

Combinatorial Auctions

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Block Course at ZIB
"Combinatorial Optimization at Work"
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Overview

- Combinatorial Auctions
 - Definition
 - Concepts
 - Examples
- Auction Types
 - Simultaneous Ascending Auction (SAA)
 - Adaptive User Selection Mechanism (AUSM)
 - Vickery-Clarke-Groves Mechanism (VCG)
 - Ascending Proxy Auction (APA)
- Railway Track Auction



Arguments for Auctions

- Auctions can ...
 - resolve user conflicts in such a way that the bidder with the highest willingness to pay receives the commodity (efficient allocation, welfare maximization)
 - maximize the auctioneer's earnings
 - reveal the bidders' willingness to pay
 - reveal bottlenecks and the added value if they are removed
- Economists argue ...
 - that a "working auctioning system" is usually superior to alternative methods such as bargaining, fixed prices, etc.



Auctions

- **Commodities/Bids**
 - Independent commodities (classical auction)/ commodity bundles (combinatorial auction)
 - Combinatorial bids (and/or/xor)
- **Bidders**
 - Cooperation forbidden/ cooperation allowed
- **Payment**
 - First price/second price (Vickrey) auction
- **Information**
 - Private Values/Common Values (winner's curse)
 - Sealed Bid/Open Bid
- **Mechanism**
 - English auction/dutch auction
 - Increment/number of rounds
 - Activity rules/taking bids back
 - Direct bidding/clock/proxy auction



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Combinatorial Auction

- Example

- $M = \{a, b, c, d\}$
- Bids

S	{a, d}	{b}	{c, d}	{a, b}	{b, c}	{a, c}	{a, b, c, d}
Bid	1	3	2	5	4	5	6

- Winning bids = {a, c}, {b}



Combinatorial Auction

▪ Setting

- M objects, N bidders
- Bid $b^j(S)$ by j for $S \subseteq M$
- Winner determination = combinatorial auction problem (CAP)
 - $y(S, j)$ 0/1-variable for giving S to j

$$\begin{aligned}
 \max \quad & \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S, j) \\
 & \sum_{S \ni i} \sum_{j \in N} y(S, j) \leq 1 \quad \forall i \in M \\
 & y(S, j) \in \{0, 1\} \quad \forall S \subseteq M, j \in N
 \end{aligned}$$

- Set Packing Problem



Combinatorial Auction 2

▪ Setting

- M objects, N bidders
- Superadditive bids $b^j(S)$, i.e., $b^j(S+S') \geq b^j(S) + b^j(S')$ for $S \cap S' = \emptyset$
- Winner determination = combinatorial auction problem (CAP2)

$$\begin{aligned} \max \quad & \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y^j(S) \\ & \sum_{S \ni i} y^j(S) \leq 1 \quad \forall i \in M \\ & y^j(S) \in \{0,1\} \quad \forall S \subseteq M \end{aligned}$$

- Every bidder gets one object
- $y^j(S)$ 0/1-variable for assigning highest bid on S
- $b(S) = \max_{j \in N} b^j(S)$, highest bid on S
- Set Packing Problem



Goals

- Auctioneer
 - Revenue maximization
 - Cost recovery
(minimum prices)
- Bidder
 - Gain maximization
 - Transparency
 - Information sealing
- General
 - Efficiency
 - Fairness
 - Equilibrium
 - Dominant Strategy
 - Truthful bidding
 - Implementability



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Complexity

- $N \times 2^M$ possible bids
- Encoding length (bid functions?)
- Set Packing Problem is NP-hard (even for polynomial number of variables)
- Difficulties
 - For the auctioneer: Solving the CAP
 - For the bidder: Stating reasonable bids
- Requirements for practical applications
 - Small number of bids
 - CAP can be solved reasonably quickly



Efficiency

■ CAP

- Valuation $v^j(S)$ by j for $S \subseteq M$
- Bid $b^j(S)$ by j for $S \subseteq M$

$$\begin{aligned} \max \quad & \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S, j) \\ & \sum_{S \ni i} \sum_{j \in N} y(S, j) \leq 1 \quad \forall i \in M \\ & y(S, j) \in \{0, 1\} \quad \forall S \subseteq M, j \in N \end{aligned}$$

- $\bar{y} = y(b) = \operatorname{argmax} \operatorname{CAP}(b)$
 $y^* = y(b=v) = \operatorname{argmax} \operatorname{CAP}(b=v)$
- Efficiency

$$\sum v_i(\bar{y}_i) / \sum v_i(y_i^*)$$



Game Theoretical Interpretation: Auction as a Non-Cooperative Game

- Non-Cooperative Game (N, S, a)
 - $N = \{1, \dots, n\}$ player
 - $S = \{(s_1, \dots, s_n)\}$ strategies
 - $a: S \rightarrow \mathbb{R}^n$ payoff
- Concepts
 - Dominant strategy s^*_i for i
 $a(s_1, \dots, s_i, \dots, s_n) \leq a(s_1, \dots, s^*_i, \dots, s_n)$
 - (Nash-)Equilibrium \hat{s}
 $a(\hat{s}_1, \dots, s_i, \dots, \hat{s}_n) \leq a(\hat{s}_1, \dots, \hat{s}_n) \quad \forall i$
 (i.g. no existence/uniqueness)
 - Matrix games: saddle point, minimax
- **Theorem (Nash):** Every finite non-cooperative n -person game has at least one equilibrium of mixed strategies.
- **Theorem (Nikaido, Isoda):** Generalization to auction frameworks.



Game Theoretical Interpretation: Auction as a Cooperative Game

- Cooperative Game (N, S, a)
 - $N = \{1, \dots, n\}$ players
 - $S = \{(s_1, \dots, s_n)\}$ strategies
 - $v: 2^N \rightarrow \mathbb{R}^n$ payoff
 - $x: N \rightarrow \mathbb{R}^n, x(N) = v(N)$ payoff vector (imputation)
- Concepts
 - Coalitions $L \subseteq N$, grand coalition $L = N$
 - Core $C = \{x: x(N) = v(N), x(L) \geq v(L) \text{ for all coalitions } L \subseteq N\}$
 - Can be empty
- Auction Game
 - Seller = Player 0
 - $v = v(S), x = v - b$



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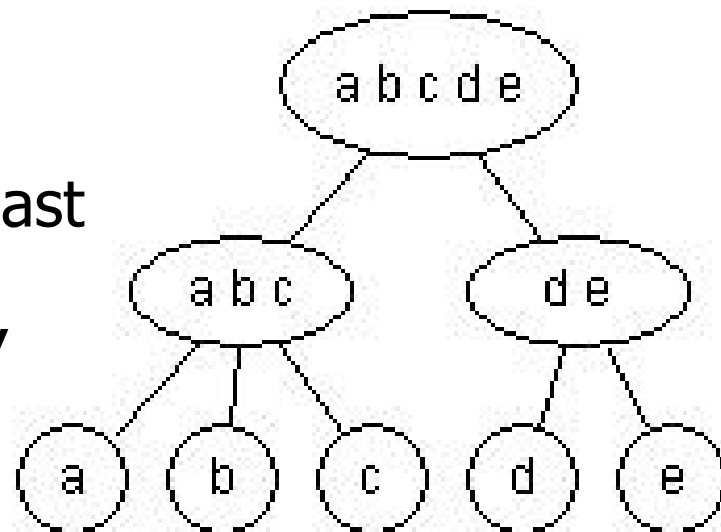


Real Estate

- Bids have consecutive ones property if they include consecutive items
- **Proposition:** If bids are c.o., the constraint matrix of CAP is totally unimodular and CAP can be solved in polynomial time.

- **Examples**

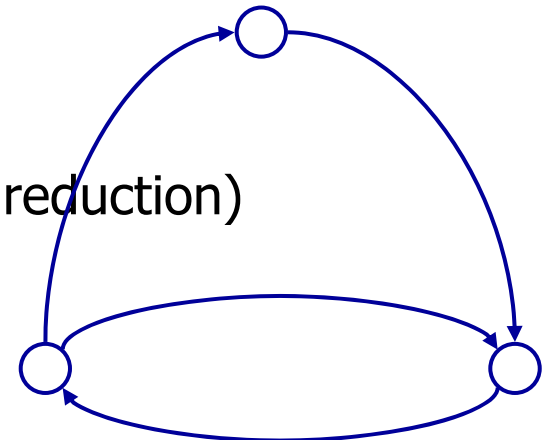
- Contiguous real estate at a coast
- Bids have tree structure, i.e., $S \cap S' \in \{\emptyset, S, S'\}$ for all S, S'





Sears, Roebuck & Co.

- 3-year contracts for transports on dedicated routes
- First auction in 1994 with 854 contracts
- Combinatorial auction
 - „And-“ and „or-“ bids allowed
 - 2^{854} ($\approx 10^{257}$) theoretically possible combinations
 - Sequential auction (5 rounds, 1 month between rounds)
- Results
 - 13% cost reduction
 - Extension to 1.400 contracts (14% cost reduction)



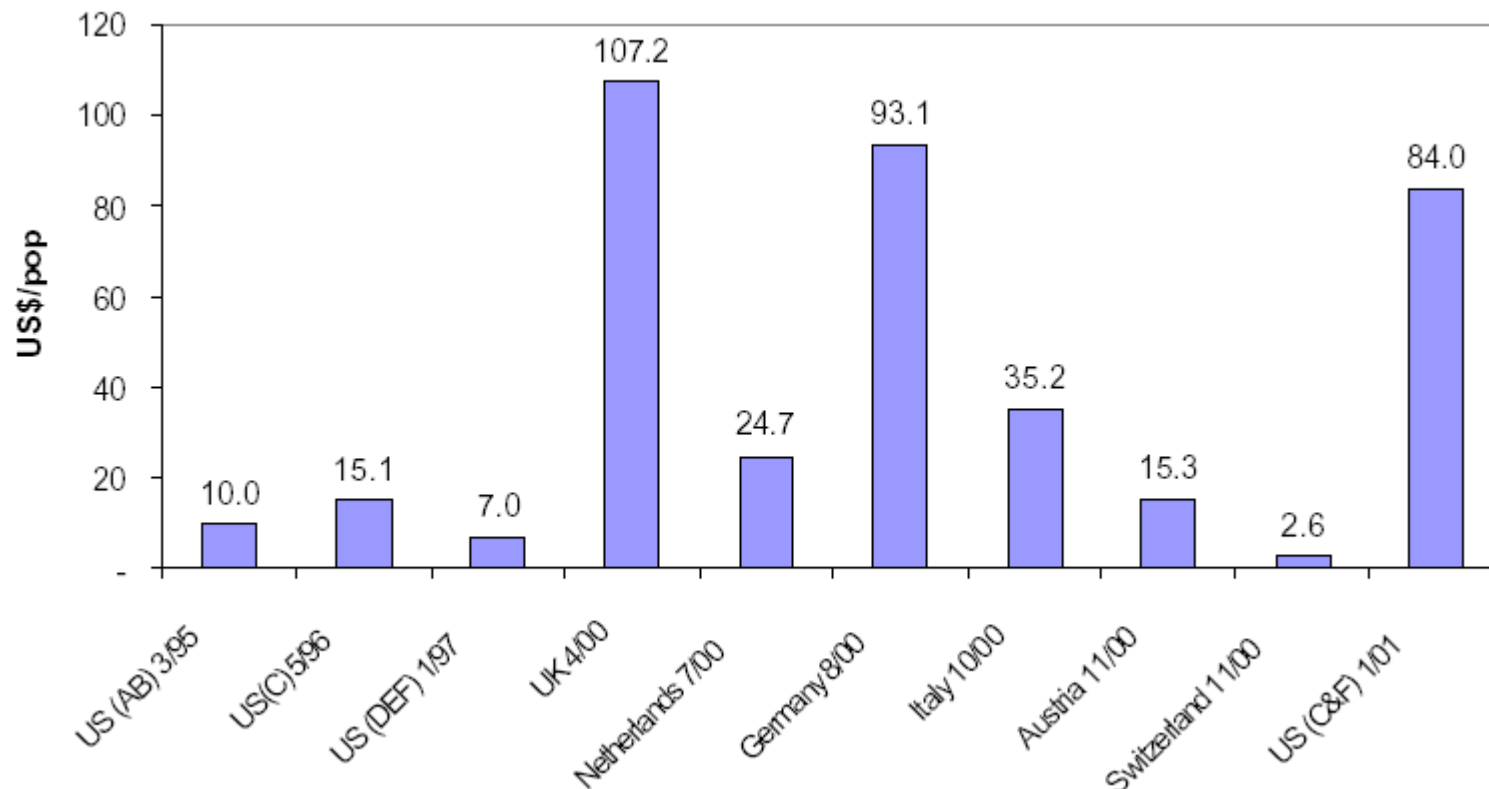
Frequency Auctions

- US
 - 1927: FCC "beauty contest,,
 - 1982: Placement of more than 1,000 licenses
 - Beauty contest too elaborat
 - Loophole: lottery
 - Consequence: company foundation to participate in the lottery
 - In both cases: no revenue for the state
 - New idea: Auctions
- New Zealand
 - License auction in 1990
 - second price sealed-bid auction for individual licenses
 - bad results
 - Example: highest bid 7 Mio NZ-\$, second highest bid: 2500 NZ-\$
 - Revenue was only 36 Mio NZ-\$ instead of the expected 250 Mio. NZ-\$
 - Change to first price sealed-bid auction brought no improvement



Frequency Auctions

(Cramton 2001, Spectrum Auctions, *Handbook of Telecommunications Economics*)



- Prices for mobile telecommunication frequencies (2x10 MHz+5MHz)
- Low earnings are not per se inefficient
- Only min. prices => insufficient cost recovery



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Simultaneous Ascending Auction

Rules

- Multiple heterogeneous objects
- Combinatorial auction, but only individual bids
- First price sealed bid
- N rounds, minimum increment
- Activity rule #objects
- Fee for taking back
- Empirical efficiency 67%
- High revenues, partly due to losses for bidders

Equilibrium

	A	B	AB
1	4	6	9
2	4	5	7
P	4	6	

Exposure problem

	A	B	AB
1	2	2	7
2	4	4	6
P	?	?	



Simultaneous Ascending Auction

- Auction #1 (USA 1994)
 - 10 licenses
 - 3 national bandwidths
 - Paging/messaging services
 - ≤ 3 licenses/bidder
 - Increment 2%
 - 47 rounds (1 week)
 - 617 Mio. USD
(50 Mio. USD expected)
- Auction #4 (USA 1994)
 - 99 licenses
 - 2 bandwidths, 51 MTAs
 - Mobile telephone services
 - Increment 5%
 - 112 rounds (3 months)
 - 7.000 Mio. USD



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Adaptive User Selection Mechanism

Rules

- Several heterogeneous objects
- Combinatorial bids
- First price open bid
- Continuous bidding
- No activity rule
- Auctioneer determines end

▪ Empirical efficiency 94%

▪ Complexity with bidders, lower revenues than SAA

Threshold problem

	A	B	AB
1	7	2	8
2	2	7	8
3	4	4	10
P	?	?	10

Proposal: Standby Q

	A	B	AB
1	7	2	8
2	2	7	8
3	4	4	10
P	6	?	10

▪ Free rider problem



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Vickrey-Clarke-Groves-Mechanism

(Nobel price in Economics 1996)

- Combinatorial auction

$$E(N, b) := \max \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S, j)$$

$$\sum_{S \ni i} \sum_{j \in N} y(S, j) \leq 1 \quad \forall i \in M$$

$$y(S, j) \in \{0, 1\} \quad \forall S \subseteq M, j \in N$$

- Private values v_j

- Mechanism

- Bids $b_j = v_j$

- Payments

$$z_j = E(N \setminus j, v) - E(N, v) | N \setminus j$$



Vickrey-Clarke-Groves-Mechanism

- Combinatorial auction

$$E(N, b) := \max \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S, j)$$

$$\sum_{S \ni i} \sum_{j \in N} y(S, j) \leq 1 \quad \forall i \in M$$

$$y(S, j) \in \{0, 1\} \quad \forall S \subseteq M, j \in N$$

- Private values v_j

- Mechanism

- Bids $b_j = v_j$

- Payments

$$z_j = E(N \setminus j, v) - E(N, v) | N \setminus j$$

- Example

	A	B	AB
1	10	5	15
2	1	6	12
P	6	5	

- Collusion

	A	B	C	ABC
1				2
2	1			
3		1		
4			1	
P	0	0	0	

- Shill bidding

- Fraud by auctioneer



Vickrey-Clarke-Groves-Mechanism

- Combinatorial auction

$$\begin{aligned}
 E(N, b) &:= \max \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S, j) \\
 \sum_{S \ni i} \sum_{j \in N} y(S, j) &\leq 1 \quad \forall i \in M \\
 y(S, j) &\in \{0, 1\} \quad \forall S \subseteq M, j \in N
 \end{aligned}$$

- Private values v_j

- Mechanism

- Bids $b_j = v_j$

- Payments

$$z_j = E(N \setminus j, v) - E(N, v) | N \setminus j$$

- Theorem:** Truthful bidding, i.e., $b=v$, is a dominant strategy in a VCG auction and leads to an efficient allocation.
- The complexity of VCG $n+1$ times that of a standard combinatorial auction.



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Ascending Proxy Auction

- Combinatorial first price sealed bid auction
- Assumptions
 - Free disposal, private values
 - Mutually exclusive bids for every bidder
- Straightforward bidding by proxy-agent (program)
 - Value of a bid set once before auction
 - Proxy increases non-winning bids in every round by ϵ (small, fixed)



Ascending Proxy Auction

- **Theorem (Ausubel, Milgrom):** An ascending proxy auction terminates in the core of the cooperative game associated with the auction.
- **Theorem (Ausubel, Milgrom):** A proxy auction, interpreted as a non-cooperative game, terminates under certain conditions in a Nash-equilibrium, in particular, if a corresponding Vickrey-Clarke-Groves-auction terminates in a Nash-equilibrium.
- Combinations with other auctions, e.g., clock-proxy, to simplify programming of the agent.



More Auctions

- Resource Allocated Design
- iBundle
- Clock proxy auctions
- ...



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- **Railway Track Auction**



Suche

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- **Einkauf & Verkauf**
- **Fahrzeuge Straße/Schiene**
- **Immobilien**
- **Infrastruktur & Energie**
 - Energie
 - Fahrweg
 - Netzzugang
 - **Trassen**
 - Leistungen
 - Trassen Güterverkehr
 - Trassen Personenverkehr
 - Trassenpreise
 - Trassenpreisauskunft
 - Besonderheiten & Fristen
 - Formulare
 - Anlagen
 - Nebenleistungen
 - Ansprechpartner
 - Internationale Verkehre
 - Baustelleninformationen
 - NetzNachrichten
 - Abgabe Infrastruktur
 - Station
- **IT/TK Infrastruktur**
- **Personaldienstleistungen**
- **Weitere Serviceleistungen**

Trassennutzung für den Personen- und Güterverkehr

Hier finden Sie detaillierte Angebote und Preisinformationen zur Nutzung von Trassen der DB Netz AG für den Personen- und Gütertransport. Die zusätzlich angebotene Software unterstützt Sie bei der Kalkulation der Preise für Ihre gewünschte Trasse.



Leistungen



Leistungsangebot der DB Netz AG zur Bereitstellung von Bahninfrastruktur

Aufgabe der DB Netz ist es leistungsfähige Eisenbahninfrastrukturen sowie technische Anlagen und Einrichtungen marktgerecht zur Verfügung zu stellen. Das Leistungsangebot setzt sich aus den Produktfeldern Trassen, Anlagen und Infrastrukturanschlüsse zusammen.

[mehr](#) ➔

Besonderheiten & Fristen

Grundsätzliches zur Trassen-Anmeldung

Bei der Anmeldung von Trassen gibt es Besonderheiten und Fristen, die Sie unbedingt beachten müssen. Alle Informationen zu diesem Thema finden Sie hier.

[mehr](#) ➔

Trassen Güterverkehr



Trassen für den Güterverkehr

Als Kunde im Güterverkehr haben Sie die Wahl zwischen mehreren Produkten. Je nach Nutzung wird zwischen Güterverkehrs-Express-Trassen, Güterverkehr-Standard-Trassen, Güterverkehr-Zubringer-Trassen und Güterverkehrs-LZ-Trassen unterschieden.

[mehr](#) ➔

Formulare

Formulare als PDF-Download

Wenn Sie als Eisenbahnverkehrsunternehmen Trassen anmelden möchten, müssen Sie für Ihre Anmeldung bestimmte Formulare verwenden. Diese Formulare nebst Erläuterung finden Sie hier.

[mehr](#) ➔

Trassen Personenverkehr

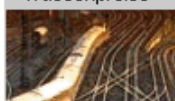


Trassen für den Personenverkehr

Die Personenverkehrs-Trassen lassen sich in vier verschiedene Kategorien einteilen. Als Kunde haben Sie die Wahl zwischen Personenverkehrs-Express-Trassen, Personenverkehrs-Takt-Trassen, Personenverkehrs-Economy-Trassen und Personenverkehrs-LZ-Trassen.

[mehr](#) ➔

Trassenpreise



Gültige Preise ab dem 12.12.2004 und 11.12.2005

Hier finden Sie das seit dem 12.12.2004 gültige und ab dem 11.12.2005 geltende Trassenpreissystem mit seinen Anlagen sowie Streckenkategoriekarten als PDF-Dateien zum Download.

Trassenanmeldung / Trassenstudie / Fahrzeitrechnung / Preisanfrage / Fahrplananpassung
(Zutreffendes bitte ankreuzen)

für Güterzugtrassen

Zugart	Zugnummer	Nutzungsdauer	bestehende Vergleichstrasse	Interne Bearbeitungs-D. Kunde	Kunden-Nr	Eingang der Anmeldung
Kunde, Bevollmächtigter laut ABN		Telefon	Fax	E-Mail		

Gewünschtes Trassenprodukt

<input type="checkbox"/> Express-Trasse	<input type="checkbox"/> Standard-Trasse gewünschte Systemtrasse:	<input type="checkbox"/> Zubringer-Trasse	Zur Zubringer-Trasse gehörende Standard-Trasse:
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Verkehrszeitraum

ab Ort	Zugart/ung	Verkehrszeitraum	Zusatztage	Ausfalltage	Konstruktionspielraum

Betrieblich-technische Angaben (Zugcharakteristik)

ab Ort	Vmax	Tfz 1	Tfz 2	Schiebel	gekuppelt	Last	Brmsstellung	BrH	Länge	EBuLa	Besonderheiten, LU, KLV, Gefahrgut	

Trassenzeiten

Kundenanmeldung							Konstruktionsergebnisse			
Ort	Clois	Ank	Halt	Art	Abf	Vorgaben/ Änderungen der Zugcharakteristik	1		2	
							Ank	Abf	Ank	Abf



Fahrplan		Bestellung EVU			Angebot GB Netz		
Abk.	Bahnhof	an	Min.-halt	ab	an	ab	Bemerkungen
	120:60 Min nach Ankunft 84079 in RLB Zwischen Lu-BASF und Lu-Gggersheim nächstmögliche Trasse rd 3 Std nach Ankunft 84079. — We 14.01.2001						
FLB	Ludwigshafen-BASF					16.42	
FLB	Ludwigshafen-BASF					14.47	
FLO	Lu-hafen-Gggersheim				16.53	17.15	Kopfmachen
FLO	Lu-hafen-Gggersheim				14.58	15.21	Kopfmachen
FWOR	Worms-Hbf					17.29	We 14.01.2002
FWOR	Worms-Hbf					15.35	We 06.02.02
FMZ	Mainz-Hbf					18.28	
Biblic					15.45.6	15.47.4	Ra 07.02.02
FKBWG	Kaiserbrücke-Gst					18.33.3	Ra 17.01.02
FKBWD	Kaiserbrücke-Gst						
—	Oberlahnstein-Gbf					20.02	
KOL G	Oberlahnstein-Gbf						
—	Linz (Rh)					20.35	He 17.01.02
KLI	Linz (Rh)						
PMB	18.56/17.12						
FKOS	Abzw Kostheim					17.20	
KOL G	Oberlahnstein-Gbf					18.57	
KLI	Linz (Rh)					19.35	He 11.02.02
KUN	Unkel				+19.42	19.48	
KG-G	Grämburg-Gbf					20.27	An 19.2.02
KDO	Dormagen	19:00					

Bemerkungen Kunde

BR 185 mit 5,6 MW

Preisgünstigster Weg gewünscht.

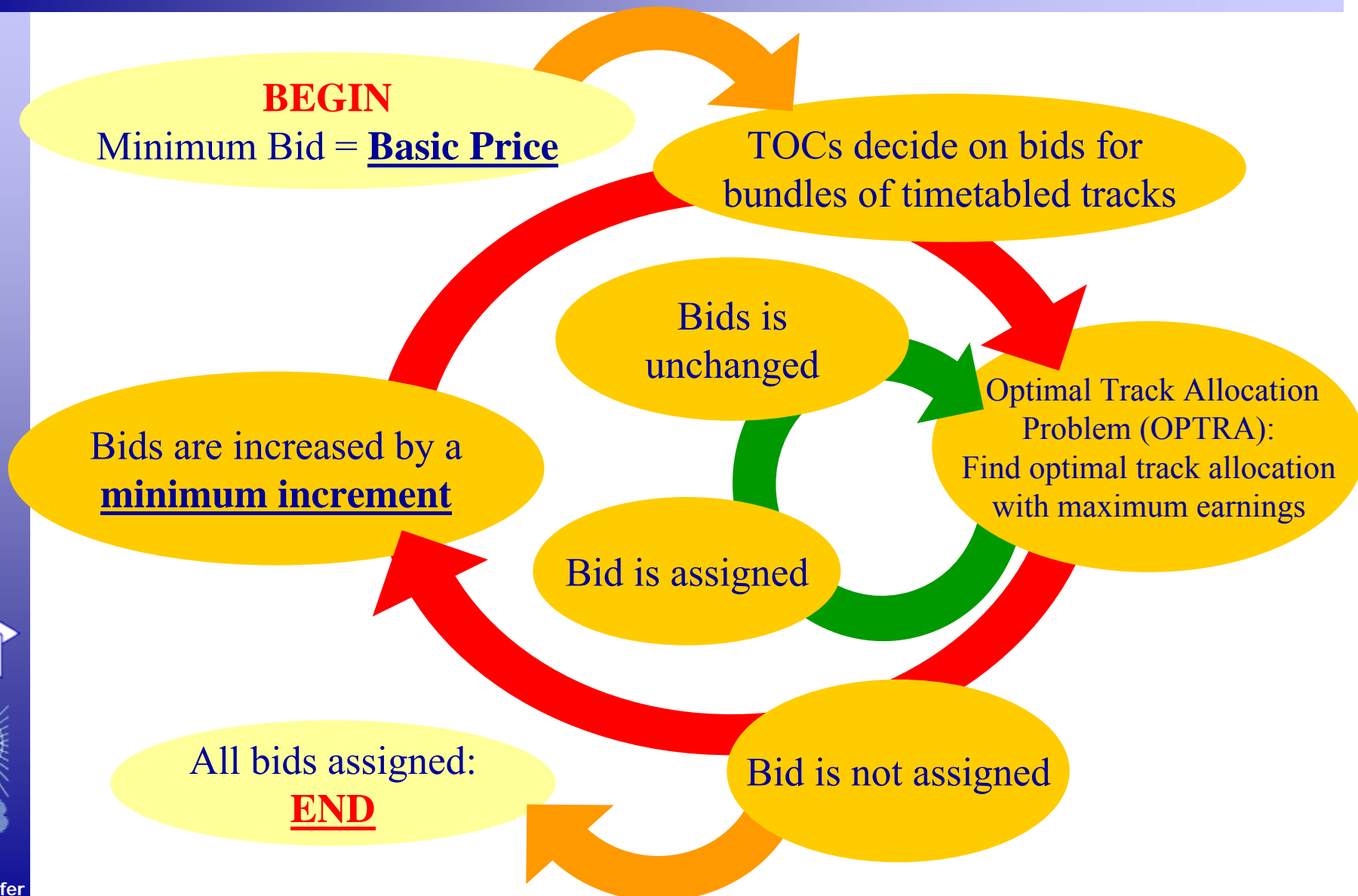


Rail Track Auctioning



- Goals
 - More traffic at lower cost
 - Better service
- How do you measure?
 - Possible answer: in terms of willingness to pay
- What is the "commodity" of this market?
 - Possible answer: timetabled track
= dedicated, timetabled track section
- How does the market work?
 - Possible answer: by auctioning timetabled tracks
- Auctions can be in-company auctions

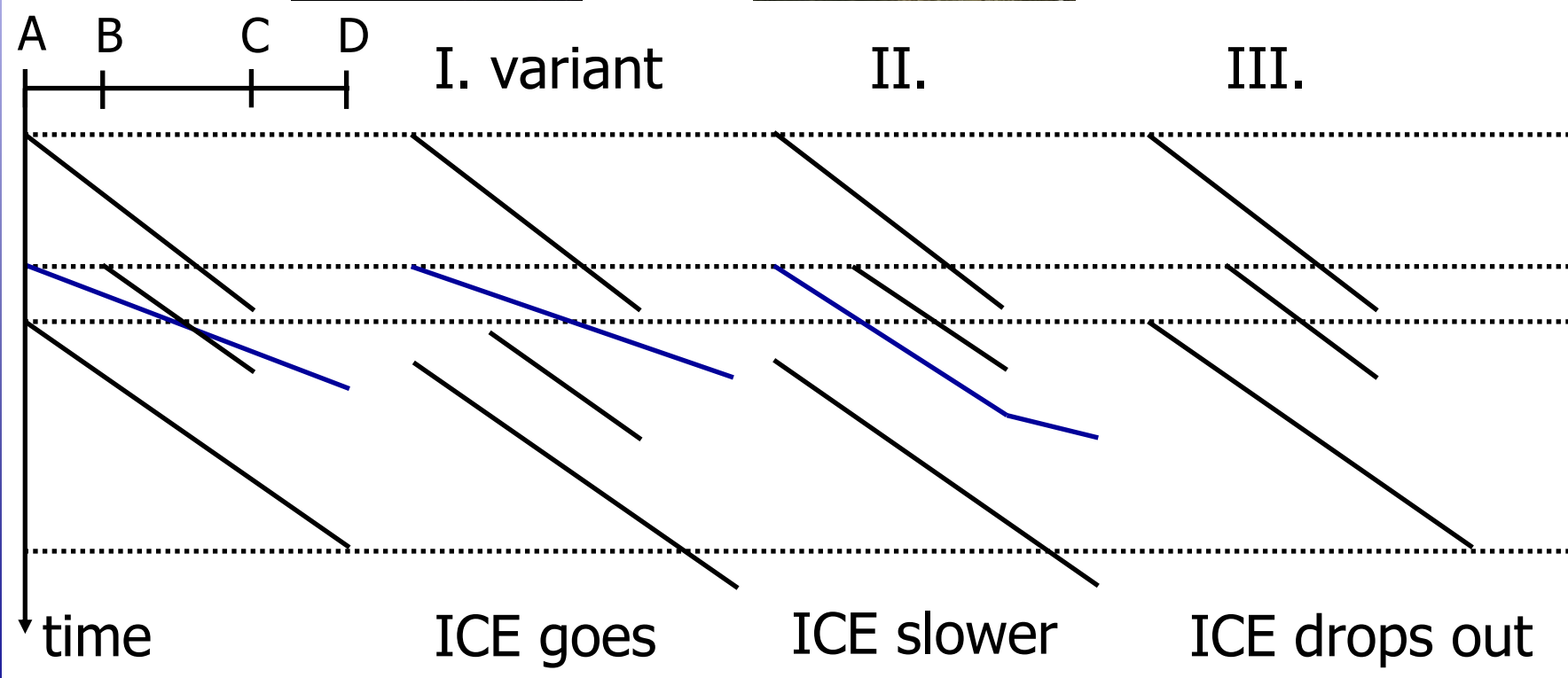


Rail Track Auction



Rail Track Auction Results

3 x  + 1 x  = ???

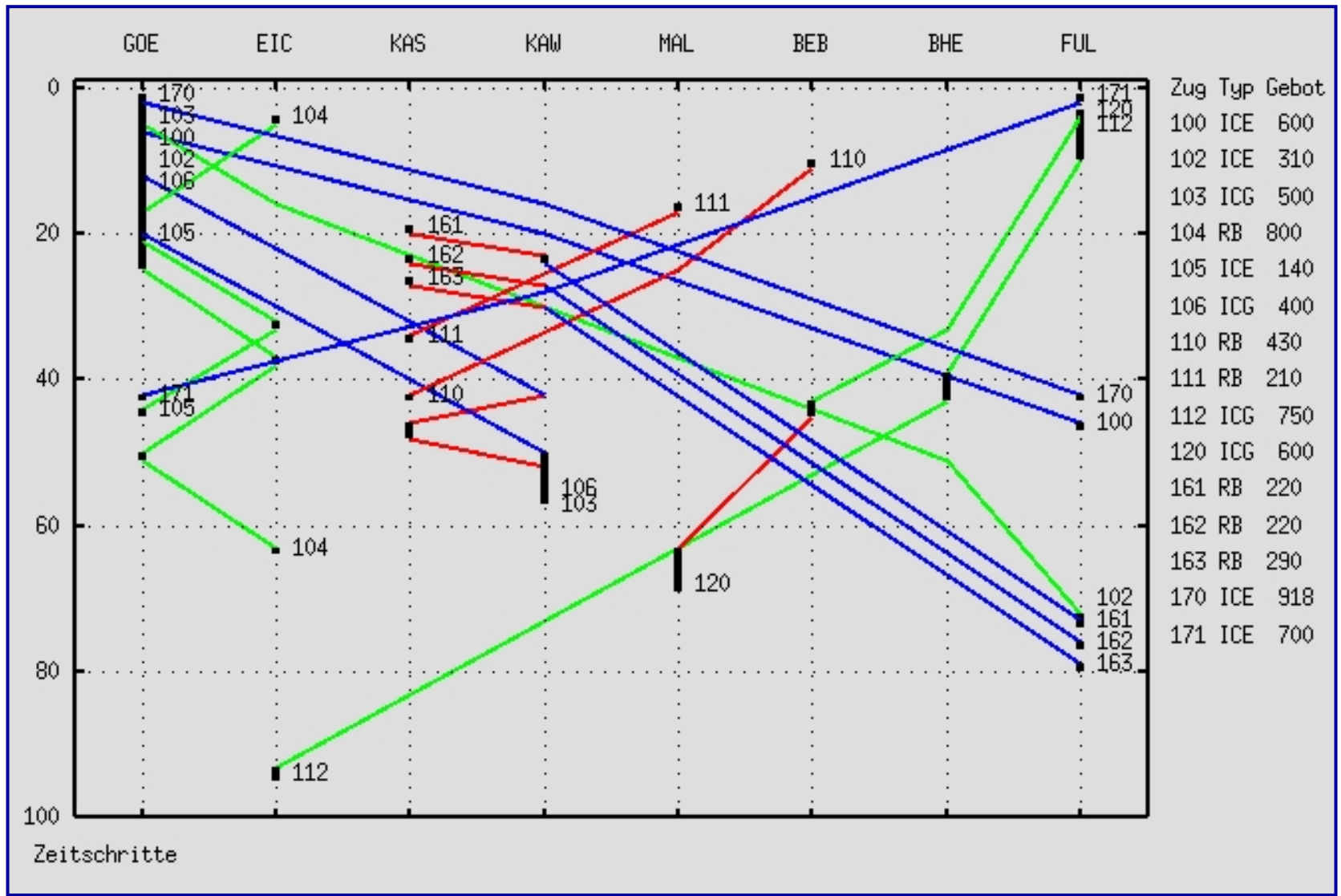


ICE goes ICE slower ICE drops out



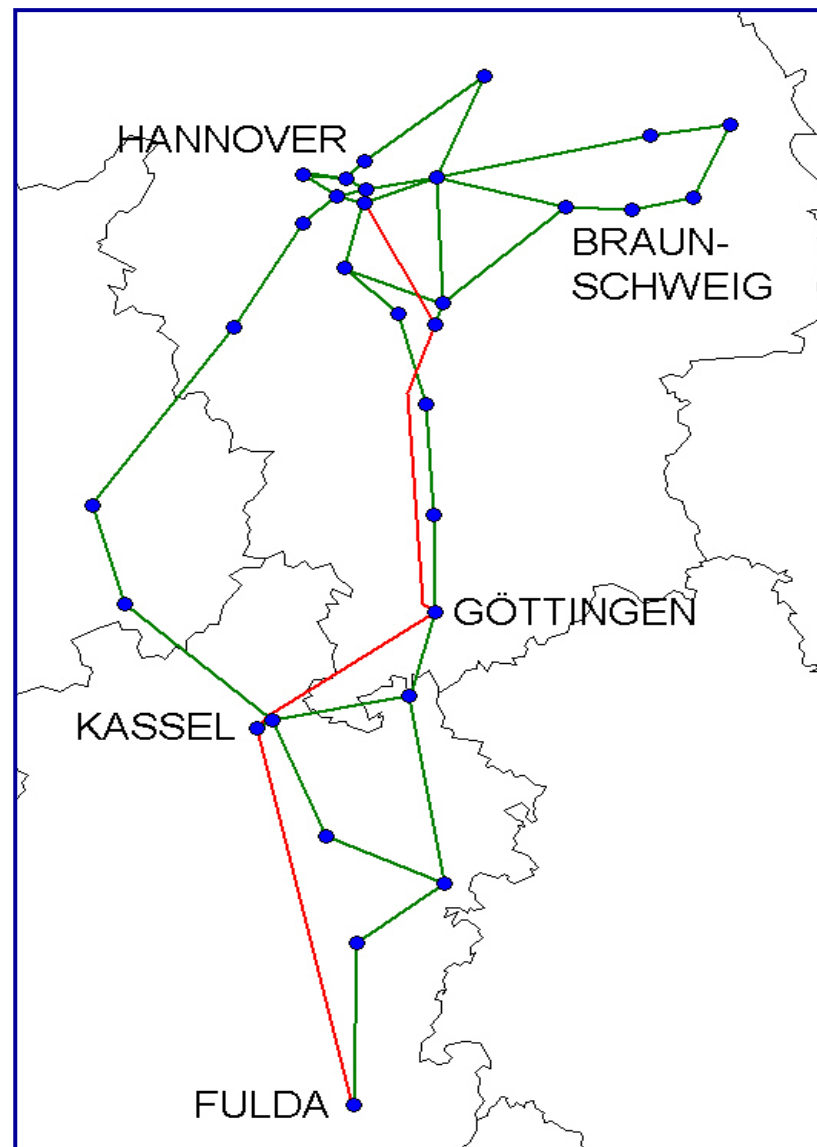
Rail Track Auction Results

(14,439 Variables, 13,408 Constraints, 48 Minutes)



Test Network

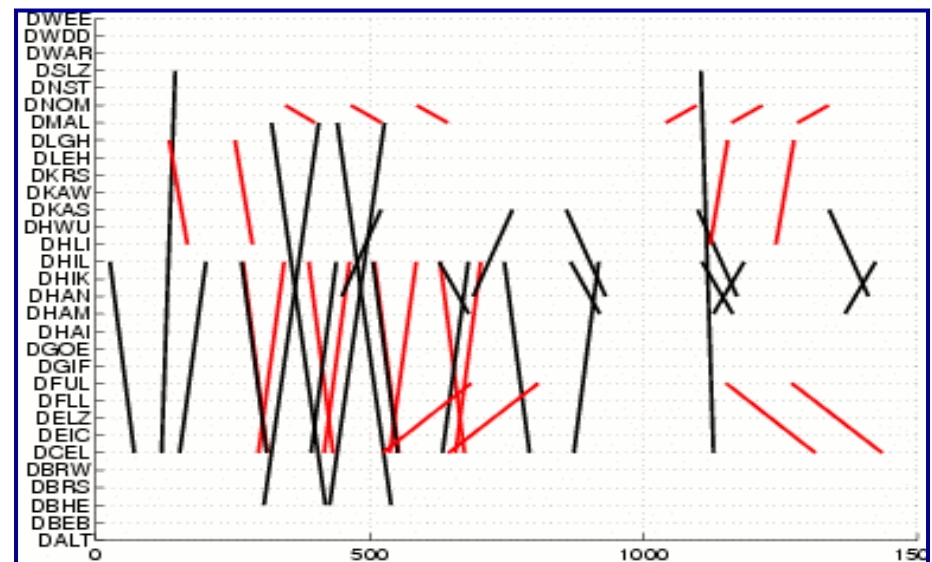
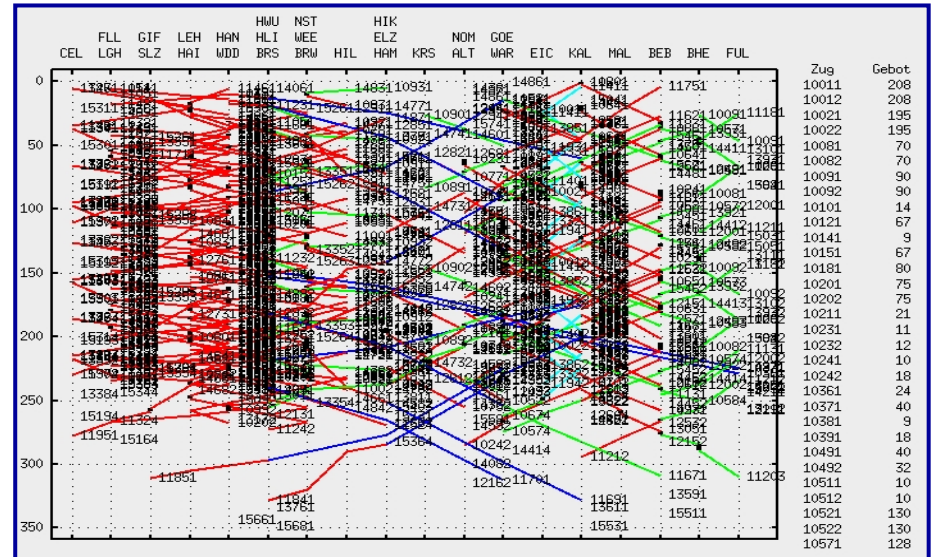
- **Criteria**
 - Important characteristics ("Hildesheimer Kurve")
 - Important subnet
 - Used in earlier studies
- **Data**
 - 45 sections = 1176 km
 - 31 nodes
 - 6 train types



Auction Experiments

(Reuter 2005, Rounds 8 and 9)

Round	Earnings	Round	Earnings
1	44563	9	46575
2	44563	10	47051
3	44698	11	48096
4	44799	12	48253
5	44799	13	48337
6	44972	14	48391
7	45551	15	48513
8	46375		



Auction Experiments

(Reuter 2005)

	<i>ICE</i>		<i>IC</i>		<i>RE</i>		<i>RB</i>		<i>S</i>		<i>ICG</i>	<i>#</i>
<i># Trains/Type</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	
Timetable	27	0	27	0	38	19	87	23	0	61	28	—
+24 IC/ICE ind	30	0	29	0	38	19	85	23	0	61	25	18
+24 IC/ICE sync	24	9	27	9	36	19	83	19	0	58	26	22
+27 R*/S ind	27	0	25	0	44	19	89	23	5	58	27	20
+27 R*/S sync	27	0	27	0	36	19	83	32	0	62	27	30
+15 ICG	27	0	27	0	38	19	87	23	0	61	42	19
+66 *	28	0	25	3	38	25	85	29	2	55	31	29



Auction Experiments

(Reuter 2005)

	<i>ICE</i>		<i>IC</i>		<i>RE</i>		<i>RB</i>		<i>S</i>		<i>ICG</i>	Σ
<i>€/km</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>sync</i>	<i>ind</i>	<i>€</i>
Timetable												
+24 IC/ICE ind	2.04		1.78		1.24	1.07	0.93	0.90		0.98	1.12	34421
+24 IC/ICE sync	1.89	1.94	1.45	3.27	1.14	1.10	0.89	0.83		0.90	1.10	36031
+27 R*/S ind	1.74		1.41		1.23	1.08	0.91	0.90	1.15	1.10	1.14	31180
+27 R*/S sync	2.31		1.34		1.02	1.04	0.88	1.41		1.06	0.98	33663
+15 ICG	1.45		1.44		1.08	1.08	0.87	0.90		0.88	1.03	32994
+66 *	2.21		1.88	2.87	1.03	1.10	0.89	1.11	1.53	1.47	1.60	41263



Tripling Experiment

<i>variation</i>	<i>cpu time (CPLEX)</i>	<i>earnings (% Status Quo)</i>	<i>trains (% Status Quo)</i>
0 mins	6 secs	52.066 (+ 84%)	420 (+ 47%)
1 mins	8 secs	60.612 (+114%)	496 (+ 74%)
4 mins	1 days	67.069 (+137%)	617 (+117%)
5 mins	3+ days	67.975 (+140%)	737 (+159%)

- **Status quo**

- 284 tracks through 6 hours in the Hannover—Braunschweig—
Fulda network, (hypothetical) total income of 28,255 €

- **Scenario**

- triple requests to 946 bids
(~15 minutes alteration, identical willingness to pay)



Thank you for your attention.

