Solving Large Scale Track Allocation Problems
CO@Work Berlin

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03.10.2009
Agenda

► The Track Allocation Problem
  ► Motivation
  ► Real World Problem
  ► Complexity

► Integer Programming Models
  ► Packing
  ► Extended Formulation
  ► Comparison

► Solution Approach
  ► Column Generation
  ► Lagrange Relaxation
Motivation

- Auction Idea
- Planning Process in „Reality“
Liberalization of railway transport
- Introducing a fair, open access, and transparent market for railway slots!
Fear of unused infrastructure!
Hope to increase efficiency (by fair access and optimization)!

(Performed trainkilometer per trackkilometer per day)

Vickrey Track Auction

BEGIN

TOCs decide on bids for single timetabled train paths

Winner Determination
Solve Track Allocation Problem:
Find optimal track allocation with maximum earnings

Vickrey Price Determination

NIP declares infrastructure

END

IP inside!
Combinatorial Bids

AND-Bids: "One for all, and all for one"

XOR-Bids: "There can be only one"
Definition 1:

A bidding strategy is called dominant, if it maximizes the utility function of the bidder no matter what any other participants submits.

Definition 2:

An auction mechanism is called incentive compatible, if truthful bidding (b=v) is a dominant strategy.

Definition 3:

An auction mechanism is called (allocative) efficient, if the winner allocation is maximizing the willingness to pay (v).
Some Pro’s & Con’s of VCG Auctions

VCG Auctions are

- incentive compatible (truthful bidding is a dominant strategy)
- efficient (the winner is the bidder with the highest valuation)

But unfortunately,

- vulnerable to collusion
- vulnerable to shill bidding
- not necessarily maximizing seller revenues
- seller's revenues are non-monotonic with regard to the bids
- ...
- rarely accepted by the participants and sellers
- hard to solve (WDP is $NP$-hard)
Combinatorial Vickrey Auction

Railway Example:

Potsdam ← A B → Berlin
Vickrey (Track) Auction

- Additional rule „Minimum bid“ (at least 3)
- Utility Matrix

<table>
<thead>
<tr>
<th>Track/ Bidder</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>CityConnex</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>DB Regi</td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>
Vickrey (Track) Auction

- Additional rule „Minimum bid“ (at least 3)
- Winner Allocation

<table>
<thead>
<tr>
<th>Track/ Bidder</th>
<th>A</th>
<th>B</th>
<th>Price</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>CityConnex</td>
<td>9</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DB Regi</td>
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<td>-</td>
<td>9</td>
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</tbody>
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Vickrey (Track) Auction

- Additional rule „Minimum bid“ (at least 3)
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<table>
<thead>
<tr>
<th>Bidder</th>
<th>Price</th>
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</tr>
</thead>
<tbody>
<tr>
<td>CityConnex</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>DB Regi</td>
<td>10</td>
<td>3 (&gt;)</td>
</tr>
</tbody>
</table>

► Additional rule “Minimum bid” (at least 3)
► Winner Allocation
Summary to Auctioning

- Combinatorial Vickrey (Track) Auction is incentive compatible, but not with rules for
  - minimum bid value
  - Limit on submitted bids
  - ...

“idealized“ Planning in Public Transport

Strategic Stage
- Stops
- Prices

Tactical Stage
- Connections

Operational Stage
- Rotations
- Duties

resource acquisition

resource allocation

Tracks
Lines/Freq.
Timetables
Vehicles
Crews
Railway Planning “Triangle“

- Lines/Freq.
- Connections
- Prices
- Cargo Routes

Timetables

- Rolling Stock
- Infrastructure
- Signaling
- Stops
- Tracks
Track Allocation/Train Timetabling Problem (TTP)

- **Strategic Stage**
  - Tracks
  - Lines/Freq.

- **Tactical Stage**
  - Timetables
  - Vehicles
  - Crews

- **Operational Stage**
  - Stops
  - Prices
  - Connections
  - Rotations
  - Duties

Flow Arrows:
- Tracks -> Lines/Freq. -> Timetables -> Vehicles -> Crews
- Strategic Stage => Tactical Stage => Operational Stage

Resource Acquisition and Allocation:
- Strategic Stage
  - Tracks
  - Lines/Freq.
  - Stops
  - Prices
  - Connections

- Operational Stage
  - Vehicles
  - Crews
  - Rotations
  - Duties

Diagram:
- PESP
- TTP
- TPP

**Notes**
- Thomas Schlechte
- 03.10.2009
- Slide 18
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- Solution Approach
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  - Lagrange Relaxation
From Microscopy to Macroscopy
From Blocks to Headways

Block & Signal System

Headways

<table>
<thead>
<tr>
<th>h</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
Micro-Macro Transformation

- **Micro-Macro-Tool**
- Based on conservative microsimulation
- Automatic conflict detection
- Automatic discretization of time

![Diagram of Micro-Macro Transformation]

Microworld (Simulation)  
**Transformation**  
Macroworld (Optimization)
Close the Cycle - Evaluation in simulated „Reality“
3D Visualization Tool TraVis by M.Kinder & B.Erol, based on JavaView
Train Timetabling Problem

Train Requests $\rightarrow$ Scheduling Digraph $\rightarrow$ Timetable
Track Allocation Literature - Railway Timetabling (TTP)

- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Semet and Schoenauer (2005),
- Caprara, Monaci, Toth and Guida (2005)
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- Fischer, Helmberg, Janßen, Krostitz (2008)
- Fischetti, Salvagnin, Zanette (2009) …

non-cyclic timetabling literature
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**Proposition** [Caprara, Fischetti, Toth (02)]:
OPTRA/TTP is $\mathcal{NP}$-hard.

**Proof:**
Reduction from Independent-Set.
Independent/Stable Set Problem

\[ S \subseteq 2^V \]
\[ s \in S \iff \forall u, v \in s : (u, v) \notin E \]

\[ G = (V, E) \]

\[ \max_{s \in S} |s| \]
Edge (1,2) $\Leftrightarrow$ Conflict on track (1,2)
Edge \((2,3) \iff \text{Conflict on track } (2,3)\)
Edge (2,4) ⇔ Conflict on track (2,4)
Edge $(3,4) \Leftrightarrow$ Conflict on track $(3,4)$
Edge (4,5) ⇔ Conflict on track (4,5)
Feasible Set of Train Routes $\iff$ Stable Set

\[ s \quad (1,2) \quad (2,3) \quad (2,4) \quad (3,4) \quad (4,5) \quad t \]
Maximize Scheduled Trains $\iff$ Maximize Independent Set

\[
\begin{align*}
\text{s} & \\
(1,2) & (2,3) & (2,4) & (3,4) & (4,5) & t
\end{align*}
\]
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Variables for Track Usage

maximize

subject to

\[ \text{Berlin} \quad \text{Potsdam} \]
Variables
- Path usage (request i uses path p)

Constraints
- Do no violated conflict sets

Objective
- Maximize utility/ proceedings

- Comparability graphs for APP and PPP

\[(PPP)\]
\[
\max \sum_{i \in I} \sum_{p \in P_i} u_p^i x_p^i
\]
\[\text{s.t.} \quad \sum_{i \in I} x_p^i \leq 1 \quad \forall i \in I \quad \text{(i)}
\]
\[
\sum_{p \in C} x_p^i \leq \kappa_c \quad \forall c \in C \quad \text{(ii)}
\]
\[
x_p^i \in \{0, 1\} \quad \forall p \in P_i, \forall i \in I \quad \text{(iii)}
\]
Maximal Conflicts Sets

- Maximal Conflict Sets
- Pairwise Conflicts

Berlin  Potsdam
Packing Models in Theory

- Conflict graph
- Maximal cliques
- Perfect graph

- Caprara, Fischetti & Toth (2002)
  - Langrangean approach
  - Conflict graphs of quadrangle-linear headway matrices
  - Conflict graphs of block occupation (interval graphs)
- Helmberg et al. (2008)
  - Bundle Method for packing formulation
Packing Models

► **Proposition:** The LP-relaxation of APP can be solved in poly. time.
  ► ... but in practice.
Alternative IP Model

“A bird in the hand is worth two in the bush !”

≡

“A sparrow in the hand is better than a dove on the roof (german) !“

Instead of forcing feasibility of flows by huge number of constraints – allow only feasible flows (inner verus outer approximation).
Variables determine Capacity on Tracks
Track Digraph
Alternative Model - Extended Formulation

- Track Digraph
- Timeline(s)
- Config paths

- Balas (2005)
  - Projection, Lifting and Extended Formulation in Integer and Combinatorial Optimization
  - construction for block conflicts
  - construction for triangle linear headway matrices
- Helmberg et al. (2009)
  - dynamic network construction for a special objective function
TTP as Path Coupling Problem

\[(PCP)\]
\[
\begin{align*}
\text{max} \quad & \sum_{p \in P} u_p x_p \\
\text{s.t.} \quad & \sum_{p \in P_i} x_p \leq 1, \quad \forall i \in I \\
& \sum_{q \in Q_j} y_q \leq 1, \quad \forall j \in J \\
& \sum_{p \in P, a \in p} x_p \quad - \quad \sum_{q \in Q, a \in q} y_q \leq 0, \quad \forall a \in A_{LR} \\
& x_p, y_q \geq 0, \quad \forall p \in P, q \in Q \\
& x_p, y_q \in \{0, 1\}, \quad \forall p \in P, q \in Q.
\end{align*}
\]

- **Variables**
  - Path and config usage (request i uses path p, track j uses config q)

- **Constraints**
  - Path and config choice
  - Path-config-coupling to ensure track feasibility (capacity)

- **Objective**
  - Maximize utility/proceedings
PCP is an extended formulation of PPP

Lemma (B., S. [2007]):
\[ \nu_{LP}(PCP) = \nu_{LP}(PPP) \quad \text{and} \quad \nu_{IP}(PCP) = \nu_{IP}(PPP). \]
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Linear Relaxation of PCP

\begin{align*}
\text{(MLP)} & \quad \max \sum_{p \in P} u_p x_p & \text{(i)} \\
\text{s.t.} & \quad \sum_{p \in P_i} x_p \leq 1, \quad \forall i \in I & \text{(ii)} \\
& \quad \sum_{q \in Q_j} y_q \leq 1, \quad \forall j \in J & \text{(iii)} \\
& \quad \sum_{p \in P, a \in p} x_p - \sum_{q \in Q, a \in q} y_q \leq 0, \quad \forall a \in A_{LR} & \text{(iv)} \\
& \quad x_p, y_q \geq 0, \quad \forall p \in P, q \in Q & \text{(vi)}
\end{align*}

<table>
<thead>
<tr>
<th>Dual Variable</th>
<th>Information About</th>
<th>Useful For</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_i$</td>
<td>Bundle price</td>
<td>Analysing request</td>
</tr>
<tr>
<td>$\pi_j$</td>
<td>Track price</td>
<td>Analysing network</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>Arc price</td>
<td>-</td>
</tr>
</tbody>
</table>
\[(DLP)\]

\[
\begin{align*}
\min & \quad \sum_{j \in J} \pi_j + \sum_{i \in I} \gamma_i \\
\text{s.t.} & \quad \gamma_i + \sum_{a \in P} \lambda_a \geq \sum_{a \in P} p_a^i \quad \forall p \in P_i, \forall i \in I \quad (i) \\
& \quad \pi_j - \sum_{a \in Q} \lambda_a \geq 0 \quad \forall q \in Q_j, \forall j \in J \quad (ii) \\
& \quad \gamma_i \geq 0 \quad \forall i \in I \quad (iii) \\
& \quad \lambda_a \geq 0 \quad \forall a \in A_I \cup A_J \quad (iv) \\
& \quad \pi_j \geq 0 \quad \forall j \in J \quad (v)
\end{align*}
\]
Pricing of $x$-variables

\[ (\text{PRICE}(x)) \quad \exists \overline{p} \in P_i : \quad \gamma_i < \sum_{a \in \overline{p}} (u_a - \lambda_a) \]

\[ c_a = -u_a + \lambda_a \]

Pricing Problem($x$) :
Acyclic shortest path problems for each slot request $i$ with modified cost function $c$ !
Pricing of $y$-variables

$\exists q \in Q_j : \pi_j < \sum_{a \in q} \lambda_a$

$c_a = -\lambda_a$

Pricing Problem($y$):
Acyclic shortest path problem for each track $j$ with modified cost function $c$!
Observation for „optimal“ Pricing

\[(\text{PRICE}(x)) \quad \exists \, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)\]
(PRICE (y)) \[ \exists \bar{q} \in Q_j : \pi_j < \sum_{a \in \bar{q}} \lambda_a \]

\[ \theta_j := \max_{\bar{q} \in Q_j} \sum_{a \in \bar{q}} \lambda_a - \pi_j, \ \forall j \in J \]

\[ \theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \ \forall j \in J, q \in Q_j \]

\[ \theta_j + \pi_j \text{ satisfies } (DLP)(ii) \]
Pricing Upper Bound

\[(\max\{\eta + \gamma, 0\}, \max\{\theta + \pi, 0\}, \lambda)\] is feasible for \((DLP)\)

\[
\beta(\gamma, \pi, \lambda) := \sum_{i \in I} \max\{\gamma_i + \eta_i, 0\} + \sum_{j \in J} \max\{\pi_j + \theta_j, 0\}
\]

**Lemma:** Given (infeasible) dual variables of PCP and let \(v_{LP}(PCP)\) be the optimum objective value of the LP-Relaxtion of PCP, then:

\[v_{LP}(PCP) \leq \beta(\gamma, \pi, \lambda)\]
PCP-Run of TS-OPT /LP Stage

scenario 570 trains

objective value

\( \beta(\gamma, \pi, \lambda) \)

\( v(\text{RPLP}) \)

column generation iterations
Two Step Approach

1. LP Solving
2. IP Solving

TS-OPT

- Column Generation
- Rapid Branching Heuristic
- Pricing by Dijkstra’s Shortest Path
- Duals by Bundle Method

Thomas Schlechte
Linear versus Lagrangean Relaxation

- solved by simplex or barrier methods
- feasible fractional solution
- strong bound but time and memory consuming

- solved by subgradient or bundle methods
- (infeasible) primal approximation
- potentially better bound and even faster
\[(LD) \min_{\lambda \geq 0} \left[ \max_{Ax=1, \atop x \in \{0,1\}^{|P|}} (u^T - \lambda^T C)x + \max_{By=1, \atop y \in \{0,1\}^{|Q|}} (\lambda^T D)y \right] \]
Idea of the Bundle Method (Kiwiel [1990], Helmberg [2000])

Problem: minimize convex function $f$

new candidate: $\lambda_{k+1} = \arg\min_{\lambda \in \mathbb{R}^m} \hat{f}_k(\lambda) + \frac{u_k}{2} \|\lambda - \hat{\lambda}_k\|^2$

$$\hat{f}(\lambda) := \max_{\mu \in J_k} \overline{f}_\mu(\lambda)$$

$$\overline{f}_\mu(\lambda) := f(\mu) + g(\mu)^T (\lambda - \mu), \quad g(\mu) \in \partial f(\mu)$$
Bundle Algorithm for PCP (B., Weider & S. [2009])

\[
(LD) \quad \min_{\lambda \geq 0} \left[ \max_{A x = 1, \atop x \in [0,1]^{|P|}} (u^T - \lambda^T C)x + \max_{B y = 1, \atop y \in [0,1]^{|Q|}} (\lambda^T D)y \right]
\]

(FE) decomposes in acyclic shortest path

(FE) decomposes in acyclic shortest path
Rapid Branching (Weider [2007])

- diving heuristic guided by primal approximation (or fractional solution)
- use perturbation to decrease integer infeasibilities and identify candidates
- try to fix large subsets of candidates $C$ at once to 1
- explore only promising nodes (after some pricing)
- avoid backtracks and ignore 0-branch
"Obvious" bottleneck in a "real world" railway system

- 15 stations, 32 tracks and 6 different train types
- Already fixed passenger traffic (63 trains per day)
- How many additional cargo trains can be scheduled?
Sensitivity Analysis

Computational results for variation of accuracy and routes

- Network construction with different units of seconds
- Scenario from 8pm to 12pm (20 passenger trains + 24 cargo trains)
- Optimization of "same" scenarios with TS-OPT (using model ACP)

<table>
<thead>
<tr>
<th>Discretization in seconds</th>
<th>6</th>
<th>10</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>#Trains with &quot;free&quot; routing through stations (24 routes)</td>
<td>38</td>
<td>38</td>
<td>37</td>
<td>26</td>
</tr>
<tr>
<td>#Trains with &quot;fixed&quot; routing through stations (12 routes)</td>
<td>27</td>
<td>24</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>Computation time</td>
<td>hours</td>
<td>minutes</td>
<td>seconds</td>
<td>seconds</td>
</tr>
</tbody>
</table>
Saturation Experiment

- Estimation of the maximum "corridor" capacity
  - Network accuracy of 6s
  - Consider complete routing through stations
  - Saturate by additional cargo trains

- Conflict free train schedules in simulation software (1s accuracy)
- Proven upper bound of capacity, at most x trains per hour/day (if ...
Thank you for your attention!

- Thomas Schlechte

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