Branching, Presolving, and Constraint Handlers

CO@Work Berlin

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Berlin Mathematical School

DFG Research Center MATHEON
Mathematics for key technologies

Outline

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How do we solve discrete optimization problems?

Mixed Integer Programming

- LP relaxation
- cutting planes
- branch-and-bound

SATisfiability problems

- conflict analysis
- periodic restarts
- branch-and-bound

Constraint Programming

- domain propagation
- symmetry handling
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Branch-and-bound

- Land, Doig [1960], Dakin [1965]
- "smart" enumeration
- use dual bounds

LP-based Branch-and-Bound (Algorithm)

Steps

1. Abort criterion
2. Node selection
3. Solve relaxation
4. Bounding
5. Feasibility check
6. Branching

$\mathcal{L} \leftarrow \{P\}, \quad U \leftarrow \infty, \quad x^{\mathrm{IP}} \leftarrow \mathrm{NULL}$

if $\mathcal{L} = \emptyset$ **then return** x^{IP} *and* U

Select $P_i \in \mathcal{L}, \quad \mathcal{L} \leftarrow \mathcal{L} \backslash \{P_i\}$

Solve LP relaxation of $P_i, \quad L_{\mathrm{loc}} \leftarrow c(x^{\mathrm{LP}})$ or ∞

if $L_{loc} \geq U$ **then goto** line 2

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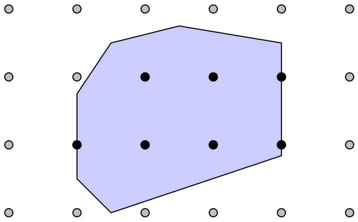

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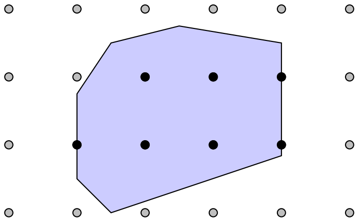

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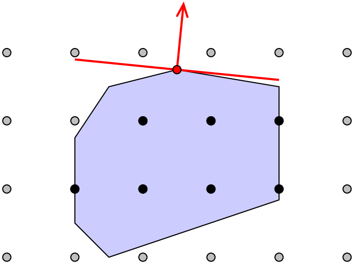

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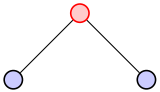

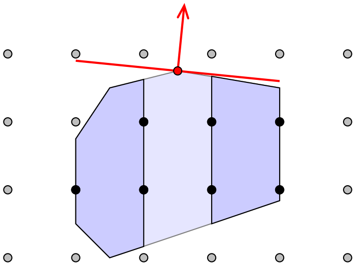

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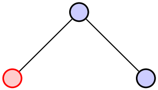

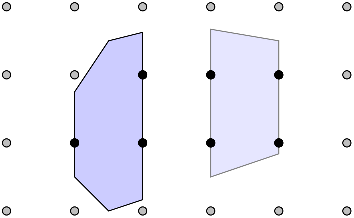

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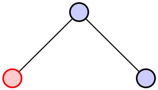

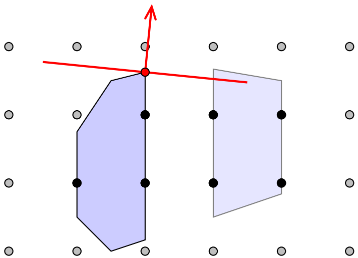

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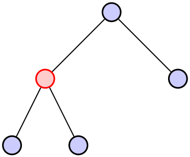

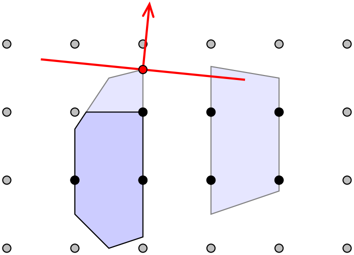

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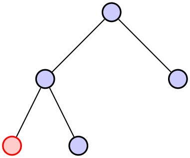

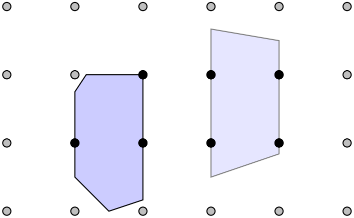

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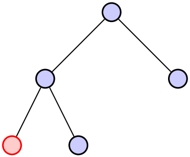

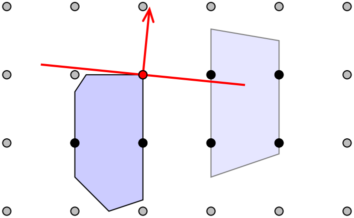

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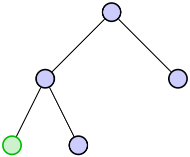

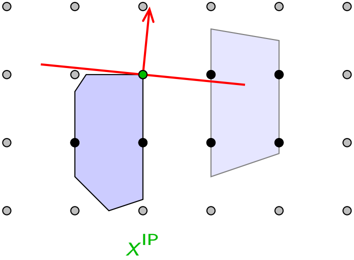

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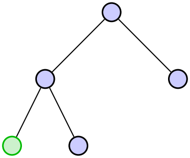

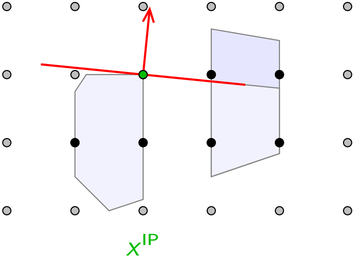

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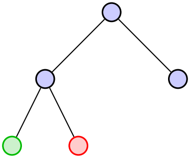

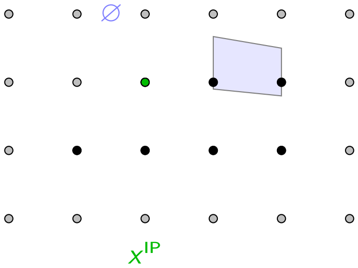

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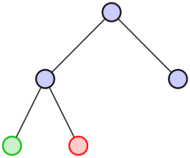

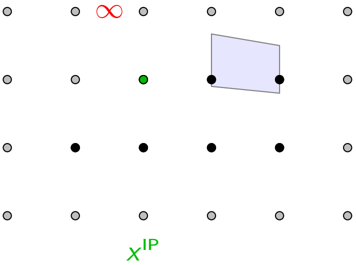

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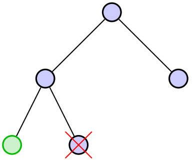

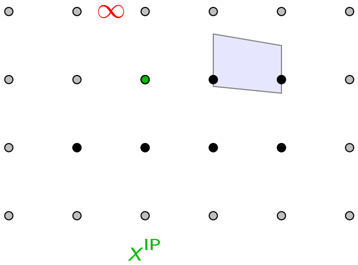

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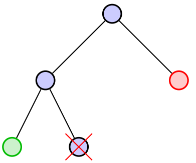

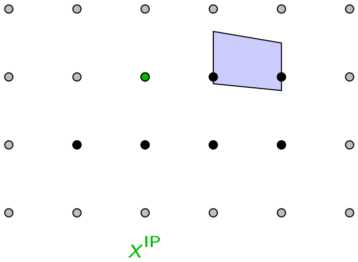

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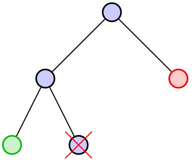

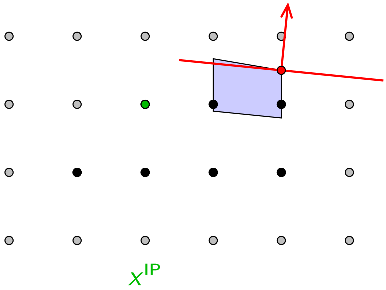

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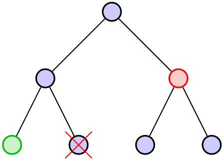

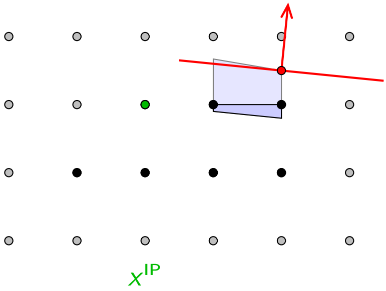

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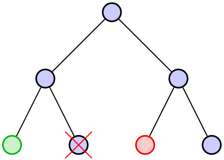

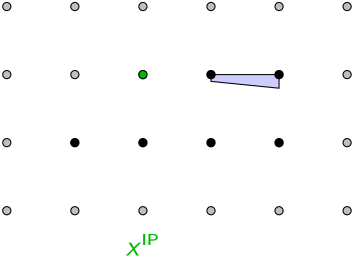

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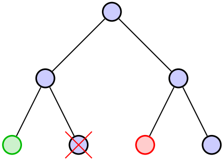

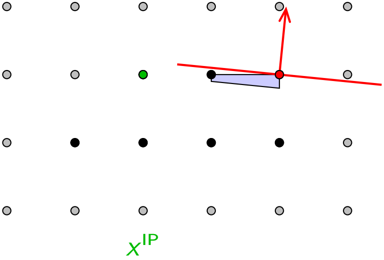

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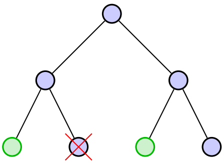

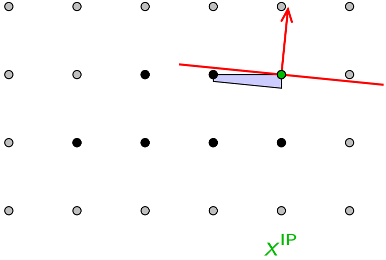

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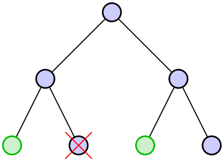
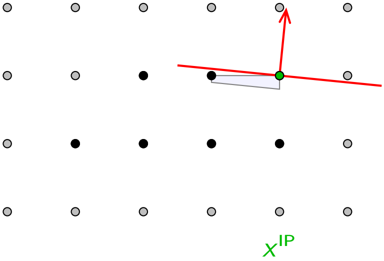
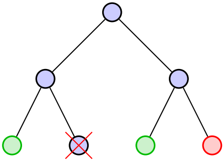
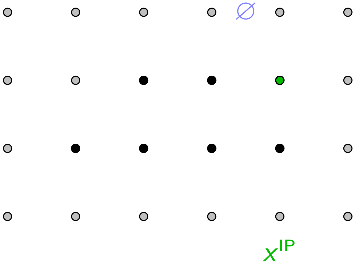




Steps

1. Abort criterion
2. Node selection
3. Solve relaxation
4. Bounding
5. Feasibility check
6. Branching





LP-based Branch-and-Bound (Colorful Picture)

Steps

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2. Node selection
3. Solve relaxation
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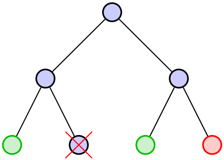

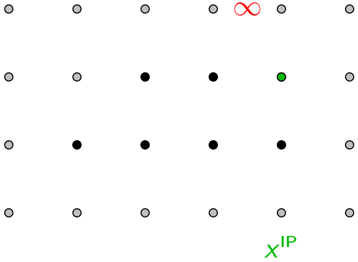

LP-based Branch-and-Bound (Colorful Picture)

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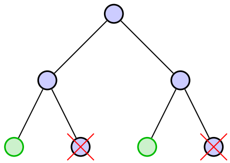

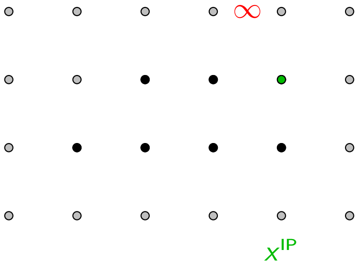

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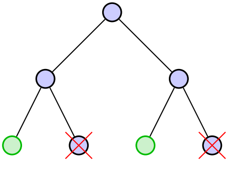

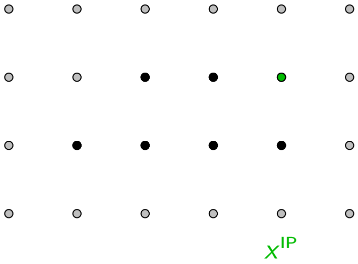

```
SCIP> read check/IP/miplib2003/air04.mps.gz
SCIP> optimize
...
presolved problem has 7370 variables (7370 bin, 0 int, 0 impl, 0 cont)
    601 constraints of type <setppc>
  node   | left   |LP iter| mem |  dualbound     | primalbound    |  gap
      1 |      0 |  2756 |  40M| 5.553544e+04 |      --        |   Inf
...
      1 |      2 |  8342 |  41M| 5.564871e+04 |      --        |   Inf
     50 |     49 | 32939 |  41M| 5.576070e+04 |      --        |   Inf
*    73 |      8 | 47394 |  41M| 5.576165e+04 | 5.641900e+04 |  1.18%
  node   | left   |LP iter| mem |  dualbound     | primalbound    |  gap
*    76 |      9 | 47533 |  37M| 5.576165e+04 | 5.640000e+04 |  1.14%
*    79 |      8 | 47636 |  37M| 5.576165e+04 | 5.638500e+04 |  1.12%
*    82 |      7 | 48134 |  37M| 5.580334e+04 | 5.636300e+04 |  1.00%
*    98 |      4 | 50423 |  36M| 5.584525e+04 | 5.613800e+04 |  0.52%
    100 |      4 | 50529 |  35M| 5.584525e+04 | 5.613800e+04 |  0.52%
*   128 |      2 | 52075 |  35M| 5.584525e+04 | 5.613700e+04 |  0.52%
    150 |      2 | 54032 |  35M| 5.593390e+04 | 5.613700e+04 |  0.36%

SCIP Status        : problem is solved [optimal solution found]
Solving Time (sec) : 59.78
Solving Nodes      : 162
```

```
Welcome to CPLEX Interactive Optimizer 12.1.0
CPLEX is a registered trademark of IBM Corp.
CPLEX> read check/IP/Bugs/Kaibel/ggt3.lp
CPLEX> optimize
Presolve time =    0.00 sec.
MIP search method: dynamic search.
Parallel mode: none, using 1 thread.
Root relaxation solution time =    0.00 sec.

   Node     Left    Objective  IInf  Best Integer  Best Node      Gap

        0      0       1.0000     1                   1.0000
*       0+     0                          3.0000      1.0000   66.67%
        0      2       1.0000     1       3.0000      1.0000   66.67%
...
```

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        0      2       1.0000     1       3.0000      1.0000    66.67%
...
224000000      2       1.0000     1       3.0000      1.0000    66.67%

Solution pool: 1 solution saved.

MIP - Time limit exceeded, integer feasible:  Objective =  3.0000000000e+00
Current MIP best bound =  1.0000000000e+00 (gap = 2, 66.67%)
Solution time = 3600.01 sec.  Iterations = 224141556  Nodes = 224141558 (2)
```

```
bzfberth@opt29 ~/projects/scip> less check/IP/Bugs/Kaibel/ggt3.lp


Minimize
obj: x3
Subject To
c1:  12 x1 + 9  x2 - x3 = 0
Bounds
-inf <= x1
-inf <= x2
   1 <= x3
General
x1
x2
x3
End
```

```
bzfberth@opt29 ~/projects/scip> less check/IP/Bugs/Kaibel/ggt3.lp


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```

Not a CPLEX issue!

Other codes do not perform better!

Branch-And-Bound (Algorithm) continued

$\mathcal{L} \leftarrow \{P\}, \quad U \leftarrow \infty, \quad x^{\mathrm{IP}} \leftarrow \mathrm{NULL}$

if $\mathcal{L} = \emptyset$ **then return** x^{IP} *and* U

Select $P_i \in \mathcal{L}, \quad \mathcal{L} \leftarrow \mathcal{L} \setminus \{P_i\}$

Solve LP relaxation of $P_i, \quad L_{\mathrm{loc}} \leftarrow c(x^{\mathrm{LP}})$ or ∞

if $L_{loc} \geq U$ **then goto** line 2

if $x^{LP} \in P$ **then** $x^{\mathrm{IP}} \leftarrow x^{\mathrm{LP}}, \quad U \leftarrow L_{\mathrm{loc}}$, **goto** line 2

Select $j \in I : x_j^{\mathrm{LP}} \notin \mathbb{Z}$. Split P_i into $P_{2i+1} := P_i \cup \{x_j \leq \lfloor x_j^{\mathrm{LP}} \rfloor\}$, and $P_{2i+2} := P_i \cup \{x_j \geq \lceil x_j^{\mathrm{LP}} \rceil\}, \quad \mathcal{L} \leftarrow \mathcal{L} \cup \{P_{2i+1}, P_{2i+2}\}$, **goto** line 2

Two degrees of freedom

1. Node selection
2. Variable selection (branching rule)

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Two degrees of freedom

1. Node selection
2. Variable selection (branching rule)

Same algorithm, different scope

Common goal: Keep branch-and-bound tree small!

MIP

- improve LP value
- reduce LP complexity

SAT

- detect infeasibilities
- generate short conflicts

CP

- enable propagations
- reduce domains

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But

Also in MIP, sometimes...

- there's no objective
- instances are infeasible
- combinatorial structure

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- instances are infeasible
- combinatorial structure

Standard MIP branching inferior in these cases

Branching rules in MIP

Most infeasible branching

- ▷ choose variable with fractional value closest to 0.5
- ▷ often referred to as a simple, standard rule
- ▷ computationally as bad as random branching!

Strong branching [ABCC '95]

- ▷ solve LP relaxations for some candidates, choose best
- ▷ effective w.r.t. number of nodes, expensive w.r.t. time

Pseudocost branching [Bénichou et al. '71]

- ▷ try to estimate LP values, based on history information
- ▷ effective, cheap, but weak in the beginning

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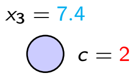
What are pseudocosts?

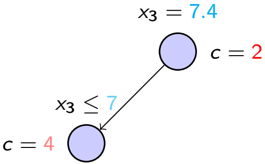
Estimating the objective



What are pseudocosts?

Estimating the objective

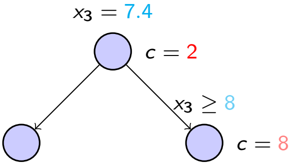
- objective gain per unit:
  - $\zeta^-(x_3) = \frac{4-2}{7.4-7} = \frac{2}{0.4} = 5$



What are pseudocosts?

Estimating the objective

- objective gain per unit:
  - $\zeta^{+}(x_3) = \frac{8-2}{8-7.4} = \frac{6}{0.6} = 10$



What are pseudocosts?

Estimating the objective

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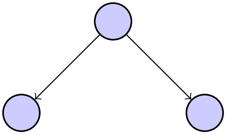
  - $\zeta_1^-(x_3) = 5$, $\zeta_1^+(x_3) = 10$



What are pseudocosts?

Estimating the objective

- objective gain per unit:
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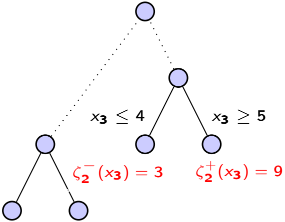
  - other values at other nodes



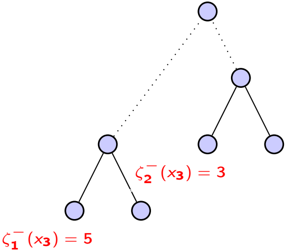
What are pseudocosts?

Estimating the objective

- objective gain per unit:
  - $\zeta_1^-(x_3) = 5$, $\zeta_1^+(x_3) = 10$
  - other values at other nodes
- pseudocosts:

  average objective gain

  $\psi^-(x_3) = \frac{\zeta_1^-(x_3)+\ldots+\zeta_n^-(x_3)}{n} = \frac{5+3}{2} = 4$



What are pseudocosts?

Estimating the objective

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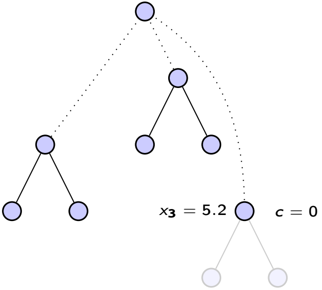
  - $\zeta_1^-(x_3) = 5$, $\zeta_1^+(x_3) = 10$
  - other values at other nodes
- pseudocosts:
  average objective gain
  $\psi^-(x_3) = 4$, $\psi^+(x_3) = 9.5$
- estimate increase of objective
  by pseudocosts and fractionality:



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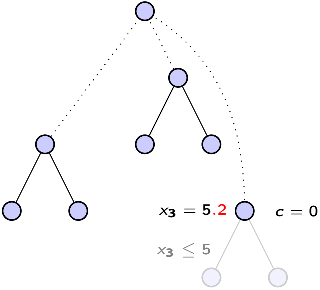
  $\psi^-(x_3) \cdot \mathrm{frac}(x_3)$



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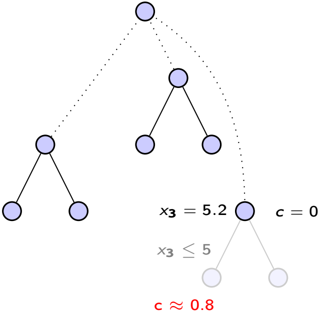
  $\psi^-(x_3) = 4$, $\psi^+(x_3) = 9.5$
- estimate increase of objective
  by pseudocosts and fractionality:
  $\psi^-(x_3) \cdot \mathrm{frac}(x_3) = 4 \cdot 0.2 = 0.8$,



What are pseudocosts?

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  - $\zeta_1^-(x_3) = 5$, $\zeta_1^+(x_3) = 10$
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- ▷ pseudocosts:
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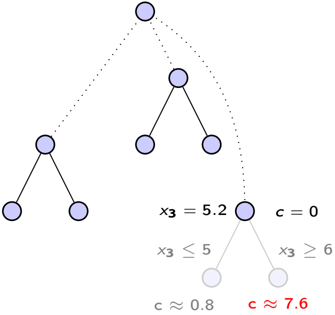
  $\psi^-(x_3) = 4$, $\psi^+(x_3) = 9.5$
- ▷ estimate increase of objective
  by pseudocosts and fractionality:
  $\psi^-(x_3) \cdot \mathrm{frac}(x_3) = 4 \cdot 0.2 = 0.8$,
  and $\psi^+(x_3)(1 - \mathrm{frac}(x_3)) = 7.6$



Reliability Branching

Early branchings are the most important ones!
Problem: In the beginning, pseudocosts are all zero

Pseudocost with strong branching initialization [Linderoth and Savelsbergh '99]

- ▷ use strong branching, if pseudocosts have not been initialized yet

Reliability branching [Achterberg et. al. '05]

- ▷ use strong branching, if pseudocosts are unreliable
- ▷ unreliable: pseudocosts have been updated less than k times
- ▷ computational results: $k = 8$

A general branching rule for CP

Inference branching: [Li and Anbulagan '97]

- average number of applied domain deductions
- history based
- captures combinatorial structure
- estimates tightening of subproblems

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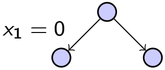
$$
\begin{aligned}
x_1 \quad\quad + x_2 &= 1 \\
x_1 + x_3 + x_4 &\leq 1 \\
-x_1 \quad\quad + z &\geq 3 \\
z &\in \mathbb{Z}_+ \\
x_i &\in \{0, 1\}
\end{aligned}
$$



$$
\begin{aligned}
x_1 = 0 \Rightarrow x_2 &= 1 \\
\Rightarrow z &\geq 3
\end{aligned}
$$

$$s_j^{\text{infer}}(-) = 2$$

A general branching rule for CP

Inference branching: [Li and Anbulagan '97]

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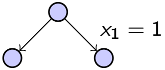
$$\begin{aligned} x_1 \quad\quad\;\; + x_2 &= 1 \\ x_1 + x_3 + x_4 &\leq 1 \\ -x_1 \quad\quad\;\; + z &\geq 3 \\ z &\in \mathbb{Z}_+ \\ x_i &\in \{0,1\} \end{aligned}$$



$$\begin{aligned} x_1 = 1 &\Rightarrow x_2 = 0 \\ &\Rightarrow x_3 = 0 \\ &\Rightarrow x_4 = 0 \end{aligned}$$

$$s_j^{\text{infer}}(+) = 3$$

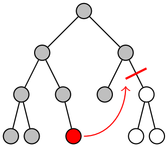
A general branching rule for CP

Inference branching: [Li and Anbulagan '97]

- average number of applied domain deductions
- history based
- captures combinatorial structure
- estimates tightening of subproblems

- analogy to pseudocost values in MIP
- one value for upwards branch, one for downwards
- initialization: probing ($\approx$ strong branching)

A general branching rule for SAT



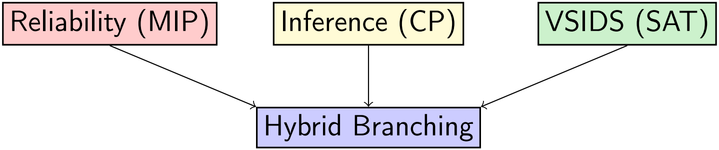
- important feature: conflict analysis / no-goods
- learning of small clauses which trigger infeasibility
- can be generalized to MIP, CP

VSIDS branching: [Moskewicz et. al. '01]

- largest number of (conflict) clauses, a variable appears in
- prefer "recent" conflicts
- "recent": exponentially decreasing importance
- works in particular well for infeasible problems
- state-of-the-art in SAT solving

Put it all together

Hybrid Branching: [Achterberg and B. '09]
Combine strategies to a new hybrid strategy for MIP



- use reliable pseudocosts, inference values, VSIDS
- additionally incorporate:
  - number of pruned subproblems
  - average length of conflict clauses

Hybrid branching

How the combination works:

- scaling: divide each value by average over all variables
- normalize each of the (scaled) values by $f : \mathbb{Q}_{\geq 0} \rightarrow [0, 1),\ x \mapsto \frac{x}{x+1}$
- use a weighted sum of all criteria

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- use a weighted sum of all criteria

In formulae:

$$s_j = \omega^{\text{pscost}} f\left(\frac{s_j^{\text{pscost}}}{s_\varnothing^{\text{pscost}}}\right) + \omega^{\text{infer}} f\left(\frac{s_j^{\text{infer}}}{s_\varnothing^{\text{infer}}}\right) + \omega^{\text{vsids}} f\left(\frac{s_j^{\text{vsids}}}{s_\varnothing^{\text{vsids}}}\right) + \omega^{\text{prune}} f\left(\frac{s_j^{\text{prune}}}{s_\varnothing^{\text{prune}}}\right) + \omega^{\text{conf}} f\left(\frac{s_j^{\text{conf}}}{s_\varnothing^{\text{conf}}}\right)$$

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Choice for the weights:

- high weight for pseudocosts: 1
- medium weight for VSIDS and conflict length: 10^{-2} and 10^{-3}, resp.
- low weight for inference and cutoff values: 10^{-4}

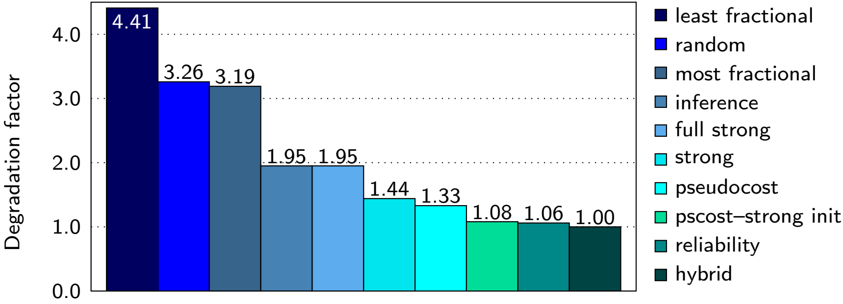
Hybrid branching

Branching score function

- yields two values: One for downwards, one for upwards branching
- need to combine them to a single value
- usually: convex sum
- includes minimum and maximum as extreme cases

- we use: multiplication
  $\text{score}(x_j) = \max\{s_j^-, \epsilon\} \cdot \max\{s_j^+, \epsilon\}$ $(\epsilon = 10^{-6})$
- computational results: 10% faster

Computational results



- ratio to hybrid branching (SCIP default)
- large test set: 575 instances
- shifted geometric time
- time limit of 1 hour

Computational results II

Test set:

- MIPLIB2003: selection of 60 quite different, difficult instances
- Cor@l: Huge collection of 350 instances
- Cor@l-BP: The 118 pure 0/1-programs of the Cor@l test set
- Infeasible: 30 infeasible graph coloring instances

Comparison:

- geometric means of overall running time and number of branch-and-bound nodes
- ratio between reliability branching and hybrid branching

Computational results III

test set	MIPLIB2003		Cor@l	
	Time	Nodes	Time	Nodes
reliability	450.4	5091	803.6	4110
hybrid	445.6	5051	735.0	3575
ratio reli/hyb	1.01	1.01	1.09	1.15

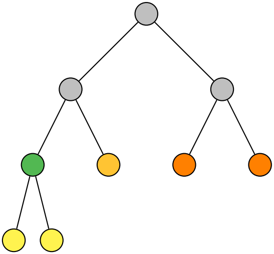
Result: No difference / slight improvement for general MIPs

test set	Cor@l-BP		Infeasible	
	Time	Nodes	Time	Nodes
reliability	672.4	2145	290.7	5612
hybrid	577.2	1681	166.0	1998
ratio reli/hyb	1.16	1.28	1.75	2.81

Result: Medium / large improvement for special MIPs

Outline

Node Selection



Task

- ▷ improve primal bound
- ▷ keep comp. effort small
- ▷ improve global dual bound

Techniques

- ▷ basic rules:
  - depth first search (DFS):
    $\longrightarrow$ early feasible solutions
  - best bound search (BBS):
    $\longrightarrow$ improve dual bound
  - best estimate search (BES):
    $\longrightarrow$ improve primal bound
- ▷ combinations:
  - BBS or BES with plunging
  - hybrid BES/BBS
  - interleaved BES/BBS

Remarks on Node Selection Rules

Computational Effort

Distinction of open leaf nodes:

- child node $\longrightarrow$ subproblem processing is less expensive
- sibling node $\longrightarrow$ subproblem processing is less expensive
- other open leaf node

Results

- best estimate faster than best bound
- interleaved BES/BBS faster than hybrid
- speed-up by plunging (interleaved DFS)

Outline

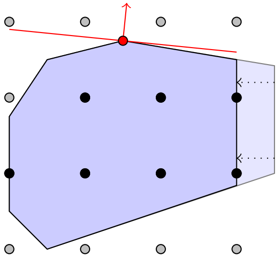
Presolving

It's a dirty job but somebody has to do it!

Presolving

Task

- ▷ reduce size of model by removing irrelevant information
- ▷ strengthen LP relaxation by exploiting integrality information
- ▷ make the LP relaxation numerically more stable
- ▷ extract useful information



Primal Reductions:
- ▷ based on feasibility reasoning
- ▷ no feasible solution is cut off

Dual Reductions:
- ▷ consider objective function
- ▷ at least one optimal solution remains

Trivial stuff

- remove empty rows, columns
  - e.g., $\mathbf{0}^T x \leq b_i,\ b_i < 0 \Rightarrow$ infeasible
- tighten fractional bounds of integer variables
- substitute fixed variables
- boundshifting of general integers
  - e.g., Replace $x_i \in \{N, N+1\},\ N \in \mathbb{Z}$ by $\tilde{x} \in \{0,1\},\ \tilde{x} = x - N$
- replace singleton rows
  - e.g., $a_{ij}x_j \leq b_i,\ a_{ij} < 0 \Rightarrow x_j \geq \frac{b_i}{a_{ij}} \Rightarrow$ new lower bound on x_j
- normalize constraints
  - e.g., if all coefficients are integral, divide by greatest common divisor
- upgrade constraints
- ...

Linear presolving

Minimal and maximal activities

Let a linear constraint $a^T x \leq b$, and bounds $\ell \leq x \leq u$ be given.

$$\alpha_{\min} \quad := \quad \begin{array}{cc} \min & a^T x \\ s.t. & \ell \leq x \leq u \end{array} \quad = \sum_{j, a_j > 0} a_j \ell_j + \sum_{j, a_j < 0} a_j u_j .$$

the minimal activity of the linear constraint.

Linear presolving

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Let a linear constraint $a^T x \leq b$, and bounds $\ell \leq x \leq u$ be given.

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First observation

- $\alpha_{\min} > b \Rightarrow$ problem infeasible
- $\alpha_{\max} \leq b \Rightarrow$ constraint redundant

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Bound strengthening

Let $a_k > 0$. For all feasible solutions x, it holds that:

$$a^T x - a_k x_k + a_k x_k \leq b$$

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$$\begin{aligned} a^T x - a_k x_k + a_k x_k \leq b \;\Leftrightarrow\; x_k &\leq \frac{b - (a^T x - a_k x_k)}{a_k} \\ \Rightarrow\; x_k &\leq \frac{b - \alpha_{\min} + a_k \ell_k}{a_k}. \end{aligned}$$

Linear presolving

Minimal and maximal activities

Let a linear constraint $a^T x \leq b$, and bounds $\ell \leq x \leq u$ be given.

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the maximal activity of the linear constraint.

Coefficient tightening

Let $a_k > 0$, $x_k \in \{0, 1\}$, $\alpha_{\max} - a_k < b$. Then

$$a_k x_k + \sum_{j \neq k} a_j x_j \leq b$$

can be reformulated as

$$(\alpha_{\max} - b) x_k + \sum_{j \neq k} a_j x_j \leq \alpha_{\max} - a_k.$$

Linear presolving

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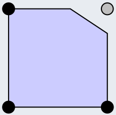
the maximal activity of the linear constraint.

Coefficient tightening

Consider $7x_1 + 8x_2 \leq 13$. $\alpha_{\max} = 15$



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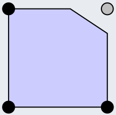
Consider $7x_1 + 8x_2 \leq 13$. $\alpha_{\max} = 15$

$(15 - 13)x_1 + 8x_2 \leq (15 - 7)$



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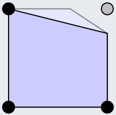
Consider $7x_1 + 8x_2 \leq 13$. $\alpha_{\max} = 15$

$2x_1 + 8x_2 \leq 8 \quad \alpha_{\max} = 10$



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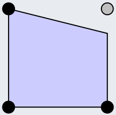
Consider $7x_1 + 8x_2 \leq 13$. $\alpha_{\max} = 15$

$$2x_1 + 8x_2 \leq 8 \quad \alpha_{\max} = 10$$

$$2x_1 + (10-8)x_2 \leq (10-8)$$



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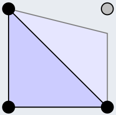
Consider $7x_1 + 8x_2 \leq 13$. $\alpha_{\max} = 15$

$2x_1 + 8x_2 \leq 8 \quad \alpha_{\max} = 10$

$2x_1 + 2x_2 \leq 2$



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Minimal and maximal activities

Let a linear constraint $a^T x \leq b$, and bounds $\ell \leq x \leq u$ be given.

$$\alpha_{\max} \;:=\; \begin{array}{ll} \max & a^T x \\ s.t. & \ell \leq x \leq u \end{array} \;=\; \sum_{j, a_j > 0} a_j u_j + \sum_{j, a_j < 0} a_j \ell_j .$$

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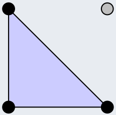
Consider $7x_1 + 8x_2 \leq 13.$ $\alpha_{\max} = 15$

$$2x_1 + 8x_2 \leq 8 \quad \alpha_{\max} = 10$$

$$2x_1 + 2x_2 \leq 2$$

$$x_1 + x_2 \leq 1$$



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$2x_1 + 8x_2 \leq 8 \quad \alpha_{\max} = 10$

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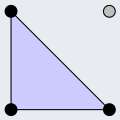
$x_1 + \ x_2 \leq 1$

$\text{setpack}(x_1, x_2)$



And so on...

- probing: tentatively fix binary variables and propagate
- dominance test: pairwise comparison of rows/columns
- aggregation of equations with only two variables
  $a_k x_k + a_j x_j = b \Rightarrow x_k = \frac{b}{ai} - \frac{a_j}{ak} x_j$
- dual fixing: If $a_{ik} \geq 0$ for all i and $c_k \geq 0$, then x_k can be fixed to its lower bound
- dual aggregation: If $c_k \geq 0$ and there is exactly one i for which $a_{ik} < 0$, we can aggregate $x_k = \frac{b_i}{a_j} - \frac{1}{a_k} \sum a_j x_j$.
- dual bound reduction: Strengthen bounds of variables to the tightest value for which all its constraints are redundant
- clique detection (e.g., for knapsack constraints)
- variable lifting
- ...

Outline

The Heart of the CIP Concept

Mixed Integer Program

- Linear objective function
- Linear constraints
- Real and integer variables

Constraint Program

- Arbitrary objective function
- Arbitrary constraints
- Arbitrary (discrete) variables

Constraint Integer Program

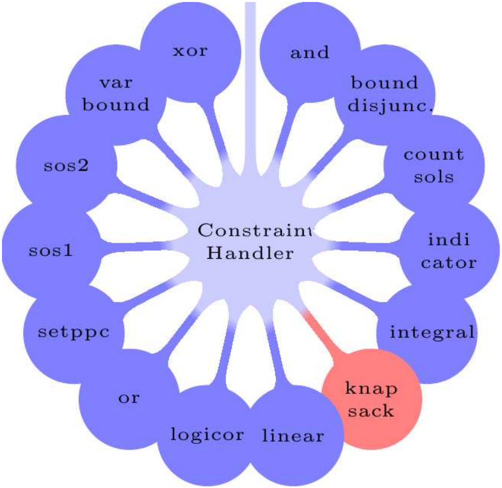
- Linear objective function
- Arbitrary constraints
- Real and integer variables
- After fixing all integer variables: CIP becomes an LP

Remark:

- Arbitrary objective or variables modeled by constraints

$\longrightarrow$ For each type of constraint, one constraint handler is responsible.

Default Plugins



Special Linear Constraints

$$\begin{aligned}
\min \quad & x_1 + x_2 + x_3 + x_4 + x_5 + y_1 + y_2 \\
\text{s.t.} \quad & y_1 \le 3 + 4\,x_1 \\
& y_2 \le 4 + 5\,x_2 \\
& 3\,x_1 + 2\,x_2 + 4\,x_3 \le 8 \\
& x_2 + x_3 + x_4 = 1 \\
& 3\,x_1 + 4.5\,x_2 + 3.2\,x_5 + 0.8\,y_1 + 1.2\,y_2 \ge 4 \\
& x_1,\, x_2,\, x_3,\, x_4 \in \{0,1\} \\
& x_5 \in \mathbb{Z}_+ \\
& y_1,\, y_2 \in \mathbb{R}_+
\end{aligned}$$

```
presolved problem has 7 variables (4 bin, 1 int, 0 impl, 2 cont) and 5
      2 constraints of type <varbound>
      1 constraints of type <knapsack>
      1 constraints of type <setppc>
      1 constraints of type <linear>
```

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       1 constraints of type <linear>
```

Knapsack Constraints

Feasible region of 0-1 knapsack problem:

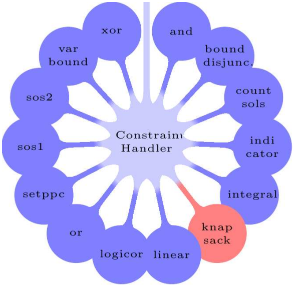
$$\{\ x \in \{0,1\}^{|N|}\ :\ \sum_{j \in N} a_j x_j \leq b\ \}$$

- weights of the variables: $a_j \in \mathbb{Z}_+$ for all $j \in N$
- capacity of the knapsack: $b \in \mathbb{Z}_+$

Ingredients of a Constraint Handler

Callback Methods

- ▷ **Fundamental:**
  - CONSLOCK
  - CONSCHECK
  - CONSENFOLP, CONSENFOPS
- ▷ **Additional:**
  - CONSINIT..., CONSEXIT...
  - CONSSEPALP, CONSSEPASOL
  - CONSPROP, CONSRESPROP, CONSPRESOL
  - CONSACTIVE, CONSDEACTIVE, CONSENABLE, CONSDISABLE, CONSTRANS, CONSDELETE, CONSFREE
  - CONSPRINT, CONSCOPY, CONSPARSE



Further Ingredients

- ▷ Private data
- ▷ Interface methods
- ▷ Properties/Parameters

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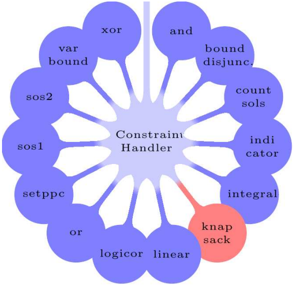

Further Ingredients

- Private data
- Interface methods
- Properties/Parameters

Private Data

Constraint data: information needed to define single constraint

```
struct SCIP_ConsData
{
  SCIP_VAR**      vars;      // variables in knapsack
  int             nvars;     // number of variables
  SCIP_Longint*   weights;   // weights of variables
  SCIP_Longint    capacity;  // capacity of knapsack
};
```

Constraint handler data: information belonging to constraint handler itself

```
struct SCIP_ConshdlrData
{
  int   maxrounds;    // max nr. of sepa rounds per node
  int   maxsepacuts;  // max nr. of cuts per sepa round
};
```

Interface Methods

Creating a single knapsack constraint.

```
SCIP_RETCODE SCIPcreateConsKnapsack(
  SCIP*          scip,       // SCIP data structure
  SCIP_CONS**    cons,       // pointer to hold created cons
  const char*    name,       // name of constraint
  SCIP_VAR**     vars,       // array with variables
  int            nvars,      // number of variables in knapsack
  SCIP_Longint*  weights,    // array with weights
  SCIP_Longint   capacity,   // capacity of knapsack
  SCIP_Bool      separate,   // should constraint be separated?
  ...
)
{
  SCIP_CONSDATA* consdata;
  SCIP_CALL( consdataCreate(scip, &consdata, nvars, vars,
                            weights, capacity) );
  SCIP_CALL( SCIPcreateCons(scip, cons, name, conshdlr,
                            consdata, separate, ...) );
  ...
}
```

Interface Methods

Including the knapsack constraint handler.

```
SCIP_RETCODE SCIPincludeConshdlrKnapsack(
  SCIP*          scip         // SCIP data structure
)
{
  SCIP_CONSHDLRDATA* conshdlrdata;

  SCIP_CALL( conshdlrdataCreate(scip, &conshdlrdata) );

  SCIP_CALL( SCIPincludeConshdlr(scip, CONSHDLR_CHECKPRIORITY,
             consCheckKnapsack, ..., conshdlrdata) );
  ...
}
```

Ingredients of a Constraint Handler

Callback Methods

- ▷ Fundamental:
  - CONSLOCK
  - CONSCHECK
  - CONSENFOLP, CONSENFOPS
- ▷ Additional:
  - CONSINIT..., CONSEXIT...
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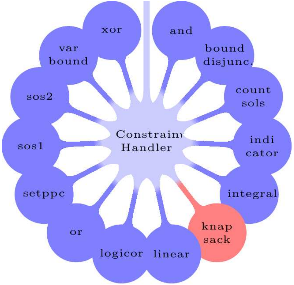


Further Ingredients

- ▷ Private data
- ▷ Interface methods
- ▷ Properties/Parameters

CONSLOCK

- Provides dual information for single constraints (useful for presolving, primal heuristics, ...)
- For each variable of a constraint, returns whether ...
  - increasing its value, and/or
  - decreasing its value

  may lead to a violation of the constraint

$$3\,x_1 - 5\,x_2 + 2\,x_3 \qquad \leq 7$$

increasing: x_1 and x_3

decreasing: x_2

CONSCHECK

Most important callback ...

- usually called by primal heuristics
- checks given solution for feasibility wrt all constraints of its type
- possible result values
  - SCIP_FEASIBLE
  - SCIP_INFEASIBLE

Given solution:

$$(x_1,\ x_2,\ x_3) = (1,\ 0,\ 1)$$

Knapsack constraints:

$$3x_1 + 6x_2 + 4x_3 \leq 8$$

$$2x_1 + \qquad + 2x_3 \leq 3$$

Result: SCIP_INFEASIBLE

CONSENFOLP and CONSENFOPS

CONSENFOLP: checks LP solution for feasibility
CONSENFOPS: checks Pseudo solution for feasibility

LP solution

- solution of LP relaxation

$$\begin{array}{ll} \min & x_1 - x_2 + x_3 \\ \text{s.t.} & 3\,x_1 + 8\,x_2 + 4\,x_3 \le 4 \\ & x_1,\, x_2,\, x_3 \in \{0,1\} \end{array}$$

LP solution: $(0, \frac{1}{2}, 0)$

Pseudo Solution

- solution of LP relaxation with only bound constraints
- used if LP solving disabled, or
- numerical difficulties occurred

$$\begin{array}{ll} \min & x_1 - x_2 + x_3 \\ \text{s.t.} & x_1,\, x_2,\, x_3 \in \{0,1\} \end{array}$$

Pseudo Solution: $(0, 1, 0)$

CONSENFOLP and CONSENFOPS

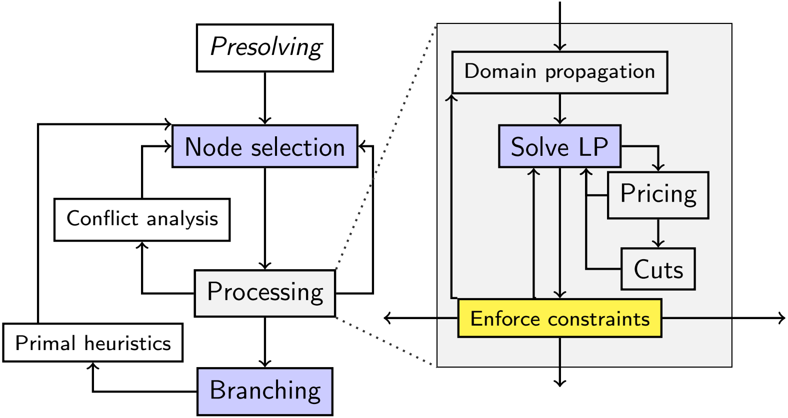
LP solution may violate a constraint not contained in the relaxation.

> Enforcement callbacks are necessary for a correct implementation!

In addition, they can resolve an infeasibility by ...

- reducing a variable's domain,
- separating a cutting plane (may use integrality),
- adding a (local) constraint,
- creating a branching,
- concluding that the subproblem is infeasible and can be cut off, or
- just saying "solution infeasible".

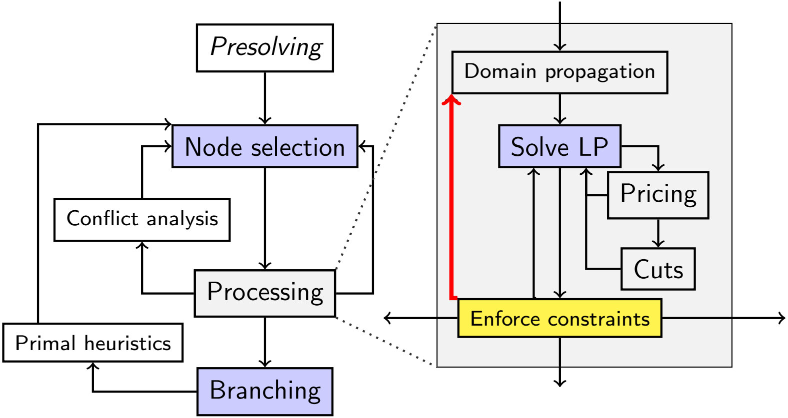
CONSENFOLP and CONSENFOPS



Enforcement result of constraint handler:

- reduced domain
- added constraint
- added cut
- branched
- cutoff
- infeasible
- feasible

CONSENFOLP and CONSENFOPS



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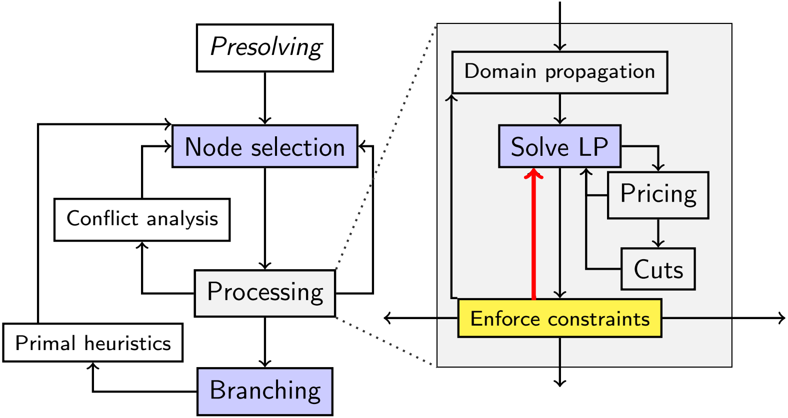

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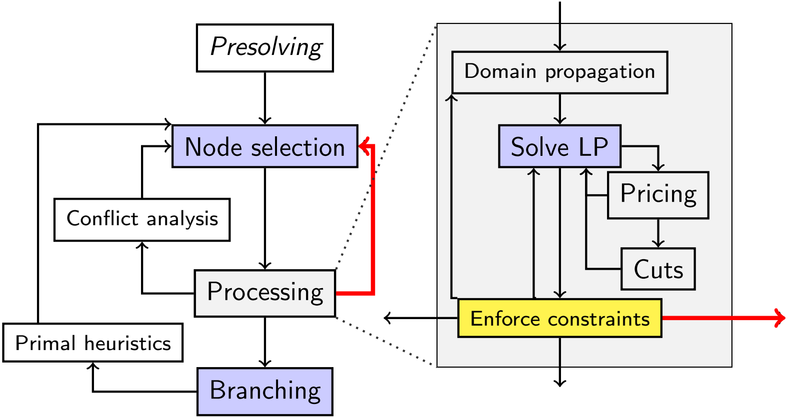

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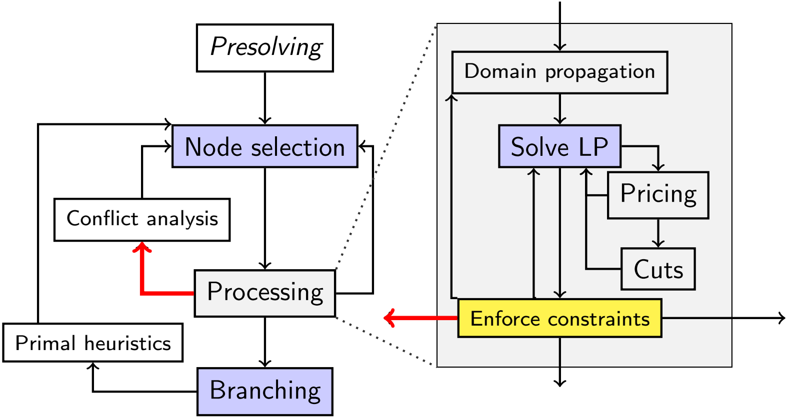

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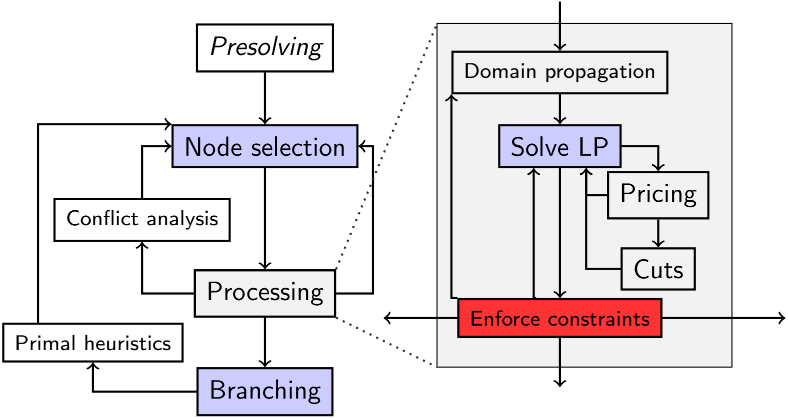

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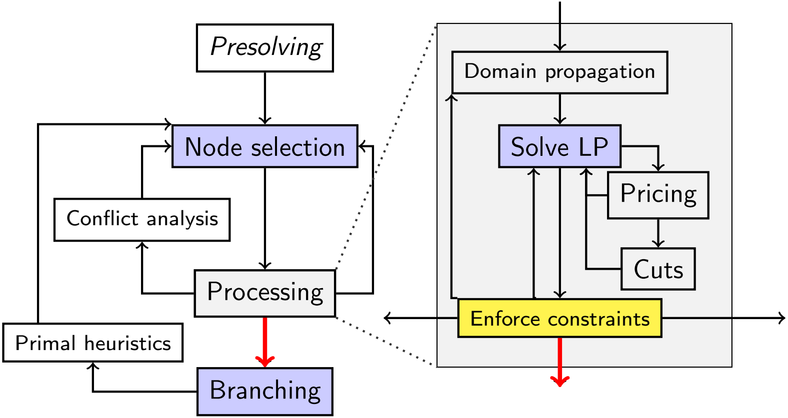

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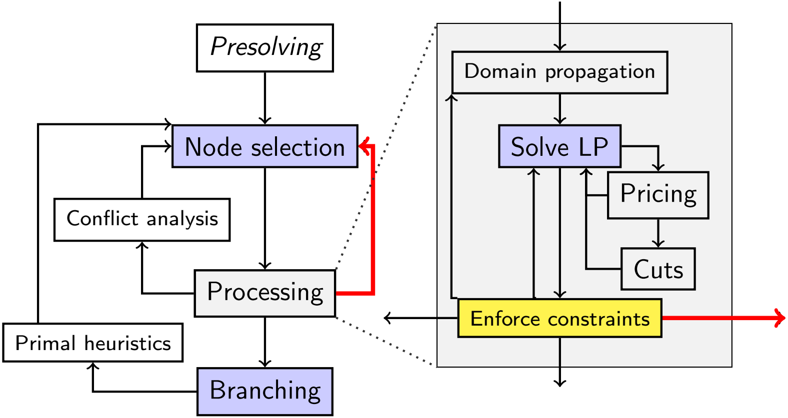
CONSENFOLP and CONSENFOPS



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Ingredients of a Constraint Handler

Callback Methods

- ▷ Fundamental:
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  - CONSENFOLP, CONSENFOPS
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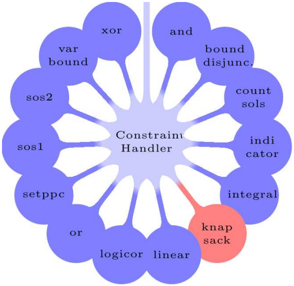
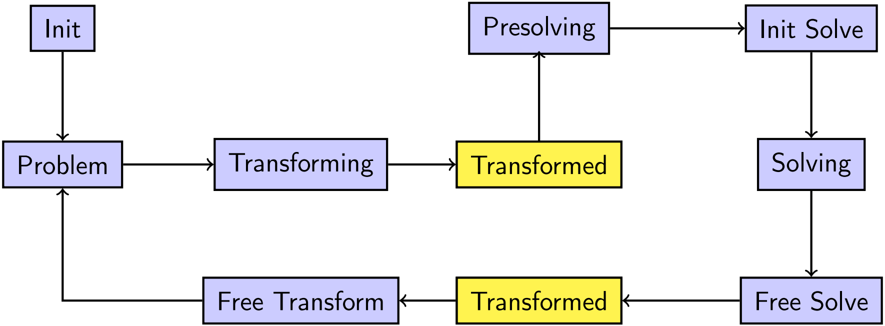


Further Ingredients

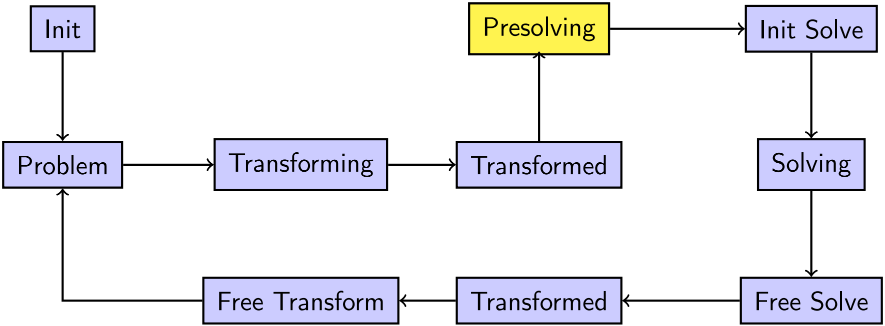
- ▷ Private data
- ▷ Interface methods
- ▷ Properties/Parameters



CONSINIT and CONSEXIT:

- called after problem was transformed /
  before transformed problem is freed
- initialize and free statistics in `SCIP_ConshdlrData`

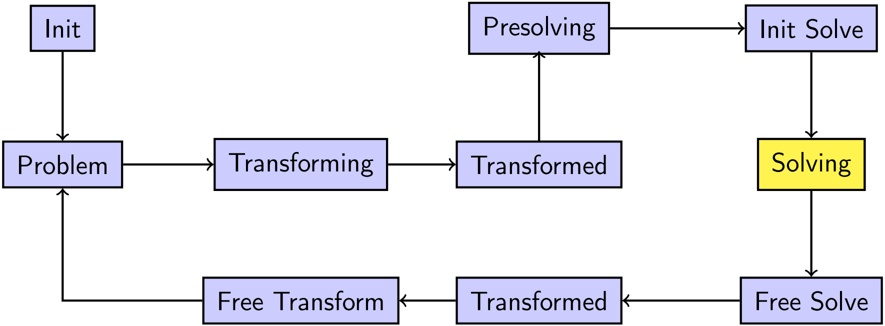
CONSINIT..., CONSEXIT...



CONSINITPRE and CONSEXITPRE:

- called before presolving starts / after presolving is finished
- initialize and free presolving data

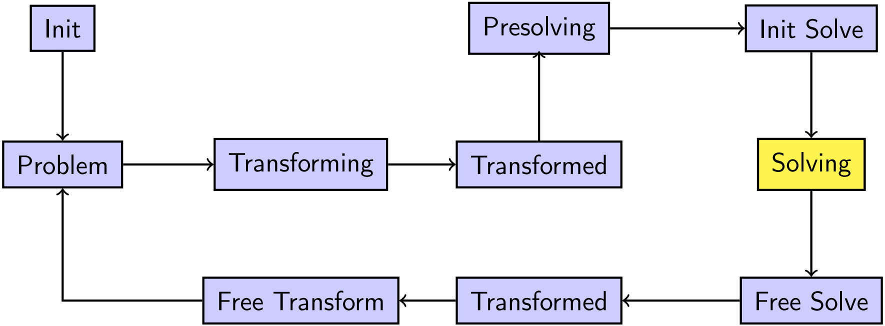
CONSINIT..., CONSEXIT...



CONSINITSOL and CONSEXITSOL:

- called before branch-and-bound process starts /
  before branch-and-bound process is freed
- initialize and release branch-and-bound specific data

CONSINIT..., CONSEXIT...



CONSINITLP:

- called before first LP relaxation is solved
- add linear relaxation of all "initial" constraints to the LP relaxation

Feasible region of 0-1 knapsack problem:

$$\{\ x \in \{0,1\}^{|N|}\ :\ \sum_{j \in N} a_j x_j \le b\ \}$$

Minimal Cover: $C \subseteq N$

- ▷ $\sum_{j \in C} a_j > b$
- ▷ $\sum_{j \in C \setminus \{i\}} a_j \le b \quad \forall\, i \in C$

Minimal Cover Inequality

$$\sum_{j \in C} x_j \le |C| - 1$$

$5x_1 + 6x_2 + 2x_3 + 2x_4 \le 8$

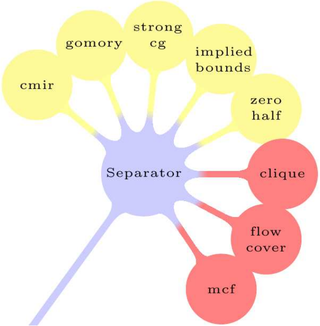
Minimal cover: $C = \{2,3,4\}$

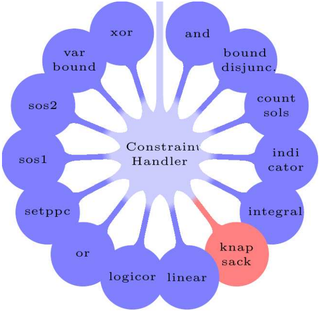
Minimal cover inequality:

$x_2 + x_3 + x_4 \le 2$

separated in
knapsack constraint handler

CONSSEPALP, CONSSEPASOL





Separation is implemented in separators and constraint handlers.

1. Separators with SEPA_PRIORITY ≥ 0 (decreasing order)
2. Constraint handlers (decreasing order of CONSHDLR_SEPAPRIORITY)
3. Separators with SEPA_PRIORITY < 0 (decreasing order)

Ingredients of a Constraint Handler

Callback Methods

- ▷ Fundamental:
  - CONSLOCK
  - CONSCHECK
  - CONSENFOLP, CONSENFOPS
- ▷ Additional:
  - CONSINIT..., CONSEXIT...
  - CONSSEPALP, CONSSEPASOL
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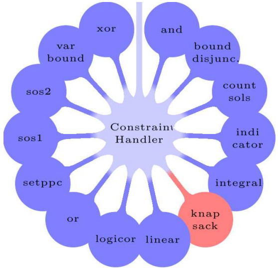

- ▷ Domain propagation (during subproblem processing)
- ▷ Reason for domain reductions (for conflict analysis)
- ▷ Presolving (before processing root node)

Ingredients of a Constraint Handler

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- **Fundamental:**
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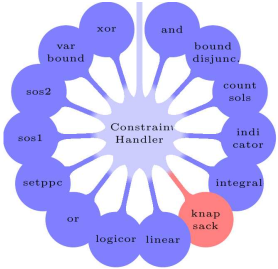

Called whenever ...

- SCIP enters/leaves subtree where local constraint exists
- constraint is enabled/disabled (no propagation, no separation)

Ingredients of a Constraint Handler

Callback Methods

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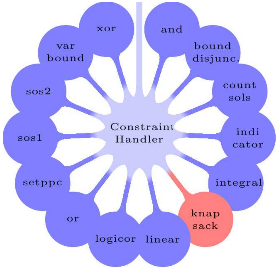

- ▷ Displaying, parsing problems
- ▷ Copying problems between different SCIP environments

Ingredients of a Constraint Handler

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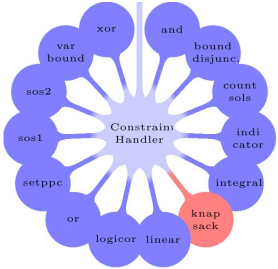


Further Ingredients

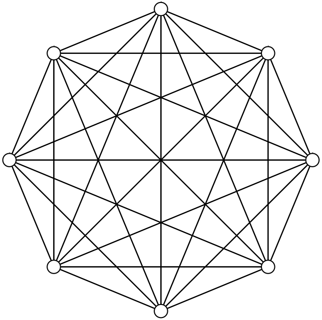
- ▷ Private data
- ▷ Interface methods
- ▷ Properties/Parameters

Exercise: Traveling Salesman Problem (TSP)

Definition

Given a complete graph $G = (V, E)$ with edge lengths c_e.

Find a Hamiltonian cycle (cycle containing all nodes, tour) of minimum length.



K_8

Exercise: Traveling Salesman Problem (TSP)

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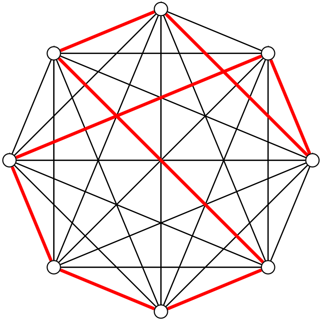


K_8

Exercise: TSP

MIP Formulation

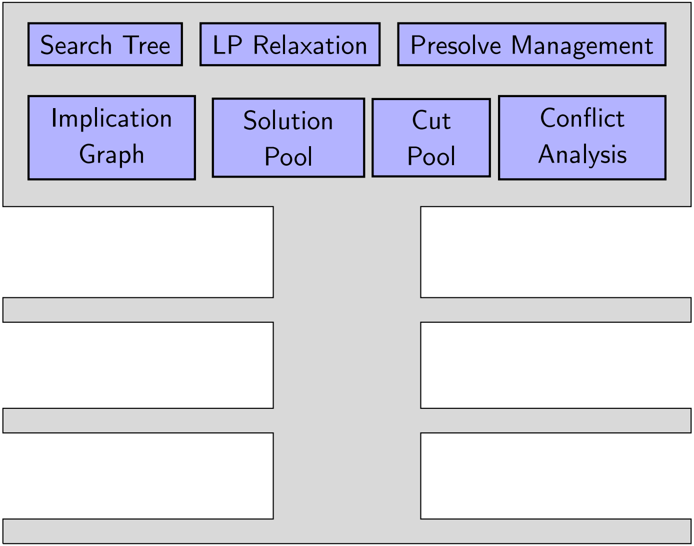
$$
\begin{aligned}
\min \quad & \sum_{e \in E} c_e \, x_e \\
s.t. \quad & \sum_{e \in \delta(v)} x_e = 2 \quad \forall\, v \in V \\
& \sum_{e \in \delta(S)} x_e \geq 2 \quad \forall\, S \subset V, S \neq \varnothing \\
& x_e \in \{0,1\} \quad \forall\, e \in E.
\end{aligned}
$$

CIP Formulation

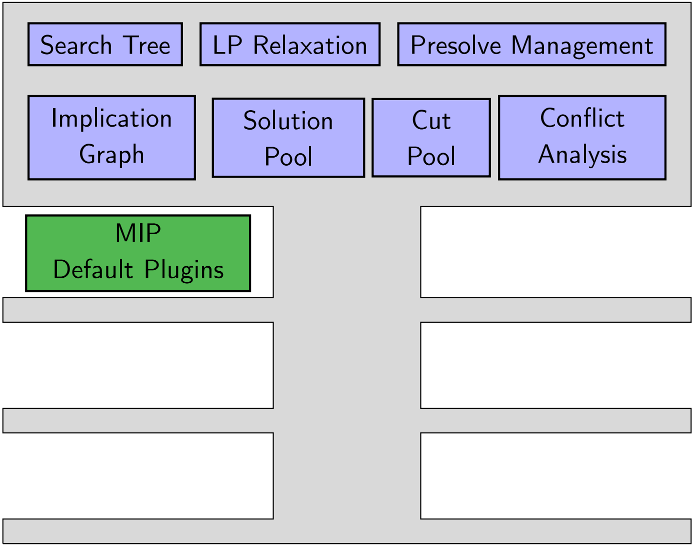
$$
\begin{aligned}
\min \quad & \sum_{e \in E} c_e \, x_e \\
s.t. \quad & \sum_{e \in \delta(v)} x_e = 2 \qquad \forall\, v \in V \\
& \mathrm{nosubtour}(G, x) \\
& x_e \in \{0,1\} \qquad \forall\, e \in E.
\end{aligned}
$$

$\mathrm{nosubtour}(G, x) \Leftrightarrow \nexists\, C \subseteq \{e \in E \mid x_e = 1\} : C$ is a cycle of length $|C| < |V|$

Exercise: TSP



Exercise: TSP



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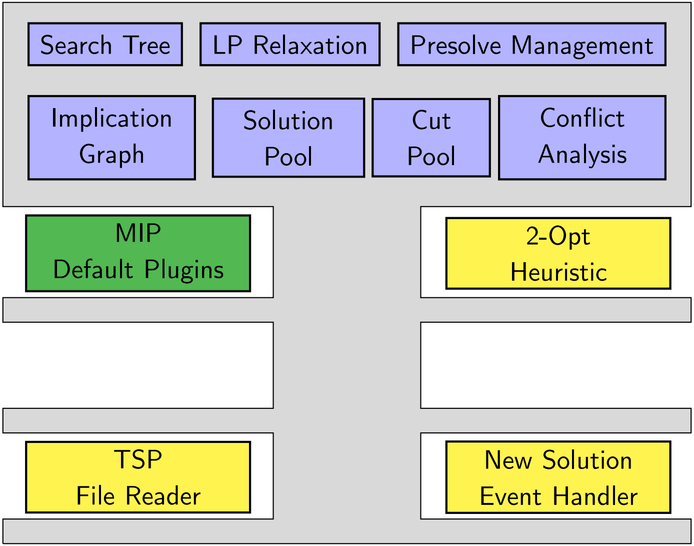

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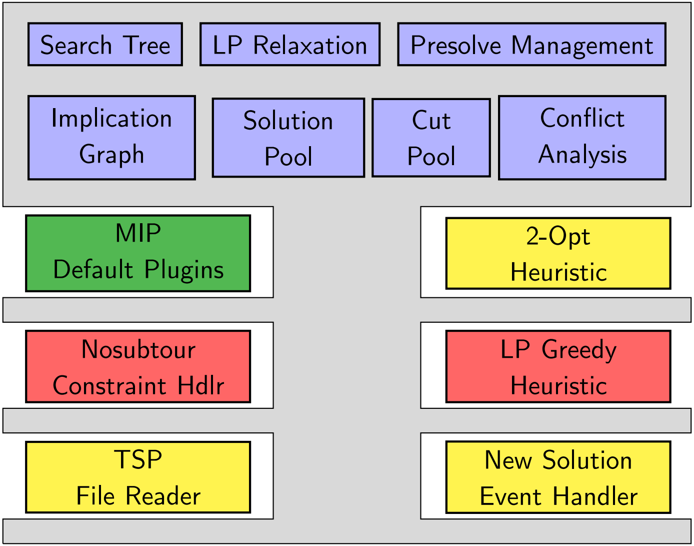

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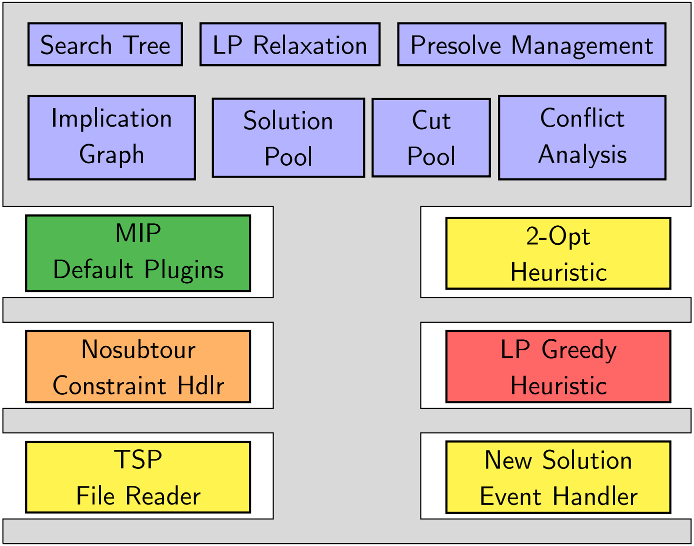

Exercise: TSP

Callback Methods

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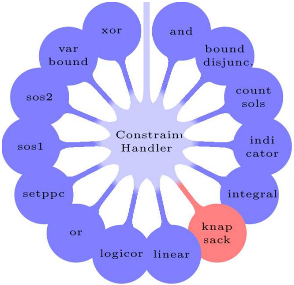

Further Ingredients

- Private data
- Interface methods
- Properties

Exercise: TSP

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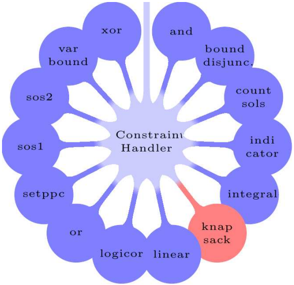

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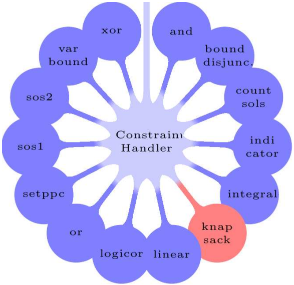

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CONSSEPALP, CONSSEPASOL

Separation Problem

Given complete graph $G = (V, E)$ and $x^* \in [0, 1]^{|E|}$.

- Decide whether x^* satisfies all subtour elimination constraints.
- If not, find violated subtour elimination constraint.

Separation Problem

Given complete graph $G = (V, E)$ and $x^* \in [0, 1]^{|E|}$.

- ▷ Decide whether x^* satisfies all subtour elimination constraints.
- ▷ If not, find violated subtour elimination constraint.

Trivial observation:

- ▷ Consider $G = (V, E)$ with edge capacities x^*_e.
- ▷ x^* violates at least one subtour elimination constraint
  $\Leftrightarrow \exists$ cut $\delta(S)$ with capacity $x^*(\delta(S)) < 2$.

Separation Problem

Given complete graph $G = (V, E)$ and $x^* \in [0,1]^{|E|}$.

- Decide whether x^* satisfies all subtour elimination constraints.
- If not, find violated subtour elimination constraint.

Idea of separation algorithm:

- $\forall\, s, t \in V$: Find (s,t)-cut $\delta(S)$ of minimum capacity
- If all cut capacities ≥ 2, all subtour elimination constraints satisfied

$\longrightarrow$ $\binom{|V|}{2}$ times MaxFlow-MinCut algo

Separation Problem

Given complete graph $G = (V, E)$ and $x^* \in [0,1]^{|E|}$.

- Decide whether x^* satisfies all subtour elimination constraints.
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Idea of separation algorithm:

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- If all cut capacities ≥ 2, all subtour elimination constraints satisfied

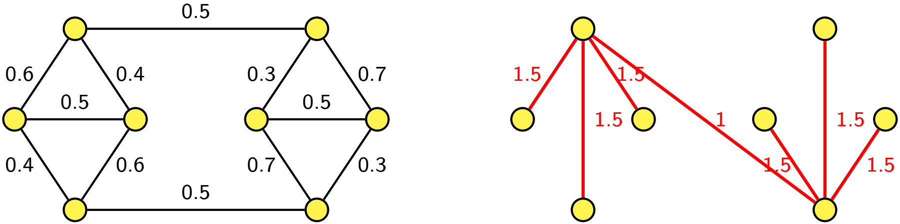
$\longrightarrow$ $|V| - 1$ times MaxFlow-MinCut algo within Gomory-Hu-Algorithm

Result of Gomory-Hu-Algorithm

Gomory-Hu-Tree T

For all $s, t \in V$:

- capacity of minimum (s, t)-cut in G:
  minimum label f_e of all edges e in unique (s, t)-path in T
- minimum (s, t)-cut in G:
  bipartition of V obtained be deleting this edge from T



Result of Gomory-Hu-Algorithm

Gomory-Hu-Tree T

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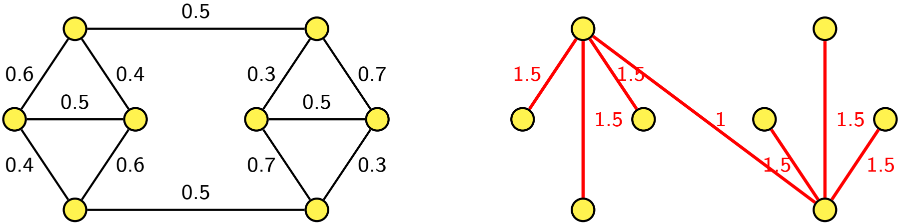

`ghc_tree()` returns all minimum (s, t)-cuts with capacity $\leq 2 - \texttt{minviol}$

Documentation

- Constraint Integer Programming
  Tobias Achterberg, Ph.D. dissertation, TU Berlin, 2007
- Primal Heuristics for Mixed Integer Programs
  Timo Berthold, Diploma thesis, TU Berlin, 2006
- Implementation of Cutting Plane Separators for Mixed Integer Programs
  Kati Wolter, Diploma thesis, TU Berlin, 2006

- http://scip.zib.de
  Doxygen documentation, HowTos, FAQ
- source code
  `scip.h`, `pub_*.h`, `type_*.h`

Please, . . .

- build groups of 3 people
  - laptop owner
  - C expert
  - IP expert
- raise your hand, if you get stuck.

Branching, Presolving, and Constraint Handlers

CO@Work Berlin

Timo Berthold and Kati Wolter

09/26/2009

Berlin Mathematical School

DFG Research Center MATHEON
Mathematics for key technologies