Solving Large Scale Track Allocation *Problems*
CO@Work Berlin

Thomas Schlechte

Ralf Borndörfer Berkan Erol Martin Grötschel

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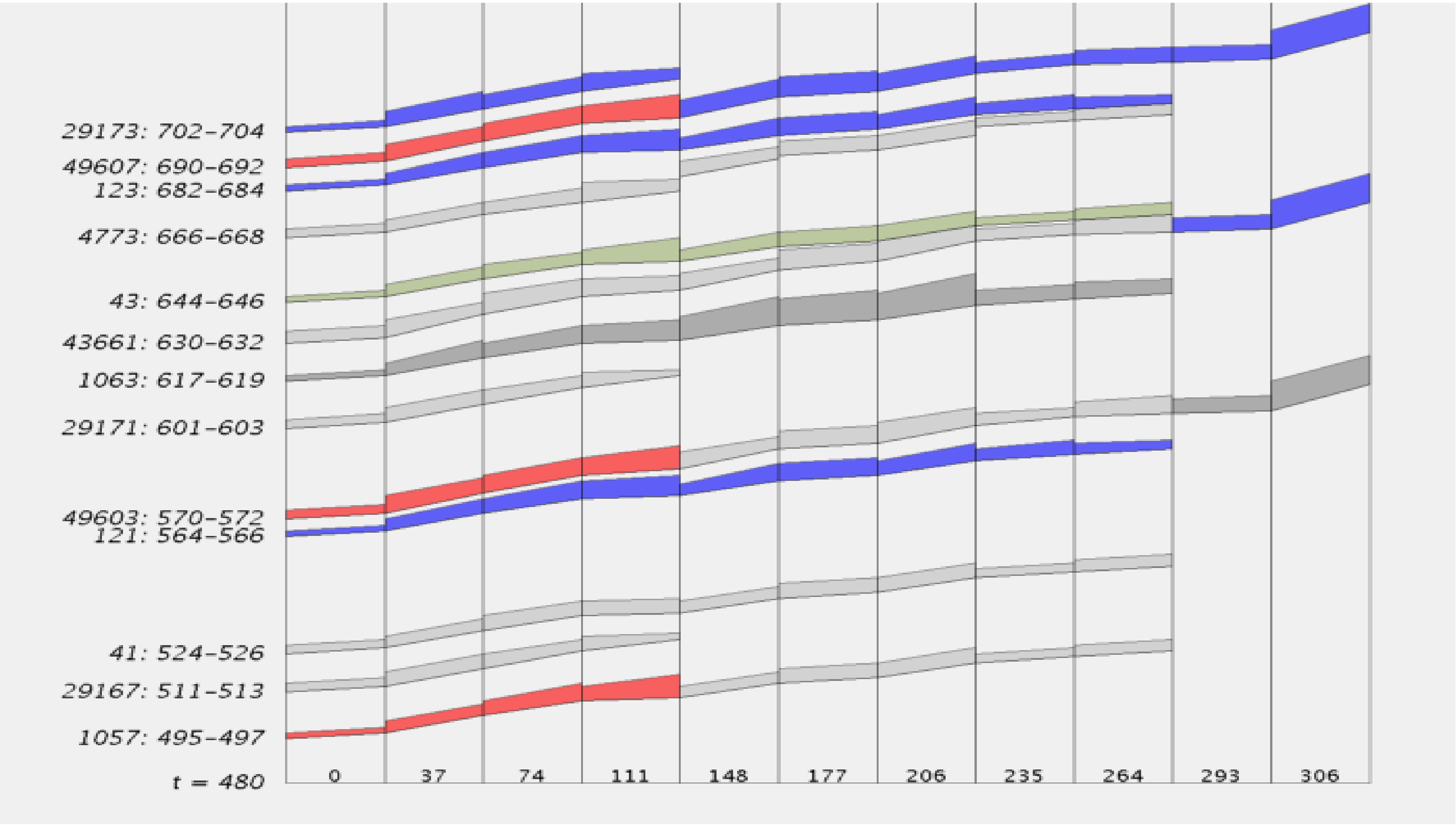
Agenda

- The Track Allocation Problem
  - Motivation
  - Real World Problem
  - Complexity
- Integer Programming Models
  - Packing
  - Extended Formulation
  - Comparison
- Solution Approach
  - Column Generation
  - Lagrange Relaxation

Motivation

- Auction Idea
- Planning Process in „Reality“



Federal Ministry of Economics and Technology

Liberalization of railway transport

-

Introducing a fair, open access, and transparent market for railway slots !

WIP

ermcon Rail Management Consultants

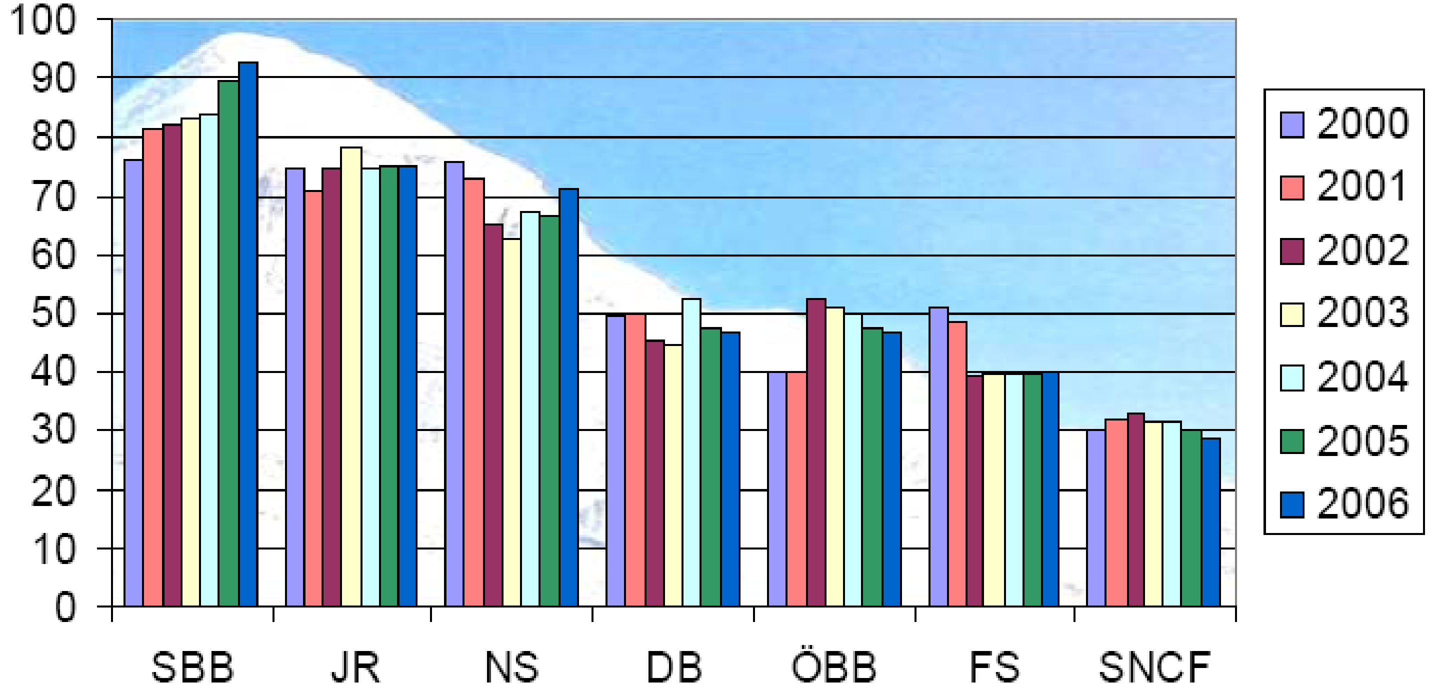
ZIB

IVE

SFWBB

Fear of unused infrastructure !

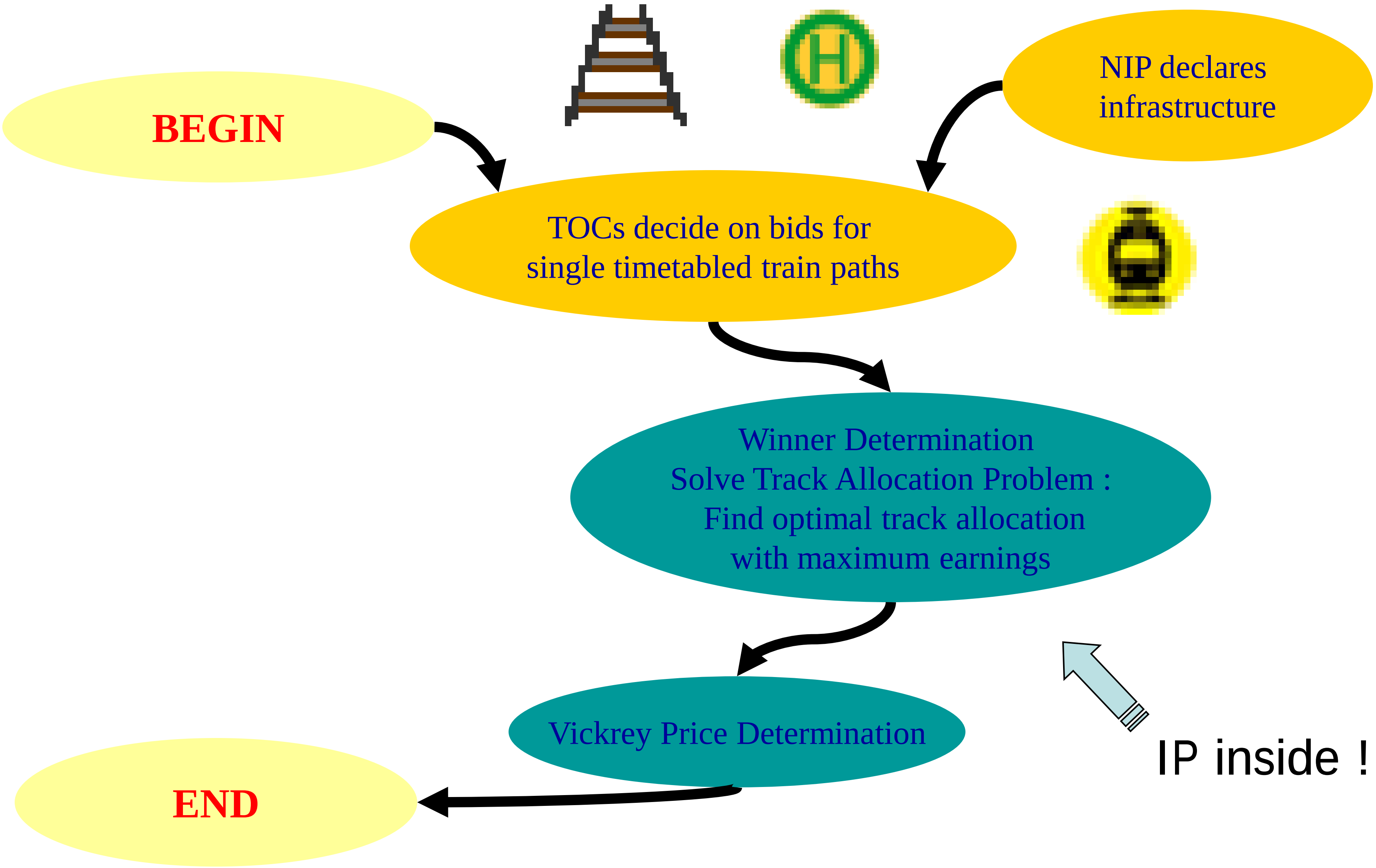
Hope to increase efficieny (by fair access and optimization) !



(performed trainkilometer per trackkilometer per day)

UIC Leaflet 406: „*Capacity*“, International Union of Railways, 2004.

Vickrey Track Auction



AND-Bids:
"One for all,
and all for one“

XOR-Bids:
"There can be only one“

Some Defintions from Auction Theory

Definition 1:

A *bidding strategy* is called *dominant*, if it maximizes the utility function of the bidder no matter what any other participants submits.

Definition 2:

An *auction* mechanism is called *incentive compatible*, if truthful bidding (b=v) is a dominant strategy.

Definition 3:

An *auction* mechanism is called *(allocative) efficient,* if the winner allocation is maximizing the willingness to pay (v).

Some Pro‘s & Con‘s of VCG Auctions

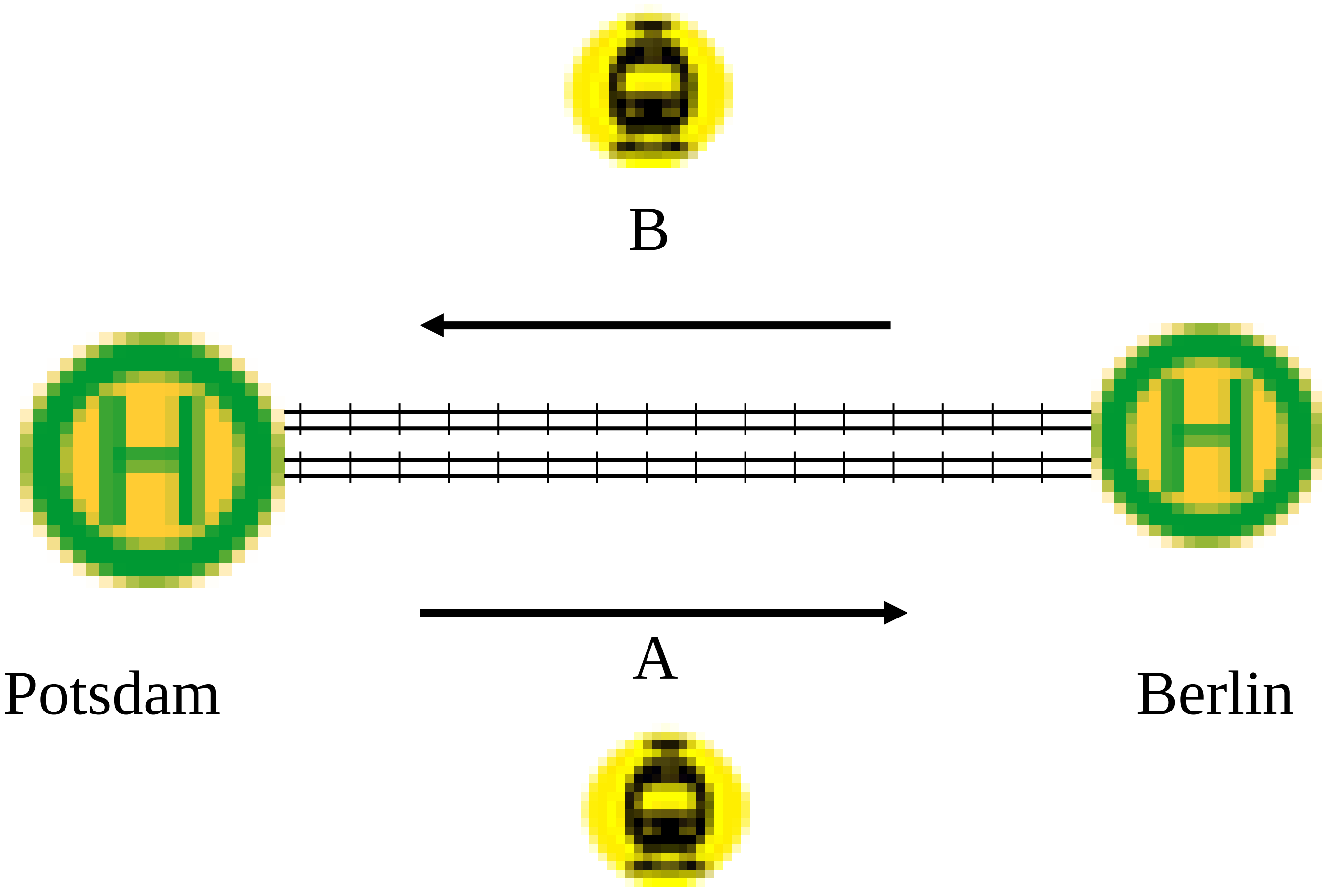
VCG Auctions are

- incentive compatible (truthful bidding is a dominant strategy)
- efficient (the winner is the bidder with the highest valuation)

But unfortunately,

- vulnerable to collusion
- vulnerable to shill bidding
- not necessarily maximizing seller revenues
- seller's revenues are non-monotonic with regard to the bids
- ...
- rarely accepted by the participants and sellers
- hard to solve ((WDP) is $\mathcal{NP}$-*hard*)

Railway Example :



Vickrey (Track) Auction

- Additional rule „Minimum bid“ (at least 3)
- Utility Matrix

Track/ Bidder	A	B
CityConnex	9	1
DB Regi	10	2

Vickrey (Track) Auction

- Additional rule „Minimum bid“ (at least 3)
- Winner Allocation

Track/ Bidder	A	B	Price	Utility
CityConnex	9	-	0	0
DB Regi	10	-	9	1

- Additional rule „Minimum bid“ (at least 3)
- Winner Allocation

ice	Utility
0	0
9	3

- Combinatorial Vickrey (Track) Auction is incentive compatible, but not with rules for
  - minimum bid value
  - Limit on submitted bids
  - …

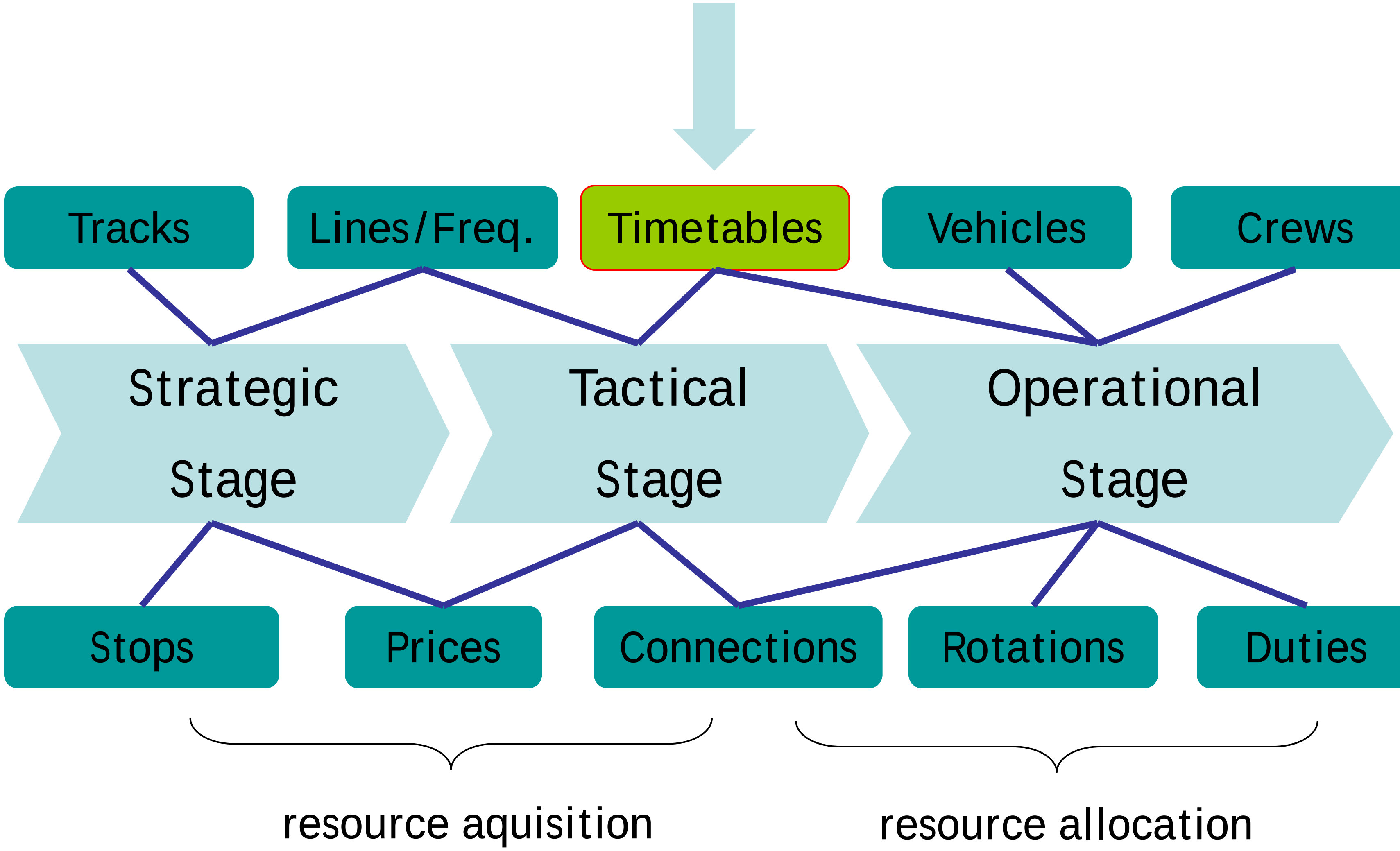
P. Milgrom: „*Putting Auction Theory to Work*“, Cambridge University Press, 2004.

A. Mura: „*Trassenauktionen im Schienenverkehr*“, Master Thesis, TU Berlin, 2006.

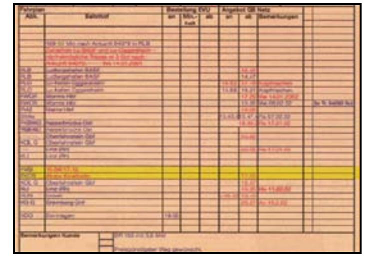
P. Cramton, Shoham & Steinberg, „*Combinatorial Auctions*“, The MIT Press, 2006.

R.Borndörfer, A.Mura & T.S.: „Vickrey Auctions for Railway Tracks“, OR-Proceedings, 2008.

"idealized" Planning in Public Transport

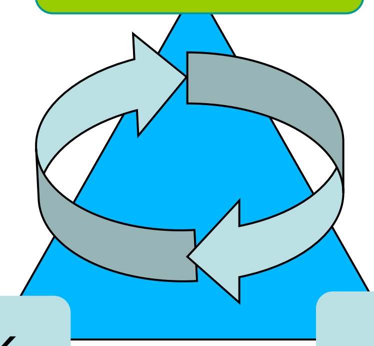


Railway Planning “Triangle“



Lines/Freq. | Connections | Prices | Cargo Routes

Timetables

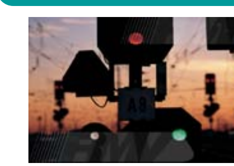


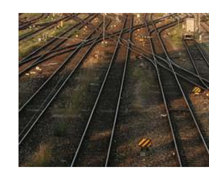
Rolling Stock

Infrastructure

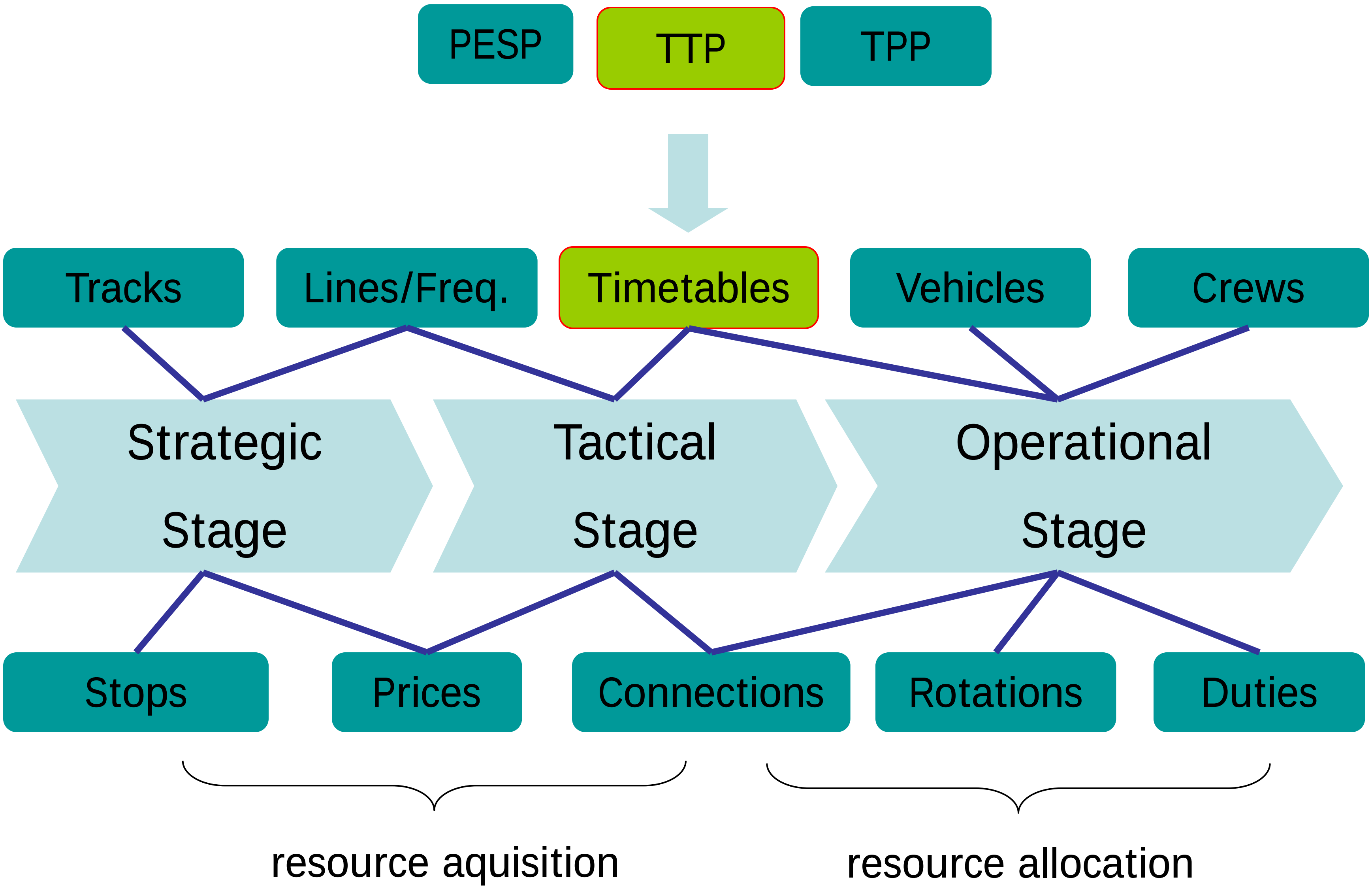
Tracks

Signaling | Stops





Track Allocation/Train Timetabling Problem (TTP)



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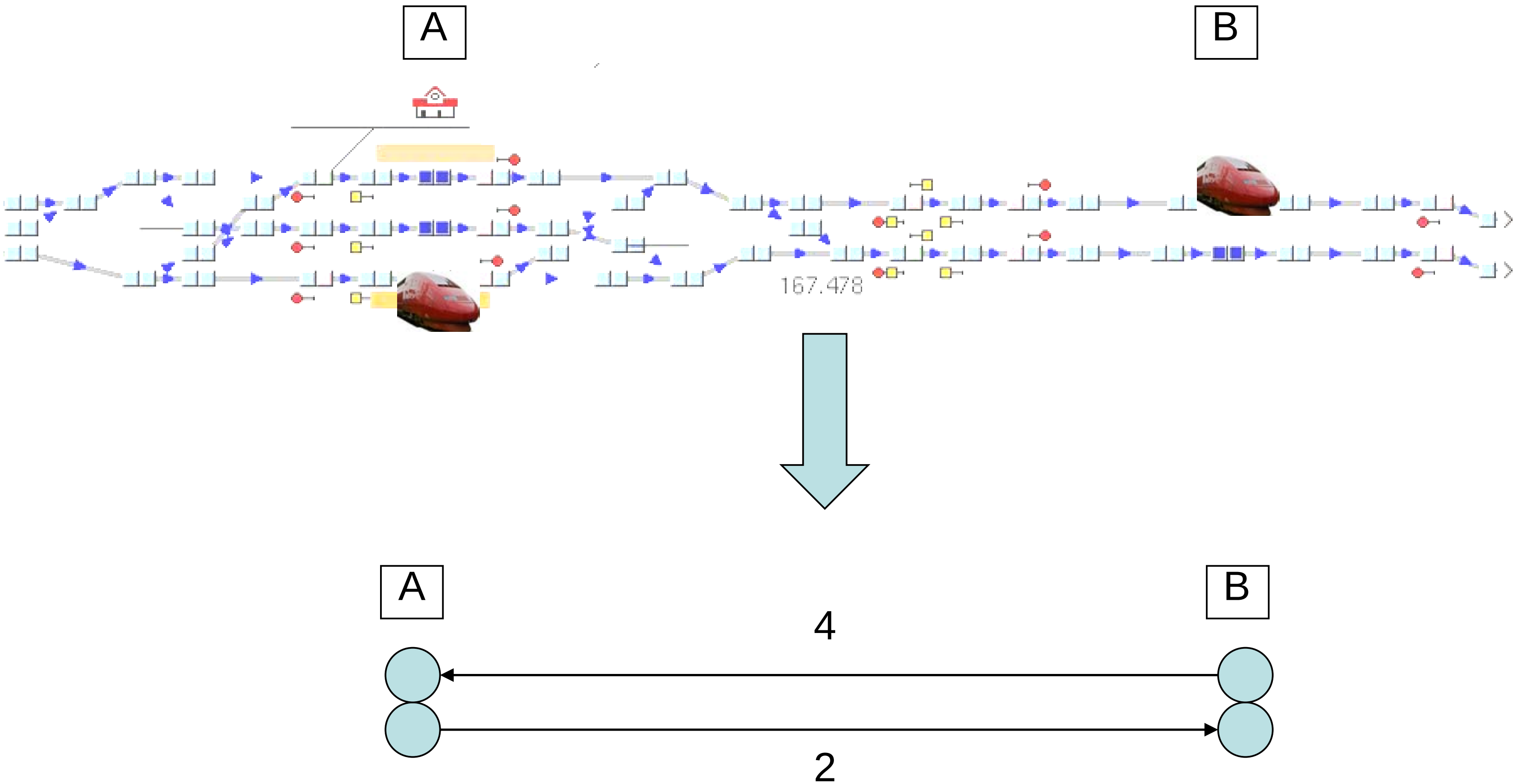
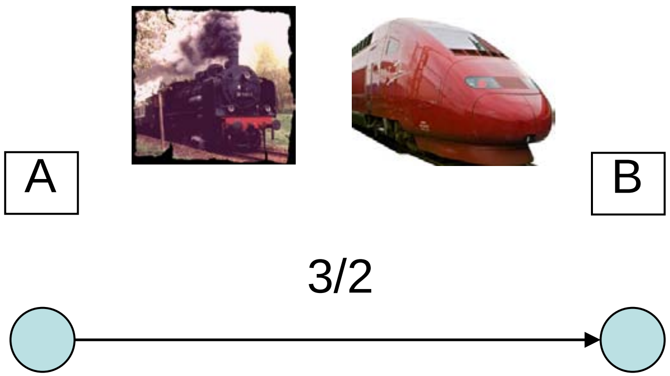
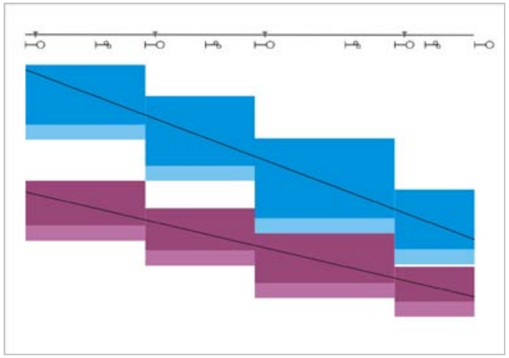
From Microscopy to Macroscopy



From Blocks to Headways





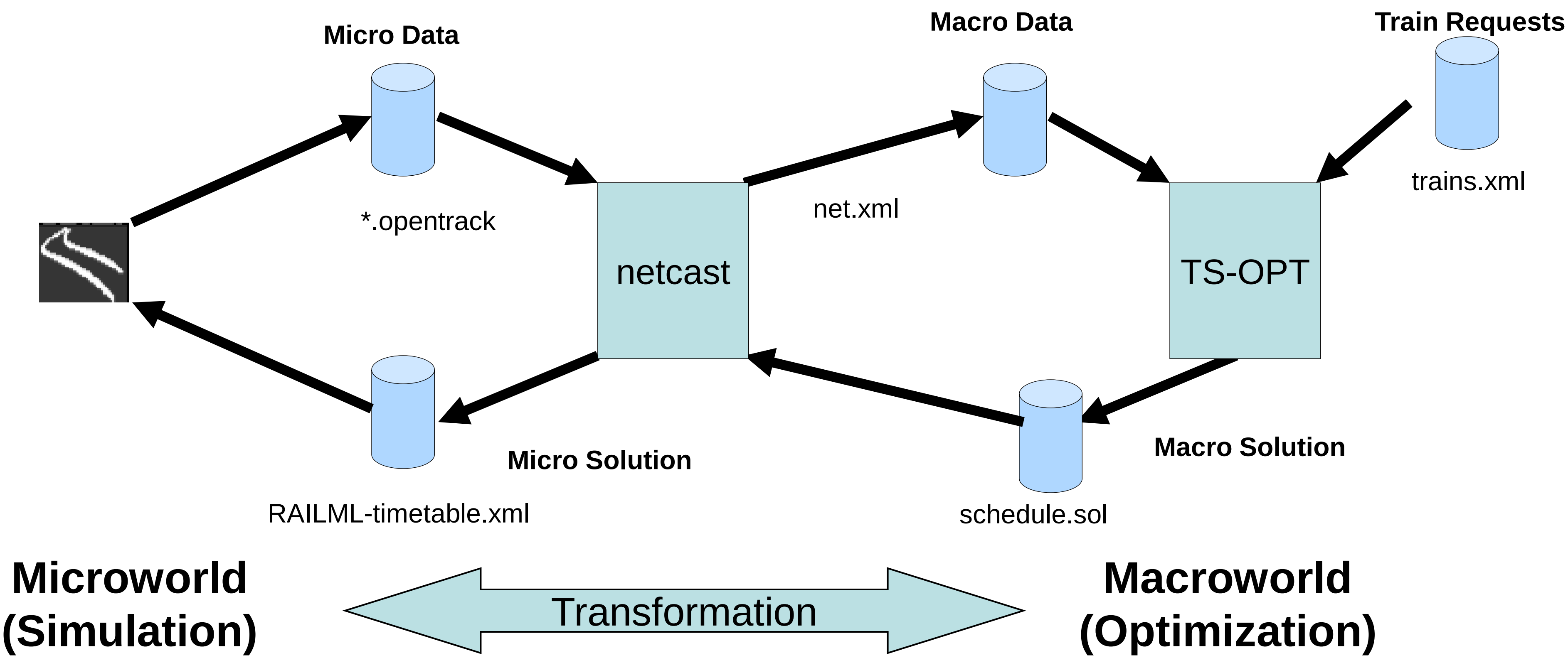
Block & Signal System

Headways

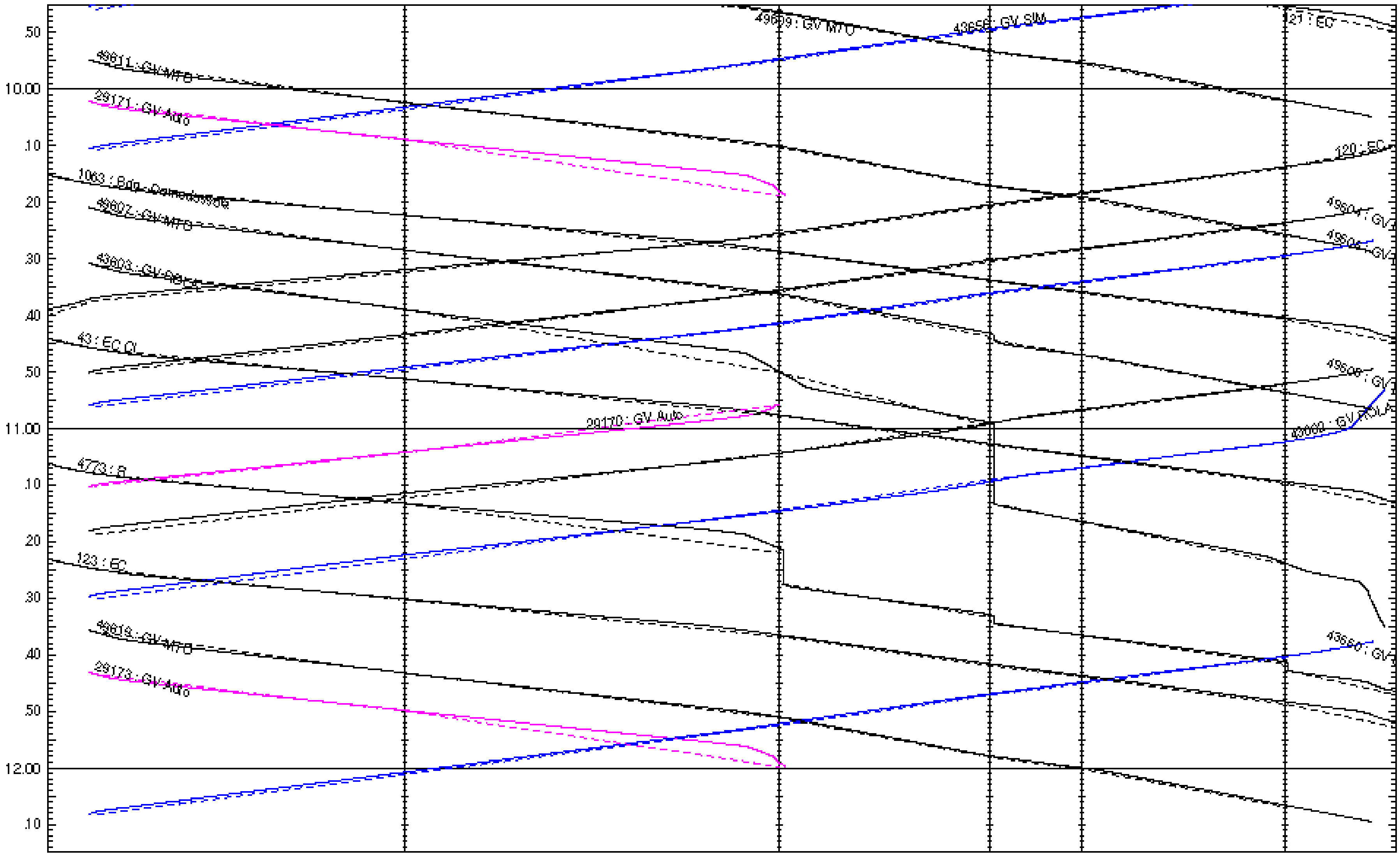
h			d
	2	3	3
	1	1	2

Micro-Macro Transformation

- Micro-Macro-Tool
- based on conservative microsimulation
- automatic conflict detection
- automatic discretization of time



Close the Cycle - Evaluation in simulated „Reality“



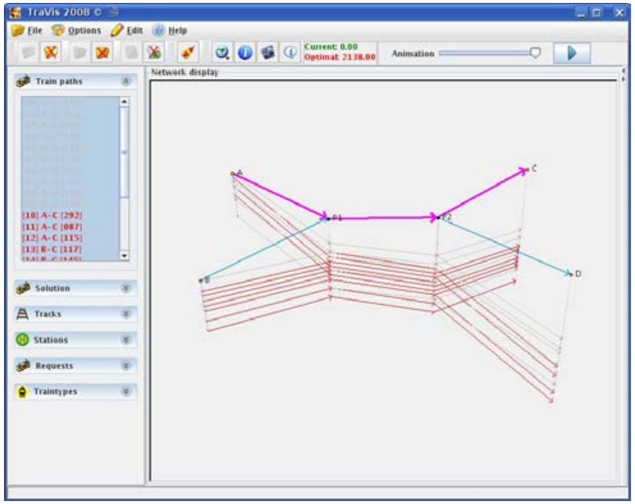
Track Allocation Problem

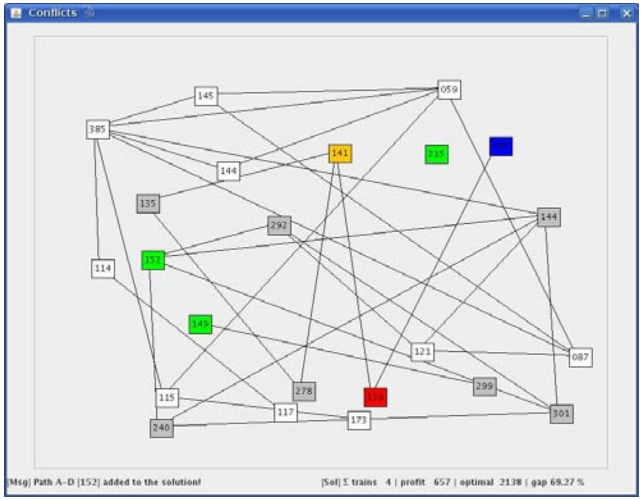
I - set of train request

P_i - set of railway slots for request $i \in I$

C - conflict sets, $\{(P_q \in 2^P, \kappa_q)\}$

u_p^i - utility of $i \in I$ for $p \in P$

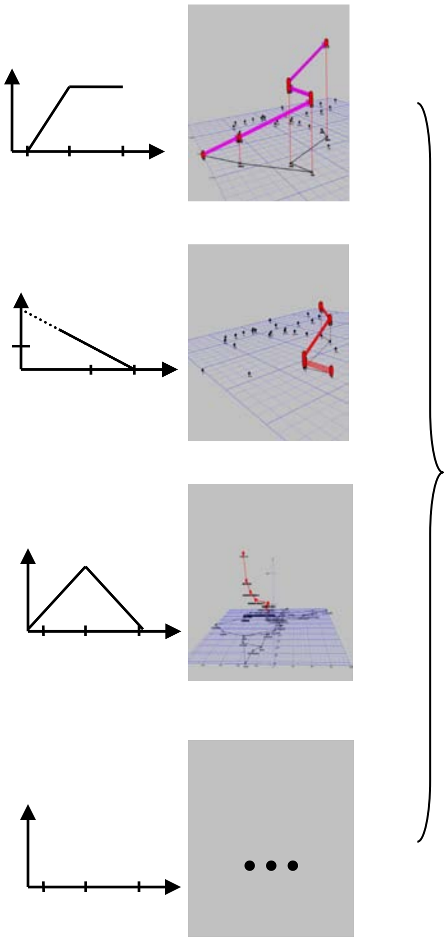


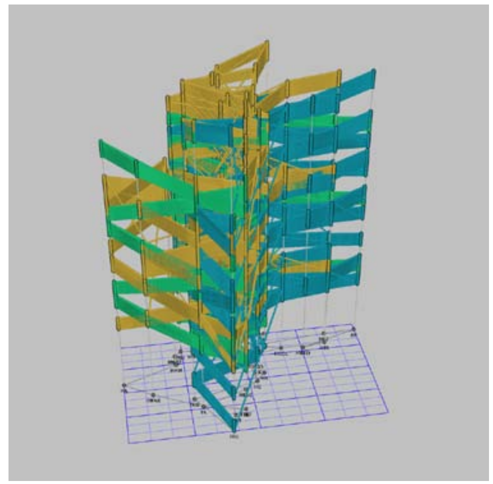


3D Visualization Tool TraVis by M.Kinder & B.Erol, based on

JavaView

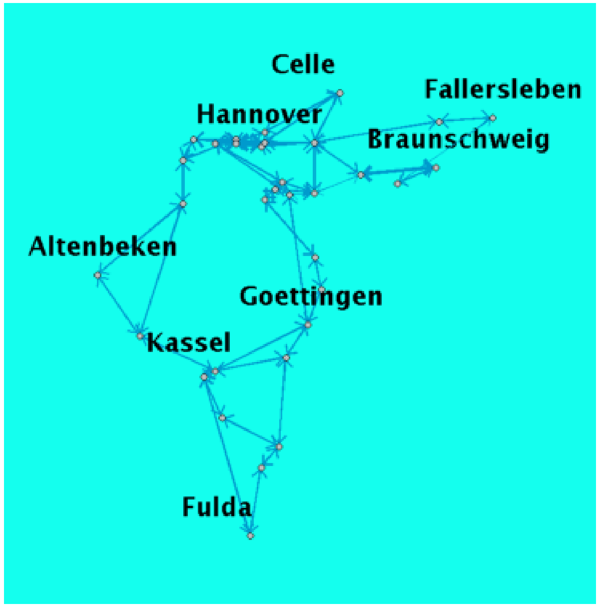
Train Timetabling Problem





Train Requests → Scheduling Digraph → Timetable

Track Allocation Literature - Railway Timetabling (TTP)



- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- **Brannlund, Lindberg, Nou, Nilsson (1998)**, Lindner (2000), Oliveira and Smith (2000)
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- **Cacchiani, Caprara, T. (2006), Cachhiani (2007)**
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- **Borndoerfer, S. (2005, 2007)** Caimi G., Fuchsberger M., Laumanns M., Schüpbach K. (2007)
- **Fischer, Helmberg, Janßen, Krostitz (2008)**
- **Fischetti, Salvagnin, Zanette (2009) …**

non-cyclic timetabling literature

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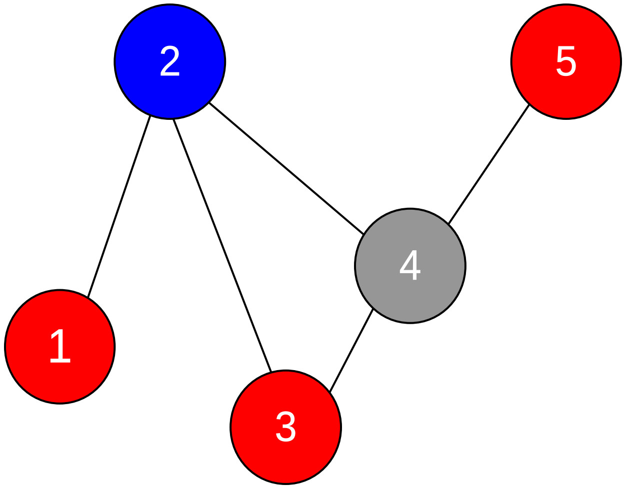
Complexity

Proposition [Caprara, Fischetti, Toth (02)]:

OPTRA/TTP is $\mathcal{NP}$-hard.

Proof:

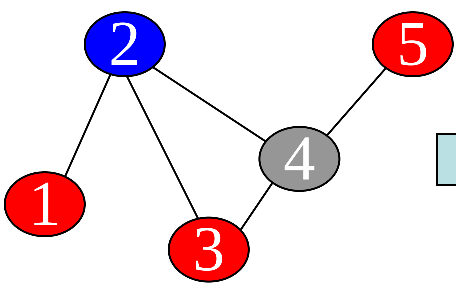
Reduction from Independent-Set.

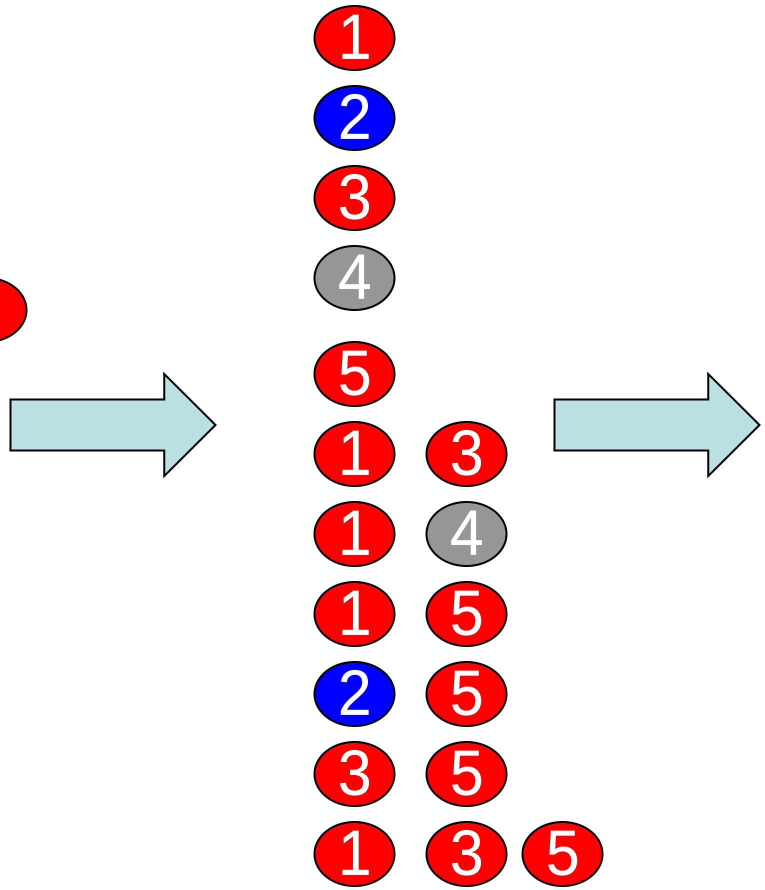


$$S \subseteq 2^V$$

$$s \in S \Leftrightarrow \forall u, v \in s \, : \, (u, v) \notin E$$

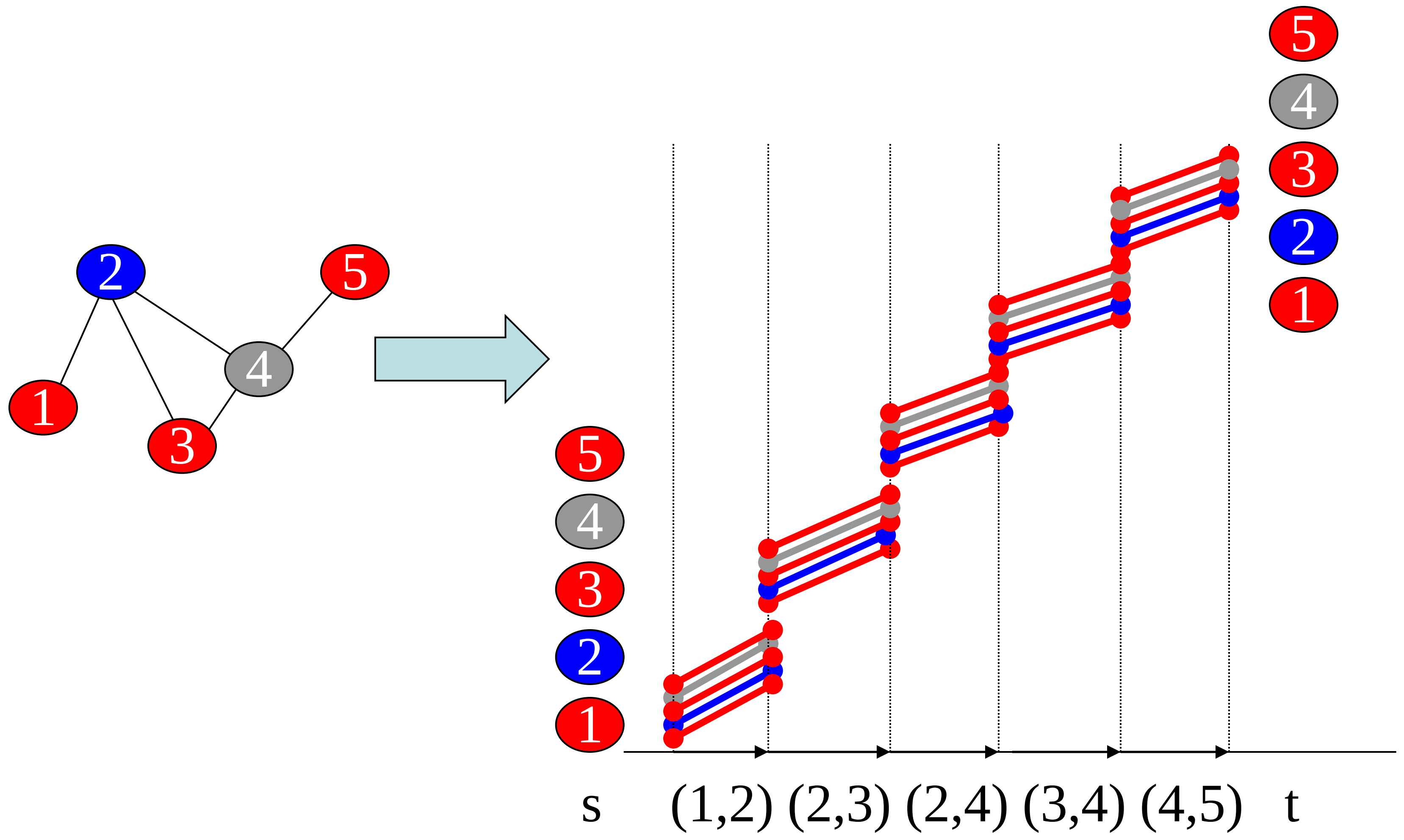
$$G = (V, E)$$



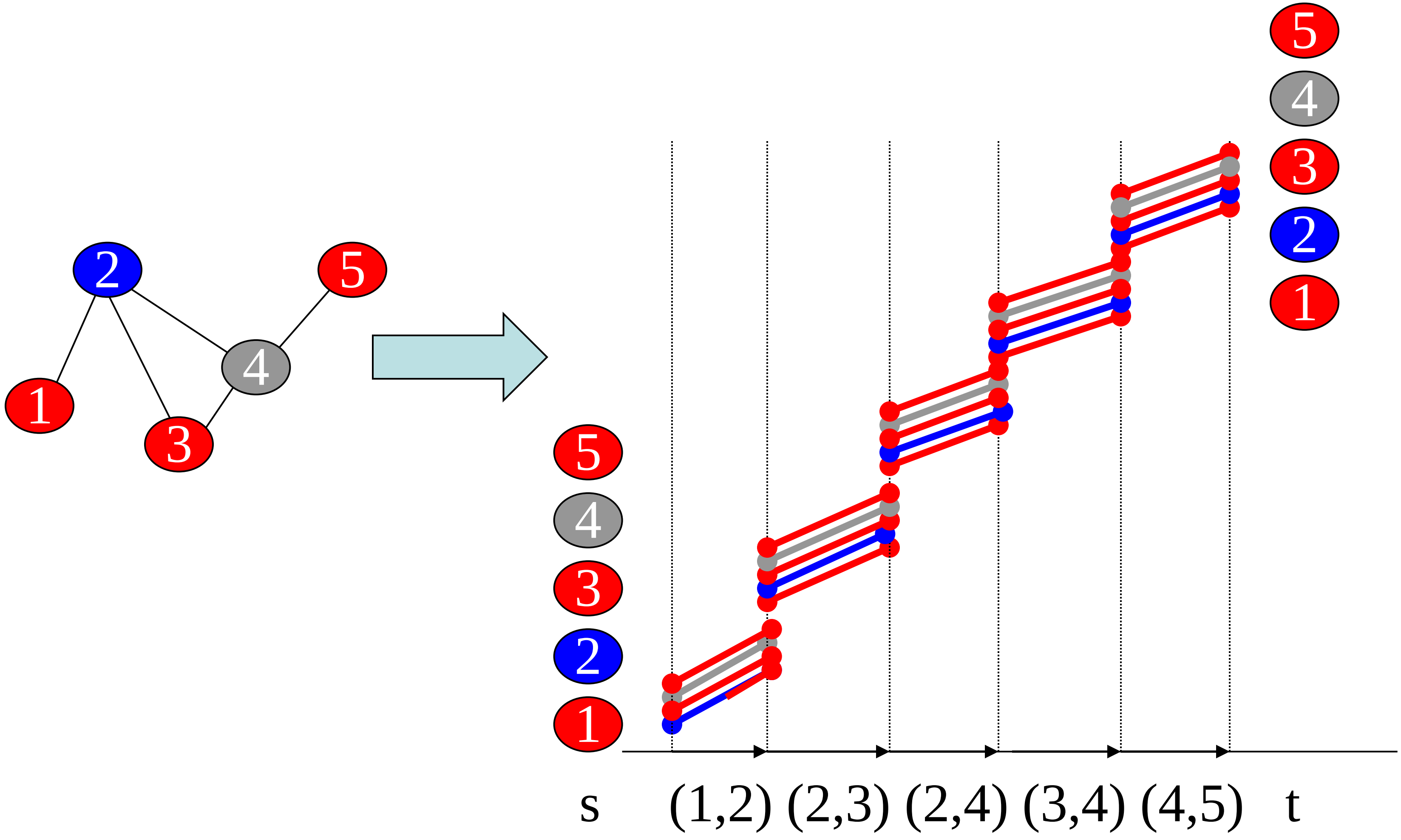


$$\max_{s \in S} |s|$$

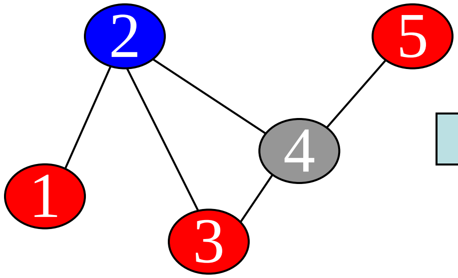
Polynomial Reduction

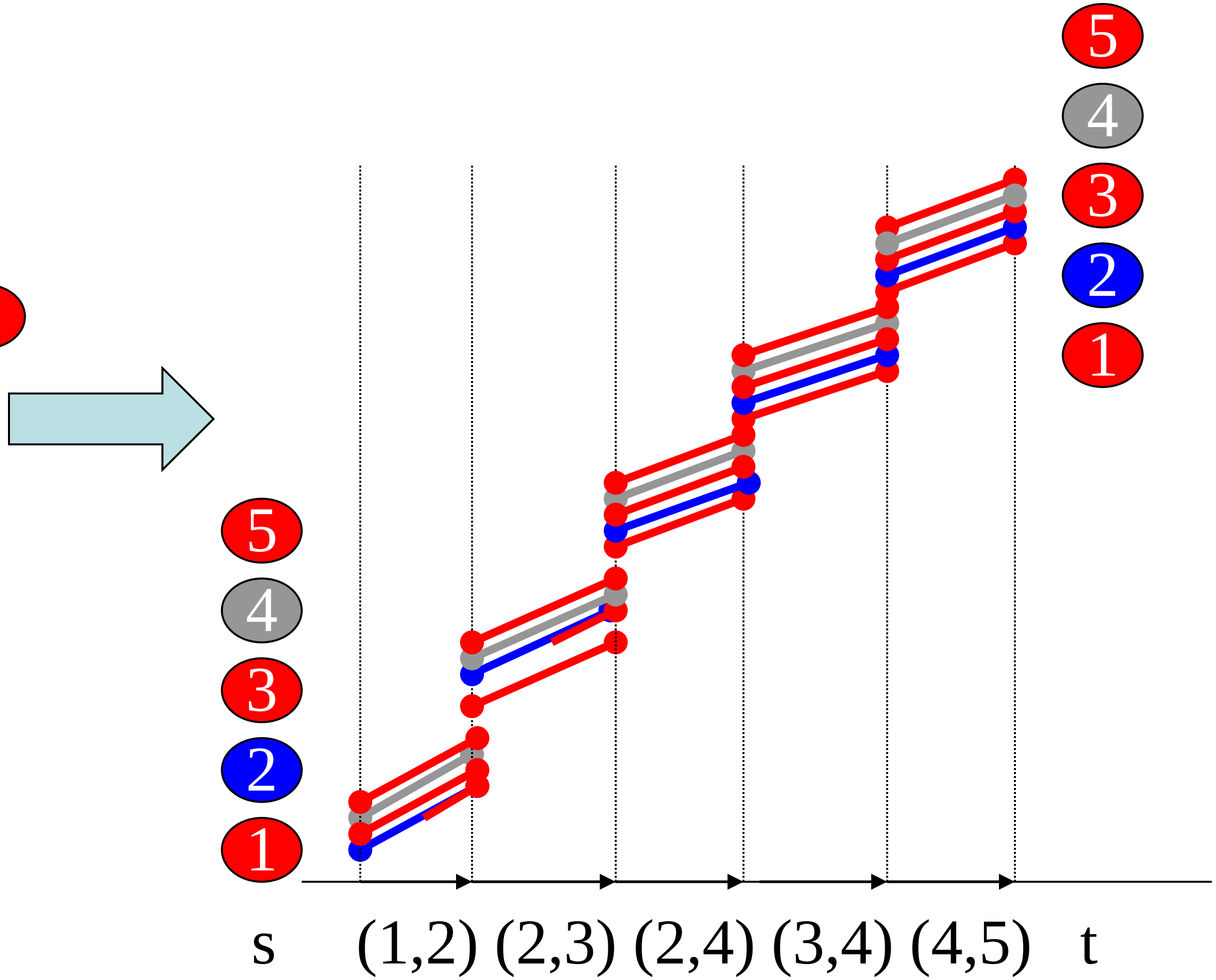


Edge (1,2) ⇔ Conflict on track (1,2)

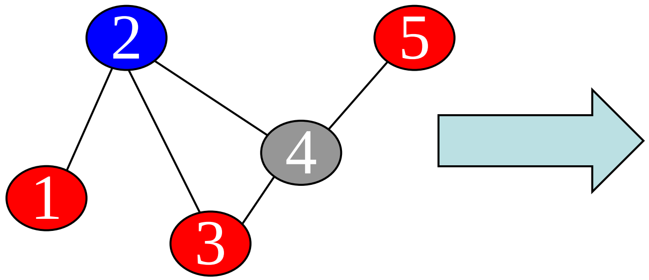


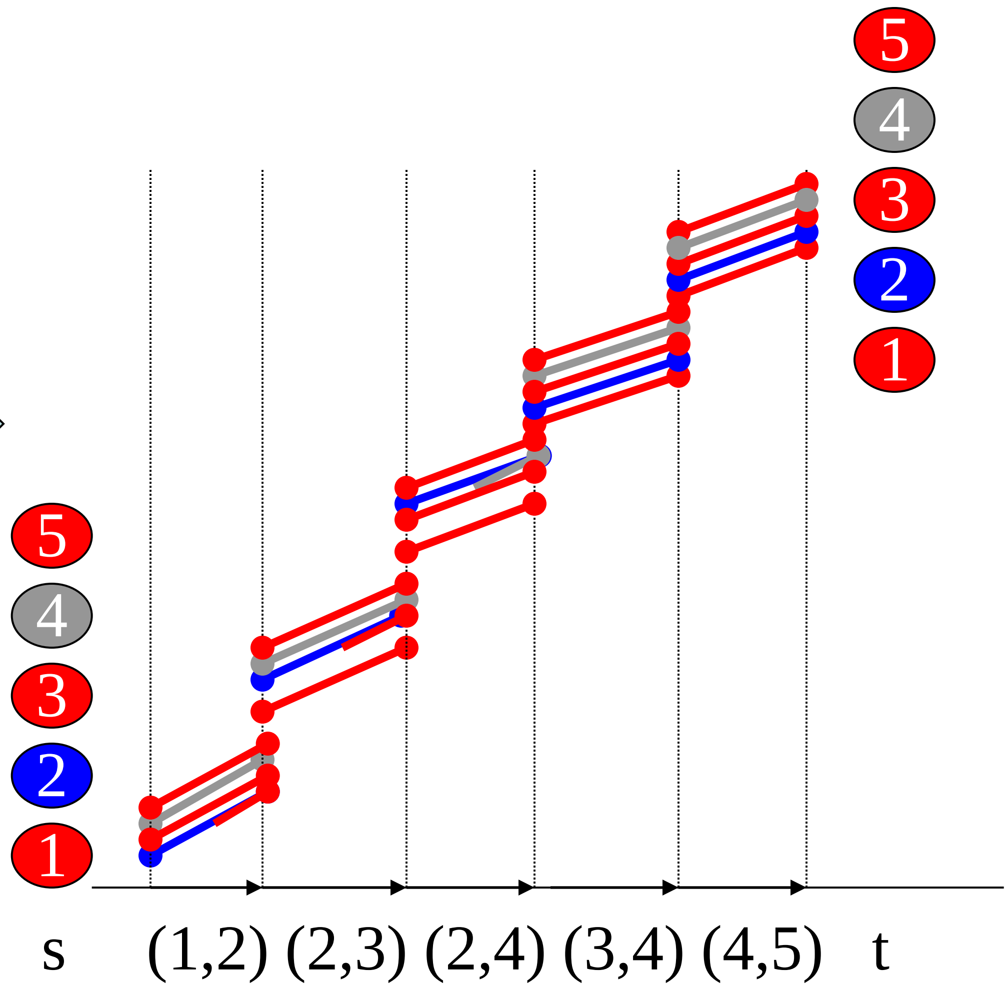
Edge (2,3) ⇔ Conflict on track (2,3)



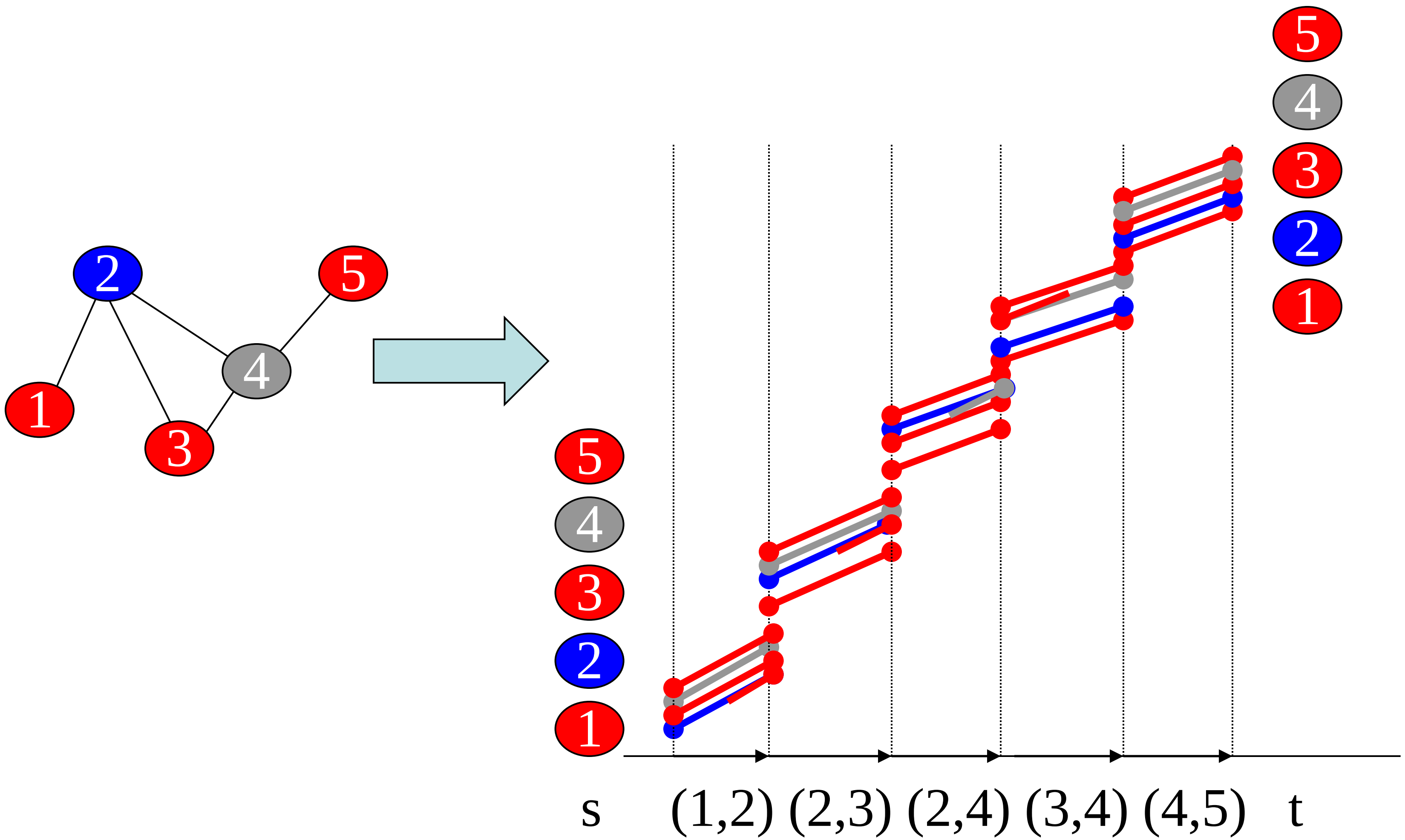


Edge (2,4) ⇔ Conflict on track (2,4)

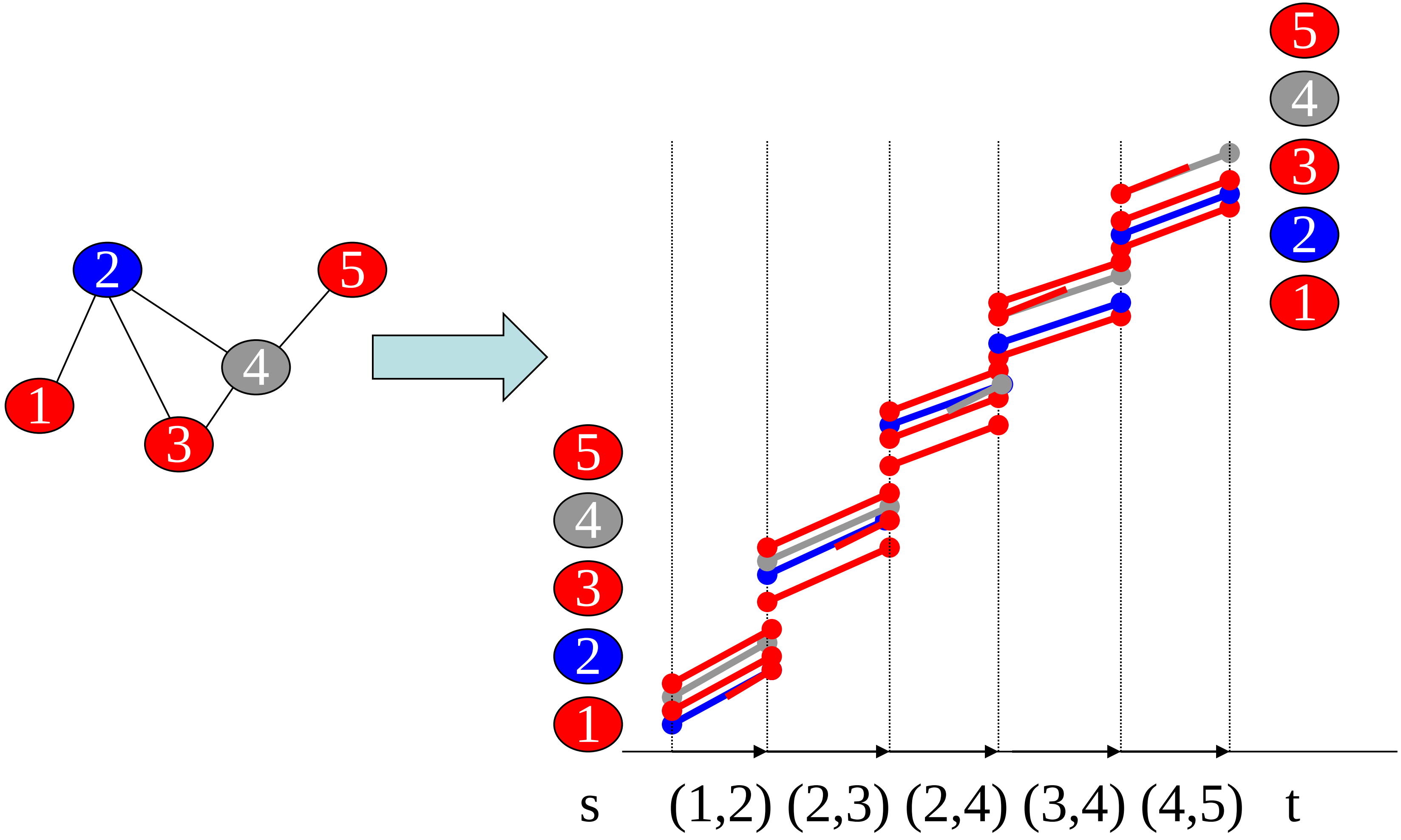




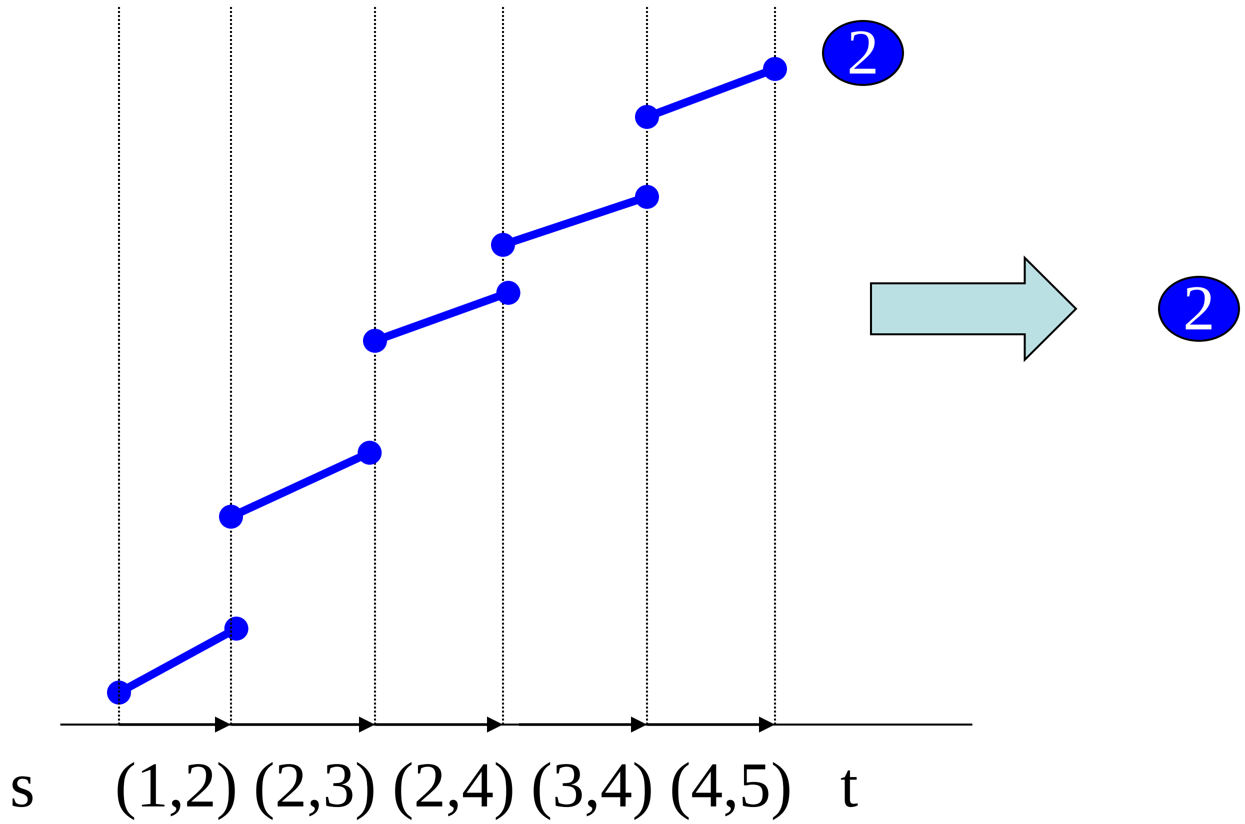
Edge (3,4) ⇔ Conflict on track (3,4)



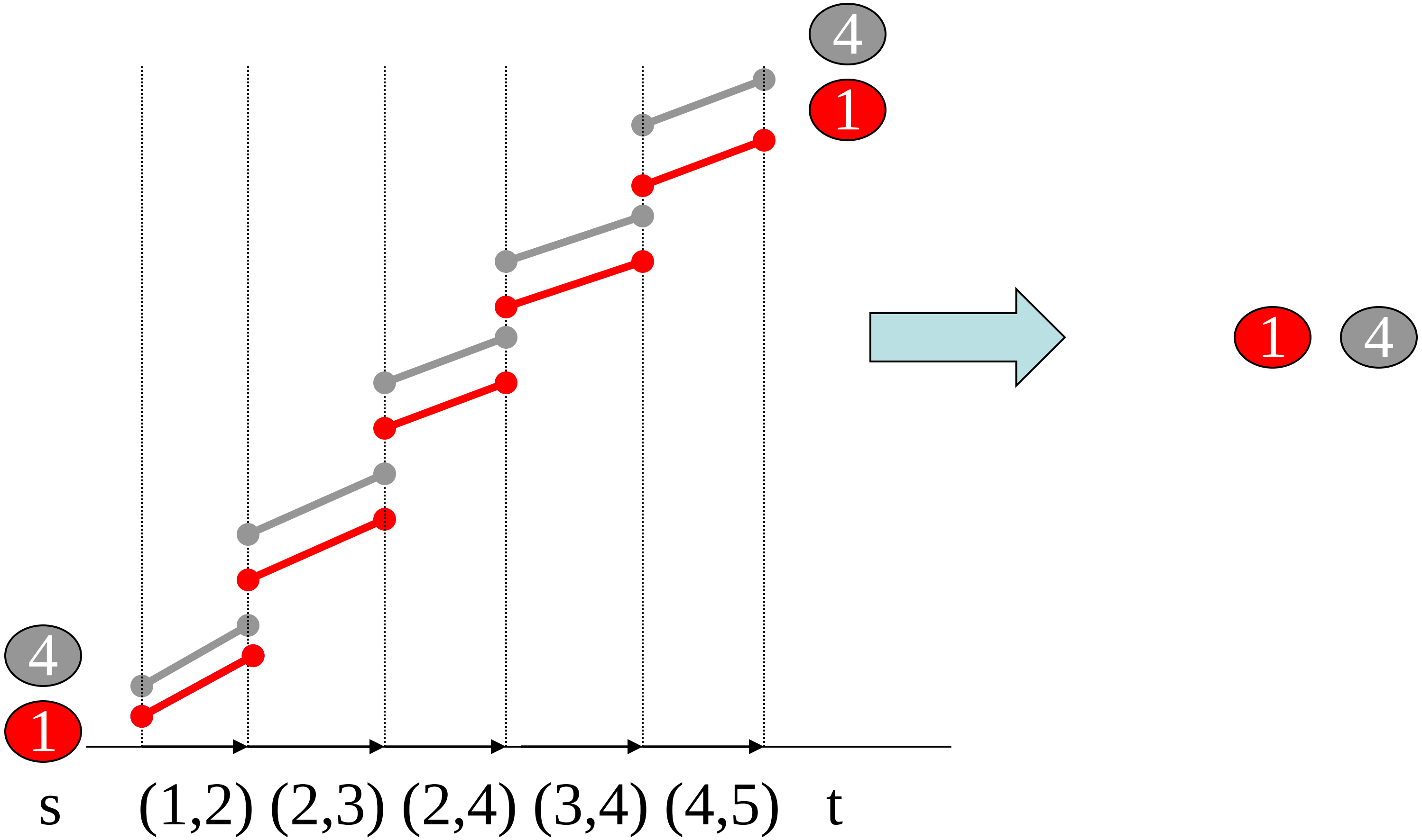
Edge (4,5) ⇔ Conflict on track (4,5)



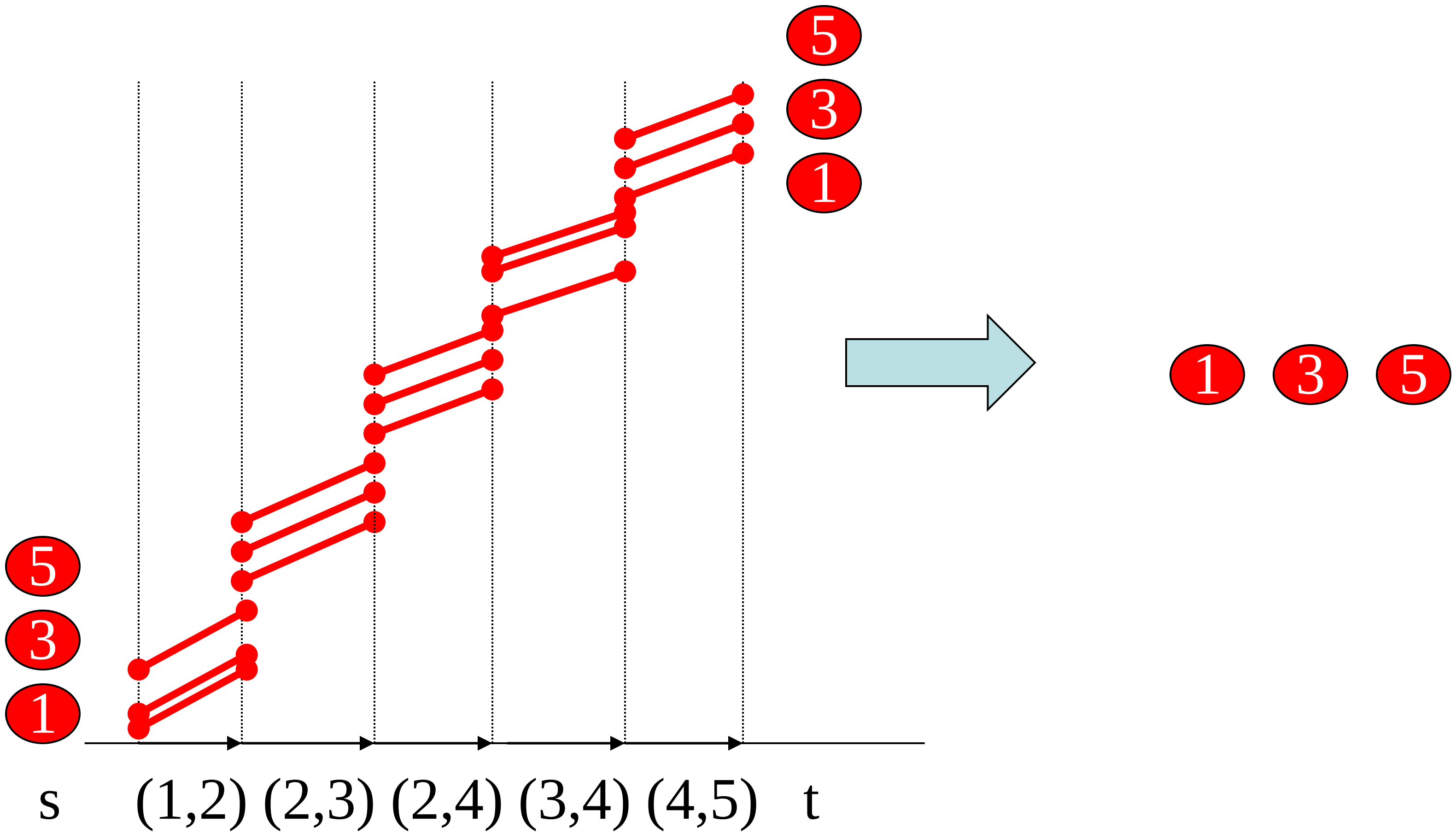
Feasible Set of Train Routes⇔ Stable Set



Maximize Scheduled Trains⇔ Maximize Independent Set



Optimal Schedule



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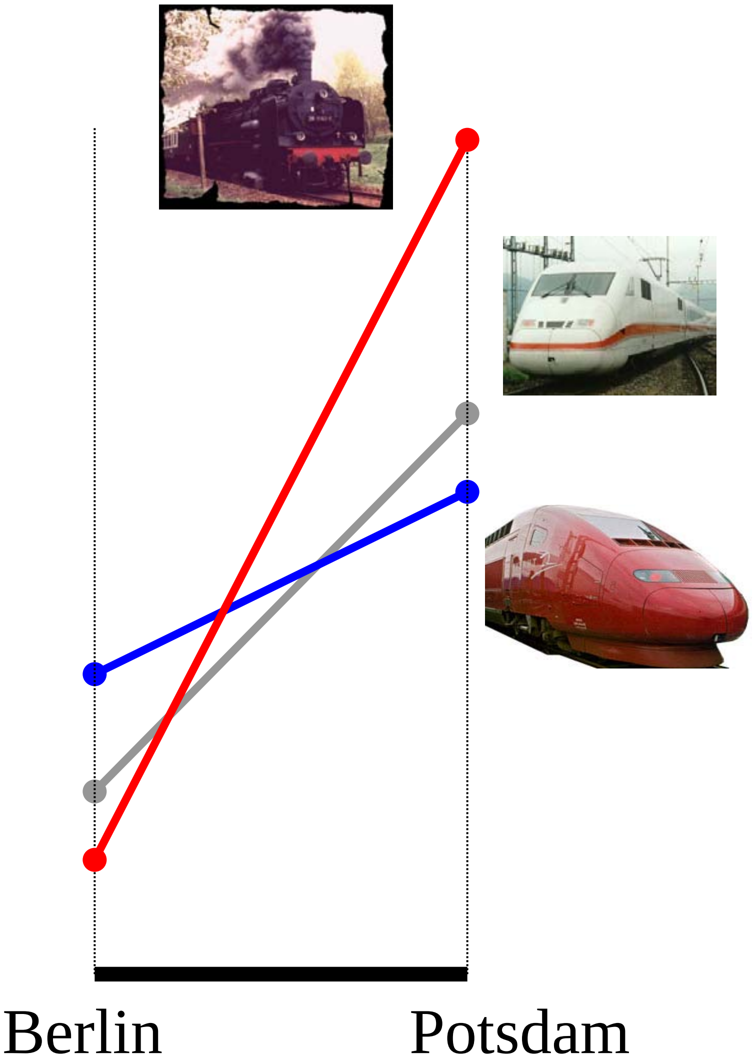
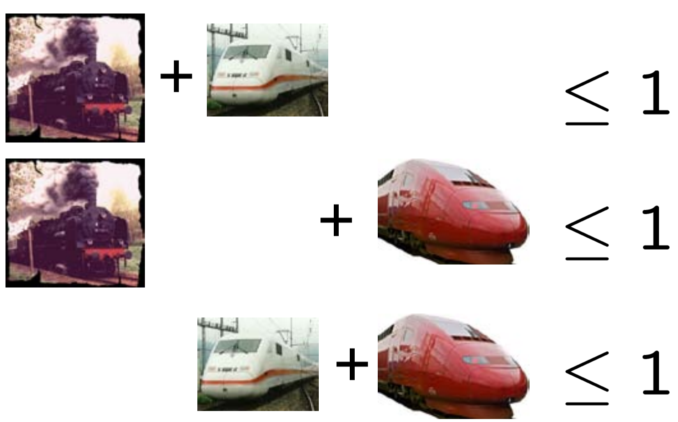


maximize

subject to



TTP as Packing Models

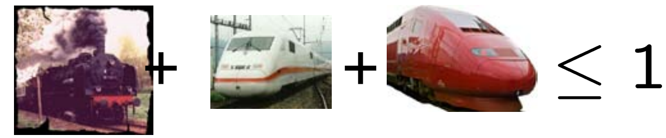
$$
\begin{array}{llll}
(PPP) & & & \\
\max & \sum_{i\in\mathcal{I}} \sum_{p\in P_i} u_p^i x_p^i & & \\
\text{s.t.} & \sum_{i\in\mathcal{I}} x_p^i \leq 1 & \forall i \in \mathcal{I} & \text{(i)} \\
 & \sum_{p\in c} x_p^i \leq \kappa_c & \forall c \in C & \text{(ii)} \\
 & x_p^i \in \{0,1\} & \forall p \in P_i, \forall i \in \mathcal{I} & \text{(iii)}
\end{array}
$$

- **Variables**
  - Path usage (request i uses path p)
- **Constraints**
  - Do no violated conflict sets
- **Objective**
  - Maximize utility/ proceedings

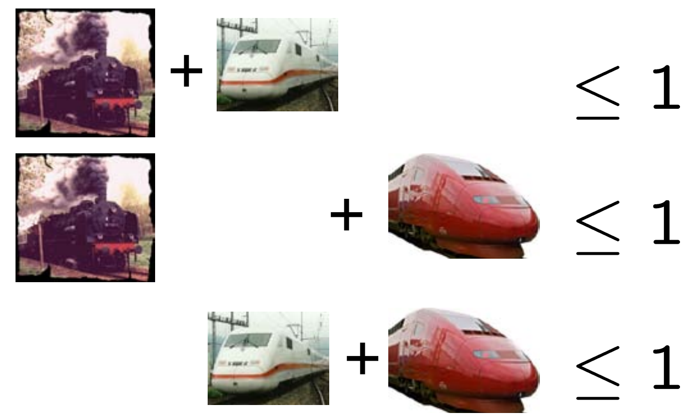
- PhD Thesis V.Cachhiani (2007)
  - Comparability graphs for APP and PPP

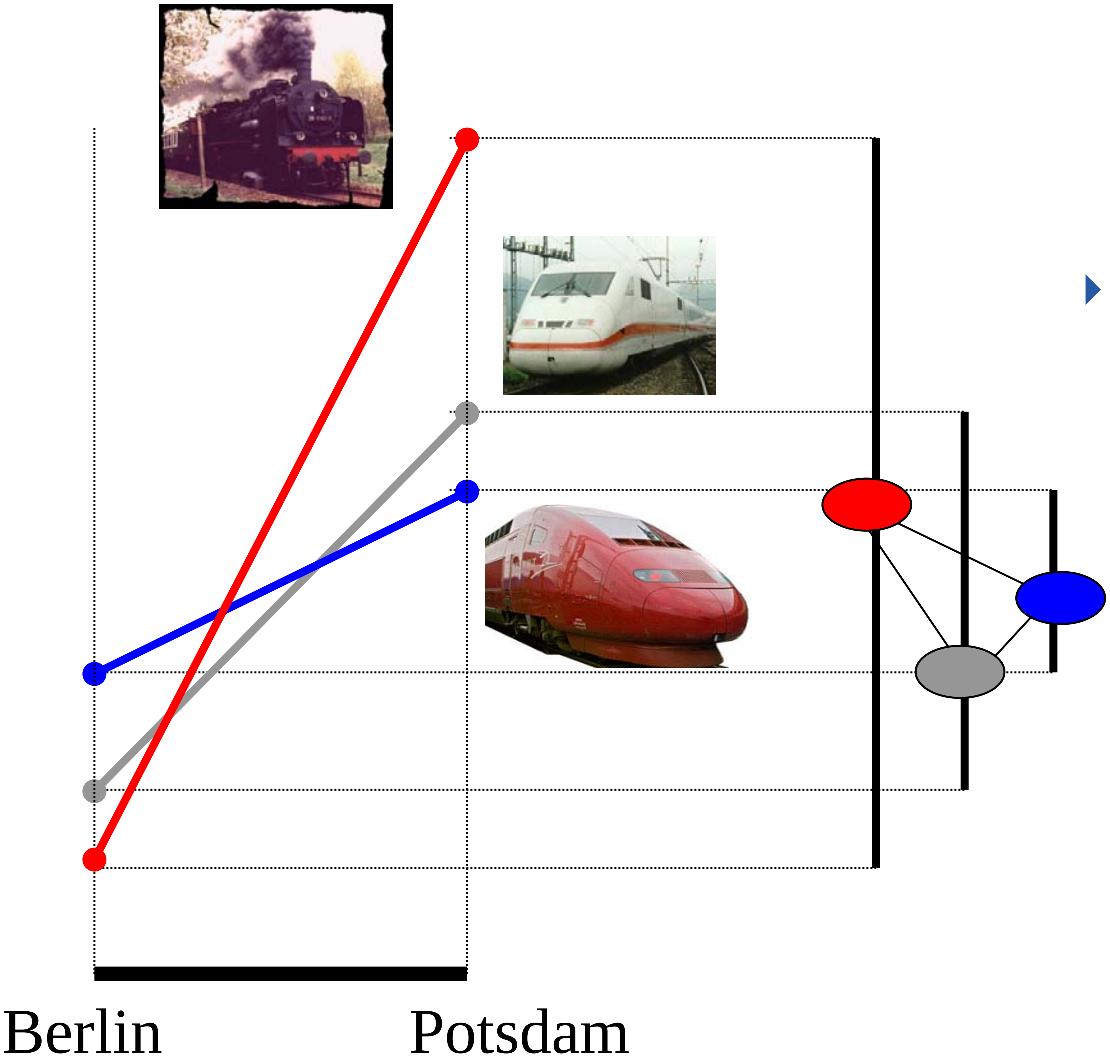
Maximal Conflicts Sets

- **Maximal Conflict Sets**

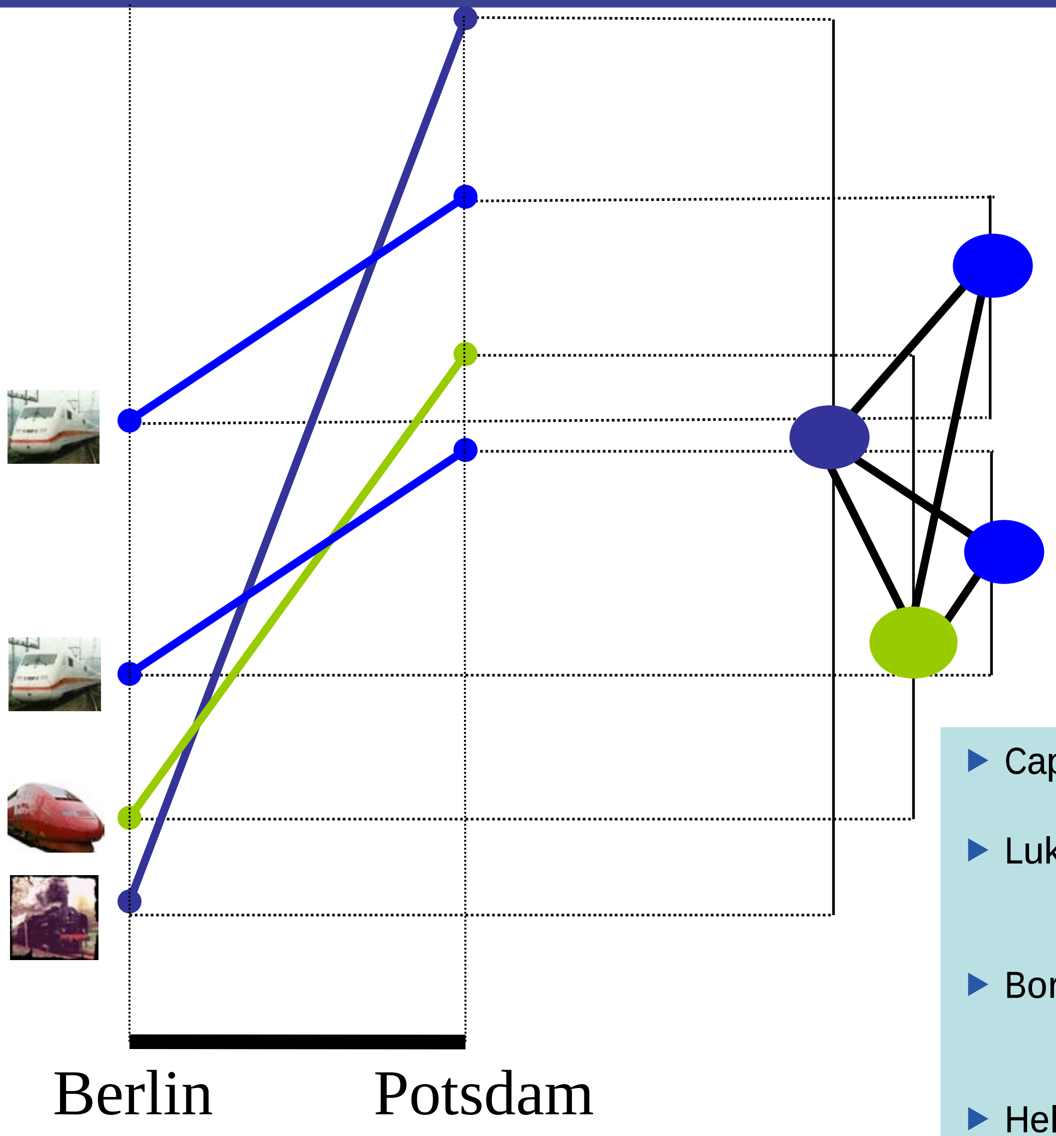


- **Pairwise Conflicts**





Packing Models in Theory

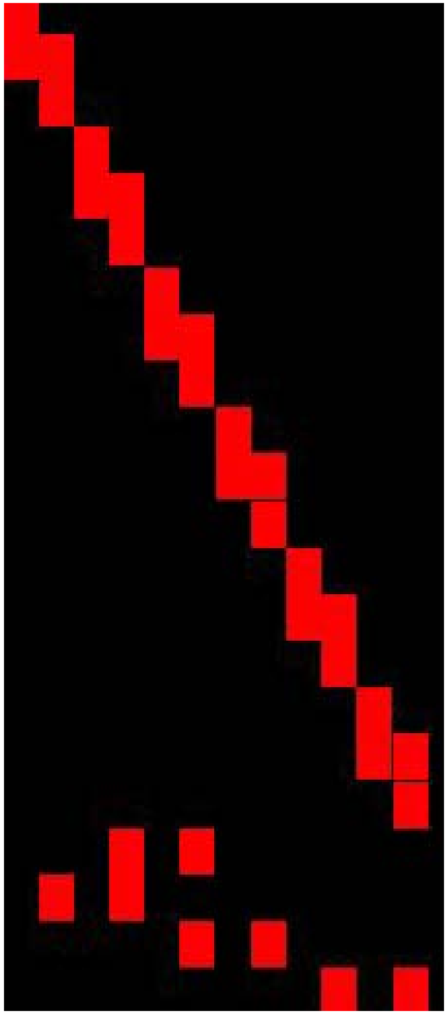
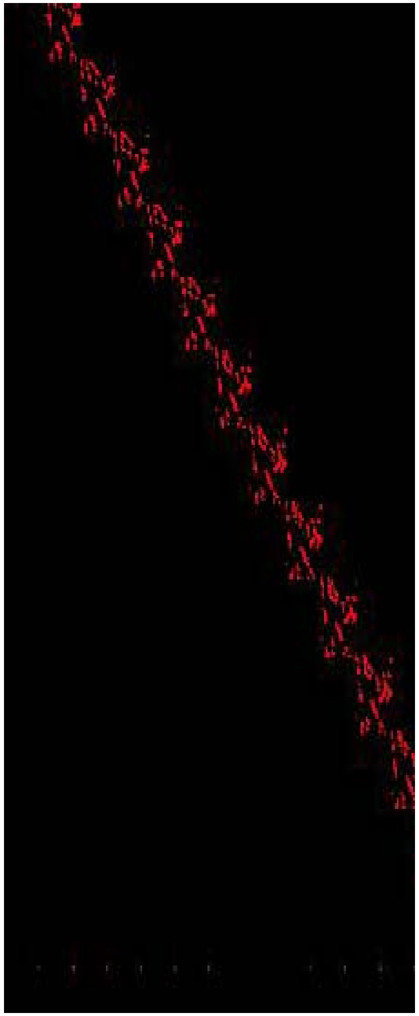
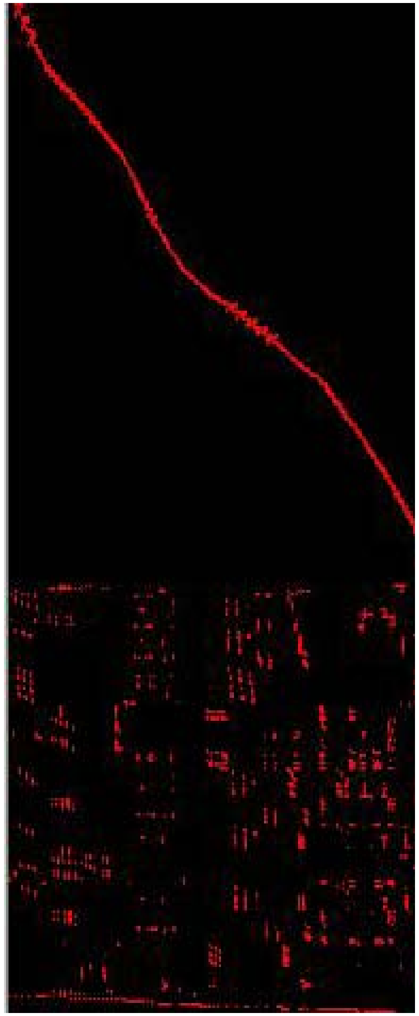
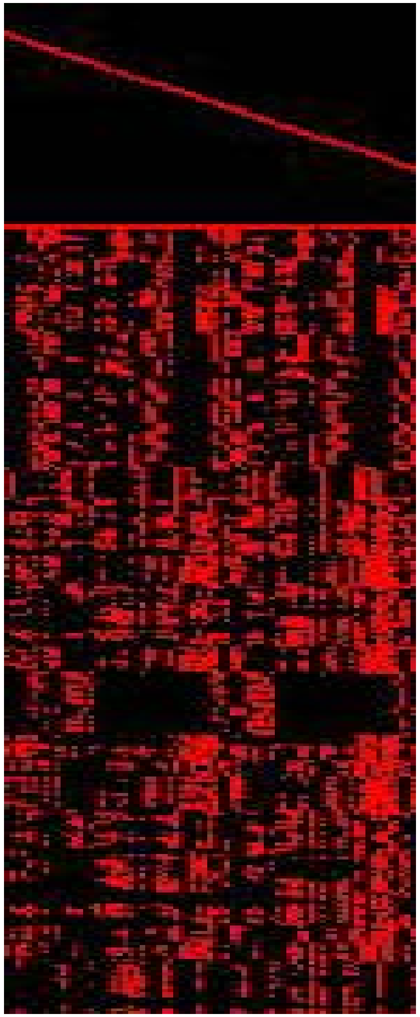


- Conflict graph
- Maximal cliques
- Perfect graph

- Caprara, Fischetti & Toth (2002)
  - Langrangean approach
- Lukac (2004)
  - Conflict graphs of quadrangle-linear headway matrices
- Borndörfer, S.(2007)
  - Conflict graphs of block occupation (interval graphs)
- Helmberg et al. (2008)
  - Bundle Method for packing formulation

Packing Models

- **Proposition:** The LP-relaxation of APP can be solved in poly. time.
  - ... but in practice.

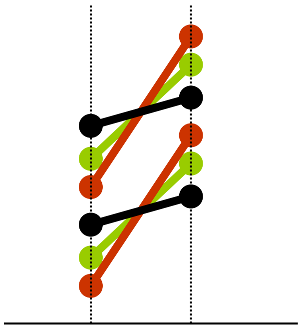





"A bird in the hand is worth two in the bush !"

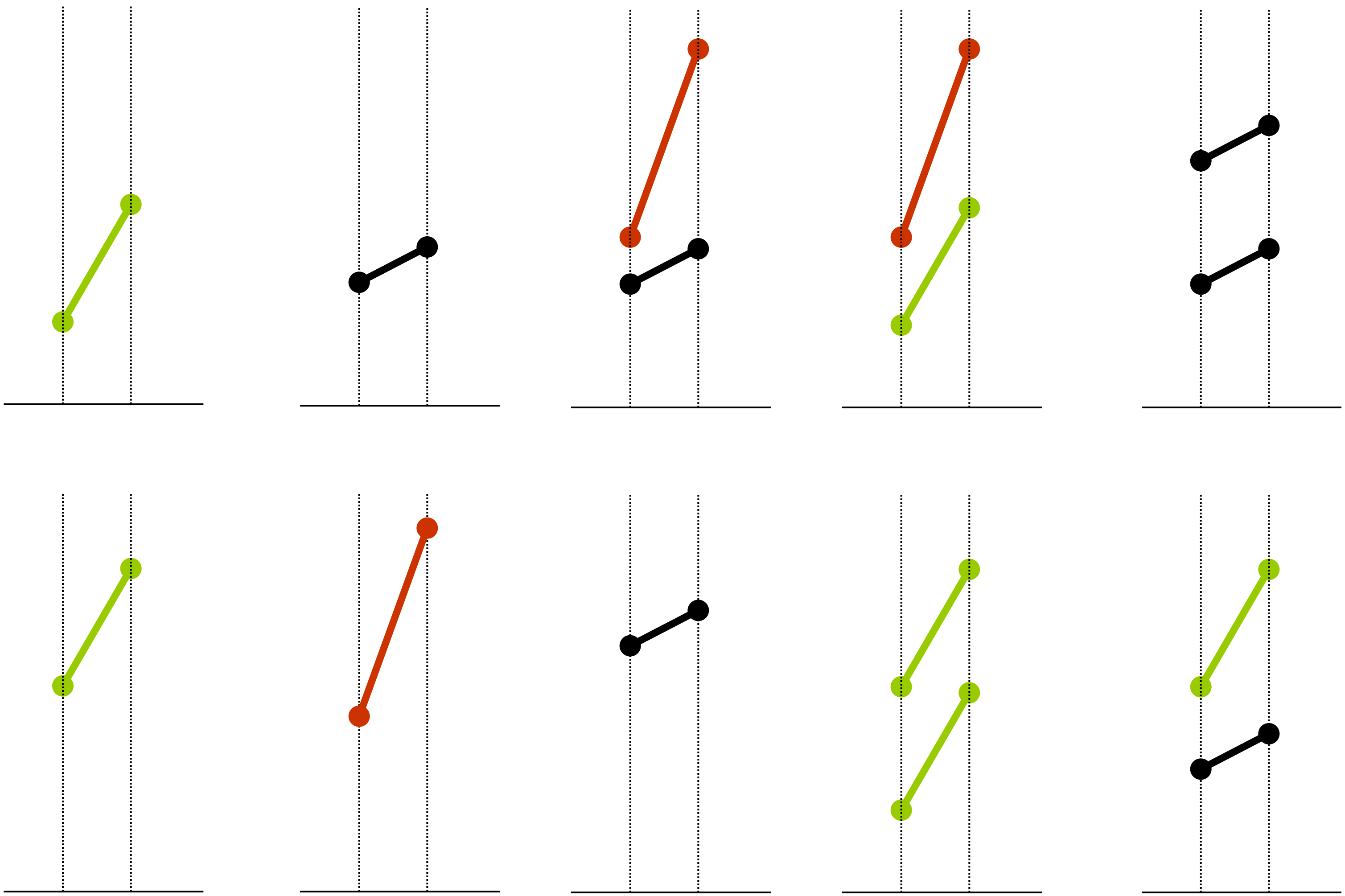
⇔

"A sparrow in the hand is better
than a dove on the roof (german) !"

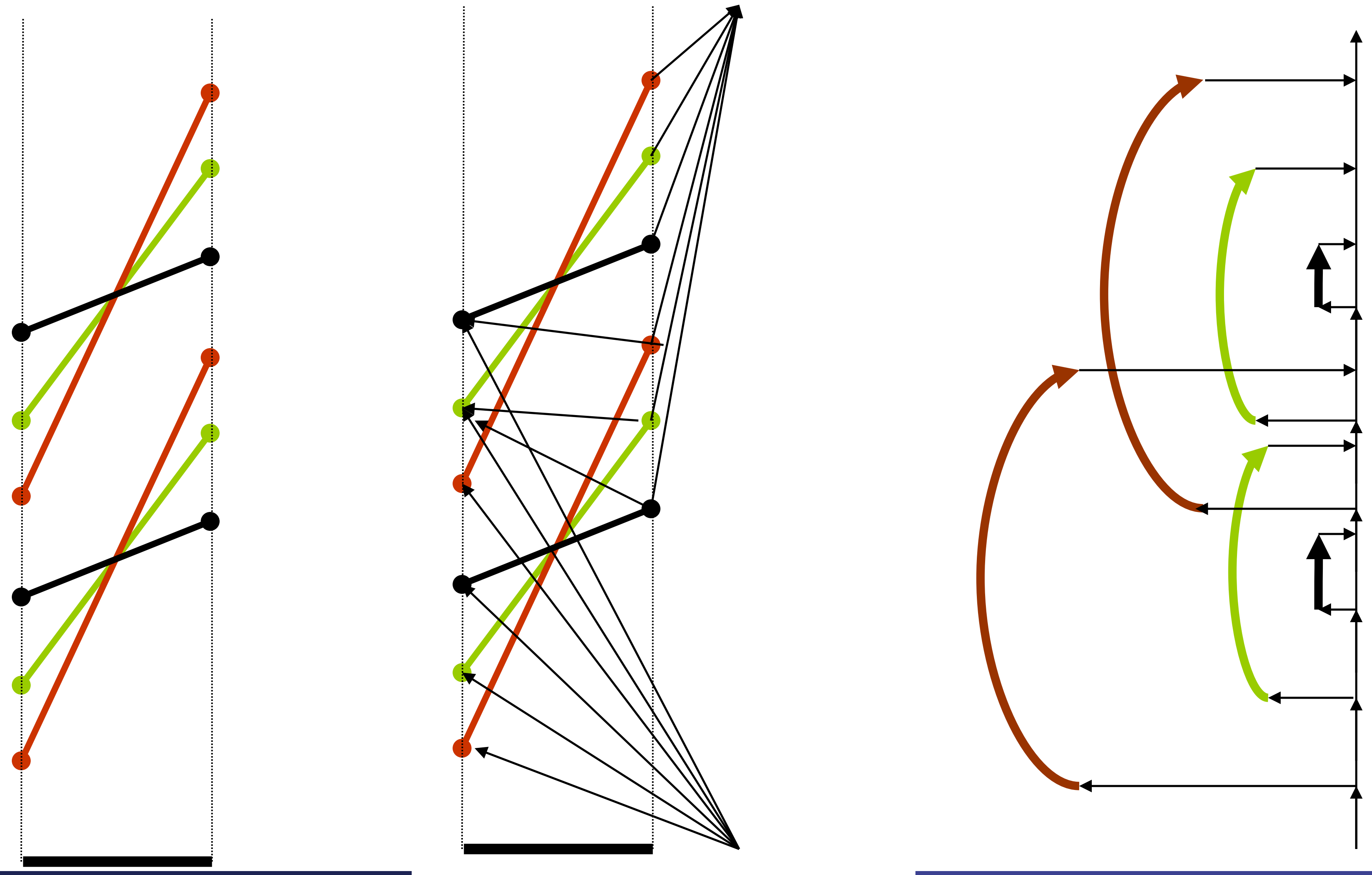
Instead of forcing feasibility of flows by huge number of constraints – allow only feasible flows (inner verus outer approximation).



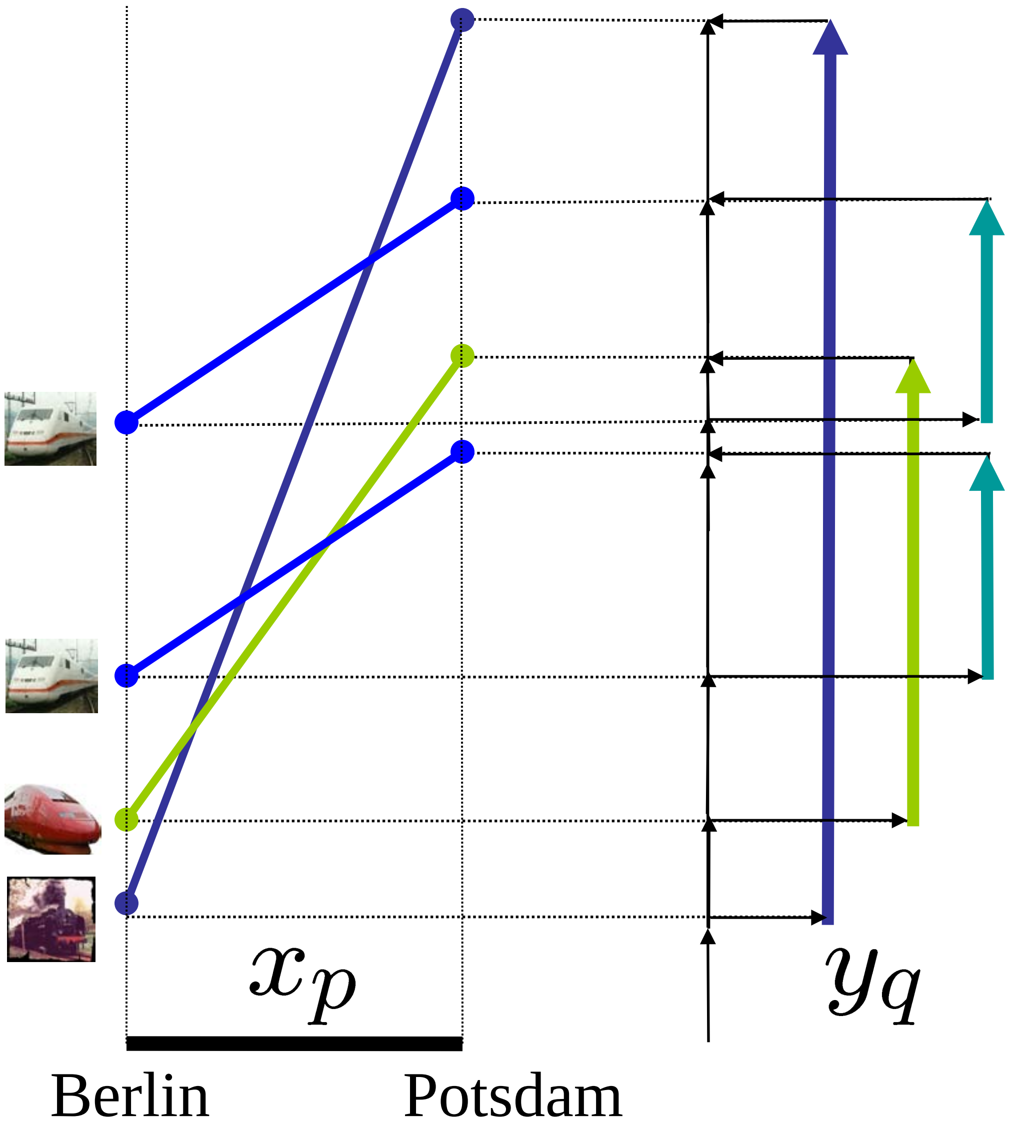
Variables determine Capacity on Tracks



Track Digraph



Alternative Model – Extended Formulation



- Track Digraph
- Timeline(s)
- Config paths

- Balas(2005)
  - Projection, Lifting and Extended Formulation in Integer and Combinatorial Optimization
- Borndörfer, S.(2007)
  - construction for block conflicts
  - construction for triangle linear headway matrices
- Helmberg et. al (2009)
  - dynamic network construction for a special objective function

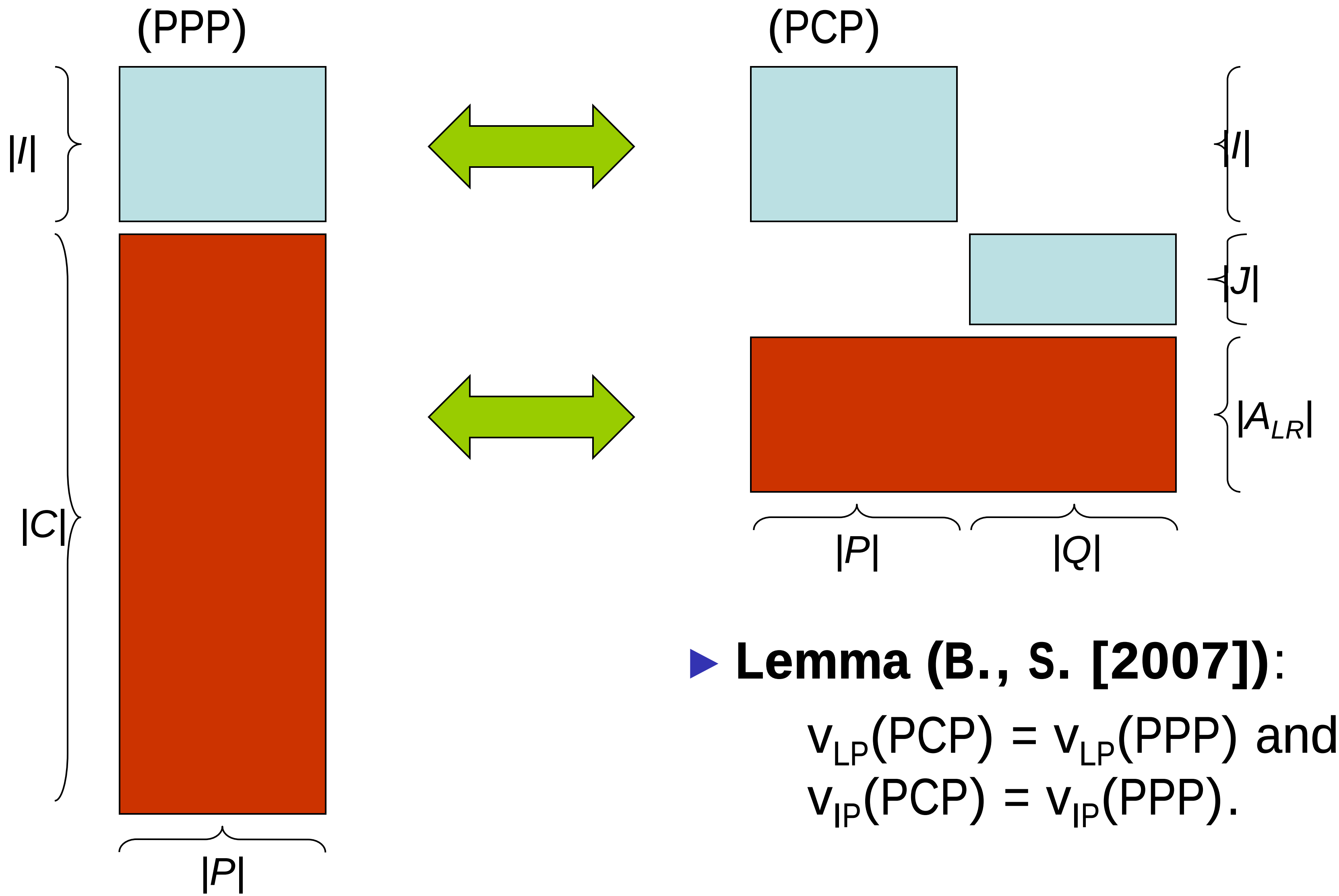
TTP as Path Coupling Problem

(PCP)

$$
\begin{array}{llllll}
\max & \displaystyle\sum_{p \in P} u_p x_p & & & & \text{(i)} \\
\text{s.t.} & \displaystyle\sum_{p \in P_i} x_p & \leq 1, & & \forall\, i \in I & \text{(ii)} \\
& \displaystyle\sum_{q \in Q_j} y_q & \leq 1, & & \forall\, j \in J & \text{(iii)} \\
& \displaystyle\sum_{p \in P, a \in p} x_p - \sum_{q \in Q, a \in q} y_q & \leq 0, & & \forall\, a \in A_{LR} & \text{(iv)} \\
& x_p, y_q & \geq 0, & & \forall\, p \in P,\ q \in Q & \text{(vi)} \\
& x_p, y_q & \in \{0, 1\}, & & \forall\, p \in P,\ q \in Q. & \text{(vii)}
\end{array}
$$

- **Variables**
  - Path and config usage (request i uses path p, track j uses config q)
- **Constraints**
  - Path and config choice
  - Path-config-coupling to ensure track feasibilty (capacity)
- **Objective**
  - Maximize utility/proceedings

PCP is an extended formulation of PPP



- **Lemma (B., S. [2007])**:

  $v_{LP}(PCP) = v_{LP}(PPP)$ and
  $v_{IP}(PCP) = v_{IP}(PPP)$.

Agenda

- The Track Allocation Problem
  - Motivation
  - Real World Problem
  - Complexity
- Integer Programming Models
  - Packing
  - Extended Formulation
  - Comparison
- Solution Approach
  - Column Generation
  - Lagrange Relaxation

(MLP)

$$
\begin{array}{llll}
\max & \sum_{p\in P} u_p x_p & & \text{(i)} \\
\text{s.t.} & \sum_{p\in P_i} x_p \le 1, & \forall\, i \in I & \text{(ii)} \\
& \sum_{q\in Q_j} y_q \le 1, & \forall\, j \in J & \text{(iii)} \\
& \sum_{p\in P, a\in p} x_p - \sum_{q\in Q, a\in q} y_q \le 0, & \forall\, a \in A_{LR} & \text{(iv)} \\
& x_p, y_q \ge 0, & \forall\, p \in P,\ q \in Q & \text{(vi)}
\end{array}
$$

γ_i
π_j
λ_a

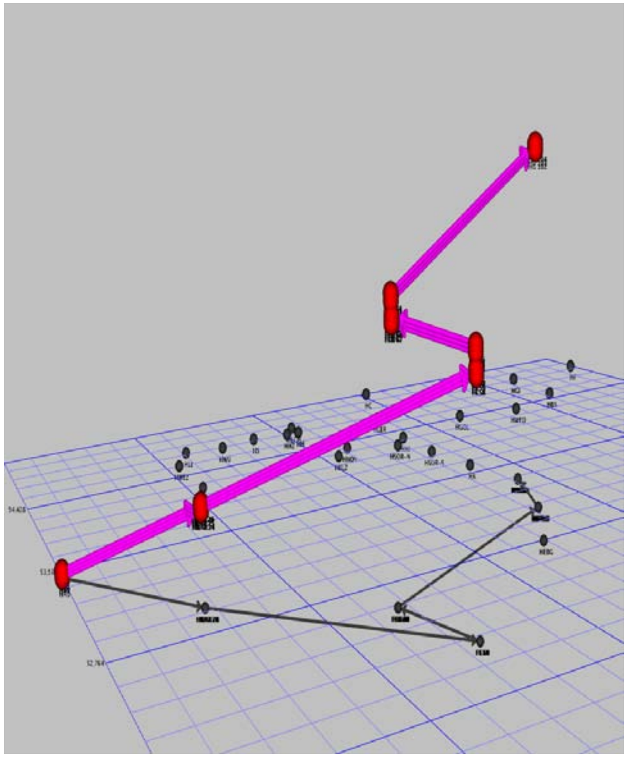
dual variable	information about	useful for
γ_i	bundle price	analysing request
π_j	track price	analysing network
λ_a	arc price	-

Dualization

(DLP)

$$
\begin{array}{llll}
\min & \sum_{j\in J} \pi_j + \sum_{i\in I} \gamma_i & & \\
\text{s.t.} & \gamma_i + \sum_{a\in p} \lambda_a \geq \sum_{a\in p} p_a^i & \forall p \in \mathcal{P}_i, \forall i \in I & \text{(i)} \\
& \pi_j - \sum_{a\in q} \lambda_a \geq 0 & \forall q \in \mathcal{Q}_j, \forall j \in J & \text{(ii)} \\
& \gamma_i \geq 0 & \forall i \in I & \text{(iii)} \\
& \lambda_a \geq 0 & \forall a \in A_I \cup A_J & \text{(iv)} \\
& \pi_j \geq 0 & \forall j \in J & \text{(v)}
\end{array}
$$

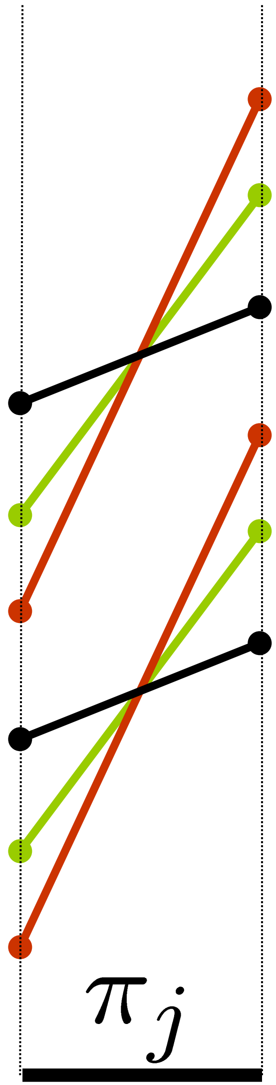
(PRICE (x)) $\exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (u_a - \lambda_a)$



$$c_a = -u_a + \lambda_a$$

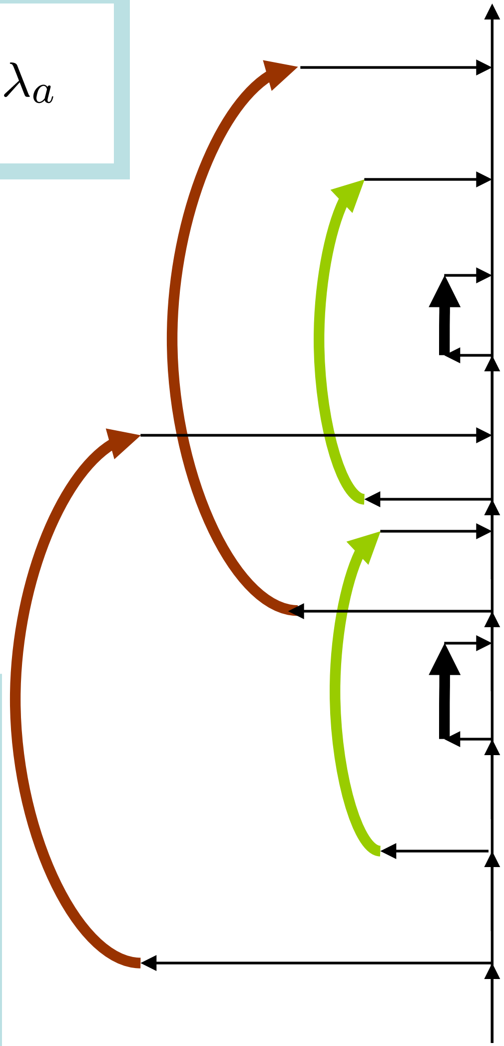
γ_i

Pricing Problem(x) :
Acyclic shortest path problems for each slot request i with modified cost function c !



(PRICE (y)) $\exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a$

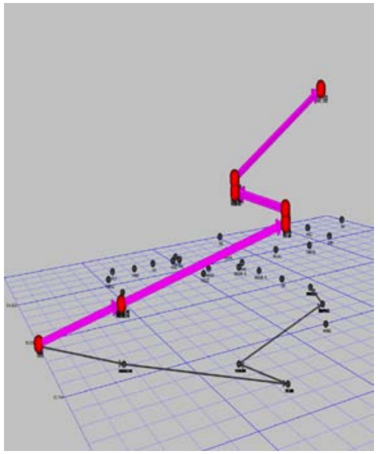
$$c_a = -\lambda_a$$



__Pricing Problem(y) :__
Acyclic shortest path problem for each track j with modified cost function c !

Observation for „optimal“ Pricing

(PRICE (x)) $\exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$



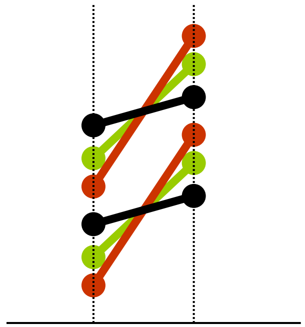
$$\eta_i := \max_{p \in P_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \ \forall i \in I$$

$$\eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \ \forall i \in I, p \in \mathcal{P}_i$$

$\eta_i + \gamma_i$ satisfies $(DLP)(i)$

And analogously …

(PRICE (y)) $\exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a$



$$\theta_j \;:=\; \max_{\overline{q} \in Q_j} \sum_{a \in \overline{q}} \lambda_a - \pi_j,\ \forall j \in J$$

$$\theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \ \forall j \in J, q \in \mathcal{Q}_j$$

$\theta_j + \pi_j$ satisfies $(DLP)(ii)$

Pricing Upper Bound

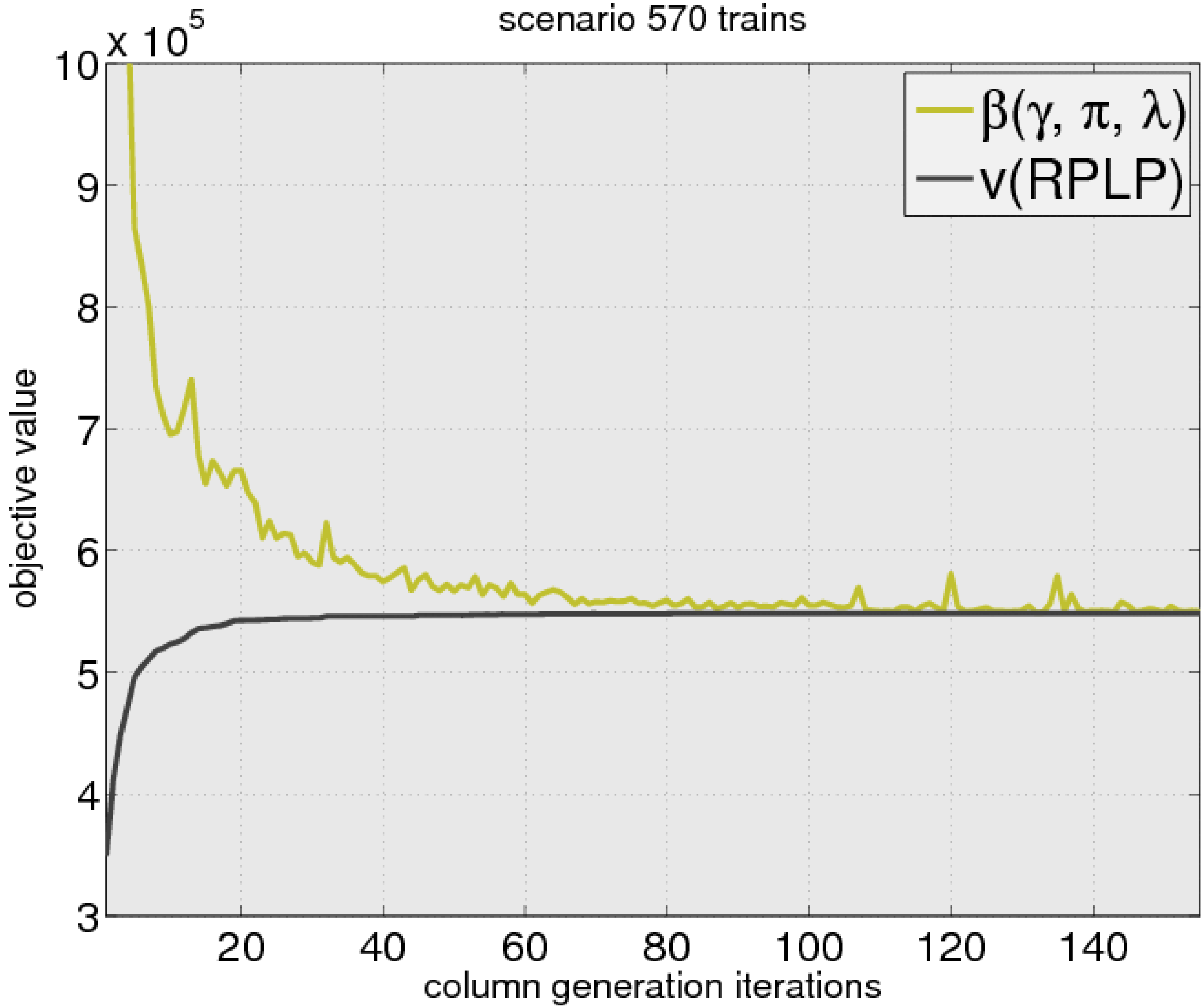
$(\max\{\eta+\gamma,0\},\max\{\theta+\pi,0\},\lambda)$ is feasible for (DLP)

$$\beta(\gamma,\pi,\lambda) := \sum_{i\in I}\max\{\gamma_i+\eta_i,0\}+\sum_{j\in J}\max\{\pi_j+\theta_j,0\}$$

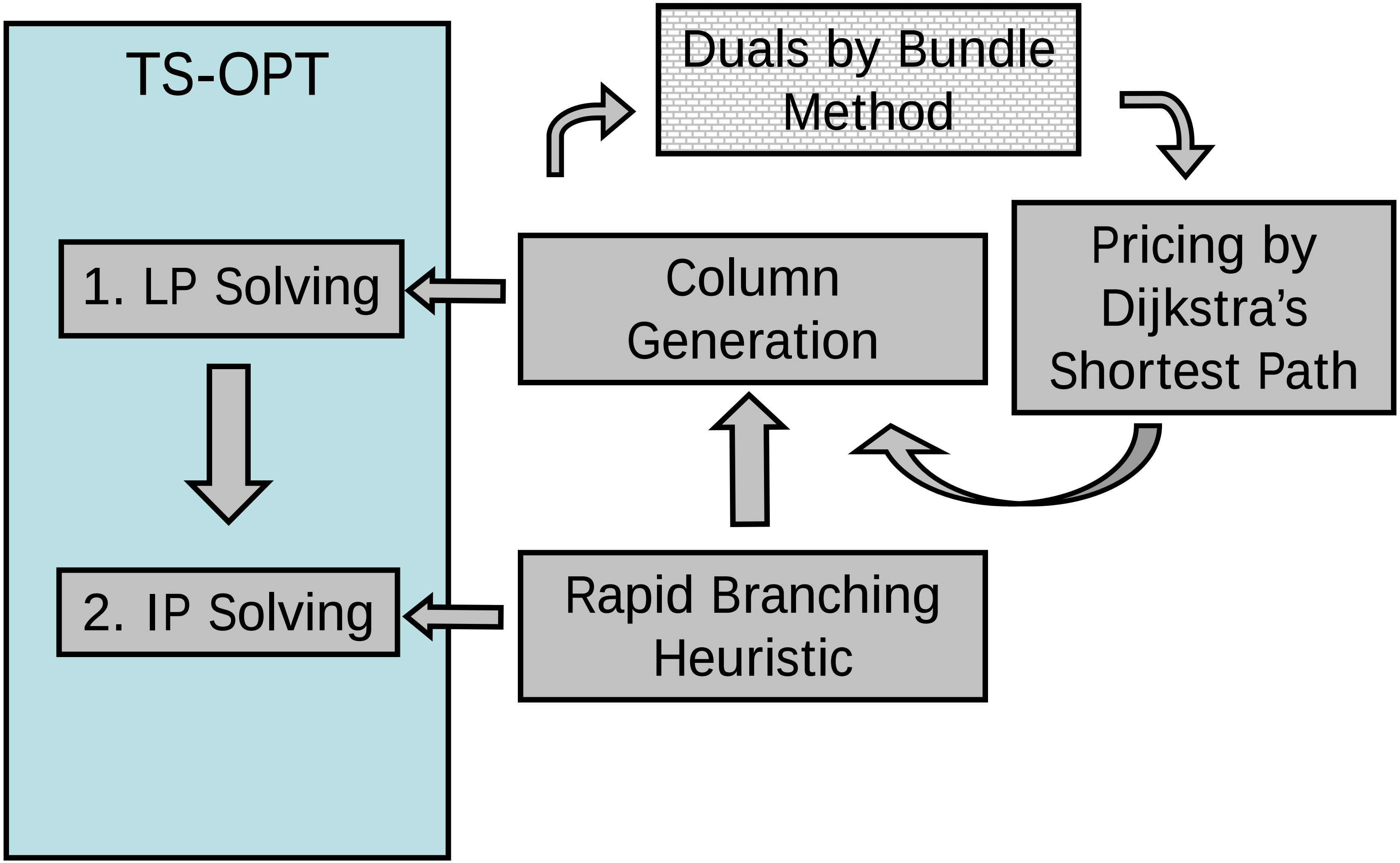
Lemma: Given (infeasible) dual variables of PCP and let $v_{LP}(PCP)$ be the optimum objective value of the LP-Relaxtion of PCP, then:

$$v_{LP}(PCP) \leq \beta(\gamma,\pi,\lambda)$$

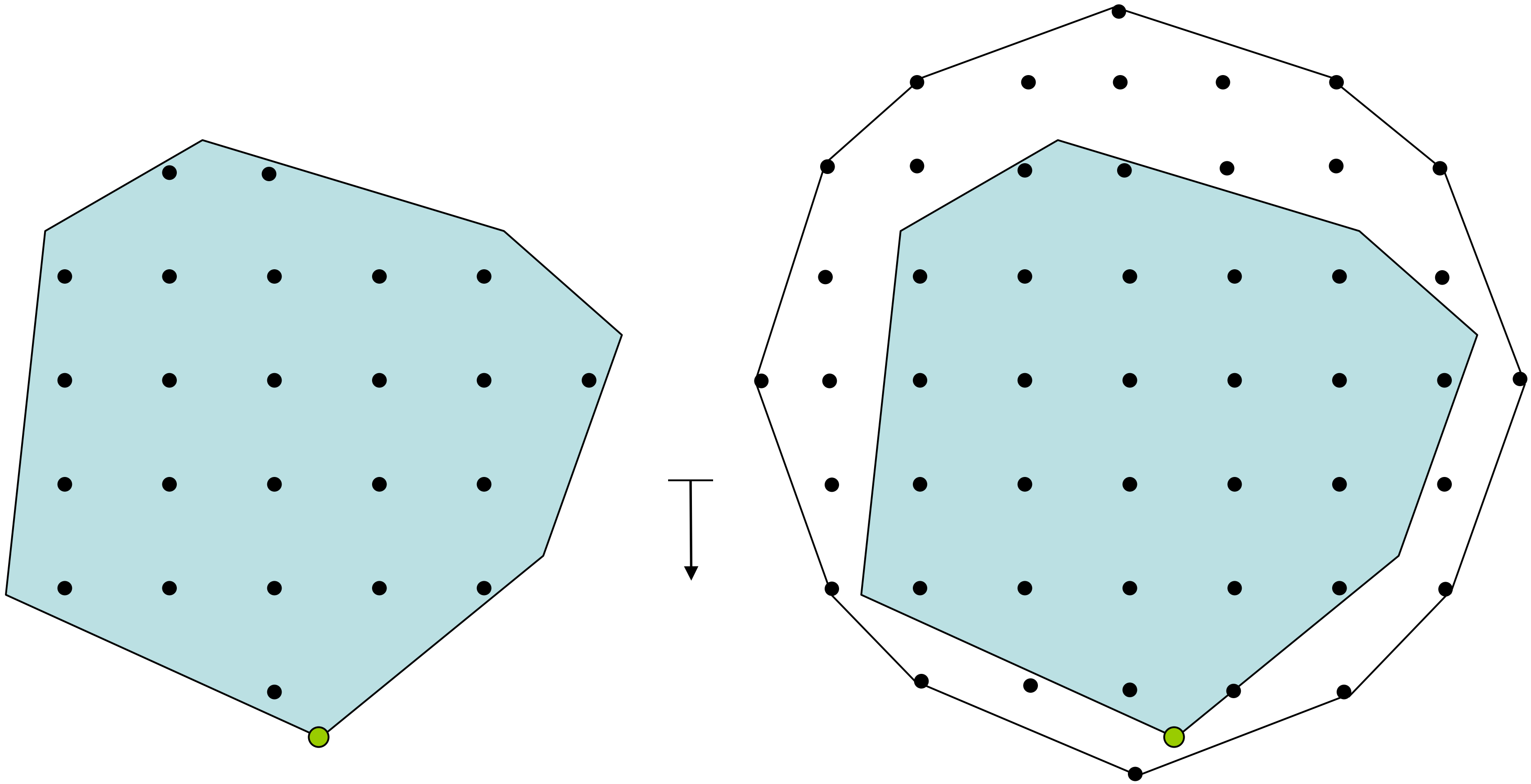
PCP-Run of TS-OPT / LP Stage



Two Step Approach



Linear versus Lagrangean Relaxation



- solved by simplex or barrier methods
- feasible fractional solution
- strong bound but time and memory consuming

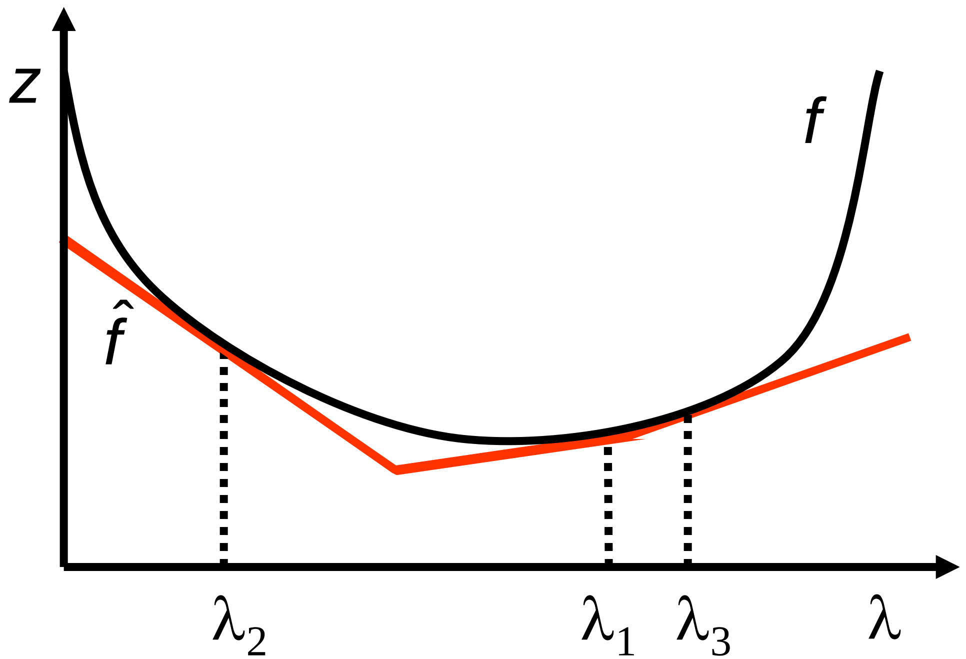
- solved by subgradient or bundle methods
- (infeasible) primal approximation
- potentially better bound and even faster

Langrangean Dual of PCP

$$\text{(LD)} \qquad \min_{\lambda \geq \mathbf{0}} \left[\max_{\substack{Ax = \mathbf{1}, \\ x \in [0,1]^{|P|}}} (u^\top - \lambda^\top C)x + \max_{\substack{By = \mathbf{1}, \\ y \in [0,1]^{|Q|}}} (\lambda^\top D)y \right]$$

Idea of the Bundle Method (Kiwiel [1990], Helmberg [2000])

Problem: minimize convex function f



$$\bar{f}_\mu(\lambda) := f(\mu) + g(\mu)^T(\lambda - \mu),$$
$$g(\mu) \in \partial f(\mu)$$

$$\hat{f}(\lambda) := \max_{\mu \in J_k} \bar{f}_\mu(\lambda)$$

new candidate: $$\lambda_{k+1} = \operatorname*{argmin}_{\lambda \in \mathbb{R}^m} \hat{f}_k(\lambda) + \frac{u_k}{2}\left\|\lambda - \hat{\lambda}_k\right\|^2$$

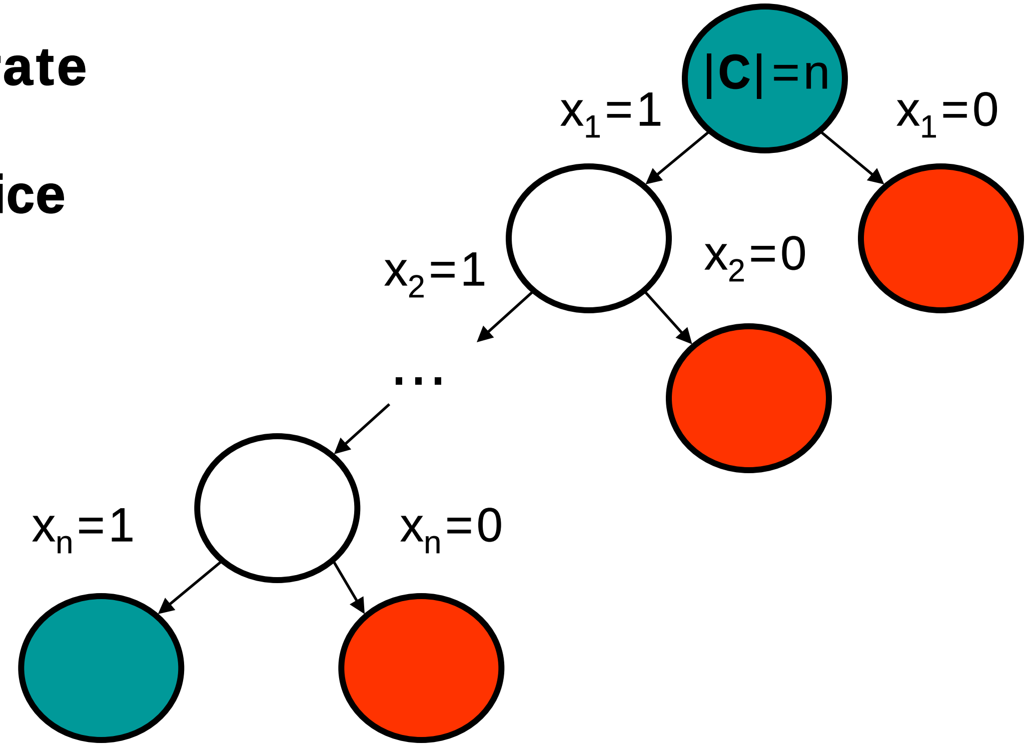
Bundle Algorithm for PCP (B., Weider & S. [2009])

$$\text{(LD)} \quad \min_{\lambda \geq \mathbf{0}} \left[\max_{\substack{Ax=\mathbf{1},\\ x\in[0,1]^{|P|}}} (u^\top - \lambda^\top C)x + \max_{\substack{By=\mathbf{1},\\ y\in[0,1]^{|Q|}}} (\lambda^\top D)y \right]$$

(FE) decomposes in acyclic shortest path

(FE) decomposes in acyclic shortest path

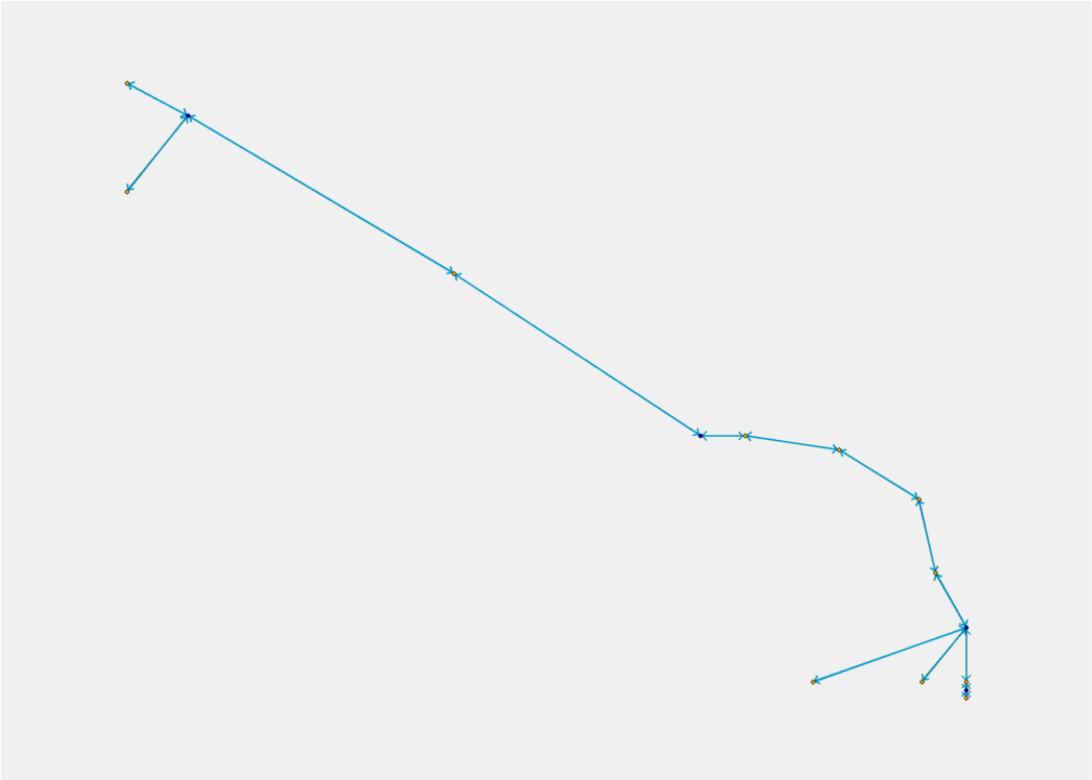
Dive & Generate versus Branch & Price



- diving heuristic guided by primal approximation (or fractional solution)
- use perturbation to decrease integer infeasibilities and identify candidates
- try to fix large subsets of candidates **C** at once to 1
- explore only promising nodes (after some pricing)
- avoid backtracks and ignore 0-branch

Computational Results - Instance "corridor"

- "Obvious" bottleneck in a „real world“ railway system
  - 15 stations, 32 tracks and 6 different train types
  - Already fixed passenger traffic (63 trains per day)
  - How many additional cargo trains can be scheduled ?



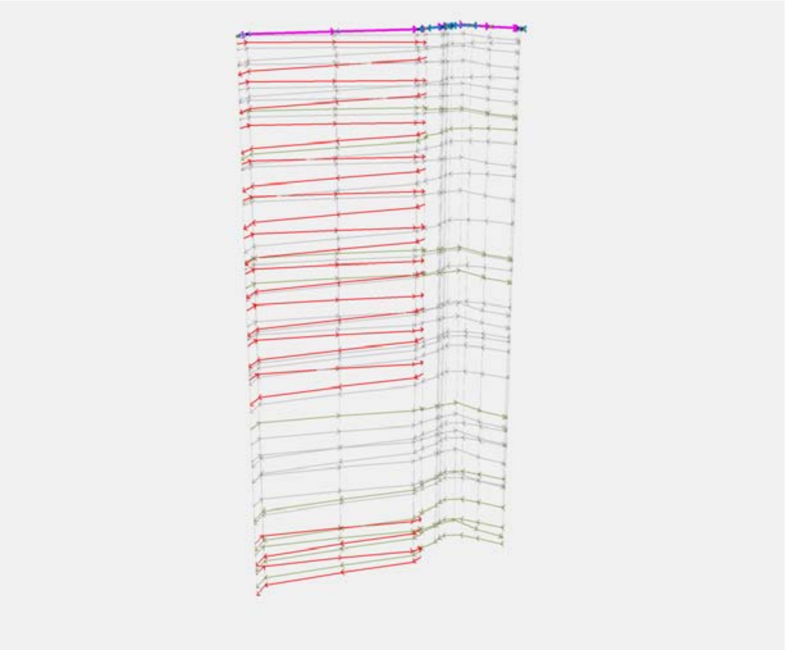
Sensitivity Analysis

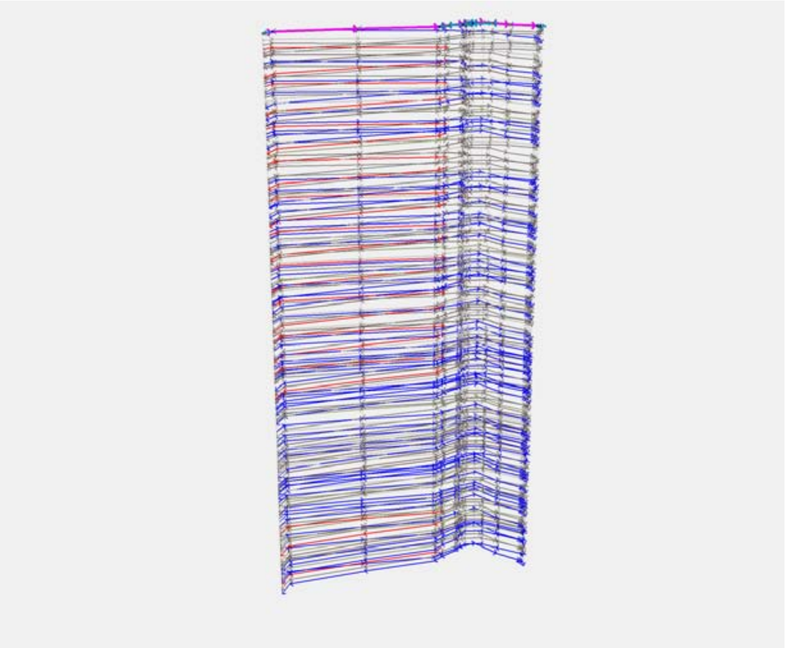
- Computational results for variation of accuracy and routes
  - Network construction with different units of seconds
  - Scenario from 8pm to 12pm (20 passenger trains + 24 cargo trains)
  - Optimization of "same" scenarios with TS-OPT (using model ACP)

Discretization in seconds	6	10	30	60
#Trains with "free" routing through stations (24 routes)	38	38	37	26
#Trains with "fixed" routing through stations (12 routes)	27	24	24	14
Computation time	hours	minutes	seconds	seconds

Saturation Experiment

- Estimation of the maximum "corridor" capacity
  - Network accuracy of 6s
  - Consider complete routing through stations
  - Saturate by additional cargo trains





  - Conflict free train schedules in simulation software (1s accuracy)
  - Proven upper bound of capacity, at most x trains per hour/day (if ....)

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Thank you for your attention !

- Thomas Schlechte

  Zuse Institute Berlin (ZIB)
  Takustr. 7, 14195 Berlin
  Fon (+49 30) 84185-317
  Fax (+49 30) 84185-269

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