Open Pit Mine Production Scheduling

Some IP modelling and solution techniques

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Outline

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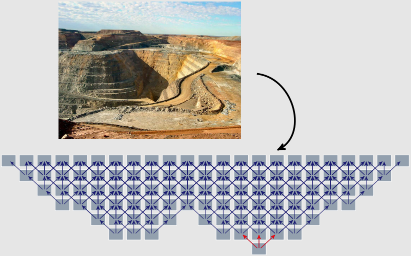
Open Pit Mining Production Scheduling
- Problem description
- Ultimate pit limits
- Time-indexed MIP formulations

OPMPSP relaxations
- Standard LP relaxation
- Full relaxation of resource constraints
- Lagrangean relaxation of resource constraints

Precedence Constrained Knapsack Substructures
- Precedence Constrained Knapsack Problem
- PCKs in open pit mining
- Valid inequalities for the Precedence Constrained Knapsack Polytope

Open Pit Mining Production Scheduling

Input



- ▷ block model with precedences
- ▷ deterministic block content
- ▷ ore prices and production costs
- ▷ equipment capacities

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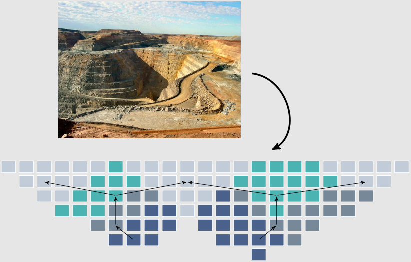

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Open Pit Mining Production Scheduling

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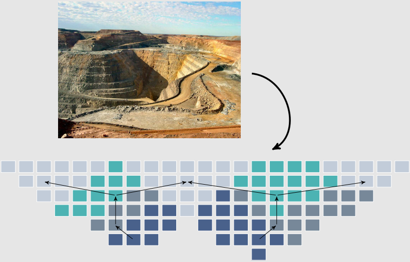

- ▷ block model with precedences
- ▷ deterministic block content
- ▷ ore prices and production costs
- ▷ equipment capacities
- ▷ possibly aggregation of blocks

Problem

Find an order of excavation with

- ▷ max. net present value

satisfying

- ▷ precedence constraints,
- ▷ resource constraints.

Simultaneously: Determine

- ▷ opt. processing decisions

for each block.

Outline

Getting started: computing ultimate pit limits

Given: Profit/negative cost w_i of mining each block $i \in \mathcal{N} = \{1, \ldots, N\}$.

Seek: Precedence-feasible set of blocks $X \subseteq \mathcal{N}$ with max. $\sum_{i \in X} w_i$.

(If $i \in X$ than all predecessors of i are in X.)

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Observation: This is a max-weight closure problem in the precedence graph

$$\big(\mathcal{N}, \mathcal{S} = \{(i,p) \in \mathcal{N} \times \mathcal{N} \mid p \text{ predecessor of } i\}\big)$$

with node weights w_i.

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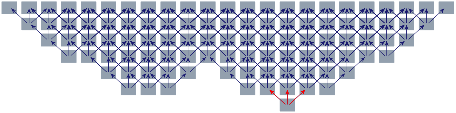
- Solve e.g. by min-cut algorithm in a slightly extended graph:



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A new MIP formulation (Boland-Dumitrescu-Froyland-G'08)

Variables

$$y_{k,t} = \text{fraction of aggregate } k \text{ mined during period } t$$

$$z_{i,t} = \text{fraction of block } i \text{ processed during period } t$$

A new MIP formulation (Boland-Dumitrescu-Froyland-G'08)

Variables

$$x_{k,t} = \begin{cases} 1 & \text{if aggregate } k \text{ may be mined} \\ & \text{during periods } t, \ldots, T \\ 0 & \text{otherwise} \end{cases}$$

$$y_{k,t} = \text{fraction of aggregate } k \text{ mined during period } t$$

$$z_{i,t} = \text{fraction of block } i \text{ processed during period } t$$

A new MIP formulation (Boland-Dumitrescu-Froyland-G'08)

Objective function

$$NetPresentValue\,(y, z) = \sum_{t=1}^{T} \left(\frac{1}{1+q}\right)^{t-1} \left(\sum_{\text{aggs } k} -m_k y_{k,t} + \sum_{\text{blocks } i} p_i z_{i,t} \right)$$

A new MIP formulation (Boland-Dumitrescu-Froyland-G'08)

$$\max \quad NetPresentValue\,(y, z)$$

s.t. (x, y) precedence feasible

$$
\begin{aligned}
x_{k,t} &\in \{0,1\} && \text{for } k = 1, \ldots, K,\ t = 1, \ldots, T\\
0 \le y_{k,t} &\le 1 && \text{for } k = 1, \ldots, K,\ t = 1, \ldots, T\\
0 \le z_{i,t} &\le y_{k,t} && \text{for } k = 1, \ldots, K,\ i \in B_k,\ t = 1, \ldots, T
\end{aligned}
$$

A new MIP formulation (Boland-Dumitrescu-Froyland-G'08)

$$\max \quad NetPresentValue(y, z)$$

$$
\begin{aligned}
\text{s.t.} \qquad & (x, y) \text{ precedence feasible} \\
& \sum_{k=1}^{K} \bar{R}_k y_{k,t} \leq M_t && \text{for } t = 1, \dots, T \\
& \sum_{i=1}^{N} R_i z_{i,t} \leq P_t && \text{for } t = 1, \dots, T \\
& x_{k,t} \in \{0, 1\} && \text{for } k = 1, \dots, K,\ t = 1, \dots, T \\
& 0 \leq y_{k,t} \leq 1 && \text{for } k = 1, \dots, K,\ t = 1, \dots, T \\
& 0 \leq z_{i,t} \leq y_{k,t} && \text{for } k = 1, \dots, K,\ i \in B_k,\ t = 1, \dots, T
\end{aligned}
$$

- integrated optimisation of processing decisions

For this talk: no aggregates, no fractional mining

$$\max \quad NetPresentValue(x, z)$$

$$\text{s.t.} \quad x_{i,t-1} \leq x_{i,t} \quad \text{for } i \in \mathcal{N},\ t = 2, \ldots, T$$

$$x_{i,t} \leq x_{p,t} \quad \text{for } (i,p) \in \mathcal{S}, t = 1, \ldots, T$$

$$x_{i,t} \in \{0, 1\} \quad \text{for } i \in \mathcal{N},\ t = 1, \ldots, T$$

$$x_{i,0} = 0 \quad \text{for } i \in \mathcal{N}$$

$$0 \leq z_{i,t} \leq x_{i,t} - x_{i,t-1} \quad \text{for } i \in \mathcal{N},\ t = 1, \ldots, T$$

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\begin{aligned}
\text{s.t.} \quad & x_{i,t-1} \leq x_{i,t} && \text{for } i \in \mathcal{N},\ t = 2, \dots, T \\
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& \sum_i R_i (x_{i,t} - x_{i,t-1}) \leq M_t && \text{for } t = 1, \dots, T \\
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$$\max \quad NetPresentValue(x, z)$$

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\end{aligned}
$$

▷ NP-hard by reduction from Precedence-Constrained Knapsack

Outline

Standard LP relaxation

$$\max \quad \mathit{NetPresentValue}(x, z)$$

$$
\begin{aligned}
\text{s.t.} \qquad & x_{i,t-1} \le x_{i,t} && \text{for } i \in \mathcal{N},\ t = 2, \dots, T \\
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▷ typically: interior point outperforms simplex (when solving from scratch)

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Full relaxation of resource constraints

$$\max \quad NetPresentValue(x, z)$$

$$\begin{aligned}
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\end{aligned}$$

Full relaxation of resource constraints

$$
\begin{aligned}
\max \quad & \mathit{NetPresentValue}(x) \\
\text{s.t.} \quad & x_{i,t-1} \le x_{i,t} && \text{for } i \in \mathcal{N},\ t = 2, \ldots, T \\
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\end{aligned}
$$

▷ substitute z according to objective function coefficients

Full relaxation of resource constraints

$$\max \quad \mathit{PresentValue}(x)$$

s.t.

$$x_i \leq x_p \qquad \text{for } (i,p) \in \mathcal{S}$$

$$x_i \in \{0,1\} \qquad \text{for } i \in \mathcal{N}$$

- substitute z according to objective function coefficients
- costs and profits decrease over time $\Rightarrow$ all blocks are mined in period 1 (if at all) $\rightsquigarrow$ equivalent to computing the **Ultimate Pit Limit**

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- substitute z according to objective function coefficients
- costs and profits decrease over time $\Rightarrow$ all blocks are mined in period 1 (if at all) $\rightsquigarrow$ equivalent to computing the **Ultimate Pit Limit**
- solve efficiently as max-weight closure problem
- use as preprocessing: all blocks in an optimal OPMPSP schedule must be contained in the Ultimate Pit – remove the others from the model

Outline

Lagrangean relaxation of resource constraints

$$\max \quad \mathit{NetPresentValue}(x, z)$$

$$
\begin{aligned}
\text{s.t.} \quad & x_{i,t-1} \le x_{i,t} && \text{for } i \in \mathcal{N},\ t = 2, \ldots, T \\
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& \sum_i R_i(x_{i,t} - x_{i,t-1}) \le M_t && \text{for } t = 1, \ldots, T \\
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\end{aligned}
$$

Lagrangean relaxation of resource constraints

$$\max \quad NetPresentValue(x,z) + \sum_t \mu_t \left[M_t - \sum_i R_i (x_{i,t} - x_{i,t-1}) \right] + \sum_t \pi_t \left[P_t - \sum_i R_i z_{i,t} \right]$$

$$\begin{aligned}
\text{s.t.} \quad & x_{i,t-1} \leq x_{i,t} && \text{for } i \in \mathcal{N},\ t = 2, \dots, T \\
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\end{aligned}$$

Lagrangean relaxation of resource constraints

$$\max \quad NetPresentValue(x,z)$$
$$+\sum_t \mu_t \left[M_t - \sum_i R_i (x_{i,t} - x_{i,t-1})\right] + \sum_t \pi_t \left[P_t - \sum_i R_i z_{i,t}\right]$$

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\end{aligned}$$

- feas. region is integral ⇒ opt. multipliers as in LP, best dual bound equal to LP bound (Geoffrion 1974)

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$$\max \quad NetPresentValue(x, z) + \sum_t \mu_t \Big[M_t - \sum_i R_i (x_{i,t} - x_{i,t-1})\Big] + \sum_t \pi_t \Big[P_t - \sum_i R_i z_{i,t}\Big]$$

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- feas. region is integral ⇒ opt. multipliers as in LP, best dual bound equal to LP bound (Geoffrion 1974)
- z-variables can be substituted according to obj. coefficient
- transformation to unconstrained project scheduling

Lagrangean relaxation of resource constraints

$$\max \quad \mathit{NetPresentValue}(x,z) + \sum_t \mu_t \big[M_t - \sum_i R_i(x_{i,t} - x_{i,t-1})\big] + \sum_t \pi_t \big[P_t - \sum_i R_i z_{i,t}\big]$$

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\end{aligned}$$

- ▷ feas. region is integral ⇒ opt. multipliers as in LP, best dual bound equal to LP bound (Geoffrion 1974)
- ▷ z-variables can be substituted according to obj. coefficient
- ▷ transformation to unconstrained project scheduling
- ▷ efficient solution by min-cut computations (Möhring et al. 2003)

Lagrangean dual (with Helmberg's ConicBundle 0.2d) vs. LP-relaxation (with CPLEX 11.0 Barrier, no crossover)

Problem instance	LP-relaxation		Lagrangean dual			
	time [s]	obj. val. $[10^8]$	time [s]	obj. val. $[10^8]$	rel. time	rel. obj. val.
ma-115-8513	11.2	7.230097	1.8	7.230097	0.158787	1.000000
ma-296-8513	27.2	7.276624	2.6	7.276631	0.094993	1.000001
ma-1038-8513	259.5	7.306851	9.5	7.306854	0.036415	1.000001
ca-121-29266 (avg)	34.9	59.898146	6.1	59.898190	0.175164	1.000001
ca-121-29266 (05)	64.7	60.899745	6.6	60.899800	0.101763	1.000001
wa-125-96821	324.4	0.508814	21.8	0.508815	0.067139	1.000001

Realistic problem instances provided by industry partner BHP Billiton, `http://www.bhpbilliton.com/`.

Outline

Precedence Constrained Knapsack Problem

Given:

- Items $i \in \mathcal{N}$ with sizes a_i and values c_i
- Knapsack capacity b
- Precedence order $\mathcal{S} \subseteq \mathcal{N} \times \mathcal{N}$ (transitively reduced)

Feasible Solution:

- set $X \subseteq \mathcal{N}$ with $\sum_{i \in X} a_i \leqslant b$ and
- X is closed under $\mathcal{S}$ (If $i \in X$ and $(i,j) \in \mathcal{S}$, then $j \in X$.)

Goal: maximize $\sum_{i \in X} c_i$

Important substructure in many applications:

- Resource constrained scheduling
- Multi-level facility location, access & distribution network design
- Routing problems: confluent flows, Internet routing, ...
- ...

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PCKs in open pit mining

$$\max \quad NetPresentValue(x, z)$$

$$
\begin{aligned}
\text{s.t.} \qquad & x_{i,t-1} \le x_{i,t} && \text{for } i \in \mathcal{N},\ t = 2, \ldots, T \\
& x_{i,t} \le x_{p,t} && \text{for } (i,p) \in \mathcal{S}, t = 1, \ldots, T \\
& \sum_i R_i (x_{i,t} - x_{i,t-1}) \le M_t && \text{for } t = 1, \ldots, T \quad \leftarrow \text{add up} \\
& \sum_i R_i z_{i,t} \le P_t && \text{for } t = 1, \ldots, T \\
& x_{i,t} \in \{0, 1\} && \text{for } i \in \mathcal{N},\ t = 1, \ldots, T \\
& x_{i,0} = 0 && \text{for } i \in \mathcal{N} \\
& 0 \le z_{i,t} \le x_{i,t} - x_{i,t-1} && \text{for } i \in \mathcal{N},\ t = 1, \ldots, T
\end{aligned}
$$

PCKs in open pit mining

$$x_{i,t} \le x_{p,t} \qquad \text{for } (i,p) \in \mathcal{S}$$

$$\sum_i R_i \; x_{i,t} \le \sum_{s=1}^{t} M_s$$

$$x_{i,t} \in \{0,1\} \qquad \text{for } i \in \mathcal{N}$$

- For each time period, the OPMPSP formulation contains one precedence constrained knapsack.

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Precedence Constrained Knapsack Polytope

$$PCKP := \text{conv}\Big\{x \in \{0,1\}^{\mathcal{N}} \mid \sum_{i \in \mathcal{N}} a_i x_i \leqslant b, \quad \text{(i)} \\ x_i \leqslant x_j \text{ for } (i,j) \in \mathcal{S} \Big\} \quad \text{(ii)}$$

Polyhedral structure of $PCKP$ well studied:

- Dimension, Conditions for (i) and (ii) to be facet-defining
- Knapsack-based inequalities [Boyd'93, Park-Park'97]
  - Induced cover inequalities
  - (Induced) k-cover inequalities
  - (Induced) (1,k)-configuration inequalities
- Sequential lifting results [v.d.Leensel-v.Hoesel-v.d.Klundert'99]

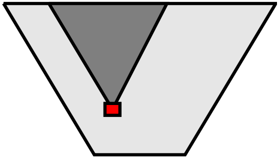
But:

- Nothing implemented in MIP solvers

Variable fixing

Notation

- $Pred(i) := \{j : j \text{ predecessor of } i\} \cup \{i\}$
  (Items that must be included with i.)
- $A(i) := \sum_{j \in Pred(i)} a_j$
  (Total size induced by i.)



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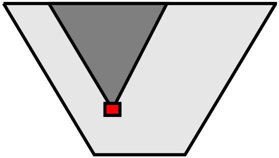

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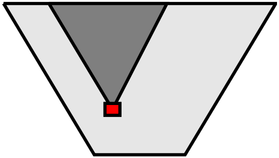

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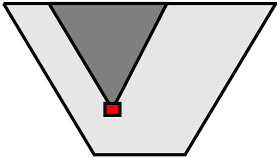

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Remarks

- Can compute *all* $A(i)$ in $\mathcal{O}(\mathcal{S})$.
- Fixing is very efficient in practice (reduces LP gaps substantially).
- CPLEX does *not* find these fixings even with most aggressive probing.

Pairwise conflicts

Apply same idea to pairs of items:

- $A(i,j) := \sum_{k \in Pred(i) \cup Pred(j)} a_k$
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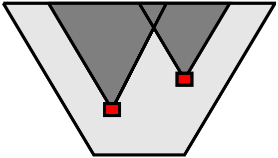

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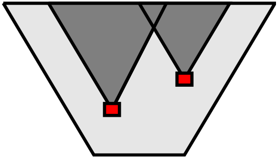

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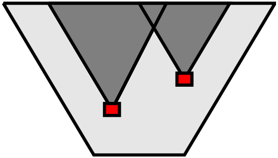

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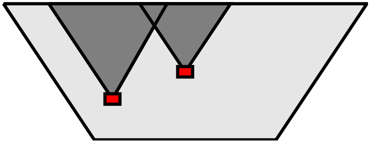
- $\{i,j\}$ with $A(i,j) > b$: Induced cover of size 2
- Induced cover: $X \subset \mathcal{N}$ with $A(X) > b$
- [Boyd'93, Park-Park'97] Conditions when induced cover inequalities are facet-defining (for low-dimensional faces of PCKP)
- [v.d.Leensel-v.Hoesel-v.d.Klundert'99] Lifting facet-defining inequalities for low-dimensional faces of PCKP to facets of PCKP

Clique inequalities

Idea

- Consider conflict graph $CG = (\mathcal{N}, E)$ with $E := \{ij \mid A(i,j) > b\}$

Precedence order



Conflict graph CG

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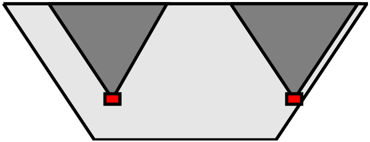

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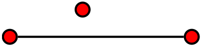

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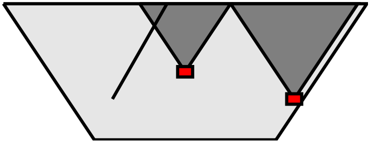

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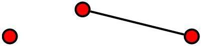

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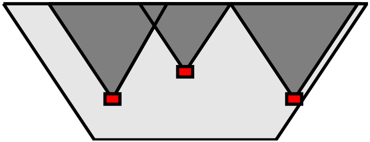

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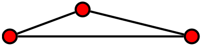

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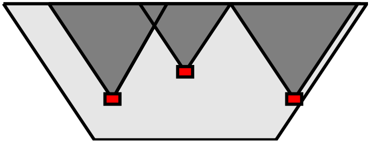

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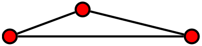

Observation: Any valid inequality for $STAB(CG)$ is valid for $PCKP$.

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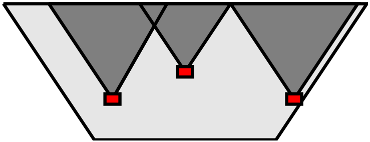

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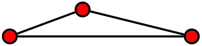


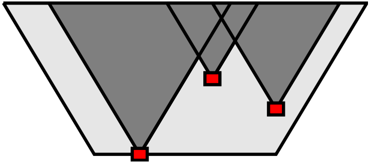
Observation: Any valid inequality for $STAB(CG)$ is valid for $PCKP$.

Corollary: Let C be a clique in CG. Then (1) is valid for $PCKP$.

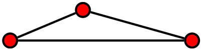
$$\sum_{i \in C} x_i \leqslant 1 \qquad (1)$$

Cliques with common predecessors

Precedence order



Extended conflict graph
CG'



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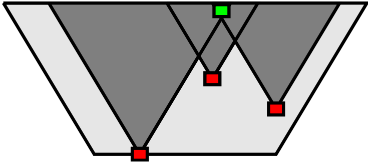

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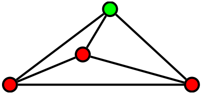

Observation: Let C be a clique in CG with $P(C) = \bigcap_{i \in C} Pred(i) \neq \emptyset$. Then, for any $i \in P(C)$,

$$\sum_{j \in C} x_j \leqslant x_i \tag{2}$$

is valid for $PCKP$.

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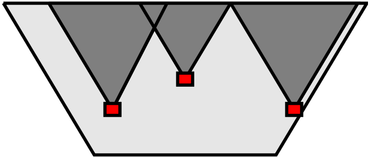

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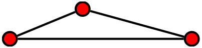

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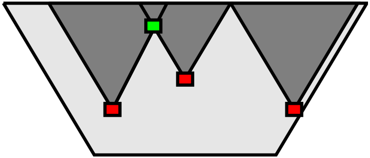

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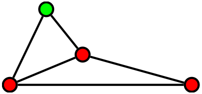

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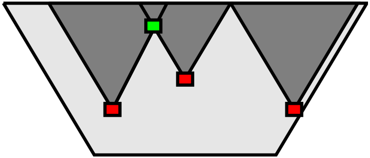

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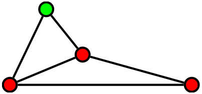

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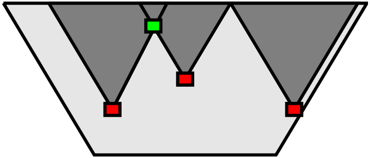

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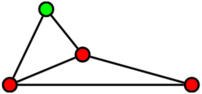

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Separation of clique-based inequalities

Given: $x \in [0,1]^{\mathcal{N}}$ fractional
Seek:

(1) clique C in CG such that $\sum_{j \in C} x_j > 1$, or

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- Separation of (1) and (2) is equivalent to finding a *maximum-weight clique* in CG' with node weights $w_k := x_k$ and $w_{k'} := 1 - x_k$ for all $k \in \mathcal{N}$
  - $w(C^*) \leqslant 1$: all inequalities (1) and (2) satisfied
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- NP-hard, but: Efficient MWC implementations exist in any state-of-the-art MIP solver.
- Ideally: extend the conflict graph of your MIP solver.

More details on facets and computational results: see e.g.
Boland-Froyland-Fricke-Sotirov'06, Bley-Boland-Fricke-Froyland'09.

Summary

Open Pit Mining Production Scheduling

- Problem description
- Ultimate pit limits
- Time-indexed MIP formulations

OPMPSP relaxations

- Standard LP relaxation
- Full relaxation of resource constraints
- Lagrangean relaxation of resource constraints

Precedence Constrained Knapsack Substructures

- Precedence Constrained Knapsack Problem
- PCKs in open pit mining
- Valid inequalities for the Precedence Constrained Knapsack Polytope

Open Pit Mine Production Scheduling

Some IP modelling and solution techniques

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Zuse Institute Berlin

09/28/2009

Berlin Mathematical School

berlin

DFG Research Center MATHEON
Mathematics for key technologies

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