Online optimization: Elevator and vehicle scheduling

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Mathematics for key technologies

Online problems

high rack warehouse

elevator scheduling

dispatching service vehicles

Outline

Recap: Online optimization

Theoretical framework: Online Dial-a-Ride problems and competitive analysis

Online Optimization in Practice: Reoptimization Algorithms
- Dispatching the service vehicles of ADAC
- Controlling cargo elevators in a distribution center
- Controlling passenger elevators in high-rise buildings

Theory again: The Online Bin Coloring problem

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What is online?

typical application: elevator control

- passenger calls arrive over time
- a new call must immediately be incorporated in the elevator schedule

Online optimization

In online optimization, we have to make decisions before all data are known.
Online problems are often real-time.

common assumption: nothing known about the future, not even stochastical information

Online algorithms

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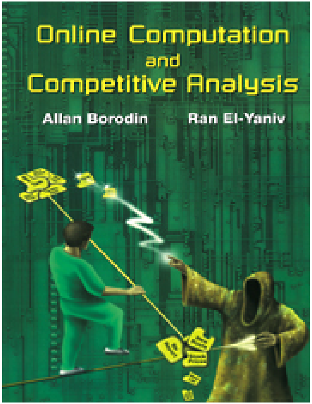

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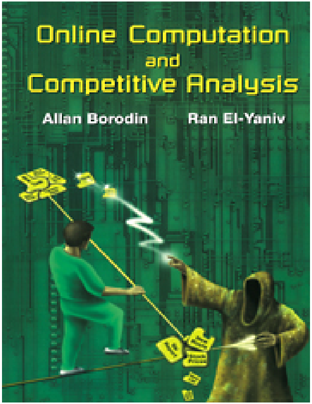

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For our real-world applications:

- Decisions are (partially) revocable.
- Running time does matter.

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  4. Every request is picked up and dropped exactly once.

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Objective

- Find a schedule with minimum completion time.

Online Dial-a-Ride Problems

- system operates continuously, requests arrive over time
- online control algorithm does not know anything about future requests, note even their number
- schedule for server may change in response to new requests
- assumption: schedule may only change when the server is at a node

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Offline Dial-a-Ride problem

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Online Dial-a-Ride problem

- An online algorithm $\textsc{Alg}$ is called a c-competitive algorithm if

$$\textsc{Alg}(G, d, R) \leq c \cdot \textsc{Opt}(G, d, R)$$

for all instances (G, d, R). Here, $\textsc{Opt}$ is the optimal offline algorithm that has access to the entire sequence, i.e. knows the future.

Construction of online algorithms

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- for complex online problems like Dial-a-Ride, one wants to make use of offline algorithms

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Ignore-strategy

- As a request arrives, serve it if the server is idle. If the server is not idle, ignore the request and complete the current schedule.
- As the server becomes idle, compute a schedule for the requests ignored so far using offline algorithm Alg and follow this schedule.

Competitive analysis results

Completion time [Ascheuer, Krumke, Rambau '00, Ausiello et al '01]

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Average/maximum flow/waiting time

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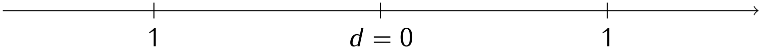

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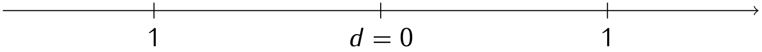

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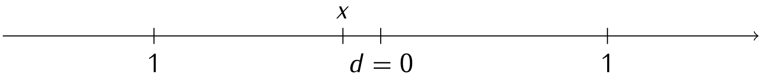

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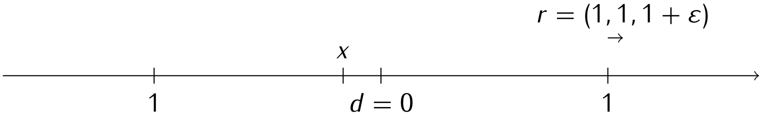

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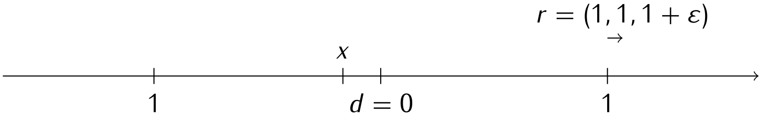

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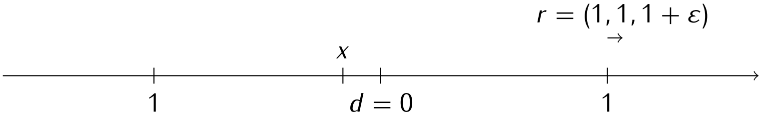

Alternatives to pure competitive analysis

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- resource augmentation
- max/max ratio
- worst order ratio
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- ...many more

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Results for Dial-a-Ride under reasonable load

Theorem ([Hauptmeier, Krumke, Rambau '00])

1. *Let σ be a Δ-reasonable request sequence. Then maximal flow time achieved by* IGNORE *on σ is **bounded by** 2Δ.*

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1. *Let σ be a Δ-reasonable request sequence. Then maximal flow time achieved by* Ignore *on σ is bounded by 2Δ.*
2. *There is a $\Delta \in \mathbb{R}$, a Δ-reasonable request sequence σ and a request r in σ such that the flow time for r achieved by* Replan *is unbounded.*

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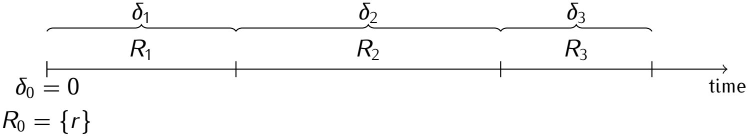

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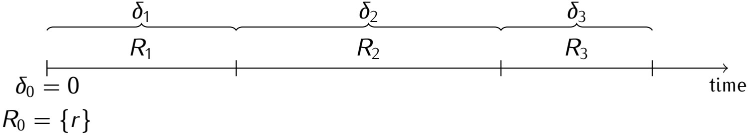

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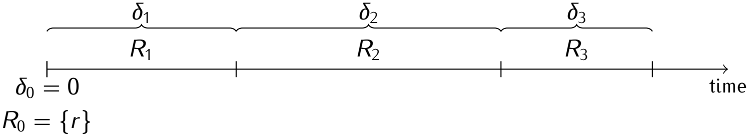

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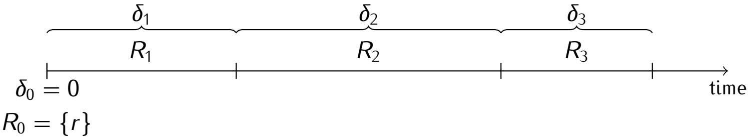
- observation: $\textsc{Ignore}^{\text{max-flow}}(R_i) \leq \delta_i + \delta_{i+1} \leq 2\Delta$
- suffices to show: $\delta_i \leq \Delta$ for all i

Proof of 2Δ bound for Ignore



- Let r be the first request arriving at time $\delta_0 = 0$.
- Define
  - $R_0 = \{r\}$
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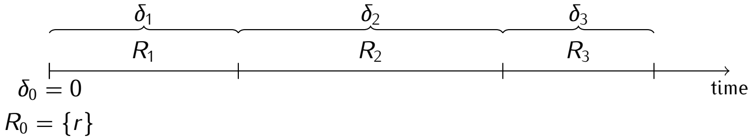

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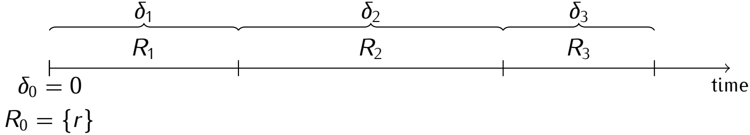

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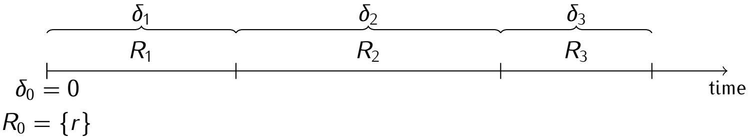

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Recap: Online optimization

Theoretical framework: Online Dial-a-Ride problems and competitive analysis

Online Optimization in Practice: Reoptimization Algorithms

- Dispatching the service vehicles of ADAC
- Controlling cargo elevators in a distribution center
- Controlling passenger elevators in high-rise buildings

Theory again: The Online Bin Coloring problem

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Application #1: Dispatching the service vehicles of ADAC

Supereinsatz. In Berlin und Brandenburg mussten die Gelben Engel letztes Jahr mehr als 240 000 Mal ausrücken, um Havaristen in der Hauptstadt und auf 1700 Autobahnkilometern wieder flottzumachen – ein Rekordeinsatz. Einen Rückgang von zehn Prozent bei den Pannen registrierten dagegen die Gelben Engel in Mecklenburg-Vorpommern. Bei insgesamt 72 389 Einsätzen schafften sie jedoch auch einen Rekord: In 84 Prozent der Fälle konnten die Autofahrer mit dem Wagen weiterfahren.

Application #1: Dispatching the service vehicles of ADAC

Application #1: Dispatching the service vehicles of ADAC

phone call →

help center

Application #1: Dispatching the service vehicles of ADAC

phone call

help center

dispatch:
human operator

Application #1: Dispatching the service vehicles of ADAC

phone call

ADAC

help center

wireless transmission

Yellow Angel

dispatch:
human operator

Application #1: Dispatching the service vehicles of ADAC

phone call

help center

service

wireless transmission

dispatch:
human operator

Yellow Angel

ADAC break down service

Situation

- $\approx$ 1,700 Yellow Angels
- $\approx$ 1,200 road service partners with $\approx$ 5,000 vehicles
- 5 help centers
- on average 10,000 requests a day; peak: 45,000 requests in 4 hours

Task

Determine assignment of the requests to units and partners. Schedule corresponding tours for the units online and in real-time (computation time $\leq$ 10 seconds).

Goal

Minimize operating cost + lateness cost.

Difficulties and solution strategies

- The ADAC dispatching problem is an online problem.
  $\Rightarrow$ Reoptimization algorithm: At certain decision times, compute a (good) dispatch for the current situation.

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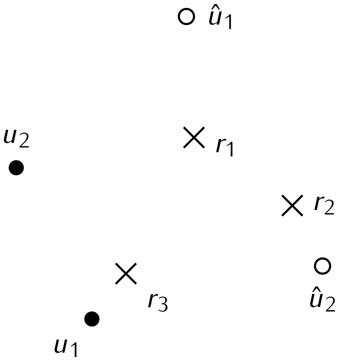
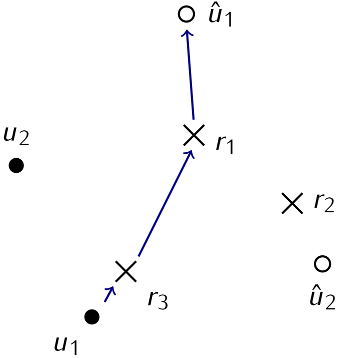
- The offline problem is NP-hard and there are real time requirements.
  $\Rightarrow$ Abandon optimality and use efficient, problem-specific approximation algorithm based on an exact approach.
- Cost structure is complex: nonlinear lateness cost, discounted partner costs, ...
  $\Rightarrow$ tour-based model

Tour-based model: Details

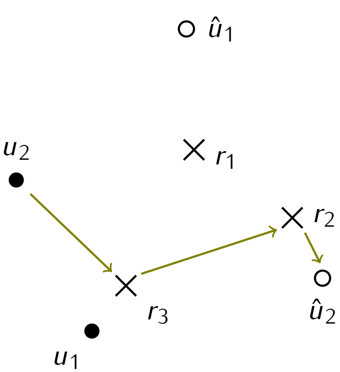


Tour-based model: Details



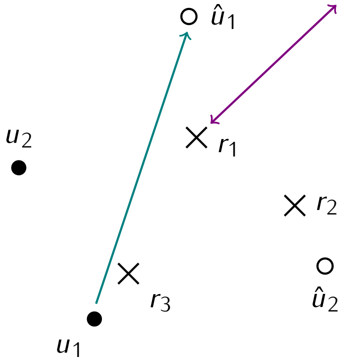
t_1	
1	r_1
0	r_2
1	r_3
1	u_1
0	u_2
c_{t_1}	

Tour-based model: Details



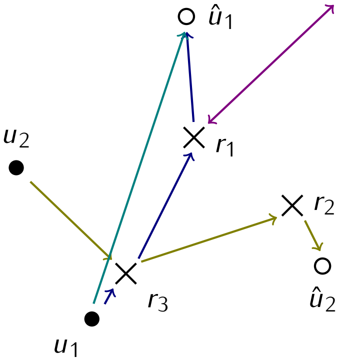
t_1	t_2	
1	0	r_1
0	1	r_2
1	1	r_3
1	0	u_1
0	1	u_2
c_{t_1}	c_{t_2}	

Tour-based model: Details



t_1	t_2	t_3	t_4	
1	0	0	1	r_1
0	1	0	0	r_2
1	1	0	0	r_3
1	0	1	0	u_1
0	1	0	0	u_2
c_{t_1}	c_{t_2}	c_{t_3}	c_{t_4}	

Tour-based model: Details



	t_1	t_2	t_3	t_4	...	t_N	
	1	0	0	1	...		r_1
	0	1	0	0	...		r_2
	1	1	0	0	...		r_3
	1	0	1	0	...		u_1
	0	1	0	0	...		u_2
	c_{t_1}	c_{t_2}	c_{t_3}	c_{t_4}	...	c_{t_N}	

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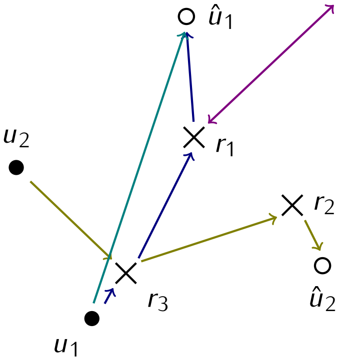

	t_1	t_2	t_3	t_4	$\dots$	t_N	
	1	0	0	1	$\dots$		r_1
	0	1	0	0	$\dots$		r_2
A	1	1	0	0	$\dots$		r_3
	1	0	1	0	$\dots$		u_1
	0	1	0	0	$\dots$		u_2
c^T	c_{t_1}	c_{t_2}	c_{t_3}	c_{t_4}	$\dots$	c_{t_N}	
x^T	x_{t_1}	x_{t_2}	x_{t_3}	x_{t_4}	$\dots$	x_{t_N}	

obtain set partitioning IP:

$$\begin{aligned} \min \quad & c^T x \\ Ax &= 1 \\ x &\in \{0,1\}^n \end{aligned}$$

The reoptimization algorithm

Online problem

REPLAN strategy: When new information becomes known, compute new dispatch by solving an instance of the offline problem (the snapshot problem).

The reoptimization algorithm

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Offline problem

- extremely many feasible tours (= columns of A)
  $\Rightarrow$ dynamic column generation

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- initial IP model contains
  - return-home tour for each unit
  - single request tour for each partner
  - a feasible dispatch from a simple best-insertion heuristic based on the current dispatch

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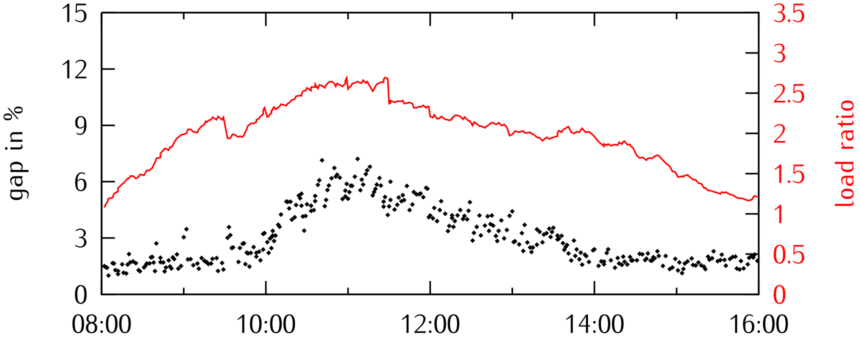
  $\Rightarrow$ reasonable dual prices

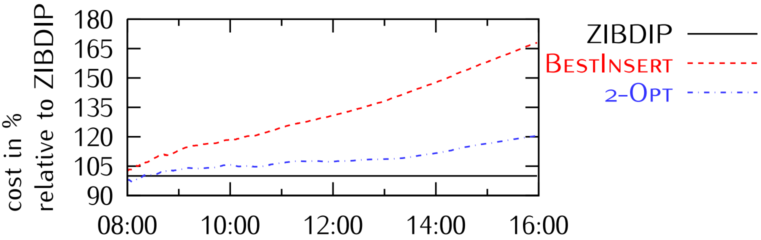
Data and simulation setup

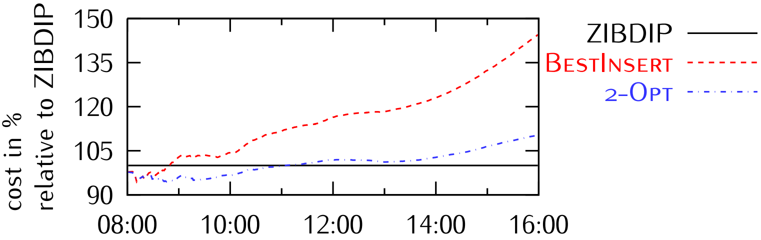
- ADAC provided production data of several days
  $\Rightarrow$ input for our online simulation
- reoptimization everytime a new request arrived
- computation time ZIBDIP: 10 seconds
- computer: Xeon 2.4 GHz, 2 GB RAM
- details and further results in [Hiller, Krumke, Rambau '06]

Snapshot solution quality over the day

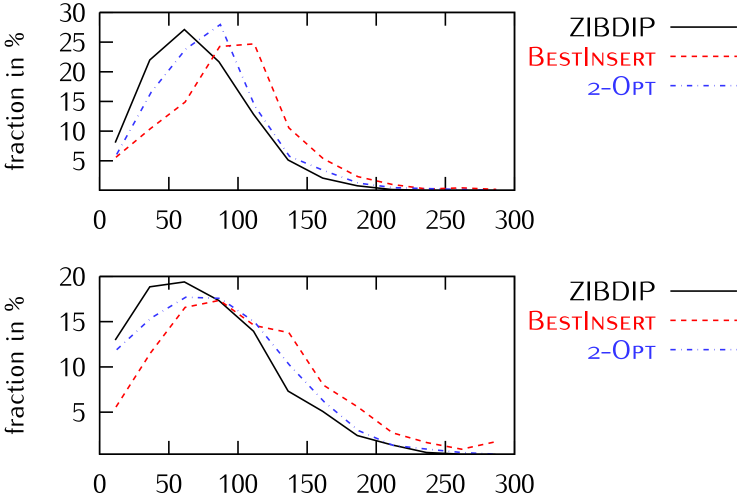


Comparison with heuristics over the day





Comparison waiting times (in minutes)



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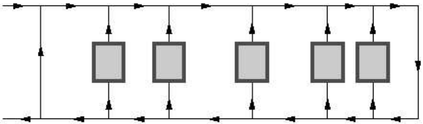
Theory again: The Online Bin Coloring problem

Movie

Application #2: Controlling cargo elevators in a distribution center

Application: elevators in a Herlitz high rack warehouse



idealized setup in [Friese, Rambau '06]:

- global queue for every floor
- local queue for every elevator on each floor
- pallet takes fixed time to travel from global to each local queue

Overview on considered control algorithms

Assign-first, route-second algorithms

FIFO elevator assignment according to round-robin; each elevator serves its requests in FIFO order

NN request is assigned to the elevator having least load including the new request; elevator serves nearest request next

REPLAN elevator assignment as NN; requests scheduled optimally w. r. t. completion time

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Integrated algorithms

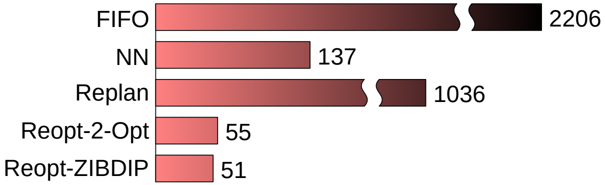
Reopt-2-OPT determines dispatch using a 2-exchange-heuristic

Reopt-ZIBDIP determines dispatch using modified ADAC algorithm ZIBDIP

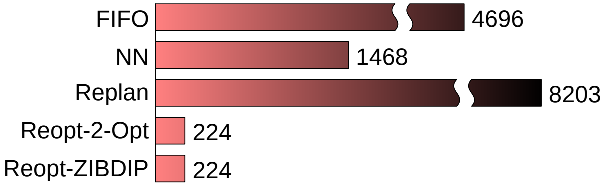
The simulated elevator system features 5 elevators and 16 floors.

Results

average waiting time (seconds)



maximum waiting time (seconds)



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  FIFO 8 elevators needed

  NN 6 elevators needed

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- for much more detailed results see [Friese, Rambau '06]:

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Theory again: The Online Bin Coloring problem

Application #3: Controlling passenger elevators in high-rise buildings

Elements: floors, elevators, passengers, travel calls

Task
Serve all travel calls.

Goal
Small waiting and travel times.

Waiting time:
time for passenger to wait

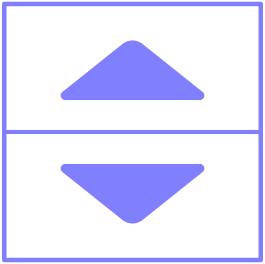
Travel time:
time for passenger to arrive at destination

Conventional System

Two-step input for each passenger:

- Outside car: landing call providing
  - start floor
  - travel direction
  - landing call release time

  unknown: number of passengers, actual passenger arrival times, destination floor(s)



Conventional System

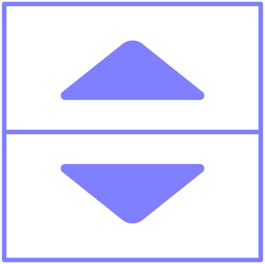
Two-step input for each passenger:

- Outside car: landing call providing
  - start floor
  - travel direction
  - landing call release time

  unknown: number of passengers, actual passenger arrival times, destination floor(s)

- Inside car: car call providing
  - destination floor

  unknown: number of passengers



Destination Call System

Outside car:
destination call providing

- start floor
- destination floor
- release time

Destination Call System

Outside car:
destination call providing

- start floor
- destination floor
- release time

Known at destination call release time:

- destination floor
- number of passengers*
- actual arrival time of each passenger*

No possibility to confirm destination inside car (no panel).

Two Variants of Destination Call Systems

Immediate Assignment (IA)

- after issuing a call, the passenger is immediately assigned to a serving elevator
- used by ThyssenKrupp, Schindler, Kollmorgen Steuerungstechnik

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Delayed Assignment (DA)

- issuing a call and assignment of an elevator are separate steps
- each elevator signals shortly before arrival the floors it is going to serve, i. e., which passenger should board
- not yet implemented

Properties of passenger elevator systems

1. A passenger never moves in opposite direction of his travel direction (no turns)
   - Passenger boards only if elevator departs in travel direction
   - Elevator control has to ensure drop stops before changing direction

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1. A passenger never moves in opposite direction of his travel direction (no turns)
   - Passenger boards only if elevator departs in travel direction
   - Elevator control has to ensure drop stops before changing direction
2. Elevator stops at a floor: no control which passengers board car
   - enough capacity: all (assigned) waiting passengers with matching travel direction / destination floor board
   - not enough capacity: an arbitrary unknown subset; remaining passengers reissue travel call

Request model for elevator schedules

We employ a general model for elevator schedules applicable to all systems (conventional, IA, DA).

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A request expresses that these calls can only be served by the same elevator.

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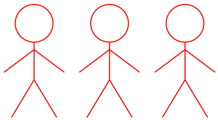

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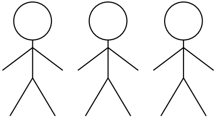

$\longrightarrow$

$$c_1 : 1 \to 5$$
$$c_2 : 1 \to 6$$
$$c_3 : 1 \to 5$$

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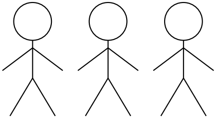

$$\longrightarrow \quad \begin{array}{l} c_1 : 1 \to 5 \\ c_2 : 1 \to 6 \\ c_3 : 1 \to 5 \end{array} \quad \xrightarrow{\text{IA}} \quad r = \{c_1, c_2, c_3\}$$

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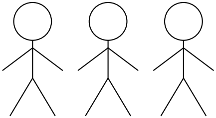

$$\longrightarrow \quad \begin{array}{l} c_1 : 1 \to 5 \\ c_2 : 1 \to 6 \\ \boxed{c_3 : 1 \to 5} \end{array} \quad \xrightarrow{\text{IA}} \quad \begin{array}{l} r_1 = \{c_1, c_2\} \\ r_2 = \{c_3\} \end{array}$$

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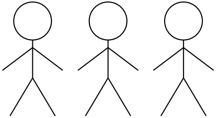

$$\longrightarrow \boxed{\begin{array}{l} c_1 : 1 \to 5 \\ c_2 : 1 \to 6 \\ c_3 : 1 \to 5 \end{array}} \xrightarrow{\text{DA}} \begin{array}{l} r_1 = \{c_1, c_3\} \\ r_2 = \{c_2\} \end{array}$$

Exact Approach via Set Partitioning

- compute dispatch by solving set partitioning formulation

$$
\begin{array}{llll}
\min & \displaystyle\sum_{S\in\mathcal{S}} c_S x_S & & \text{cost } c_S \text{ of a schedule } S \text{ is weighted sum of waiting and travel times} \\
& \displaystyle\sum_{S\in\mathcal{S}\colon r\in S} x_S = 1 & r \in \mathcal{R}_u & \text{every unassigned request } r \in \mathcal{R}_u \text{ is served} \\
& \displaystyle\sum_{S\in\mathcal{S}_e} x_S = 1 & e \in E & \text{every elevator } e \text{ gets a feasible schedule } S \in \mathcal{S}_e \\
& x_S \in \{0,1\} & S \in \mathcal{S} & \text{feasible schedule } S \text{ can be used or not}
\end{array}
$$

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$$
\begin{array}{rll}
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\displaystyle\sum_{S \in \mathcal{S}\colon r \in S} x_S = 1 & r \in \mathcal{R}_u & \\
\displaystyle\sum_{S \in \mathcal{S}_e} x_S = 1 & e \in E & \\
x_S \in \{0,1\} & S \in \mathcal{S} &
\end{array}
$$

cost c_S of a schedule S is weighted sum of waiting and travel times

every unassigned request $r \in \mathcal{R}_u$ is served

every elevator e gets a feasible schedule $S \in \mathcal{S}_e$

feasible schedule S can be used or not

- pricing problem: find a minimum cost feasible schedule, serving all requests assigned to this elevator plus some subset of the unassigned requests (reward: dual price)

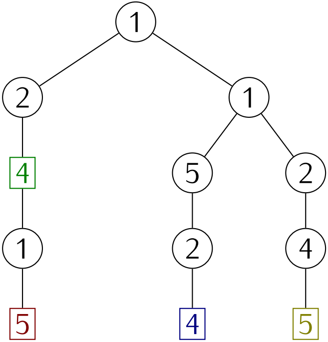
Branch & Bound Pricing Algorithm

$r_1 = \{c_1 \colon 2 \to 4\}, r_2 = \{c_2 \colon 1 \to 5\}$

Branch & Bound Pricing Algorithm

$r_1 = \{c_1 : 2 \to 4\}, r_2 = \{c_2 : 1 \to 5\}$

$A = \{r_1\}, O = \{r_2\}$



Node labels:

- A Set of not yet picked up assigned requests.
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Branch & Bound Pricing Algorithm

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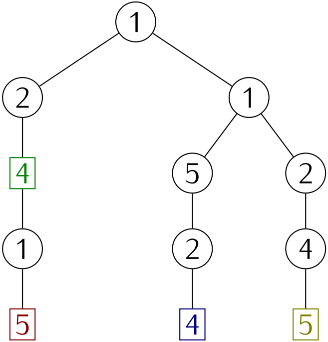

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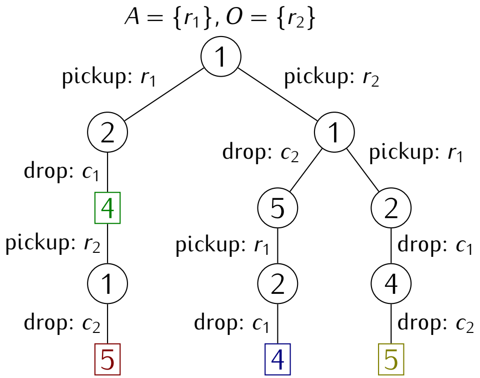

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Branching

- drop call
- pick up request

Branch & Bound Pricing Algorithm

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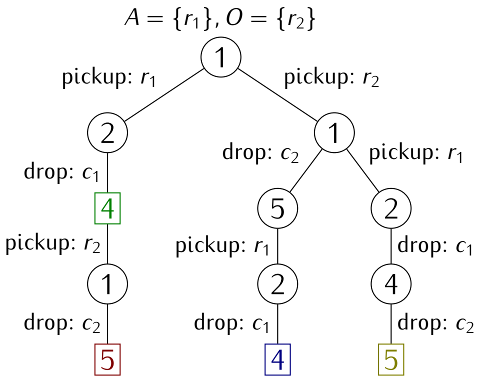

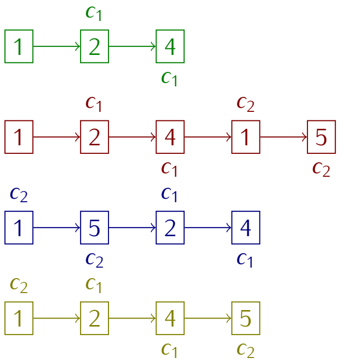

Greedy Lower Bounds

Example partial schedule in some node in the search tree:

$$\overset{s_1}{\boxed{1}} \longrightarrow \overset{s_2}{\boxed{15}} \longrightarrow \overset{s_3}{\boxed{10}} \longrightarrow \overset{s_4}{\boxed{8}} \longrightarrow \overset{s_5}{\boxed{7}} \longrightarrow \overset{s_6}{\boxed{1}}$$

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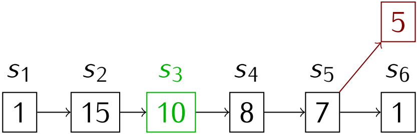

request 1: $5 \rightarrow 2$:

$$t^{+}(1) \geq s_5.\mathit{ArrivalTime} + \tau_{\mathrm{drv}}(7, 5)$$

Greedy Lower Bounds

Example partial schedule in some node in the search tree:

$$\begin{array}{ccccccccccccc} s_1 & & s_2 & & s_3 & & s_4 & & s_5 & & s_6 & & \\ \boxed{1} & \to & \boxed{15} & \to & \boxed{10} & \to & \boxed{8} & \to & \boxed{7} & \to & \boxed{1} & \to & \boxed{8} \end{array}$$

request 2: $8 \to 9$:

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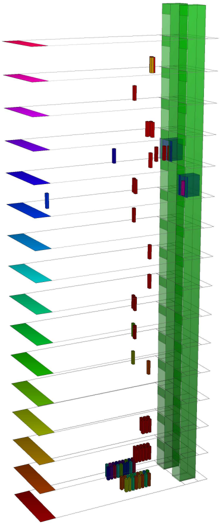
- similar bounds for the travel time of each call
  $\rightsquigarrow$ bounds on cost for each request
- comparing cost bound with dual price allows additional pruning

Simulation Model

- realistic time model: acceleration, maximum speed, ...
- waiting time of elevator at a floor depends on the number of passengers entering/leaving
- elevators stay at floor visited last if no further calls are assigned
- elevators cannot stop or reverse direction halfway between floors
- first-come-first-served boarding: at each stop, passengers enter elevator car in order of increasing waiting time

Simulation Data

- building with a group of 5 elevators serving 25 floors
- three entrance floors, two of which are only served by two elevators



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  Up traffic (U): all calls start from entrance floors

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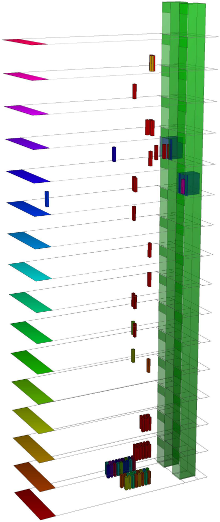

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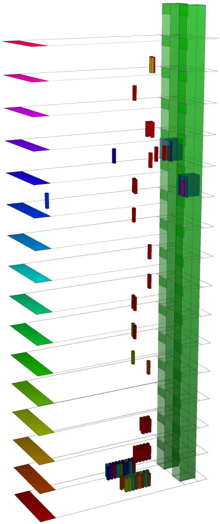
- here: one hour Real Up peak traffic with varying intensities



System Comparison: Waiting Times

evaluate control algorithms using median $\alpha_{0.5}$, 90% quantile $\alpha_{0.9}$, and average $\emptyset$ of waiting time

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scenario	conventional			IA system			DA system		
	$\alpha_{0.5}$	$\alpha_{0.9}$	$\emptyset$	$\alpha_{0.5}$	$\alpha_{0.9}$	$\emptyset$	$\alpha_{0.5}$	$\alpha_{0.9}$	$\emptyset$
Real Up 80%	6	22	9	11	26	12	10	25	11
Real Up 100%	7	25	10	12	34	15	9	28	12
Real Up 144%	176	436	202	16	47	20	8	37	15
Real Up 168%	493	1181	535	42	161	65	–	–	–

waiting times in seconds

System Comparison: Travel Times

scenario	conventional			IA system			DA system		
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Real Up 80%	61	113	65	45	73	47	44	71	45
Real Up 100%	71	129	75	54	86	55	50	84	52
Real Up 144%	260	515	279	72	116	74	69	112	71
Real Up 168%	566	1253	611	106	235	127	–	–	–

travel times in seconds

Snapshot Solution Quality

Solution quality for snapshot problems solved for each new call

gap integrality gap between LP solution after complete pricing and final cost of the dispatch

time total solution time including pricing and solving IP to optimality

$|\mathcal{R}_u|$ number of unassigned requests

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quantile	IA system *(Real Up 168%)* gap [%]	time [s]	$\lvert\mathcal{R}_u\rvert$	DA system *(Real Up 144%)* gap [%]	time [s]	$\lvert\mathcal{R}_u\rvert$
$\alpha_{0.5}$	0.0	0.01	1	0.0	0.13	7
$\alpha_{0.75}$	0.0	0.02	1	0.0	0.27	10
$\alpha_{0.9}$	0.0	0.15	2	0.0	2.09	13
$\alpha_{1.0}$	0.1	1.21	5	0.7	2540.52	21

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- proven optimal dispatch often found in seconds, even for high intensity traffic
  $\Longrightarrow$ real-time compliant for IA systems
- for DA systems computation times still too long for practical use; but already useful for assessing quality of heuristics

Outline

Recap: Online optimization

Theoretical framework: Online Dial-a-Ride problems and competitive analysis

Online Optimization in Practice: Reoptimization Algorithms

- Dispatching the service vehicles of ADAC
- Controlling cargo elevators in a distribution center
- Controlling passenger elevators in high-rise buildings

Theory again: The Online Bin Coloring problem

Online Bin Coloring [Krumke et al '01]

Task

Put a sequence of n colored items into m bins, subject to:

- As soon as a bin has B items it is replaced by an empty one (but no earlier).
- Items must be packed without knowledge of future items, i. e., online.
- Items may not be rearranged between the bins.

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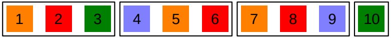
Goal

Minimize the maximum number of colors in a bin (*colorfulness*).

Three Algorithms

sequence for $m = 2$, $B = 3$: 1 2 3 4 5 6 7 8 9 10 sequence for $m = 2$, $B = 3$: 

OneBin Put all items in first bin.

1: 

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Three Algorithms

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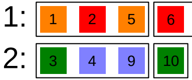
FixedColors Assign each bin equally many colors. Put only items with one of those colors in the bin.



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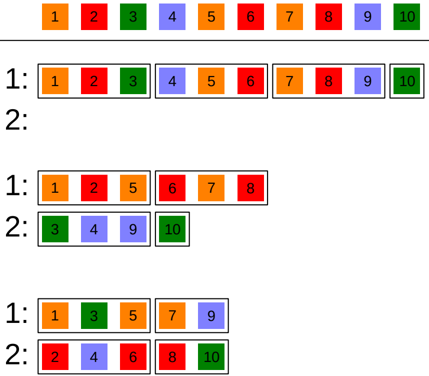

OneBin Put all items in first bin.

FixedColors Assign each bin equally many colors. Put only items with one of those colors in the bin.

GreedyFit Put an item with a new color in a bin which currently has least colorfulness.

Competitive Analysis Results [Krumke et al '01]

- GreedyFit has competitive ratio not greater than $3m$, but greater or equal to $2m$.
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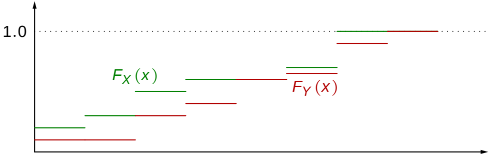
- OneBin has competitive ratio at most $2m - 1$.
- simulations (random data): GreedyFit significantly better than OneBin
- from now on: sequence is constructed by choosing each color independently according to distribution γ

New idea for analysis: Use stochastic dominance

Stochastic Dominance

Let X and Y be random variables with distribution functions F_X and F_Y. X is stochastically dominated by Y, written $X \leq_{st} Y$, if

$$F_X(x) \geq F_Y(x) \quad \text{for all } x \in \mathbb{R}.$$



Example: Stochastic Dominance for Bin Colorfulness

Colorfulness distribution for $m = 3$, $B = 5$, $C = 15$

#Items	FixedColors	GreedyFit
5	0.028, 0.657, 0.971, 1.000, 1.000	0.094, 1.000, 1.000, 1.000, 1.000

Shown is the cumulative distribution function!

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10	0.054, 0.619, 0.975, 1.000	0.189, 0.981, 1.000, 1.000
20	0.001, 0.192, 0.896, 1.000	0.009, 0.725, 0.999, 1.000
40	0.021, 0.766, 1.000	0.429, 0.999, 1.000
80	0.560, 1.000	0.149, 0.998, 1.000
160	0.299, 1.000	0.018, 0.993, 1.000
1000	1.000	0.956, 1.000

Shown is the cumulative distribution function!

A Stochastic Dominance Result

Theorem ([Hiller, Vredeveld '08])

Assume that the color sequence Σ is generated by choosing each color independently at random according to color distribution γ. Then

$$GF(\Sigma) \leq_{\mathrm{st}} OB(\Sigma).$$

Proof involves some probability tools ...

- Markov chains
- stopping times
- mixtures of probability distributions
- couplings

...but it is straightforward.

Consequences of the result

- $X \leq_{\text{st}} Y \Longrightarrow \mathbb{E}[X] \leq \mathbb{E}[Y]$

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	GF	OB
σ_1:	3	4
σ_2:	2	3
σ_3:	3	4
σ_4:	4	3
σ_5:	2	2

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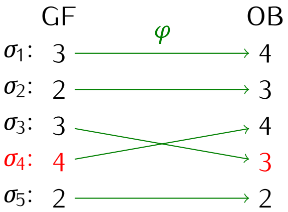

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- stochastic dominance for the uniform distribution is equivalent to a bijective analysis [Angelopoulos et al '07] result: There is a bijective map φ on the sequences s. t.

$$\text{GF}(\sigma) \leq \text{OB}(\varphi(\sigma)) \quad \forall \sigma$$

$$\begin{tikzcd}[row sep=small]
 & \text{GF} & & \text{OB} \\
\sigma_1: & 3 \arrow[rr, "\varphi"] & & 4 \\
\sigma_2: & 2 \arrow[rr] & & 3 \\
\sigma_3: & 3 \arrow[rrd] & & 4 \\
\sigma_4: & 4 \arrow[rru] & & 3 \\
\sigma_5: & 2 \arrow[rr] & & 2
\end{tikzcd}$$

Summary

Theory

- Competitive analysis and its variants are the main tool to study online algorithms.
- Different measures and ways of analysis might provide more useful / interesting results.

Practice

- Exact reoptimization algorithms perform well in practice.
- Mathematical programming techniques can sometimes be used to obtain really fast algorithms.

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