# Line Planning in Public Transport

## CO@Work Berlin

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10/02/2009

# Outline

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Introduction

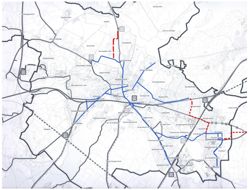
Line Planning with Fixed Passenger Routes

A Column Generation Approach to Line Planning

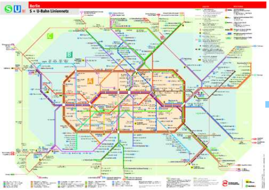
Line Planning in Practice

# Planning Steps in Public Transport

network design



line planning



timetabling

| | Halt | Ankunft | Abfahrt | Fahrt |
|---|---|---|---|---|
| | S+U Alexanderplatz Bhf | | 12:06 | Bus 100 |
| | Spandauer Str./Marienkirche (Berlin) | 12:07 | 12:07 | |
| | Lustgarten (Berlin) | 12:09 | 12:09 | |
| | Staatsoper (Berlin) | 12:11 | 12:11 | |
| | Unter den Linden/Friedrichstr. (Berlin) | 12:13 | 12:13 | |
| | S Unter den Linden | 12:15 | 12:15 | |
| | Reichstag/Bundestag (Berlin) | 12:18 | 12:18 | |
| | Platz der Republik (Berlin) | 12:19 | 12:19 | |
| | Haus der Kulturen der Welt (Berlin) | 12:21 | 12:21 | |
| | Schloß Bellevue (Berlin) | 12:23 | 12:23 | |
| | Großer Stern (Berlin) | 12:24 | 12:24 | |
| | Nordische Botschaften/Adenauer-Stiftung (Berlin) | 12:26 | 12:26 | |
| | Lützowplatz (Berlin) | 12:27 | 12:27 | |
| | Schillstr. (Berlin) | 12:28 | 12:28 | |
| | Bayreuther Str. (Berlin) | 12:29 | 12:29 | |
| | Breitscheidplatz (Berlin) | 12:30 | 12:30 | |
| | S+U Zoologischer Garten Bhf | 12:32 | | |

Verkehrstage:
fährt Mo - Fr, nicht an allg. Feiertg; auch 31. Okt 2006

vehicle scheduling

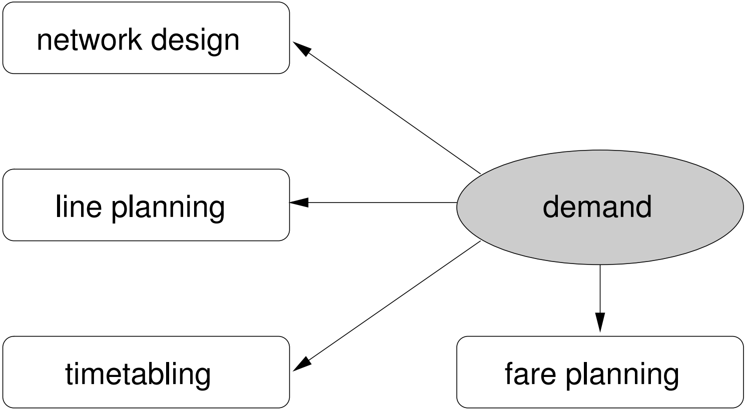
duty scheduling

fare planning

| Einzelfahrausweis [2] | Regeltarif Preis in € | Ermäßigt Preis in € |
|---|---|---|
| Berlin | | |
| AB | 2.10 | 1.40 |
| BC | 2.30 | 1.60 |
| ABC | 2.60 | 1.90 |
| Brandenburg a. d. H., Cottbus, Frankfurt (Oder) | | |
| AB, BC | 1.20 | 0.90 |
| ABC | 2.10 | 1.60 |
| Potsdam | | |
| AB | 1.60 | 1.20 |
| BC | 1.50 | 1.10 |
| ABC | 2.30 | 1.70 |
| Orte mit Stadtlinienverkehr | | |
| Typ I | 1.00 | 0.70 |
| Typ II | 1.10 | 0.80 |
| Typ IV | 0.90 | 0.60 |
| Landkreise | | |
| bis 2 Waben | 1.30 | 1.00 |
| 3 Waben | 1.80 | 1.40 |
| 4 Waben | 2.30 | 1.80 |
| 5 Waben | 3.10 | 2.30 |
| über 5 Waben oder bis 25 km | 3.60 | 2.70 |
| bis 35 km | 4.80 | 3.60 |
| bis 255 km | 31.20 | 23.40 |

2) Einzelfahrausweise gelten nur für eine Fahrt in Richtung auf das Fahrtziel. Umsteigen ist erlaubt.

# Service Design



# Passenger Demand

## Data acquisition

- passenger interviews (purpose of trip, start and end,...)
- automatic counting in vehicles
  $\rightarrow$ reflects aggregated passenger demand of *current* transportation system

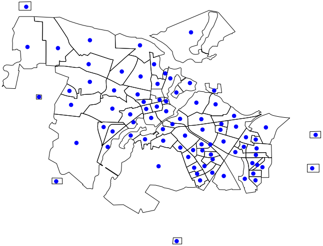
## Data representation

- Origin Destination Matrix (OD Matrix)
- passenger volume for each edge

There are statistical and mathematical programming methods for estimating OD matrices from edge counts.

# OD Matrix

- public transport area divided into different districts
- district represented by OD node
- OD Matrix – number of passengers traveling between each two OD nodes



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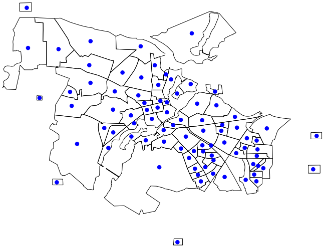
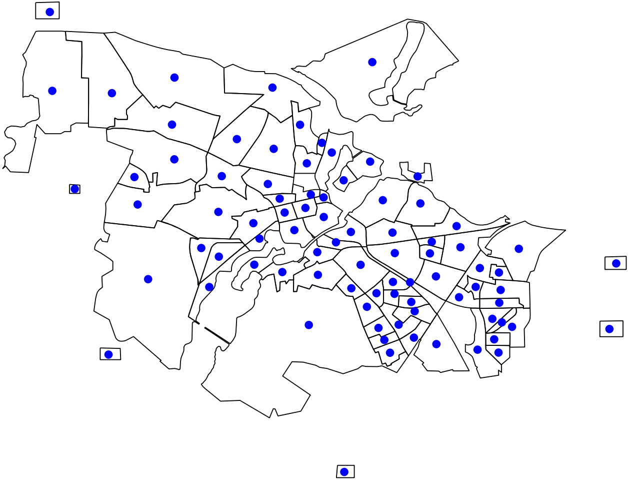


## Features

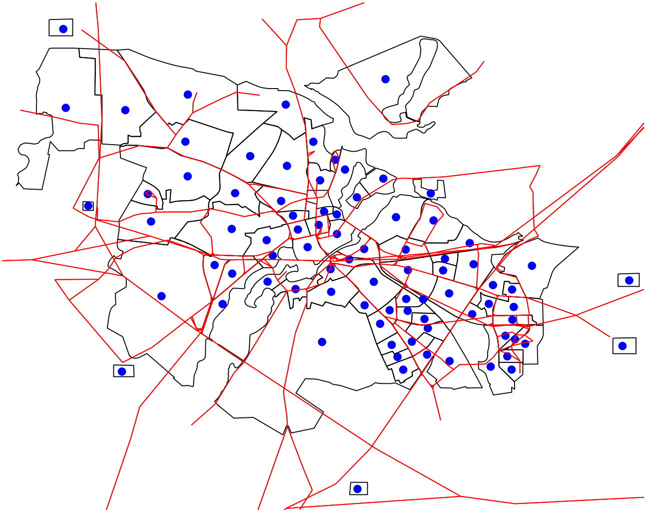
- aggregated (usually given for a whole day)
- give snapshot type of view
- representation of reality questionable
- industry standard for estimating transportation demand
- no relevant alternative in sight

# OD Matrix – Potsdam

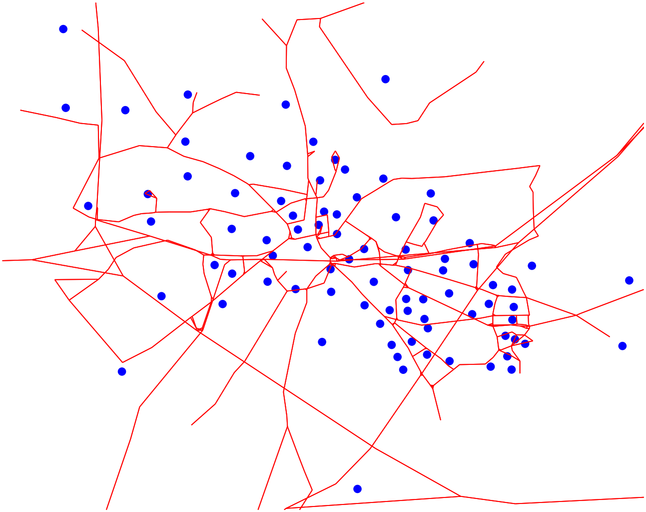
# Traffic Area Divided in Districts (OD-Nodes)



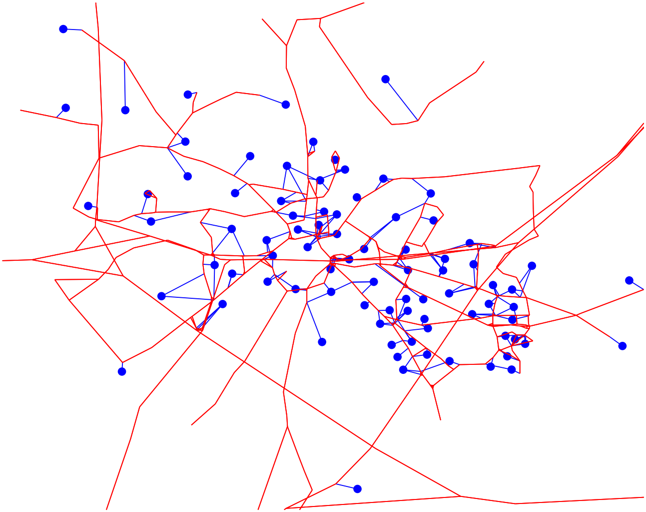
# OD-Nodes, Districts, and Network



# OD-Nodes and Network



# OD-Nodes and Connection to Network



# Line Planning Problem (LPP)

Given:

- network
- demands $d_{st}$ (OD-matrix)
- operating costs and traveling times

***Line:*** path (list of stations) with special start and end nodes, frequency

Problem: Design lines to satisfy demand.

Goals:

- minimize traveling times or number of transfers
- minimize costs of line plan

# Literature Overview I

Heuristics:

- Build lines from smaller pieces
  Remove lines from a “complete” line plan:
  *Patz, 1925*
  *Lampkin and Saalmans 1967*;
  *Dubois, Bel, and Llibre 1979*;
  *Sonntag 1979*
- Enumeration of lines:
  *Ceder and Wilson 1986*
- Local search:
  *Mandl 1980*
- Quadratic covering model:
  *Ceder and Israeli 1992, 1995*

# Literature Overview II

Mixed integer programming methods:

- Fixed Passenger Routes – System Split (for rail transport)
  - Minimize cost:
    *Claessens, van Dijk, and Zwaneveld 1995*;
    *Goossens, van Hoesel, and Kroon 2001, 2002*;
    *Bussieck, Lindner, and Lübbecke 2002*
  - Maximize direct travelers:
    *Bouma, Oltrogge 1994*;
    *Bussieck, Kreuzer, and Zimmermann 1997*
- Free Routing of Passengers
  - Minimize transfers/transfer time:
    *Scholl 2005*;
    *Schöbel and Scholl 2005*
  - Minimize travel time and cost (weighted sum):
    *Borndörfer, Grötschel, Pfetsch, 2005, 2007*

# Literature Overview – System Split

*Bouma and Oltrogge 1994*

Idea: Split network into different means of transport (fast train, local train; bus, tram, subway)
$\rightsquigarrow$ find line plan for each network, independently

Assumptions on behavior of passengers:

- "choose shortest path",
- "change to faster system as early as possible",
- "change to slower system as late as possible".

$\rightsquigarrow$ distribution of passengers to different paths

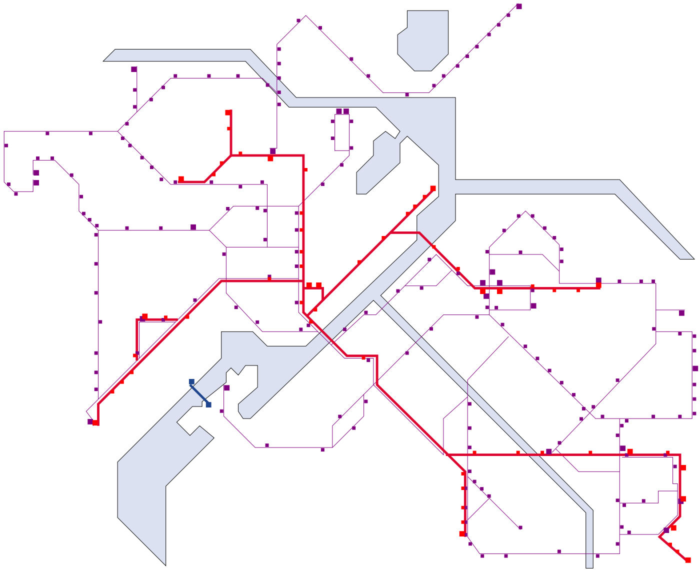
$\rightsquigarrow$ passenger traveling paths are known

# Some Notation

Notation for line planning problems (LPP)

- number $M$ of transportation modes (bus, tram, subway,...)
- undirected multigraph $G = (V, E) = (V, E_1 \cup, \ldots, \cup, E_M)$
- terminals: set of nodes $\mathcal{T}_1, \ldots, \mathcal{T}_M$ where lines can start and end
- OD matrix $d_{st} \in \mathbb{Q}_+^{V \times V}$
- $D = \{(s, t) \in V \times V \mid d_{st} > 0\}$ set of OD pairs
- $\mathcal{L}$ set of lines (simple paths)
- $\mathcal{F}_\ell$ set of frequencies for each line

# Potsdam Network



# Outline

# Line Planning with System Split

## Assumptions

- number of passengers $\rho_e$ for each edge known
- only one mode given, i.e.,capacity for all lines equal
- $\mathcal{L}$ pool of predefined lines

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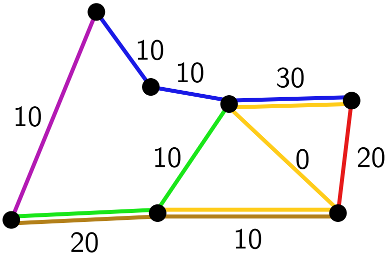
**Definition** Let

- $G = (V, E)$ be a public transportation graph,
- $\mathcal{L}$ a set of simple line paths in $G$ with capacity $K$
- $\mathcal{F}$ a set of possible frequencies
- $\rho_e$ transportation demand for each edge $e \in E$.

The *Feasible Line Plan Problem* is to find a set of lines $\mathcal{L}' \subseteq \mathcal{L}$ and frequencies $f_\ell \in \mathcal{F}$ for all $\ell \in \mathcal{L}'$ such that

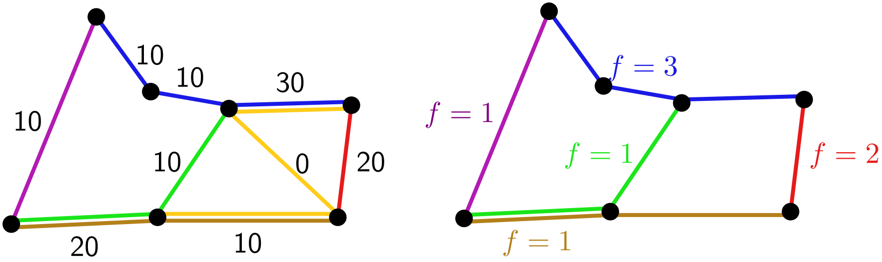
$$\sum_{\ell \in \mathcal{L}', e \in \ell} K \cdot f_\ell \geq \rho_e \quad \forall e \in E.$$

# Example




- public transport network with given demand on edges
- capacity of a line $K = 10$
- possible frequencies $\mathcal{F} = \{1, 2, 3\}$

# Example

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- possible frequencies $\mathcal{F} = \{1, 2, 3\}$

# Subproblem: Minimum Line Cover

## Definition

Given

- a public transportation graph $G = (V, E)$,
- a set of simple paths $\mathcal{L}$ defined in $G$,
- and a set of edges $E' \subseteq E$ with positive transportation demand.

The *Minimum Line Cover Problem* is to find a minimum set of lines $\mathcal{L}' \subseteq \mathcal{L}$ that cover all "demand edges" $E'$.

# Subproblem: Minimum Line Cover

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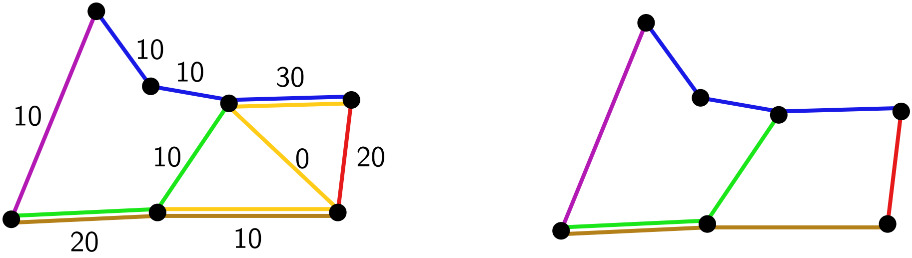

# Subproblem: Minimum Line Cover

The minimum line cover problem can be formulated as a set covering problem.

$$\begin{aligned} \min \quad & \sum_{\ell \in \mathcal{L}} x_\ell & & \\ \text{s.t.} \quad & \sum_{\ell : e \in \ell} x_\ell & \geq 1 \quad & \forall\, e \in E' \\ & x_\ell & \in [0,1] \quad & \forall\, \ell \in \mathcal{L} \end{aligned}$$

# Subproblem: Minimum Line Cover

The minimum line cover problem can be formulated as a set covering problem.

$$
\begin{array}{llll}
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\text{s.t.} & \sum_{\ell : e \in \ell} x_\ell & \geq 1 & \forall\, e \in E' \\
& x_\ell & \in [0,1] & \forall\, \ell \in \mathcal{L}
\end{array}
$$

### Proposition

*The minimum line cover problem is NP-hard.*

Proof: Reduction from set covering problem.

# Complexity – Proof (Idea: Schöbel, Scholl, 2005)

$S = \{a, b, c, d, e\}, \quad (\{a, c\}, \{b, d\}, \{b, c\}, \{c, e\}, \{a, d, e\})$

# Complexity – Proof (Idea: Schöbel, Scholl, 2005)

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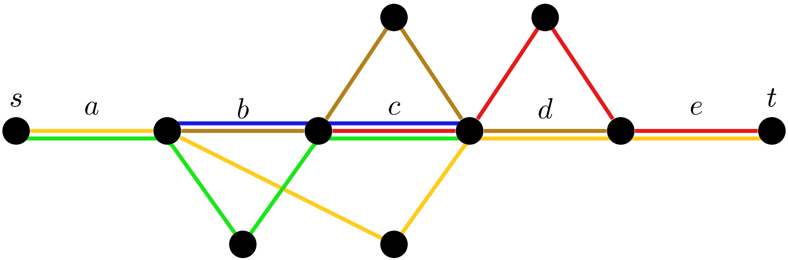

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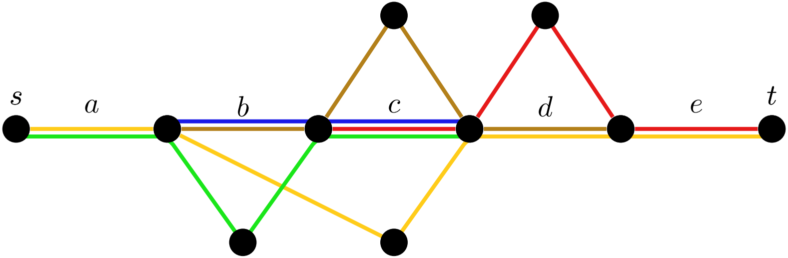


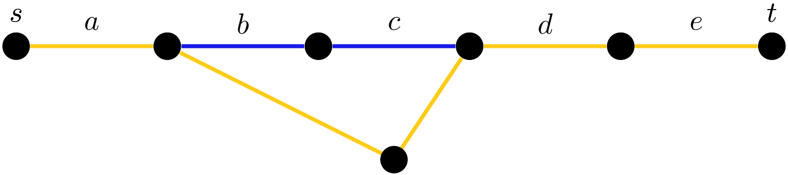


□

# Cost Minimization Model (Claessens et al.)

$$\min \sum_{\ell \in \mathcal{L}} \left\lceil \frac{f_\ell \, T_\ell}{T} \right\rceil (C^t + C^c \, z_\ell) + d_\ell \, f_\ell \, (c^t + c^c \, z_\ell)$$

$$s.t. \quad \underline{\Lambda}_e \leq \sum_{\ell \in \mathcal{L}_e} f_\ell \leq \overline{\Lambda}_e \qquad \forall \, e \in E$$

$$\sum_{\ell \in \mathcal{L}_e} K \, f_\ell \, z_\ell \geq \rho_e \qquad \forall \, e \in E$$

$$\underline{z} \leq z_\ell \leq \overline{z} \qquad \forall \, \ell \in \mathcal{L}$$

$$f_\ell, z_\ell \in \mathbb{Z}_+ \qquad \forall \, \ell \in \mathcal{L}$$

| | | |
|---|---|---|
| Variables: | $z_\ell$ | number of carriages of line $\ell$ |
| | $f_\ell$ | frequency of line $\ell$ |
| Parameter: | $K$ | capacity of one carriage |
| | $\underline{\Lambda}_e, \overline{\Lambda}_e$ | lower, upper bound on frequency ($\underline{\Lambda}_e = \lceil \frac{\rho_e}{\overline{z} \cdot K} \rceil$) |
| | $\underline{z}, \overline{z}$ | lower, upper bound on number of carriages |

# Objective Function

$$\underbrace{\left\lceil \frac{f_\ell \, T_\ell}{T} \right\rceil}_{\#\text{ trains}} \underbrace{(C^t + C^c \, z_\ell)}_{\text{fixed costs}} + \underbrace{d_\ell \, f_\ell \, (c^t + c^c \, z_\ell)}_{\text{operating costs}}$$

Parameter: $T$ time horizon

$T_\ell$ turn around time for line $\ell$

# Objective Function

$$\underbrace{\left\lceil \frac{f_\ell \, T_\ell}{T} \right\rceil}_{\#\text{ trains}} \underbrace{(C^t + C^c \, z_\ell)}_{\text{fixed costs}} + \underbrace{d_\ell \, f_\ell \, (c^t + c^c \, z_\ell)}_{\text{operating costs}}$$

Parameter:

| | |
|---|---|
| $T$ | time horizon |
| $T_\ell$ | turn around time for line $\ell$ |
| $C^t$ | fixed cost for one train |
| $C^c$ | fixed cost for one carriage |

# Objective Function

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Parameter:

| | |
|---|---|
| $T$ | time horizon |
| $T_\ell$ | turn around time for line $\ell$ |
| $C^t$ | fixed cost for one train |
| $C^c$ | fixed cost for one carriage |
| $d_\ell$ | length of line $\ell$ |
| $c^t$ | operating cost for one train per distance |
| $c^c$ | operating cost for one carriage per distance |

# Cost Minimization Model

$$\min \sum_{\ell \in \mathcal{L}} \left\lceil \frac{f_\ell \, T_\ell}{T} \right\rceil (C^t + C^c \, z_\ell) + d_\ell \, f_\ell \, (c^t + c^c \, z_\ell)$$

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s.t. \; & \underline{\Lambda}_e \leq \sum_{\ell \in \mathcal{L}_e} f_\ell \leq \overline{\Lambda}_e & \forall e \in E \\
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& \underline{z} \leq z_\ell \leq \overline{z} & \forall \ell \in \mathcal{L} \\
& f_\ell, z_\ell \in \mathbb{Z}_+ & \forall \ell \in \mathcal{L}
\end{aligned}$$

## Linearization

- $\mathcal{F}$ set of feasible frequencies, e.g., $\mathcal{F} = \{1, \ldots, F\}$
- $\mathcal{C}$ set of feasible numbers of carriages, e.g., $\mathcal{C} = \{3, 4, 5\}$
- $\mathcal{R} = \mathcal{L} \times \mathcal{F} \times \mathcal{C}$

# Linearized Cost Model (Goossens et al.)

$$
\begin{aligned}
\min \; & \sum_{r \in \mathcal{R}} \left( \left\lceil \tfrac{f_{r_\ell} \, T_{r_\ell}}{T} \right\rceil (C^t + C^c \, r_z) + d_{r_\ell} \, r_f \, (c^t + c^c \, r_z) \right) \cdot y_r \\
s.t. \; & \underline{\Lambda}_e \le \sum_{r \in \mathcal{R} \colon e \in r_\ell} r_f \, y_r \le \overline{\Lambda}_e && \forall \, e \in E \\
& \sum_{r \in \mathcal{R} \colon e \in r_\ell} K \, r_f \, r_z \, y_r \ge \rho_e && \forall \, e \in E \\
& \sum_{r \in \mathcal{R} \colon r_\ell = \ell} y_r \le 1 && \forall \, \ell \in \mathcal{L} \\
& y_r \in \{0, 1\} && \forall \, r \in \mathcal{R}
\end{aligned}
$$

Variables: $y_r$ choosing combination of $r = (r_\ell, r_f, r_z) \in \mathcal{R}$ (line frequency and number of carriage)

Solving with preprocessing and branch-and-cut methods.

# Cost Minimization – Complexity

## Proposition

*The cost minimizing line planning approach is NP-hard.*

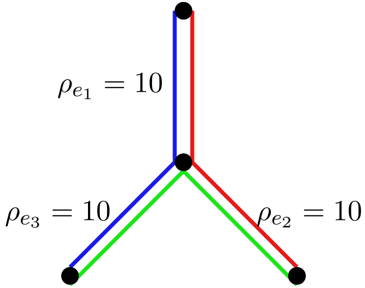
## Proof.

Setting

- $\underline{z} = \overline{z}$, (i.e., fixed number of carriages),
- $F = 1$, (i.e., fixed frequency),
- $K = \max\{\rho_e \mid e \in E\}$,
- $\underline{\Lambda}_e = 1$, $\overline{\Lambda}_e = \infty$
- $C^t = 1$, $C^c = c^c = c^t = 0$

leads to a minimum line cover problem. □

# Cutting Plane – Example




- $\mathcal{F} = \mathcal{C} = \{1\}$, $\mathcal{L} = \{1, 2, 3\}$, $K = 10$
- consider capacity constraint

$$\sum_{r \in \mathcal{R}:\, e \in r_\ell} K\, r_f\, r_z\, y_r \geq \rho_e \quad \forall\, e$$

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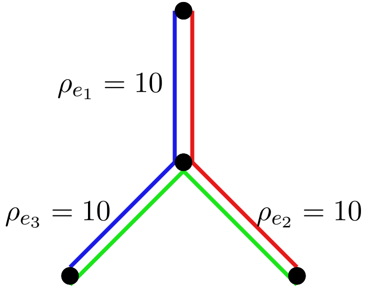


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$$\begin{aligned} 10 \cdot y_1 + 10 \cdot y_2 &\geq 10 \quad \{e_1\} \\ 10 \cdot y_1 + 10 \cdot y_3 &\geq 10 \quad \{e_2\} \\ 10 \cdot y_2 + 10 \cdot y_3 &\geq 10 \quad \{e_3\} \end{aligned}$$

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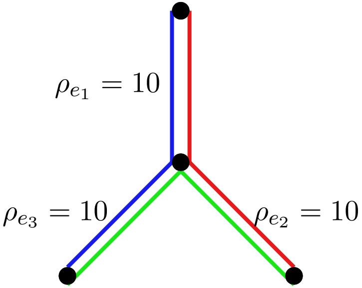


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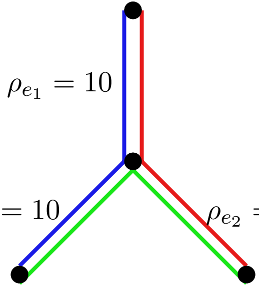
- $y_1 = y_2 = y_3 = 0.5$ is solution
- $\rightsquigarrow y_1 + y_2 + y_3 \geq 2$ valid

# Cutting Plane – Example




**Proposition**

*Let*

- $E' \subseteq E$ *such that* $\rho_e > 0 \; \forall \, e \in E'$,
- $\alpha^{\ell}_{E'} := |\ell \cap E'|$, *and*
- $\alpha^{\max}_{E'} := \max\{\alpha^{\ell}_{E'} \,|\, \ell \in \mathfrak{L}\}$.

*Then*

$$\sum_{r \in \mathcal{R}, \alpha^{r_\ell}_{E'} \geq 1} y_r \geq \left\lceil \frac{|E'|}{\alpha^{\max}_{E'}} \right\rceil$$

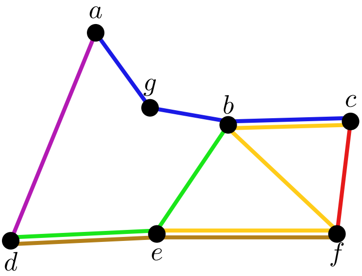
*is a valid inequality.*

# Discussion of the Model

- detailed cost function
  based on following assumption
  - no switching of rolling stock between lines
  - line is operated by same trains (same number of carriages)
  - timetable is periodic (e.g. repeated every hour)
- many variables
  (every possible combination of frequency and number of carriages)
  however, reduction by preprocessing
- only one transportation mode considered
- passenger paths are fixed
- line pool

# Outline

# Example – Free Passenger Routing

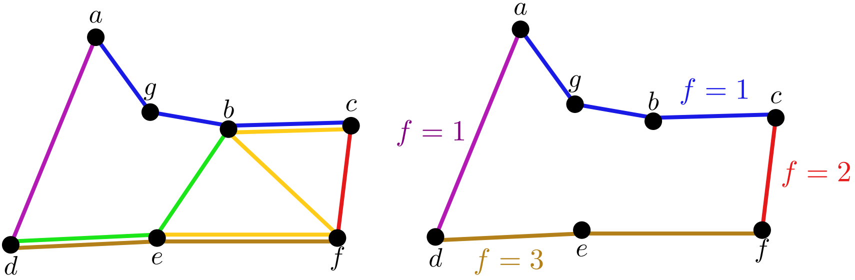



- public transport network with following demand (OD pairs):
  - $a \to c$: 10
  - $a \to d$: 10
  - $d \to c$: 10
  - $c \to f$: 20
  - $d \to f$: 20

| | $a$ | $c$ | $d$ | $f$ |
|---|---|---|---|---|
| $a$ | 0 | 10 | 10 | 0 |
| $c$ | 0 | 0 | 0 | 20 |
| $d$ | 0 | 10 | 0 | 20 |
| $f$ | 0 | 0 | 0 | 0 |

- capacity of a line $K = 10$, frequencies $\mathcal{F} = \{1, 2, 3\}$

# Example – Free Passenger Routing




- public transport network with following demand (OD pairs):
  - $a \to c$: 10
  - $a \to d$: 10
  - $d \to c$: 10
  - $c \to f$: 20
  - $d \to f$: 20

| | $a$ | $c$ | $d$ | $f$ |
|---|---|---|---|---|
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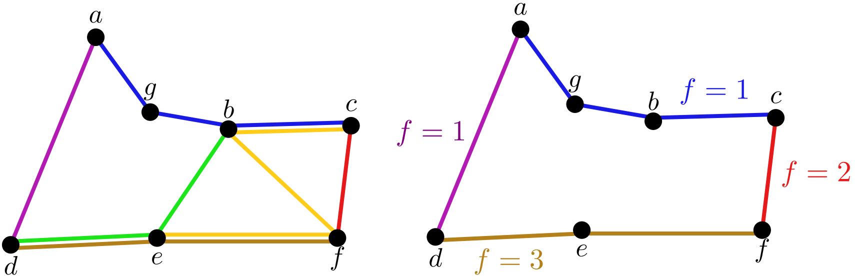


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  - $a \to d$: 10
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| | $a$ | $c$ | $d$ | $f$ |
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| $f$ | 0 | 0 | 0 | 0 |

- capacity of a line $K = 10$, frequencies $\mathfrak{F} = \{1, 2, 3\}$
- $\rightsquigarrow$ directed graph $G = (V, A)$ for passenger paths

# Multi Commodity Flow Model (Grötschel, Borndörfer, Pfetsch)

$$\min \quad \lambda \sum_{\ell} (C_\ell \, x_\ell + c_\ell \, f_\ell) \; + \; (1-\lambda) \sum_{p} \tau_p \, y_p$$

| | | | |
|---|---|---|---|
| i) | $\sum_{p \in \mathcal{P}_{st}} y_p = d_{st}$ | $\forall \, (s,t) \in D$ | transport all passengers |
| ii) | $\sum_{p \ni a} y_p \leq \sum_{\ell : e(a) \in \ell} \kappa_\ell \, f_\ell$ | $\forall \, a \in A$ | arc capacity constraints |
| iii) | $\sum_{\ell \ni e} f_\ell \leq \Lambda_e$ | $\forall \, e \in E$ | frequency bounds |
| iv) | $f_\ell \leq F x_\ell$ | $\forall \, \ell \in \mathcal{L}$ | coupling constraints |
| | $y_p \in \mathbb{R}_+$ | $\forall \, p \in \mathcal{P}$ | passenger flow |
| | $x_\ell \in \{0,1\}$ | $\forall \, \ell \in \mathcal{L}$ | choose line $\ell$ |
| | $f_\ell \in \mathbb{R}_+$ | $\forall \, \ell \in \mathcal{L}$ | frequency of line $\ell$ |

# Discussion of the Model

Properties of the model:

- no system split
- continuous frequencies
- other linear constraints possible
- system optimum = user equilibrium

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- Traveling paths of passengers are not fixed a priori.
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# Discussion of the Model

Properties of the model:

- no system split
- continuous frequencies
- other linear constraints possible
- system optimum = user equilibrium

Advantages of the model:

- Traveling paths of passengers are not fixed a priori.
- Lines can be generated dynamically (column generation).

Disadvantages of the model:

- Some passengers may use long paths.
  possible solution: length constraints
- Transfers between lines of same type cannot be controlled.

# Transfers

Lines of different type:

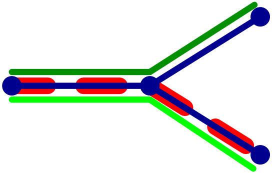
Time penalties for transfers

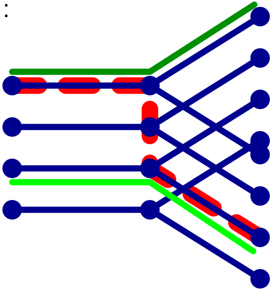
Lines of same type:

Capacity constraints do not distinguish between these lines:

$$\sum_{p \ni a} y_p \leq \sum_{\ell : e(a) \in \ell} \sum_{f \in \mathcal{F}_\ell} \kappa_\ell^f \, x_\ell^f$$

Solution: expansion of the graph:





Problem: Symmetries

# LP Relaxation

$$
\begin{aligned}
\min \quad & \sum_{\ell \in \mathcal{L}} (C_\ell \, x_\ell + c_\ell \, f_\ell) + \sum_{p \in \mathcal{P}} \tau_p \, y_p \\
\text{s.t.} \quad & \sum_{p \in \mathcal{P}_{st}} y_p = d_{st} && \forall \, (s,t) \in D \\
& \sum_{p : a \in p} y_p \le \sum_{\ell : e(a) \in \ell} \kappa_\ell \, f_\ell && \forall \, a \in A \\
& \sum_{\ell : e \in \ell} f_\ell \le \Lambda_e && \forall e \in E \\
& f_\ell \le F \, x_\ell && \forall \ell \in \mathcal{L} \\
& x_\ell \in [0,1] && \forall \, \ell \in \mathcal{L} \\
& f_\ell \ge 0 && \forall \, \ell \in \mathcal{L} \\
& y_p \ge 0 && \forall \, p \in \mathcal{P}.
\end{aligned}
$$

# LP Relaxation

$$
\begin{aligned}
\text{(LP)} \quad \min \quad & \sum_{\ell \in \mathcal{L}} \gamma_\ell \, f_\ell + \sum_{p \in \mathcal{P}} \tau_p \, y_p \\
\text{s.t.} \quad & \sum_{p \in \mathcal{P}_{st}} y_p = d_{st} && \forall \, (s,t) \in D \\
& \sum_{p : a \in p} y_p - \sum_{\ell : e(a) \in \ell} \kappa_\ell \, f_\ell \leq 0 && \forall \, a \in A \\
& \sum_{\ell : e \in \ell} f_\ell \leq \Lambda_e && \forall \, e \in E \\
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& y_p \geq 0 && \forall \, p \in \mathcal{P}.
\end{aligned}
$$

Solve LP relaxation with column generation.

## Proposition

*The computation of the optimal value of (LP) is NP-hard.*

# Pricing of Passenger Paths

The dual model is:

$$
\begin{array}{llrll}
\text{(DLP)} & \min & \displaystyle\sum_{(s,t)\in D} d_{st}\,\pi_{st} - \sum_{e\in E} \Lambda_e\,\eta_e & & \\
& \text{s.t.} & \displaystyle \pi_{st} - \sum_{a\in p} \mu_a & \le \tau_p & \forall\, p \in \mathcal{P}_{st},\ (s,t)\in D \\
& & \displaystyle\sum_{e\in\ell} (\kappa_\ell(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e) & \le \gamma_\ell & \forall\, \ell \in \mathcal{L} \\
& & \eta_e & \ge 0 & \forall\, e \in E \\
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$$\overline{\tau}_p = \tau_p - \pi_{st} + \sum_{a\in p} \mu_a = -\pi_{st} + \sum_{a\in p} (\mu_a + \tau_a)$$

$$\overline{\tau}_p < 0 \Leftrightarrow \sum_{a\in p} (\mu_a + \tau_a) < \pi_{st}$$

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$$

$$
\overline{\tau}_p < 0 \Leftrightarrow \sum_{a\in p} (\mu_a + \tau_a) < \pi_{st}
$$

$\rightsquigarrow$ shortest path problem

# Pricing of Line Paths

$$
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$$

Reduced cost $\overline{\gamma}_\ell$ for $f_\ell$, $\ell \in \mathcal{L}$, $(s,t) \in D$:

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& \sum_{e\in\ell} (\kappa_\ell(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e) \le \gamma_\ell \quad \forall\, \ell \in \mathcal{L} \\
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\end{aligned}
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# Pricing of Line Paths

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# Pricing of Line Paths

$$\begin{aligned}\overline{\gamma}_\ell &= \gamma_\ell - \sum_{e\in\ell}(\kappa_\ell(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e) \\ &= \frac{C_\ell}{F} + c_\ell - \sum_{e\in\ell}(\kappa_\ell(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e)\end{aligned}$$

# Pricing of Line Paths

$$\begin{aligned}
\overline{\gamma}_\ell &= \gamma_\ell - \sum_{e\in\ell}(\kappa_\ell(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e) \\
&= \frac{C_\ell}{F} + c_\ell - \sum_{e\in\ell}(\kappa_\ell(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e) \\
&= \frac{C_i}{F} + \sum_{e\in\ell} c_e^i - \sum_{e\in\ell}(\kappa_i(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e)
\end{aligned}$$

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&= \tfrac{C_i}{F} - \sum_{e\in\ell}\big(\kappa_i(\mu_{a(e)} + \mu_{\bar{a}(e)}) - \eta_e - c_e^i\big)
\end{aligned}$$

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&= \tfrac{C_i}{F} - \sum_{e\in\ell}\big(\kappa_i\big(\mu_{a(e)} + \mu_{\bar{a}(e)}\big) - \eta_e - c_e^i\big)
\end{aligned}$$

$$0 > \overline{\gamma}_\ell \quad \Leftrightarrow \quad \sum_{e\in\ell}\big(\kappa_i\big(\mu_{a(e)} + \mu_{\bar{a}(e)}\big) - \eta_e - c_e^i\big) > \tfrac{C_i}{F}$$

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$$

$\rightsquigarrow$ longest path problem (NP-hard)

# Pricing of Line Paths

Let $n$ be the number of nodes.

**Theorem**

*If the lengths of paths are $O(\log n)$, one can solve the longest path problem in polynomial time.*

**Corollary**

*If the lengths of lines are $O(\log n)$, one can solve the LP relaxation in polynomial time.*

Alternative method: Find lines by *enumeration*.

# Outline

Introduction

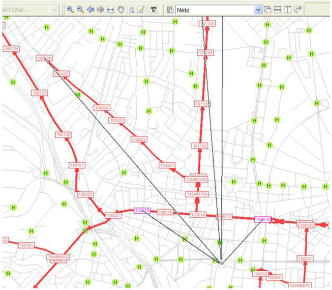
Line Planning with Fixed Passenger Routes

A Column Generation Approach to Line Planning

Line Planning in Practice

# Current Practice

- sophisticated simulation tools, e.g., VISUM (but no mathematical optimization methods)
- experience of practitioners
- political requirements



# Computational Experiments for Potsdam

Cooperation with: ViP Potsdam

network of Potsdam:

| | | | |
|---|---|---|---|
| inhabitants: | | | 150,000 |
| travels in morning traffic: | | | 42 973 |
| number of bus lines: | | | 15 + 8 |
| number of tram lines: | | | 6 |
| nodes: | 872 (1643) | edges: | 2462 (5470) |
| OD-nodes: | 385 | nonzeros: | 12 787 |

# Variation of (Grötschel, Borndörfer, Pfetsch)

$$\min \quad \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in F} c_\ell^f \, x_\ell^f + (1 - \lambda) \sum_{p \in \mathcal{P}} \tau_p \, y_p$$

| | | | |
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| i) | $\sum_{p \in \mathcal{P}_{st}} y_p = d_{st}$ | $\forall \, (s,t) \in D$ | transport all passengers |
| ii) | $\sum_{p \ni a} y_p \leq \sum_{\ell : e(a) \in \ell} \sum_{f \in \mathcal{F}_\ell} \kappa_\ell^f \, x_\ell^f$ | $\forall \, a \in A$ | arc capacity constraints |
| iii) | $\sum_{f \in \mathcal{F}_\ell} x_\ell^f \leq 1$ | $\forall \, \ell \in \mathcal{L}$ | one frequency per line |
| | $y_p \in \mathbb{R}_+$ | $\forall \, p \in \mathcal{P}$ | passenger flow |
| | $x_\ell^f \in \{0, 1\}$ | $\forall \, \ell \in \mathcal{L}$ | line and frequency |

- pricing problem for passenger paths similar as before
- pricing of line paths? (exercise)

# Things to Do Besides Optimization

How to present the solution?

# Things to Do Besides Optimization

How to present the solution?

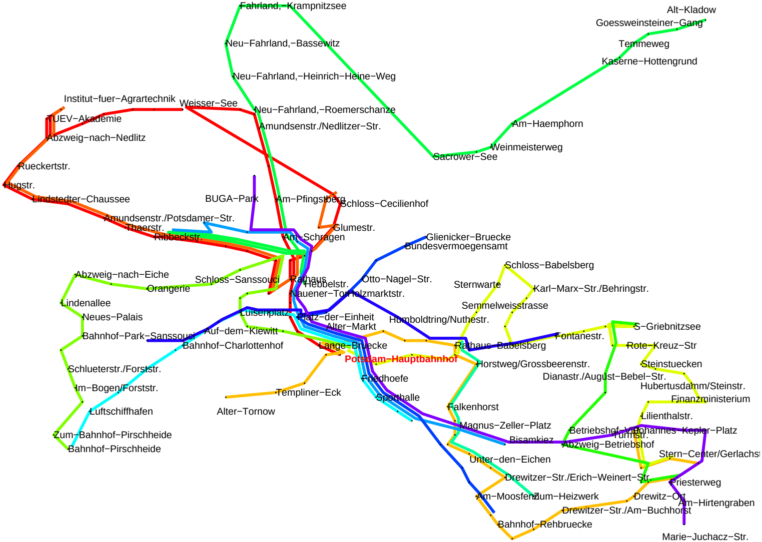
- list of lines with stations and frequencies, basic visualization

# Solution – List

line name cycle time in minutes:
BVB134R;32088;U 30
BVB234H;10723;U 30
BVB534R;35166;U 30
HVG612;180287;U 30
N01RB20H;612362;U 60
N01RB21R;610169;U 60
VIB692H;530239;U 30
VIT92;92_KA-MJ;U 20

list of stations for each line:
BVB134R;32088;U Hottengrund (Berlin)
BVB134R;32088;U Kaserne Hottengrund (Berlin)
BVB134R;32088;U Temmeweg (Berlin)
BVB134R;32088;U Gösweinsteiner Gang (Berlin)
BVB134R;32088;U Parnemannweg (Berlin)
BVB134R;32088;U Alt-Kladow (Berlin)
BVB134R;32088;U Finnenhaus-Siedlung (Berlin)
BVB134R;32088;U Neukladower Allee (Berlin)
BVB134R;32088;U Krankenhaus Havelhöhe (Berlin)
BVB134R;32088;U General-Steinhoff-Kaserne (Berlin)
BVB134R;32088;U Weg nach Breitehorn (Berlin)
BVB134R;32088;U Breitehornweg (Berlin)
BVB134R;32088;U Helleberge (Berlin)
BVB134R;32088;U Am Graben (Berlin)
BVB134R;32088;U Alt-Gatow (Berlin)
BVB134R;32088;U Gatow Kirche (Berlin)
BVB134R;32088;U Pfirsichweg (Berlin)
BVB134R;32088;U Emil-Basdeck-Str. (Berlin)
BVB134R;32088;U Biberburg (Berlin)
BVB134R;32088;U Zur Haveldüne (Berlin)
BVB134R;32088;U Gatower Str./Weinmeisterhornweg (Berlin)
BVB134R;32088;U Sandheideweg (Berlin)
BVB134R;32088;U Gatower Str./Heerstr. (Berlin)

# Basic Visualization



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How to present the solution?

- list of lines with stations and frequencies, visualization with matlab
  advantage: easy
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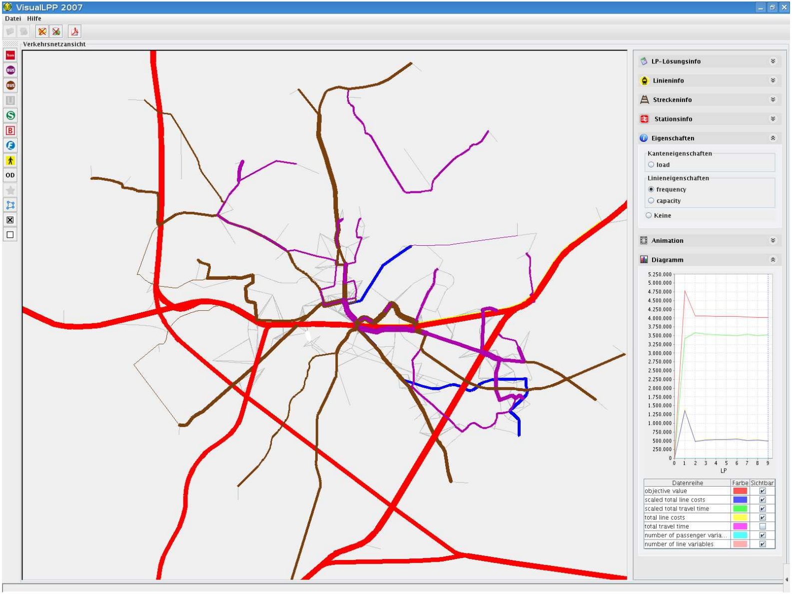
# VisualLPP (M. Kinder)



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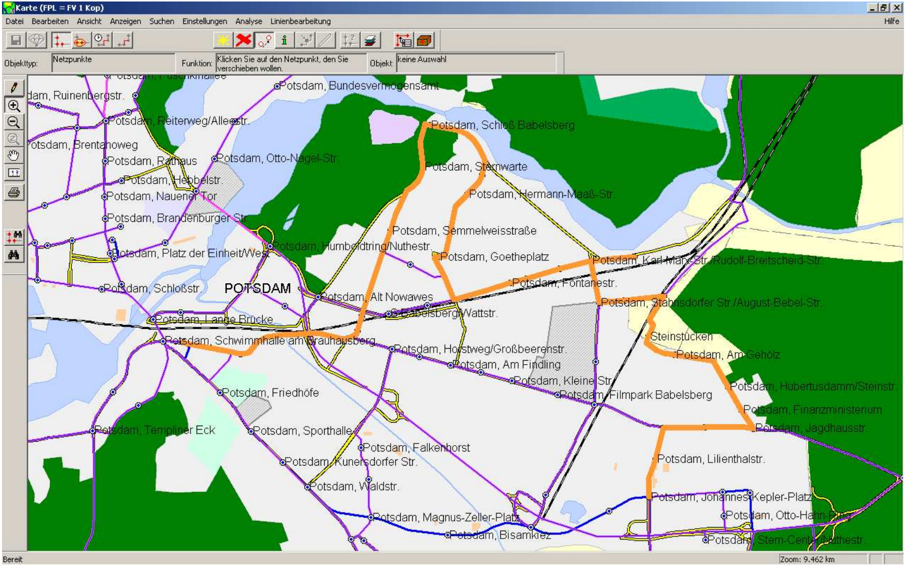
- visualization with map

# Map


Karte (FPL = FV 1 Kop)
Datei Bearbeiten Ansicht Anzeigen Suchen Einstellungen Analyse Linienbearbeitung
Hilfe
Objekttyp: Netzpunkte
Funktion: Klicken Sie auf den Netzpunkt, den Sie verschieben wollen.
Objekt: keine Auswahl
Potsdam, Bundesvermögensamt
Potsdam, Reiterweg/Alleestr.
Potsdam, Schloß Babelsberg
Potsdam, Rathaus
Potsdam, Otto-Nagel-Str.
Potsdam, Sternwarte
Potsdam, Hebbelstr.
Potsdam, Nauener Tor
Potsdam, Hermann-Maaß-Str.
Potsdam, Brandenburger Str.
Potsdam, Semmelweisstraße
Potsdam, Humboldtring/Nuthestr.
Potsdam, Platz der Einheit/West
Potsdam, Goetheplatz
Potsdam, Karl-Marx-Str./Rudolf-Breitscheid-Str.
Potsdam, Fontanestr.
Potsdam, Schloßstr.
POTSDAM
Potsdam, Alt Nowawes
Potsdam, Stahnsdorfer Str./August-Bebel-Str.
Babelsberg Wattstr.
Potsdam, Lange Brücke
Steinstücken
Potsdam, Schwimmhalle am Brauhausberg
Potsdam, Horstweg/Großbeerenstr.
Potsdam, Am Gehölz
Potsdam, Am Findling
Potsdam, Kleine Str.
Potsdam, Hubertusdamm/Steinstr.
Potsdam, Friedhöfe
Potsdam, Filmpark Babelsberg
Potsdam, Finanzministerium
Potsdam, Templiner Eck
Potsdam, Sporthalle
Potsdam, Jagdhausstr.
Potsdam, Falkenhorst
Potsdam, Kunersdorfer Str.
Potsdam, Lilienthalstr.
Potsdam, Waldstr.
Potsdam, Johannes-Kepler-Platz
Potsdam, Magnus-Zeller-Platz
Potsdam, Bisamkiez
Bereit
Zoom: 9.462 km

# Things to do besides Optimization

How to present the solution?

- list of lines with stations and frequencies, visualization with matlab
  advantage: easy
  disadvantage: very rudimental, e.g., no switching through lines
- visualization tool implemented by M. Kinder (student at ZIB)
  advantage: tool was ready
  disadvantage: no geographic map, restricted evaluation of solution
- visualization with map
  advantage: geographic map, part of software used by Potsdam
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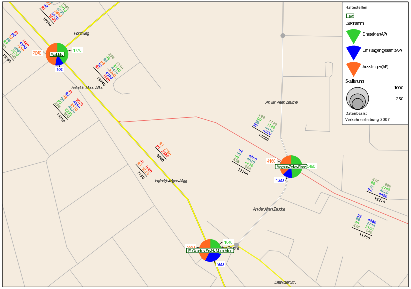
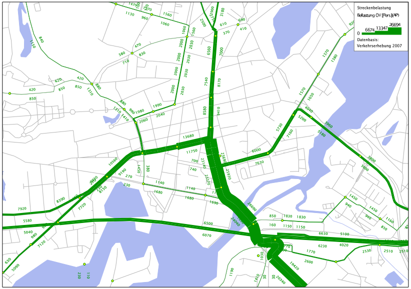
- visualization with map
  advantage: geographic map, part of software used by Potsdam
  disadvantage: restricted evaluation of solution
- visualization with VISUM
  advantage: geographic map, evaluation of solution possible
  disadvantage: expensive

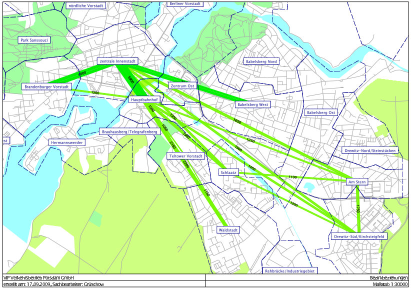
# VISUM



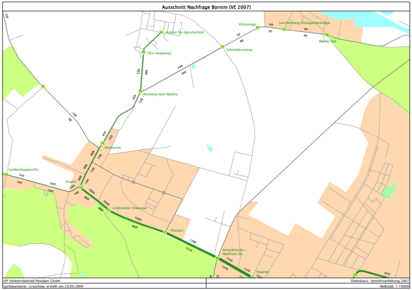
Haltestellen
Text
Diagramm
Einsteiger(AP)
Umsteiger gesamt(AP)
Aussteiger(AP)
Skalierung
1000
250
Datenbasis:
Verkehrserhebung 2007
Horstweg
Heinrich-Mann-Allee
An der Alten Zauche
Drewitzer Str.

Streckenbelastung
Belastung ÖV [Pers](AP)
Datenbasis:
Verkehrserhebung 2007

nördliche Vorstadt
Berliner Vorstadt
Park Sanssouci
zentrale Innenstadt
Babelsberg Nord
Brandenburger Vorstadt
Zentrum Ost
Hauptbahnhof
Babelsberg West
Babelsberg Ost
Brauhausberg/Telegrafenberg
Hermannswerder
Drewitz-Nord/Steinstücken
Teltower Vorstadt
Schlaatz
Am Stern
Waldstadt
Drewitz-Süd/Kirchsteigfeld
Rehbrücke/Industriegebiet
ViP Verkehrsbetrieb Potsdam GmbH
Bezirksbeziehungen
erstellt am: 17.09.2009, Sachbearbeiter: Grüschow
Maßstab 1:30000

Ausschnitt Nachfrage Bornim (VE 2007)
Institut für Agrartechnik
TÜV-Akademie
Kläranlage
Lerchensteig/Kleingartenanlage
Weißer See
Schneiderremise
Abzweig nach Nedlitz
Rückertstr.
Landeshauptarchiv
Hugstr.
Lindstedter Chaussee
Florastr.
Amundsenstr./Nedlitzer Str.
Thaerstr.
ViP Verkehrsbetrieb Potsdam GmbH
Datenbasis: Verkehrserhebung 2007
Sachbearbeiter: Grüschow, erstellt am 29.09.2009
Maßstab: 1:10000

# Other Things to Do

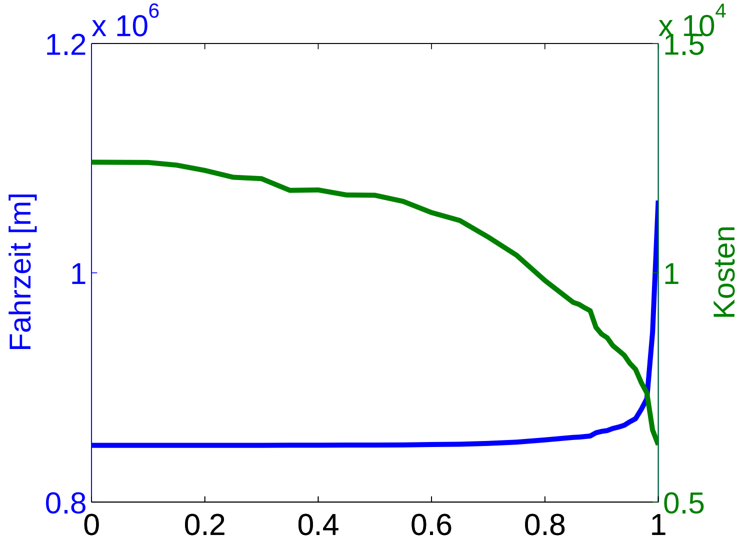
## Data

- some data missing, incomplete, contains errors
  $\rightsquigarrow$ long iterative process to get needed data

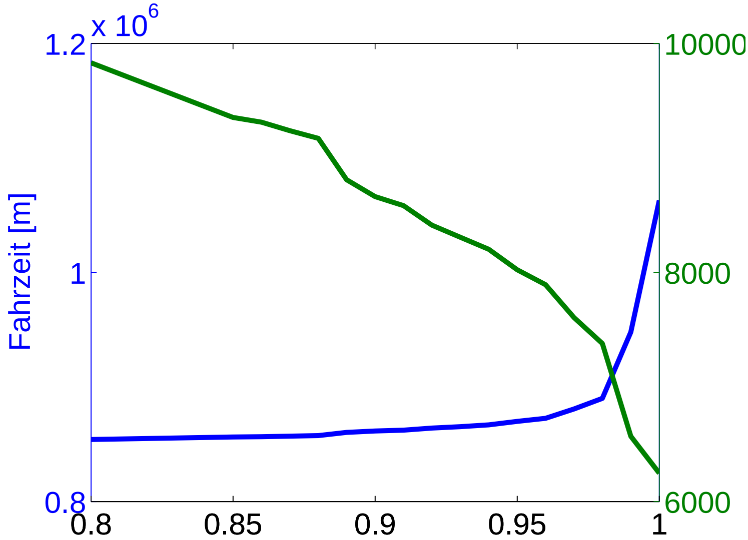
## Parameter

- cost function, operating cost for each line
- capacities of lines
- choose terminal nodes (endpoint of line)
- weighting of cost and travel time (choosing $\lambda$)
  $\rightsquigarrow$ computation of Pareto curve

# Pareto Curve



# Pareto Curve



# Computations

column generation of passenger paths, predefined line pool (computations in chronological order):

| Instance | $\|\mathcal{L}\|$ | $\|\mathcal{F}\|$ | #line var | #constr. | time[1] |
|---|---|---|---|---|---|
| potsdam1 | 30 232 | 4 | 120 592 | 44 577 | 349 |
| potsdam2 | 29 142 | 2 | 58 268 | 43 419 | 125 |
| potsdam3 | 1172 | 3 | 3 486 | 15 491 | 24 |
| potsdam4 | 623 | 3 | 1 755 | 14 939 | 20 |
| potsdam5 | 3861 | 3 | 11 471 | 21 199 | 377[2] |

[1] in minutes, after solving root node including separators and heuristics; gap $< 5\%$

[2] for adjusted network

# Problems

## General Problems

- data contains information relevant for transportation company but not relevant for optimization tools
  (e.g., nodes of network not only stops but also crossings, track switches, turnouts,..)

## Problems concerning our solutions

- too many too short lines
- service (frequency) on some stations to small
- the importance of tram not represented by our solution
  (important for tourism, environment, prestige)
- lines over a train crossing – no robust timetable
- "curious" bus lines (no stations shared with tram, regional traffic)

# Problems – Solutions

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  Idea: high cost for using crossing in line; prohibit the use of crossing
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  $\rightsquigarrow$ bound on minimal frequency for serving a station
- the importance of the tram not represented by our solution
  Idea: condition of covering all tracks of tram
- lines over train crossing
  Idea: high cost for using crossing in line; prohibit the use of crossing
- "curious" bus lines (no stations shared with tram, regional traffic)
  Idea: generate only bus lines that contain a station shared with tram or regional traffic

# Line Plan – Solutions

Work in Progress...

# Line Plan – Solutions

Work in Progress...

- first step to establish optimization methods for line planning in practice
- line planning optimization not completely solved
- hope for future: optimization in (service design of) public transport as decision support

# Line Planning in Public Transport

## CO@Work Berlin

Marika Neumann
Ralf Borndörfer, Marc Pfetsch

10/02/2009

Berlin Mathematical School

DFG Research Center MATHEON
Mathematics for key technologies