Optimization Problems in Health Care

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CO@Work - October 7, 2009

Outline

- Introduction
- Canadian Health Care System
- Some general comments on using optimization models
- Surgeon Scheduling
- Priority Scheduling
- Radiotherapy Scheduling
- Workforce Planning
- Concluding Remarks

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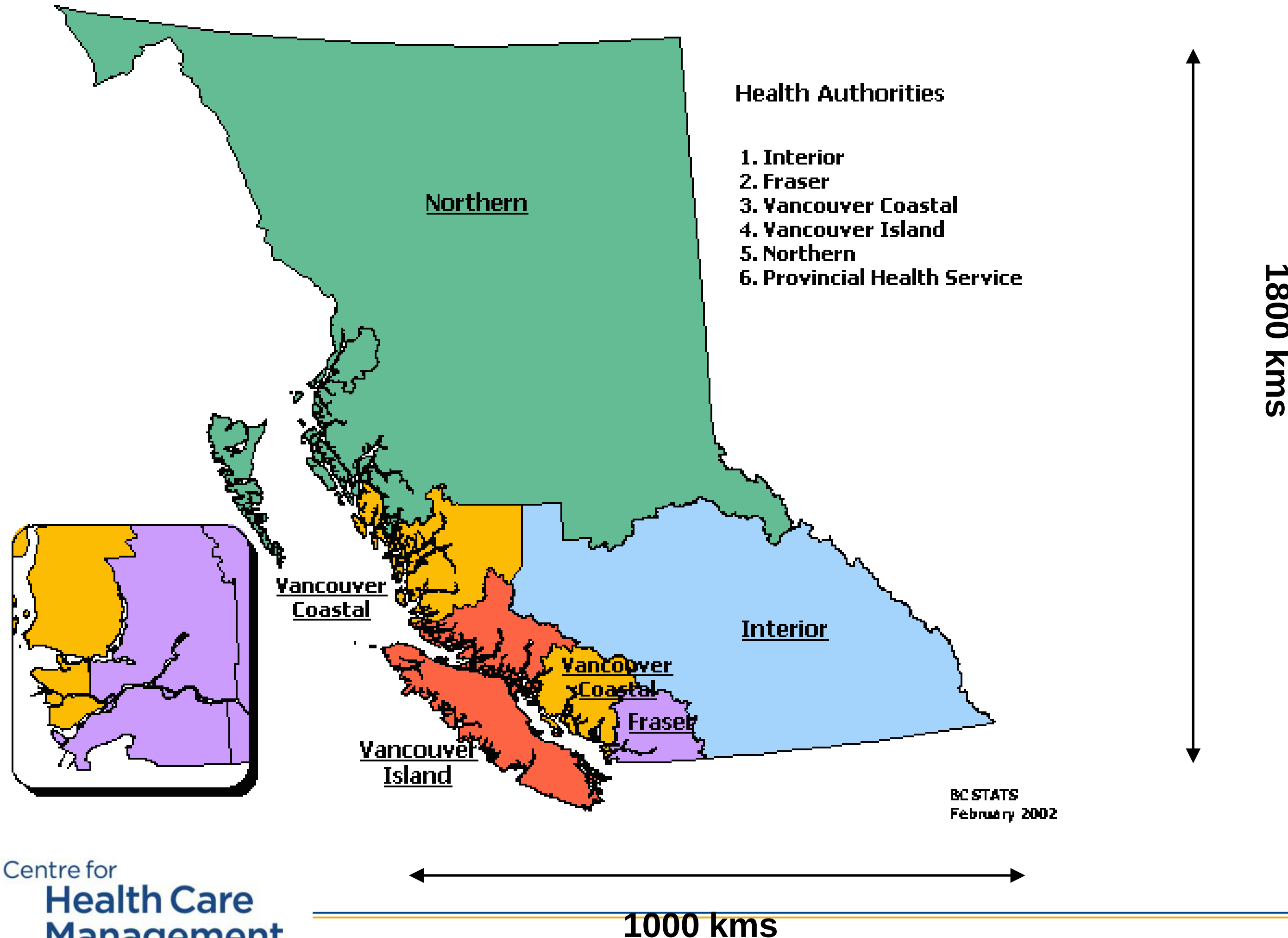
Health Care in BC and Canada

A Comparison

- British Columbia
  - Area 944,735 sq-km
  - Population 4.4 million
- Germany
  - Area 357,021 sq-km
  - Population 82 million

British Columbia Health Authorities



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Canada Health Act - Principles

- **Public Administration**
  - Health care insurance plans are to be administered and operated on a non-profit basis by a public authority, responsible to the provincial governments.
- **Comprehensiveness**
  - The health insurance plans of the provinces and territories must insure all hospital, physician, surgical-dental health services and, where permitted, services rendered by other health care practitioners.
- **Universality**
  - One hundred percent of the insured residents of a province or territory must be entitled to the insured health services provided by the plans on uniform terms….
- **Portability**
  - Residents moving from one province or territory to another must continue to be covered for insured health care services by the "home" province.
- **Accessibility**
  - Health care delivery must be provided on uniform terms and there must be no discrimination between clients based on age, lifestyle, or health status.

Canadian Health Care Background
CIHI – Health Care in Canada 2006

- In 2006, Canada spent $148 billion on health care or $4,411 per person
  - This represents approximately 10% of Canada's GDP
  - 30% is spent on hospitals; 17% on retail drugs
- 1.5 million people work in health care
  - 1 out of 10 Canadians work in health care
  - Nurses and Physicians are the largest groups

International Comparisons
OECD – 2006-2007 data

Country	Per capita expenditures (PP adjusted)	Government share of spending	Life expectancy (years) (2006)	Infant mortality rate (per 1000 births) (2006)
US	$7,290	47%	78.1	6.7
Canada	$3,895	70%	80.7	5.0
Germany	$3,588	77%	80.7	3.8
UK	$2,992	82%	79.3	5.0

Canadian Health System Challenges

- Wait times for services
- Aging population
- Aging and declining workforce
- Costly new technologies and therapies

Levers to Address These Challenges

- Add capacity
  - Physical Space
  - Staff
- Reduce demand
- Work more efficiently
  - Use existing capacity better
  - Allocate resources better
- Reduce variability in processes and arrivals

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OR in Health Care

Operations Research Methods

- OR Tools help to address these challenges
- Most widely used OR tools in health care
  - Simulation
  - Optimization
  - Statistical Models
  - Queuing Models
  - Dynamic Programming

Some problems where we have used optimization

- Surgical Scheduling and Bed Utilization
- Priority Patient Scheduling
  - Single Appointment (CT Scans)
  - Multiple Appointment (RT Treatment)
- Staff Planning
  - Long Term
  - Skill Acquisition
- Chemotherapy Scheduling

Other Health Care Optimization Problems

- Facility Location
- Radiotherapy Beam Direction Optimization
- Shift Scheduling
- Surgical Path Control
- ???

Optimization Challenges

- Identifying the problem
- Model formulation
  - Variables
  - Constraints
  - Objective function choice
- Data
- Solving to optimality
- Multiple optima
- Robust codes for applications

Overview of our modelling approach

- Observe and map process
- Formulate stochastic problem
  - Identify sources of variability
  - Constraints
  - Decision variables
  - Performance metrics
- Abstract key features and develop a deterministic optimization problem based on “averages”
- “Solve” optimization problem
- Evaluate through simulation
- Revise and repeat previous two steps
- Communicate results to practitioners

Reducing Surgical Ward Congestion Through Improved Surgical Scheduling

Challenges

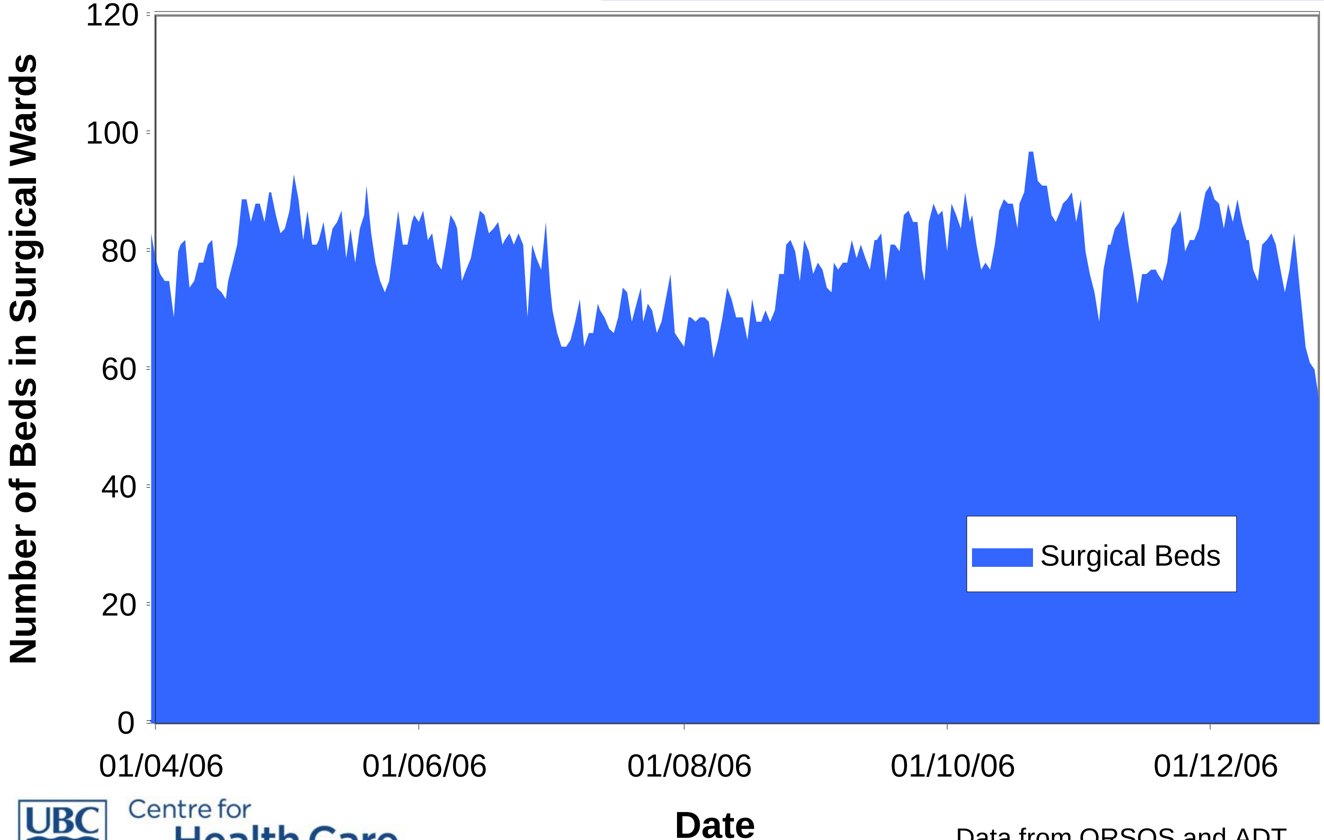
- Increasing surgical demand
- Fluctuating patient arrivals
- Cancellations due to lack of bed availability
- Competition for beds between “surgical” and “medical” patients
- Systematic variability in ward occupancy attributable to planned cases
- Surgery schedules designed and managed “by hand”

Painting the Picture…

Royal Jubilee Hospital, Victoria BC

(Interaction with medical patients

Variability within and across weeks / s



Data from ORSOS and ADT

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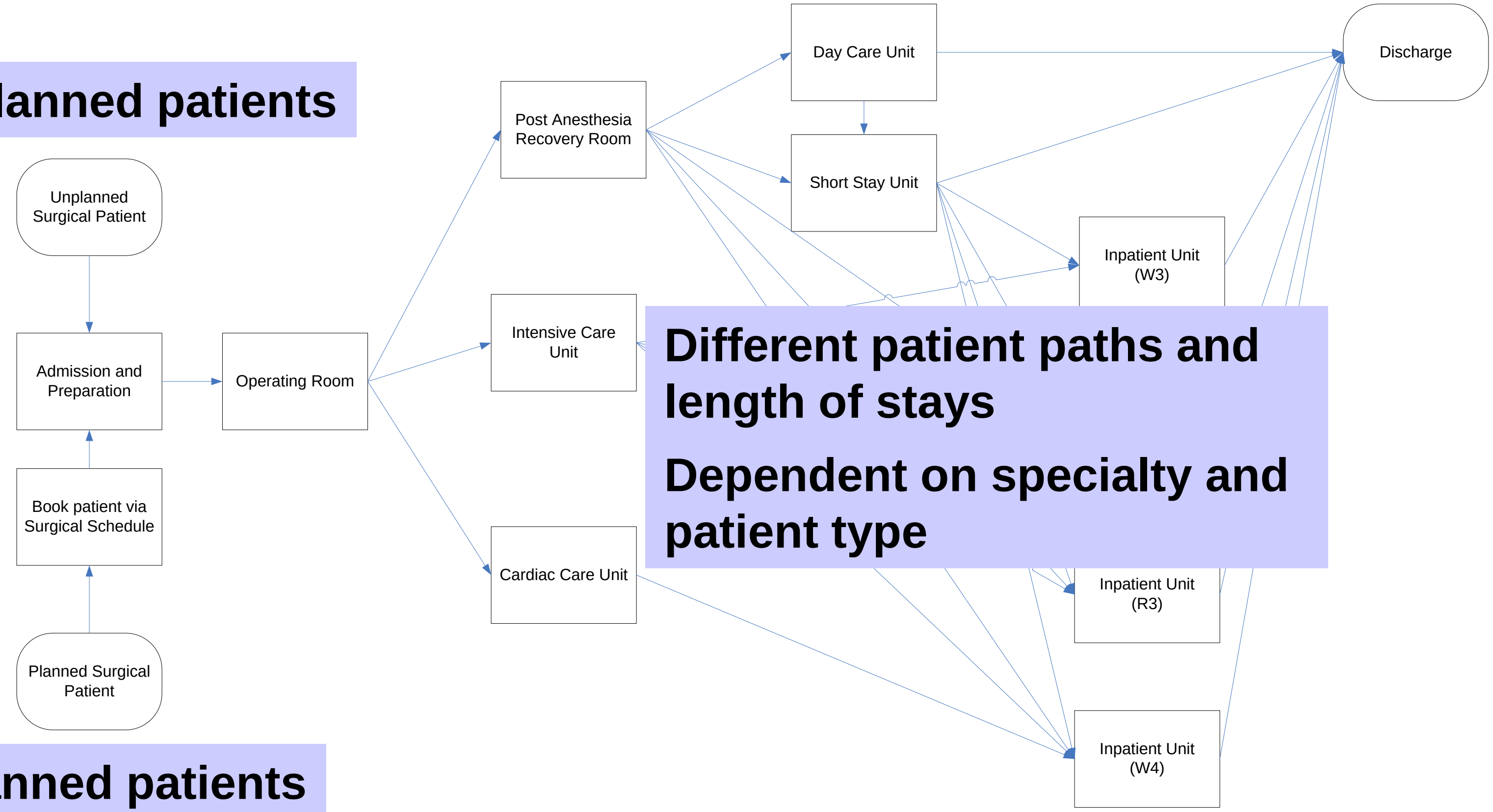
Patient Flow Map

Unplanned patients



Different patient paths and length of stays

Dependent on specialty and patient type

Planned patients

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Surgical Schedules

<table>
<tr><th colspan="2" rowspan="2">RJH</th><th colspan="5">WEEK 1</th></tr>
<tr><th>MONDAY</th><th>TUESDAY</th><th>WEDNESDAY</th><th>THURSDAY</th><th>FRIDAY</th></tr>
<tr><td rowspan="2">OR 1</td><td>AM</td><td rowspan="2">SURGEON 1</td><td rowspan="2">SURGEON 2</td><td rowspan="2">SURGEON 3</td><td rowspan="2">SURGEON 4</td><td rowspan="2"></td></tr>
<tr><td>PM</td></tr>
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<tr><td>PM</td><td>SURGEON 15</td></tr>
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- Downstream bed utilization depends on the surgeon and the type of cases selected (by the surgeon)
- Changing when surgeons operate can alter downstream ward utilization patterns (base model)
- Changing the mix of cases within a surgical block can further alter downstream ward utilization patterns (slate model)

Our Solution

- Bed Utilization Simulator (BUS)
  - Excel based
  - Uses historical patient flow patterns and cases
  - Uncapacitated
    - Given a surgical schedule it computes downstream bed utilization assuming all cases are assigned to appropriate wards
  - Potentially usable by client
- Surgical Schedule Optimizer (SSO)
  - Assigns surgeons (and slates) to day-of-week and week within cycle
  - Mixed integer program
  - Requires expert input
- Evaluate SSO output through BUS

Bed Utilization Simulator: Model Logic

Preserves the “within patient” correlations and through random sampling of cases, eliminates “between patient” correlations

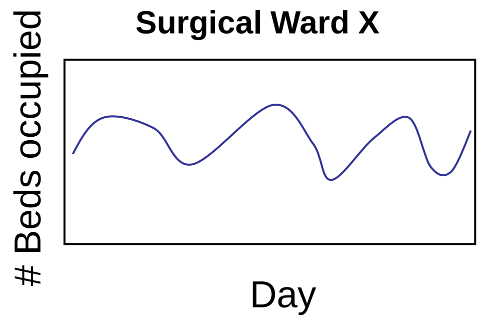
Create planned arrivals

SURGICAL SCHEDULE 01/01/2007 - 05/30/2007		WEEK 1				
		MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
OR 1	AM	SURGEON 1	SURGEON 2	SURGEON 3	SURGEON 4	SURGEON 1
	PM	SURGEON 1	SURGEON 2	SURGEON 3	SURGEON 4	SURGEON 1
OR 2	AM	SURGEON 5	SURGEON 7	SURGEON 8	SURGEON 9	SURGEON 10
	PM	SURGEON 5	SURGEON 7	SURGEON 8	SURGEON 9	SURGEON 10
OR 3	AM	SURGEON 11	SURGEON 12	SURGEON 13	SURGEON 14	SURGEON 15
	PM	SURGEON 11	SURGEON 12	SURGEON 13	SURGEON 14	SURGEON 15

Randomly select from historical records

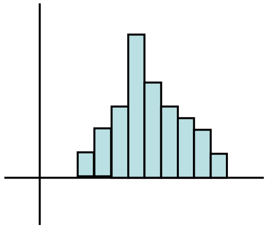
Patient path…
Length of stay…

Generate Output



Estimate the Daily Demand for Beds in each Ward

Create unplanned arrivals



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BUS Screenshots

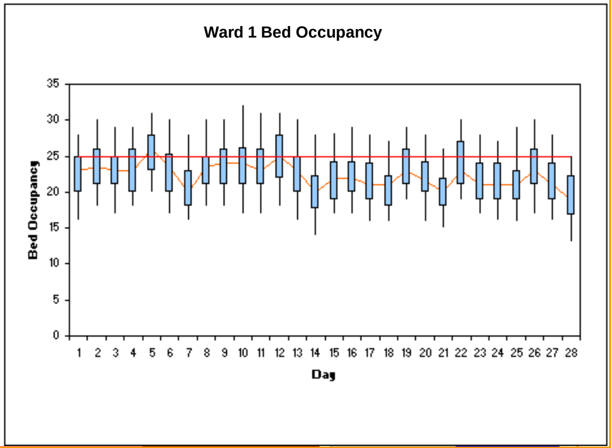
Schedule Input Interface

Simulation Output

Main Menu

Ward 1 — Capacity: 25

Days	Min	0.25 Percentile	0.05 Percentile	0.5 Percentile	0.95 Percentile	0.75 Percentile	Max	Average
1	13	20	16	23	28	25	30	22.24
2	15	21	17.95	23.5	30	26	32	23.56
3	14	21	17	23	29	25	32	23.17
4	13	20	18	23	29.05	26	34	23.15
5	17	23	20	26	31	28	38	25.79
6	15	20	16.95	23.5	30	25.25	33	23.3
7	12	18	16	20	28	23	32	20.84
8	12	21	17.95	23.5	30	25	36	23.32
9	14	21	18	24	30.1	26	35	24.05
10	10	21	17	24	32	26.25	35	24.11
11	12	21	16.95	23	31	26	36	23.32
12	15	22	18	25	31	28	38	24.76
13	14	20	16	23	30	25	34	22.83
14	11	17.75	14	20	28	22.25	30	20.33
15	12	19	16.95	22	28.05	24.25	32	21.83
16	12	20	17	22	29	24.25	34	22.55
17	13	19	15.95	21	28	24	30	21.5
18	13	18	15.95	21	27	22.25	29	20.61
19	17	21	18.95	23	29	26	31	23.55
20	13	20	15.95	21.5	28	24.25	30	21.97
21	12	18	15	20	26	22	28	19.94
22	17	21	18.95	23	30.05	27	34	23.84
23	16	19	17	21	28	24	30	21.54
24	15	19	16	21	27.05	24	33	21.74
25	13	19	15.95	21	29	23	34	21.41
26	15	21	17	23	30.05	26	35	23.52
27	14	19	16	21	28	24	34	21.74
28	12	16.75	13	19	25	22.25	34	19.8
Bed Days Over Capacity	0	0	2	11	26.25	72.25	109	19.4
Range (Min to Max)	5	9	11	13	14	18.05	21	12.65
Max - Average	2.39	3.885714286	5.321428571	6.21428571	7.22321429	8.905357143	11.96	6.31893
Median	17.5	18.975	21	22.5	24	27	29.5	22.485



28		3997	7726	26097	23478	5212		4532
29	DC	1	4	0	8	0	DC	
30	SS	0	1	3	1	0	SS	
31	SDSA	2	0	1	0	3	SDSA	
32	OR6	LEE SHUNG	PATHAK IRVIN	PIERCY G BRUCE	O'NEILL MICHAEL O s	POMMERVILLE PETER J	OR6	O'NEILL MICHAEI

Interface

Draw AutoShapes

Ready

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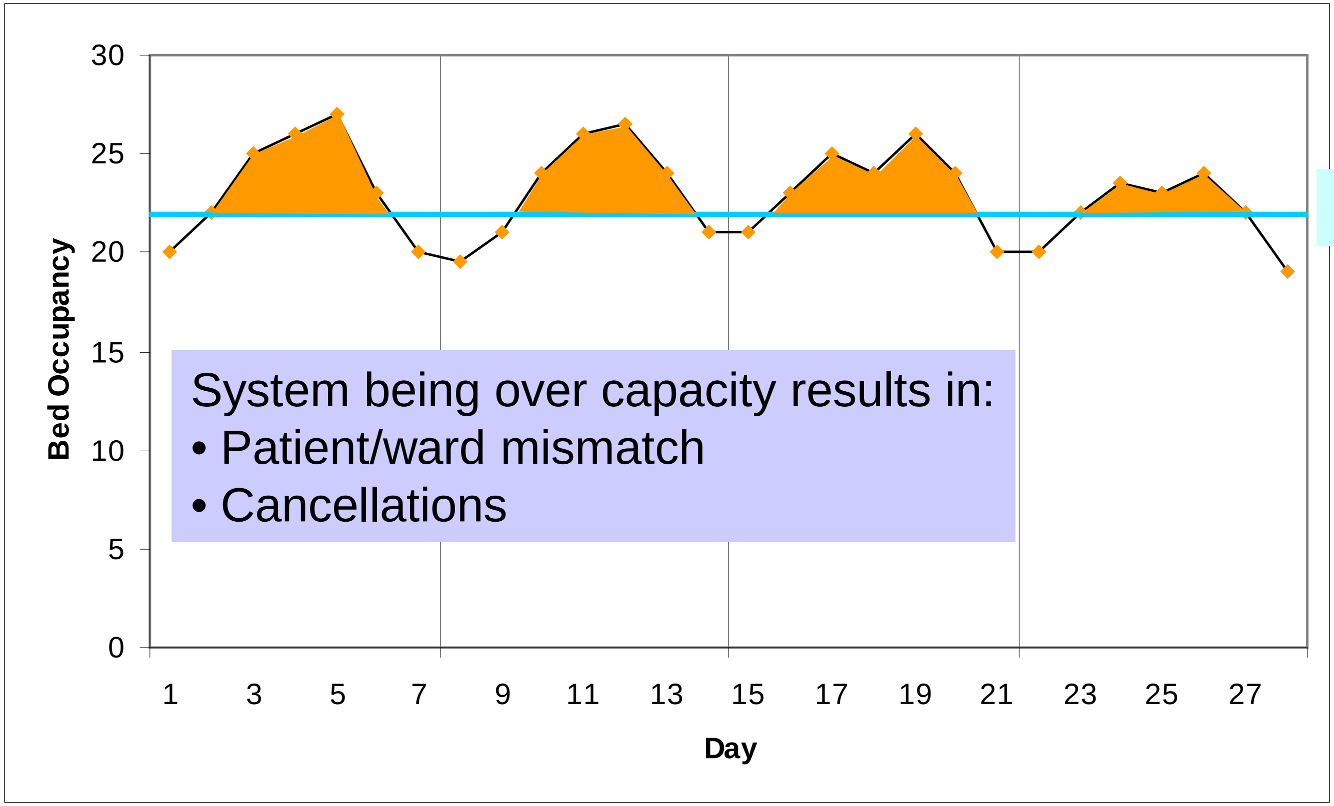
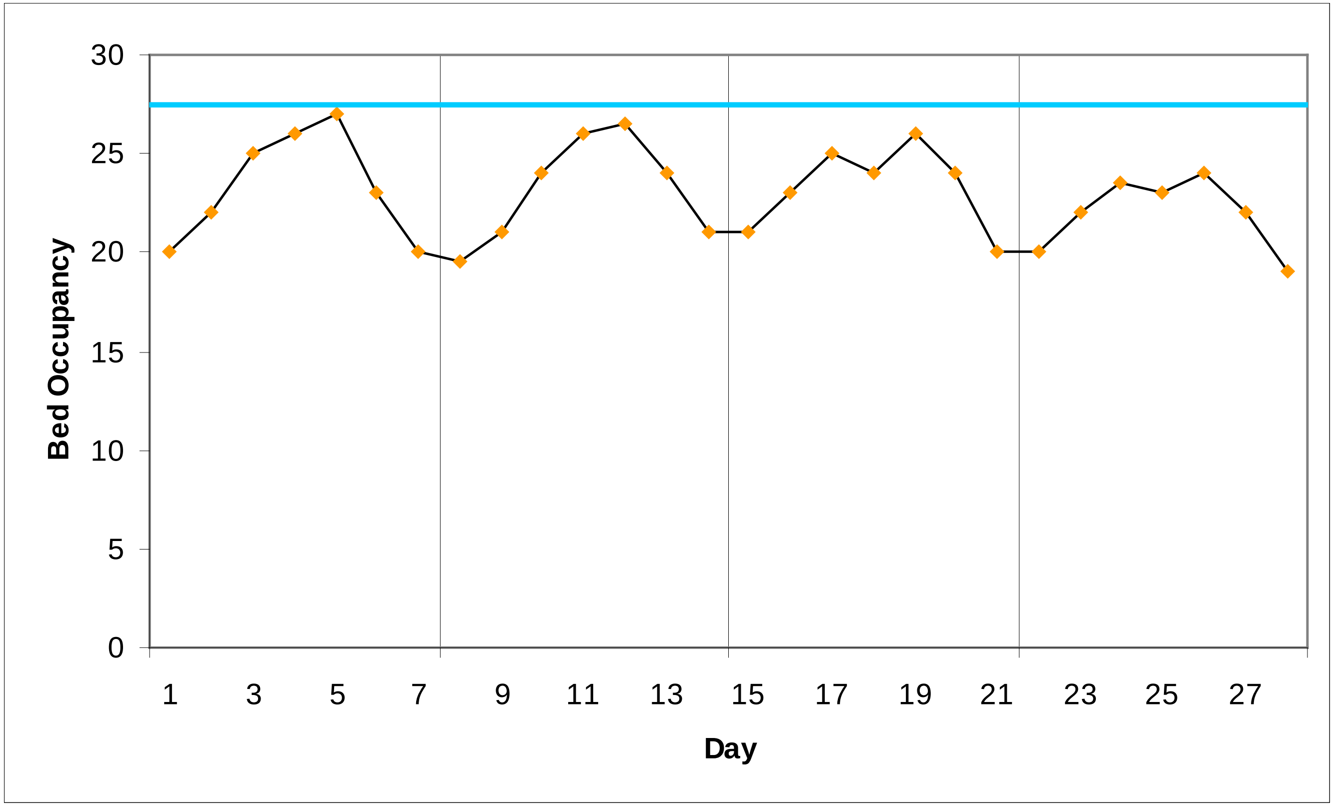
A potential objective: bed days over capacity

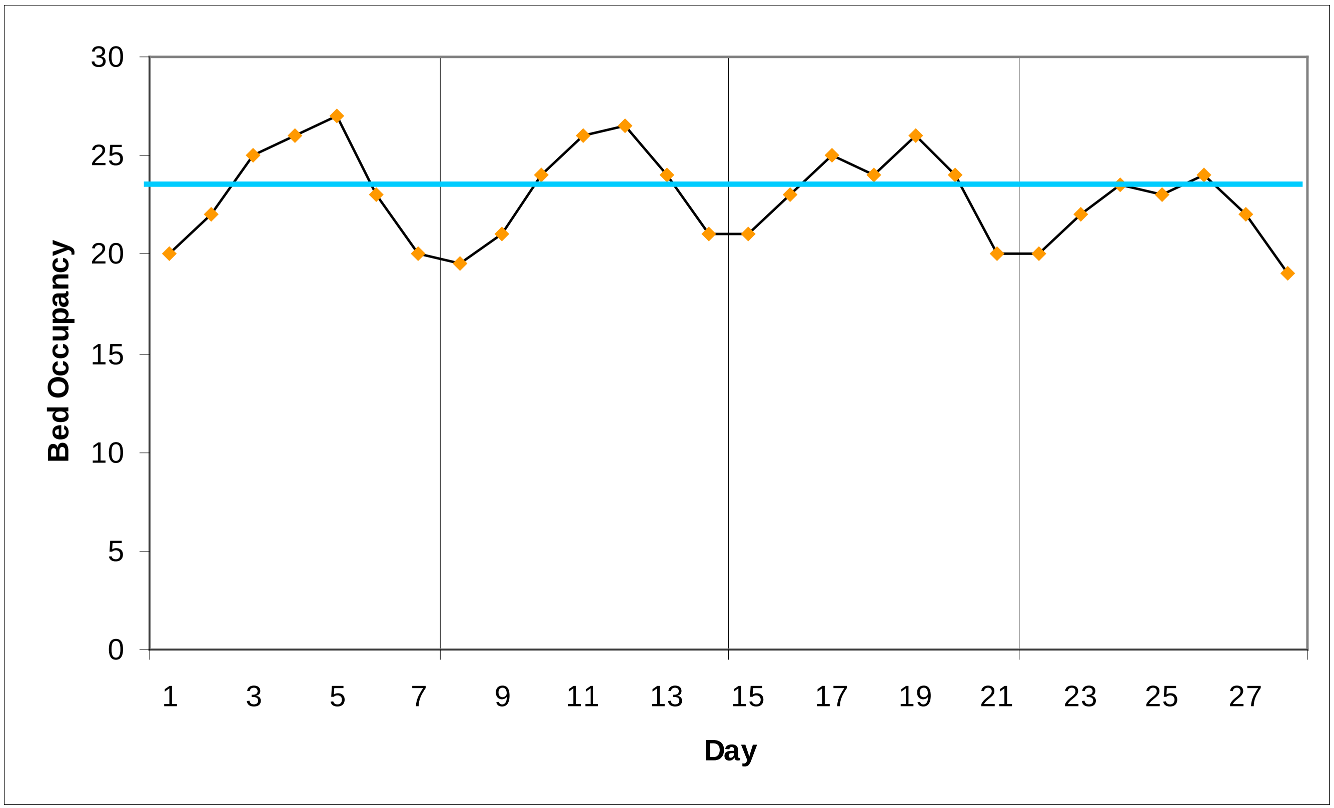


Another objective – maximum ward utilization



Maximum Utilization

Another objective – variance in ward utilization



Mean Utilization

Surgical Scheduling Optimizer

RJH		WEEK 1				
		MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
OR 1	AM	SURGEON 1	SURGEON 2	SURGEON 3	SURGEON 4	
	PM					
OR 2	AM	SURGEON 5	SURGEON 6	SURGEON 8	SURGEON 9	SURGEON 10
	PM		SURGEON 7			
OR 3	AM	SURGEON 11		SURGEON 12	SURGEON 13	SURGEON 14
	PM					SURGEON 15

- Assigns surgeons to blocks to smooth bed occupancies over wards
- Optimizes over a four week block schedule

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Optimization Model Formulation: Base Model

- Mixed Integer Optimization
- Decision
  - Which surgeon block to schedule to which day on which week over a multi-week cycle.
- Objective
  - Minimize the total over wards of the maximum bed occupancy per ward
- Constraints
  - Daily OR capacity
  - Daily specialty capacity
  - Daily and weekly surgeon capacity
  - Ensure all blocks are allocated

Optimization model inputs

- Number of surgeons (47), blocks (74)
- Number of ORs available each day (determined by policy)
- Blocks per week per surgeon and speciality (determined by policy)
- Number of ORs required per surgeon (1/2, 1 or 2)
- Number of cases per block for a surgeon (historical records)
- **Average** ward bed utilization pattern (number of days and which wards) for a patient type for a particular surgeon (8 wards)
- In the model a *block* refers to a surgeon – OR duration mix
  - Block 27 might correspond to surgeon 13 being assigned to an OR for a whole day while block 28 might correspond to surgeon 13 being assigned to an OR for a half day.

Model Formulation (base model)

Decision Variables

$X_b^{i,w}$: $\begin{cases} 1 \text{ if block (surgeon) } b \text{ is scheduled on day } i \text{ of week } w \\ 0 \text{ otherwise} \end{cases}$

MD_u : Maximum number of beds in use in ward *u* over the scheduling period

Constraints

Total OR capacity: $\sum_b X_b^{i,w} \cdot NumOR_b \le ORperDay^{i,w} \qquad \forall i,w$

OR capacity for each surgeon: $\sum_{b \in B(d)} X_b^{i,w} \cdot NumOR_b \le ORperDaySurgeon_d^{i,w} \qquad \forall i,w,d$

Surgical blocks for each week: $\sum_i X_b^{i,w} \le WeekBlock_b^w \qquad \forall b,w$

Maintain same number of blocks: $\sum_w \sum_i X_b^{i,w} = TotalBlock_b \qquad \forall b$

Maximum bed utilization across the scheduling period in each ward:

$$\sum_w \sum_i \sum_b \sum_p X_b^{i,w} \cdot NumCases_b^p \cdot Bed_b^{i,j,w,p,u} \le MD_u \qquad \forall j,u$$

Objective

Minimize the summation of the maximum bed occupancy in each surgical ward: $Min \sum_u MD_u$

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SSO Modification: Slate Optimization

RJH		WEEK 1				
		MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
OR 1	AM	SURGEON 1	SURGEON 2	SURGEON 3	SURGEON 4	
	PM					
OR 2	AM	SURGEON 5	SURGEON 6	SURGEON 8	SURGEON 9	SURGEON 10
	PM		SURGEON 7			
OR 3	AM	SURGEON 11		SURGEON 12	SURGEON 13	SURGEON 14
	PM					SURGEON 15

DC	2
SS:	0
SDSA:	1

or

DC	3
SS:	1
SDSA:	0

An additional choice of "Slates" for each block

Model Formulation (slate model)

- Additional decision
  - What slate to select in which block
- Objective
  - Remains the same
- Additional constraints
  - Select one slate per block
  - Total patient volume for each surgeon must be greater than equal to historical volumes
- Our approach to modeling specifies only a few (2) possible slates per surgeon
- Note: the model selects slates and blocks for a surgeon

Model Formulation (Slate model)

Decision Variables Changes

$X_b^{i,w,s}$: Replaces $X_b^{i,w}$ $\begin{cases} 1 & \text{if block } b \text{ is scheduled on day } i \text{ of week } w \text{ with slate } s \\ 0 & \text{otherwise} \end{cases}$

Constraint Changes

Addition: Choose a at most one slate: $\sum_s X_b^{i,w,s} \leq 1 \qquad \forall i, w, b$ (8)

Addition: Number of cases must be at least equal to historical volumes:

$$\sum_w \sum_i \sum_{b \in B(d)} \sum_s X_b^{i,w,s} \cdot NumCases_b^{p,s} \geq TotalCases_d^{\,p} \qquad \forall p, d \tag{9}$$

Replaces (1): $\sum_b \sum_s X_b^{i,w,s} \cdot NumOR_b \leq ORperDay^{i,w} \qquad \forall i, w$ (10)

Replaces (2): $\sum_{b \in B(d)} \sum_s X_b^{i,w,s} \cdot NumOR_b \leq ORperDaySurgeon_d^{\,i,w} \qquad \forall i, w, d$ (11)

Replaces (3): $\sum_{b \in B(y)} \sum_s X_b^{i,w,s} \cdot NumOR_b \leq ORperDayUrol^{i,w} \qquad \forall i, w$ (12)

Replaces (4): $\sum_i \sum_s X_b^{i,w,s} \leq WeekBlock_b^{\,w} \qquad \forall b, w$ (13)

Replaces (5): $\sum_w \sum_i \sum_s X_b^{i,w,s} = TotalBlock_b \qquad \forall b$ (14)

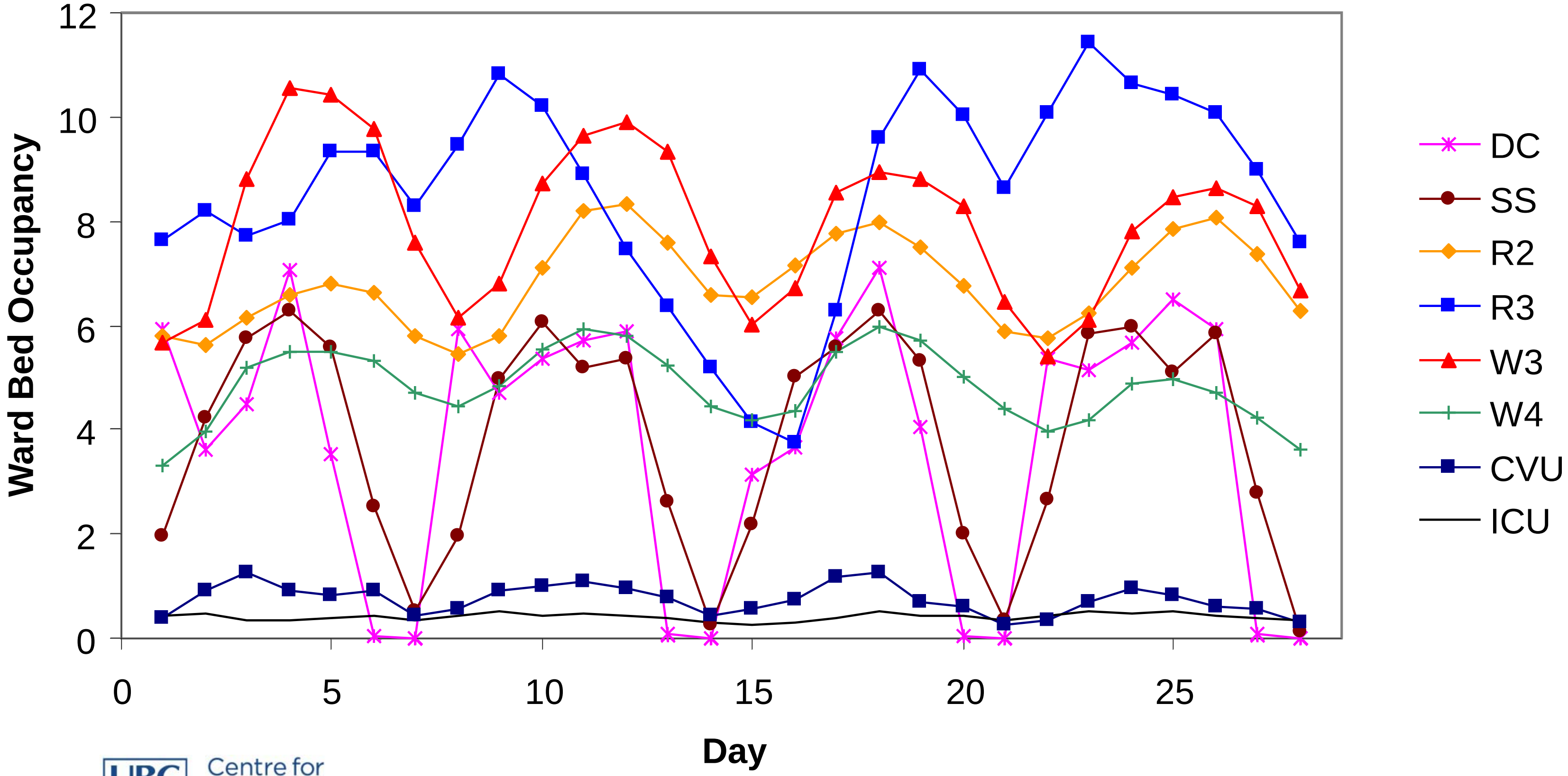
Replaces (6): $\sum_w \sum_i \sum_b \sum_p \sum_s X_b^{i,w,s} \cdot NumCases_b^{p,s} \cdot Bed_b^{i,j,w,p,u} \leq MD_u \qquad \forall j, u$ (15)

Objective

Remains the same as (7) $Min \sum_u MD_u$

Base Model Result

Average peak occupancy decreases by 8.5 beds



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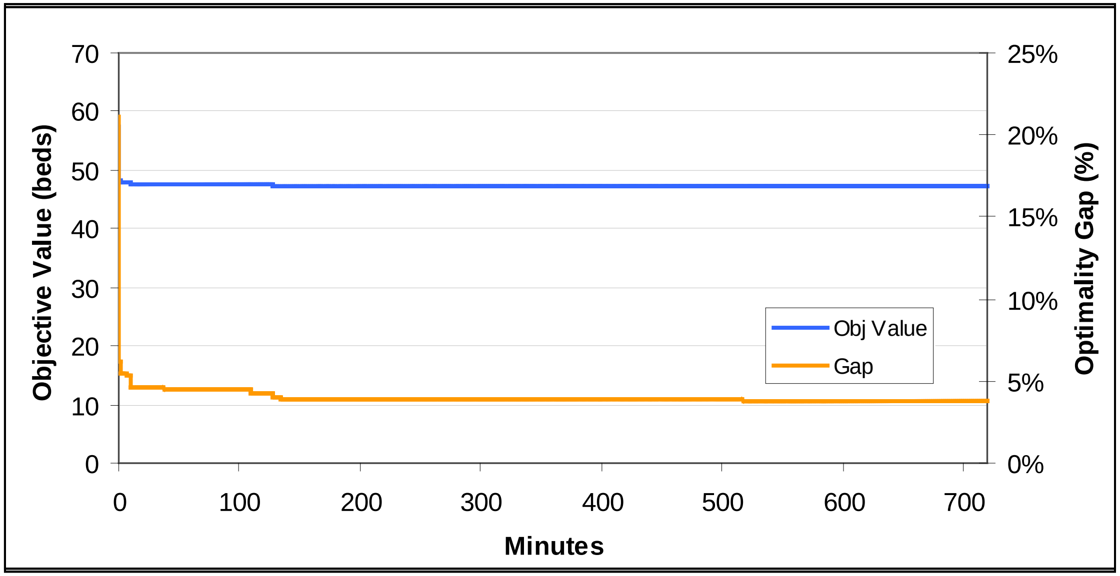
Computation

- Base model -1500 decision variables and 1600 constraints
- Slate model – 3000 decision variables and 3200 constraints
- Tested model on two platforms:
  - Frontline Premium Solver Platform with Xpress Solver Engine
  - GAMS and Cplex 11

Computation: Cplex



Mipemphasis = 2 (for optimality)
Probing = 3 (full probing)
Intel Q6600 (2.4ghz - 4 threads)

- Feasible and good schedules with smoothed bed utilization obtained within minutes
- Optimal solution: ??
- Similar results for slate model

Some results based on BUS evaluation

- Base Model
  - Reduced bed-days over capacity by 16% or 13 cases over a four week period on average.
    - Consequence – avoid up to 13 patient redirections or cancellations
- Slate Model
  - Increased surgical throughput by 15 cases per 4 week period
  - Reduced bed days over capacity by 10%.
  - Note there was additional constraint on volumes

Useful Scheduling Guidelines

- SSO challenges
  - Difficult for non technical users
    - Non-optimality
    - Infeasibility?
  - Considerable coordination, upkeep, and re-optimization
  - Long computation time – cannot reach true optima
- Developed scheduling guidelines to immediately impact practice and ensure sustainability
  1. Schedule blocks based on both specialty and patient mix
  2. For inpatient wards: schedule blocks with high patient volumes and long stay requirements at the beginning and end of the week
  3. For short stay wards (closed on weekends) schedule blocks with high demand for ward beds on Mondays and Wednesdays

Reference

- Vincent S. Chow, Martin L. Puterman, Neda Salehirad, Wenhai Huang and Derek Atkins “Reducing Surgical Ward Congestion through Improved Surgical Scheduling ” POMS – Under 2nd review

Priority Scheduling

Vancouver General Hospital

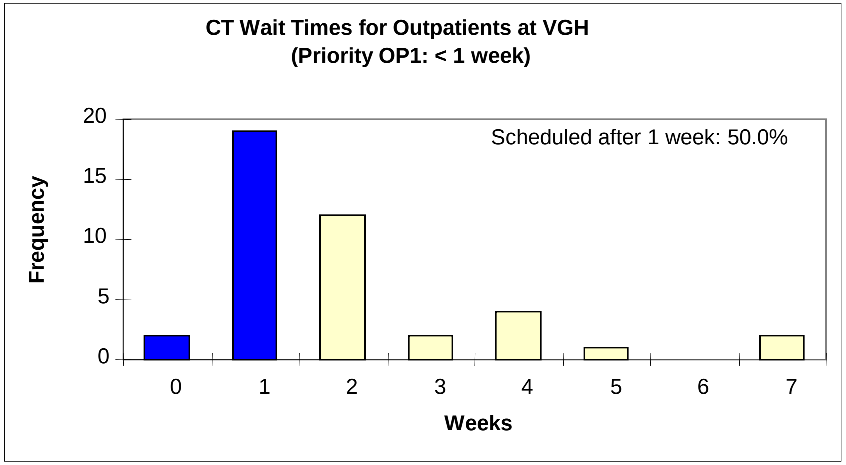
- 950 Bed Tertiary Care Hospital
- Specialties
  - Alzheimer Disease
  - Arthritis
  - Bone Marrow Transplant and Leukemia
  - Burns and Plastics
  - Epilepsy Surgery Program
  - Immunology
  - Magnetic Resonance Imaging (MRI)
  - Multiple Sclerosis
  - Oncology
  - Ophthalmology
  - Organ Transplant
  - Orthopedics
  - Psychiatry
  - Rehabilitation
  - Sleep Disorders
  - Spinal Cord Injury
  - Sports Medicine
  - Trauma Services

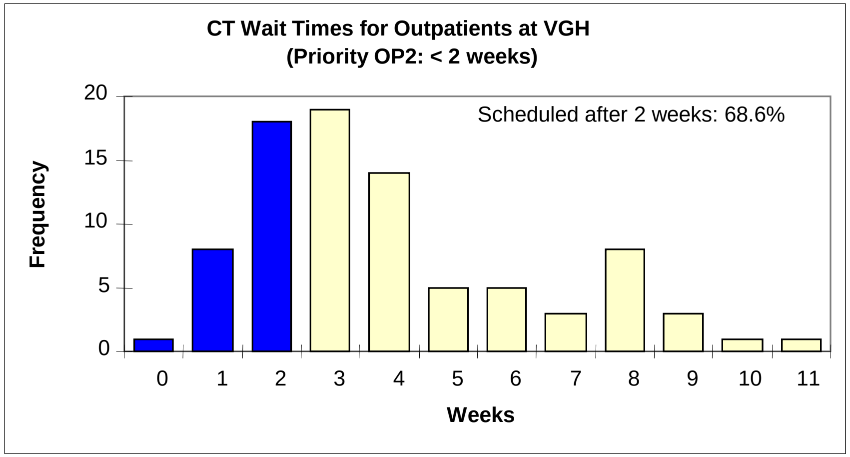
Reducing Wait Times for CT Scans

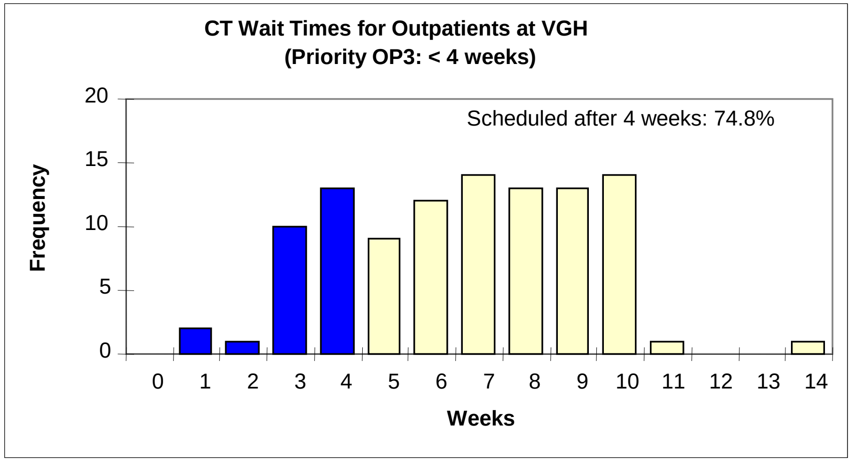
- Research motivated by a project with Vancouver General Hospital.
- Purpose of the project:
  - Determine current waiting times for CT scans
  - Recommend methods to reduce wait times
- Outcomes:
  - Determined current waiting times
  - Cost-benefit analysis of possible options for increasing throughput
  - Identified the need to improve scheduling of CT scan appointments
  - Reviewed and suggested improvements to patient transport system operations

The problem; Outpatient Waiting Times

Outpatient Categories	OP1	OP2	OP3
Recommended WT (RWT)	< 1 wk	< 2 wks	< 4 wks
Actual WT			
Average (wks)	1.6	3.6	6.3
Max	6.6	10.4	13.9
Min	0.0	0.0	0.1
Sample Size	42	86	103
% scanned after RWT	50.0%	68.6%	74.8%







Data source: VGH 2-week data, based on scheduled wait time

Scheduling Diagnostic Imaging

Demand for a diagnostic resource comes from multiple patient priority classes

- Emergency Demand
  - Immediate service
- Inpatient Demand
  - Maximum 24 hour waiting time
- Outpatient Demand
  - Can be scheduled days or weeks in advance depending on priority

Dynamic Scheduling Problem Formulation

- Demand for a resource comes from *I* different priority classes with maximum recommended waiting time, T*(i).*
- Demand arrives randomly with known probability distribution.
- Resource manager may book up to *N* days in advance (booking horizon).
- Books once a day for all accumulated demand.
- Daily capacity C_1 patients. *(Base Capacity)*
  - Related research question – how to set C_1
- At most, C_2 patients can be served through overtime. (*Surge Capacity*)
  - Here we assume $C_2=\infty$

Given the number of available slots each day in the future and the total amount of observed demand from each priority class, at what future day should each patient be scheduled?

Modeling Approach

Formulate as an infinite horizon discounted MDP

Convert MDP to an LP

Problem: too many variables and constraints

Approximate value function

Reduces # of variables

Problem: Still too many constraints

Solve dual via column generation

Obtain optimal approximate value function

Obtain optimal actions as needed

MDP Components

- Primitives
  - Decision Epochs
  - States
  - Actions
  - Rewards
  - Transition Probability
  - Planning Horizon
- Derived Quantities
  - Decision Rules
  - Policies
  - Stochastic Processes
- Comparing Policies - Optimality Criterion
  - Expected Total (discounted) Reward
  - Long Run Average Reward

Solving MDPs

- Solving = finding optimal policy
  - This is accomplished through finding the value function *v(s)*
- The Bellman (optimality) Equation

$$v(s) = \max_{a \in A_s}\{r(s,a) + \lambda \sum_{s \in S} p(j \mid s,a) v(j)\}$$

- Algorithms
  - Policy Iteration and variants
  - Value Iteration
  - Linear Programming
- Challenge v(s) may have many^n components

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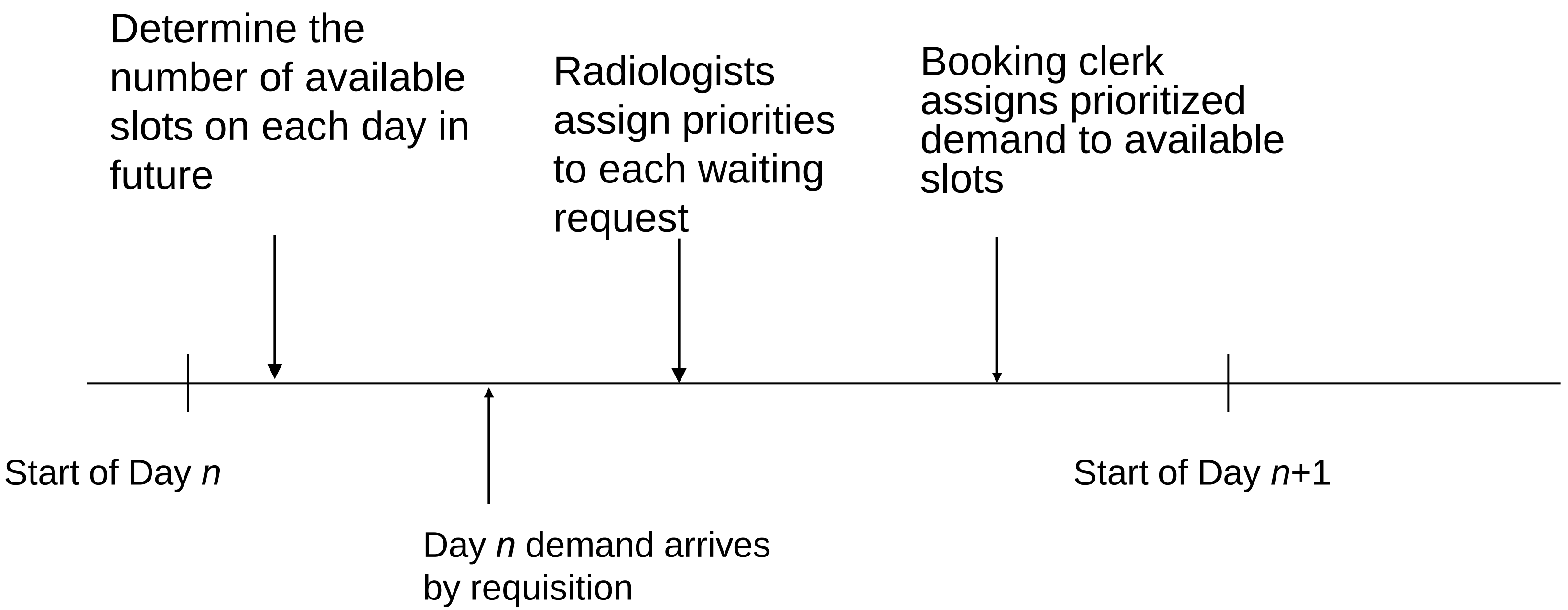
MDP vs. Approximate DP

- **MDP**
  - Produces policy in terms of a lookup table.
  - Obtain directly from dual LP solution.
  - Gives optimal action in each state in advance.
  - Not practical if state space is large.
- **Approximate DP**
  - Produces an approximate value function.
  - Only determine "optimal" action as needed.
  - Must solve optimization problem in each state.
  - Action need not be optimal

Application revisited: Decision Epochs



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States

$$(\vec{x}, \vec{y}) = (x_1, \ldots, x_N, y_1, \ldots, y_I)$$

where

- x_n is the number of patients already booked on day n ($x_n \leq C_1$, where C_1 is the daily base capacity)
- y_i is the number of patient waiting to be booked from priority class i

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Actions

- Which day to book unscheduled demand from each priority class (represent by an I x N matrix a).
- How many patients of each priority class to "remove" from the queue (represented by a vector z) through overtime
  - With no capacity on overtime we assume it is done ASAP.
- These can be expressed as a series of linear constraints

Costs

$$c(\vec{a},\vec{z}) = \sum_{i,n} c(i,n)a_{in} + \sum_{i} h(i)z_i$$

where $c(i,n) = k(i)[n - T(i)]^+$

$k(i) =$ the cost per day for delaying the scan of a priority class i patient.

$h(i) =$ the per unit overtime cost.

- $k(i)$ is decreasing in i.
  - It is more costly to delay the scan of a higher priority patient
- $h(i)$ is constant for all i independent of the priority class
  - *Note in other formulations* $h(i)$ might be decreasing in i.

Transition Probabilities

- No patient booked more than N days in advance
- No patient choice
- Rolling N day horizon

Only stochastic element in the transition is the new demand which arrives according to known probability distribution.

Transition Probabilities

$$(\vec{x},\vec{y}) = (x_1,\ldots,x_N, y_1,\ldots,y_I)$$

$$\rightarrow \left(x_2 + \sum_{i=1}^{I} a_{i2},\ldots., x_N + \sum_{i=1}^{I} a_{iN}, 0; d_1,\ldots., d_I \right)$$

Where *d* is the new demand vector.

The only stochastic element is the new demand.

LP Formulation of the MDP

The optimal value, $v(x,y)$, function of an MDP can be obtained by solving the LP:

$$\max \sum_{\vec{x},\vec{y} \in S} \alpha(\vec{x},\vec{y}) v(\vec{x},\vec{y})$$

subject to

$$c(\vec{x},\vec{a},\vec{z}) + \gamma \sum_{\vec{d} \in D} \left[p(\vec{d}) v \left(x_2 + \sum_{i=1}^{I} a_{i2}, \ldots, x_N + \sum_{i=1}^{I} a_{iN}, 0; d_1, \ldots, d_I \right) \right]$$
$$\geq v(\vec{x},\vec{y}) \quad \forall (\vec{a},\vec{z}) \in A_{\vec{x},\vec{y}} \text{ and } (\vec{x},\vec{y}) \in S$$

Solution independent of α!

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‘Curse of Dimensionality’

For any reasonably sized facility, size of the state space is too large to allow an exact numerical solution!

$$\textit{Number of states} \quad = C_1^N I^M$$

For example N=30, C_1=15, I = 3, M=10 (maximum daily demand in each class) has $15^{30}3^{10}$ states!

An Affine Approximation

We reduce the number of variables in the LP by approximating the value function.

We consider an affine approximation:

$$v(\vec{x},\vec{y}) = W_0 + \sum_{n=1}^{N} V_n x_n + \sum_{i=1}^{I} W_i y_i$$

- V_n can be interpreted as the marginal cost associated with an additional booked slot on day *n*.
- W_i can be interpreted as the marginal cost associated with having one more priority *i* patient in the queue.

Approximate LP

Assuming α is a probability distribution and $C_2 = \infty$:

$$\max_{\vec{V},\vec{W}} \left\{ W_0 + \sum_{n=1}^{N} E_\alpha[X_n] V_n + \sum_{i=1}^{I} E_\alpha[Y_i] W_i \right\}$$

subject to

$$(1-\gamma)W_0 + \sum_{n=1}^{N} V_n \left(x_n - \gamma x_{n+1} - \gamma \sum_{i=1}^{I} a_{i,n+1} \right) + \sum_{i=1}^{I} W_i \left(y_i - \gamma E_\alpha[Y_i] \right)$$
$$\leq c(\vec{a}, \vec{z}) \quad \forall (\vec{a}, \vec{z}) \in A_{\vec{x},\vec{y}} \text{ and } (\vec{x}, \vec{y}) \in S$$

$$\vec{V}, \vec{W} \geq 0$$

Solution depends on α!

Column Generation Overview

1. Obtain an initial feasible set, *S'*, of state-action pairs for the dual
2. Solve the dual over *S'*
3. Solve the Reduced Costs problem (i.e. find the most violated constraint in the primal)
4. Add the state-action pair that produces the most negative reduced cost into *S'*
5. Repeat steps 2-4 until the reduced costs are all positive (minimum in 3. is positive) or you are close enough.

The Form of the Optimal Approximate Value Function

- Solved dual numerically through column generation. (numerous times)
- ***Surprising Result***: We determined the optimal form of the approximate value function in terms of the problem data without solving the approximate LP.
  - Especially important since costs are arbitrary

Optimal Solution to Approximate LP

Case 1: $h(i) = h$ **for all** i:

$$V_n = \begin{cases} h & n \le T(1) \\ \gamma V_{n-1} & \forall\, T(1) < n < N \\ 0 & n = N \end{cases}$$

$$W_i = V_{T(i)} \quad \forall\, i$$

$$W_0 = h\left(\gamma \sum_{i=1}^{n} \frac{\gamma^{T(i)-T(1)}}{1-\gamma} \lambda_i - T(1) C_1 - \frac{\gamma C_1}{1-\gamma} \right)$$

The optimal solution form holds under the following conditions

(1) $$k(i)[n-T(i)]^{+} + \gamma^{n-T(1)}h > \gamma^{T(i)-T(1)}h$$

(2) $$\sum_{i=1}^{I} \frac{\gamma^{T(i)-n}}{1-\gamma} I_{T(i)>n}\lambda_i + \sum_{m=n}^{N} \gamma^{|m-n|^{+}} E_{\alpha}[X_n] < \frac{C_1}{1-\gamma}$$

(3) $$0 < \sum_{i=1}^{I} \frac{\gamma^{T(i)-T(1)}}{1-\gamma}\lambda_i + \sum_{n=1}^{N} \gamma^{[n-T(1)]^{+}} E_{\alpha}[X_n] - T(1)C_1 - \frac{\gamma C_1}{1-\gamma} < \frac{C_2}{1-\gamma}$$

for $n > T(1)$; $i \geq 1$

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Interpreting the conditions

1. It is more costly to schedule a priority *i* patient n-T(i) days late *and then* do OT than it is to do OT immediately
2. On any day n, there is sufficient capacity to meet the average demand of any priority.
3. Total expected demand and initial bookings exceeds total available capacity (over infinite horizon) and there is sufficient OT capacity to stabilize demand.
   - This means base capacity is constrained.

Note if constraints are violated; LP still has solution but not of the form above!

- If lower bound (3) is violated, V(x,y) = 0 for all (x,y).

Outline of Proof

- Assume optimal primal solution has the proposed form under the given conditions
- Establish primal feasibility
- Construct a feasible dual solution that, together with proposed primal solution, satisfies complementary slackness

Finding the Optimal Action

- If we had been able to solve the LP directly then the optimal policy could have been derived from the dual solution
- Since we solved an approximate LP via column generation we don't know values of all dual variables.
    - Find optimal action *as needed* by solving a one step greedy integer program using the right hand side of the Bellman equation with the optimal approximate value function obtained from the LP substituted in (see next slide).
    - Can be evaluated in each state as needed in practice or in a simulation.
    - When system has above structured solution the optimal policy has a nice form.

Optimal Action Optimization Problem

$$\min_{(a,z)\in A_{x,y}} \left\{ \sum_{i,n} B_{i,n} a_{i,n} + D_i z_i + \text{constant} \right\}$$

- Clearly only choose $a_{i,n} > 0$ if $B_{i,n} < 0$ and $z_i > 0$ if $D_i < 0$
- Under Conditions (1) - (3)
  - $B_{i,n}$ is increasing in i
  - $B_{i,n}$ is convex in n with minimum at n=T(i)
  - $B_{i,n} \geq 0$, n > T(i)+1 and $B_{i,T(i)} = 0$
  - D_i increasing in i
  - And the paper provides additional conditions which identify when coefficients above are negative.

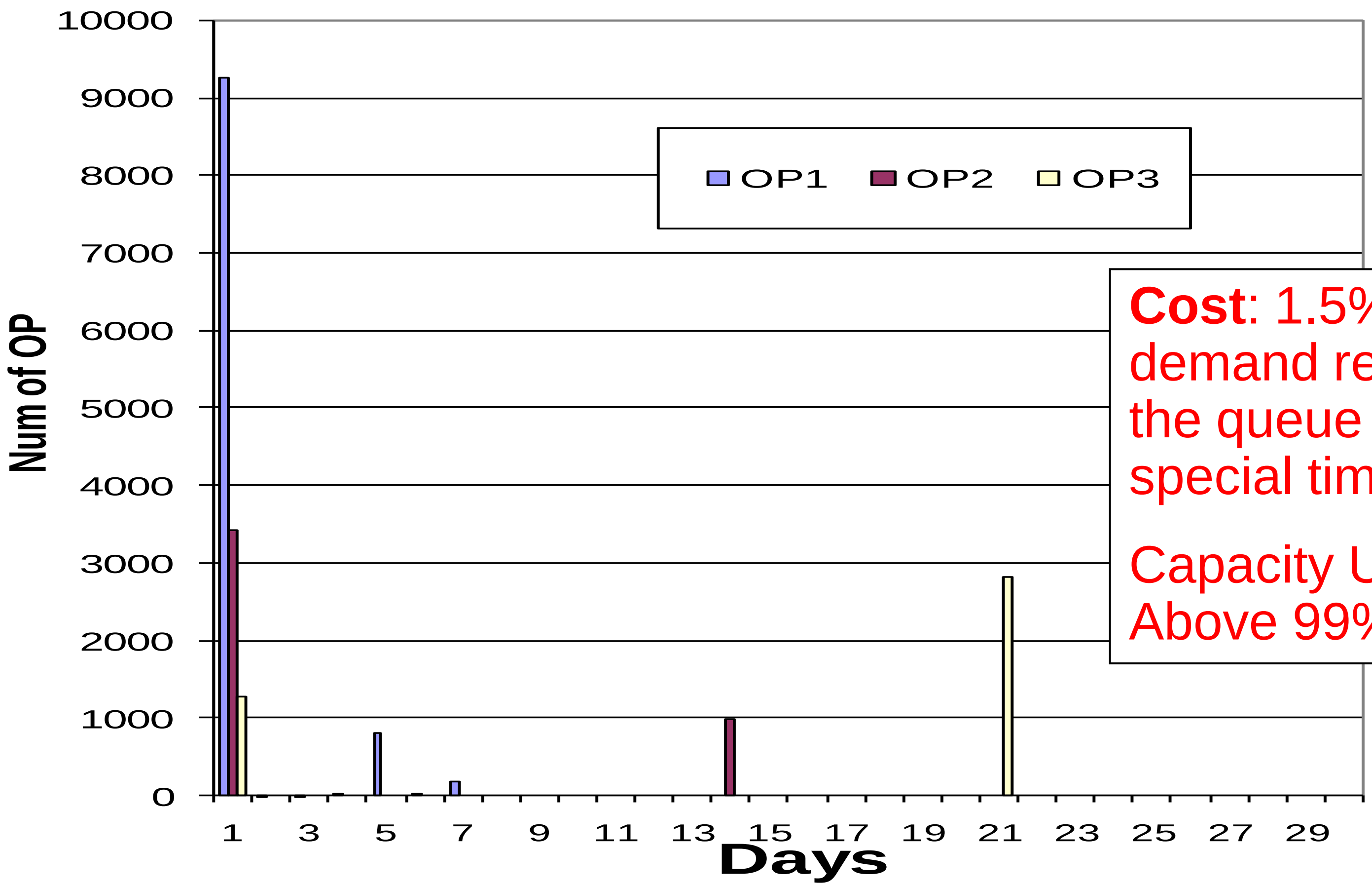
Optimal Scheduling Policy

- Schedule demand in priority order.
- Fill any open slots in tomorrow's slate.
- Priority 1
  - Schedule as much Priority 1 (P1) demand as possible no later than day *T(1).*
  - Book any outstanding P1 demand through OT
- Priority j; j >1.
  - Do for j = 2, …,I
  - Book outstanding Pj demand prior to day T(j)
  - Book any outstanding Pj demand through OT

A Simulation Study

- Length 20,000 days
- Warm up period of 5000 days
- Initial system empty (or randomly generated)
- Coded in AMPL in order to be able to solve the optimization problem
- Initial Parameter Values
  - 3 outpatient priority classes
    - Arrival rates of 5, 3 and 2.
    - Maximum recommended waiting times of 7, 14 and 21.
    - Daily late penalties of 3, 2 and 1.
    - Overtime cost 37.
    - C_1=10, $C_2 = \infty$
  - Booking horizon, *N* = 30.
  - Discount rate, $\gamma = 0.99$

Policy Performance: Simulation



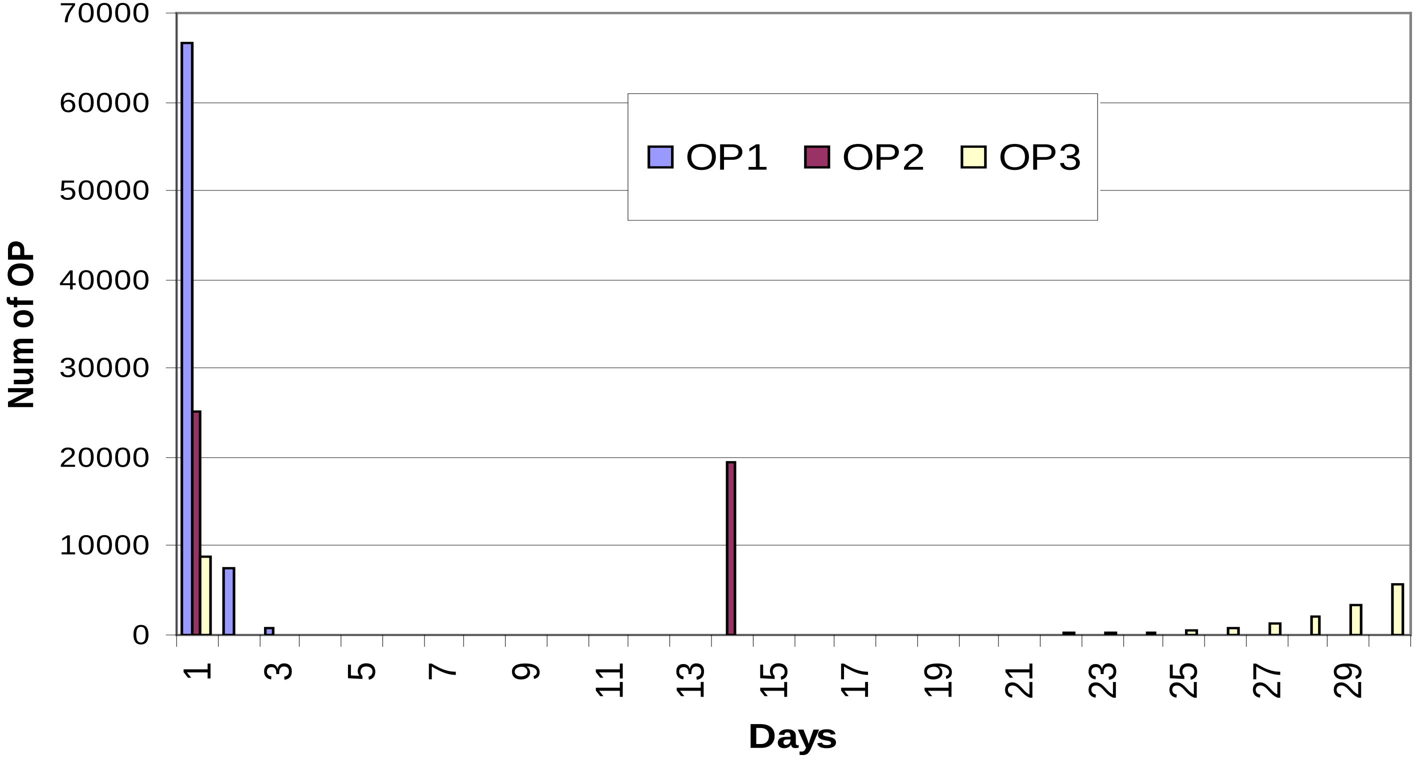
Cost: 1.5% of OP1 demand removed from the queue but only at special times

Capacity Utilization: Above 99%

Comparative Policies

- Fixed Reservation Policy
  - Apply a static daily booking limit to the lower priority classes (i.e. strict reservation policy)
  - Determine optimal booking limits through search
- "Overtime as a Last Resort":
  - Initially assign appointments in same way as "optimal policy"; then schedule patients at earliest day possible over reminder of booking horizon; then use overtime only if absolutely necessary.
  - Somewhat similar to current practice

Waiting Time by Priority Class: Booking Limit Policy



Cost: 21% of OP3 demand overtime, 50% of booked OP3 late.

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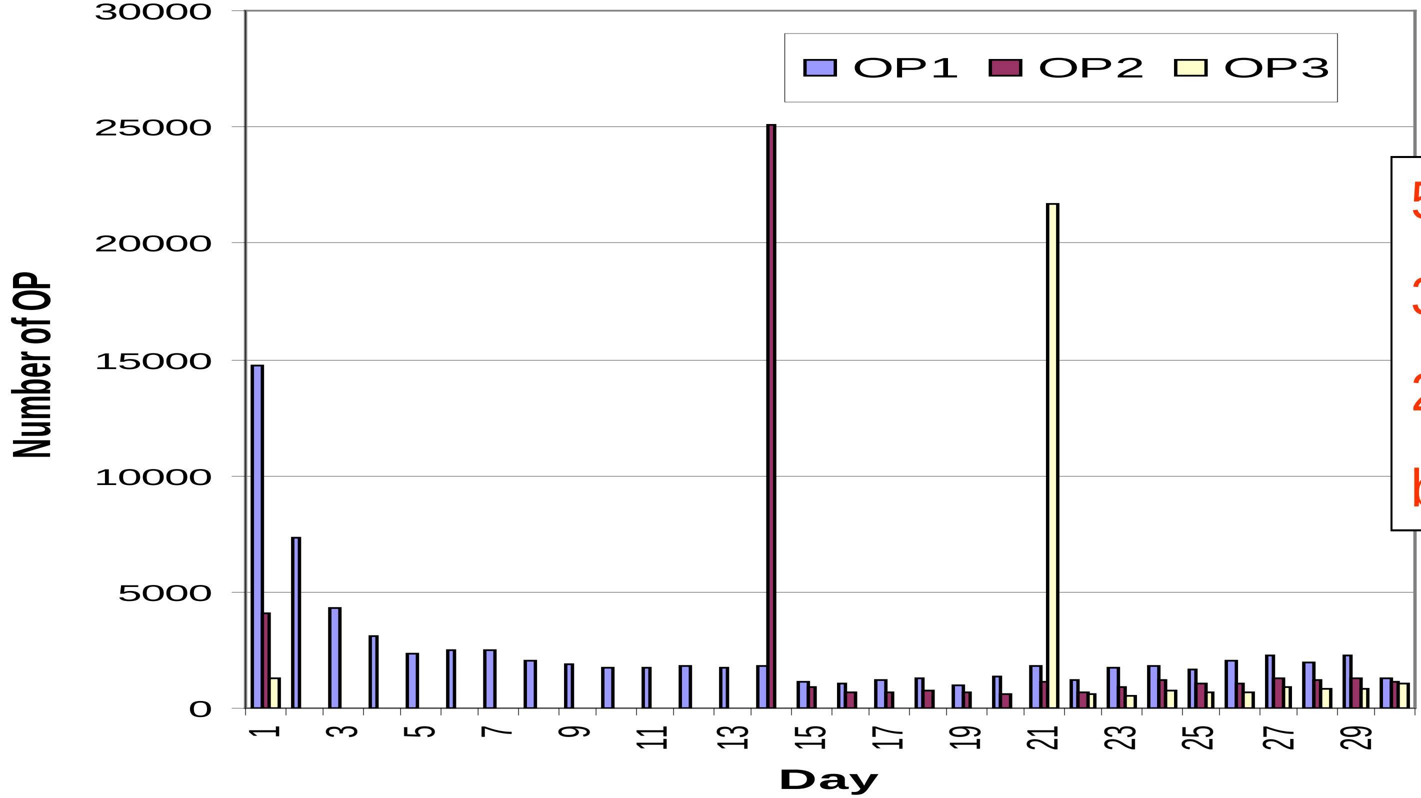
Waiting Time:
'Overtime as Last Resort' Policy



53% of OP1,

36% of OP2,

25% of OP3

booked late!

Comparative Results

	% of Patients with late scans				% of Patients served through OT			
	OP1	OP2	OP3	Total	OP1	OP2	OP3	Total
Overtime Policy	0	0	0	0	1.44	0	0	0.72
Booking Limit	0	0.02	49.52	9.94	0	0	21	4.13
Remove as last resort	53.17	35.77	24.85	42.29	0.08	0.43	0	0.17

Measuring the Optimality Gap

- Available bounds are generally not tight
- Objective function in the approximate LP is a lower bound on total discounted cost; value depends on initial system configuration
  - If initial system full, results within 18% (±3.08%) of bound
  - If initial system half full, results within 30% (±8.87%) of bound
  - True optimality gap likely less
- Convex approximation yields the same linear approximation
- Exact solution of small model and then regression on state variables suggests
  - Linear approximation does not fit perfectly
  - Some interaction terms are significant
  - Linear approximation underestimates value in low capacity states and over-estimates it in high capacity states.

Policy Insights

- In a system where demand is approximately equal to capacity, the judicious use of a small amount of overtime coupled with a good patient scheduling policy can have a significant impact on maintaining reasonable waiting times
- OT gives the resource manager the ability to deal with spikes in demand
  - Without this ability the next spike in demand simply compounds the problem
  - Booking demand later and later merely compounds the problem: best to confront problem directly through judicious use of overtime

Further Research: Extensions of the Model

- Variations of the model
  - No shows
  - Multiple service times
  - Random service times
  - Piece-wise linear OT costs
  - Customer choice
  - Demand dependent on expected waiting time
- Applicability to other settings
  - Multiple hospital setting
  - Cancer radiotherapy treatment
  - Surgery
- Probabilistic Analysis of System
- Determining appropriate Base and Surge Capacity

Further Research: Theoretical Issues in ADP

- Effect of α on policy
  - Varying α leads to what other policies?
- Bounds on “cost” of approximation
  - Traditional DP bounds?
  - Temporal difference methods?
  - LP geometry?
- Determining the “best” approximation
  - Can we iteratively update the chosen form?

Extension to RT Treatment Scheduling

RT treatment

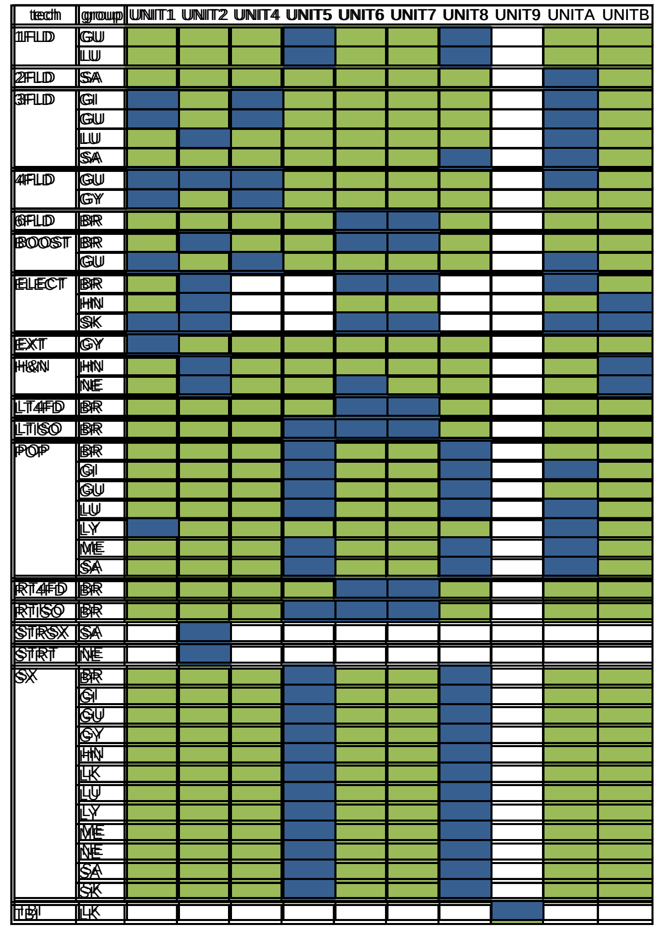
- Characteristics:
  - Delivered in daily fractions (1 to 35 consecutive sessions)
  - Each treatment fraction usually lasts from 12 to 36 minutes
  - All fractions are delivered in the same treatment unit
  - Each unit provides a set of treatment techniques

Treatment specifications

Specifications
Cancer Group
Treat. Technique
Urgency Level
Treat. Dist.
Earliest start date

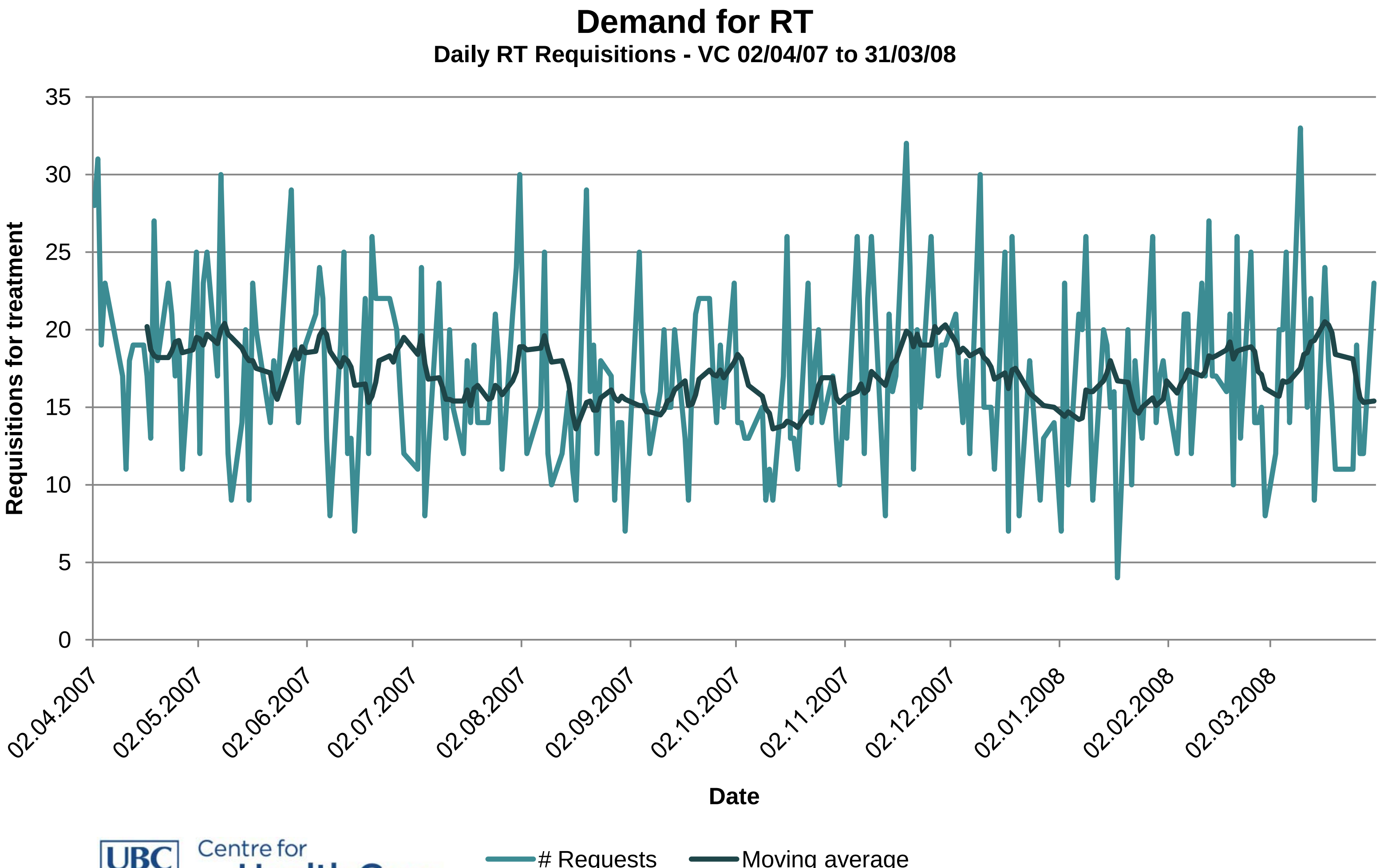
↓

Specifications
Treat. Units
Urgency Level
Treat. Dist.
Earliest start date



Technical Constraints
Preferences

Demand for treatment



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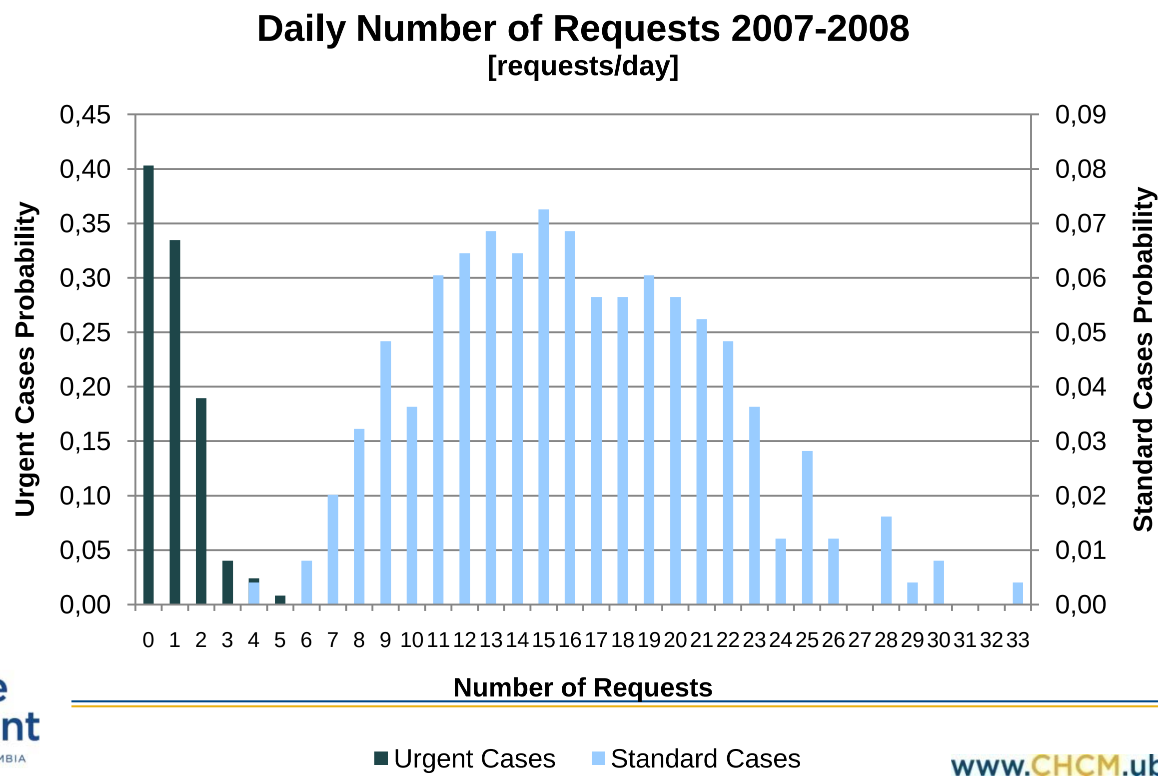
Source: Scheduling Warehouse, BCCA

Treatment specifications

Urgency level	Agency target	Provincial target
Urgent	24 to 48 hours	--
Emergent	3 days	--
Standard	wait list	--
Overall	2 weeks	4 weeks

Specifications
Cancer Group
Treat. Technique
Urgency Level
Treat. Dist.
Earliest start date



≈ 1 [req./day] ≈ 16 [req./day]

Treatment specifications

Specifications
Cancer Group
Treat. Technique
Urgency Level
Treat. Dist.
Earliest start date

Fraction	Date	Duration
1	05-Dec-07	36
2	06-Dec-07	24
3	07-Dec-07	24
4	10-Dec-07	24
5	11-Dec-07	24
6	12-Dec-07	24
7	13-Dec-07	24
8	14-Dec-07	24
9	17-Dec-07	24
10	18-Dec-07	24
11	19-Dec-07	24
12	20-Dec-07	24
13	21-Dec-07	24
14	27-Dec-07	24
15	28-Dec-07	24

Fraction	Date	Duration
1	10-Apr-07	24
2	11-Apr-07	12
3	12-Apr-07	12
4	13-Apr-07	12
5	16-Apr-07	12
6	17-Apr-07	12
7	18-Apr-07	12
8	19-Apr-07	12
9	20-Apr-07	12
10	23-Apr-07	12
11	24-Apr-07	12
12	25-Apr-07	12
13	26-Apr-07	12
14	27-Apr-07	12
15	30-Apr-07	12
16	01-May-07	12

Two breast cancer treatments
(LT4FD and LTISO)

Source: Scheduling Warehouse, BCCA

Treatment specifications

0
12
15
24
30
36
45
48
60
84
96
24-12
30-15
24-24
36-12
12-12-12
24-12-12
30-15-15
36-12-12
12-12-12-12
15-15-15-15
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45
48
60
84
96
24-12
30-15
24-24
36-12
12-12-12
24-12-12
30-15-15

Looking only at 2008 data (VC):

• 880 different patterns for a total of 2,511 treatments

• 18 patterns capture 50.7% of records (378 for 80%)

36-12-12-12-12-12-12-12-12-12
15-15-15-15-15-15-15-15-15-15
30-15-15-15-15-15-15-15-15-15
24-24-24-24-24-24-24-24-24-24
36-24-24-24-24-24-24-24-24-24
24-12-12-12-12-12-12-12-12-12-12
24-12-12-12-12-12-12-12-12-12-12-12
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24-12-12-12-12-12-12-12-12-12-12-12-12
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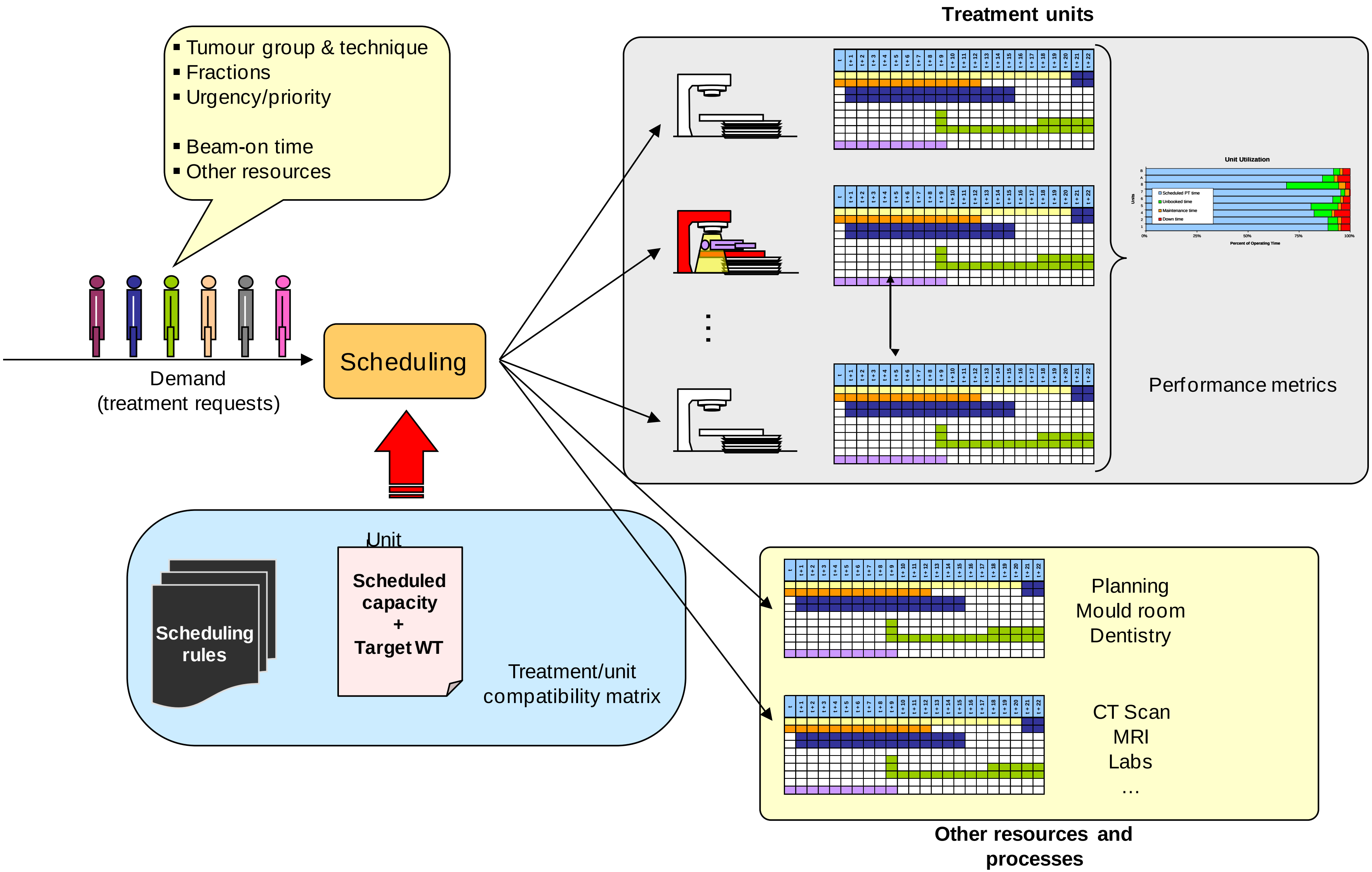
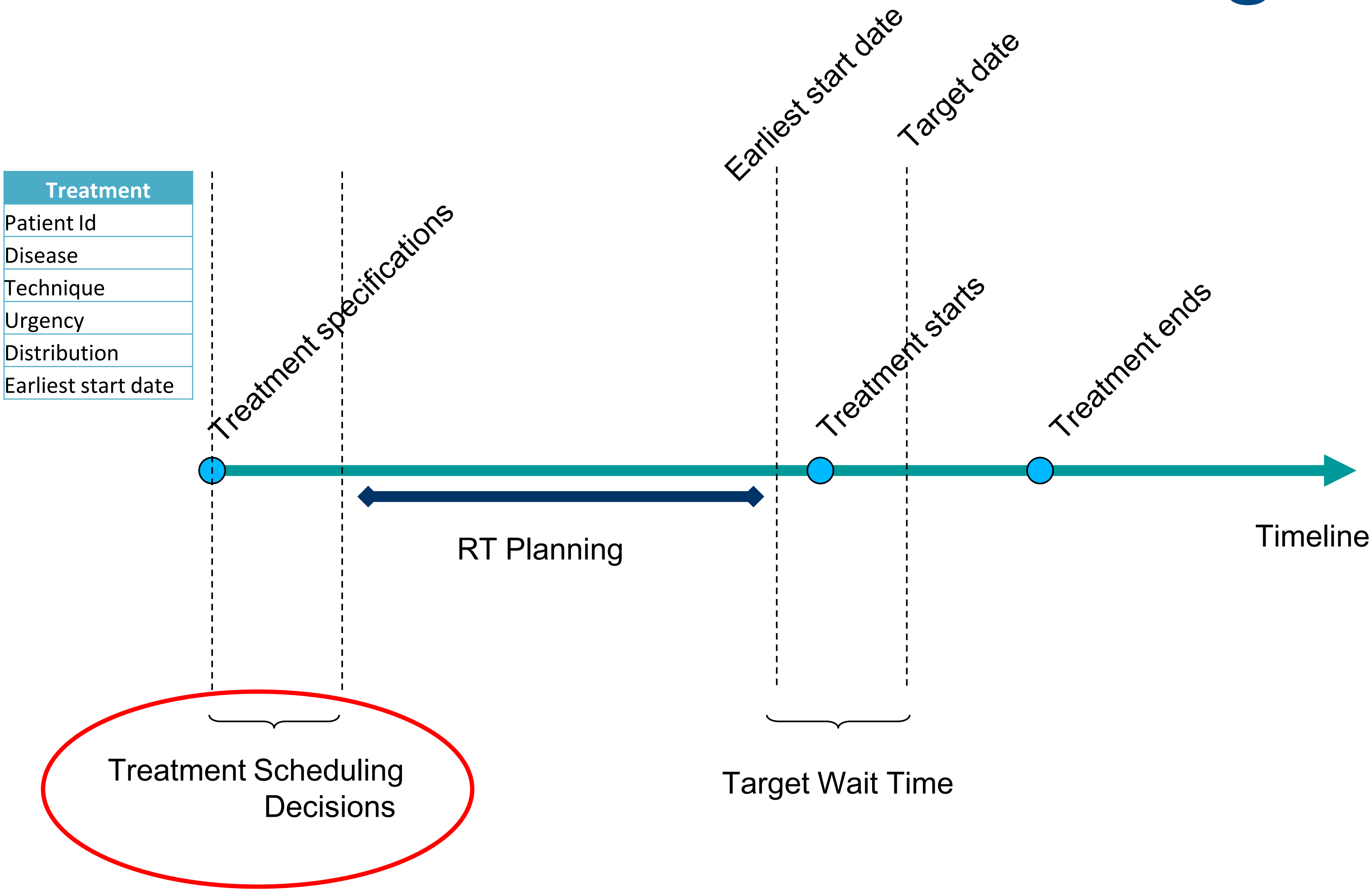
Source: Scheduling Warehouse, BCCA

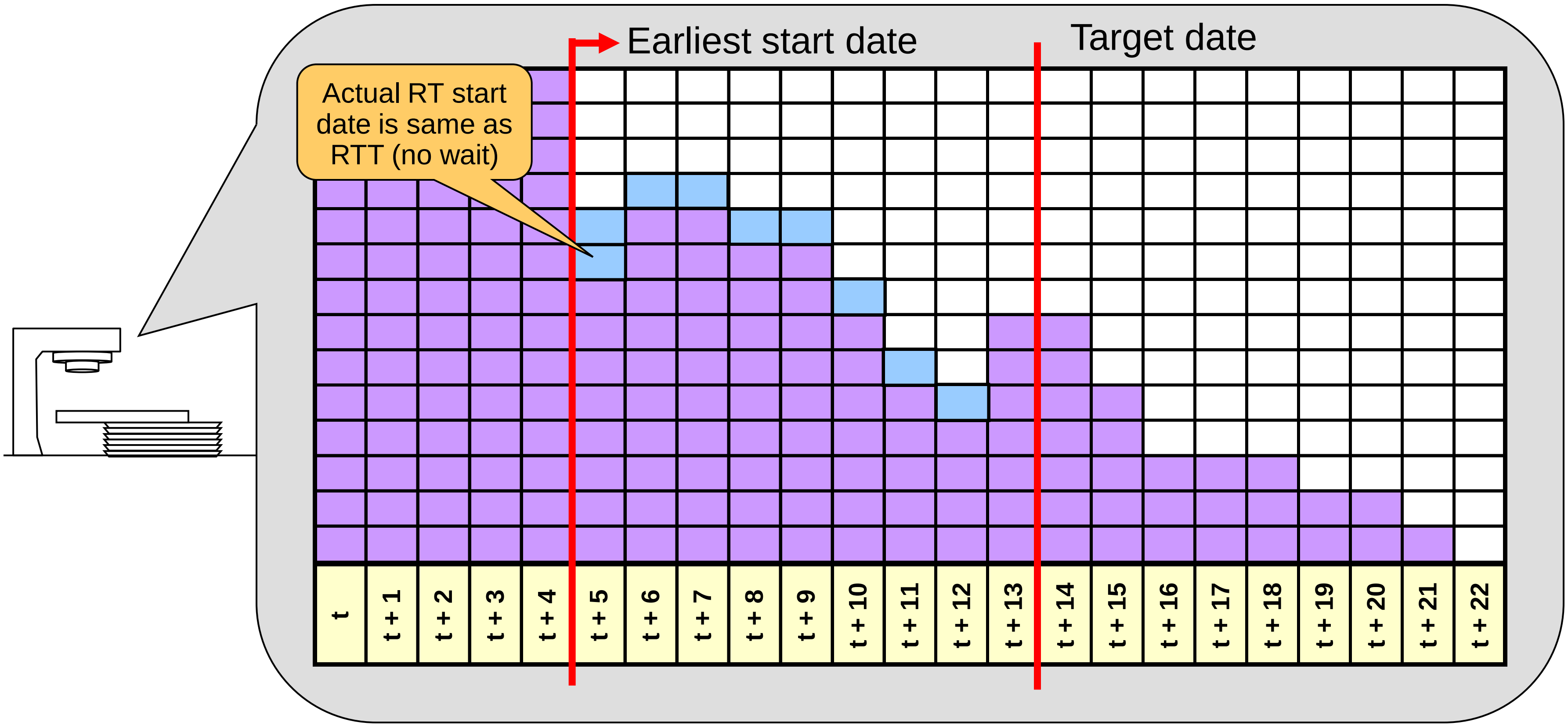
RT treatment scheduling



RT treatment scheduling

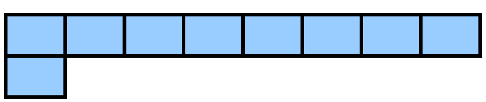


Scheduling decisions

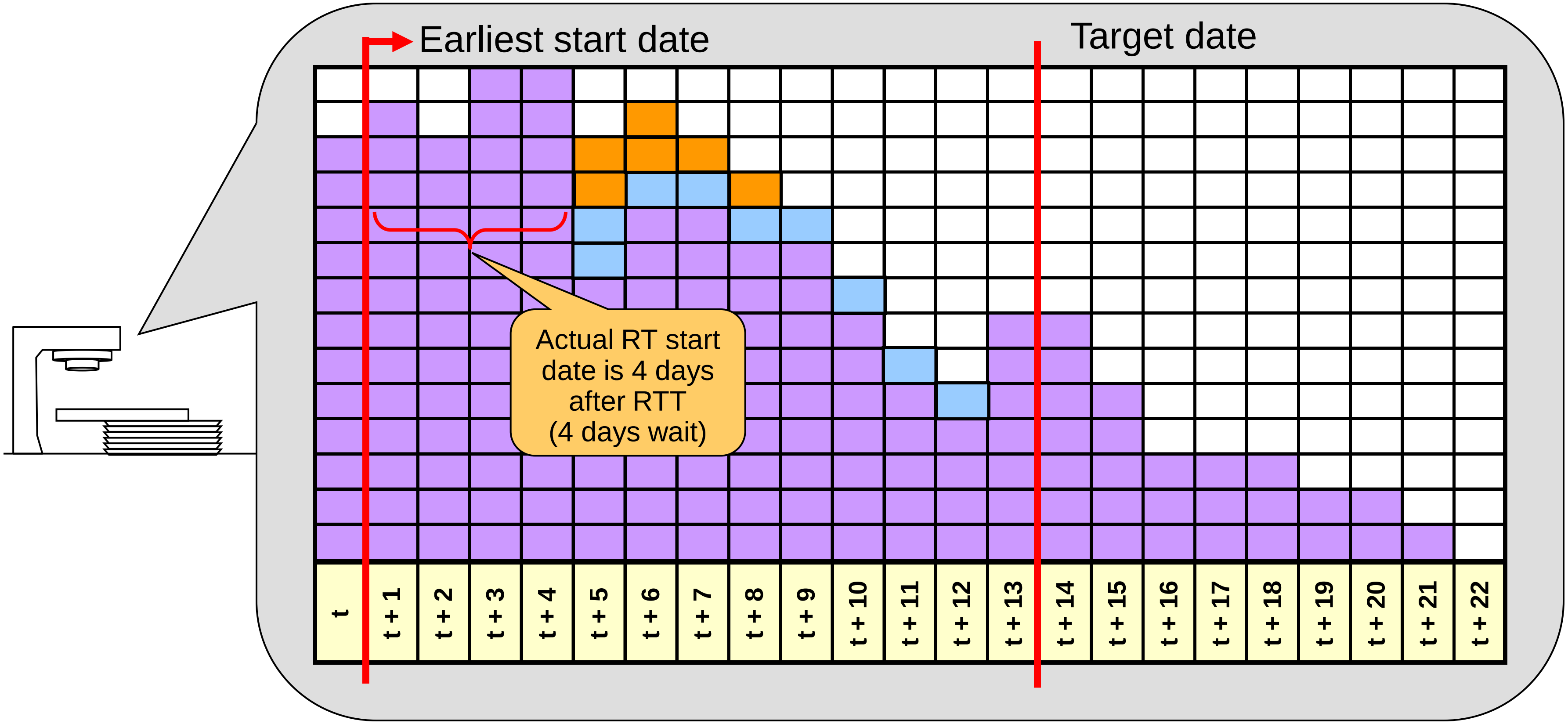


RTT = 5; Planning = 3

1600 cGy, 8 fractions

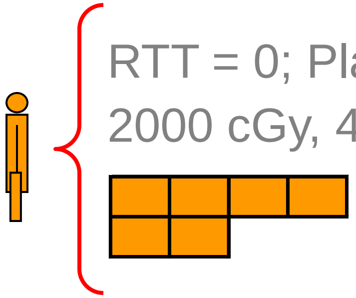


Scheduling decisions



RTT = 0; Planning = 1

2000 cGy, 4 fractions



Problem Characteristics

RT at the BCCA

- 15 cancer groups
- 17 treatment techniques
- 10 treatment units
- “3” urgency levels
- Hundreds of treatment patterns

RT Treatment

- Provided on consecutive days
- All fractions in the same treatment unit
- Unit compatibilities and preferences
- Ethical considerations

Problem Challenges

Others…

- Variability in demand for treatment
- Limited treatment capacity
- Cancellations and no shows
- Corrective and preventive maintenance
- Large number of possible booking decisions and resulting schedules
- Multiple alternative actions (diversions, overtime, and postponements)

Required Modifications of Priority Scheduling Problem

- Multiple machines
- Multi day appointment schedule patterns
- These add significant computational challenges to the problem
- We focused on the second problem
  - Primary change is to the transition probabilities since a decision uses resources on multiple days.
  - But actions are also constrained due to daily capacity constraints.

References

- Patrick, J., Puterman, M.L. and Queyranne, M. “Dynamic Multi-Priority Patient Scheduling” *Operations Research, 56, 1507-152, 2008*
- Patrick, J. and Puterman, M. Reducing Wait Times Through Operations Research: Optimizing the Use of Surge Capacity, *Healthcare Policy* **3**, 75-88, 2008.