Long Term Workforce Planning
BC Nurse Population

Based on work of
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Pamela A. Ratner PhD, RN, Professor
Workforce planning, training and regulation is the dominant immediate and long term issue for policy makers, managers, and clinical organizations.

Listening for Direction: A national consultation on health services and policy issues

- Registered nurses account for just over one-third of the health workforce.
- Today’s decision have long term implications.
- Decisions in the 1990’s:
  - decrease education seats,
  - decrease nurse leadership
- In 2001, the CNA president noted “Canada should be graduating about 10,000 nurses annually to replenish the workforce in the next 10 years”.
Planners Require

- **Precise** short and long term plans that provide the annual number of:
  - nursing education seats
  - nurses to recruit and at what level
  - nurses and managers to promote

- **Models:**
  - Inputs and assumptions can be varied and implications seen

- **Flexible unified** frameworks that apply to any workforce:
  - Regional
  - Provincial
  - National
  - Sub-specialty
Commonly Used Approach to Health Workforce Planning

- Develop a plan
- Project or simulate
- Determine if needs are met
- Revise and repeat

Limitations
  - Tedious
  - Sub-optimal
  - Not suitable for “what if?” analyses

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Our Approach

- A flexible **computer-based** optimization model that provides a minimum cost **health human resource plan** that achieves workforce targets over a multi-year planning horizon
- LP Based
- Formulated in Excel and solved using the Frontline solver add on
# A Sample Plan

<table>
<thead>
<tr>
<th>Period</th>
<th>Number admitted</th>
<th>Number of recruits</th>
<th>Number promoted</th>
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<td></td>
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<td>Entry level management</td>
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<td>2009</td>
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<td>1742</td>
<td>873</td>
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<td>2011</td>
<td>2021</td>
<td>1013</td>
<td>190</td>
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<tr>
<td>2012</td>
<td>2344</td>
<td>1175</td>
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<td>2013</td>
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<td>3154</td>
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<td>2015</td>
<td>3158</td>
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<td>2016</td>
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<td>820</td>
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<td>2017</td>
<td>2784</td>
<td>565</td>
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<td>2018</td>
<td>2561</td>
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<td>2019</td>
<td>2356</td>
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<tr>
<td>2020</td>
<td>2168</td>
<td>500</td>
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<tr>
<td>2021</td>
<td>2049</td>
<td>500</td>
<td>10</td>
</tr>
</tbody>
</table>
Model Overview

Data:
- Nurse workforce size
- Student body
- Population projection

Assumptions:
- Attrition rates
- Nurse/Pop.
- Costs

Run model:
- Minimum cost plan

Outputs:
- Optimum costs
- Yearly student population
- Yearly workforce size

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Total Cost
- $1,694,786,393 4%
- $1,665,612,117 4%
- $40,138,704,506 92%

Cost of training
- Immigration cost
- Annual salary

Student(year 1)
Student(year 2)
Student(year 3)
Student(year 4)

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Scope

- Registered Nurses in British Columbia
- 20 year planning horizon
- Decisions made once each year
- Ages 17 – 65+
- Full time equivalent basis
Workforce Flowchart

- Student (year 1)
  - Student (year 2)
  - Student (year 3)
  - Student (year 4)
  - Direct Care Nurse
  - Entry Level Management
  - Senior Level Management

Cost of education
Do other work/retire
New students/recruits
Workforce flow again
Model

- Decision Variables:
  - In each year
    - How many to admit
      - Students (year 1)
      - Students (year 3)
    - How many to recruit
      - Direct care nurses
      - Entry level management
    - How many to promote
      - Direct care nurses
      - Entry level management
This model tracks the number of:

- Students(year 1)
- Students(year 2)
- Students(year 3)
- Students(year 4)
- Practicing nurses
- Nurse leaders
- Senior nurse leaders of age $i$ that are in the system in year $j$
Minimize:
Cost of training + Recruitment cost + Salary

In order to:
- Have sufficient resources (FTEs) to achieve quality of care targets.
- Achieve balance of “recruited” and BC educated nurses.
- Only promote nurses after they have been in their position a specified number of years.
Data Sources

Costs:
- Education:
  - Student (annual) (1)
  - Management training (one shot) (1)
- Recruitment and turnover cost (2)
- Salaries (by level) (3)

Continuation and attrition rates:
- Attrition rates (4), (5)
- Age distribution (6), (7), (8)
- Probability of continuing the degree (6), (9)
- Probability of passing the RN exam (7)
- Probability of leaving BC after graduation (7)
- FTE per full time person per age per experience time in position (7)

Initial Conditions:
- Distribution of current workforce by position (8)
- Ratio of RN to population (8)
- BC population projections (11)
<table>
<thead>
<tr>
<th>Variables</th>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probabilities</td>
<td>$Ps(1)_i, Ps(3)_i$</td>
<td>Probability that a student admitted into the program is of age $i$</td>
</tr>
<tr>
<td></td>
<td>$Prn(1)_i, Prn(2)_i$</td>
<td>Age distribution of nurses recruited</td>
</tr>
<tr>
<td></td>
<td>$Ppn(1)_i, Ppn(2)_i$</td>
<td>Age distribution of nurses promoted</td>
</tr>
<tr>
<td></td>
<td>$P_{continuing}(1), P_{continuing}(2), P_{continuing}(3), P_{continuing}(4)$</td>
<td>Probability of continuing in education</td>
</tr>
<tr>
<td></td>
<td>$P_{pass}$</td>
<td>Probability of passing the provincial exam</td>
</tr>
<tr>
<td></td>
<td>$P_{stay}$</td>
<td>Probability of practicing in the province</td>
</tr>
<tr>
<td></td>
<td>$P_{retire}(1)<em>i, P</em>{retire}(2)<em>i, P</em>{retire}(3)_i$</td>
<td>Attrition rate of nurses by age $i$</td>
</tr>
<tr>
<td>Initial conditions</td>
<td>$initial_s(2)_i, initial_s(3)_i, initial_s(4)_i$</td>
<td>Number of students of age $i$ in the first period of the model</td>
</tr>
<tr>
<td></td>
<td>$initial_n(1)_i, initial_n(2)_i, initial_n(3)_i$</td>
<td>Number of nurses of age $i$ in the first period of the model</td>
</tr>
<tr>
<td></td>
<td>$pfraction(1)_x, pfraction(2)_x, pfraction(3)_x$</td>
<td>Initial fraction of workers that have been in their position at least $x$ years</td>
</tr>
<tr>
<td>Costs</td>
<td>$tsCost$</td>
<td>Cost of funding a university seat per year</td>
</tr>
<tr>
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<td>$tn(2)Cost, tn(3)Cost$</td>
<td>Cost of promoting a nurse into a managerial position</td>
</tr>
<tr>
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<td>$rn(1)Cost_j, rn(2)Cost_j$</td>
<td>Recruitment cost for each nurse in year $j$</td>
</tr>
<tr>
<td></td>
<td>$sn(1)Cost, sn(2)Cost, sn(3)Cost$</td>
<td>Annual salaries</td>
</tr>
<tr>
<td>Bounds</td>
<td>$mins(1)_j, mins(3)_j$</td>
<td>Lower bound on the number of students admitted into the programs in year $j$</td>
</tr>
<tr>
<td></td>
<td>$maxs(1)_j, maxs(3)_j$</td>
<td>Upper bound on the number of students admitted into the programs in year $j$</td>
</tr>
<tr>
<td>Demand</td>
<td>$BCpop_j$</td>
<td>Population projection for year $j$</td>
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<tr>
<td></td>
<td>$n(1)ratio_j, n(2)ratio_j, n(3)ratio_j$</td>
<td>Minimum ratios of nurses to meet population demand in year $j$</td>
</tr>
</tbody>
</table>
Student admission and nurse demand constraints

max $s(1)_j \geq s(1)_{\text{admitted}}_j \geq \min s(1)_j \quad \forall j \quad (21)$

max $s(3)_j \geq s(3)_{\text{admitted}}_j \geq \min s(3)_j \quad \forall j \quad (22)$

$\sum_i FTE(n(1)_{i,j}) \geq \frac{BCpop_j}{n(1)_{\text{ratio}}_j}$

$\sum_i FTE(n(2)_{i,j}) \geq \frac{\sum_i FTE(n(1)_{i,j})}{n(2)_{\text{ratio}}} \quad \forall j \quad (23)$

$\sum_i FTE(n(3)_{i,j}) \geq \frac{\sum_i FTE(n(1)_{i,j})}{n(3)_{\text{ratio}}} \quad \forall j \quad (25)$
Full Time Equivalency

- Important practical consideration
- Not all nurse work full time
  - Maternity leave
  - Other leaves
  - Reduced productivity when starting out (ratio)
- FTE definition for maternity leave
  \[ FTE(n(\cdot)_{i,j})_{pl} = n(\cdot)_{i,j} \times (1 - \text{Fertility\_rate}_{i} \times \text{female\_ratio} \times \text{Maternity\_leave}/12) \]
Balance Constraints

- Balance First and Third Year students

\[ s(1)_{\text{admitted}} j \geq s(3)_{\text{admitted}} j \quad \forall j \quad (26) \]

- Limit number of recruits by number of graduating students

\[ \sum_{i} s(4)_{i} * P_{\text{continuing}(4)} * P_{\text{pass}} * P_{\text{stay}} \geq n(1)_{\text{recruited}} j \quad j = 1 \quad (27) \]

\[ \sum_{i} s(4)_{i,j-1} * P_{\text{continuing}(4)} * P_{\text{pass}} * P_{\text{stay}} \geq n(1)_{\text{recruited}} j \quad \forall j \geq 2 \quad (28) \]

- This maintains a 50-50 mix of BC trained and non-BC trained nurses
- This ratio is arbitrary and can be modified for policy reasons
Only promote nurses with $x$ years of experience

\[
\begin{align*}
n(1)_{\text{promoted}}_j & \leq pfraction(1)_x \times \sum_{i} n(1)_{i,0} & j = 1 \quad (29) \\
n(1)_{\text{promoted}}_j & \leq \sum_{i} \left[ n(1)_{i,0} \times pfraction(1)_{x-j} \times \prod_{k=0}^{j-2} (1 - P_{\text{retire}}(1)_{i+k}) \right] & x \geq j \geq 2 \quad (30) \\
n(1)_{\text{promoted}}_j & \leq \sum_{i} \left[ n(1)_{i,j-x} \times \prod_{k=0}^{x-1} (1 - P_{\text{retire}}(1)_{i+k}) \right] & j > x \quad (31) \\
n(2)_{\text{promoted}}_j & \leq pfraction(2)_x \times \sum_{i} n(2)_{i,0} & j = 1 \quad (32) \\
n(2)_{\text{promoted}}_j & \leq \sum_{i} \left[ n(2)_{i,0} \times pfraction(2)_{x-j} \times \prod_{k=0}^{j-2} (1 - P_{\text{retire}}(2)_{i+k}) \right] & x \geq j \geq 2 \quad (33) \\
n(2)_{\text{promoted}}_j & \leq \sum_{i} \left[ n(2)_{i,j-x} \times \prod_{k=0}^{x-1} (1 - P_{\text{retire}}(2)_{i+k}) \right] & j > x \quad (34)
\end{align*}
\]
Given $65 \geq i \geq 16 = \text{age}$, $j = 1\ldots N = \text{time period}$

- **Total number of first year students of age $i$ at time period $j = s(1)_{i,j}$**
  
  \[ s(1)_{i,j} = s(1)_{\text{admitted},j} \cdot Ps(1)_{i} \quad \forall \ i, j \]

- **Total number of second year students of age $i$ at time period $j = s(2)_{i,j}$**
  
  \[ s(2)_{i,j} = \begin{cases} 
  \text{initial } s(2)_{i,j} & \text{for } j = 1, 16 \leq i \\
  s(1)_{i,j-1} \cdot P\text{continuing}(1) & \text{for } 2 \leq j \leq N, 16 \leq i < 65 \\
  [s(1)_{i,j-1} + s(1)_{i,j-1}] \cdot P\text{continuing}(1) & \text{for } 2 \leq j \leq N, i = 65 
  \end{cases} \]

- **Total number of third year students of age $i$ at time period $j = s(3)_{i,j}$**
  
  \[ s(3)_{i,j} = \begin{cases} 
  \text{initial } s(3)_{i,j} + s(3)_{\text{admitted},j} \cdot Ps(3)_{i} & \text{for } j = 1, 16 \leq i \\
  s(2)_{i,j-1} \cdot P\text{continuing}(2) + s(3)_{\text{admitted},j} \cdot Ps(3)_{i} & \text{for } 2 \leq j \leq N, 16 \leq i < 65 \\
  [s(2)_{i,j-1} + s(2)_{i,j-1}] \cdot P\text{continuing}(2) + s(3)_{\text{admitted},j} \cdot Ps(3)_{i} & \text{for } 2 \leq j \leq N, i = 65 
  \end{cases} \]

- **Total number of fourth year students of age $i$ at time period $j = s(4)_{i,j}$**
  
  \[ s(4)_{i,j} = \begin{cases} 
  \text{initial } s(4)_{i,j} & \text{for } j = 1, 16 \leq i \\
  s(3)_{i,j-1} \cdot P\text{continuing}(3) & \text{for } 2 \leq j \leq N, 16 \leq i < 65 \\
  [s(3)_{i,j-1} + s(3)_{i,j-1}] \cdot P\text{continuing}(3) & \text{for } 2 \leq j \leq N, i = 65 
  \end{cases} \]
• Total number of practicing nurses of age $i$ at time period $j = n(1)_{i,j}$

\[
\begin{align*}
\text{initial } n(1)_{i,j} & \\
+ [n(1)\text{recruited}_j - n(1)\text{promoted}_j] * Pn(1)_t & \text{ for } j = 1, 16 \leq i \\
n(1)_{t-1,j-1} * (1 - \text{Pretire}(1)_{t-1}) & \\
+ s(4)_{t-1,j-1} * P\text{continuing}(4) * P\text{pass} * P\text{stay} & \\
+ [n(1)\text{recruited}_j - n(1)\text{promoted}_j] * Pn(1)_t & \text{ for } 2 \leq j \leq N, 16 \leq i < 65 \\
n(1)_{t-1,j-1} * (1 - \text{Pretire}(1)_{t-1}) & \\
+ n(1)_{t,j-1} * (1 - \text{Pretire}(1)_t) & \\
+ s(4)_{t-1,j-1} * P\text{continuing}(4) * P\text{pass} * P\text{stay} & \\
+ s(4)_{t,j-1} * P\text{continuing}(4) * P\text{pass} * P\text{stay} & \\
+ [n(1)\text{recruited}_j - n(1)\text{promoted}_j] * Pn(1)_t & \text{ for } 2 \leq j \leq N, i = 65
\end{align*}
\]

• Total number of nurse leaders of age $i$ at time period $j = n(2)_{i,j}$

\[
\begin{align*}
\text{initial } n(2)_{i,j} & \\
+ n(1)\text{promoted}_j * Pn(1)_t & \\
+ [n(2)\text{recruited}_j - n(2)\text{promoted}_j] * Pn(2)_t & \text{ for } j = 1, 16 \leq i \\
n(2)_{t-1,j-1} * (1 - \text{Pretire}(2)_{t-1}) & \\
+ n(1)\text{promoted}_j * Pn(1)_t & \\
+ [n(2)\text{recruited}_j - n(2)\text{promoted}_j] * Pn(2)_t & \text{ for } 2 \leq j \leq N, 16 \leq i < 65 \\
n(2)_{t-1,j-1} * (1 - \text{Pretire}(2)_{t-1}) & \\
+ n(2)_{t,j-1} * (1 - \text{Pretire}(2)_t) & \\
+ n(1)\text{promoted}_j * Pn(1)_t & \\
+ n(2)\text{promoted}_j * Pn(2)_t & \text{ for } 2 \leq j \leq N, i = 65
\end{align*}
\]

• Total number of senior nurse leaders of age $i$ at time period $j = n(3)_{i,j}$

\[
\begin{align*}
\text{initial } n(3)_{i,j} & \\
+ n(2)\text{promoted}_j * Pn(2)_t & \text{ for } j = 1, 16 \leq i \\
n(3)_{t-1,j-1} * (1 - \text{Pretire}(3)_{t-1}) & \\
+ n(2)\text{promoted}_j * Pn(2)_t & \text{ for } 2 \leq j \leq N, 16 \leq i < 65 \\
n(3)_{t-1,j-1} * (1 - \text{Pretire}(3)_{t-1}) & \\
+ n(3)_{t,j-1} * (1 - \text{Pretire}(3)_t) & \\
+ n(2)\text{promoted}_j * Pn(2)_t & \text{ for } 2 \leq j \leq N, i = 65
\end{align*}
\]
Objective function

- Training Costs
  \[ \sum_i [s(1)_{i,j} + s(2)_{i,j} + s(3)_{i,j} + s(4)_{i,j}] \times tsCost + n(1) promoted_j \times tn(2)Cost + n(2) promoted_j \times tn(3)Cost \]

- Recruitment Costs
  \[ n(1) recruited_j \times rn(1)Cost_j + n(2) recruited_j \times rn(2)Cost_j \]

- Salaries
  \[ \text{Annual Salary}_j = \sum_i n(1)_{i,j} \times sn(1)Cost + \sum_i n(2)_{i,j} \times sn(2)Cost + \sum_i n(3)_{i,j} \times sn(3)Cost \]

- These are summed over years \( j \)
- They ensure that equalities hold where possible so they force workforce to just meet demand (on an FTE) basis
Outputs – Baseline

Practicing nurse population

- Total practicing nurses
- New immigrant practicing nurses

Year


Workforce population

2020| 1360| 1360| 2125| 0| 391| 40|
Outputs – Baseline

Entry level management population

- Total entry level management
- Newly recruited entry level management
- Newly promoted entry level management

Yearly total student population

- Student(year 1)
- Student(year 2)
- Student(year 3)
- Student(year 4)
Outputs – Baseline

Nurse manager population

- Total nurse managers
- New immigrant nurse managers
- New promoted nurse managers

Senior nurse manager population

- Total senior nurse managers
- New promoted senior nurse managers
Possible Scenarios for “What if?” Analysis

- Vary nurse to population ratio
- Vary proportion of students that continue the program after each year of study
- Vary nurse to manager ratios and nurse attrition rates
- Change maternity leave policies
- Change demand for nurses
- Change attrition rates
- ...

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Effect of Attrition on Recruitment Volumes

Average Number of Nurses Recruited per Year as a Function of Attrition Rates

<table>
<thead>
<tr>
<th>Attrition from the profession</th>
<th>Baseline scenario</th>
<th>10% decrease</th>
<th>20% decrease</th>
<th>30% decrease</th>
<th>40% decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry level management</td>
<td>300</td>
<td>200</td>
<td>120</td>
<td>75</td>
<td>46</td>
</tr>
<tr>
<td>Direct care nurse</td>
<td>900</td>
<td>1000</td>
<td>840</td>
<td>720</td>
<td>630</td>
</tr>
</tbody>
</table>

- Baseline scenario: 300 nurses
- 10% decrease: 200 nurses
- 20% decrease: 120 nurses
- 30% decrease: 75 nurses
- 40% decrease: 46 nurses

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Outputs – Decreasing Nurse/Population ratios over time

Yearly total student population

Practicing nurse population

Nurse leader population

Senior nurse leader population
Outcomes – Comparing Scenarios

Number of Students Admitted per Year

Student (year 1) admitted:
- Scenario 1 (Baseline)
- Scenario 2 (School Attrition)
- Scenario 3 (Nurses to Manager Ratio)
- Scenario 4 (Maternity Leave)
- Scenario 5 (Nurses to Population Ratio)

Student (year 3) admitted:
- Scenario 1 (Baseline)
- Scenario 2 (School Attrition)
- Scenario 3 (Nurses to Manager Ratio)
- Scenario 4 (Maternity Leave)
- Scenario 5 (Nurses to Population Ratio)
Research Challenges

- **End Conditions**
  - How do we ensure that finite horizon model is optimal for infinite planning horizon?

- **Stochasticity**
  - Changes in retirements; maternity leaves, student attrition, unfilled demand

- **Feedback**
Effect of End Conditions – 20 year model

Yearly Total Student Population

- s(1)
- s(2)
- s(3)
- s(4)
Changing Age Distribution (20 year Model)

Age Distribution of Direct Care Nurses per Year (First and Last Years)

- Age group
  - <25
  - 25-29
  - 30-34
  - 35-39
  - 40-44
  - 45-49
  - 50-54
  - 55-59
  - 60-64
  - 65+

Proportion of direct care nurses (%)

- Comparison between 2007 (yellow) and 2027 (striped)

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We have developed a tool for decision makers to examine HHR planning in the broadest context. The modeling approach extends to other healthcare professionals or levels of granularity.

References

Strategic Planning of Radiation Therapists at the BC Cancer Agency

Greg Werker, Martin Puterman, Mike Darud

- This study is funded by the Canadian Institute of Health Research (CIHR).
Motivation from an HR Perspective

- RTs want more predictability, involvement
- HR push to create longer-term plan
- In practice, not so easy to do
- Currently planning done by hand
How do we create a long-term plan for all radiation therapists for the next few years?
Radiation Therapy Task Areas

Mould Rm.
- Mould Rm.

Imaging
- CTSim1
- CTSim2
- CTSim3

Treatment
- Treatment
- Sim

Planning
- Dos1
- Dos2
- Dos3
- Dos4

Booking Room
- Booking
Sample Paths

**Sample path #1**
- Treatment (12)
- CTSim1 (1)
- CTSim2 (1)
- Mould Rm. (2)
- Dos1 (1)
- Dos2 (2)
- Dos3 (4)

**Sample path #2**
- Treatment (4)
- Sim (1)
- Treatment (8)
- CTSim1 (1)
- Mould Rm. (2)
- CTSim2 (1)
- CTSim3 (2)
Variables

\[ X_{T,A,P} = \begin{cases} 
1 & \text{if } T \text{ is assigned to } A \text{ in } P \\
0 & \text{otherwise} 
\end{cases} \]

\[ Y_{T,A,P} = \begin{cases} 
1 & \text{if } T \text{ begins a sequence in } A \text{ in } P \\
0 & \text{otherwise} 
\end{cases} \]

\[ H_{T,A,B,P} = \begin{cases} 
1 & \text{if } T \text{ has the necessary experience in area } B \text{ for } A \text{ in } P \\
0 & \text{otherwise} 
\end{cases} \]

Domains:

- \( T \): Therapist
- \( A \): Area
- \( B \): Area required Before
- \( P \): Period
Constraints

Initial Position
Coverage
One Area Only
Min Duration
Has Experience
**Initial Position**

- **$INIT_{T,A} = 1$ if $T$ is initially (currently) in $A$; 0 otherwise**

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(Excel interface only displays the 1s, by showing which A each T is initially in.)
Constraint: Initial Position

\[ X_{T,A,\text{"Q0"}} = INIT_{T,A} \quad \forall T, A \]
**Total Requirements**

- $TOTREQ_A = \text{total number therapists required in area A}$

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(Excel interface and GAMS model actually allow total requirements to be specified by period)
Constraint: Coverage

\[ \sum_{T} X_{T,A,P} \geq TOTREQ_{A} \quad \forall A, P \]

Note; this is only constraint between therapists
Constraint: Therapist can work in only one area each period

\[ \sum_{A} X_{T,A,P} = 1 \quad \forall T, P \]
Minumum (and Maximum) Duration

- $MINDUR_A = \text{minimum number of consecutive periods a therapist must work in an area } A \text{ if begins work in that area}$
- $MAXDUR_A = \text{used in soft constraint for maximum number consecutive periods}$
Constraint: Min Duration

\[ \sum_{q=P}^{P+MINDUR_A-1} X_{T,A,q} \geq MINDUR_A \times Y_{T,A,P} \quad \forall T, A, P \]

\[ X_{T,A,P} - X_{T,A,P-1} \leq Y_{T,A,P} \quad \forall T, A \quad \forall P > 0 \]
Total Requirements

- $STEXP_{T,B} = \text{starting experience levels for each therapist in area B}$

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(12 is the maximum required quarters of treatment experience; similarly 1 for CTsim1, 1 for CTsim2, 3 for CTsim3...)
Total Requirements

- $\text{EXP}_{A,B} = \text{experience required in area B before being allowed to work in area A}$

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(e.g., to work in Dos3, one must have 12 quarters treatment experience, 1 quarter in Dos1, and 2 quarters in Dos2)
Constraint: Has Experience

\[ H_{T,A,B,P} \times EXP_{A,B} \leq STEXP_{T,B} + \sum_{q=1}^{P} X_{T,B,q} \]

\[ STEXP_{T,B} + \sum_{q=1}^{P} X_{T,B,q} - (EXP_{A,B} - 0.5) \leq H_{T,A,B,P} \times M \]

\[ X_{T,A,P} \leq H_{T,A,B,P} \]

all three constraints are defined...

∀ T, A, P and B (for which B is required for A)
Initial Position

\[ X_{T,A,\text{"Q0"}} = INIT_{T,A} \quad \forall T, A \]

Coverage

\[ \sum_T X_{T,A,P} \geq TOTREQ_A \quad \forall A, P \]

One Area Only

\[ \sum_A X_{T,A,P} = 1 \quad \forall T, P \]

Min (and Max) Duration

\[ \sum_{q=P}^{P+MINDUR_A-1} X_{T,A,q} \geq MINDUR_A \cdot Y_{T,A,P} \quad \forall T, A, P \]

\[ X_{T,A,P} - X_{T,A,P-1} \leq Y_{T,A,P} \quad \forall T, A \quad \forall P > 0 \]

Has Experience

\[ H_{T,A,B,P} \cdot EXP_{A,B} \leq SEXP_{T,B} + \sum_{q=1}^{P} X_{T,B,q} \]

\[ SEXP_{T,B} + \sum_{q=1}^{P} X_{T,B,q} - (EXP_{A,B} - 0.5) \leq H_{T,A,B,P} \cdot M \]

\[ X_{T,A,P} \leq H_{T,A,B,P} \]

the above 3 constraints are defined...

\[ \forall T, A, B, P \quad B \text{ is required for } A \]
Objective Function

- Possibly just a constant – i.e. find a feasible solution, or
- A combination of penalties and rewards – goal is to violate as few soft constraints as little as possible
- Weights can be tweaked (*ultimately through Excel interface*)
More Components

• **Additional functionality:**
  • New hires
  • Part-time therapists
  • Maternity leave; sick leave

• **Soft constraints:**
  • Forced assignments
  • Max duration
### Excel Input

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#### Staff Planning Model

![Staff Planning Model](image)

(model will automatically be saved first)
Size of Problem and Solution

- 8-quarter model:
  - Total solution time < 30 seconds

- 12-quarter model:
  - 82,406 binary variables
  - 169,827 constraints
  - 782,023 Non-zeros
  - Total solution time = 34 minutes
    - GAMS using Cplex solver
    - 3.00 GHz Intel Core 2 (running on single CPU of a quad chip)
    - 16 GB ram
Model Overview
Challenges

- **Size of the model:**
  - *more than 12 quarters takes a very long time to solve*
  - *Necessary to avoid “end effects”*

- **Keeping IP formulation tight** (closer to LP relaxation):
  - The addition of part-time RTs caused a couple constraints to become less tight, which increases solution time.

- **Users are confused when minor changes to inputs lead to completely different plans**
  - Currently testing an objective function that minimizes changes compared to last model run.
Conclusions

- Tool enables planning of RTs over next several years
- Can be used on a rolling horizon basis
- It has been used this quarter!
- IP balances various inputs and finds “best” solution that violates as few soft constraints as possible
- Current focus is on getting the tool ready for use
- Two papers in preparation