The Gurobi Solver V1.0

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Overview

- Background
- Rethinking the MIP solver
  - Introduction
  - Tree-of-Trees
  - Parallel MIP
  - Other
    - Heuristics
    - Cutting Planes
- Where we are now
  - Performance

Gurobi Optimization

- Gurobi Optimization, Inc. founded July, 2008
  - Founders: Gu, Rothberg, Bixby
- Building a algorithmic base
  - LP simplex and MIP in Version 1.0
- Development timeline
  - March 2008
    - Development began
  - October 2008
    - Version 1.0 development completed
  - 18 May 2009
    - Version 1.0 released
  - 2 October 2009
    - Version 2.0 to be released

Redesigning the MIP Solver

MIP problem

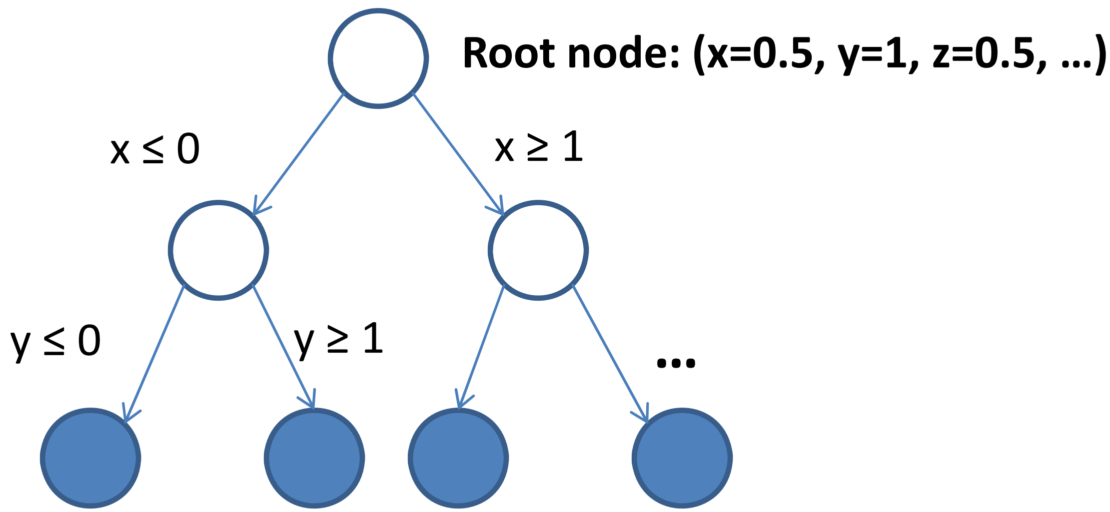
- Mixed Integer Programming

Minimize	$c^T x$
Subject To	$Ax = b$
	$l \le x \le u$

Some x_j must be integral

MIP Solution Approach

- Branch and bound [Land & Doig, 1960]
  - Explore a tree of *relaxations* (ignore integrality)



Key Ingredients

- For an effective MIP solver, you need …
  - Heuristics
    - Explore solutions near the relaxation quickly
    - Find feasible solutions at nodes other than leaf nodes
  - Parallelism
  - Presolve
    - Tighten formulation before starting branch & bound
    - Once you start branching, mistakes replicate
  - Branch-variable selection
  - Cutting planes
  - LP dual-simplex solver

A Fresh Look

A Fresh Look

- Things have changed:
  - Two examples:
    - “Sub-MIP” as a pervasive approach
    - Ubiquitous parallel processing

Sub-MIP As A Paradigm

- Key recent insight for heuristics:
  - Can use MIP solver recursively as a heuristic
  - Solve a related model:
    - Hopefully smaller and simpler
  - Examples:
    - Local cuts [Applegate, Bixby, Chvátal & Cook, 2001]
    - Local branching [Fischetti & Lodi, 2003]
    - RINS [Danna, Rothberg, Le Pape, 2005]
    - Solution polishing [Rothberg, 2007]

RINS

- Relaxation Induced Neighborhood Search
  - Given two “solutions”:
    - x^*: any integer feasible solution (not optimal)
    - x^R: optimal relaxation solution (not integer feasible)
  - Fix variables that agree
  - Solve the result as a MIP
    - Possibly requiring early termination
- Extremely effective heuristic
  - Often finds solutions that no other technique finds

Why Is RINS So Effective?

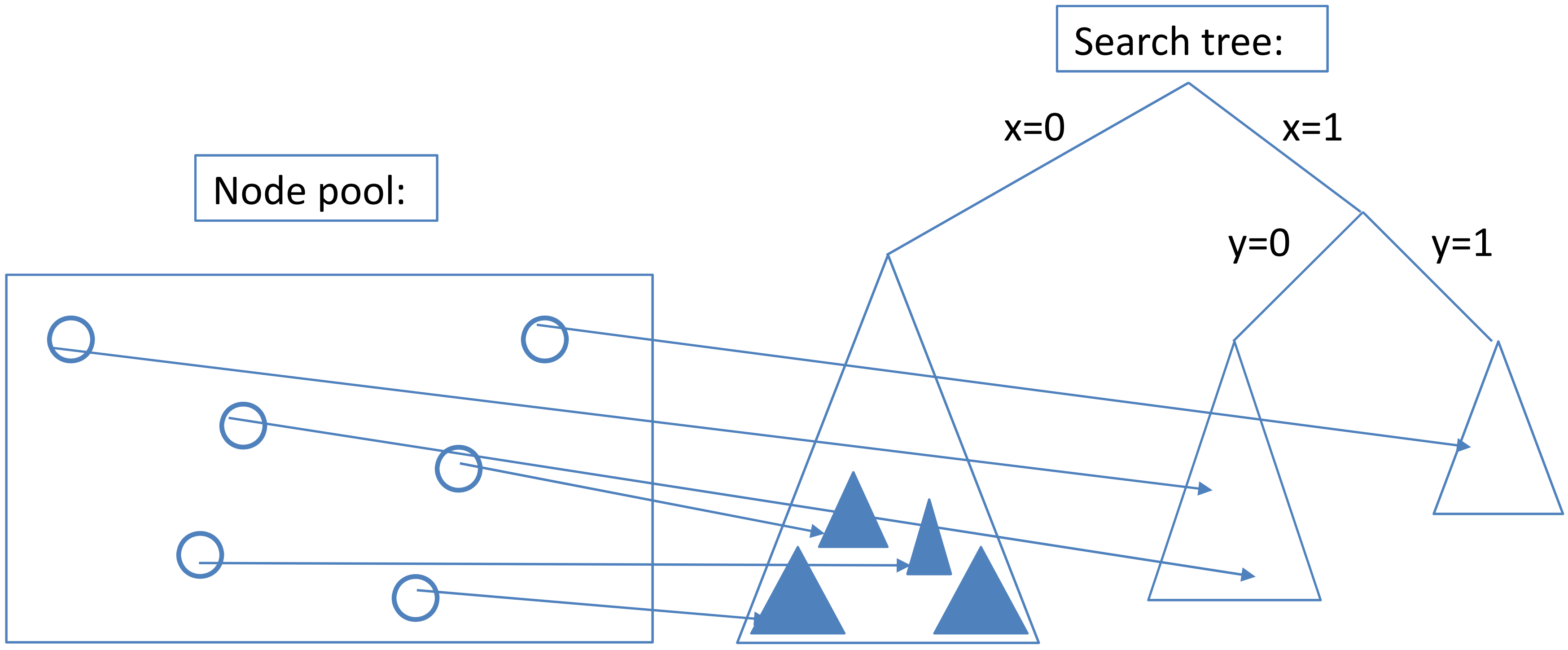
- MIP models often involve a hierarchy of decisions
  - Some much more important than others
- Fixing variables doesn't just make the problem smaller
  - Often changes the nature of the problem
    - Extreme case:
      - Problem decomposes into multiple, simple problems
    - More general case:
      - Resolving few key decisions can have a dramatic effect
  - Strategies that worked well for the whole problem may not work well for RINS sub-MIP
    - More effective to treat it as a brand new MIP

Rethinking MIP Tree Search

Tree-of-Trees

- Gurobi MIP search tree manager built to handle multiple related trees
  - Can transform any node into the root node of a new tree
- Maintains a pool of nodes from all trees
  - No need to dedicate the search to a single sub-tree

Tree-of-Trees



Tree-of-Trees

- Each tree has its own relaxation and its own strategies...
  - Presolved model for each subtree
  - Cuts specific to that subtree
  - Pseudo-costs for that subtree only
  - Symmetry detection on that submodel
  - Etc.
- Captures structure that is often not visible in the original model

Parallel MIP

Why Parallel?

- Microprocessor trends have changed
- Transistors are:
  - Still getting smaller
  - But not faster
- Implications:
  - New math for CPUs: more transistors = more cores
    - 4 cores now, 8 cores in late 2009, …
  - *Sequential software won't be getting significantly faster in the foreseeable future*
- Gurobi MIP solver built for parallel from the ground up
  - Sequential is just a special case

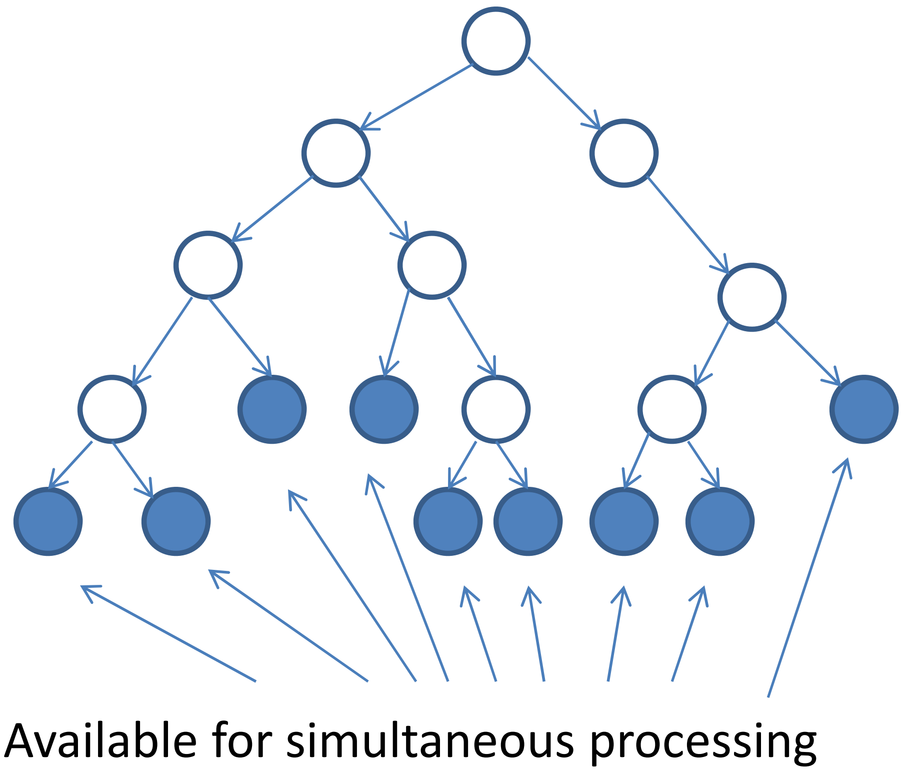
Need Deterministic Behavior

- Non-deterministic parallel behavior:
  – Multiple runs with the same inputs can give different results
- "Insanity: doing the same thing over and over again and expecting different results."
  – Albert Einstein
- Conclusion: non-deterministic parallel behavior will drive you insane

Building Blocks

Building Blocks

- Parallel MIP is parallel branch-and-bound:



Deterministic Parallel MIP

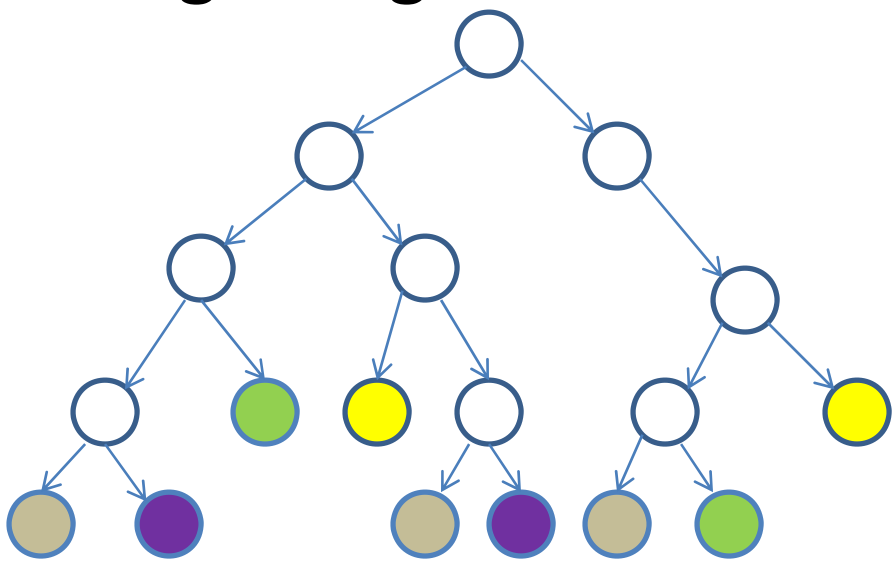
- Multiple phases
- In each phase, on each processor:
  - Explore nodes assigned to processor
  - Report back results
    - New active nodes
    - New solutions
    - New cuts
    - Etc.
- One approach to node assignment:
  - Assign a subtree to each processor
  - Limit amount of exploration in each phase

Subtree Partitioning

- Problem:
  - Subtree may quickly prove to be uninteresting
    - Poor relaxation objectives
      - May want to abandon it
    - Pruned quickly
      - Leaves processor idle

More Global Partitioning

- Node coloring: assign a color to every node



  - Processor can only process nodes of the appropriate color
  - New child node same color as parent node
  - Perform periodic re-coloring

More Dynamic Node Processing

- Allows much more flexibility
  - Processor can choose from among many nodes of the appropriate color
- *Deterministic priority queue* data structure required to support node coloring
  - Single global view of active nodes
  - Support notion of node color
    - Processor only receives node of the appropriate color
  - Efficient, frequent node reallocation

Heuristics

Summary of Heuristics

- 5 heuristics prior to solving root LP
  - 5 different variable orders, fix variables in this order
- 15 heuristics within tree (9 primary, several variations)
  - RINS, rounding, fix and dive (LP), fix and dive (Presolve), Lagrangian approach, pseudo costs, Hail Mary (set objective to 0)
- 3 solution improvement heuristics
  - Applied whenever a new integer feasible is found

Cutting Planes

Cutting Planes

- Gomory
  - Gu ISMP 2006 , strengthen by lifting in GUBs
- Flow covers
  - Strengthened by lifting before un-transforming
- MIR
  - Different aggregation for MIR and flow covers
- Knapsack covers
- Clique
- Implied bound
- Flow paths
  - Results fed into flow covers
- GUB covers

Performance

Performance Benchmarks

- Performance test sets:
  - Mittelmann optimality test set:
    - 55 models, varying degrees of difficulty
    - http://plato.asu.edu/ftp/milpc.html
  - Mittelmann feasibility test set:
    - 34 models, difficult to find feasible solutions
    - http://plato.asu.edu/ftp/feas_bench.html
  - Our own broader test set:
    - A set of 263 models that require between one minute and one hour to solve on one core
      - Publicly available models, plus a few customer models
- Test platform:
  - Q9450 (2.66 GHz, quad-core system)

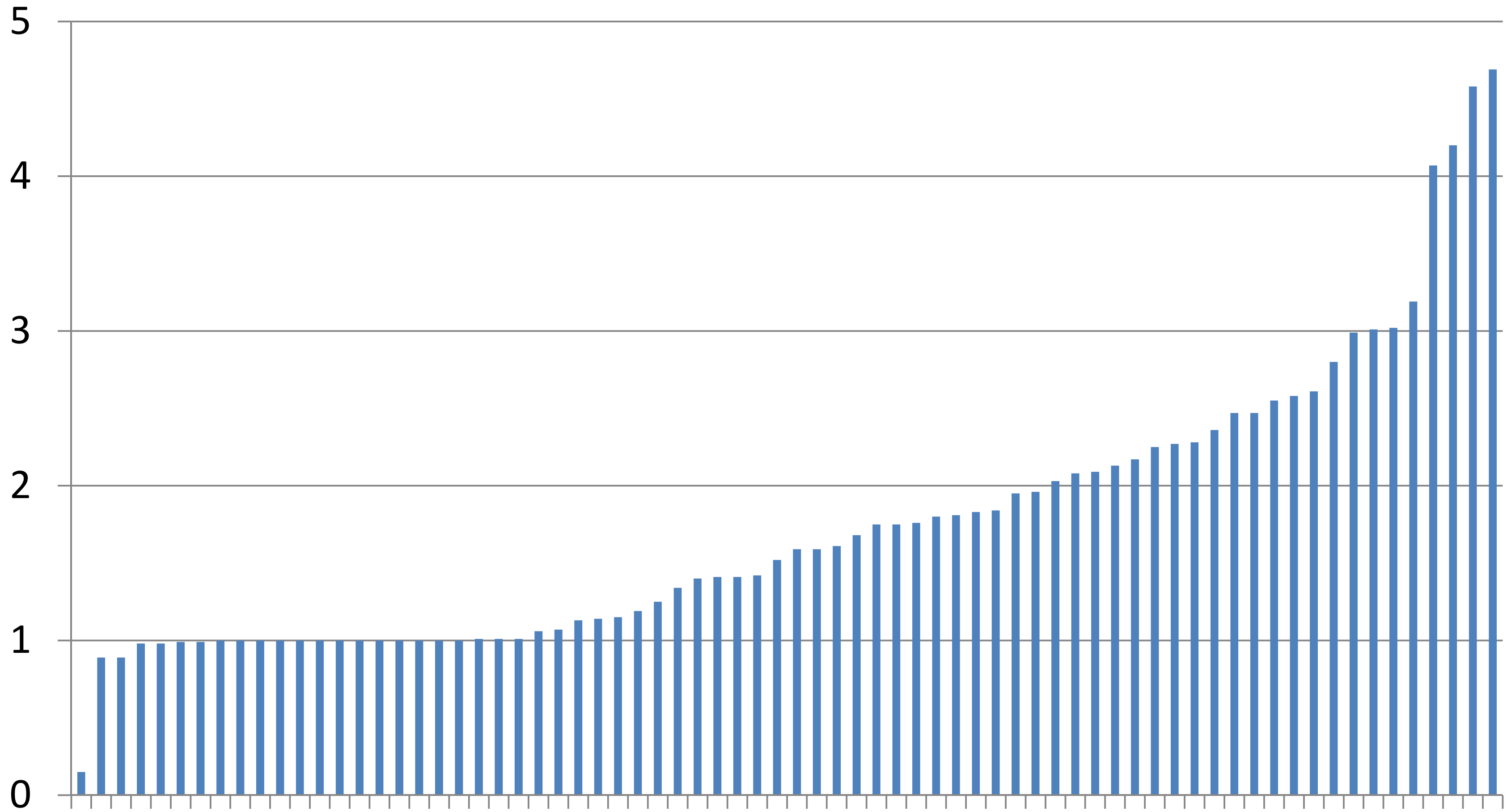
Performance Results

- Two basic questions:
  - Is P=1 efficient?
  - How much does performance improve with P>1?
- P=1 efficiency (geometric means):
  - Mittelmann optimality test set
    - Gurobi 1.1 is 1.18X **slower** than CPLEX 12.0
  - Mittelmann feasibility test set
    - Gurobi 1.1 is 2.3X **faster** than CPLEX 12.0

Parallel Performance

- Comparisons:
  - Gurobi
    - 1.45X **faster** than CPLEX 12.0 for p=4 on Mittelmann
    - Gurobi p=1 versus p=4
      - 1.73X speedup on Mittelmann
      - 1.61X speedup on broader set
  - CPLEX p=1 versus p=4
    - 1.21X speedup for CPLEX 12 on Mittelmann (non-deterministic)
    - 1.29X speedup for CPLEX 11 on their broader set

Speedups for P=4 (By Model)



Gurobi 2.0

- Improve simplex solver
- Cutting planes
  - Add missing cutting planes
- Add missing presolve reductions

Thank You