Surface mining	Industry	Modelling	Objective function	Constraints	Implementation	Extensions

### Equipment selection for surface mines

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Equipment selection for surface mines

C. Burt and Y. Chan

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Surface r	nining					

Mining is a thriving industry in Australia. We are the world's:

- Second largest producer of uranium and zinc;
- Third largest producer of iron ore, nickel, and diamond;
- Fourth largest producer of gold and coal.
- We are also the world's largest exporter of coal.

Mining contributes 5.6% of Australia's GDP. Mineral exports form 35% of our total exports.

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How a m	ine wo	rks				

- There are several mining locations located inside the pit.
- Ore is mined at these mining locations using machinery.
- This process also produces waste materials.

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### How a mine works



(Image from Kalgoorlie Consolidation Gold Mining Australia)

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How a mine works								

- At each location a loader loads ore and waste onto trucks.
- The trucks then move the ore and waste to stockpiles and dump-sites respectively.
- Then they return to get loaded again.
- This process is called materials handling.
- Materials handling costs are considered to account for 50% of the costs of the mine.

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How a m	ine wor	kc				



Equipment selection for surface mines

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Loaders						

Loaders include:

- Electric rope shovels;
- Hydraulic loaders;
- Hydraulic backhoe excavators;
- Front-end loaders.
- Capacities vary from 18 to 110 tonnes.

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Loaders						



#### A Hitachi EX2500 excavator (image from jcccw.com)

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Trucks						

- We use off-highway trucks or dump trucks.
- Truck capacities vary from 36 tonnes to 215 tonnes.
- It is also important to note the **truck cycle time**, which is the time it takes for the truck to be loaded, travel from the loader to the dump-site, dump its contents and travel back again.
- This time varies as the mine grows!

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Trucks						



# A CAT 777d dump truck (image from immersivetechnologies.com)

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Optimiza	tion					

There are many different parts of the mining process that we can optimize, including:

- Pit shape optimisation;
- Mining schedule (which Ambros will talk about later);
- Mining method.

We concentrate on equipment selection.

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Equipme	nt selec	tion				

Equipment selection involves choosing trucks and loaders to operate the materials handling process. There are three primary considerations:

- Productivity: We must be able to produce enough ore to satisfy requirements.
- Cost: We want to keep purchasing and operating costs as low as possible.
- Compatibility: The trucks and loaders must be able to work together. Not all trucks and loaders are compatible!

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Complica	tions					

- We want to optimize over a long period of time. During this time, trucks and loaders may be bought, grow old, overhauled, sold, and/or simply expire, possibly many times over!
- We want to allocate loaders to mining sites and trucks to loaders/sites. Based on the layout of the mine, this can have a significant effect on the process.

The combination of all these factors can lead to a very large number of possibilities.



During a meeting with a mining company, the equipment selection problem was presented with the following parameters and constraints:

#### Parameters

- swell factor
- diggability
- bunching
- rimpull curves
- rolling resistance
- haul grade and distance

 cycle times (spotting, travel empty)

#### Constraints

- match factor
- blending
- compatibility
- life cycle cost

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Simplifica	ations					

In order to derive a solvable model, we make some simplifying assumptions.

- Known mine schedule, operating hours and mining method;
- Single mining location and truck cycle time (for simplicity);
- We can salvage old equipment;
- We do not account for auxiliary equipment.

We do allow equipment to not be worked to its fullest capacity.

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### What model should we use?

The selection or non-selection of trucks and loaders is best modelled by binary variables.

Utilisation of trucks and loaders are continuous variables (scaled to be between 0 and 1).

Productivity and cost are generally linear functions of utilisation.

Therefore we will use a mixed-integer linear program.

### What objective should we use?

In this problem there are two common approaches:

- Maximise productivity while constraining costs;
- Minimise costs while constraining productivity (from below).

We choose to go with the latter (although the former is also acceptable).

Surface mining	Industry 0	Modelling 000€00000	Objective function	Constraints 0000000000	Implementation	Extensions 0
Discretiza	ation					

It would complicate things enormously to be able to buy, sell or utilise at any time. As is typical in this type of modelling, we discretize.

- The time frame is broken down into a number of **time periods**. Everything is measured with reference to these time periods.
- We can only buy or sell equipment at the beginning of each period.
- Utilisation of equipment is calculated according the proportion of available time each period that a piece of equipment actually works.

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Discustion						

The other non-linearity that we discretize is the cost of equipment utilisation.

- We break down the age of the equipment into a number of **age brackets**.
- The cost of running the equipment is constant over the age brackets.
- Cost traditionally has a 'sawtooth' shape, as the operating cost steadily goes up over time, and then decreases after an overhaul.

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Discretiza	ation					



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Discretiza	ation					



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Surface mining	Industry O	Modelling 0000000●0	Objective function	Constraints 00000000000	Implementation	Extensions 0
Keeping	track o	f trucks				

To account for utilisation we keep track of individual trucks/loaders. Each piece of equipment has two identifiers:

- A truck/loader type;
- An ID, which is unique for that type.

For accounting purposes, we must also note the age (bracket) of the equipment.

Surface mining	Industry 0	Modelling 00000000●	Objective function	Constraints 0000000000	Implementation 000000	Extensions 0
Variables						



- $\mathbf{x}_{e,i}^{k,l}$  1 if we own a truck/loader of type e with ID j, in time period k, which is in age bracket l at the start of the period; 0 otherwise.
- $\mathbf{f}_{a_i}^{k,l}$
- the amount of time (as a proportion of total operating hours) that this truck works in period k.



 $\mathbf{s}_{e,i}^{k,l}$  1 if we sell a truck/loader of type e with ID j at the start of time period k, when it was in age bracket l; 0 otherwise.

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Objective	functio	on				
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There are three sources of cost that we take into account:

- Capital expense (purchasing costs);
- Operating cost;
- Salvage 'cost' (really profit).



First we have to figure out when a truck/loader is bought. To do so we look at the previous period to see if it was there. So if

$$\sum_{l} \mathbf{x}_{e,j}^{k,l} - \sum_{l} \mathbf{x}_{e,j}^{k-1,l}$$

is 1, equipment of type e with ID j was bought in period k. However if it was sold in period k, this will be -1! So we add a term of

$$\sum_{l} \mathbf{s}_{e,j}^{k,l}.$$

Surface mining	Industry 0	Modelling 000000000	Objective function	Constraints 000000000000	Implementation	Extensions 0
Capital ex	kpense					

Each truck/loader type has its own cost  $F_e$ , and we also discount all costs to the present day using the interest rate *I*:

Capital expense = 
$$\sum_{e,j,k} \frac{F_e}{(1+I)^k} \left( \sum_{l} \mathbf{x}_{e,j}^{k,l} - \sum_{l} \mathbf{x}_{e,j}^{k,l} + \sum_{l} \mathbf{s}_{e,j}^{k,l} \right).$$

This is still linear because the coefficient is not a variable (and can be pre-calculated).

Surface mining	Industry O	Modelling 000000000	Objective function	Constraints 0000000000	Implementation	Extensions 0
Operating	g cost					

The cost of operating a piece of equipment depends on:

- The amount of time the equipment actually spends operating (it is proportionate to this);
- The equipment type;
- The age of the equipment (counted in brackets);
- The time period (which affects external variables like truck cycle time, etc.).

Surface mining	Industry 0	Modelling 000000000	Objective function	Constraints 0000000000	Implementation	Extensions 0
Operatin	or cost					

We denote the cost of operating equipment of type e in period k at age bracket l at full capacity to be  $V_e^{k,l}$ . Again all costs are discounted to the present day:

Operating cost = 
$$\sum_{e,j,k,l} \frac{V_e^{k,l}}{(1+l)^k} \mathbf{f}_{e,j}^{k,l}$$
.

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Salvage p	orofit					

We apply a depreciation term based on the age of the equipment (D per bracket) to the purchase cost to calculate the salvage cost. It is also discounted to present day value.

Salvage profit = 
$$(-) \sum_{e,j,k,l} \frac{(1-D)^l F_e}{(1+l)^k} \mathbf{s}_{e,j}^{k,l}$$
.

Surface mining	Industry 0	Modelling 000000000	Objective function	Constraints •000000000	Implementation 00 0000	Extensions 0
Constrair	nts					

The constraints fall into three main categories:

- Productivity;
- Compatibility;
- Internal compatibility (make sure that the variables mean what we want them to mean!)

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Productiv	vitv					

We only impose production constraints on the loaders; the truck constraints will follow from the compatibility constraints below.  $P_{i'}^{k,l}$  is the full production capacity of loader type i' in period k and age bracket l;  $T^k$  is the total production requirement in period k.

$$\sum_{i',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \geq T^k.$$

Surface mining	Industry 0	Modelling 000000000	Objective function	Constraints Im	plementation	Extensions 0
Compatik	oility					

We want our trucks and loaders to be compatible with each other. Otherwise the mine cannot work!

However, not **all** the trucks need to be compatible with **all** the loaders. We just need enough to be compatible to enable the loaders to do their job!

We denote the set of trucks compatible with loader type i' as X(i').

Surface mining	Industry 0	Modelling 000000000	Objective function	Constraints 000000000	Implementation	Extensions 0
Compatik	oility					

$$\sum_{i \in X(i'), j, l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \ge \sum_{j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l}.$$

However, this runs the risk of double-counting. Even if a truck is compatible with two loaders, it cannot service both at the same time!

This is the same case for loader sets of size 3, 4, ....

Surface mining	Industry O	Modelling 000000000	Objective function	Constraints 00000000000	Implementation 0 0000	Extensions 0
Compatik	oility					

Taking A' to be any set of loaders, this gives

$$\sum_{i\in X(A'),j,l} \mathcal{P}_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq \sum_{i'\in A',j,l} \mathcal{P}_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l}.$$

Notice how taking A' to be the full set of all possible loaders ensures that the production requirements are satsified by trucks.

This produces a **lot** of constraints and is best implemented by a separation algorithm.

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Internal of	compat	ibility				

We must make sure that the variables are 'aligned', so that if we own a truck/loader in one period, we either own it or salvage it in the next period.

By selecting the lengths of our discretizations correctly, we make sure that a piece of equipment cannot advance by more than 1 age bracket over any time period.

$$\mathbf{x}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k+1,l} + \mathbf{x}_{e,j}^{k+1,l+1} + \mathbf{s}_{e,j}^{k+1,l} + \mathbf{s}_{e,j}^{k+1,l}$$

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Internal o	compat	ibility				

Conversely, if we own or salvage a truck/loader that is not newly bought in a period, we must have owned it in the previous period.

$$\mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l} + \mathbf{x}_{e,j}^{k-1,l-1},$$

assuming  $l \ge 1$  (age brackets start from 0).



We have some more internal compatibility constraints:

• Each equipment owned is represented by one x variable:

$$\sum_{l} \mathbf{x}_{e,j}^{k,l} \leq 1.$$

• We can only salvage equipment once:

$$\sum_{k,l} \mathbf{s}_{e,j}^{k,l} \leq 1.$$

• We do not re-use ID numbers once the equipment is salvaged:

$$\sum_{k' < k,l} \mathbf{s}_{e,j}^{k',l} + \sum_{l} \mathbf{x}_{e,j}^{k,l} \le 1.$$

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Internal of	compat	ibility				

We must also make sure that we cannot use a piece of equipment (f variable) unless we own it (x variable). We let  $a_e^{k,l}$  be the availability of equipment type e in time period k when in age bracket l.

$$\mathbf{f}_{e,j}^{k,l} \leq a_e^{k,l} \mathbf{x}_{e,j}^{k,l}.$$

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Internal of	compat	ibility				

It is important that the age brackets are 'in sync' with the usage of the equipment specified by the  $\mathbf{f}$  variables in previous periods. We do this by calculating the age of each piece of equipment in hours, and then converting to brackets.

Age in hours at beginning of period  $k = \sum_{k' < k,l} O^{k'} \mathbf{f}_{e,j}^{k',l}$ .

 $O^k$  is the operating hours of the mine in time period k. To calculate the age bracket, we divide this by the bracket size  $B_0$ .

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Internal c	compat	ibility				

We then want to ensure that the age bracket is as specified:

$$l \leq \sum_{k' < k, l} \frac{O^{k'}}{B_0} \mathbf{f}_{e, j}^{k', l} \leq l + 1 - \epsilon,$$

but only if the equipment is recorded as being owned in this period at the proper age bracket, i.e. only if  $\mathbf{x}_{e,i}^{k,l}$  is 1.

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Internal of	Internal compatibility							

To do this we use big-M constraints.

$$\sum_{k' < k,l} \frac{O^{k'}}{B_0} \mathbf{f}_{e,j}^{k',l} \le M + (l+1-\epsilon-M) \mathbf{x}_{e,j}^{k,l}$$
$$\sum_{k' < k,l} \frac{O^{k'}}{B_0} \mathbf{f}_{e,j}^{k',l} \ge l + M(\mathbf{x}_{e,j}^{k,l} - 1).$$

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Implemer	ntation					

We implemented this model using data given to us by our industry partner.



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Implemer	ntation	]				

- We were able to optimise 4 periods in 27 minutes.
- We were able to optimise 5 periods to within 1.1% of optimality.
- This involved 95550 variables and 174037 constraints, and took 20 hours!
- Our solution had a total cost of \$70.9 million.
- There was no direct comparison with the actual equipment selection used (the mine ran for 13 periods), but a simpler model was able to improve the selection used by 24%, or \$40.4 million!

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## Implementation



Equipment selection for surface mines

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#### Equipment selection for surface mines

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Extension	IS					

Possible extensions to our model include:

- A more accurate way to calculate cost and utilisation bounds. The current way assumes that all discretizations hold throughout a time period (not true!). This has already been modelled and we will be running calculations soon.
- A model which takes into account multiple mining locations. A less complicated model which does not account for utilisation has been made.
- Better calculational tools to allow solving over a longer time-frame.