The MIP Landscape

Example 1: LP still can be HARD

SGM: Schedule Generation Model

157323 rows, 182812 columns, 6348437 nzs

- LP relaxation at root node:
  - 18 hours
- Branch-and-bound
  - 1710 nodes, first feasible
  - 3.7% gap
  - Time: 92 days!!
- MIP does not appear to be difficult: *LP is a roadblock*

Example 2: MIP really is HARD

A customer model: 44 cons, 51 vars, 167 nzs, maximization
51 general integer variables (*and no bounds*)

Branch-and-bound: Initial integer solution -2186.0
Initial upper bound -1379.4
…after 1.4 days, 32,000,000 B&B nodes, 5.5 Gig tree
Integer solution and bound: UNCHANGED

What's wrong? Bad modeling. Free GIs chase each other off to infinity.

Example 2 (cont.): Here’s what’s wrong

```
Maximize
  x + y + z
Subject To
  2 x + 2 y ≤ 1
  z = 0
  x free y free
  x,y integer
```

Note: This problem can be solved in several ways

- Removing z=0, objective is integral [*Presolve*]
- Euclidean reduction on the constraint [*Presolve*]

However: Branch-and-bound cannot solve!

Example 3: A typical situation today – Supply-chain scheduling

- Model description:
  - Weekly model, daily buckets: Objective to minimize end-of-day inventory.
  - Production (single facility), inventory, shipping (trucks), wholesalers (demand known)
- Initial modeling phase
  - Simplified prototype + complicating constraints (production run grouping req't, min truck constraints)
  - RESULT: Couldn't get good feasible solutions.
- Decomposition approach
  - Talk to current scheduling team: They first decide on "producibles" schedule. Simulate using heuristics.
  - Fixed model: Fix variables and run MIP

Supply-chain scheduling (continued):
Solving the fixed model

CPLEX 5.0:

```
Integer optimal solution (0.0001/0):  Objective =    1.5091900536e+05
Current MIP best bound =    1.5090391809e+05 (gap = 15.0873)
Solution time = 3465.73 sec.  Iterations = 7885711  Nodes = 489870 (2268)
```

Gurobi 2.0:

```
Cutting planes:
  Gomory: 16
  Implied bound: 33
  MIR: 1
  Flow cover: 70

Explored 9 nodes (4517 simplex iterations) in 0.49 seconds

Optimal solution found (tolerance 1.00e-04)
Best objective 1.5091900536e+05, best bound 1.5090900608e+05, gap 0.0066%
```

Original model: Now solves to optimality in 1 hour (20% improvement in solution quality)

Example 4: Unit-Commitment Model

Electrical Power Industry, ERPI GS-6401, June 1989:
Mixed-integer programming (MIP) is a powerful modeling tool, “They are, however, theoretically complicated and computationally cumbersome”

In Other Words: MIP is an interesting “toy”, but it just isn’t going to work it practice.

California 7-Day Model

UNITCAL_7: 48939 constraints, 25755 variables (2856 binary)

Reported Results 1999 – machine unknown

2 Day model: 8 hours, no progress

7 Day model: 1 hour to solve initial LP

Desktop PC -- ran full 7-day model

CPLEX 6.5 (1999): 22 minutes, optimal

California 7-Day Model

```
Gurobi Optimizer version 2.0.0

Read MPS format model from file unitcal_7.mps.bz2
Optimize a model with 48939 Rows, 25755 Columns and 127595 NonZeros
Presolved: 38804 Rows, 19960 Columns, 105627 Nonzeros

Root relaxation: objective 1.945018e+07, 18340 iterations, 0.60 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
     0     0 1.9450e+07    0  772          - 1.9450e+07     -      -    2s
     0     0 1.9582e+07    0  673          - 1.9582e+07     -      -    8s
     0     0 1.9590e+07    0  543          - 1.9590e+07     -      -   13s
     0     0 1.9592e+07    0  530          - 1.9592e+07     -      -   15s
     0     0 1.9592e+07    0  521          - 1.9592e+07     -      -   16s
     0     0 1.9592e+07    0  531          - 1.9592e+07     -      -   16s
H    0     0                    2.0669e+07 1.9592e+07  5.21%     -   18s
     0     2 1.9592e+07    0  531 2.0669e+07 1.9592e+07  5.21%     -   19s
     3     3 1.9595e+07    2  581 2.0669e+07 1.9593e+07  5.20%   567   20s
    32    35 1.9665e+07    8  394 2.0669e+07 1.9605e+07  5.15%  1043   30s
H   94    92                    1.9849e+07 1.9605e+07  1.23%   565   35s
H  122    51                    1.9646e+07 1.9605e+07  0.21%   477   39s
   385    56 1.9638e+07   31   33 1.9646e+07 1.9605e+07  0.21%   200   42s
H  747   167                    1.9636e+07 1.9608e+07  0.14%   146   48s
H  818   199                    1.9636e+07 1.9610e+07  0.13%   142   52s
   936   248 1.9630e+07   21  113 1.9636e+07 1.9611e+07  0.13%   137   57s
H 1053   304                    1.9636e+07 1.9611e+07  0.12%   130   60s
  1182   379 1.9622e+07   16  531 1.9636e+07 1.9612e+07  0.12%   124   70s
  1193   386 1.9632e+07   26  520 1.9636e+07 1.9612e+07  0.12%   122   90s
  1199   390 1.9623e+07   18  520 1.9636e+07 1.9612e+07  0.12%   122   95s
  1208   398 1.9612e+07   15  640 1.9636e+07 1.9612e+07  0.12%   157  199s
  1209   399 1.9612e+07   15  614 1.9636e+07 1.9612e+07  0.12%   159  205s
  1285   370 1.9635e+07   27  525 1.9636e+07 1.9613e+07  0.11%   169  227s
  2266   300 1.9634e+07   33  329 1.9636e+07 1.9633e+07  0.01%   113  235s

Cutting planes:
  Gomory: 32
  Cover: 22
  Implied bound: 599
  Clique: 60
  Flow cover: 391

Explored 2379 nodes (298817 simplex iterations) in 235.33 seconds

Optimal solution found (tolerance 1.00e-04)
Best objective 1.9635558244e+07, best bound 1.9633702475e+07, gap 0.0095%
```

Computational History: 1950 –1998

- **1954 Dantzig, Fulkerson, S. Johnson:** 42 city TSP
  - Solved to optimality using LP and cutting planes
- **1957 Gomory**
  - Cutting plane algorithms
- **1960 Land, Doig, 1965 Dakin**
  - B&B
- **1971 MPSX/370**
- **1972 UMPIRE**
  - LP-based B&B
  - MIP became commercially viable

- **1972 – 1998** Good B&B remained the state-of-the-art in commercial codes, in spite of ….
  - Edmonds, polyhedral combinatorics
  - 1973 Padberg, cutting planes
  - 1973 Chvátal, revisited Gomory
  - 1974 Balas, disjunctive programming
  - 1983 Crowder, Johnson, Padberg: PIPX, pure 0/1 MIP
  - 1987 Van Roy and Wolsey: MPSARX, mixed 0/1 MIP
  - TSP, Grötschel, Padberg, …

1998 … A New Generation of MIP Codes

- Linear programming
  - Stable, robust dual simplex
- Variable/node selection
  - Influenced by traveling salesman problem
- Primal heuristics
  - 12 different tried at root
  - Retried based upon success
- Node presolve
  - Fast, incremental bound strengthening (very similar to Constraint Programming)
- Presolve – numerous small ideas
  - Probing in constraints:

    $\sum x_j \leq (\sum u_j)\, y,\ \ y = 0/1$

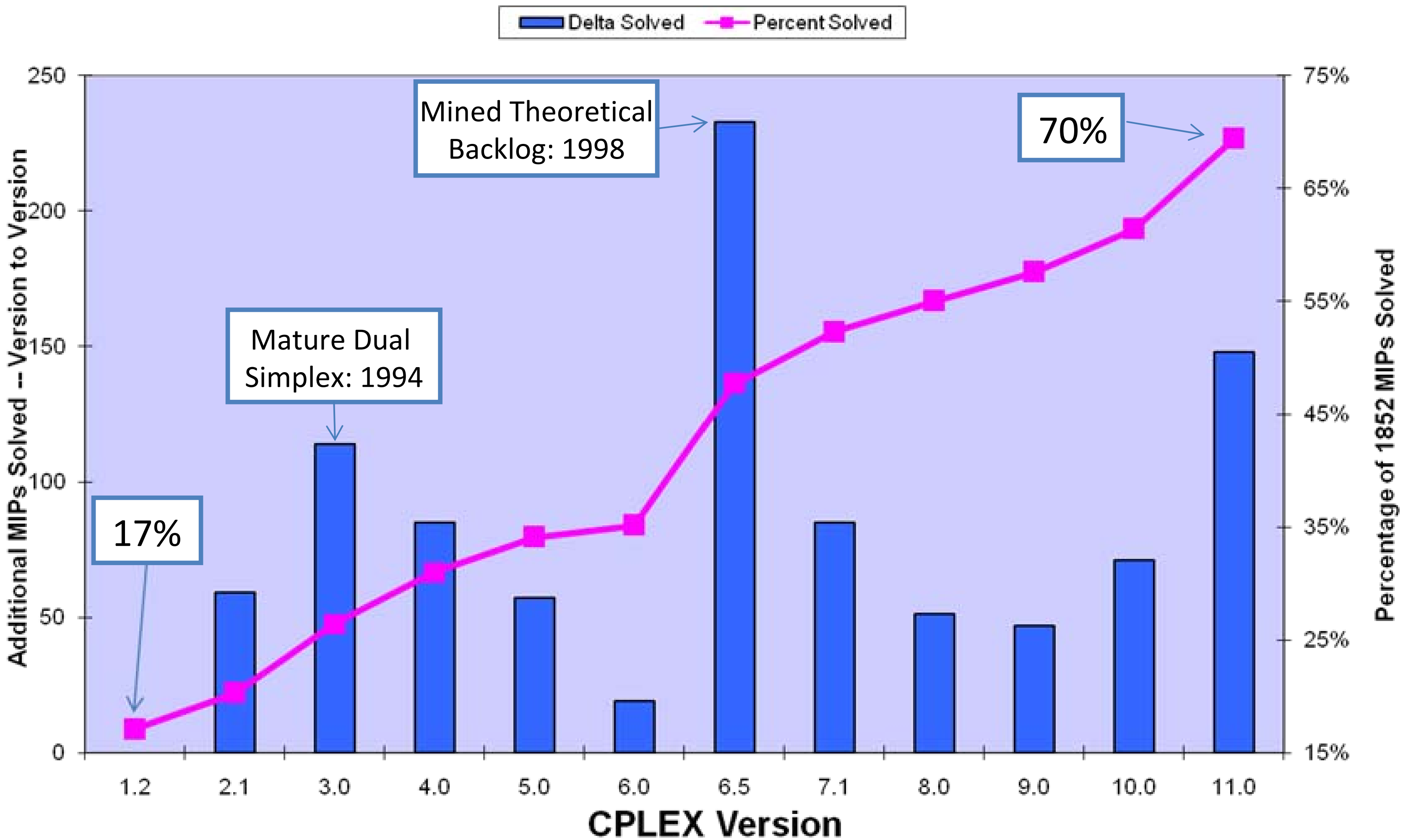
    $\Rightarrow x_j \leq u_j y$ (for all j)
- Cutting planes
  - Gomory, mixed-integer rounding (MIR), knapsack covers, flow covers, cliques, GUB covers, implied bounds, zero-half cuts, path cuts

Some Test Results

- Test set: **1852 real-world MIPs**
  - Full library
    - 2791 MIPs
  - Removed:
    - 559 “Easy” MIPs
    - 348 “Duplicates”
    - 22 “Hard” LPs (0.8%)
- Parameter settings
  - Pure defaults
  - 30000 second time limit
- Versions Run
  - CPLEX 1.2 (1991) -- CPLEX 11.0 (2007)

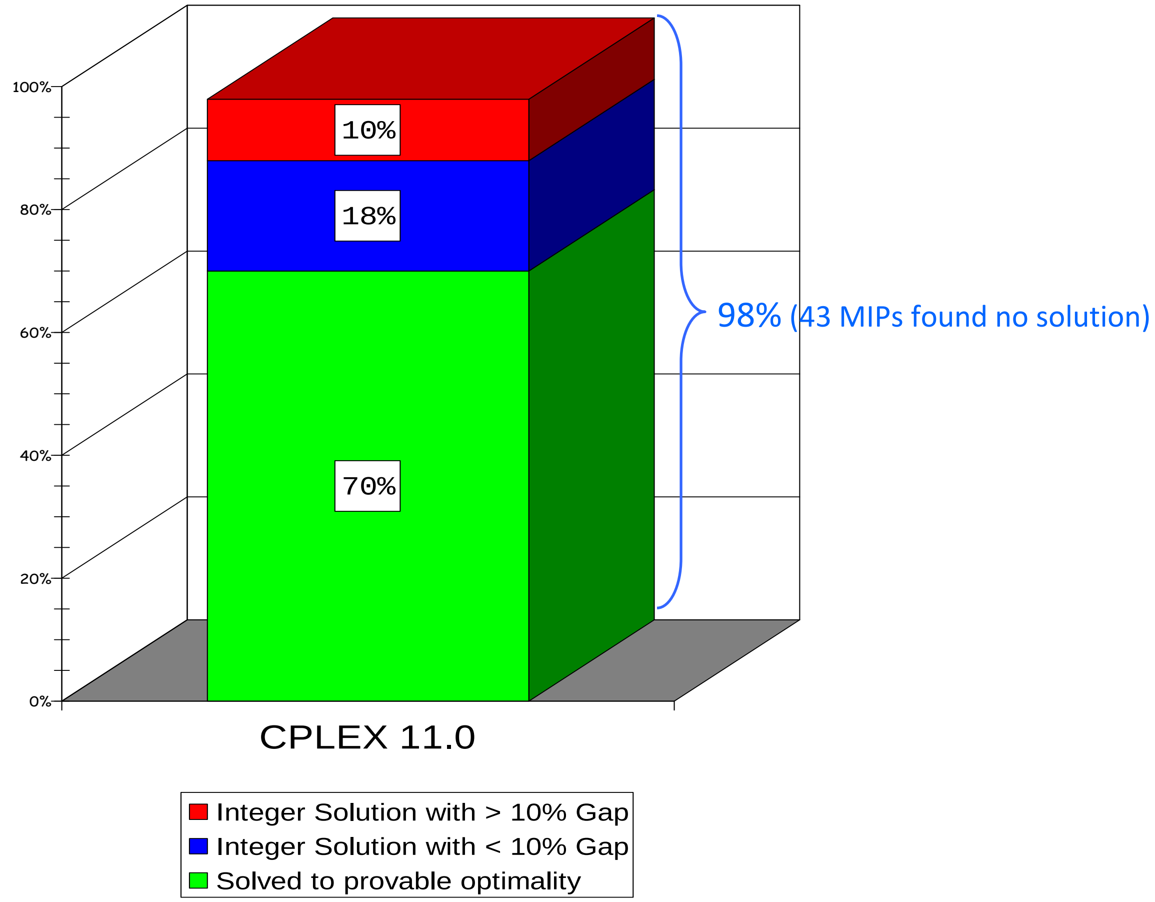
MIP Solvability

Solvability of MIPS 1991-Present



Solvability of MIPs – CPLEX 11.0

1852 MIPs, 30000 second time limit



MIP Speedups

Speedups 1991-Present