Optimization of Telecommunication Networks

Axel Werner, Fabio D'Andreagiovanni, Frank Pfeuffer, Jonad Pulaj

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Research Center Matheon
Mathematics for Key Technologies
TU berlin
ZIB
Berlin Mathematical School
MODAL
Mathematical Optimization and Data Analysis Laboratories

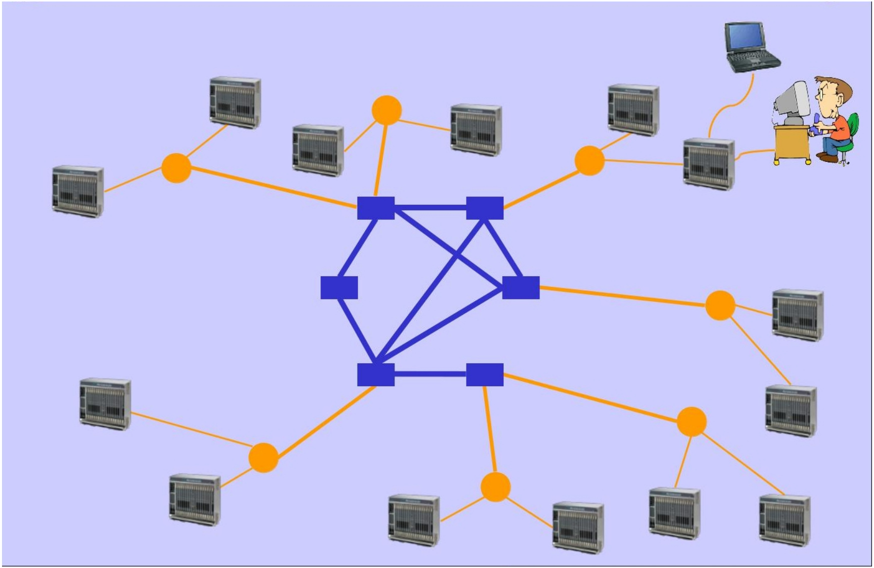
Introduction

Network Layout Planning

Network Dimensioning and Routing

Further Topics

A Telecommunication Network



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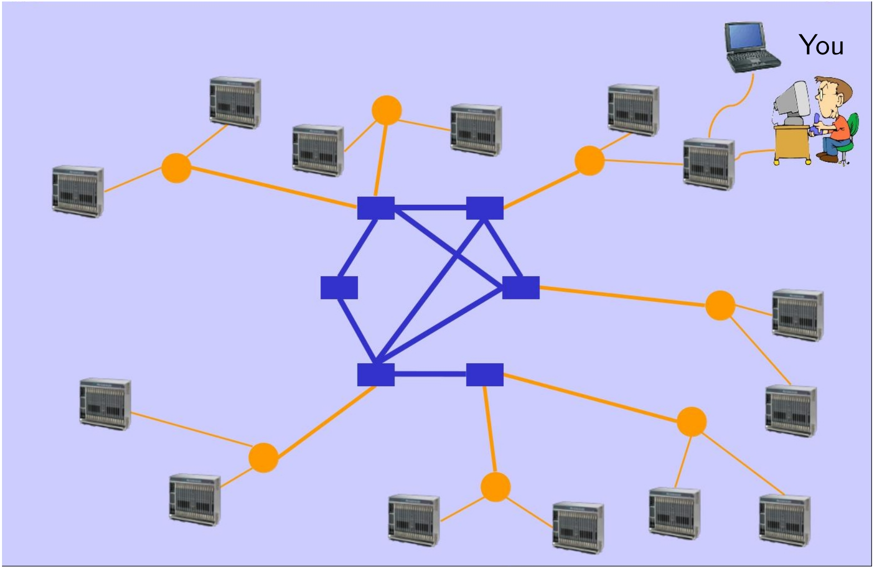

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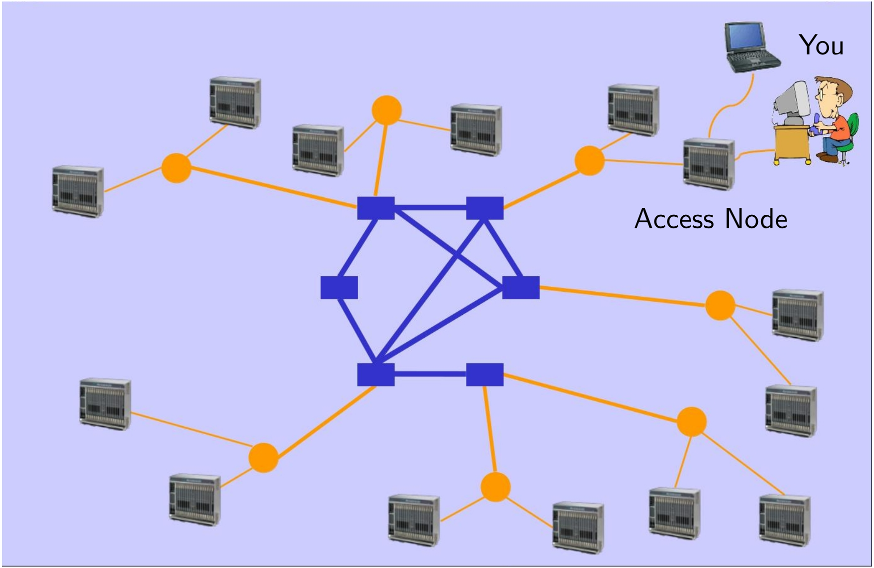

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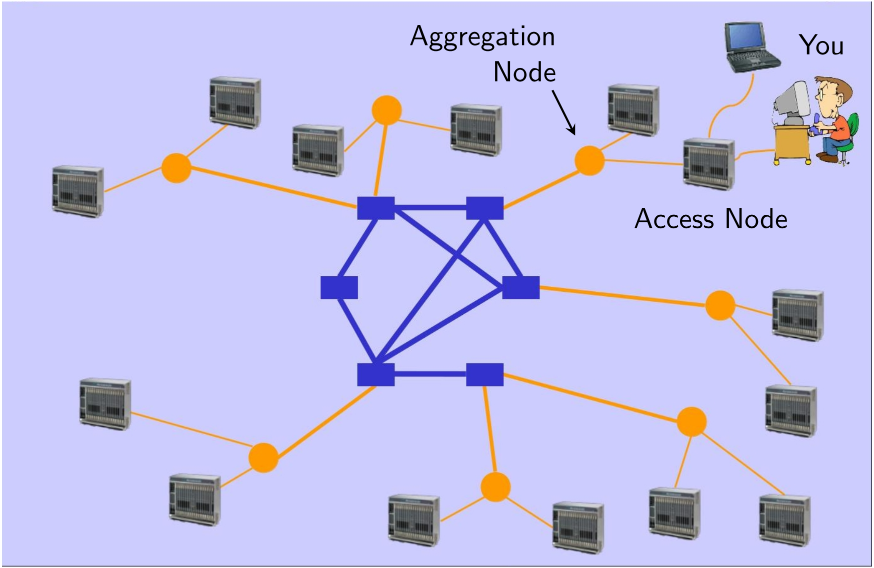

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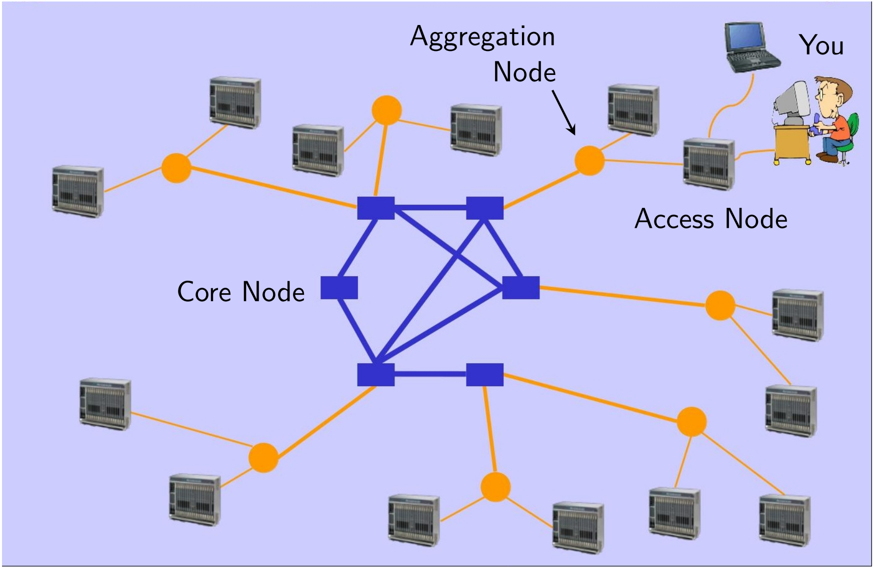
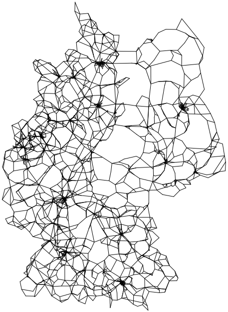
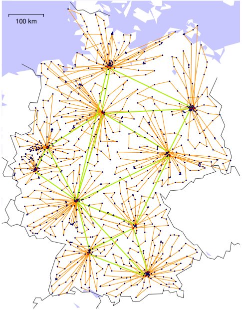
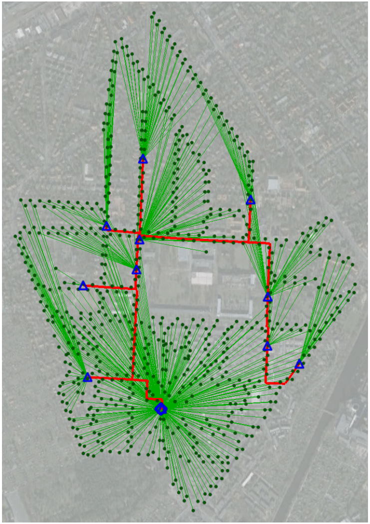



STTLng
DNVRng
SNVAng
LOSAng
KSCYng
HSTNng
CHINng
IPLSng
NYCMng
WASHng
ATLAng
ATLAM5




100 km



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Optimization Problems in Telecommunications

- Network layout planning
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- Further stuff
  - Wavelength assignment
  - Virtual networks
  - Dynamic traffic
  - Wireless networks

Overview

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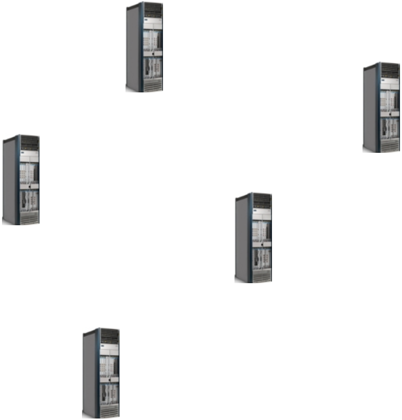
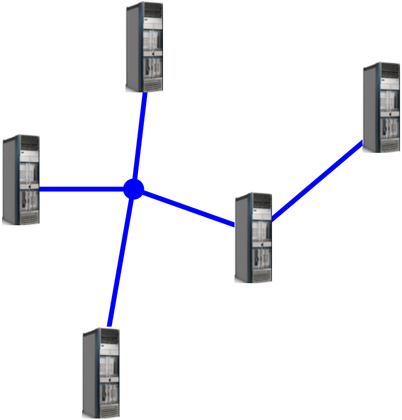
Network Dimensioning and Routing

Further Topics

Network Layout Planning: Connectivity





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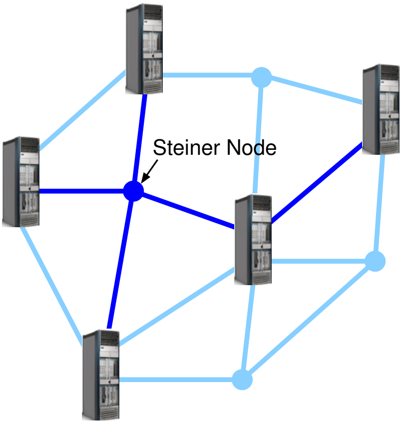
Steiner Node

Definition

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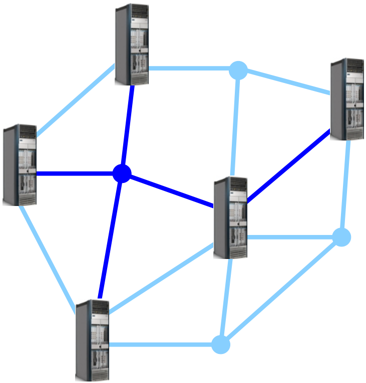
Given a graph $G = (V, E)$ with terminals $T \subseteq V$, a Steiner tree is a tree $S \subseteq E$ that connects all terminals in T.



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Definition (Minimum Steiner Tree Problem)

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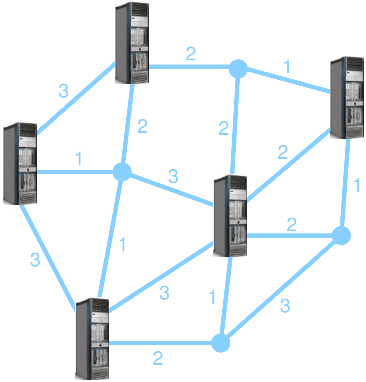

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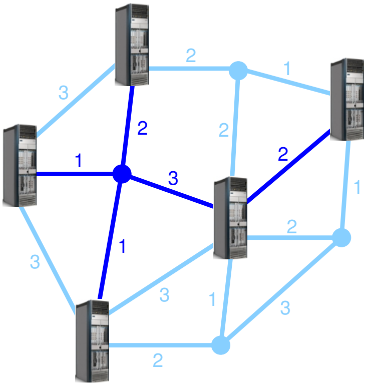

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Theorem (Garey & Johnson, 1979)

The Steiner Tree Problem is NP-complete.

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Simple special cases:

- $|T| = 2$: Shortest Path Problem
- $T = V$: Minimum Spanning Tree Problem

Undirected Cut Formulation (Aneja 1980)

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\text{s.t.} \quad & x(\delta(U)) \geq 1 && \forall U \subset V \text{ with } \emptyset \neq U \cap T \neq T \\
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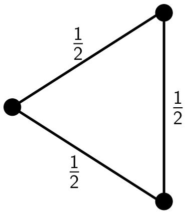

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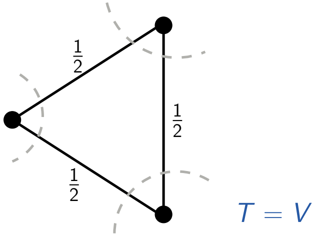

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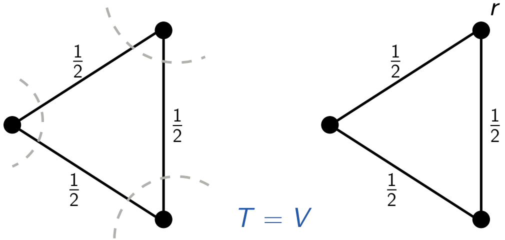

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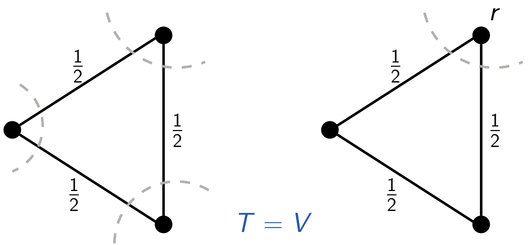

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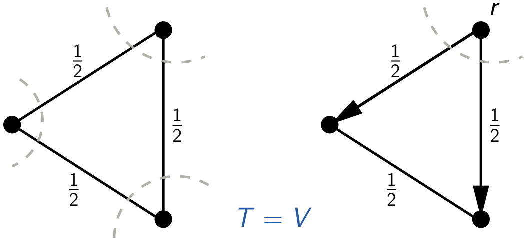

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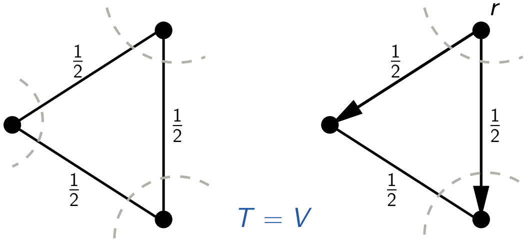

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- (Multicommodity) flow formulation:

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\text{s.t.} \quad & \sum_{a \in \delta^+(v)} f_{t,a} - \sum_{a \in \delta^-(v)} f_{t,a} = \begin{cases} 1, & v = r \\ 0, & \text{else} \end{cases} && \forall v \in V, t \in T', v \neq t \\
& f_{s,(i,j)} + f_{t,(j,i)} \leq x_e && \forall s, t \in T', e = \{i,j\} \in E \\
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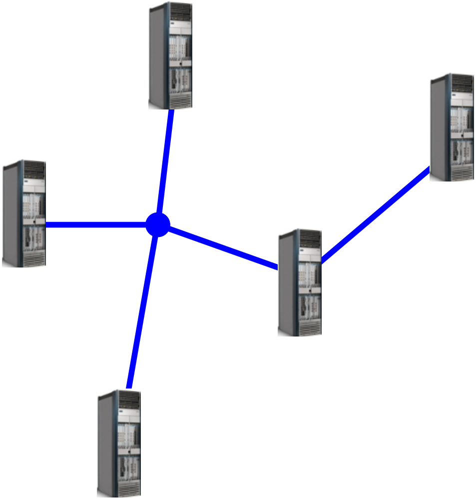
Network Layout Planning: Failures happen...



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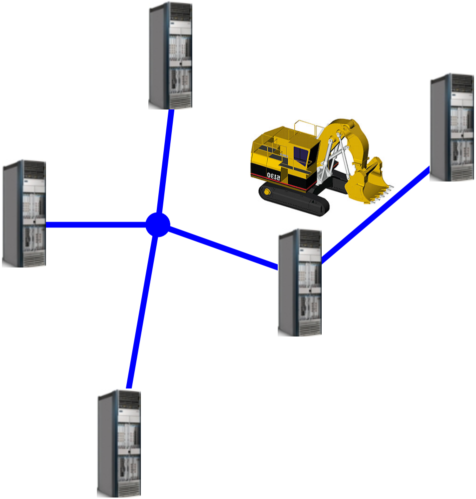

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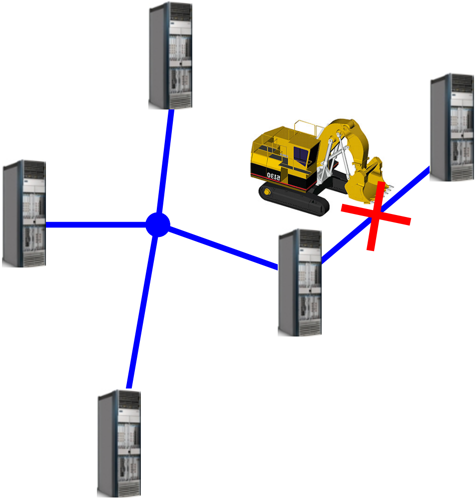

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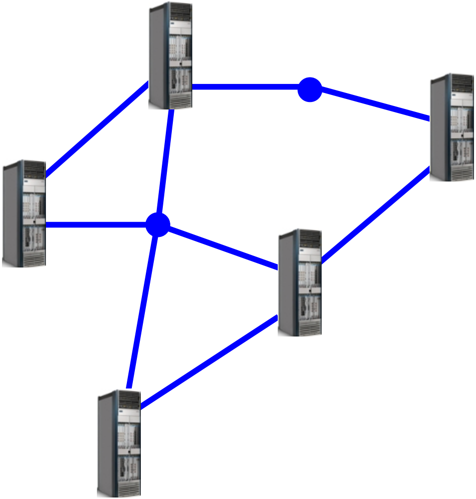

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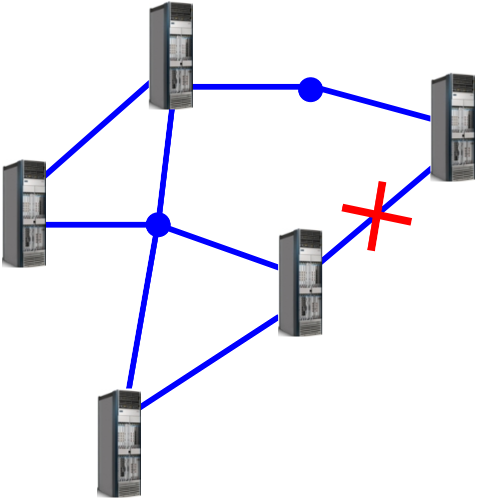

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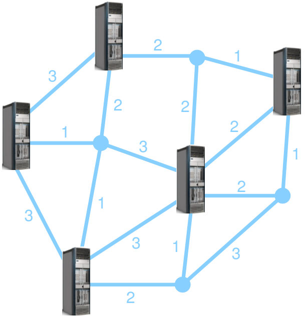
Given: $G = (V, E)$, terminals $T \subseteq V$, edge weights c_e for all $e \in E$



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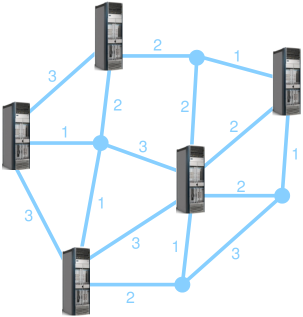
Find: Minimum cost subgraph containing all $v \in T$
that has at least two edge-disjoint s-t-paths for all $s, t \in T$



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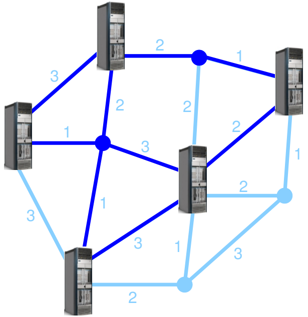

Definition (Edge-Survivable Network Design Problem)

Given: $G = (V, E)$, terminals $T \subseteq V$, edge weights c_e for all $e \in E$
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& f_{st,(i,j)} + f_{st,(j,i)} \leq x_e && \forall s, t \in T, e = \{i, j\} \in E \\
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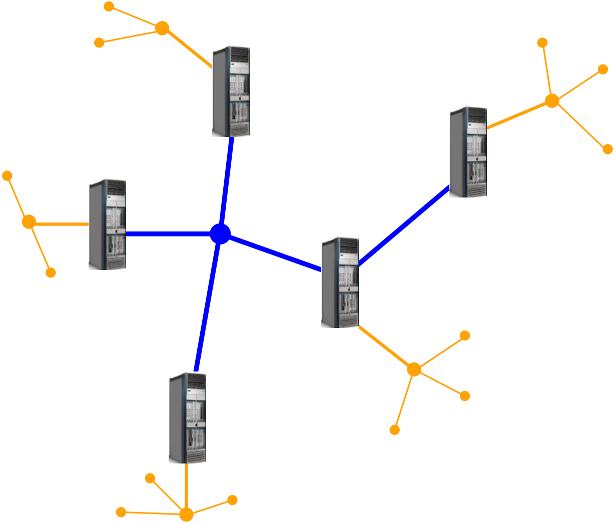
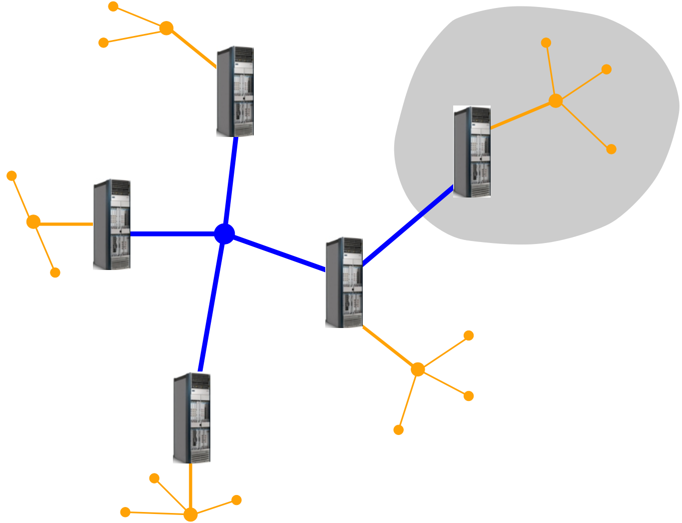


Network Layout Planning: Access Networks



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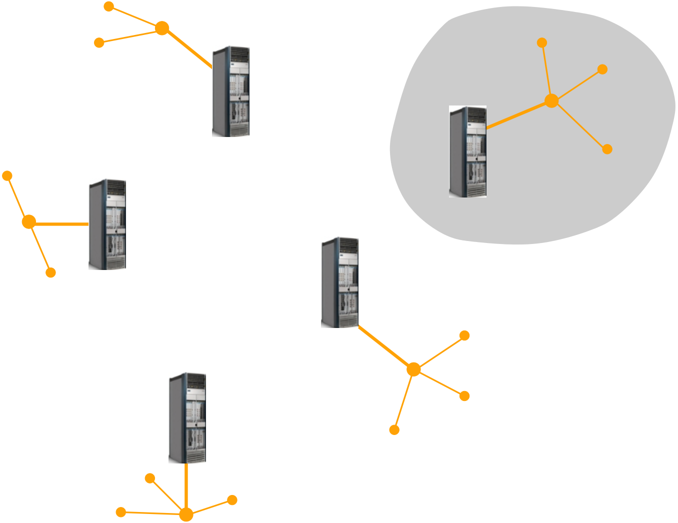

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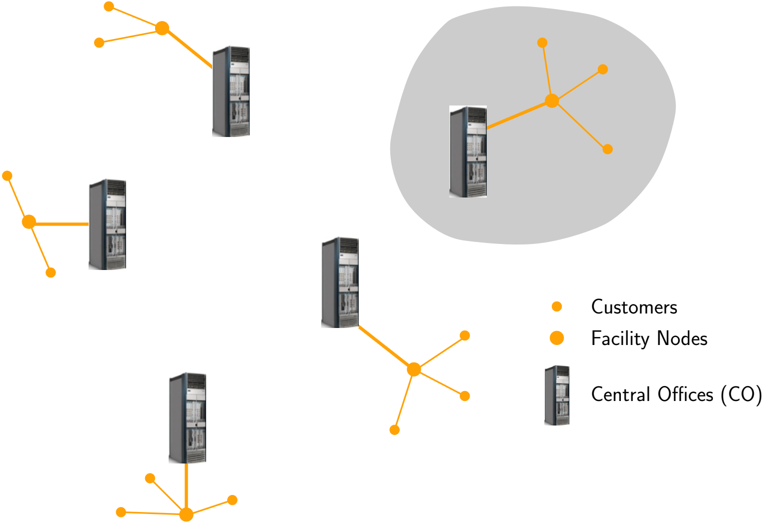
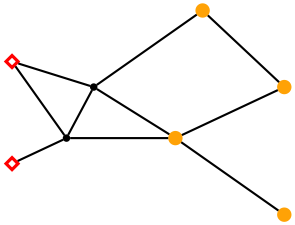


Definition (Connected Facility Location Problem)

Given: $G = (V, E)$, edge costs c_e for all $e \in E$;



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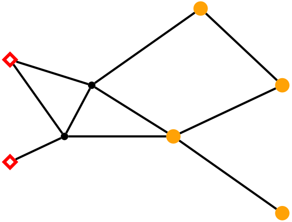
Given: $G = (V, E)$, edge costs c_e for all $e \in E$; facilities $F \subseteq V$, opening costs c_v for all $v \in F$;



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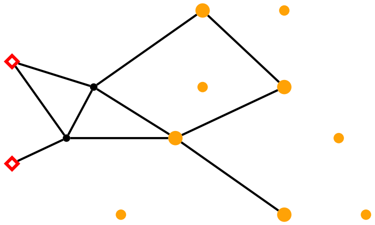

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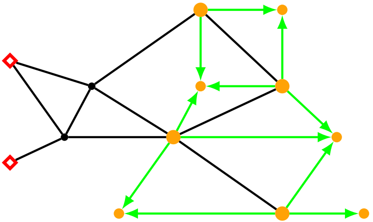

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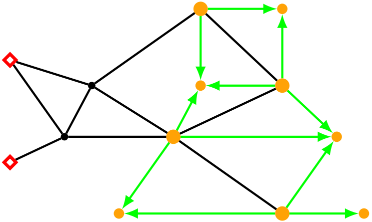
Find: Subset F' of opened facilities, assignment in $D := (F' \cup C, A')$ where $A' := A \cap (F' \times C)$, and a Steiner tree with respect to terminals F', such that the total cost is minimized



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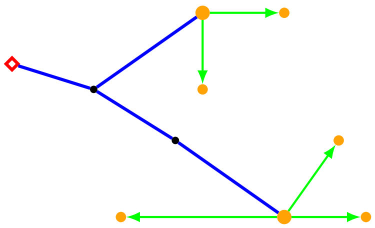

- Connectivity using undirected cuts:

$$
\begin{aligned}
\min \quad & \sum_{v \in F} c_v y_v + \sum_{(v,j) \in A} c_{vj} x_{vj} + \sum_{e \in E} c_e x_e \\
\text{s.t.} \quad & x(\delta(U)) \geq y_v && \forall v \in F, v \in U \subset V \\
& x_{vj} \leq y_v && \forall v \in F, (v,j) \in A \\
& \sum_{(v,j) \in A} x_{vj} = 1 && \forall j \in C \\
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- Variants and improvements: directed cuts, customer cuts, flow, ...

IP models for ConFL

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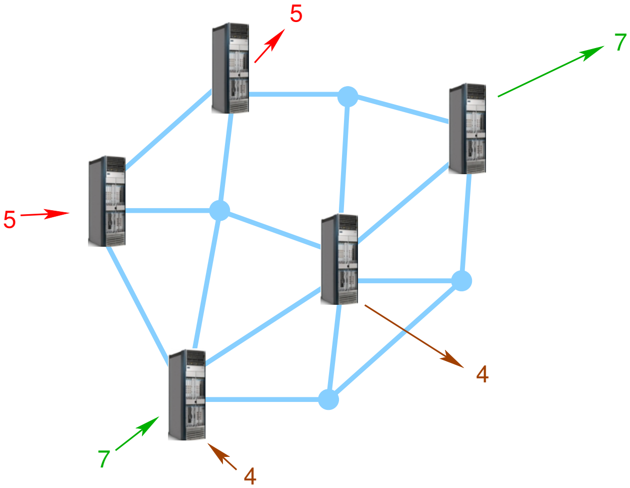
\end{aligned}$$

- Variants and improvements: directed cuts, customer cuts, flow, ...
- Extensions: different types of facilities, coverage requirements, customer demands, capacities, ...

Network Planning: Demands and Capacities



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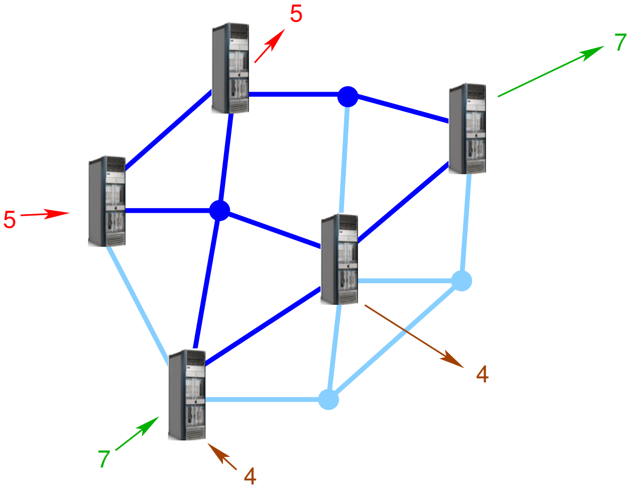

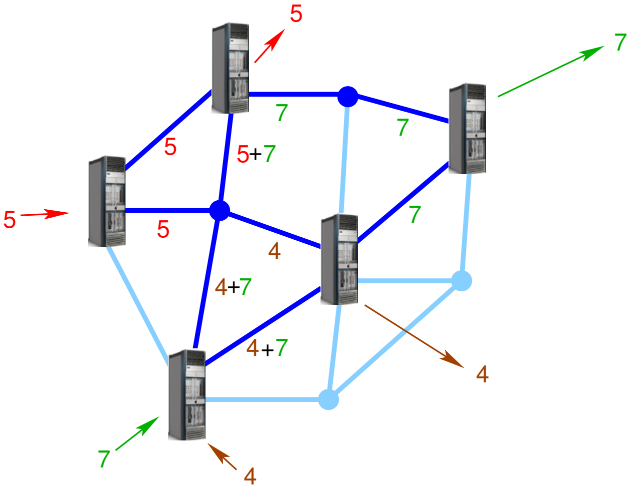
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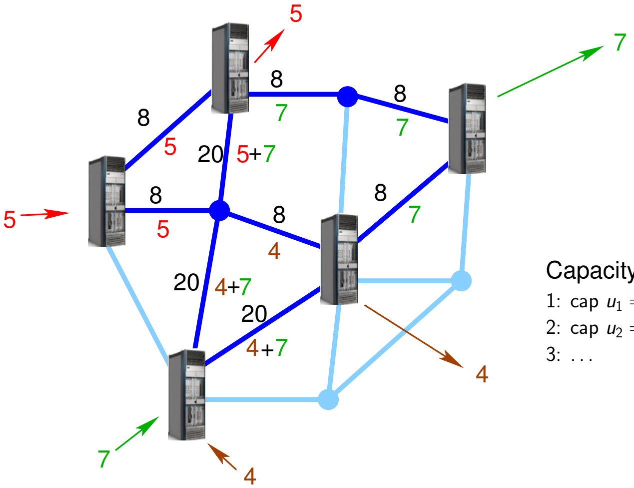


Capacity modules:

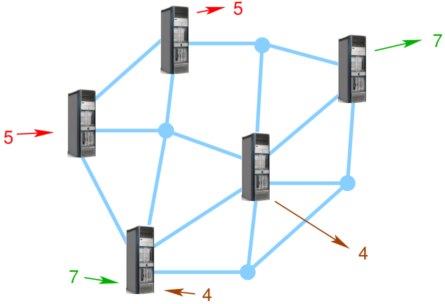
1: cap $u_1 = 8$, cost $c_1 = 1$

2: cap $u_2 = 20$, cost $c_2 = 2$

3: ...

Definition (Capacitated Network Design Problem)

Given: $G = (V, E)$, commodities $K \subseteq V \times V$, demands d_{st} for all $(s, t) \in K$,



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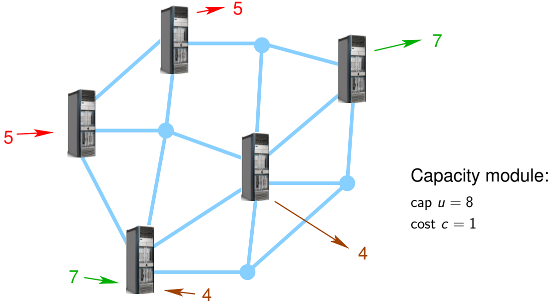

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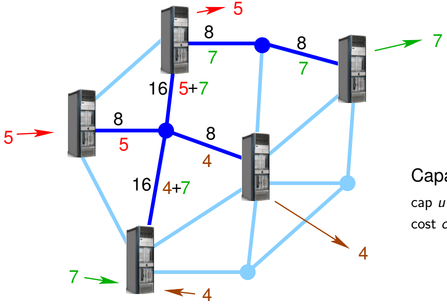
Find: Find edge capacities as multiples of u of minimum cost and a routing of all demands that does not exceed the capacities



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& f_{st,a} \in \{0,1\} \quad \text{➛ Routing} && \forall (s,t) \in K, a \in A \\
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\end{aligned}
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Network Planning: Requirements in Practice

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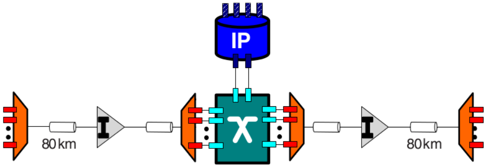
- Multilayer networks

IP-over-WDM (*Internet Protocol/Wavelength Division Multiplexing)



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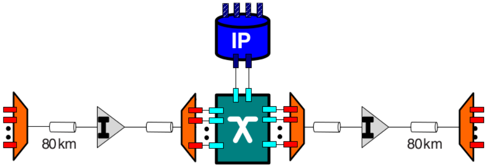
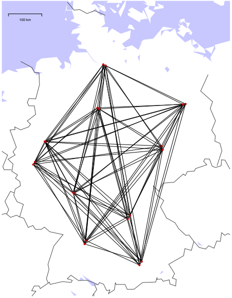
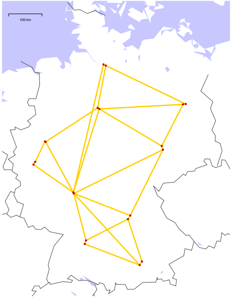


IP layer

logical links between IP routers

data encoded and routed as IP packets





WDM layer

fiber connections between optical devices

IP packets transmitted as optical signals

$$\min \quad \sum_{i \in V} E_P y_i + E_S x_i + E_L z_i + E_F w_i + \sum_{e \in E} E_e z_e$$

$$\sum_{\{i,j\} \in L} f^s_{i,j} - f^s_{j,i} = \begin{cases} \sum_{t \in T(s)} d_{(s,t)}, & i = s \\ -d_{(s,i)}, & i \in T(s) \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in V,\ s \in V$$

$$\sum_{s \in V} f^s_{i,j} + f^s_{j,i} \leq c\, y_{\{i,j\}} \qquad \forall \{i,j\} \in L$$

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$$\sum_{\{i,j\} \in L} y_{\{i,j\}} = y_i \qquad \forall i \in V$$

$$y_i \leq M_P\, x_i \qquad \forall i \in V$$

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$$z_i \leq (M_L - 1/M_F) w_i + 1 \qquad \forall i \in V$$

$$w_i \leq M_F \qquad \forall i \in V$$

$$\mathbf{f} \geq 0$$

$$\mathbf{g}, \mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{Z}_+$$

$$\min \quad \sum_{i \in V} E_P y_i + E_S x_i + E_L z_i + E_F w_i + \sum_{e \in E} E_e z_e$$

routing & dimensioning IP layer

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routing & dimensioning WDM layer

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routing & dimensioning IP layer

$$\sum_{\{i,j\}\in L} f^s_{i,j} - f^s_{j,i} = \begin{cases} \sum_{t\in T(s)} d_{(s,t)}, & i = s \\ -d_{(s,i)}, & i \in T(s) \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in V,\ s \in V$$

$$\sum_{s\in V} f^s_{i,j} + f^s_{j,i} \le c\, y_{\{i,j\}} \qquad \forall \{i,j\} \in L$$

routing & dimensioning WDM layer

$$\sum_{\{u,v\}\in E} g^i_{v,u} - g^i_{u,v} = \begin{cases} \sum_{j\in J(i)} y_{\{i,j\}}, & v = i \\ -y_{\{i,v\}}, & v \in J(i) \\ 0, & \text{otherwise} \end{cases} \qquad \forall v \in V,\ i \in V$$

$$\sum_{i\in V} g^i_{u,v} + g^i_{v,u} \le M\, z_{\{u,v\}} \qquad \forall \{u,v\} \in E$$

counting hardware

$$\sum_{\{i,j\}\in L} y_{\{i,j\}} = y_i \qquad \forall i \in V$$

$$y_i \le M_P\, x_i \qquad \forall i \in V$$

$$x_i \le M_S\, z_i \qquad \forall i \in V$$

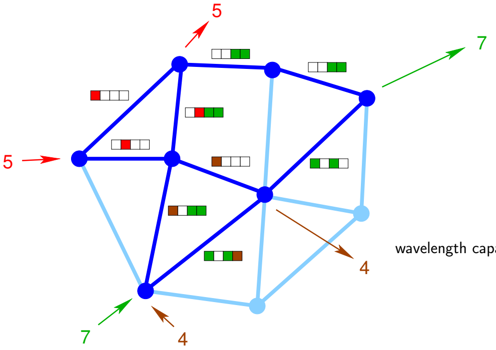
$$z_i \le (M_L - 1/M_F) w_i + 1 \qquad \forall i \in V$$

$$w_i \le M_F \qquad \forall i \in V$$

$$\mathbf{f} \ge 0$$

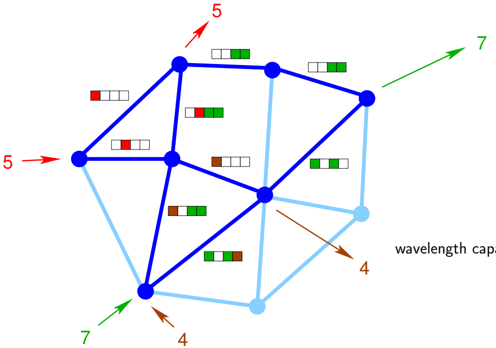
$$\mathbf{g}, \mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{Z}_+$$

Optical signals on a WDM system are multiplexed



wavelength capacity 5

Optical signals on a WDM system are multiplexed



wavelength capacity 5

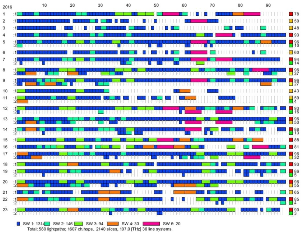
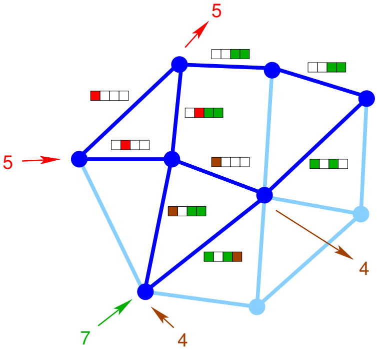


figure: ADVA

Optical signals on a WDM system are multiplexed



wavelength capacity 5

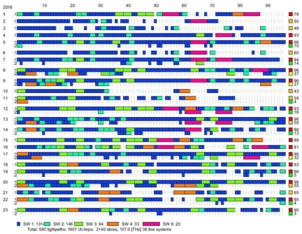
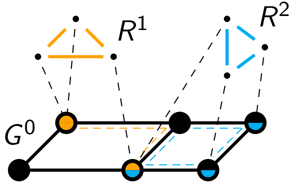


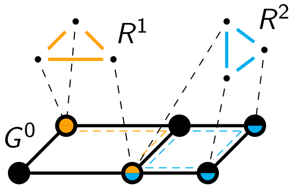
figure: ADVA

➛ Wavelength Assignment Problem: For each optical link, find an assignment of optical connections using this link to 96 color slots!

Virtual network requests instead of demands:

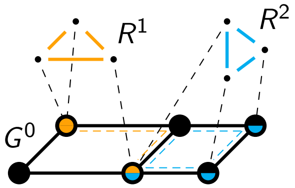


Virtual network requests instead of demands:



Virtual Network Embedding Problem: Given a substrate network G^0 and network requests $R^1, \ldots, R^n$, find an optimal embedding of as many requests as possible into G^0!

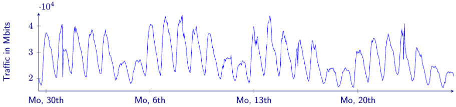
Virtual network requests instead of demands:



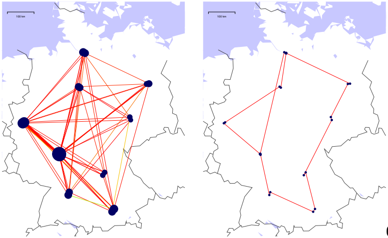
Virtual Network Embedding Problem: Given a substrate network G^0 and network requests $R^1, \ldots, R^n$, find an optimal embedding of as many requests as possible into G^0!

- Requests are dynamically changing over time
  - ➛ reoptimization necessary
  - ➛ online problem

Network demands fluctuate:

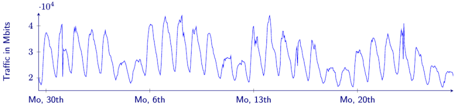


➛ Networks should adapt dynamically to work efficiently, reduce energy consumption etc.



0000

Network demands fluctuate:



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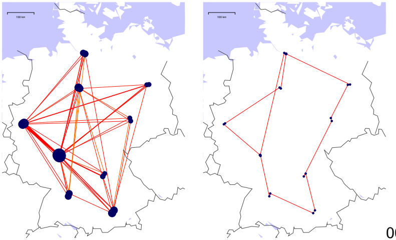
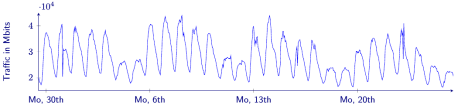


0030

Network demands fluctuate:



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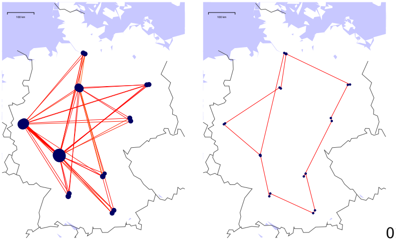
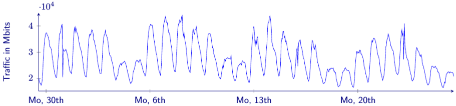


0100

Further Topics: Dynamic Network Operation

Network demands fluctuate:



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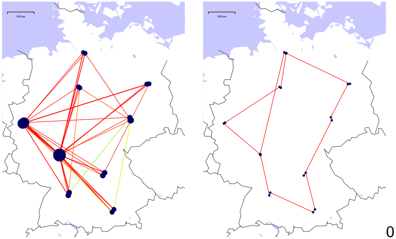
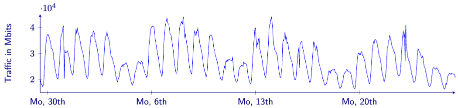


Further Topics: Dynamic Network Operation

Network demands fluctuate:



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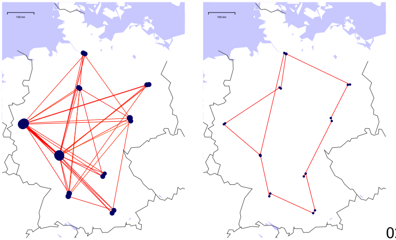
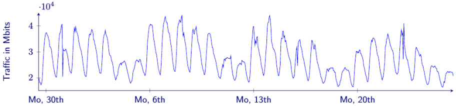


0200

Further Topics: Dynamic Network Operation

Network demands fluctuate:



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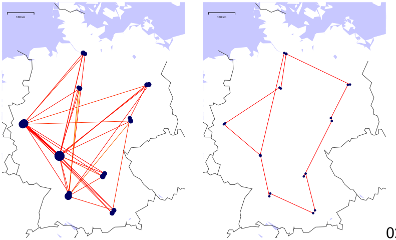
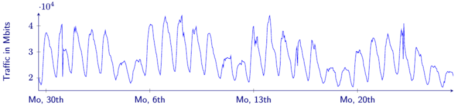


0230

Further Topics: Dynamic Network Operation

Network demands fluctuate:



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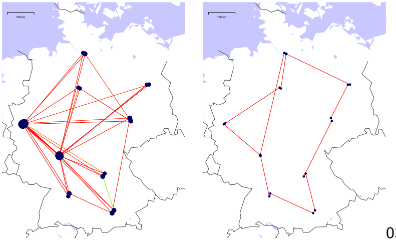
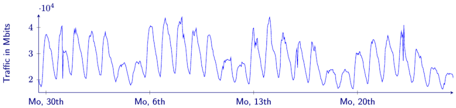
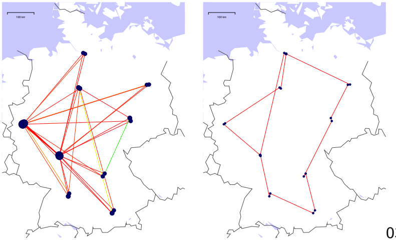


Further Topics: Dynamic Network Operation

Network demands fluctuate:



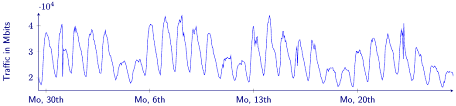


0330

➛ Networks should adapt dynamically to work efficiently, reduce energy consumption etc.

Further Topics: Dynamic Network Operation

Network demands fluctuate:



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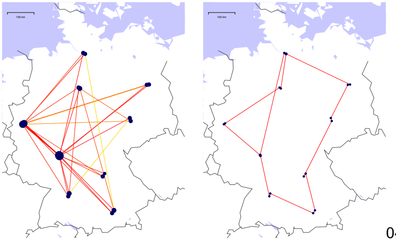
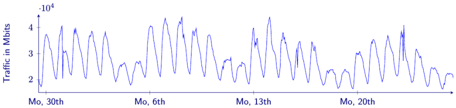


0400

Network demands fluctuate:



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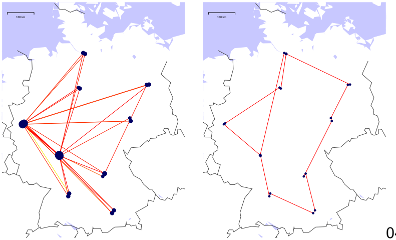
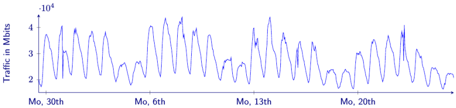


0430

Network demands fluctuate:



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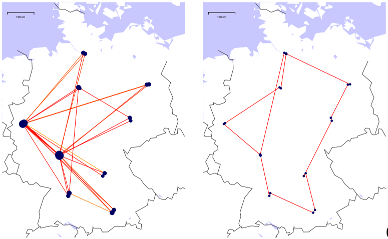
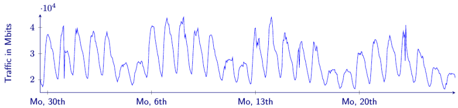


0500

Network demands fluctuate:



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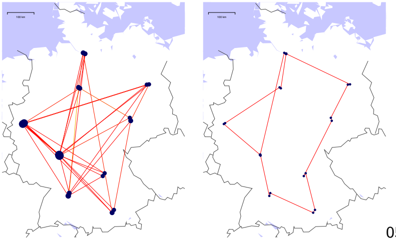
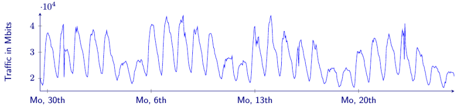


0530

Further Topics: Dynamic Network Operation

Network demands fluctuate:



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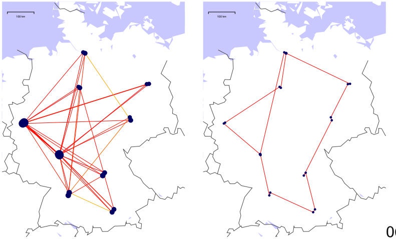

0600

Further Topics: Dynamic Network Operation

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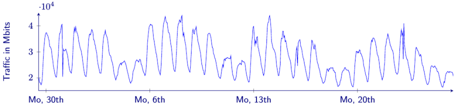

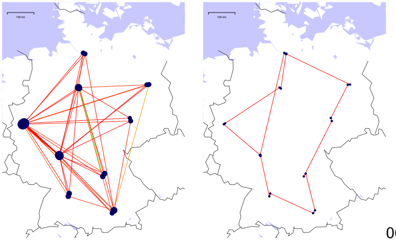

0630

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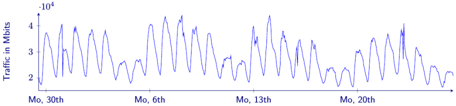
Further Topics: Dynamic Network Operation

Network demands fluctuate:



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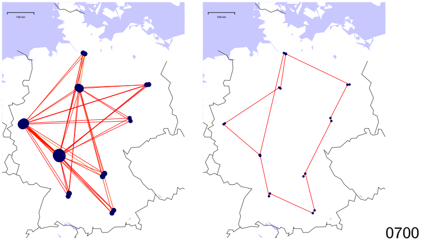
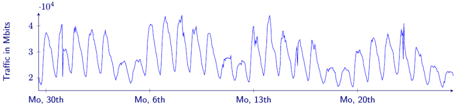


Further Topics: Dynamic Network Operation

Network demands fluctuate:



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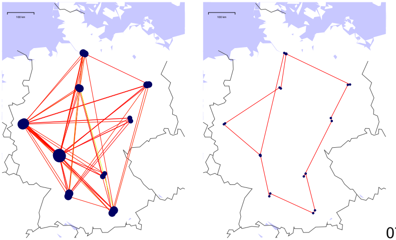
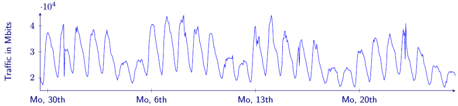


0730

Network demands fluctuate:



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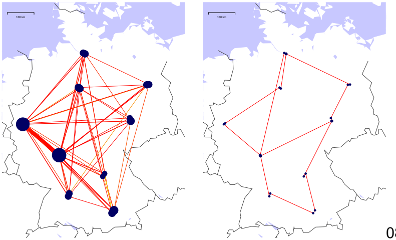
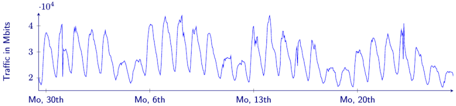


0800

Network demands fluctuate:



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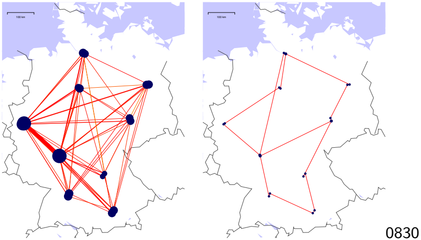
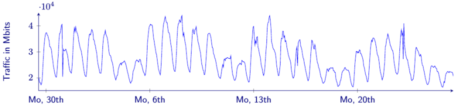


Network demands fluctuate:



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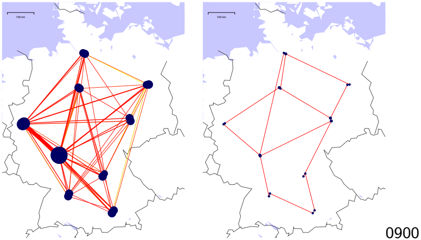
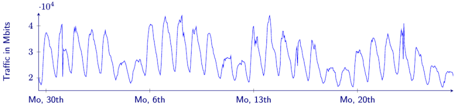


Network demands fluctuate:



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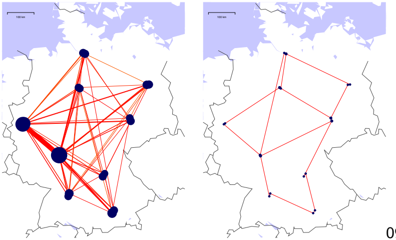
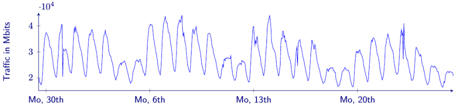


0930

Network demands fluctuate:



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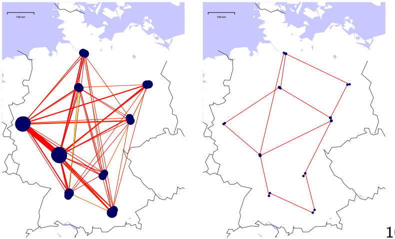
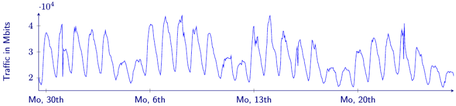


1000

Further Topics: Dynamic Network Operation

Network demands fluctuate:



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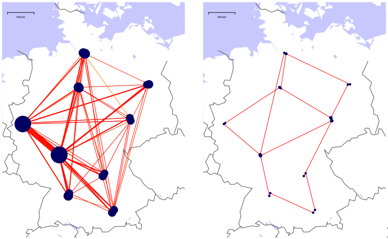
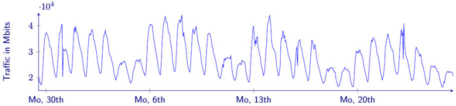


1030

Network demands fluctuate:



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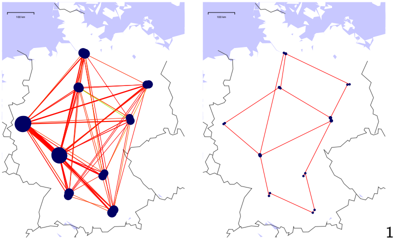
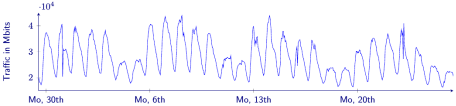


1100

Network demands fluctuate:



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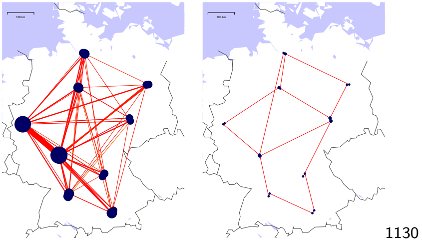
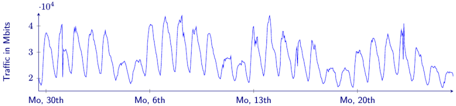
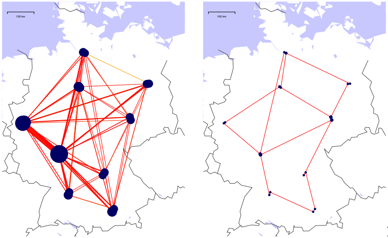


Further Topics: Dynamic Network Operation

Network demands fluctuate:

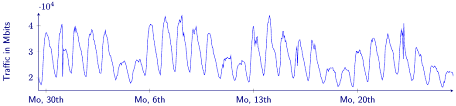




1200

➛ Networks should adapt dynamically to work efficiently, reduce energy consumption etc.

Network demands fluctuate:



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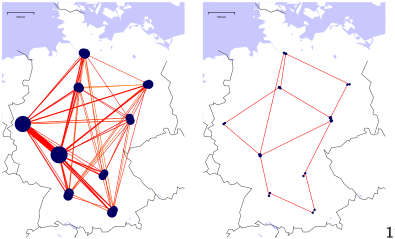

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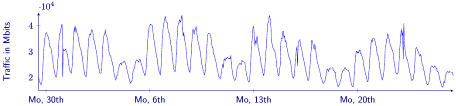

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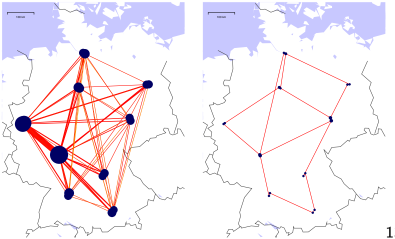
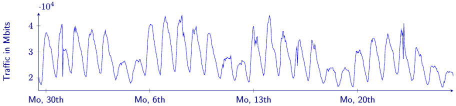
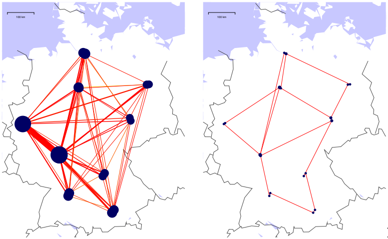


Further Topics: Dynamic Network Operation

Network demands fluctuate:

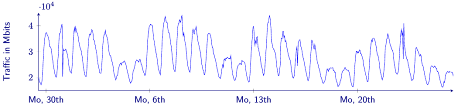


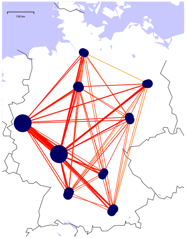
➛ Networks should adapt dynamically to work efficiently, reduce energy consumption etc.

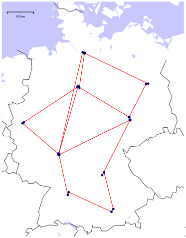


1330

Network demands fluctuate:





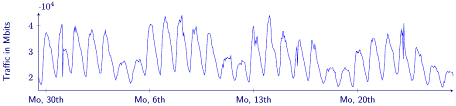


1400

➛ Networks should adapt dynamically to work efficiently, reduce energy consumption etc.

Further Topics: Dynamic Network Operation

Network demands fluctuate:



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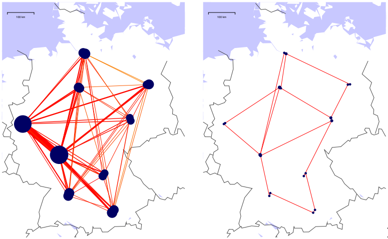

1430

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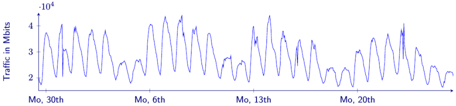

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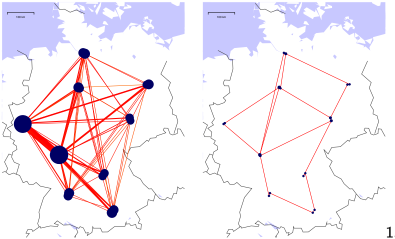

1500

Further Topics: Dynamic Network Operation

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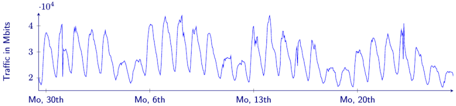

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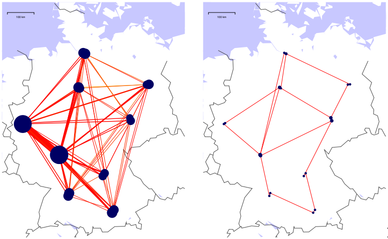
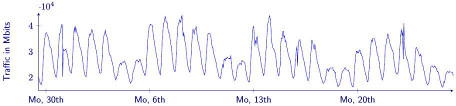


1530

Network demands fluctuate:



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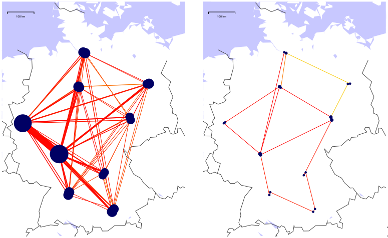
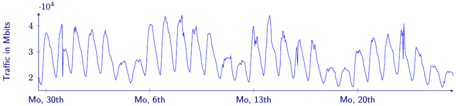


1600

Network demands fluctuate:



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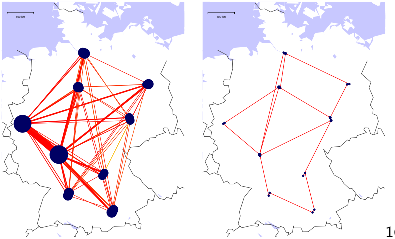
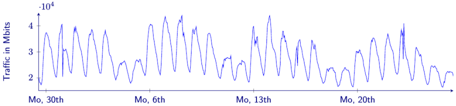


1630

Network demands fluctuate:



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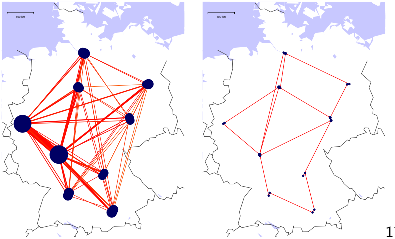
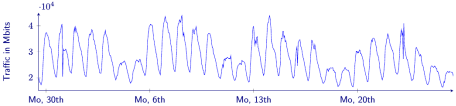


Further Topics: Dynamic Network Operation

Network demands fluctuate:



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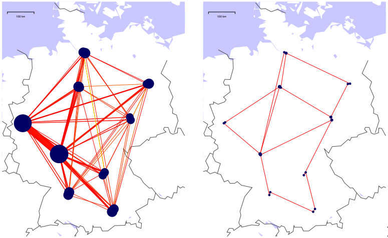

1730

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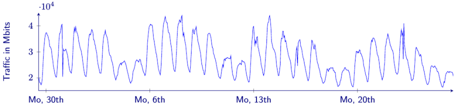

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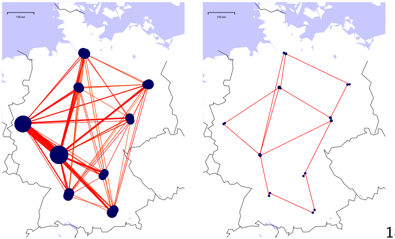
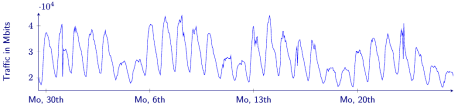
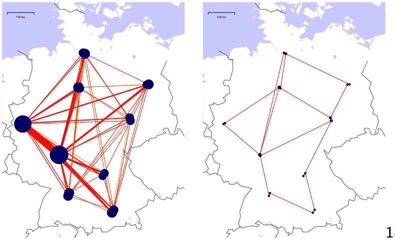


1800

Further Topics: Dynamic Network Operation

Network demands fluctuate:





1830

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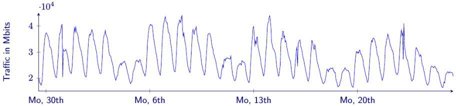

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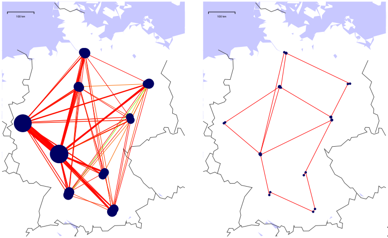

1900

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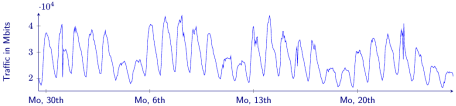

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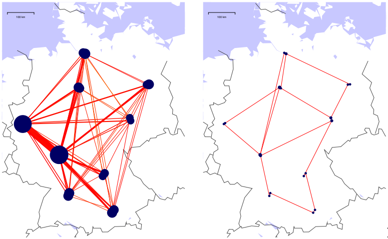

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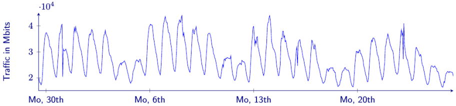

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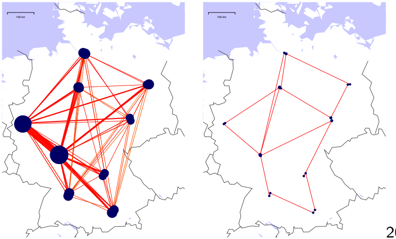

2000

Further Topics: Dynamic Network Operation

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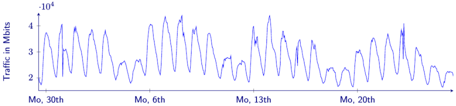

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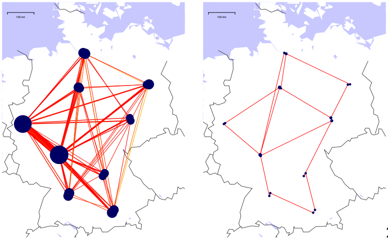

2030

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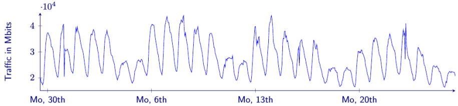

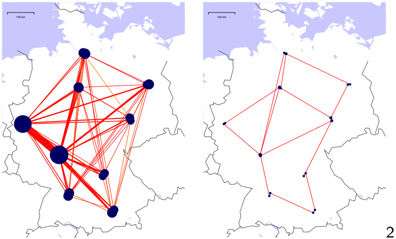

2100

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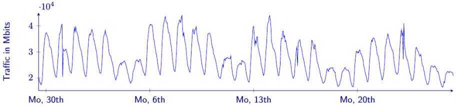

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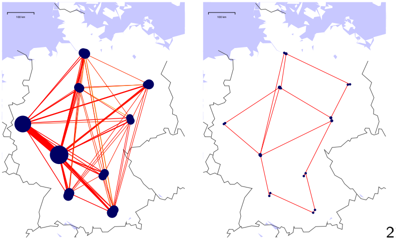

2130

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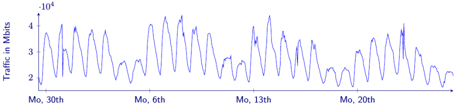

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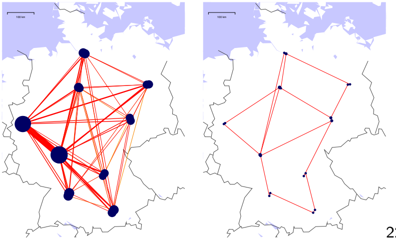

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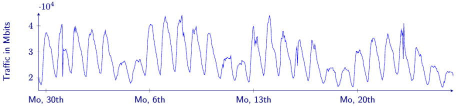

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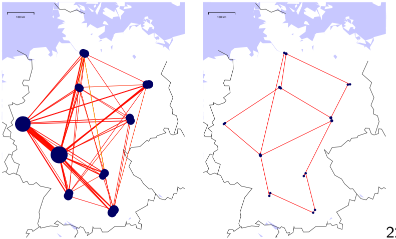

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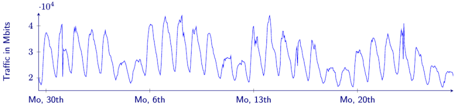

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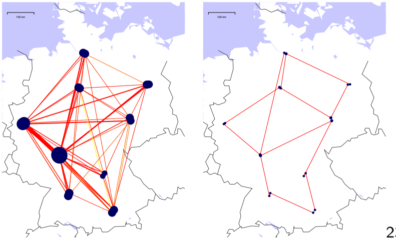
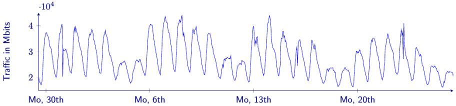


2300

Network demands fluctuate:



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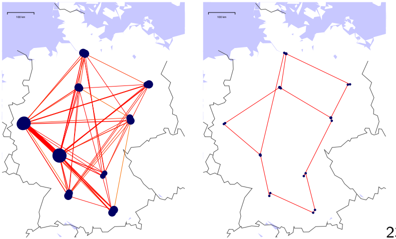
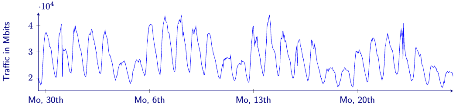
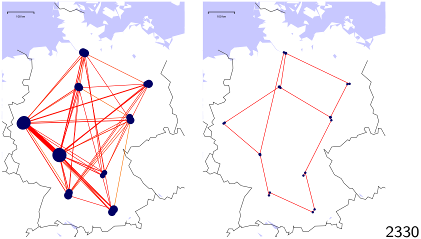


2330

Network demands fluctuate:





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➛ Dynamic Network Design Problem:
Find optimal networks for each time interval!

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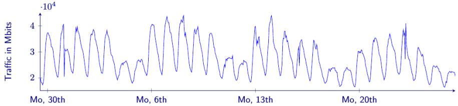

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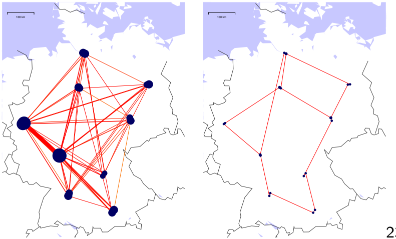
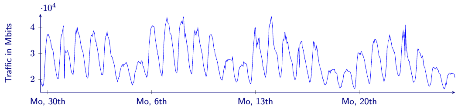


2330

➛ Dynamic Network Design Problem:
Find networks for each time interval that are globally optimal!

Network demands fluctuate:



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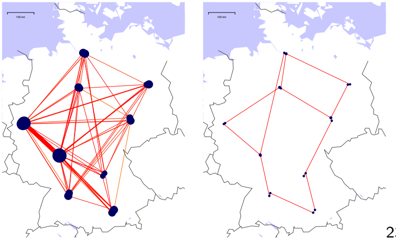

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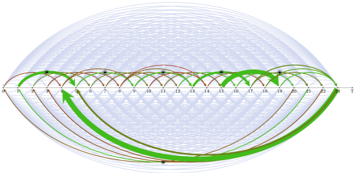
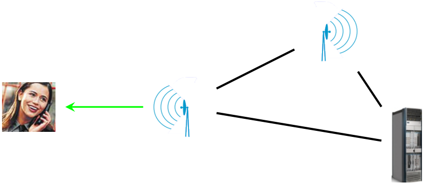


Links can also be wireless:



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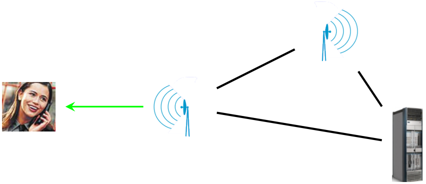

➛ Signal-to-interference ratio (SIR) has to be high enough for every user:

$$\frac{a_{jb} p_b}{\sum_{\beta \in B \setminus \{b\}} a_{j\beta} p_\beta + N} \geq \delta$$

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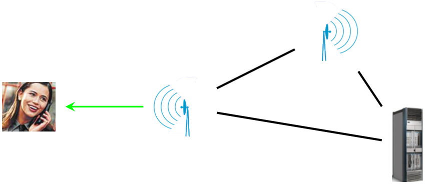

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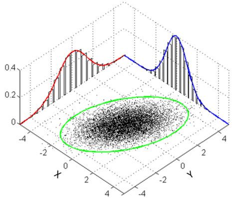
➛ SIR constraints:

$$a_{jb}p_b - \delta \sum_{\beta \in B \setminus \{b\}} a_{j\beta}p_\beta + M(1 - z_j^k) \geq \delta N \qquad \forall j \in C,\ b \in B$$

Input data might deviate from given (nominal) values



- Examples: demand values, fading coefficients, cost values, ...

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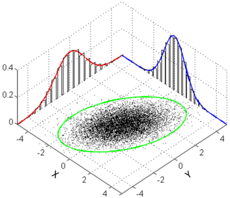

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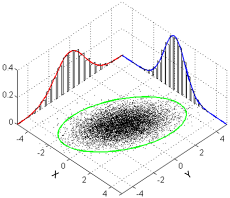

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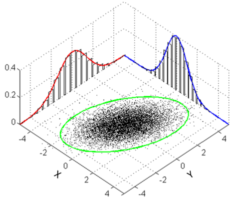

- Examples: demand values, fading coefficients, cost values, ...
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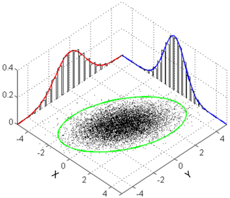
  - feasible set enlarged according to the robustness scheme
    ➛ trade-off between conservativism (too large feasible set) and inflexibility (too small feasible set)



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- Solutions must still be feasible if some parameters change "slightly"!
- Robust counterpart of the respective model
  - feasible set enlarged according to the robustness scheme
    ➛ trade-off between conservativism (too large feasible set) and inflexibility (too small feasible set)
  - robustification blows up the model, making it harder to solve
    ➛ make sure the model size stays polynomial

Large networks are usually not built overnight...

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➛ Multiperiod planning for a time horizon T (usually several years)

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➛ Time-expanded models:

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  - Discretization of the planning horizon: $\{1, \ldots, T\}$

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  - Objective can include cost changes (due to inflation, technological development, etc.), operational costs, projected revenue

Further Topics: Multi-Objective Optimization

Sometimes not only minimizing costs counts...

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$$\min \quad \sum_{(i,j)\in A_{rc}} c_{ij}x_{ij} + \sum_{l=1}^{k} \sum_{(i,j)\in A^l} c_{ij}^l x_{ij}^l + \sum_{l=1}^{k} \sum_{i\in F^l} c_i^l y_i^l$$

$$\max \quad \sum_{j\in C} d_j z_j^1$$

$$\max \quad \sum_{\ell=1}^{k} \sum_{j\in C} d_j z_j^\ell$$

$$\begin{aligned}
\sum_{l=1}^{k} z_j^l &\le 1 && \forall j \in C \\
\sum_{i\in F_j^l} x_{ij}^l &= z_j^l && \forall j \in C,\ l = 1,\dots,k \\
x_{ij}^l &\le y_i^l && \forall j \in C,\ i \in F_j^l,\ l = 1,\dots,k \\
x(\delta^-(W)) &\ge y_i^l && \forall W \subseteq V \setminus C,\ i \in F^l \cap W,\ l = 1,\dots,k \\
x_a, y_i^l, z_j^l &\in \{0,1\} && \forall a \in A_c,\ i \in F^l,\ j \in C,\ l = 1,\dots,k \\
x_{ij}^l &\in \{0,1\} && \forall i \in F_j^l,\ j \in C,\ l = 1,\dots,k
\end{aligned}$$

Sometimes not only minimizing costs counts...

$$\min \sum_{(i,j)\in A_{rc}} c_{ij}x_{ij} + \sum_{l=1}^{k}\sum_{(i,j)\in A^l} c_{ij}^l x_{ij}^l + \sum_{l=1}^{k}\sum_{i\in F^l} c_i^l y_i^l \qquad \text{(total cost)}$$

$$\max \sum_{j\in C} d_j z_j^1$$

$$\max \sum_{\ell=1}^{k}\sum_{j\in C} d_j z_j^\ell$$

$$\begin{aligned}
\sum_{l=1}^{k} z_j^l &\leq 1 && \forall j \in C\\
\sum_{i\in F_j^l} x_{ij}^l &= z_j^l && \forall j \in C,\ l = 1,\dots,k\\
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Sometimes not only minimizing costs counts...

$$\min \quad \sum_{(i,j)\in A_{rc}} c_{ij}x_{ij} + \sum_{l=1}^{k} \sum_{(i,j)\in A^l} c^l_{ij}x^l_{ij} + \sum_{l=1}^{k} \sum_{i\in F^l} c^l_i y^l_i \qquad \text{(total cost)}$$

$$\max \quad \sum_{j\in C} d_j z^1_j \qquad \text{(coverage with preferred technology)}$$

$$\max \quad \sum_{\ell=1}^{k} \sum_{j\in C} d_j z^\ell_j$$

$$\sum_{l=1}^{k} z^l_j \le 1 \qquad \forall j \in C$$

$$\sum_{i\in F^l_j} x^l_{ij} = z^l_j \qquad \forall j \in C,\ l = 1,\dots,k$$

$$x^l_{ij} \le y^l_i \qquad \forall j \in C,\ i \in F^l_j,\ l = 1,\dots,k$$

$$x(\delta^-(W)) \ge y^l_i \qquad \forall W \subseteq V \setminus C,\ i \in F^l \cap W,\ l = 1,\dots,k$$

$$x_a, y^l_i, z^l_j \in \{0,1\} \quad \forall a \in A_c,\ i \in F^l,\ j \in C,\ l = 1,\dots,k$$

$$x^l_{ij} \in \{0,1\} \quad \forall i \in F^l_j,\ j \in C,\ l = 1,\dots,k$$

Sometimes not only minimizing costs counts...

$$\min \quad \sum_{(i,j)\in A_{rc}} c_{ij}x_{ij} + \sum_{l=1}^{k} \sum_{(i,j)\in A^l} c^l_{ij}x^l_{ij} + \sum_{l=1}^{k} \sum_{i\in F^l} c^l_i y^l_i \qquad \text{(total cost)}$$

$$\max \quad \sum_{j\in C} d_j z^1_j \qquad \text{(coverage with preferred technology)}$$

$$\max \quad \sum_{\ell=1}^{k} \sum_{j\in C} d_j z^\ell_j \qquad \text{(total coverage)}$$

$$\begin{aligned}
\sum_{l=1}^{k} z^l_j &\le 1 && \forall j \in C\\
\sum_{i\in F^l_j} x^l_{ij} &= z^l_j && \forall j \in C,\ l = 1,\dots,k\\
x^l_{ij} &\le y^l_i && \forall j \in C,\ i \in F^l_j,\ l = 1,\dots,k\\
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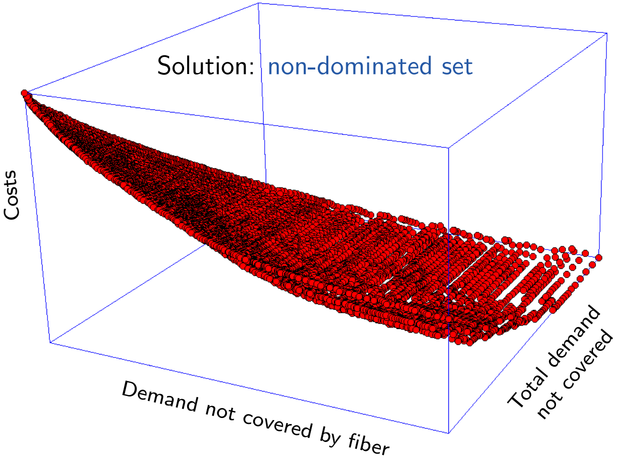
x^l_{ij} &\in \{0,1\} && \forall i \in F^l_j,\ j \in C,\ l = 1,\dots,k
\end{aligned}$$

Sometimes not only minimizing costs counts...



Exercises

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On the CO @ Work website:

- Exercise Sheet
- ZIMPL file for capacitated network design
- Some instances that can be read with the ZIMPL example
  ➛ Create new ones to play with and try out models!

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- Exercise Sheet
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Have fun!

Get Together

Restaurant Cum Laude,
Platz der Märzrevolution, 10117 Berlin

in the building of Humboldt University
close to S+U Bhf Friedrichstraße

Meeting Point after this lecture in front of the ZIB lecture hall.