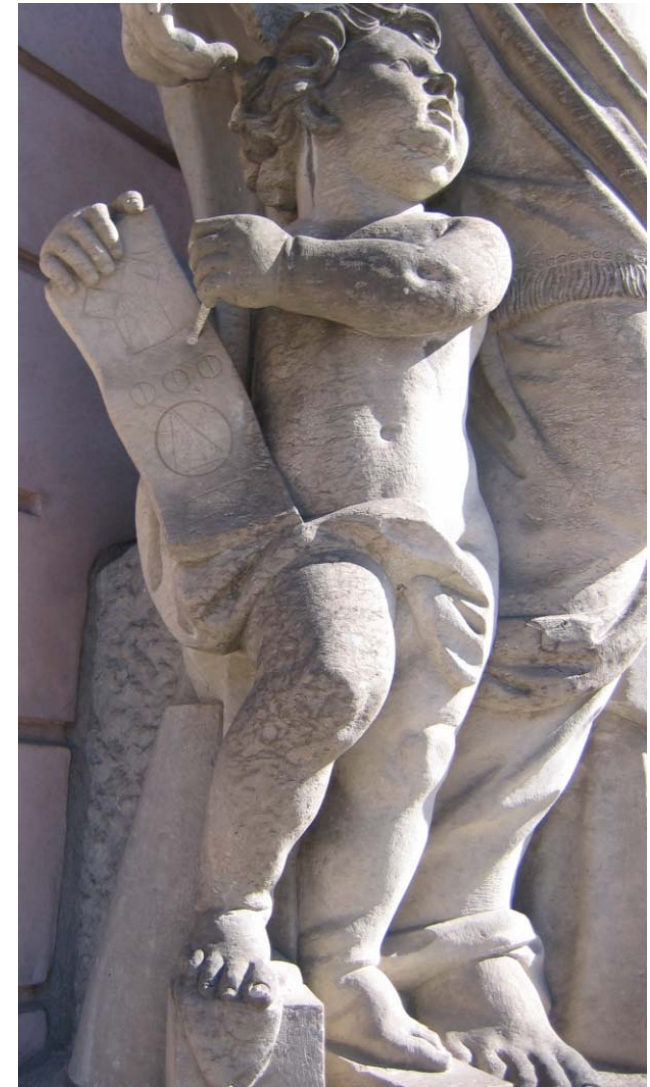


My favourite photo, ICM 2002



Considering maths as useful?

- In front of the Zeughaus in Berlin



Considering maths as useful?

- In front of the Zeughaus in Berlin



What do I mean by „useful“?

- Karl Weierstrass (1860): Every continuous function on a compact set assumes a maximum and a minimum.
- Important theorem, but useless in practice.
- Claus Scheiderer (1989): The stability index of an affine real variety of dimension $n > 0$ is $n(n+1)/2$.
- Do you understand it?
- With the Dijkstra algorithm of 1959 (and suitable data structures) a shortest path between two nodes of a weighted directed graph with n nodes and m arcs can be computed in $O(n \log(n) + m)$ time.
- A result that everyone employs every day (without knowing it).

What is efficiency?

- Complexity Theory
- Polynomial time solvability (fast): Dijkstra's algorithm
- NP-completeness (difficult)
- Practical efficiency



What is optimization?

The general standard example:

- Given a set S and a function f from S to an ordered set T . Find a point x in S such that $f(x)$ is as large or as small as possible (according to the ordering of T).
- Usually, S is a subset of the **n -dimensional vector space** over the reals (in theory); in practice we can only compute with rational numbers, though.
- And also usually, the ordered set T is the set of real or **rational numbers**.



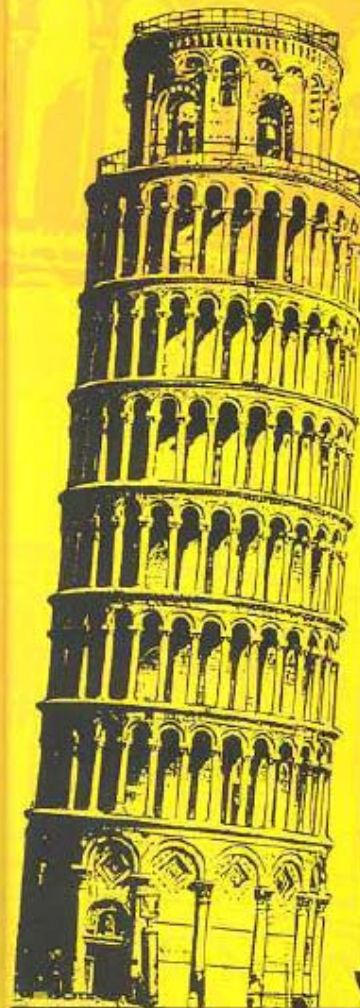
Proceedings of a
meeting of the DMV,
German Mathematical Society,
1993



Mathematik in der Praxis

Fallstudien aus
Industrie, Wirtschaft,
Naturwissenschaften
und Medizin

A. Bachem
M. Jünger
R. Schrader
(Herausgeber)



Springer

BMBF

Support Program of the
Federal Ministry of
Education and Research
Strengthening of Applied
Math Research in Germany

So far 6 periods:

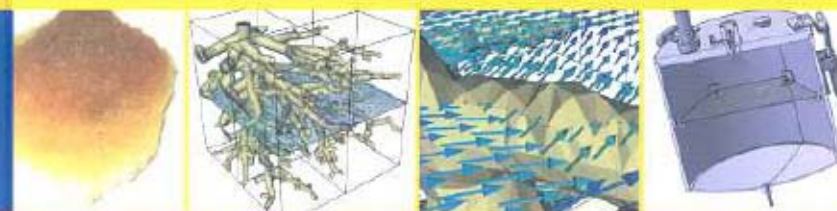
1993-1997, 1997-2001

2001-2004, 2004-2007

2007-2010, 2010-2013

Current 2014-2017:

Mathematik für Innovationen in
Industrie- und Dienstleistungen



Willi Jäger · Hans-Joachim Krebs
Editors

MATHEMATICS

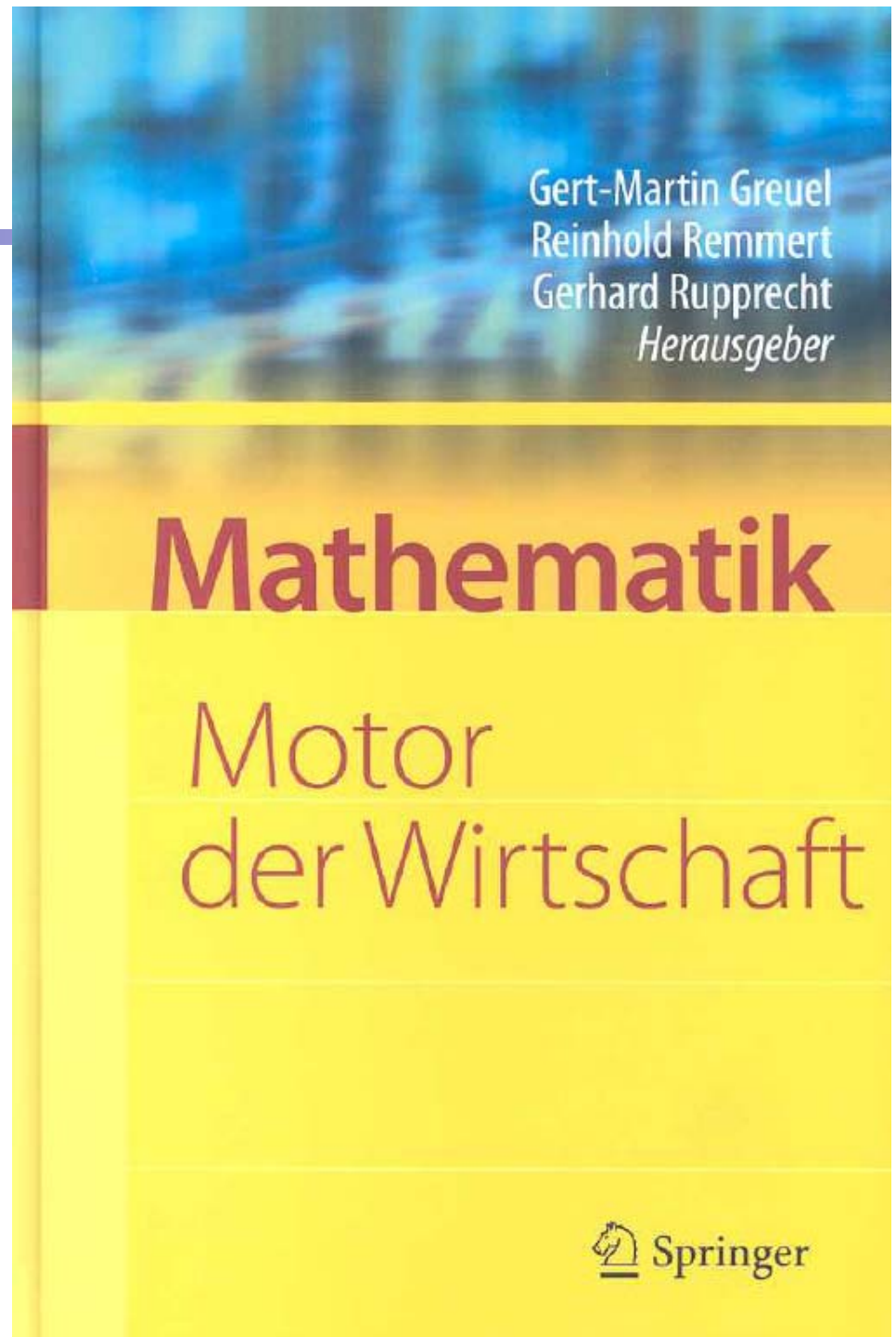
Key Technology for the Future

Joint Projects Between Universities
and Industry 2004–2007

Springer

DFG - Förderkreis
Mathematische Verbünde zur
Lösung von Problemstellungen
in Industrie und Wirtschaft

An initiative of the
German Industry for the
Year of Mathematics 2008



Statements about mathematics by DAX CEOs

- Die **Denkweise** der Allianz ist **mathematisch geprägt**.
(Rupprecht, Allianz)
- Mathematisches Können ist ein **Wettbewerbsvorteil**. (Zetsche, Daimler)
- Universelle Anwendbarkeit macht Mathematik zur **Königsdisziplin**.
(Zetsche, Daimler)
- Unternehmensführung ohne Mathematik ist wie Raumfahrt ohne Physik.
(Kagermann, SAP)
- Gerade bei einem chemisch-pharmazeutischen Unternehmen wie Bayer sind Numerik, Optimierung, Differentialgleichungen und Statistik in den wichtigen Bereichen Wirkstoffsuche und klinische Studien, Prozessoptimierung und Logistik nicht mehr wegzudenken.
(Wenning, Bayer)
- **„Ohne Mathematik tappt man doch immer im Dunkeln.“**
(Werner von Siemens)



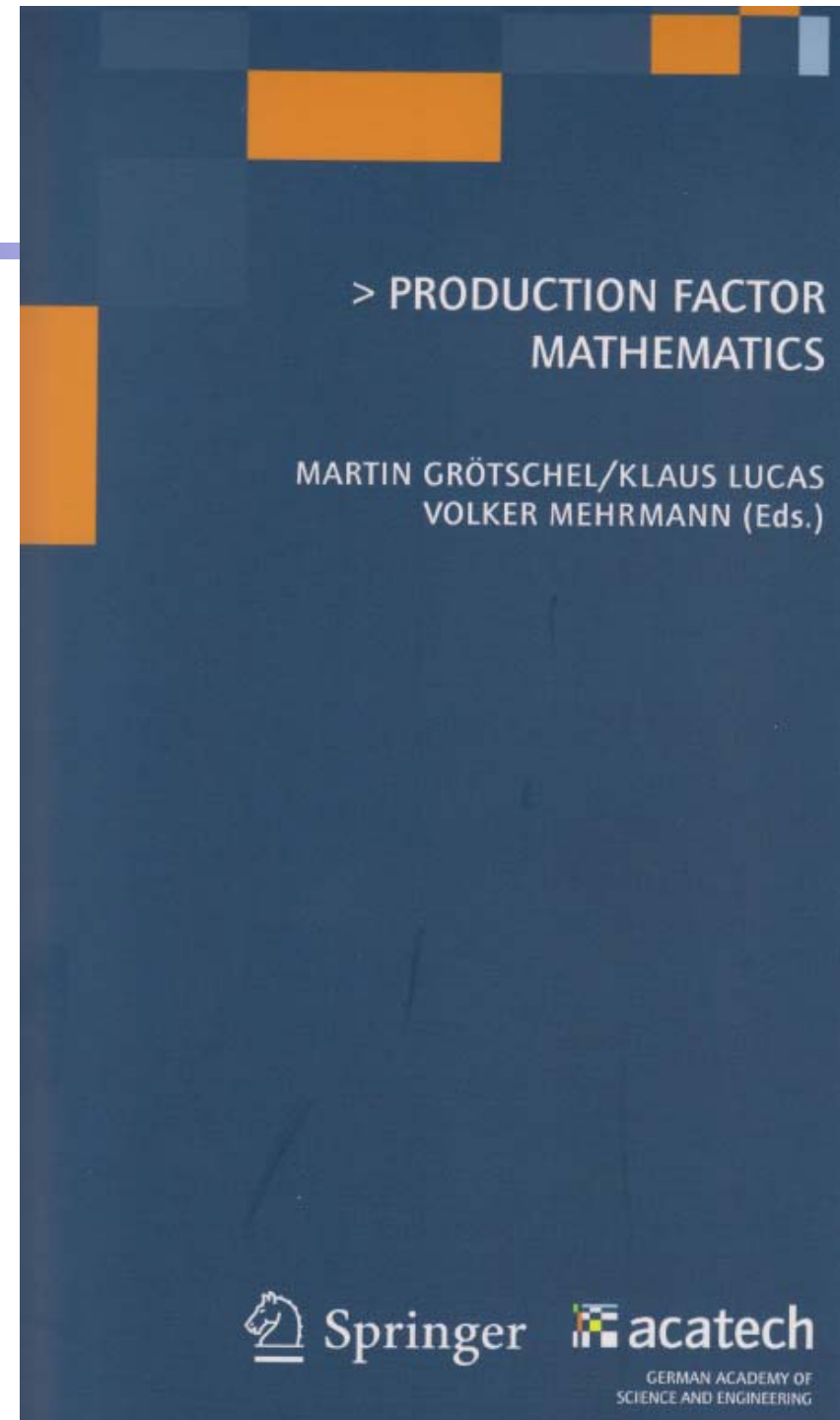
Some ZIB/MATHEON partners from industry



acatech German National Academy of Engineering

**(The Book appeared 2010,
German version in 2008)**

downloadable from my Web site



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8. Conclusions





SEARCH



AA  Impressum

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The Zuse Institute Berlin (ZIB)

is an interdisciplinary research institute for applied mathematics and data-intensive high-performance computing. Its research focuses on modeling, simulation and optimization with scientific cooperation partners from academia and industry.

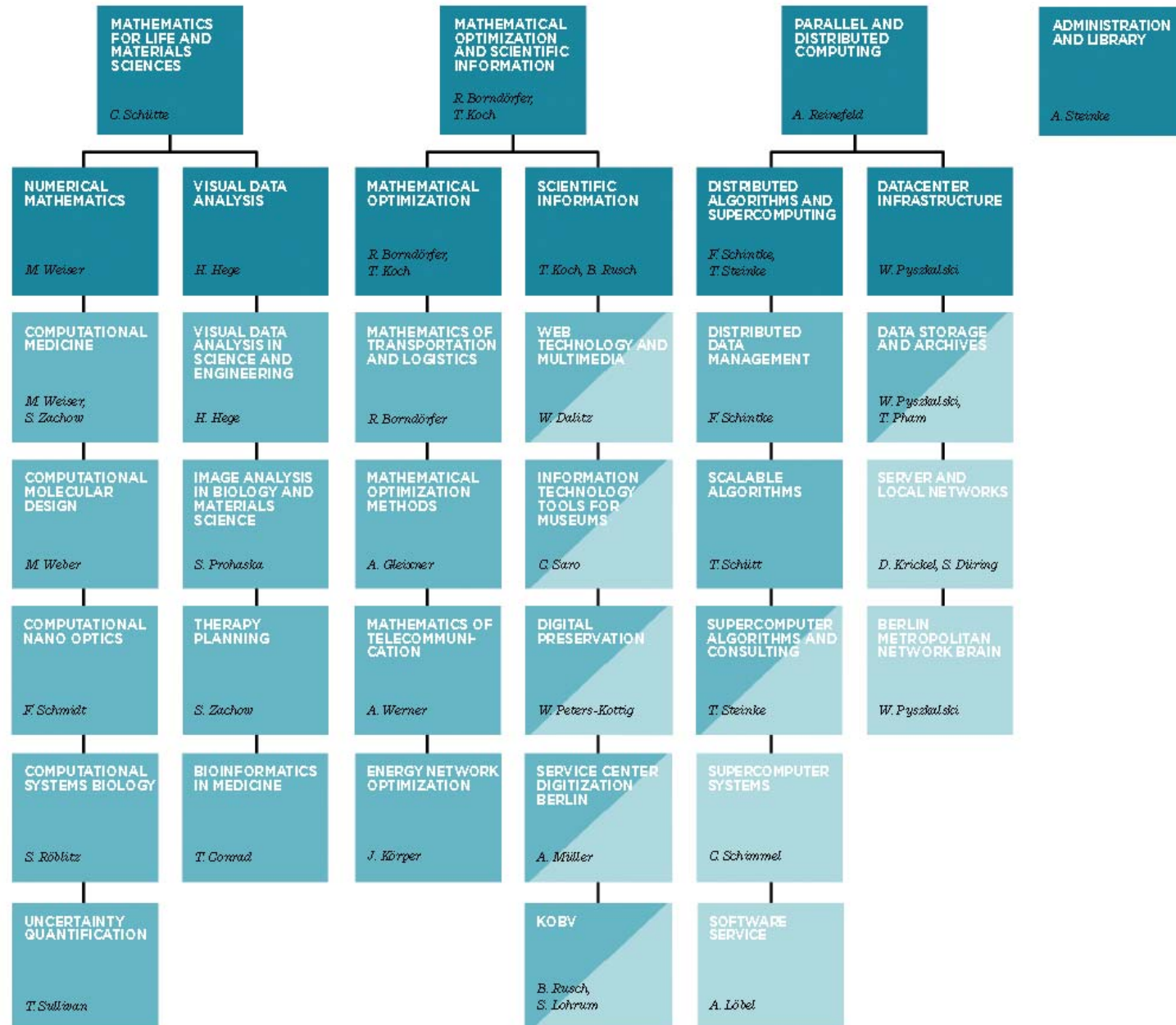
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Our latest news

EVENTS ARCHIVE

Upcoming events

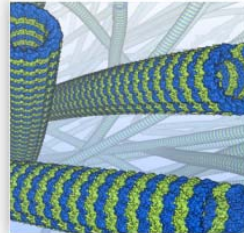
Organisational structure of ZIB



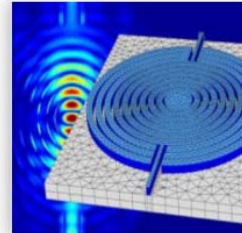
ZIB's Research Focus Areas



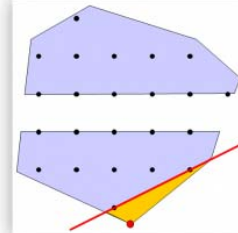
-- Research Focus Area
-- Solutions for
Individualized Medicine



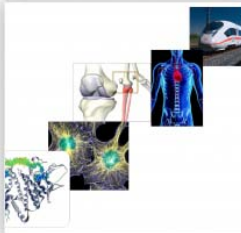
-- Research Focus Area
-- Molecular and
Biological Processes



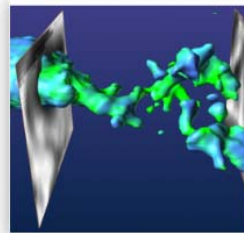
-- Research Focus Area
-- Materials and Optical
Processes



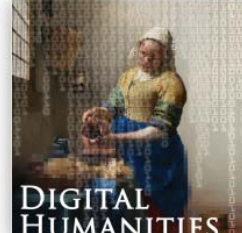
-- Research Focus Area
-- Combinatorics and
MINLP



-- Research Focus Area
-- MSO for Multiscale
Systems



-- Research Focus Area
-- Robust Solutions
under Uncertainty



-- Research Focus Area
-- Digital Humanities



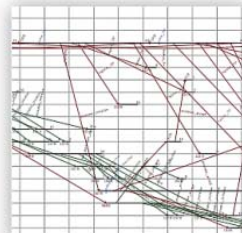
-- Research Focus Area
-- Efficient Infrastructure
and Logistics



-- Research Focus Area
-- Large-Scale Data
Management, Curation &
Analysis



-- Research Focus Area
-- Highly Parallel
Computing



-- Research Focus Area
-- Scalable and Fault-
Tolerant Algorithms

<http://www.zib.de/research>

TU Berlin, Mathematisches Institut



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Institut für Mathematik**

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Fax: 314-21754

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Gebäude MA
Str. des 17. Juni 136
10623 Berlin

TU Berlin



Technische Universität Berlin
Faculty II – Mathematics and Natural Sciences
Institute of Mathematics



Deutsche Version

Wissenschaftsjahr 2008

Mathematik
Alles, was zählt



Local MathNet Guide

Visitors Guide

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- Service for Engineers

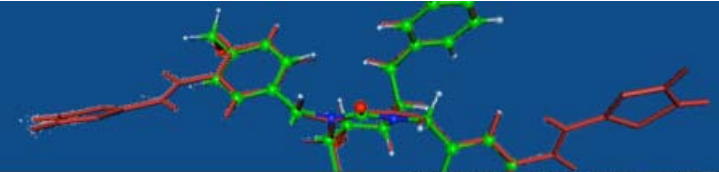
News & Events

- Positions Available
- Events, Meetings and Dates
- Mathematics Calendar Berlin-Potsdam

Sorry, but pages marked with are currently only available in German language.



Forschungszentrum MATHEON
Mathematik für Schlüsseltechnologien



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Prof. Thorsten Koch receives Google Faculty Research Award: Für Forschungsarbeiten zu „Mixed Integer Optimization as a Service“ erhält Prof. Dr. Thorsten Koch (ZIB/TUB) einen Google Faculty Research Award. Prof. Koch leitet die Abteilung Wissenschaftliche Information am ZIB und ...



Das Forschungszentrum MATHEON

entwickelt Mathematik für Schlüsseltechnologien und unterstützt Partner in Industrie, Wirtschaft und Wissenschaft. Schule und Öffentlichkeit bilden einen weiteren Fokus unserer Aktivitäten. MATHEON wird von den drei Berliner Universitäten (FU, HU und TU) und den Forschungsinstituten (WIAS und ZIB) gemeinsam getragen.



GAMM-Preis für Joscha Gedicke: Dr. Joscha Gedicke vom WIAS und MATHEON-Mitglied hat einen der vier diesjährigen Preise der Dr.-Körper-Stiftung der Gesellschaft für Angewandte Mathematik und Mechanik (GAMM) erhalten. ...



Mathematischer Modellbau leicht verständlich aufbereitet: Dr. Jannik Matuschke von der TU Berlin wird mit dem Klaus Tschira Preis für verständliche Wissenschaft ausgezeichnet. ...

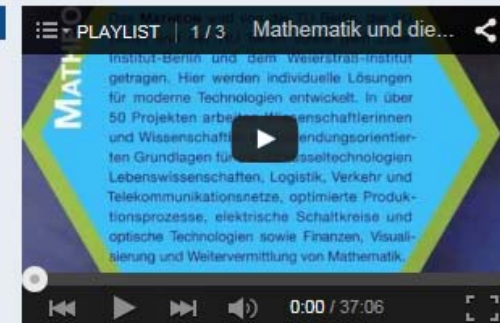


10. Vektoria Award, : dem Kreativpreis für Schüler und Schülerinnen von CASIO! ...

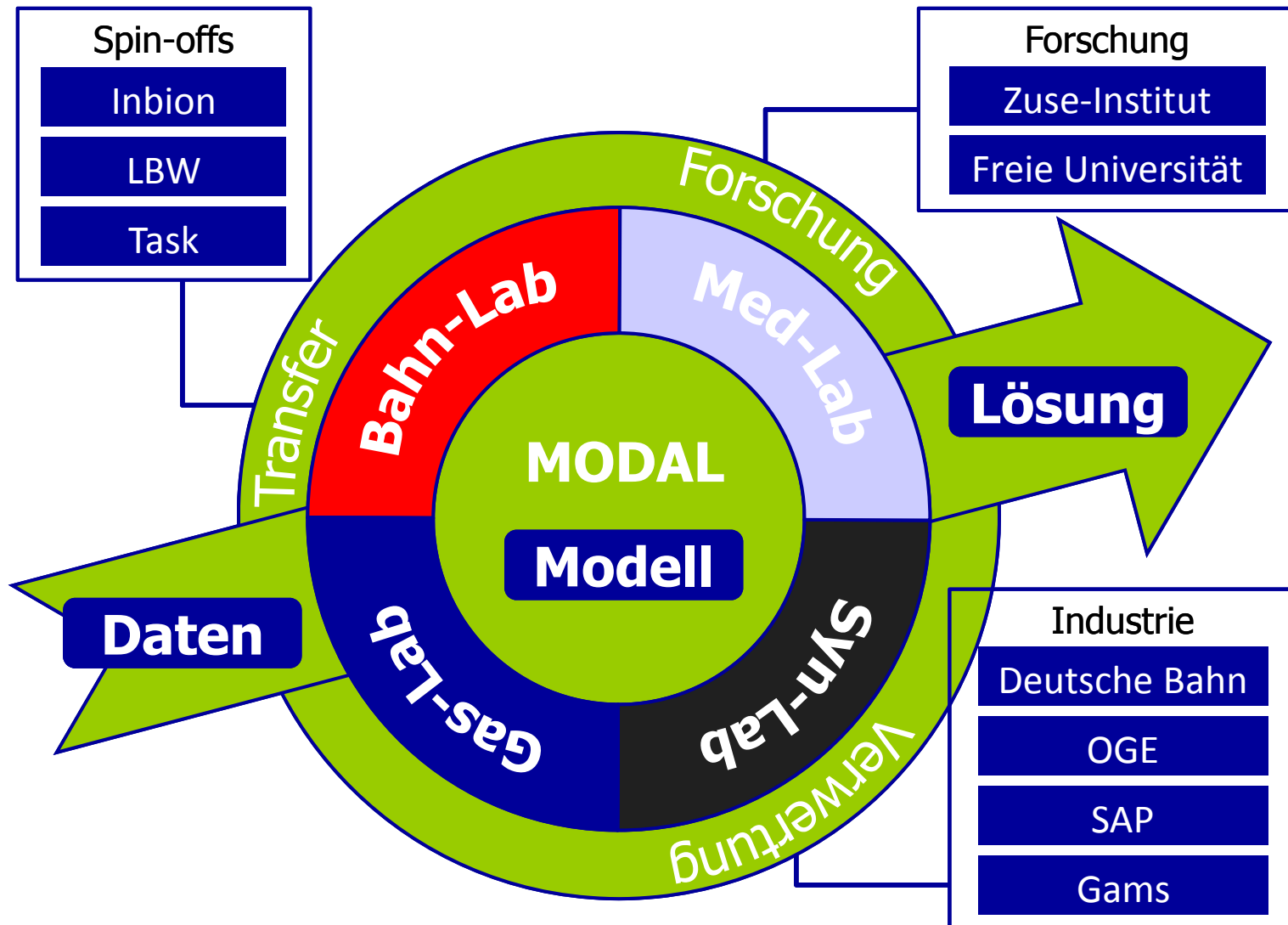


Aufgelesen

- Spätbabylonische Liebe zur Mathematik (Spiegel online)
- Die Mathematik des Riechens (Bild der Wissenschaft)
- Mathematikerin Hélène Esnault (Tagesspiegel)
- Zahlenplattform Imaginary.org (Spiegel online)
- Rechenschwäche. (Berliner Zeitung)
- Die neue „Miss Germany“ ist gekürt (Berliner Zeitung)
- Neue Methoden im Mathe-Unterricht (Arte TV)
- Die Optimierung der Welt (Tagesspiegel)
- Mathe-Prof als YouTube-Star (Spiegel Online)



Forschungscampus MODAL: BMBF and industry supported

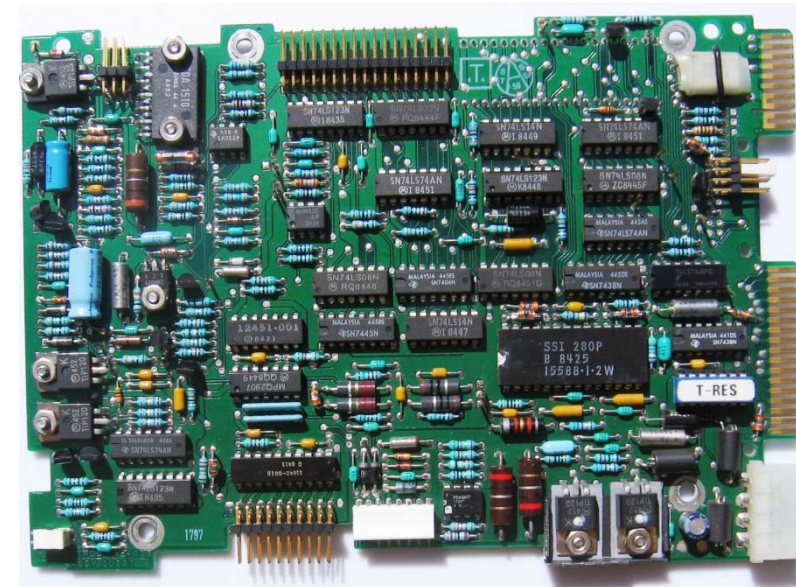
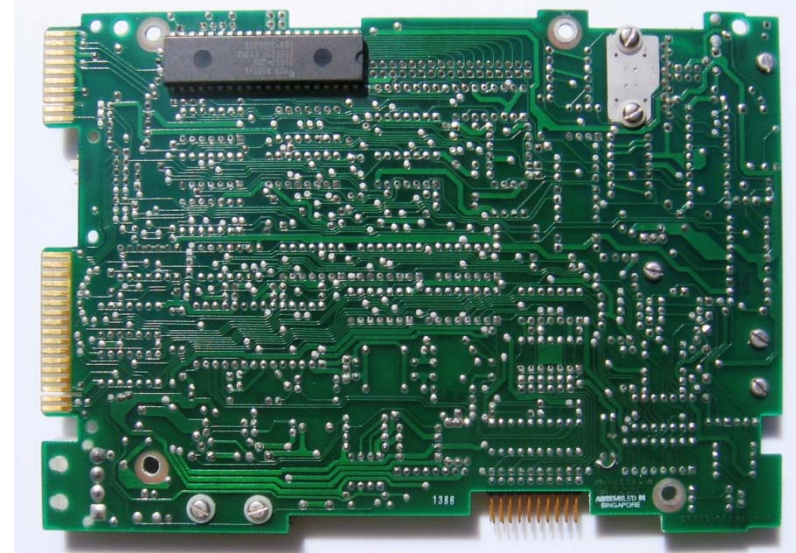


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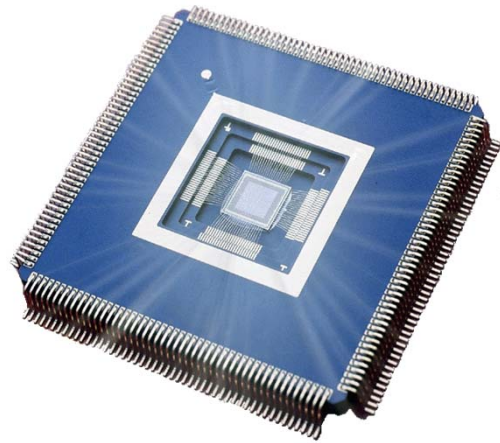
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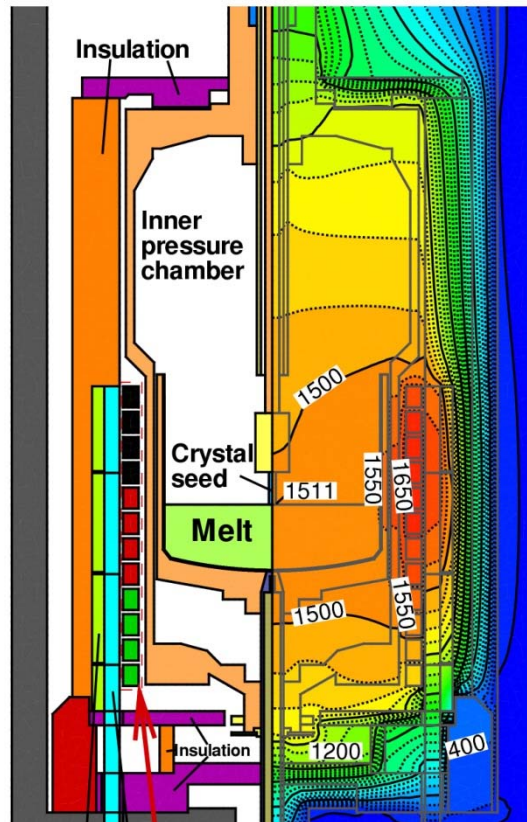
Printed Circuit Board Drilling and Assembly Machines



Design, Verification and Production of Chips



Crystal growth (MATHEON C9)



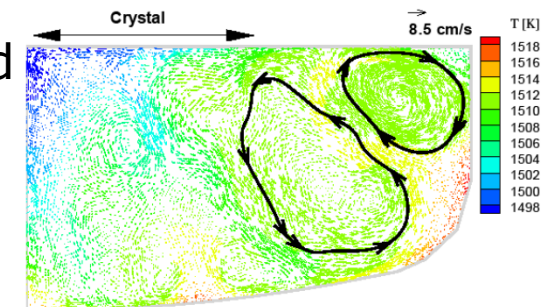
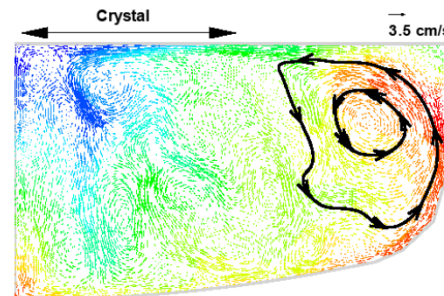
Control of the process via magnetic fields

to optimize

- temperature
- flow of the molten mass
- form of solidification surface

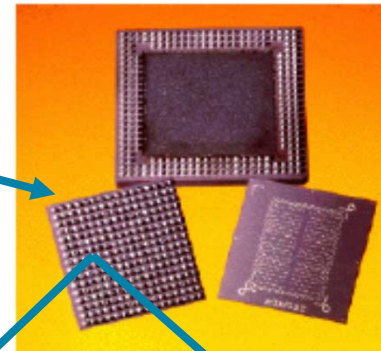
without and with

magnetic field



Soldering: Trouble on Chips and Printed Circuit Board

Phase separation and coarsening

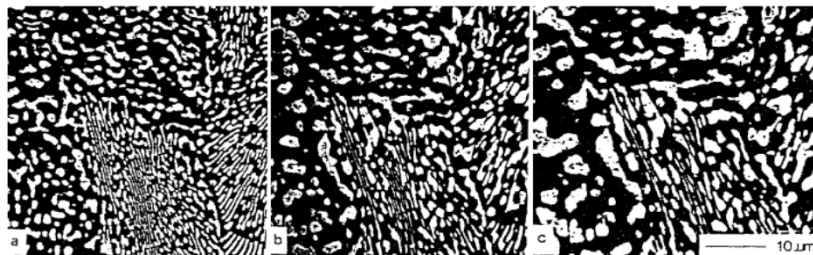


Balls of solder (e.g. tin-lead-alloy) establish electrical contact to the chip.

switched on

room temperature

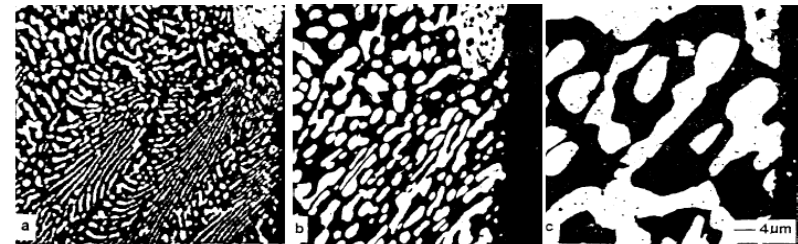
125 °C



2 hs

17 days

63 days



0 hs

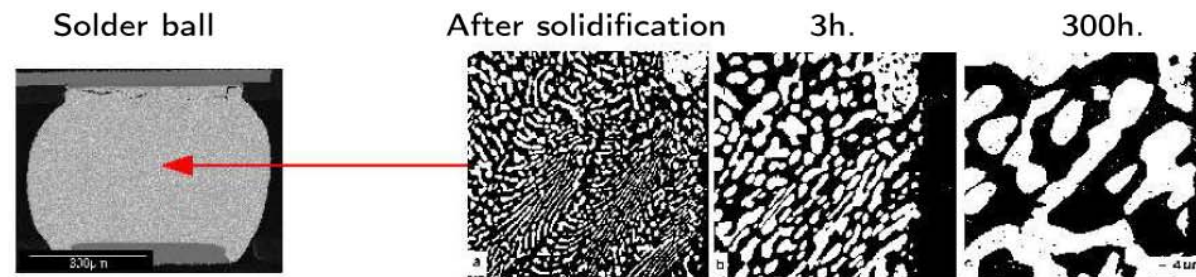
3 hs

13 days

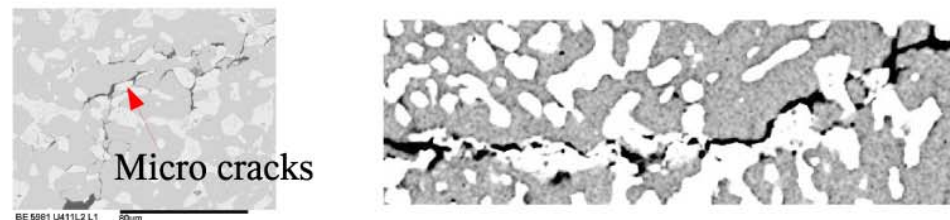
Background

Morphology in solder joints

▷ Phase separation and Coarsening



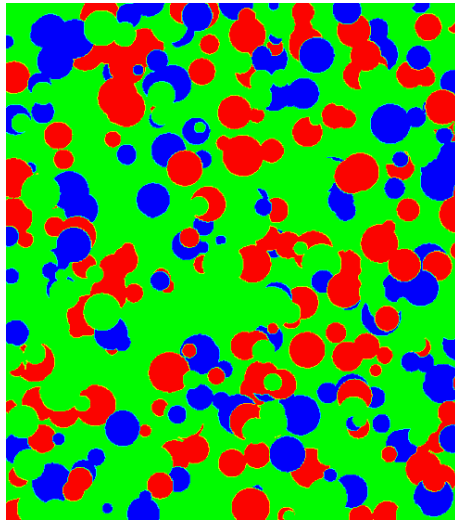
▷ Crack initiation and propagation



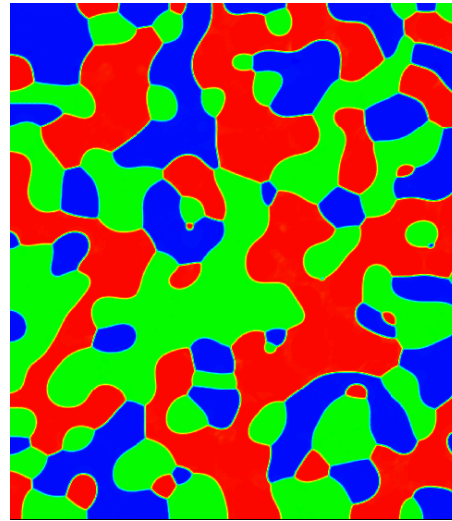
ELFNET Roadmap 2007:

High priority: Models for microstructure evolution + cracking

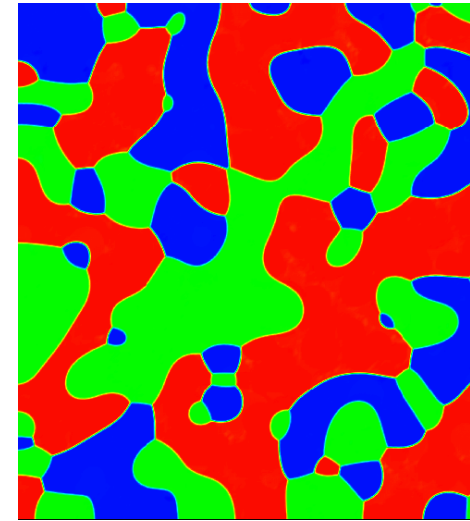
MATHEON-Projekt: Modelling and Simulation of Phase Separation



$t=0$



$t=100$



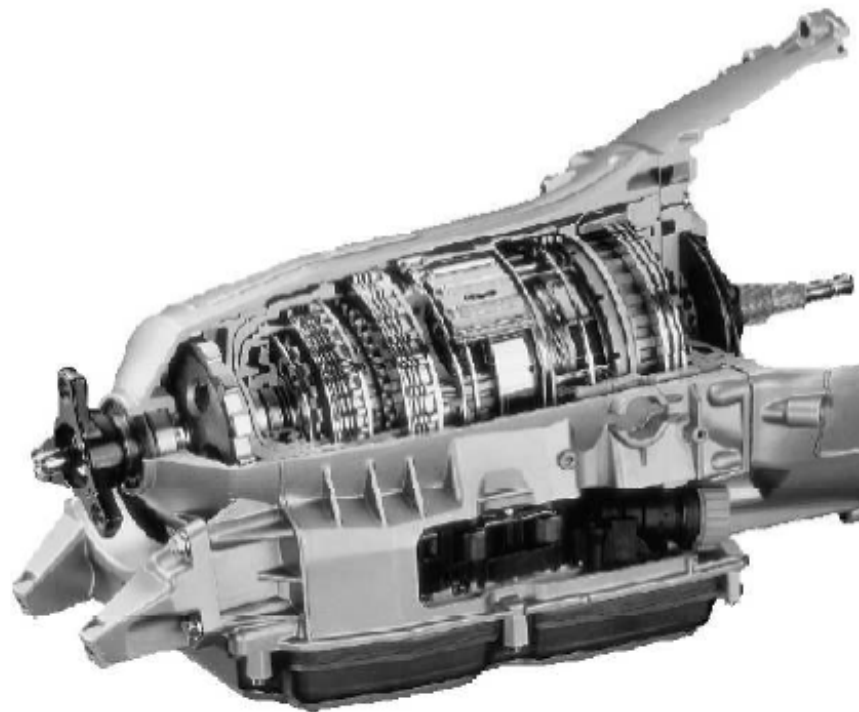
$t=600$



Optimization of automatic gear boxes

Modellbasierte Regelung von Automatikgetrieben

Dissertation P. Hamann, Daimler AG



Herlitz at Falkensee (Berlin)



Optimization and control of transport devices (such as elevators, stacker cranes) in factories



Herlitz, Falkensee

Traffic and Transport

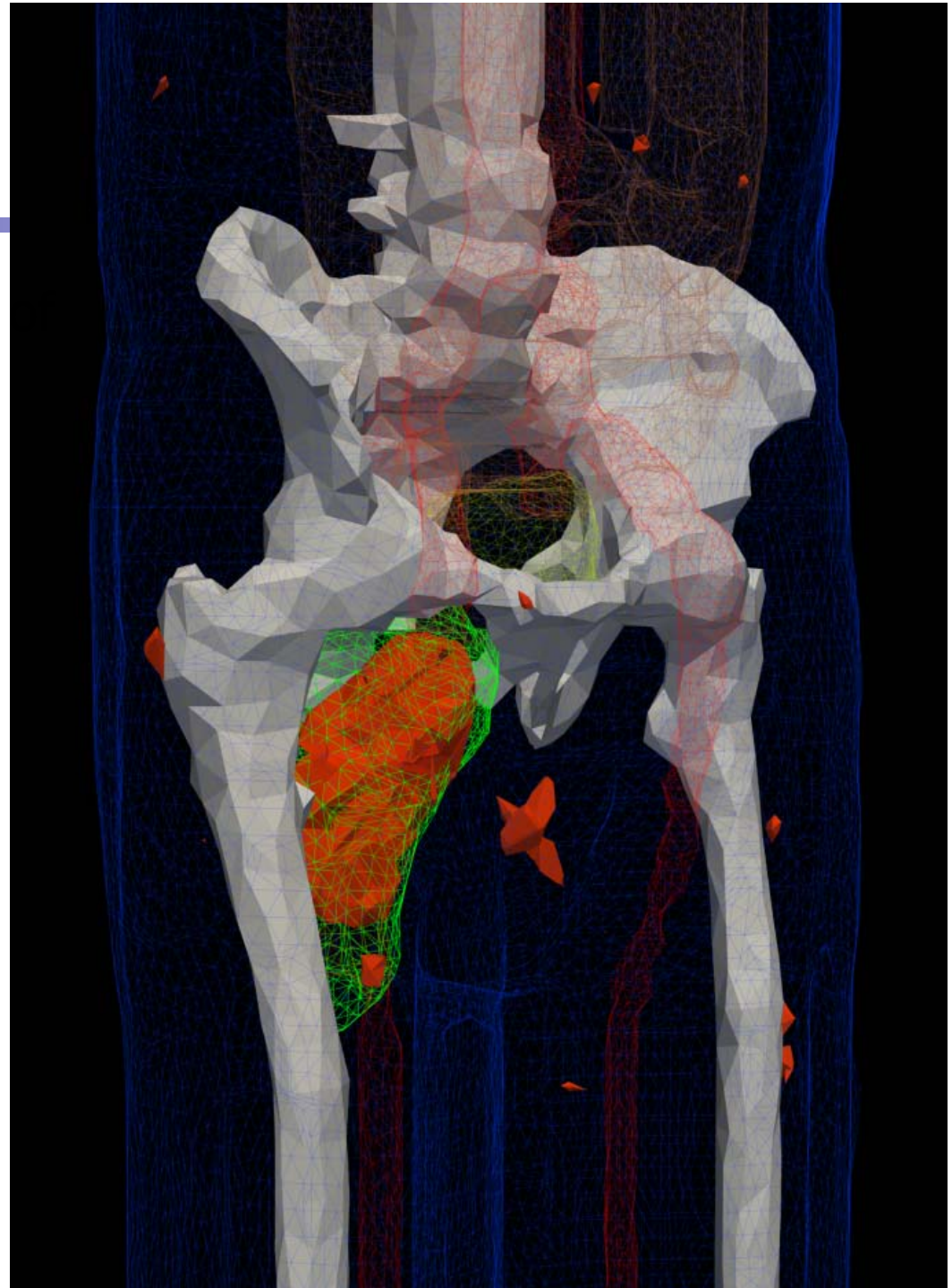


Supereinsatz. In Berlin und Brandenburg mussten die Gelben Engel letztes Jahr mehr als 240 000 Mal ausrücken, um Havaristen in der Hauptstadt und auf 1700 Autobahnkilometern wieder flottzumachen – ein Rekoreinsatz. Einen Rückgang von zehn Prozent bei den Pannen registrierten dagegen die Gelben Engel in Mecklenburg-Vorpommern. Bei insgesamt 72 389 Einsätzen schafften sie jedoch auch einen Rekord: In 84 Prozent der Fälle konnten die Autofahrer mit dem Wagen weiterfahren.



ZIB-Project Hyperthermia

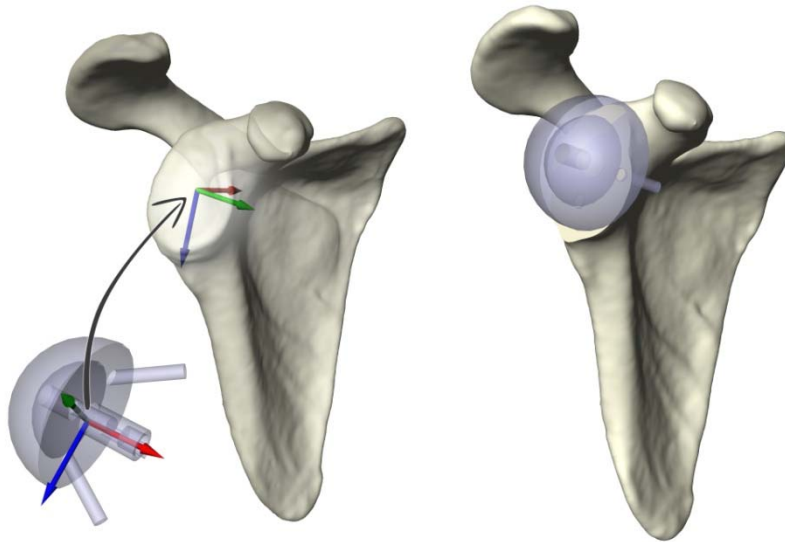
- Discovery of the location tumors (cancer zones).
- Positioning of antennas for radiation therapy
- Computation of the strength of the radiation and “ray tracing” to discover the heat distribution in the body
- Concentrating the heat in the tumor
- Visualization



ZIB-Project Hyperthermia (Deuflhard group)



Implantation & Automatic FE-Mesh Generation

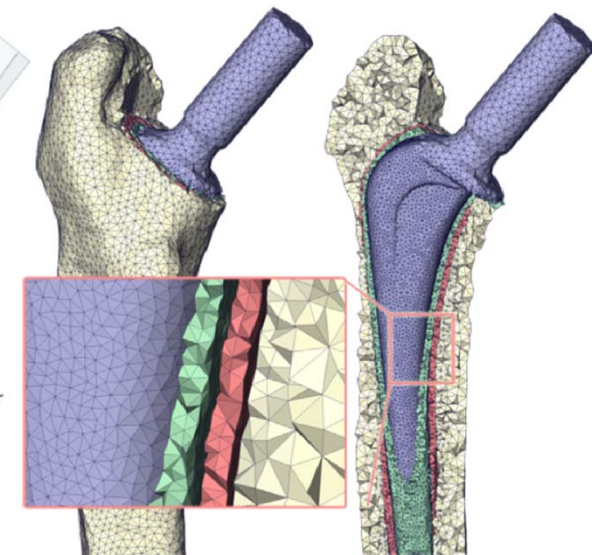
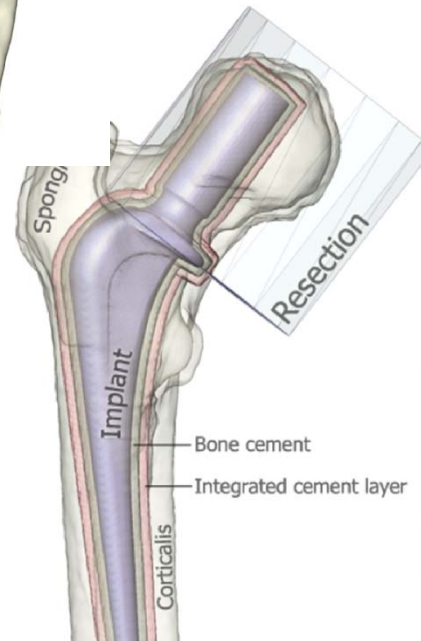


Virtual Implantation

- Extraction of joint coordinate systems
- Automatic implant selection and positioning

FE-mesh generation

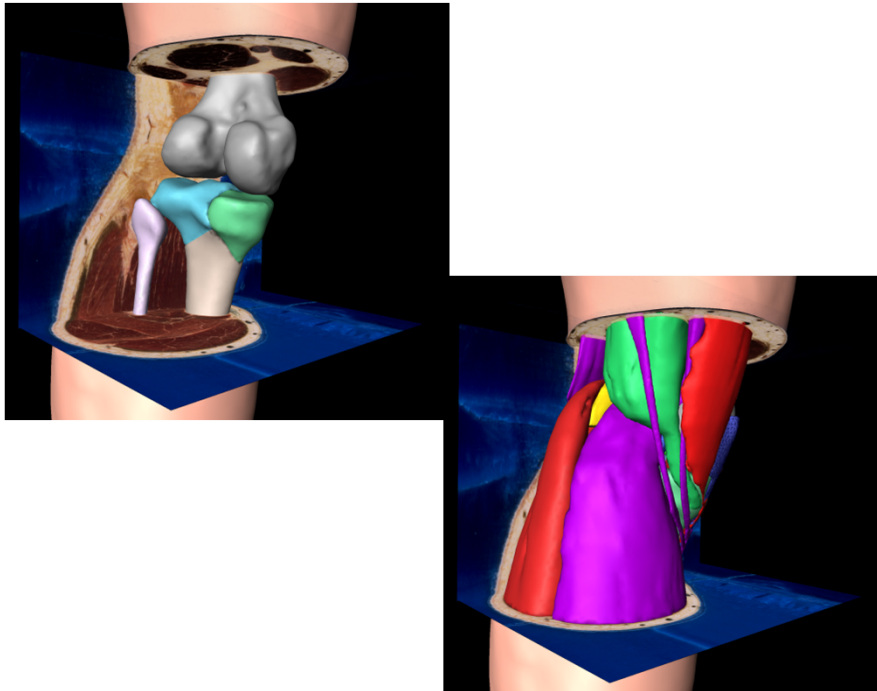
- Feature-preserving, multi-material tetrahedral meshes
- Large scale studies
- Delaunay grid



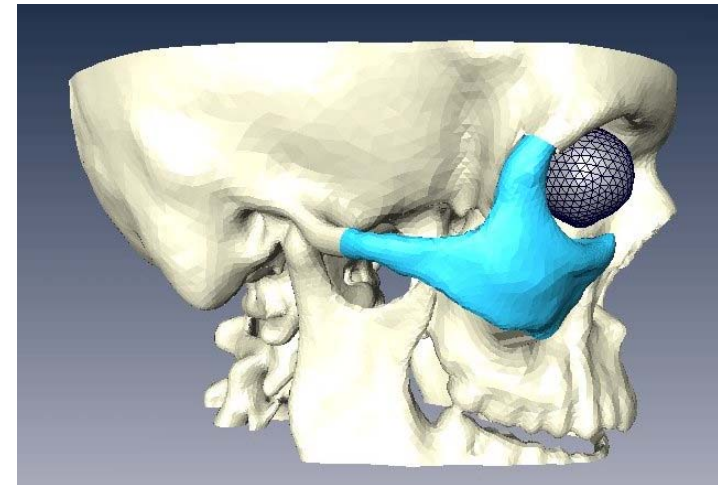
MATHEON A17

Computational Surgery Planning

Moving joints



Bone implant design

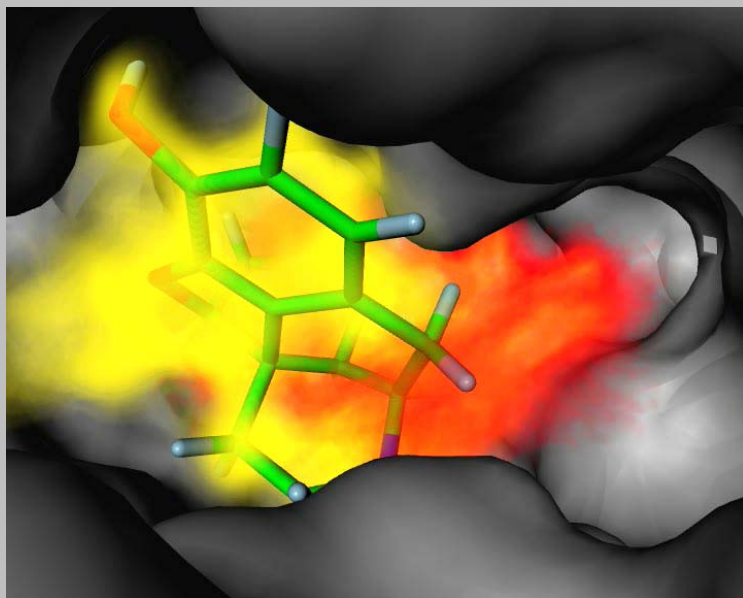


State constrained optimization, biomechanical modelling, dynamical contact problems

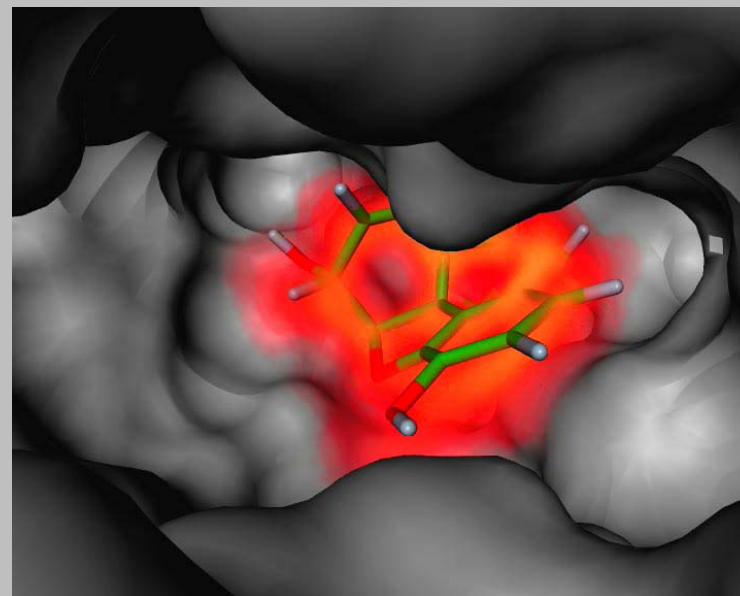
Application Area A – Life Sciences

Novel Pain Relief Drugs

Selective pain relief drug in binding pocket of μ -opioid receptor



inactive in CNS



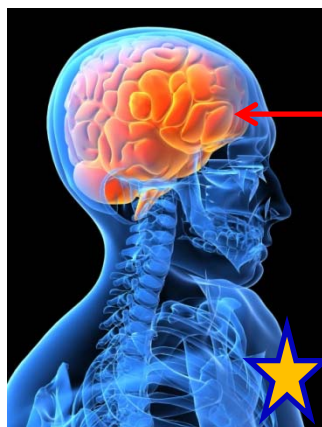
active in inflamed tissue



novel in silico design strategy
two patents
ongoing clinical tests

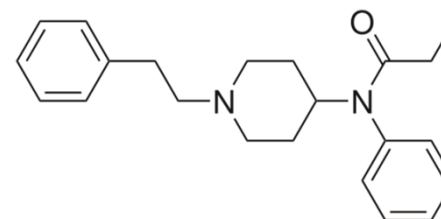


Pain relief drugs: opioids



side effects
respiratory depression
constipation, tolerance,
addiction...

pain relief effect



Opioid Fentanyl (Fen),
EUR 271 Mio. p.a. in Germany
EUR 4 Mrd p.a. world-wide (mainly USA)

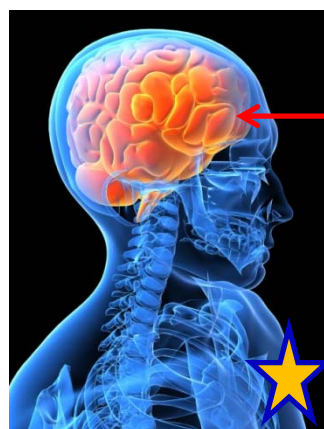


...prescription pharmaceuticals are
now involved in more overdose
deaths than either heroin or cocaine
in North America

B. Fischer, J. Rehm
Deaths related to the use of prescription opioids
CMAJ. Dec 8, 2009; 181(12): 881–882.

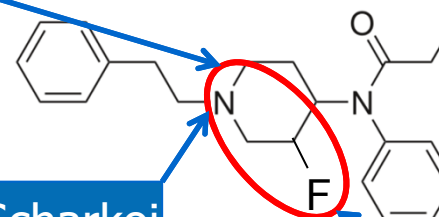
pH-dependent opioids tests: in vitro & vivo

Conformation dynamics & quantum chemistry



side effects
healthy tissue pH 7

pain relief effect
inflammation pH 5



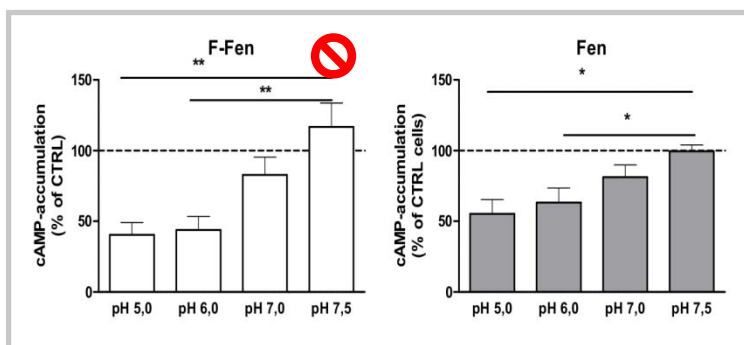
O. Scharkoi

F-Fentanyl

M. Weber

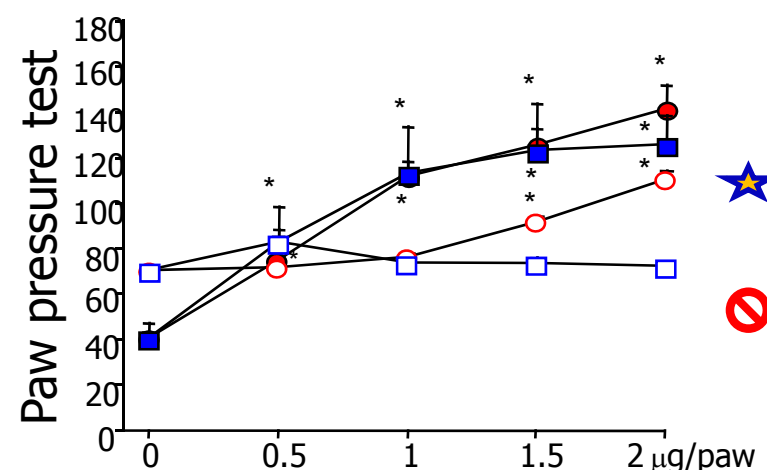
ZIB, Charité

in-vitro

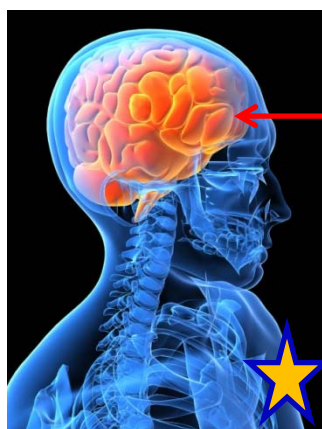


in-vivo

Injection into inflamed paw (i.pl.)



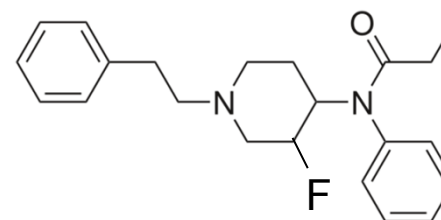
Pain relief drugs:patents



side effects
healthy tissue pH 7



pain relief effect
inflammation pH 5



F-Fentanyl

ZIB, Charité

Patents in nationalization phase (EU, USA) after PCT

C. Stein, C. Zöllner, M. Weber, O. Scharkoi: Fentanyl derivatives as pH-dependent opioid receptor agonists. EP2559685

C. Stein, M. Weber, O. Scharkoi, P. Deuflhard: Method and system for indentifying compounds that bind and preferably activate a target opioid receptor in a pH-dependent manner. EP2613277

Estimated value (including risk to fail clinical phases): EUR 6,7 Mio

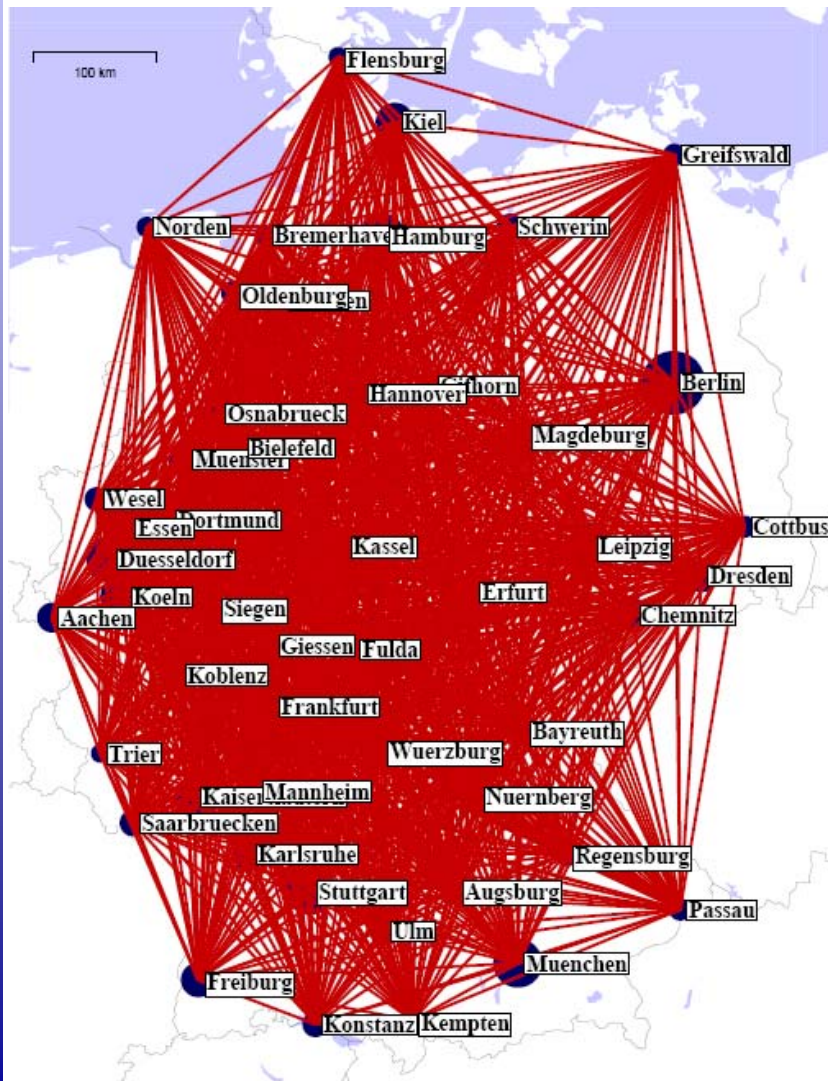
Visualization, Animation: Spider Man (spin-off mental images)



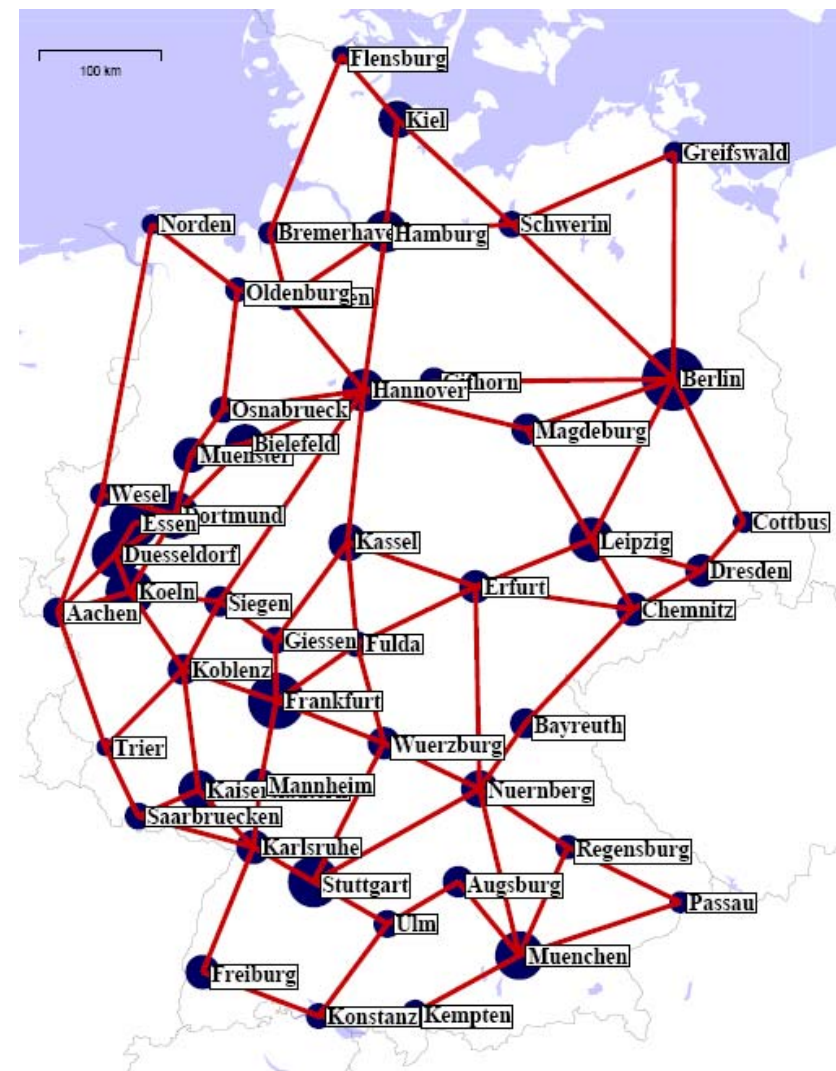
Telecommunication network design

MATHEON B3

Logical connections



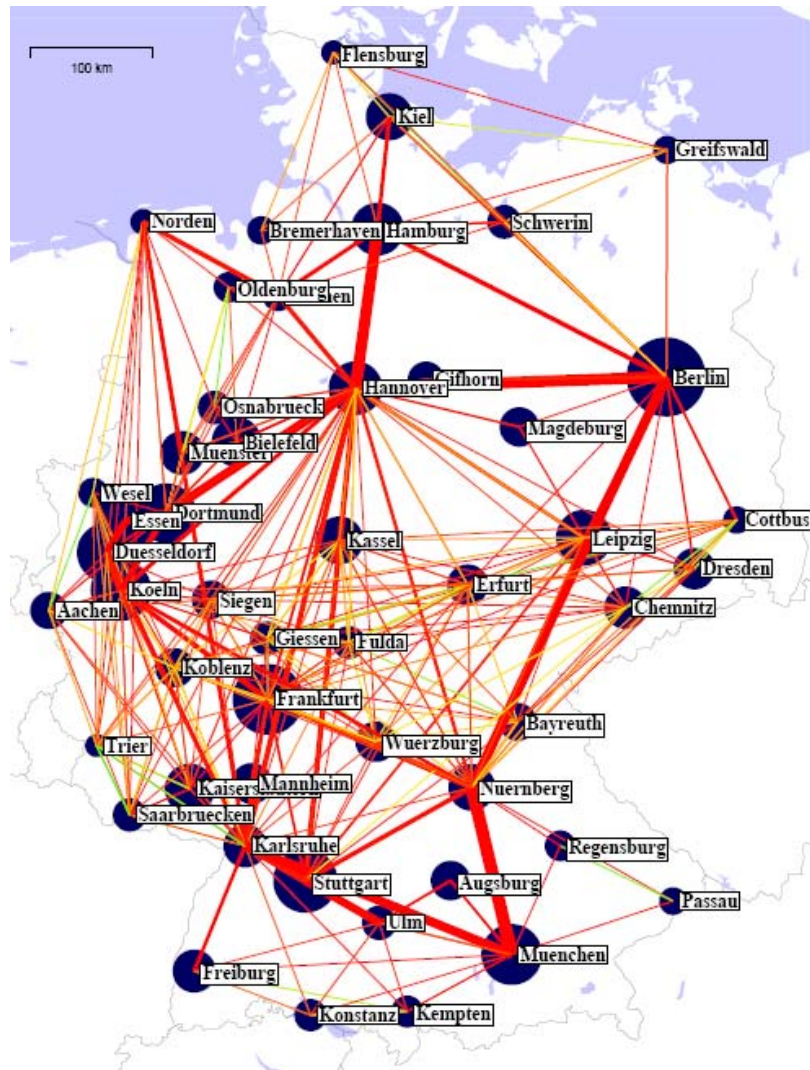
Physical connections



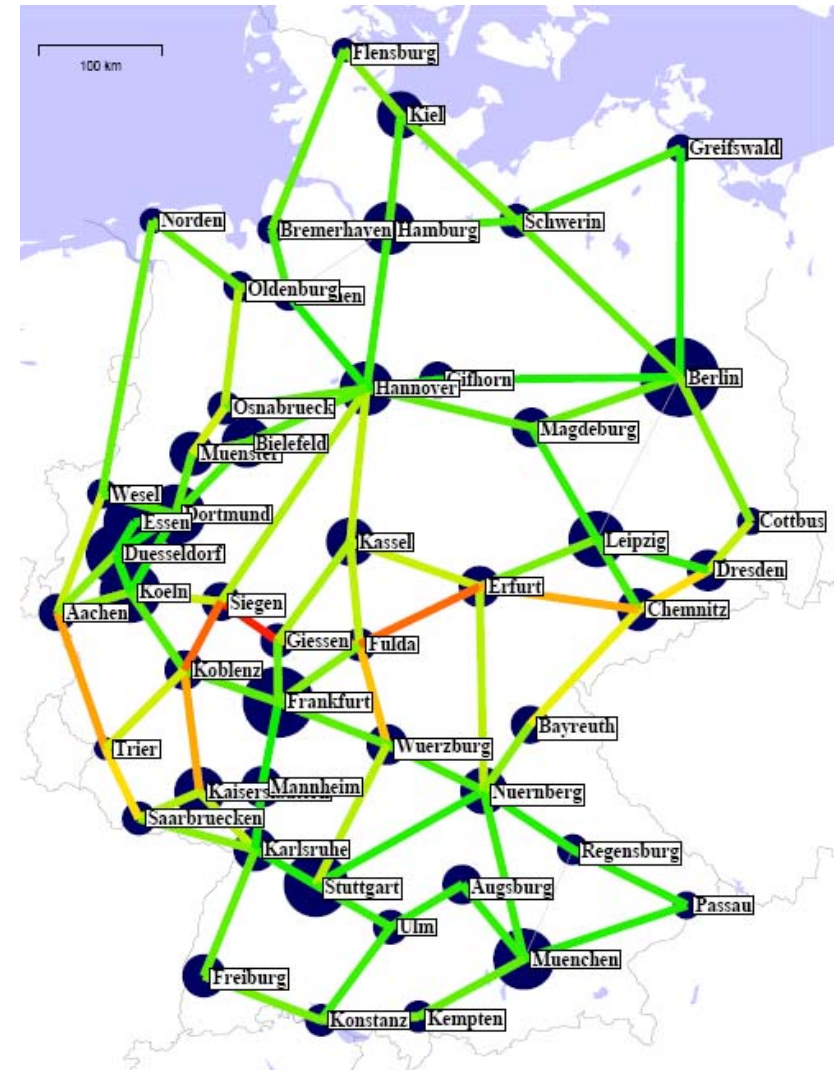
Network design

MATHEON B3

Logical connections: solution



Physical connections: solution



FTTH: Fiber To The Home



A Project with BHP Billiton

Production scheduling for an open pit mine with a stockpile

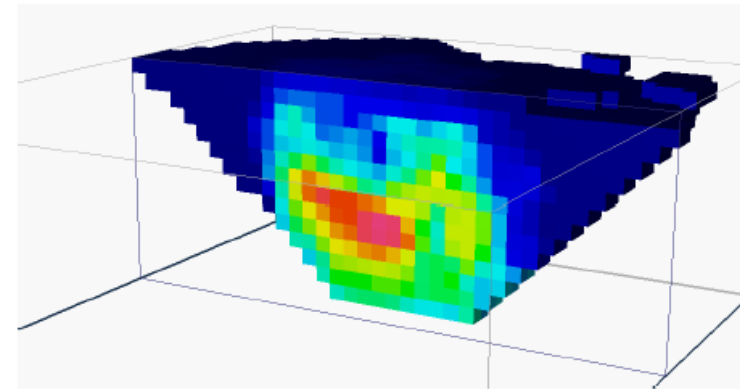
Goal Maximize net present value

Decisions

- Which block is mined when?
- What material is processed immediately?
- What material is stockpiled?
- What fraction of stockpile is processed when?

Constraints

- Block mining order
- Mining and processing capacities
- *Mixing of material in stockpile*



Block model of an open pit mine
blue = low grade ore, red = high grade ore



Stockpiles at Yandi mine, Australia.

Cooperation



The University of Melbourne



University of New South Wales



BHP Billiton

Applications

Areas with significant mathematics demand:

- Industrial production (control of CNC machines, assembly line optimization, robot control,...)
- Mining (Scheduling, rock damage and fracture models, ...)
- Health care & medicine (support for operations, drug design,...)
- Energy (optimization of energy production and mix, unit commitment,...)
- Financial mathematics (modelling of risk,...)
- Infrastructure planning (public transport, water, street, gas,... networks, harbor design)
- Telecommunication (network design, channel assignment)
- Logistics & Traffic and Transport (vehicle circulation,...)
- Sports
- ...
- I can go on forever.....



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7. Examples
 - a) MATHEON
 - b) Traffic
 - c) Telecom



Modelling?



Mathematical models of integer/combinatorial optimization

- travelling salesman problem (the prototype problem)
- routing
- location
- set-packing, -partitioning, -covering
- cut problems (max and min)
- scheduling
- node and edge colouring
- shortest paths (with resource constraints)
- various flow problems
- general integer and mixed-integer programs
- etc.



Typical optimization problems

$$\begin{aligned} &\max f(x) \text{ or } \min f(x) \\ &g_i(x) = 0, \quad i = 1, 2, \dots, k \\ &h_j(x) \leq 0, \quad j = 1, 2, \dots, m \\ &x \in \mathbb{R}^n \text{ (and } x \in S) \end{aligned}$$

„general“
(nonlinear)
program
NLP

$$\begin{aligned} &\min c^T x \\ &Ax = a \\ &Bx \leq b \\ &x \geq 0 \end{aligned}$$

linear
program
LP

$$\begin{aligned} &\min c^T x \\ &Ax = a \\ &Bx \leq b \\ &x \geq 0 \\ &x \in \mathbb{Z}^n \\ &(x \in \{0, 1\}^n) \end{aligned}$$

(linear)
integer
program
IP, MIP

program = optimization problem



Industry requirements grow

$\max f(x)$ *or* $\min f(x)$

$g_i(x) = 0, \quad i = 1, 2, \dots, p$

$h_j(x) \leq 0, \quad j = 1, 2, \dots, q$

$x_i \in \mathbb{Z} \quad \text{for some } i$

$x_j \in \{0, 1\} \quad \text{for some } j$

$x_k \in \mathbb{R}^n \quad \text{for some } k$

Plus:

Stochastic aspects

Robust optimization

Real time/online optimization

Several objectives

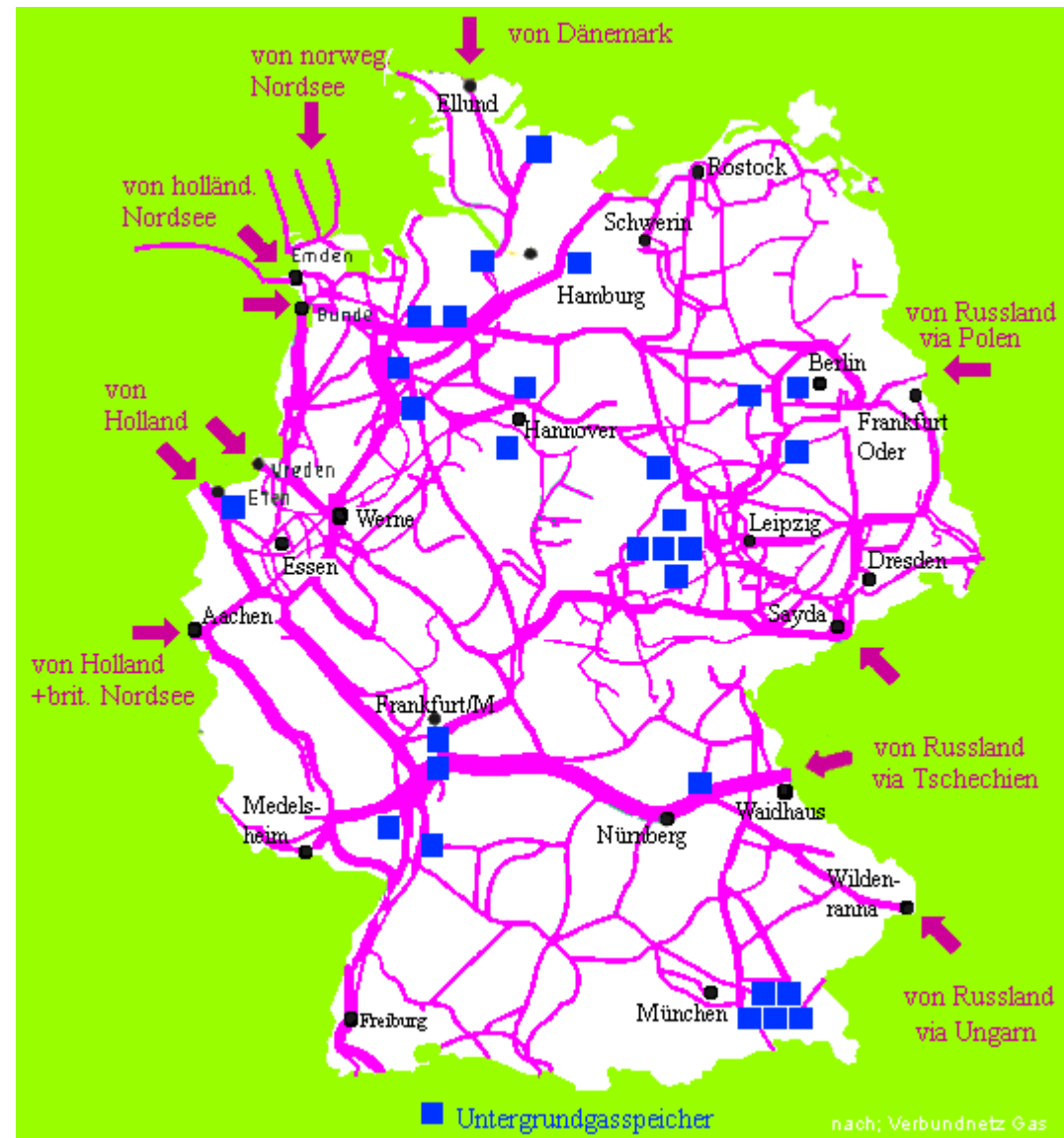
Example:

gas pipeline optimization

More general:
mixed-integer nonlinear
program
MINLP



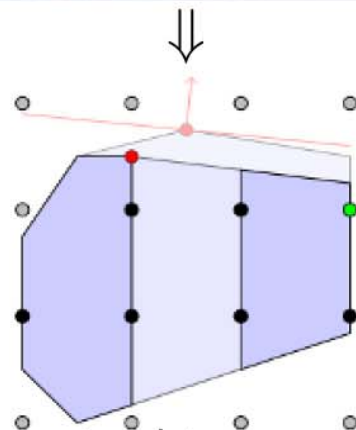
Simulating, Optimizing, Extension Planning of a Gas Pipeline Network



MATHEON B20 & FORNE Project: Optimization of Gas Transport

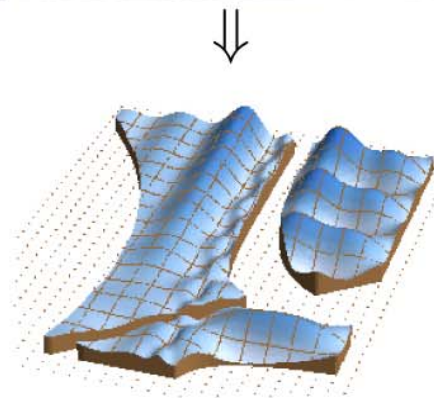
new challenges by E.ON Gastransport and the Bundesnetzagentur:

network
configuration



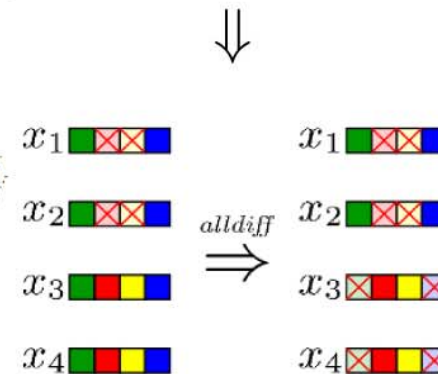
mixed-integer
programming

properties of gas



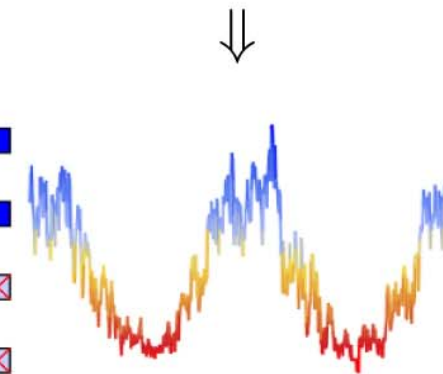
nonlinear
programming

legal requirements



constraint
programming

uncertain demand



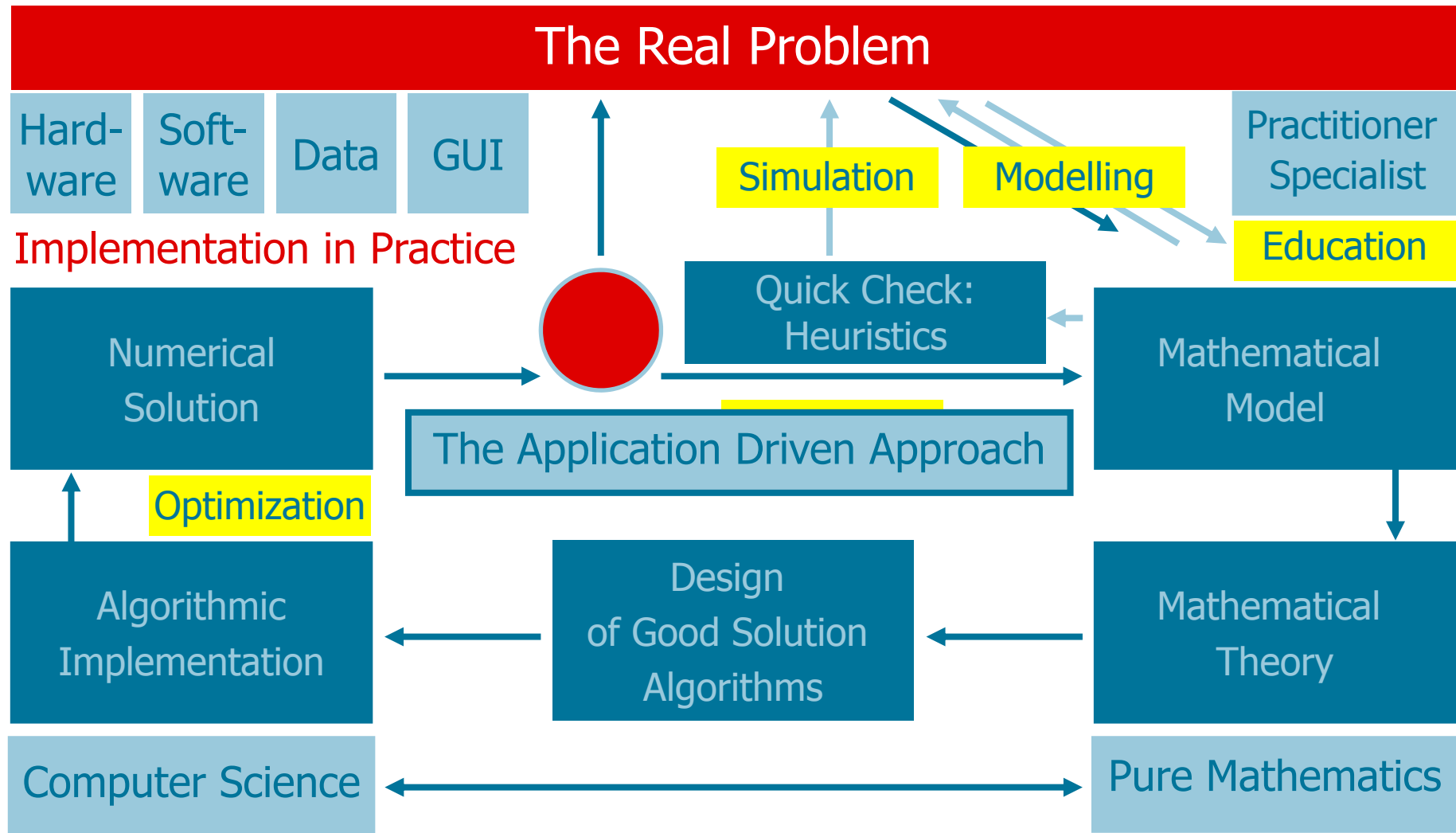
stochastic
programming

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The problem solving cycle in modern applied mathematics



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(linear)
integer
program
IP, MIP

program = optimization problem



Progress in LP: 1988—2004

Operations Research,
Jan 2002, pp. 3—15, updated in 2004)

Courtesy Bob Bixby

- Algorithms (*machine independent*):
Primal *versus* best of Primal/Dual/Barrier 3,300x
- Machines (workstations → PCs): 1,600x
- NET: Algorithm × Machine 5,300,000x

(2 months/5300000 $\approx$ 1 second)



SoPlex sequential object-oriented simplex

SoPlex is an implementation of the revised simplex algorithm. It features primal and dual solving routines for linear programs and is implemented as a C++ class library that can be used with other programs.



Roland Wunderling,
*Paralleler und Objektorientierter
Simplex-Algorithmus*,
Dissertation, TU Berlin, 1997

now employed by IBM, developing
CPLEX's LP technology

Zimpl

Zimpl is a little language to translate the mathematical model of a problem into a linear or (mixed-) integer mathematical program expressed in .lp or .mps file format which can be read and (hopefully) solved by a LP or MIP solver.



- Thorsten Koch, *Rapid Mathematical Programming*, Dissertation, TU Berlin 2004
(awarded with the Dissertation Prize 2005 of the Gesellschaft für Operations Research)



SCIP <http://scip.zib.de/>

Tobias Achterberg, Tobias, *Constraint Integer Programming*, Dissertation, TU Berlin, 2007

- Dissertation Prize 2008 of the Gesellschaft für Operations Research (GOR)
- George B. Dantzig Dissertation Award 2008 of the Institute of Operations Research and the Management Sciences (INFORMS), 2nd prize)
- Beale-Orchard-Hays Prize 2009 of the Mathematical Optimization Society for the paper: Tobias Achterberg, "SCIP: Solving constraint integer programs", *Mathematical Programming Computation*, 1 (2009), pp. 1-41.
- Started is "business career" at ILOG developing CPLEX's MIP technology, moved to IBM when ILOG was acquired by IBM, and is now with Gurobi.

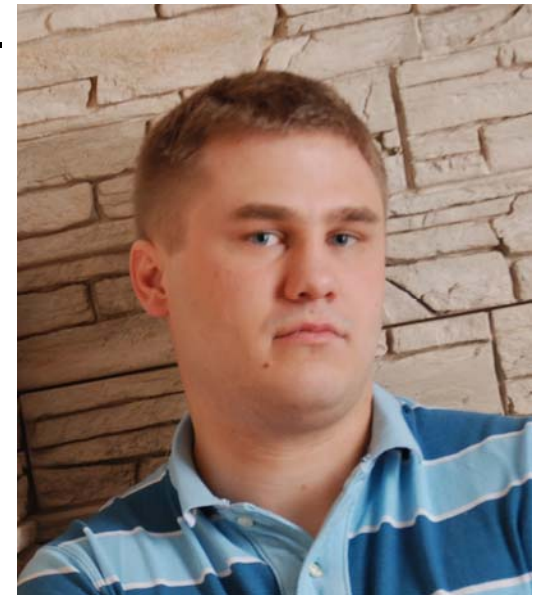


GCG Generic Column Generation

GCG extends the branch-cut-and-price framework SCIP to a generic branch-cut-and-price solver.

- performs Dantzig-Wolfe decomposition for detected or provided structure
- Solves reformulation with branch-and-price approach
- pricing problems solved as MIPs
- generic branching rules for branch-and-price

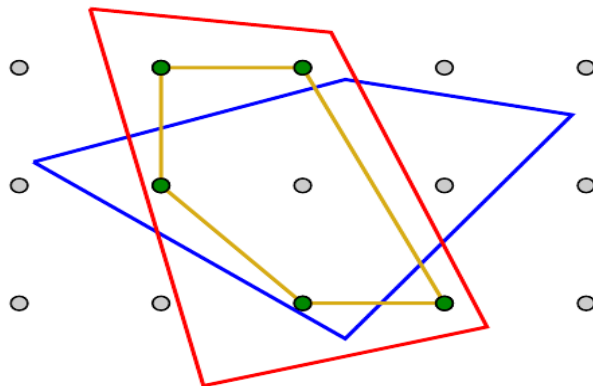
Provides easy access to another state-of-the-art MIP solving technology.



Gerald Gamrath

Generic Branch-Cut-and-Price, Diploma Thesis, TU Berlin, 2010

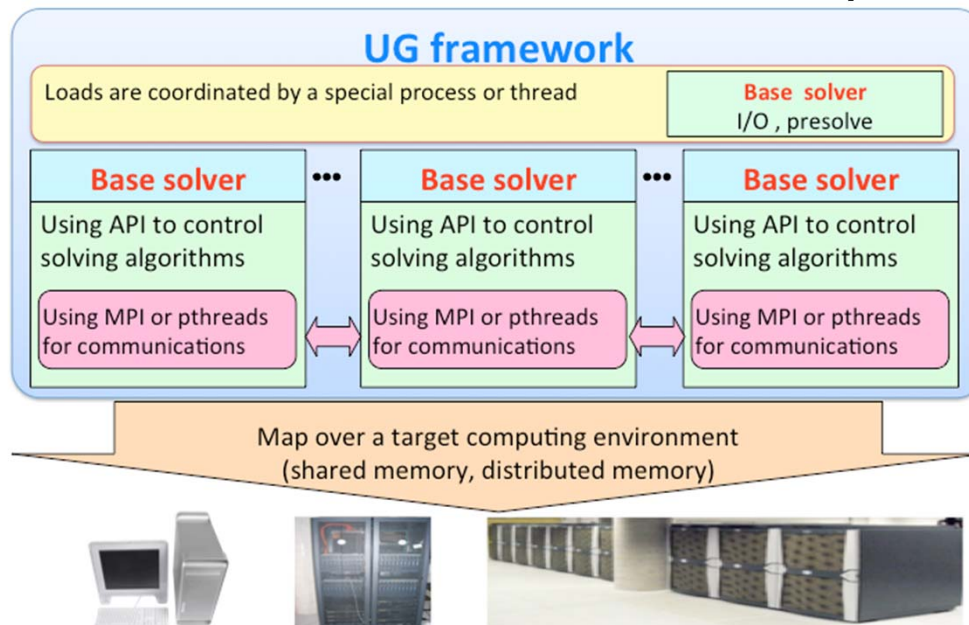
Currently developed in cooperation with RWTH Aachen, also funded by SPP 1307



UG Ubiquity Generator Framework

UG is a generic framework to parallelize branch-and-bound based solvers (e.g., MIP, MINLP, ExactIP) in a distributed or shared memory computing environment.

- Exploits powerful performance of state-of-the-art "base solvers", such as SCIP, CPLEX, etc.
- Without the need for base solver parallelization



Yuji Shinano,
*A Generalized Utility for
 Parallel Branch-and-Bound
 Algorithms,*
 Dissertation, Tokyo
 University of Science, 1997

The travelling salesman problem

Given n „cities“ and „distances“ between them. Find a tour (roundtrip) through all cities visiting every city exactly once such that the sum of all distances travelled is as small as possible. (TSP)

The TSP is called **symmetric** (STSP) if, for every pair of cities i and j , the distance from i to j is the same as the one from j to i , otherwise the problem is called **aysmmetric** (ATSP).

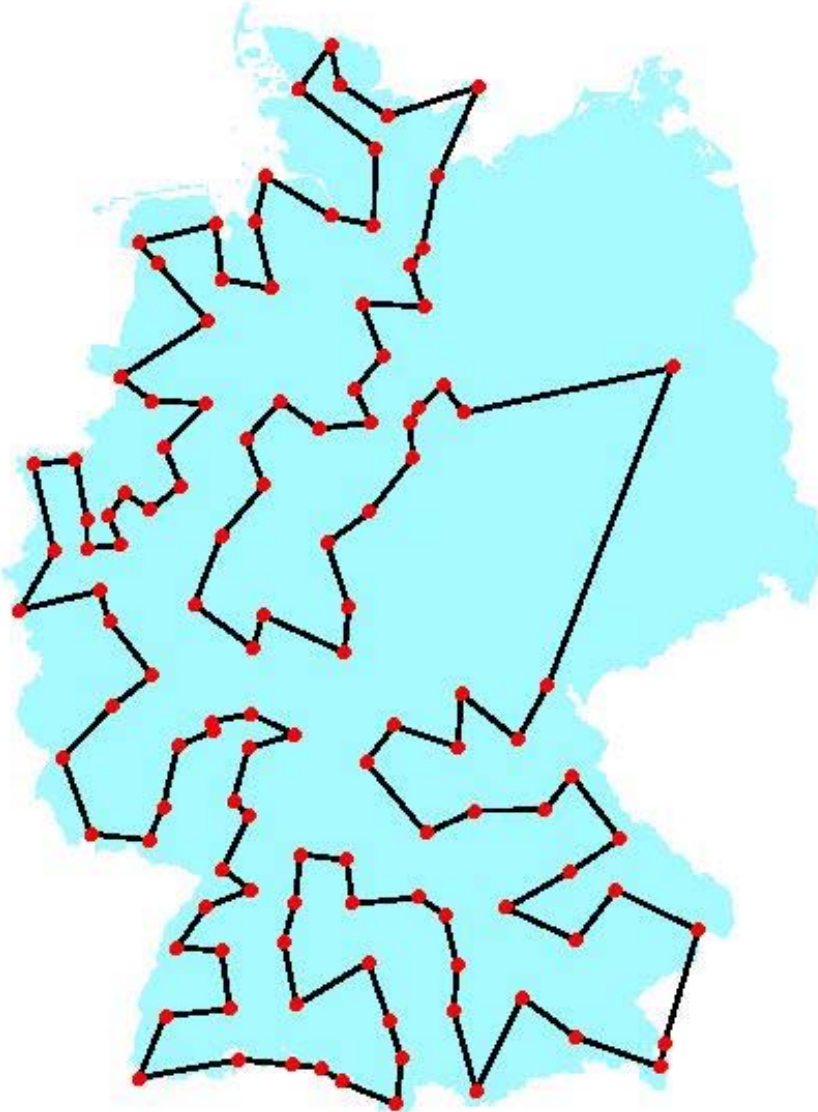


West-Germany and Berlin

120 cities
7140 variables

1975

M. Grötschel



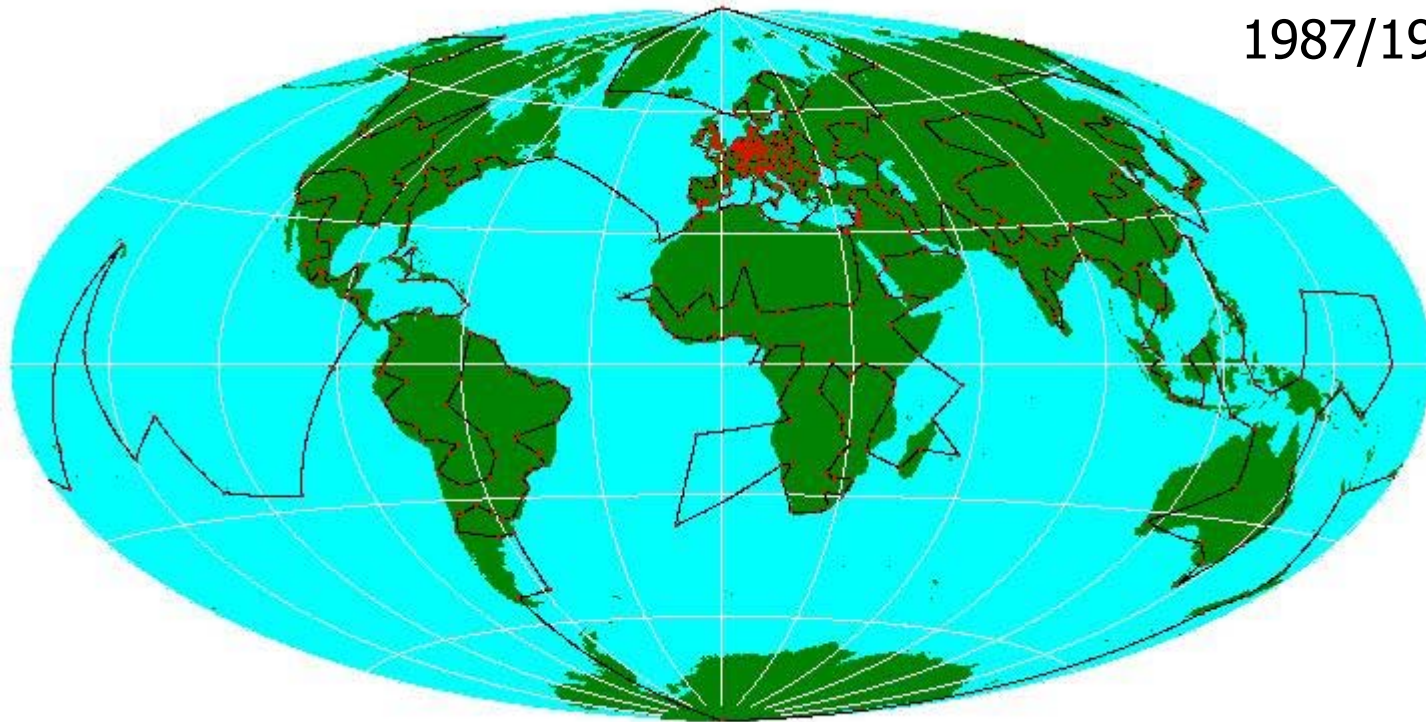
Martin
Grötschel

A trip around the world

666 cities

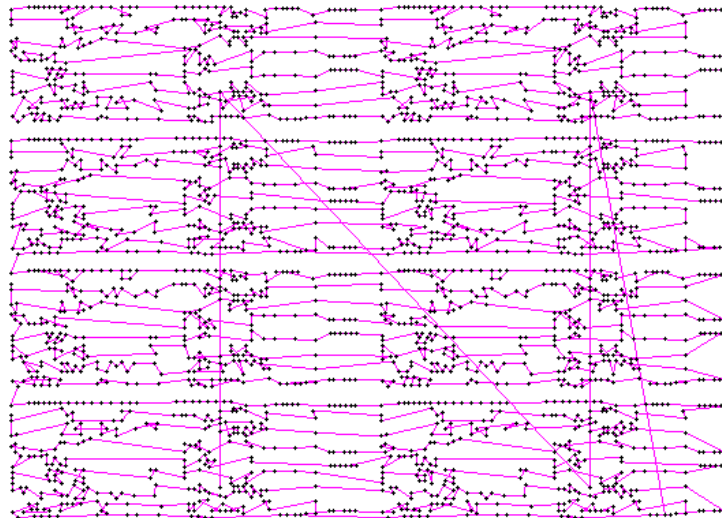
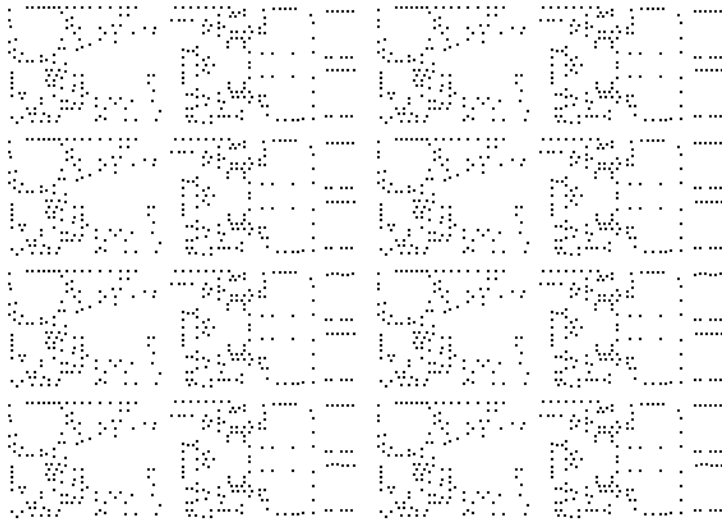
221,445 variables

1987/1991



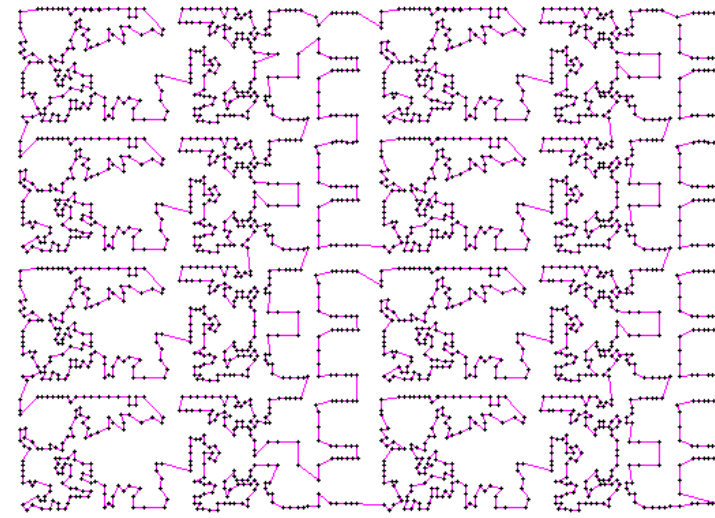
M. Grötschel, O. Holland

Drilling 2103 holes into a PCB



industry solution

Significant Improvements
via TSP

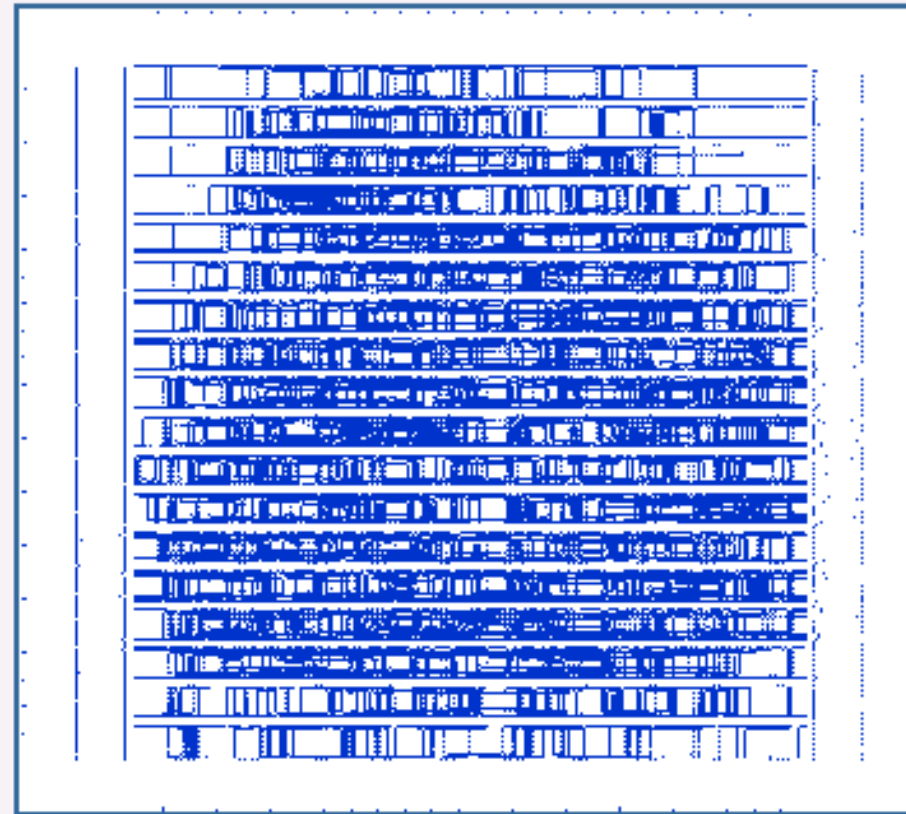


optimal solution

Some TSP World Records

2006 pla 85,900 solved 3,646,412,050 variables	year	authors	# cities	# variables
	1954	DFJ	42/49	820/1146
	1977	G	120	7140
	1987	PR	532	141,246
number of cities 2000x increase 4,000,000 times problem size increase in 52 years	1988	GH	666	221,445
	1991	PR	2,392	2,859,636
	1992	ABCC	3,038	4,613,203
	1994	ABCC	7,397	27,354,106
	1998	ABCC	13,509	91,239,786
	2001	ABCC	15,112	114,178,716
	2004	ABCC	24,978	311,937,753
2005 W. Cook, D. Epsinoza, M. Goycoolea			33,810	571,541,145

The current world record



85,900 Locations in a VLSI Application
Solved in 2006

<http://www.tsp.gatech.edu/optimal/index.html>

<http://www.tsp.gatech.edu/pla85900/index.html>

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 - b) Telecom
 - c) MATHEON



Public Transport Projects (ZIB and MATHEON)

Busses (Berlin and elsewhere)

- Telebus (Transportation of disabled persons)
- Bus Circulation
- Bus driver Scheduling
- Integrated Vehicle and Driver Scheduling
- Timetable Exchange

Subways and Light Railways

- Subway Time Tabling
- Vehicle Scheduling

Infrastructure Planning

- Line Planing
- Network Planning (Potsdam)
- Fare Planing

Airlines

- Airline Crew Scheduling
- Tail Assignment: Robustness

Railways

- Railway Track Allocation
- ICE Circulation

Spin-Offs : LBW, Intranetz



Solution “Technology”

- “good” mathematical modelling in cooperation with partners from practice
- linear programming
- integer and mixed-integer programming
- non-differentiable optimization
- lots of heuristics
- “verification” by simulation



BVG (Berlin): bus circulation

Systematisierter Einsatz

Die neuen Optimierungsmethoden, die die BVG jetzt nach und nach nutzen will, stammen vom Konrad-Zuse-Zentrum für Informationstechnik und garantieren nach Roß' Angaben Einsparungen von maximal **100 Millionen Mark im Jahr**. „Sie sind nötig, um unser Angebot in dieser schweren Lage stabilisieren und dem Einsparungsdruck überhaupt standhalten zu können.“

Bereits 1991 beauftragte die BVG die Berliner Software-Firma IVU, ein EDV-System zur Betriebsplanung zu entwickeln. IVU steht für „Gesellschaft für Information, Verkehrs- und Umweltplanung GmbH“, ein Unternehmen mit 120 Mitarbeitern, das auf Verkehrsplanung und Logistik spezialisiert ist. Der BVG ging es bei dem Auftrag vor allem darum, die Einsatzplanung ihrer Fahrzeuge zu systematisieren.

WISSENSCHAFT UND PRAXIS

Rheinischer Merkur
Nummer 39 · 26. September 1997 **37**

INFORMATIK / Ein Lehrbeispiel, wie sich Mathematik und Wirtschaft ergänzen

Auf Sparkurs zum Ziel

Das Berliner Busnetz kostet jährlich Millionen. Mit Hilfe moderner Software könnte man auf gewaltige Zuschüsse verzichten.

■ YASCO ALEXANDER SCHMIDT

Die Mathematik ist die Wissenschaft abstrakter Probleme. Deshalb erscheint sie so oft als weltabgewandte Spielerei. Doch das ist nur die halbe Wahrheit. Längst mischt sich die Mathematik in die Praxis ein. Überall, wo in der Wirtschaft gespart und verbessert werden muß, kann sie helfen. Professor Martin Grötschel, Vizepräsident des Konrad-Zuse-Zentrums für Informationstechnik in Berlin, steht für das neue Selbstverständnis der angewandten Mathematik. Er ist Experte für die kombinatorische Optimierung. Seine Botschaft: „Wer heute die großen Verkehrsnetze und komplizierten Fabriken optimieren will, muß sich mit Mathematik beschäftigen, um unnötige Kosten zu vermeiden.“

Nun gibt es in Berlin ein Lehrbeispiel, wie Mathematik und Wirtschaft zueinander kommen können. Die Berliner Verkehrsberie (BVG) haben vor wenigen Wochen begonnen, die Einsatzplanung ihrer Fahrzeuge zu verbessern. Bisher wurden die Pläne, wenn auch mit Computerhilfe, per Hand erstellt. Von einer kostengünstigen Planung war man weit entfernt.

Um ihre Kosten zu decken, hat die BVG für ihr Jahr große Zuschüsse vom Berliner Senat bekommen. Doch damit ist nun Schluss: dem Land Berlin geht das Geld aus, so daß auch die BVG unter einem enormen Sparzwang steht. „Wir müssen jährlich dreistellige Millionenbeträge einsparen“, erklärt Jürgen Roß, Planungingenieur bei der BVG. „Bis zum Jahr 2000 können wir von den heute rund 20 000 Mitarbeitern nur noch 15 000 beschäftigen.“

Systematisierter Einsatz

Die neuen Optimierungsmethoden, die die BVG jetzt nach und nach nutzen will, stammen vom Konrad-Zuse-Zentrum für Informationstechnik und garantieren nach Roß' Angaben Einsparungen von maximal 100 Millionen Mark im Jahr. „Sie sind nötig, um unser Angebot in dieser schweren Lage stabilisieren und dem Einsparungsdruck überhaupt standhalten zu können.“

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Stadt aushängen, auf Knopfdruck und ohne Umwege über eine Druckerlei im eigenen Haus produzieren. Nach und nach wurden diese Werkzeuge eingeführt. Das jetzt installierte Optimierungsmodell für die Umlaufplanung von Straßenbahnen und Bussen ist der bisher bedeutendste Schritt der Zusammenarbeit.

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Komfortable Hilfe

Zwar werden in Deutschland für die Verkehrsplanung meist schon Computer schon seit Ende der sechziger Jahre ein, um Näherungslösungen für optimale Pläne zu berechnen. Die Forscher am Konrad-Zuse-Zentrum konnten noch einen Schritt weiter gehen: Ihre Computer berechnen das Optimum der Kostenersparnis nicht nur annähernd, sondern ganz exakt. Ein wissenschaftlicher Durchbruch.

Die mathematische Grundlage ihrer Rechner bildet eine komplexe Gleichungssystem, eine sogenannte Matrix, mit mehr als 100 000 Zeilen und 70 Millionen Spalten. Die Zahlen in der gigantischen Tabelle geben an, wie die Buse eingesetzt werden sollen. Die Größe der Matrix weist auf ungeahnte Details hin, die beachtet werden müssen: Der Computer muß garantieren, daß jeder der

man sie noch nicht einmal vollständig im Computer speichern könnte“, verrät Andreas Löbl, wissenschaftlicher Mitarbeiter von Martin Grötschel. „Pro Spalte gibt es aber nur drei Einträge, die von Null verschieden sind.“ Das bedeutet, daß sich die Matrix stark vereinfachen läßt. Und genau diese Eigenschaft machen sich die Mathematiker zunutze: So versucht ihr Algorithmus zuerst mit einem Teil der Matrix zu operieren. Findet er für diesen kleinen Teil eine Lösung, so vergrößert er nach und nach das Problem, bis er eine Lösung für die gesamte Matrix gefunden hat.

Pfiffige Programme

Von der Mathematik spielen die Planer bei der BVG wenig. Nicht einmal Großrechner werden gebraucht – so pfiffige wurde das System programmiert. Je nach Teilproblem dauert die Bearbeitung wenige Stunden oder sogar nur Minuten. „Das System wurde gut angenommen, da die Pläne in kürzester Zeit verschiedene Szenarien testen und vergleichen können“, berichtet Uwe Strubbe, der als Projektleiter bei der IVU für die Entwicklung des fertigen Softwareprodukts zuständig ist.

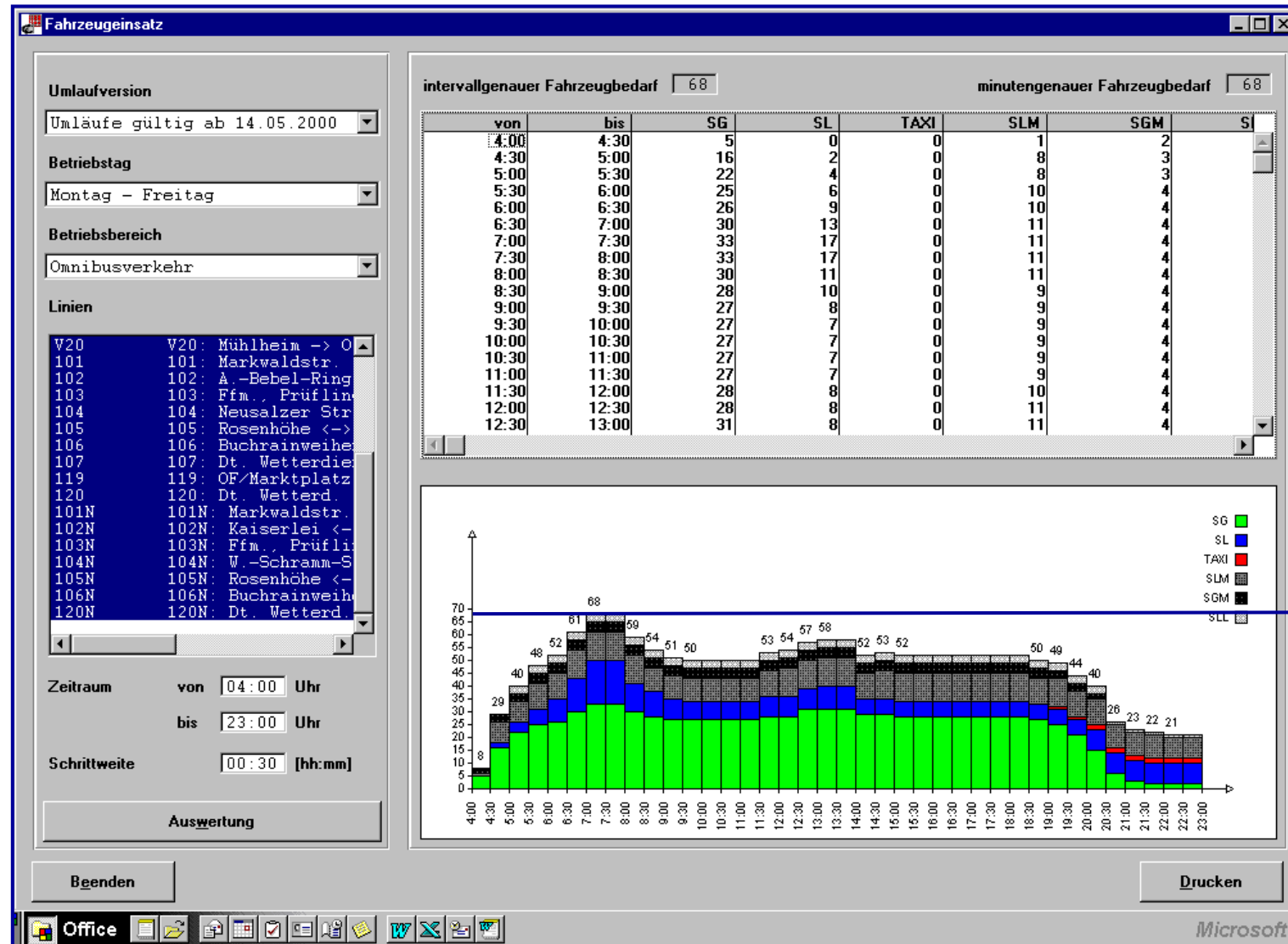
Auch Martin Grötschel betont den Nutzen für den Planer mehr als das Einsparungspotential. „Wir geben den Leuten Hilfsmittel in die Hand, die Kosten zu sparen. Ob dabei am Ende etwas eingespart oder der Service kostenneutral verbessert wird, ist nicht unsere, sondern eine politische Frage.“

Die Politik freilich erwartet zur Zeit nur die größtmögliche Einsparung. In der Mathematik hat sie dafür ein passendes Instrument gefunden. □



ZAHLENWERK: Nicht nur die Fahrpläne der Busse, sondern auch Dienst- und Einsatzpläne lassen sich programmieren. Foto: Bonn-Magazin

Vehicle Scheduling: The "Camel Curve"



Savings in Berlin public transport

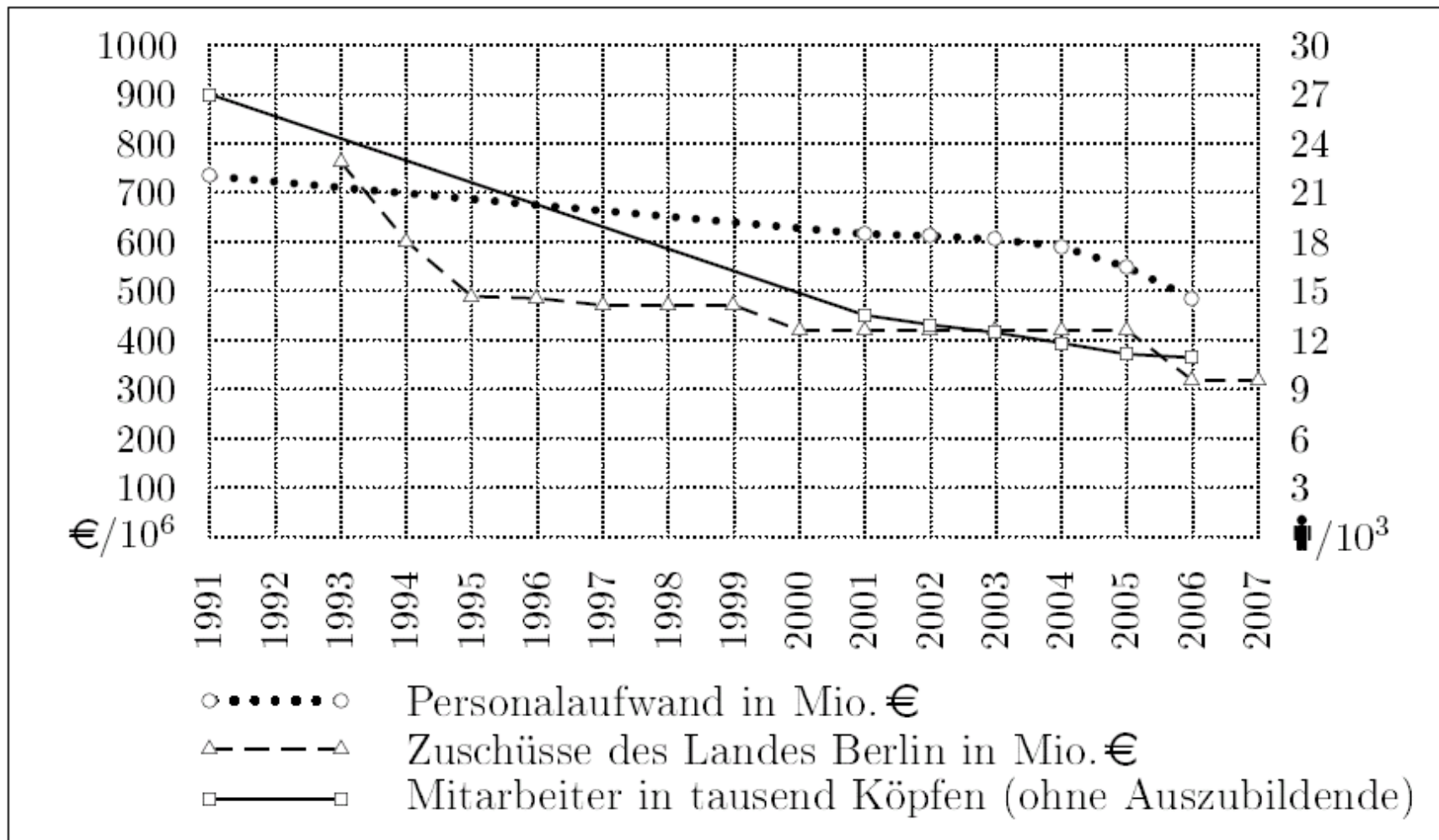
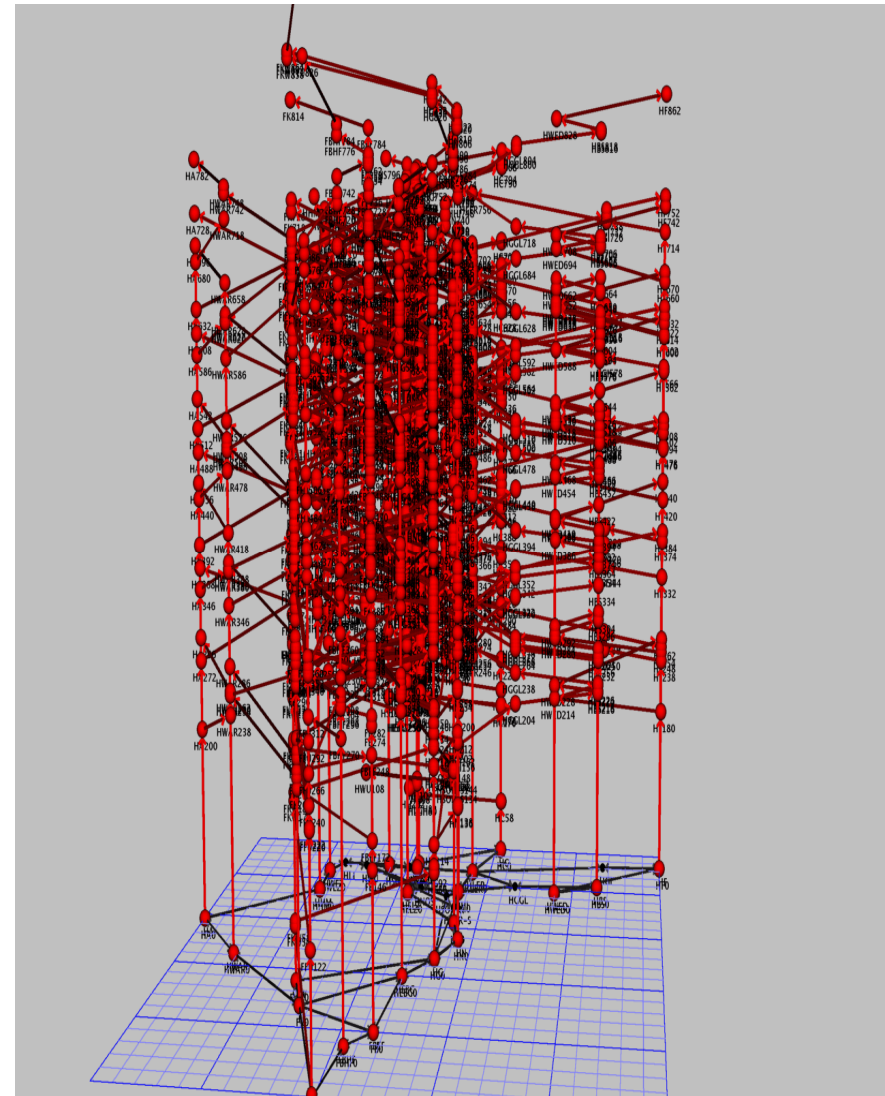
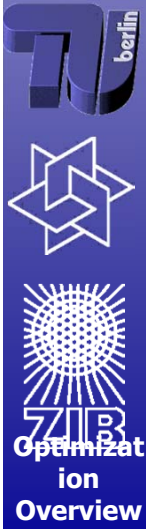


Abb. 3: Die Berliner Verkehrsbetriebe in Zahlen; Quelle: [36].





Optimization of Botany Bay a Container Terminal in Sydney, Australia



Gary Froyland, Thorsten Koch, Nicole Megow, Emily Duane, Howard Wren:
Optimizing the Landside Operation of a Container Terminal, OR Spektrum 2007

Port Botany – doubling throughput



Example: Routing automated guided vehicles in a container terminal in Hamburg Harbor

Cooperation with

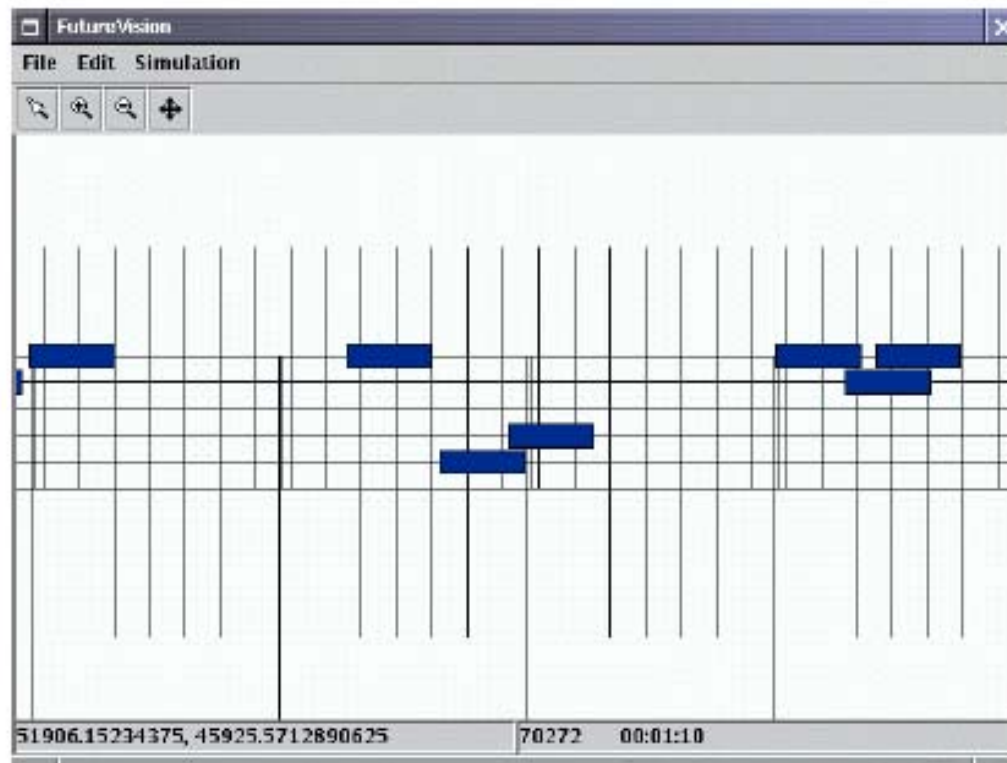


Rolf Möhring's
group

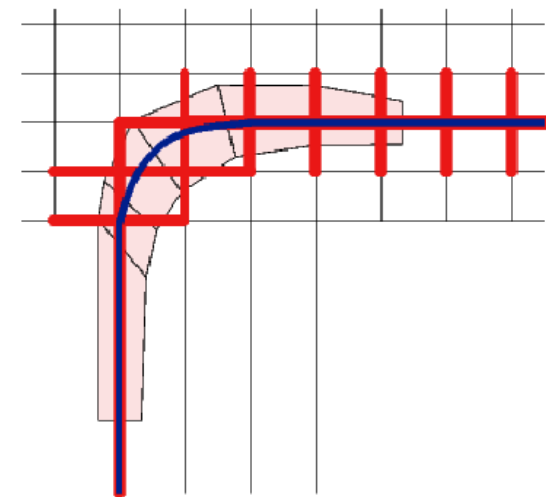


Details of AGV routing

Simple grid-like graph



Complex turns of AGVs



„Unit“ of the service fleet:

Yellow Angel
gelber Engel



Supereinsatz. In Berlin und Brandenburg mussten die Gelben Engel letztes Jahr mehr als 240 000 Mal ausrücken, um Havaristen in der Hauptstadt und auf 1700 Autobahnkilometern wieder flottzumachen – ein Rekorderinsatz. Einen Rückgang von zehn Prozent bei den Pannen registrierten dagegen die Gelben Engel in Mecklenburg-Vorpommern. Bei insgesamt 72 389 Einsätzen schafften sie jedoch auch einen Rekord: In 84 Prozent der Fälle konnten die Autofahrer mit dem Wagen weiterfahren.



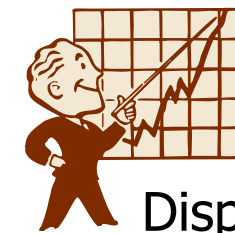
Service vehicle planning at ADAC



Help center



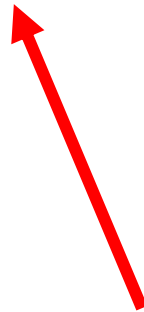
Wireless data transmission



Dispatch:
human operator



Yellow Angel

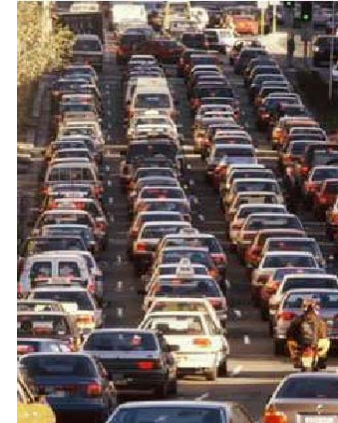


Matheon B8:

Flows over Time, a general issue



Routing in a container terminal



Control of traffic lights

Applications



Shunting of trains



Flow of materials in storage areas

Contents

1. Introduction
2. ZIB, TU, Matheon and Modal
3. Modern Industry Problems
4. Modelling of Real-World Problems
5. The Solution Cycle of Modern Mathematics
6. Software Advances
7. Examples
 - a) Traffic
 - b) **Telecom**
 - c) MATHEON



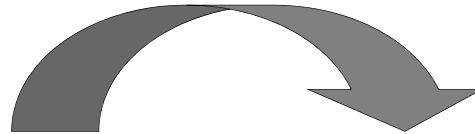
Telecommunication topics: Hardware and logistics

- **Designing mobile phones**
 - Task partitioning
 - Chip design (VLSI)
 - Component design
- **Producing Mobile Phones**
 - Production facility layout
 - Control of CNC machines
 - Control of robots
 - Cutting and welding
 - Printed Circuit Boards
 - Via minimization
 - Component Placement
 - Mounting Devices
 - Routing
 - Lot sizing
 - Scheduling
 - Logistics
- **Marketing and Distributing Mobile Phones**

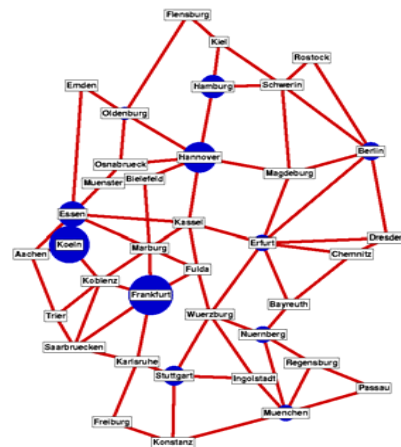


The Basic Question

Telecommunication



network design:



Input:

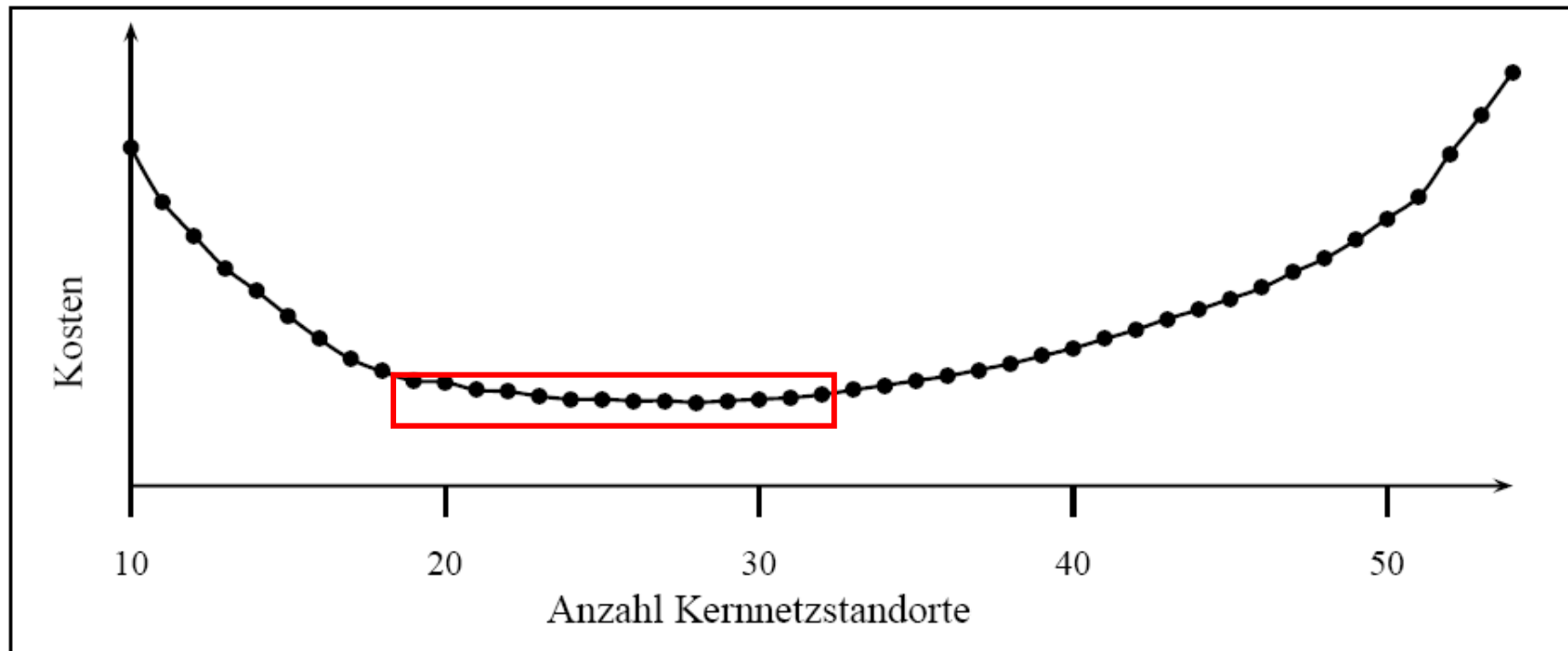
- ▷ potential network
- ▷ demands
- ▷ cost values
- ▷ various additional constraints

Output: **cost-minimal**

- ▷ subset of nodes and links
- ▷ discrete capacities
- ▷ survivable routing
- ▷ satisfying all constraints

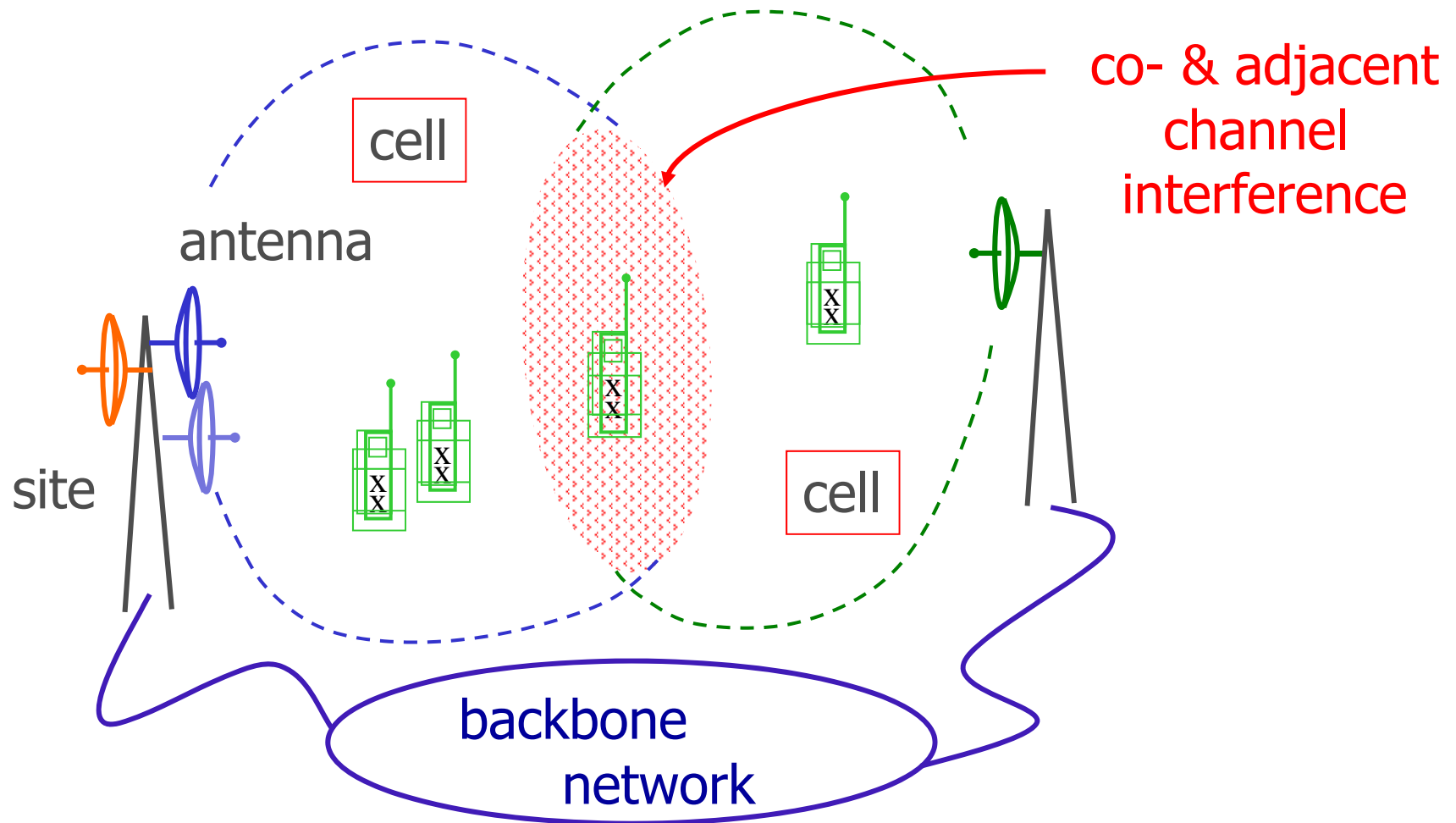


Number of Nodes in the Core Network





Antennas & Interference



Minimum Interference Frequency Assignment Problem (FAP)

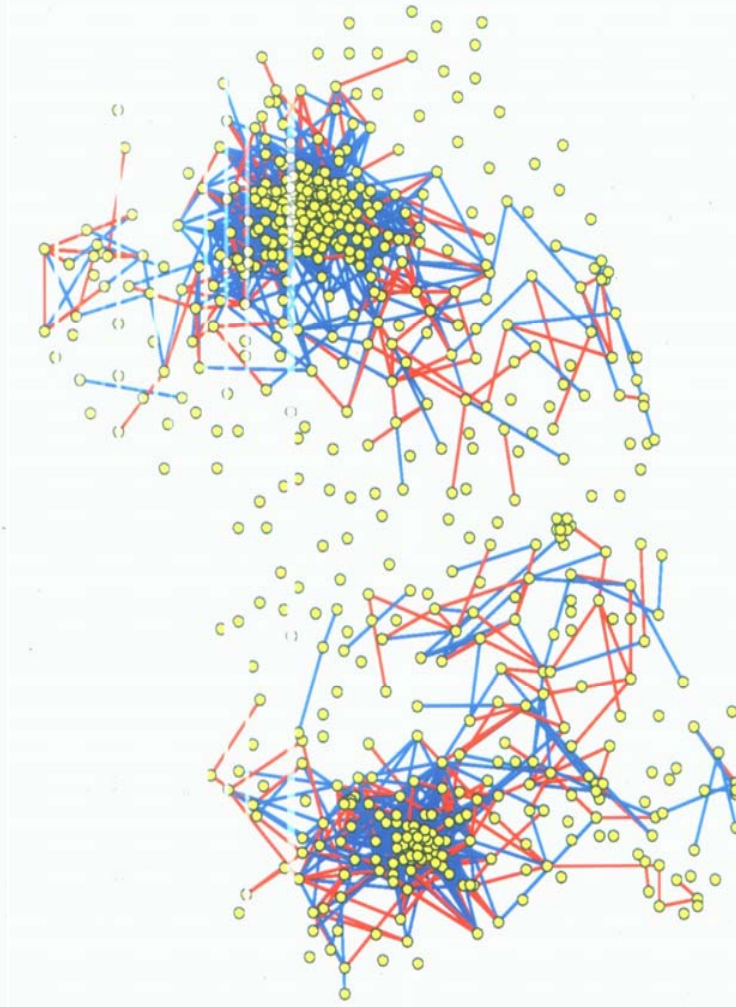
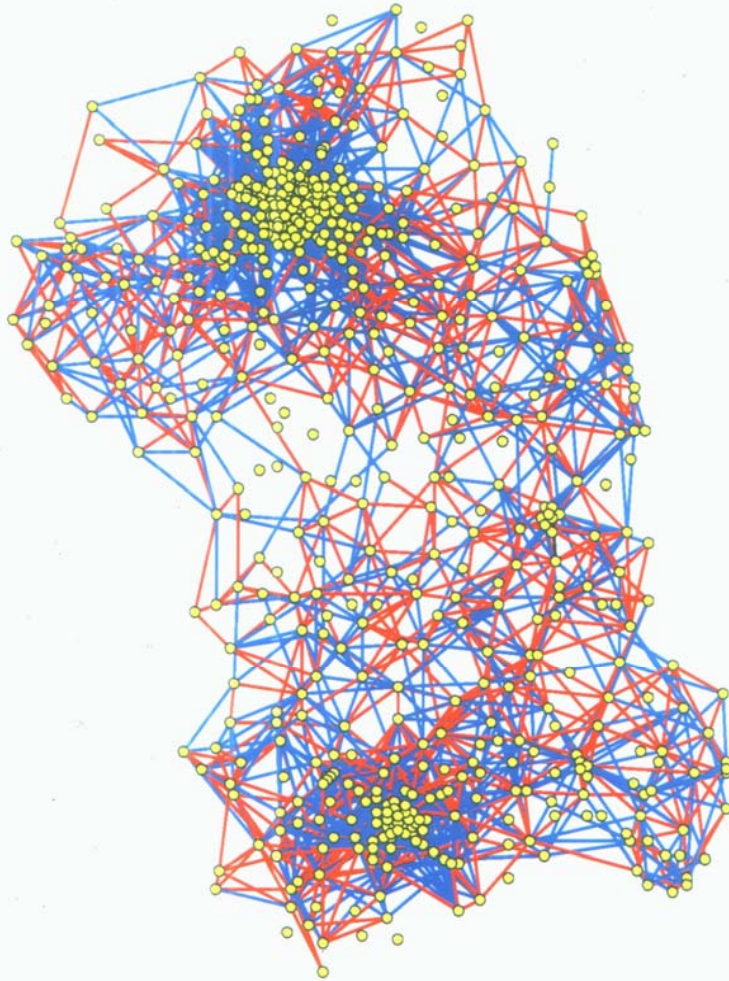
FAP is an Integer Linear Program:

$$\begin{aligned}
 \min \quad & \sum_{vw \in E^{co}} c_{vw}^{co} z_{vw}^{co} + \sum_{vw \in E^{ad}} c_{vw}^{ad} z_{vw}^{ad} \\
 s.t. \quad & \sum_{f \in F_v} x_{vf} = 1 & \forall v \in V \\
 & x_{vf} + x_{wg} \leq 1 & \forall vw \in E^d, |f - g| < d(vw) \\
 & x_{vf} + x_{wf} \leq 1 + z_{vw}^{co} & \forall vw \in E^{co}, f \in F_v \cap F_w \\
 & x_{vf} + x_{wg} \leq 1 + z_{vw}^{ad} & \forall vw \in E^{ad}, |f - g| = 1 \\
 & x_{vf}, z_{vw}^{co}, z_{vw}^{ad} \in \{0, 1\}
 \end{aligned}$$

that is very difficult to solve.



Region Berlin - Dresden

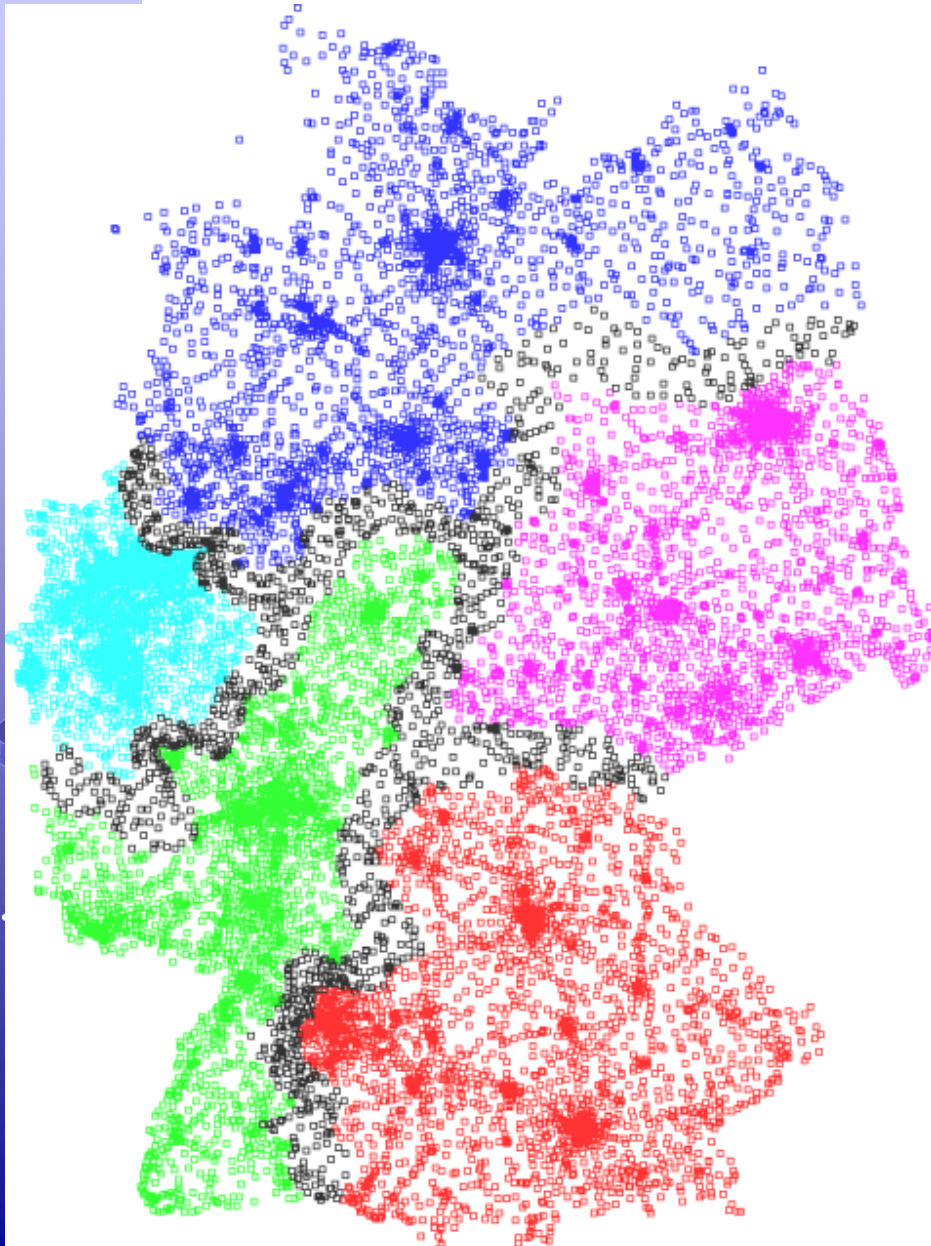


2877
carriers

50 channels

Interference
reduction:
83.6%

GSM 900-Optimization in Germany



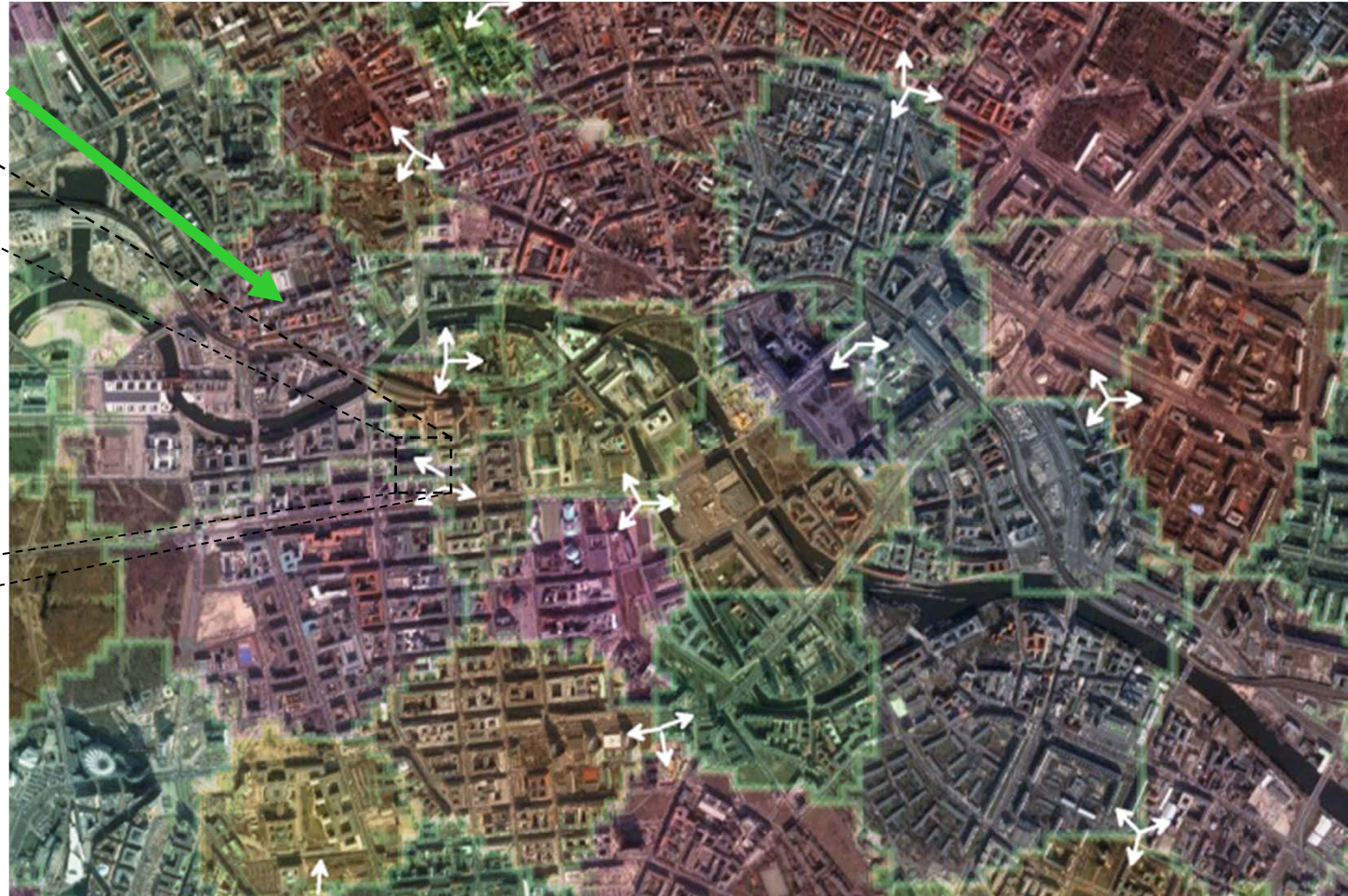
1. Optimierung je Region aller
 - Standorte
 - Sektoren
 - Bänder
2. Zusammenführung der Ergebnisse aller Regionen
3. Optimierung eines Streifens entlang der Regionsgrenzen
4. Optimierung des 1800 MHz-Anteils von Dualband-Sektoren

UMTS-Network

Cell



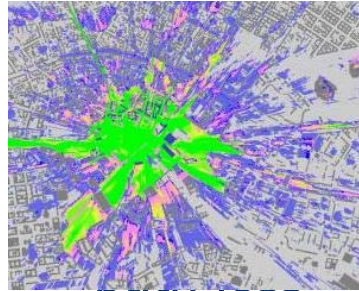
Antenna



Configuration of Antennas

Isotropic Prediction

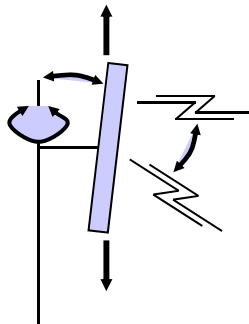
- Available for each potential antenna location



© Digital Building Model Berlin (2002),
E-Plus Mobilfunk GmbH & Co. KG

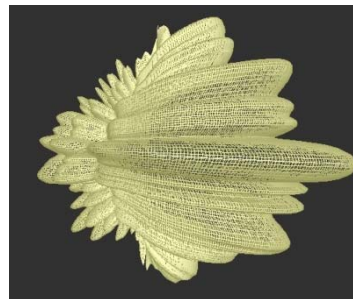
Antenna Configuration

- Azimuth
- Tilt
- Height

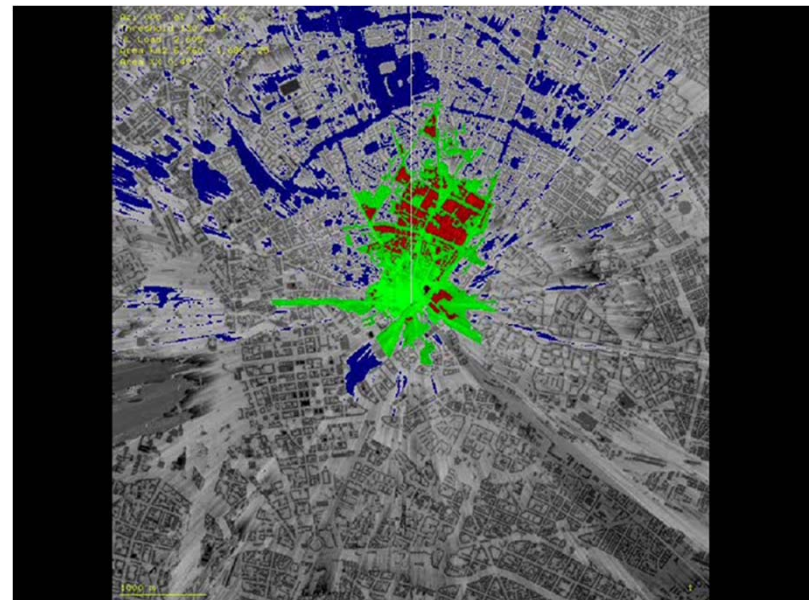


Antenna Diagram

- Signal propagation in different directions



Antenna Prediction

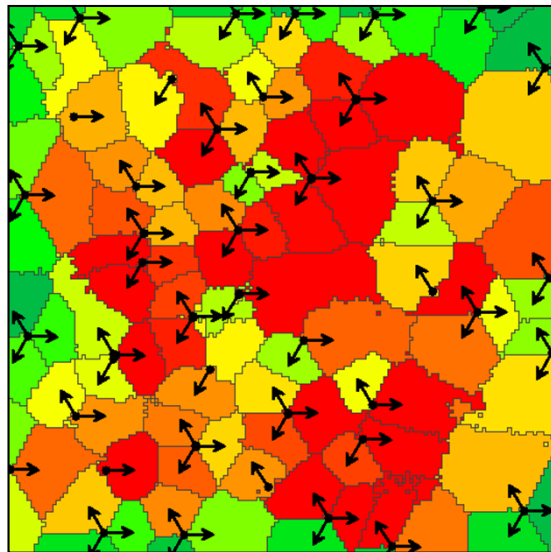
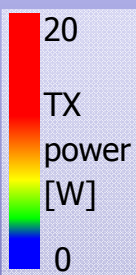


© Digital Building Model Berlin (2002), E-Plus Mobilfunk GmbH & Co. KG, Germany

height: 41m, electrical tilt: 0-8°, azimuth 0-120°

Optimization: Reduction of Network Load

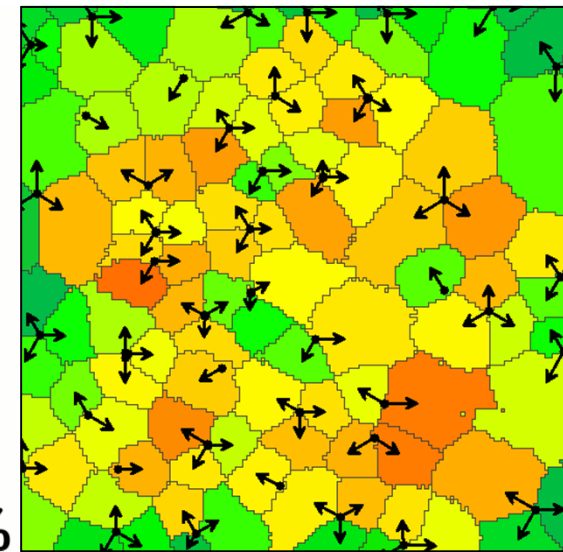
Start- Configuration



Adjustment:

- Azimuth
- Direction

Optimized Configuration



Reduction
–16.3 %



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MATHEON: A Large Competence Pool

Scientists of MATHEON

- **members** >200
- **DFG-financed researchers** ~60
- **PhD students** ~70
- **professors** ~40
- **number of projects** ~60

Most members from applied mathematics!



Mathematical Fields and Application Areas

A: Life Sciences

B: Networks

C: Production

D: Electronic and Photonic Devices

E: Finance

F: Visualization

ZE: Education

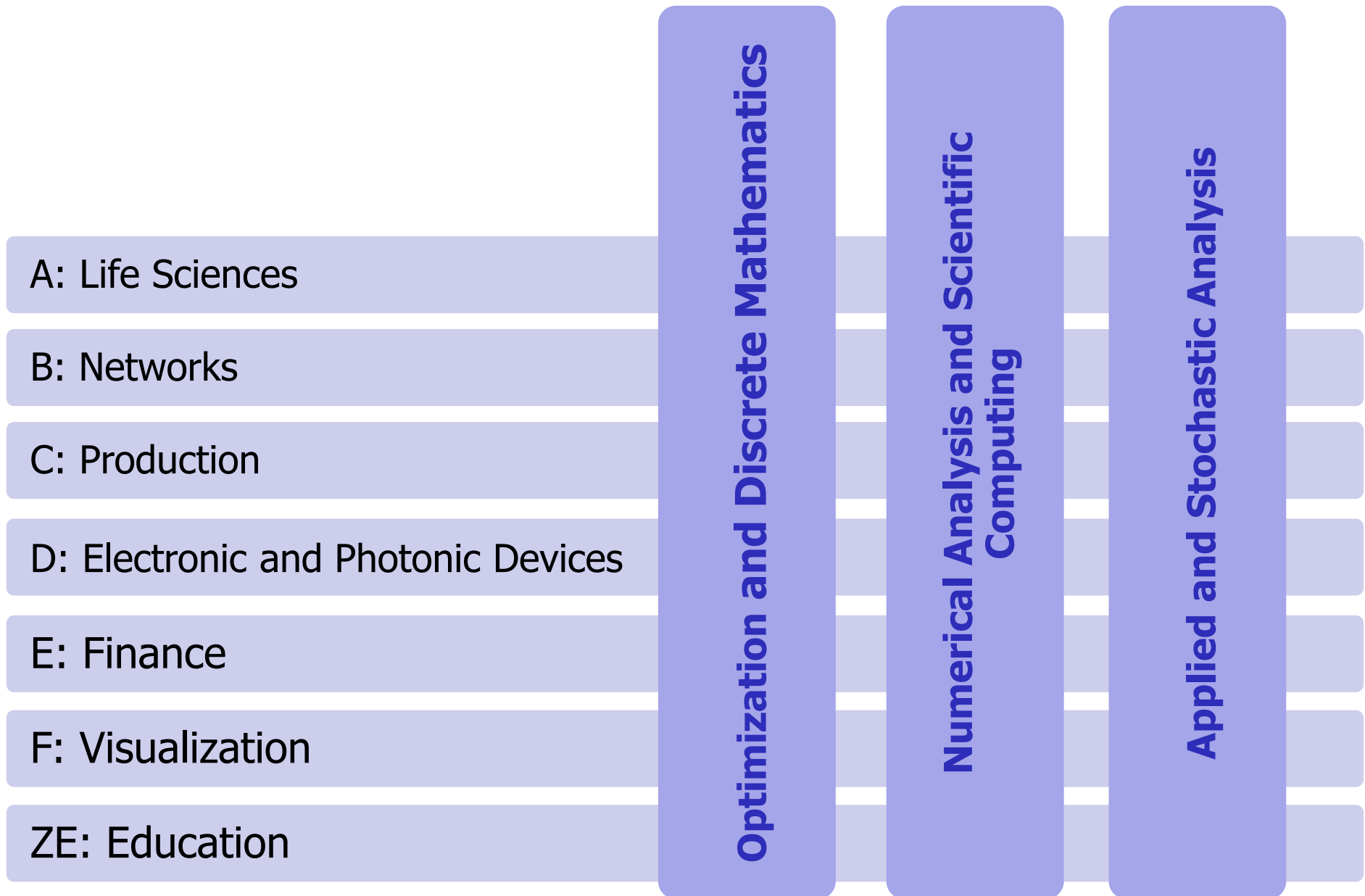
Optimization and Discrete Mathematics

**Numerical Analysis and Scientific
Computing**

Applied and Stochastic Analysis

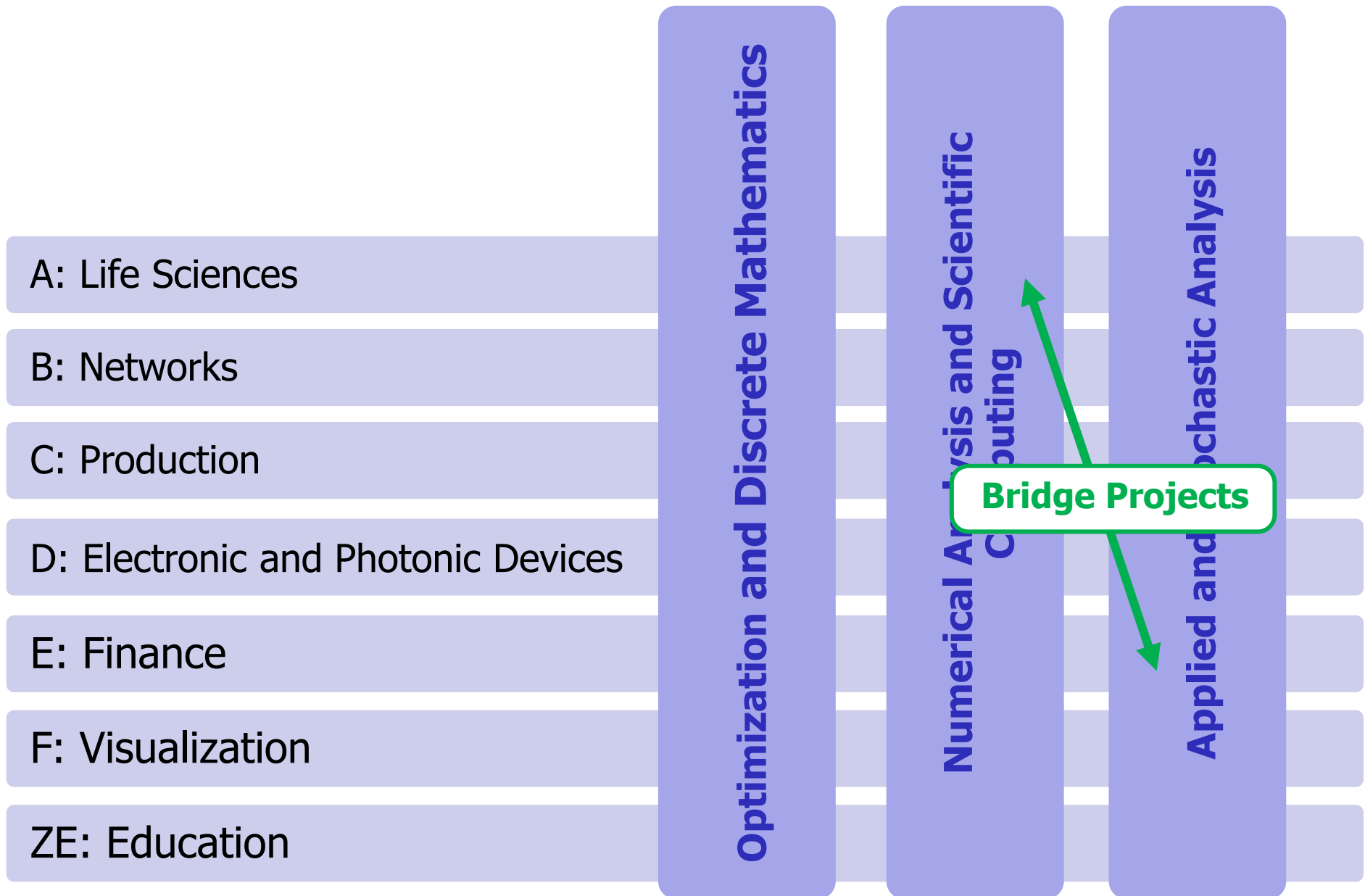


Added Value through Cooperation across Fields





Added Value through Cooperation across Fields





Industry cooperation - examples

- ❑ BASF, Bayer-Schering, Merck, Sanofi-Aventis, ...
- ❑ Siemens, IBM, Infineon, Microsoft, NEC, SAP, Philips, ...
- ❑ Deutsche Telekom (T-Systems), E-Plus, ...
- ❑ Daimler, BMW, Volkswagen, Tebis, Bosch, ...
- ❑ Deutsche Bahn, BVG, IVU, Lufthansa, ABB, ADAC, Herlitz, ...
- ❑ EdF, DREWAG, Vattenfall, E.ON, OGE, BHP Billiton, ...
- ❑ AMRO Bank, Bankgesellschaft Berlin, Commerzbank,
Deutsche Bank London, IKB, Reuters Financial Software, ...
- ❑ Numerous small and medium enterprises (SME)

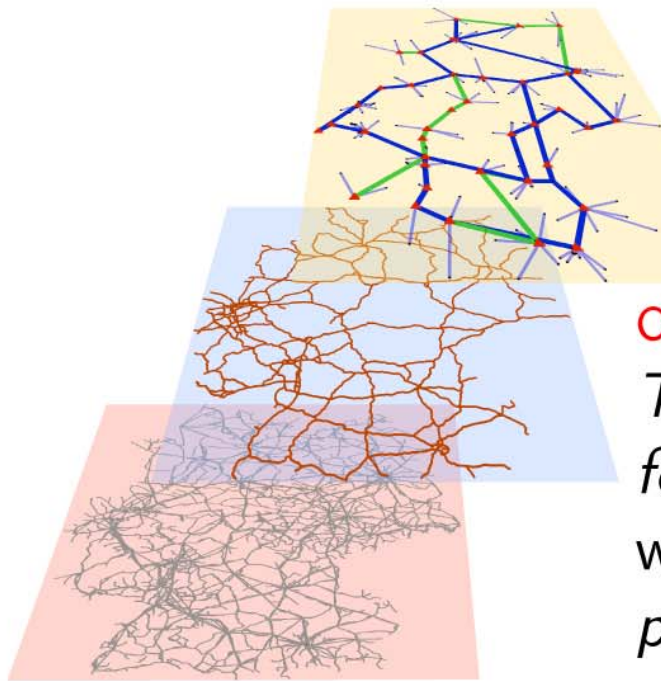


Spin-Off Companies

- ❑ **Indeed Visual Concepts** (www.amiravis.com):
3D data visualization (Amira), today: Mercury
- ❑ **JCMwave** (www.jcmwave.com):
Simulation software for nano-optical components
- ❑ **atesio** (www.atesio.de):
Plan, configure, and optimize telecommunication networks
- ❑ **Löbel, Berndörfer & Weider GbR** (www.lbw-berlin.de)
Optimization and consulting for public transport
- ❑ **Synoptio** (www.synoptio.net)
Mathematical consulting and software
- ❑ **Lenné 3D** (www.lenne3d.de)
Digital botany and real-time landscapes
- ❑ **inbion GmbH** (inbion.math.fu-berlin.de)
Bioinformatics solutions for life sciences



Networks: Our Vision



guiding question:
what constitutes a good network?

overall goal:
To develop theory, algorithms, and software for a new, advanced level of integrated network optimization that addresses network planning problems as a whole.

examples:

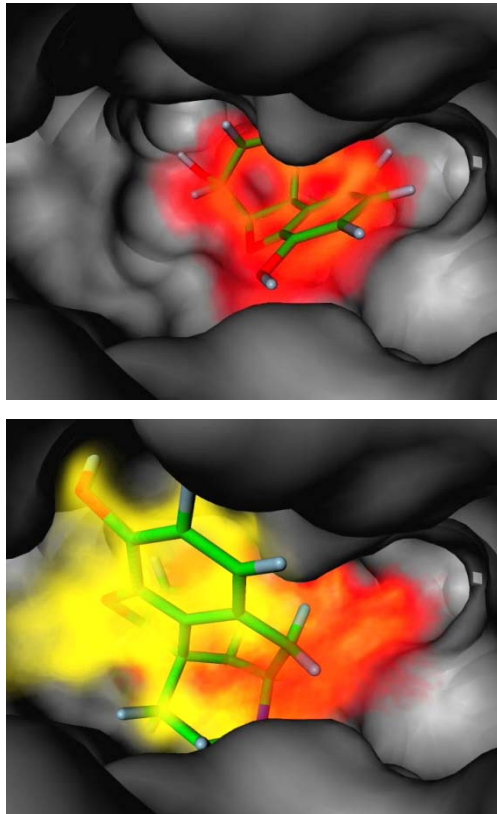
- ▷ line planning and timetabling in public transport
- ▷ fiber and UMTS telecommunication network design
- ▷ harbor and factory logistics





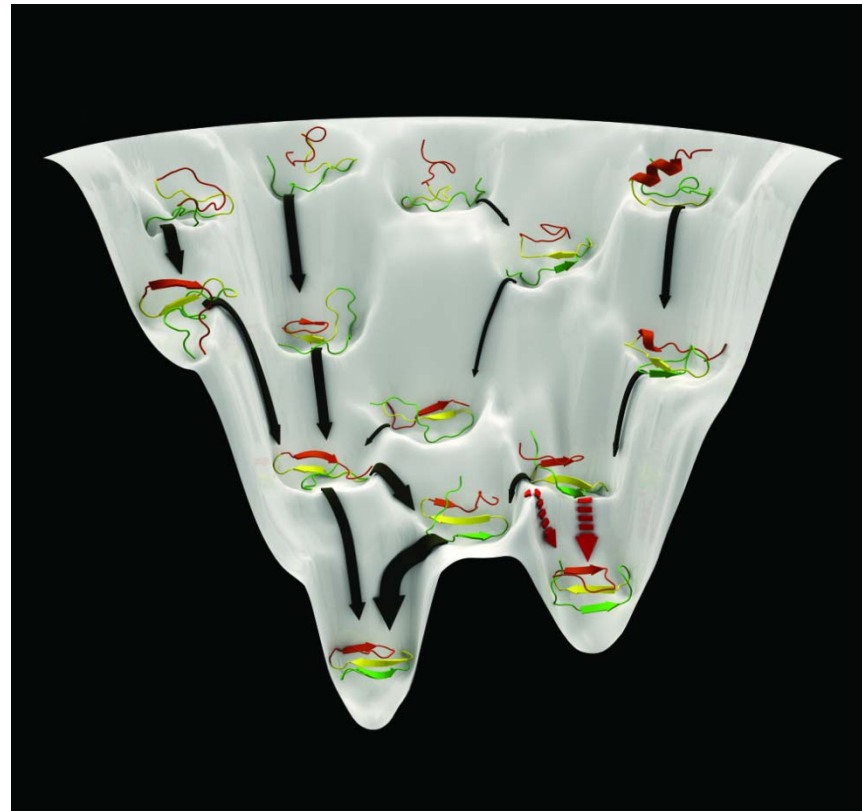
A19 Molecular Design

Drug Design



new pain relief drug

Protein folding



conformation dynamics, meshless methods, transition path theory

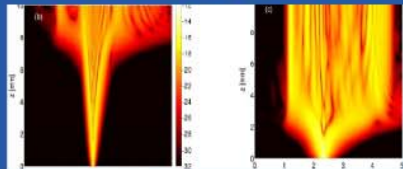


Domains of Expertise in MATHEON AA D

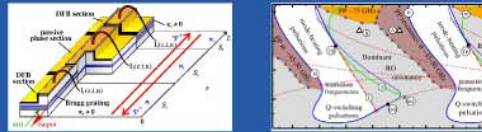
Photonic Devices

Electronic Devices

D14 Bandelow
Mielke:
Nonlinear fibers

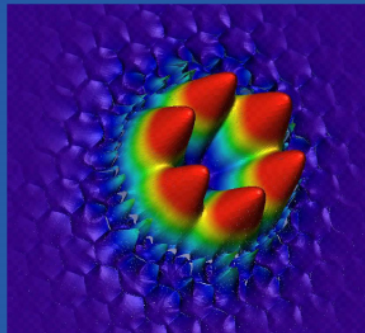


D8 Recke Wolfrum:
Laser dynamics

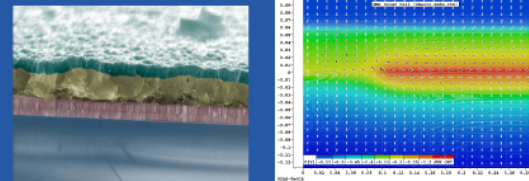


D21 Yanchuk:
Synchronization

D23 Schmidt:
Nanostructures



D22 Glitzky Mielke:
Interfaces in solar cells



**Electric
cu**

D2 Mehrmann
Reis:
Passivation and
model reduction
~> next talk

Light



Record Tandem-Cell with 14% Efficiency

Goal ZIB

Simulation and optimization of solar cell design

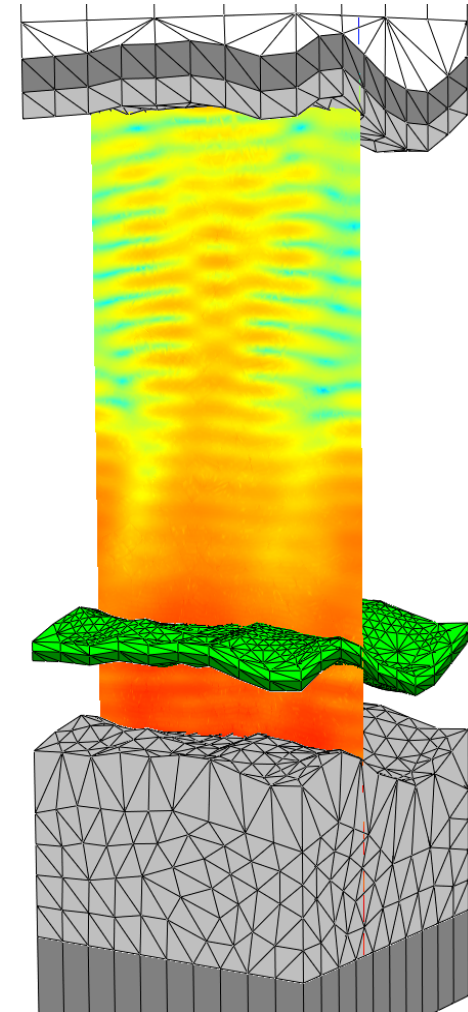
Challenge

- 3D simulation of large structures (>5 million unknowns)
- large parameter space
(materials, thicknesses, roughness, textures)

Strategies

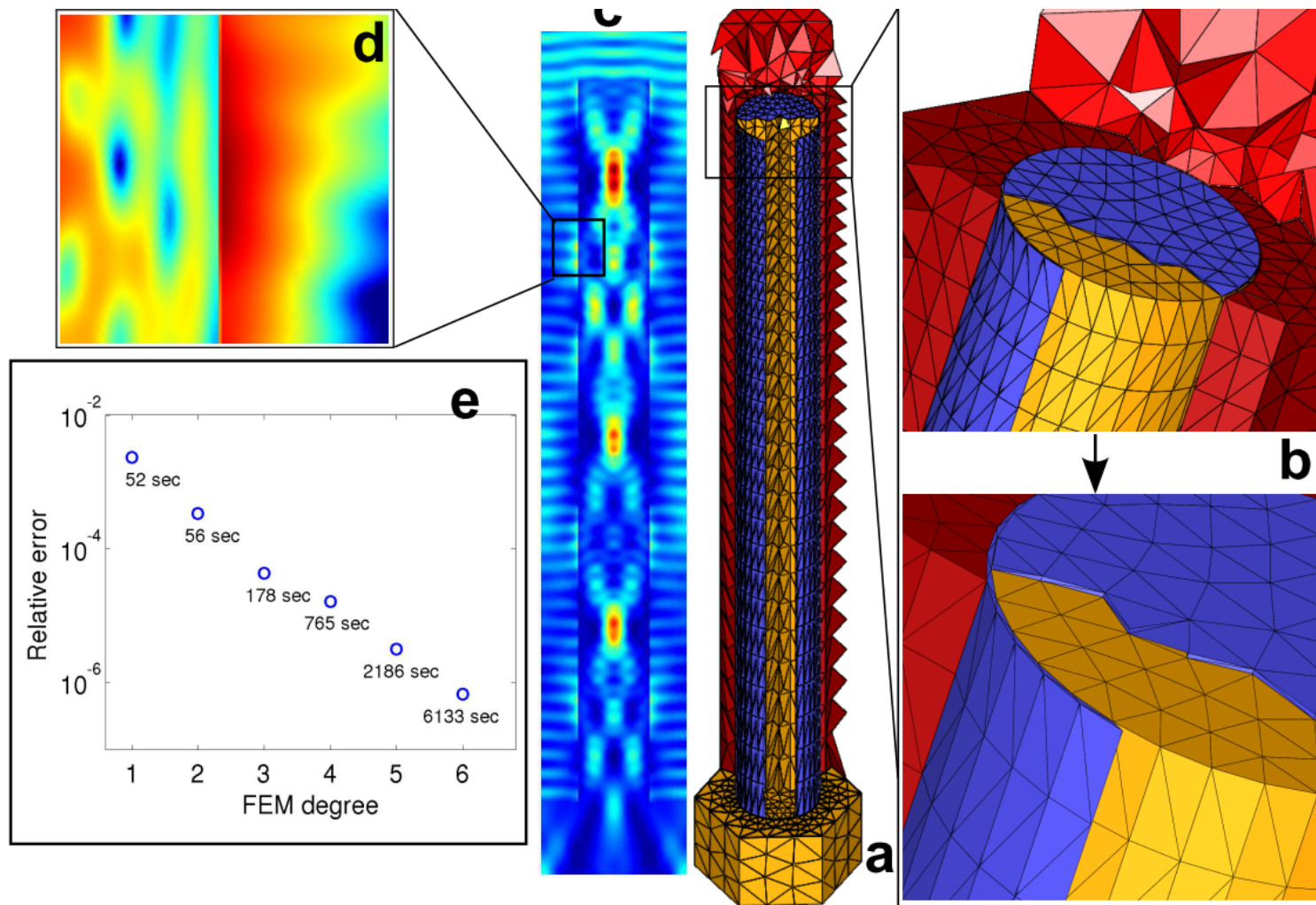
- hp-adaptivity
- domain decomposition
- adaptive spectral sampling
- reduced basis algorithms

Partners



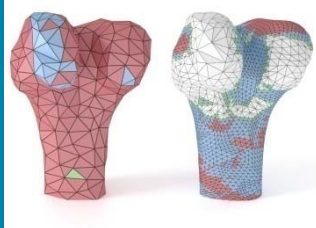
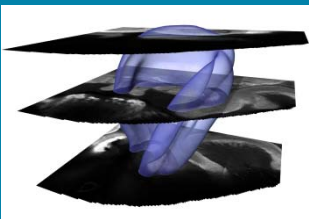
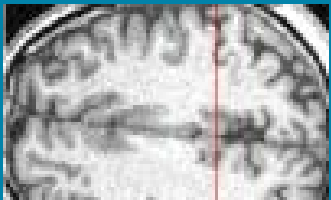
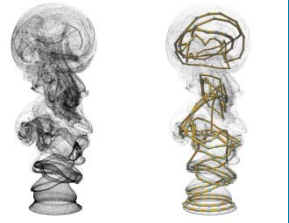


Numerical Study of Nanowires





Visualization: Projects and Domains of Expertise

		Geometry Processing F1 – F4 – F9	Image Processing F2 – F10	Interactive Graphics F5 – F6	
Industrial CAD					
	Computer Graphics	F1 Discrete Surface Parametrizations Bobenko/Ziegler	F4 Geometric Shape Optimization Polthier	F6 Multilevel Methods Kornhuber/ Polthier/Yserentant	
Medicine & Physics					
		F9 (C25) Trajectory Compression Polthier/Weiser	F2 Atlas-based Image Segmentation Hege/Weiser	F10 (A3) Image and Signal Processing in Medicine & Biosciences Polzehl/Spokoiny	F5 Physics for Interactive Graphics Pinkall/Sullivan

And so on...



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Have you encountered mathematics within the last days? Have I?

I am discussing only mathematics produced in my group:

- mobile phone channel assignment
- e-mail: (communication network and data packages)
- bus circulation in Berlin
- bus driver assignment in Berlin
- subway timetable in Berlin
- Lufthansa plane tail assignment of my flight
- car navigation system
- medical imaging



What to do to be successful in practice: Advice

- Learn (get acquainted with) many branches of mathematics, you never know what you are going to need.
- Make colleagues who know other types of maths better than you your friends. You may need them.
- Don't be arrogant! Other people are also intelligent, and they may know some things much better than you.
- Learn the language of your customers. They will not have the time and patience to learn your mathematics. But you have to communicate with them. And it is better for everyone that mathematicians learn the "other stuff".



New „Paradigm“: Understanding & utilizing big and complex data sets



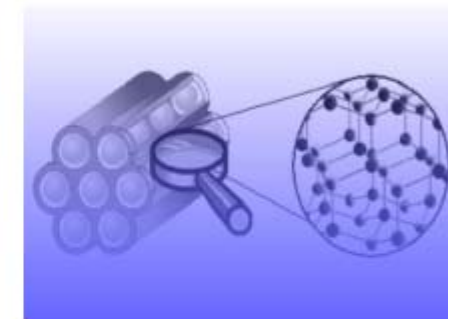
Verkehr und Logistik



Hausautomatisierung



Lebenswissenschaften



Materialforschung



Marktforschung



Digital Humanities



Ressourcenmanagement
(„smarter cities/planet“)

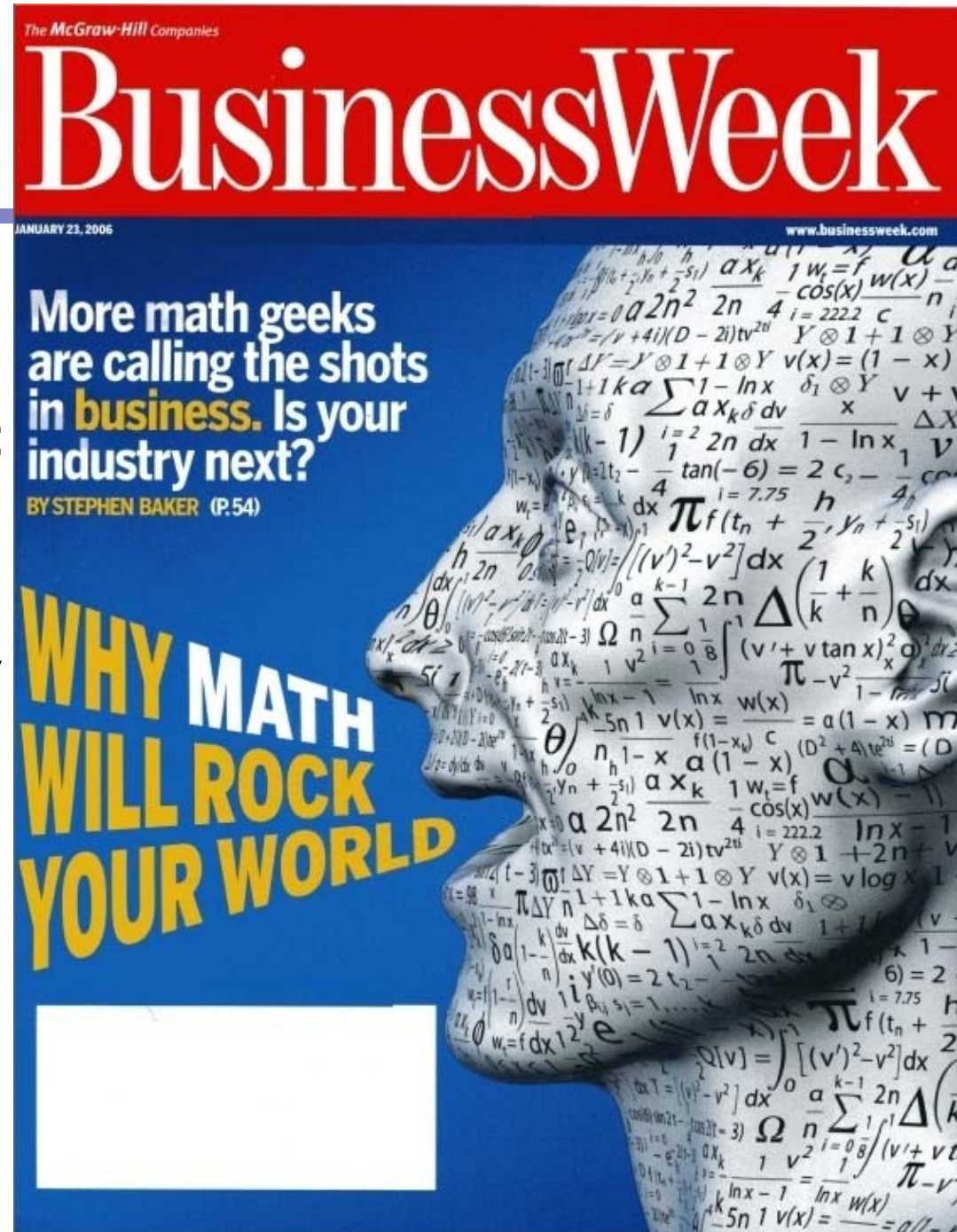


Unternehmenssteuerung

From the **IBM homepage**: "Every day, we create 2.5 quintillion bytes of data — so much that 90% of the data in the world today has been created in the last two years alone."

Business Week of January 23, 2006

For me, mathematics is going to become the most important scientific endeavour of the 21st century, because every other science and every technology needs more and more maths. Our industry will not remain competitive without significant utilization of mathematics



Suppose you would eliminate (modern) mathematics from technology

- Would the motor of a modern car run efficiently?
- Would an airplane like A380 fly?
- Would your car navigation system operate?
- Would medical imaging systems (X-ray, MRT, ...) show a tumor in the right spot?
- Would public transport be affordable?
- Would your smartphone work?
- Would high communication networks exist?
- Would data transfer be secure?

Mathematics does not provide the services and technologies, but is an essential part of them.



Grand challenges as identified by the US National Academies

THE CHALLENGES

- ◆ Make solar energy affordable
- ◆ Provide energy from fusion
- ◆ Develop carbon sequestration methods
- ◆ Manage the nitrogen cycle
- ◆ Provide access to clean water
- ◆ Restore and improve urban infrastructure
- ◆ Advance health informatics
- ◆ Engineer better medicines
- ◆ Reverse-engineer the brain
- ◆ Prevent nuclear terror
- ◆ Secure cyberspace
- ◆ Enhance virtual reality
- ◆ Advance personalized learning
- ◆ Engineer the tools for scientific discovery



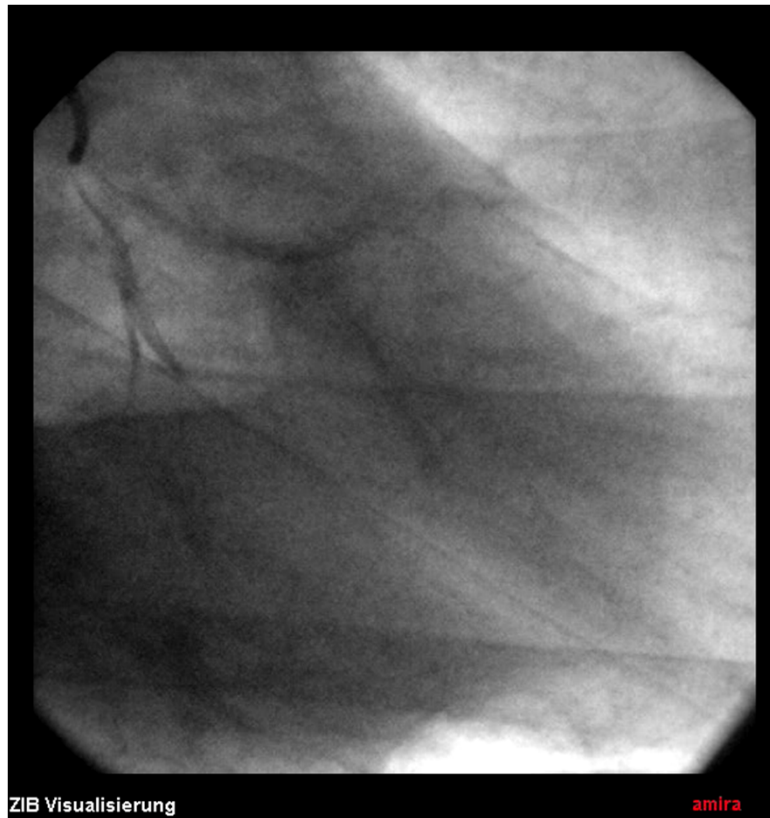
Personal health

- Diseases are going to hit me and some of you in the audiences in the future or have already done so in the past – though I hope they will not.
- Two examples

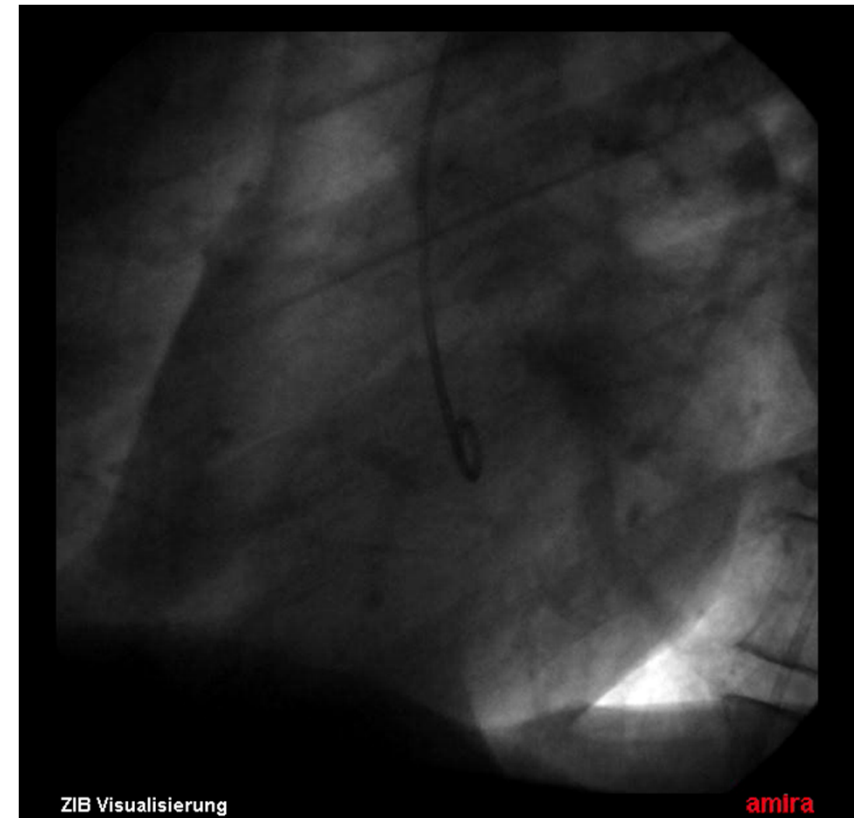


My Heart

Vorhof-Flattern
atrial fibrillation



Ablation



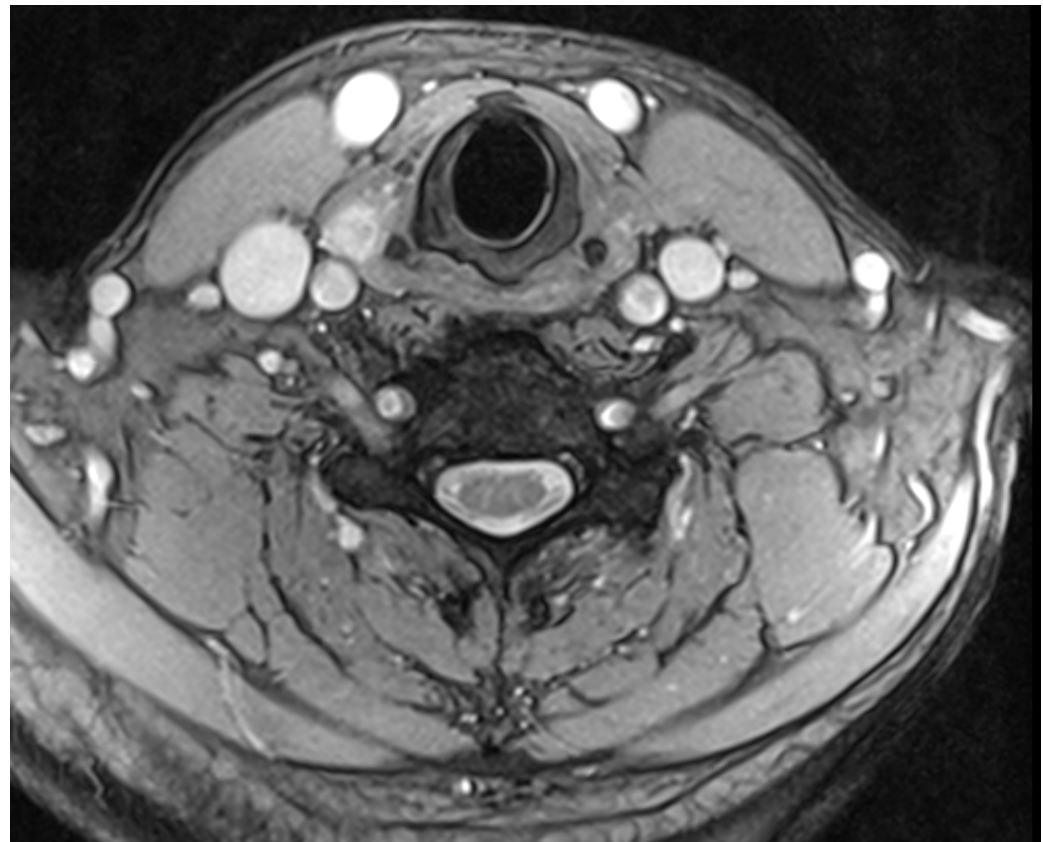
Cardio Fibrillation



Peter Deuflhard's group at ZIB

Magnetic Resonance Tomography (MRT)

My own spine 28.08.2013
operation on 03.09.2013



Overview on Applications: Is (advanced) mathematics useful? CO@W Berlin

**Thanks for your
attention**



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- Forschungszentrum MATHEON
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

<http://www.zib.de/groetschel>