Linear and Integer Programming: an Introduction

CO@W Berlin

Martin Grötschel

21.9.2009

11:00 – 12:30

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Contents

1. Linear, integer, nonlinear programming, optimization: What‵s that?
2. Historic roots
3. LP-Theory
4. Algorithms for the solution of linear programs
   1) Fourier-Motzkin Elimination
   2) The Simplex Method
   3) The Ellipsoid Method
   4) Interior-Point/Barrier Methods
5. Algorithms for the solution of integer programs
   1) Branch&Bound
   2) Cutting Planes
6. Where are we today?
   1) State of the art in LP
   2) State of the art in IP
   3) Examples

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typical optimization problems

$$\max\ f(x)\ \ or\ \ \min\ f(x)$$
$$g_i(x) = 0, \quad i = 1, 2, \ldots, k$$
$$h_j(x) \le 0, \quad j = 1, 2, \ldots, m$$
$$x \in \mathbf{R}^n\ (and\ \ x \in S)$$

„general“
(nonlinear)
program
NLP

$$\min c^T x$$
$$Ax = a$$
$$Bx \le b$$
$$x \ge 0$$
$$(x \in \mathbf{R}^n)$$
$$(x \in \mathrm{k}^n)$$

linear
program
LP

$$\min c^T x$$
$$Ax = a$$
$$Bx \le b$$
$$x \ge 0$$
$$x \in \mathbf{Z}^n$$
$$(x \in \{0,1\}^n)$$

(linear)
integer
program
IP, MIP

program = optimization problem

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will be covered by Bob Bixby on Saturday

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Linear Programming

$$\max\ c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$\textit{subject to}$$

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$\cdot$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

$$\max\ c^T x$$

$$Ax = b$$

$$x \geq 0$$

linear program
in standard form

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Linear Programming: a very brief history

- 1826/1827 Jean Baptiste Joseph Fourier (1786-1830): rudimentory form of the simplex method in 3 dimensions.
- 1939 L. V. Kantorovitch (1912-1986): Foundations of linear programming (Nobel Prize 1975)
- 1947 G. B. Dantzig (1914-2005): Invention of the simplex algorithm

$$\max c^T x$$
$$Ax = b$$
$$x \geq 0$$

- Today: In my opinion and from an economic point of view, linear programming is the most important development of mathematics in the 20th century.

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Optimal use of scarce ressources

foundation and economic interpretation of LP

Leonid V. Kantorovich Tjalling C. Koopmans

Nobel Prize for Economics 1975

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Stiglers „Diet Problem“: „The first linear program“

$\min x1 + x2$	costs
$2x1 + x2 \geq 3$	protein
$x1 + 2x2 \geq 3$	carbohydrates
$x1 \geq 0$	potatoes
$x2 \geq 0$	beans

minimizing the cost of food

George J. Stigler
Nobel Prize in economics 1982

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Sets n nutrients / calorie thousands , protein grams , calcium grams , iron milligrams vitamin-a thousand ius, vitamin-b1 milligrams, vitamin-b2 milligrams, niacin milligrams , vitamin-c milligrams /

f foods / wheat , cornmeal , cannedmilk, margarine , cheese , peanut-b , lard liver , porkroast, salmon , greenbeans, cabbage , onions , potatoes spinach, sweet-pot, peaches , prunes , limabeans, navybeans /

Parameter b(n) required daily allowances of nutrients / calorie 3, protein 70 , calcium .8 , iron 12 vitamin-a 5, vitamin-b1 1.8, vitamin-b2 2.7, niacin 18, vitamin-c 75 /

Table a(f,n) nutritive value of foods (per dollar spent)

	calorie	protein	calcium	iron	vitamin-a	vitamin-b1	vitamin-b2	niacin	vitamin-c
	(1000)	(g)	(g)	(mg)	(1000iu)	(mg)	(mg)	(mg)	(mg)
wheat	44.7	1411	2.0	365		55.4	33.3	441	
cornmeal	36	897	1.7	99	30.9	17.4	7.9	106	
cannedmilk	8.4	422	15.1	9	26	3	23.5	11	60
margarine	20.6	17	.6	6	55.8	.2			
cheese	7.4	448	16.4	19	28.1	.8	10.3	4	
peanut-b	15.7	661	1	48		9.6	8.1	471	
lard	41.7				.2		.5	5	
liver	2.2	333	.2	139	169.2	6.4	50.8	316	525
porkroast	4.4	249	.3	37		18.2	3.6	79	
salmon	5.8	705	6.8	45	3.5	1	4.9	209	
greenbeans	2.4	138	3.7	80	69	4.3	5.8	37	862
cabbage	2.6	125	4	36	7.2	9	4.5	26	5369
onions	5.8	166	3.8	59	16.6	4.7	5.9	21	1184
potatoes	14.3	336	1.8	118	6.7	29.4	7.1	198	2522
spinach	1.1	106		138	918.4	5.7	13.8	33	2755
sweet-pot	9.6	138	2.7	54	290.7	8.4	5.4	83	1912
peaches	8.5	87	1.7	173	86.8	1.2	4.3	55	57
prunes	12.8	99	2.5	154	85.7	3.9	4.3	65	257
limabeans	17.4	1055	3.7	459	5.1	26.9	38.2	93	
navybeans	26.9	1691	11.4	792		38.4	24.6	217	

Positive Variable x(f) dollars of food f to be purchased daily (dollars)

Free Variable cost total food bill (dollars)

Equations nb(n) nutrient balance (units), cb cost balance (dollars) ;

nb(n).. sum(f, a(f,n)*x(f)) =g= b(n); cb.. cost=e= sum(f, x(f));

Model diet stiglers diet problem / nb,cb /;

http://www.gams.com/modlib/libhtml/diet.htm

Solution of the Diet Problem

Goal: find the cheapest combination of foods that will satisfy the daily requirements of a person

motivated by the army's desire to meet nutritional requirements of the soldiers at minimum cost

Army's problem had 77 unknowns and 9 constraints.
Stigler solved problem using a heuristic: $39.93/year (1939)
Laderman (1947) used simplex: $39.69/year (1939 prices)

→ first "large-scale computation"
took 120 man days on hand operated
desk calculators (10 human "computers")

http://www.mcs.anl.gov/home/otc/Guide/CaseStudies/diet/index.html

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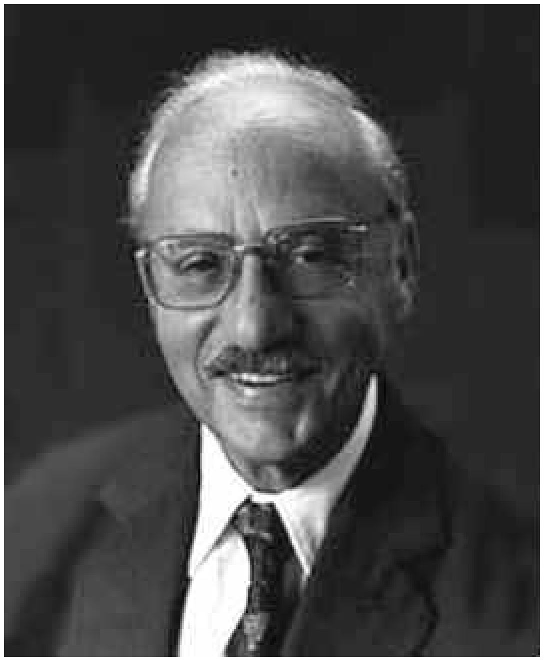
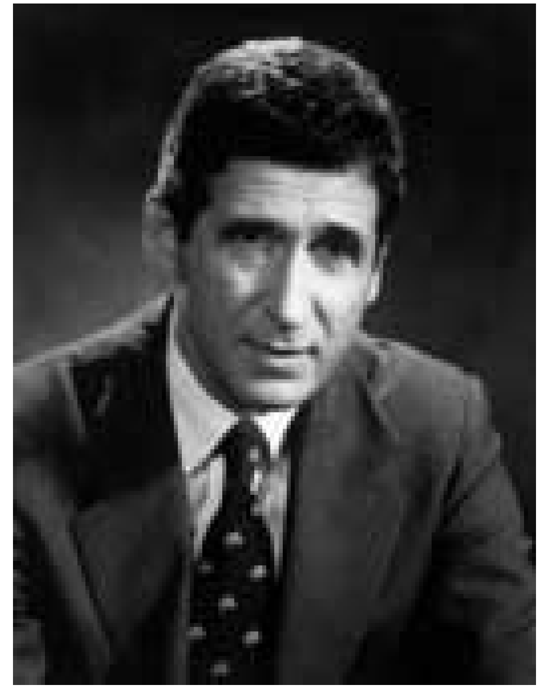
Milton Friedman on George J. Stigler

An early example of the latter is an article on "The Cost of Subsistence" (1945), which starts, "Elaborate investigations have been made of the adequacy of diets at various income levels, and a considerable number of 'low-cost,' 'moderate,' and 'expensive' diets have been recommended to consumers. Yet, so far as I know, no one has determined the minimum cost of obtaining the amounts of calories, proteins, minerals, and vitamins which these studies accept as adequate or optimum." George then set himself to determine the minimum cost diet, in the process producing one of the earliest formulations of a linear programming problem in economics, for which he found an approximate solution, explaining that "there does not appear to be any direct method of finding the minimum of a linear function subject to linear constraints." Two years later George Dantzig provided such a direct method, the simplex method, now widely used in many economic and industrial applications.

http://www.nap.edu/readingroom.php?book=biomems&page=gstigler.html

George Dantzig and Ralph Gomory





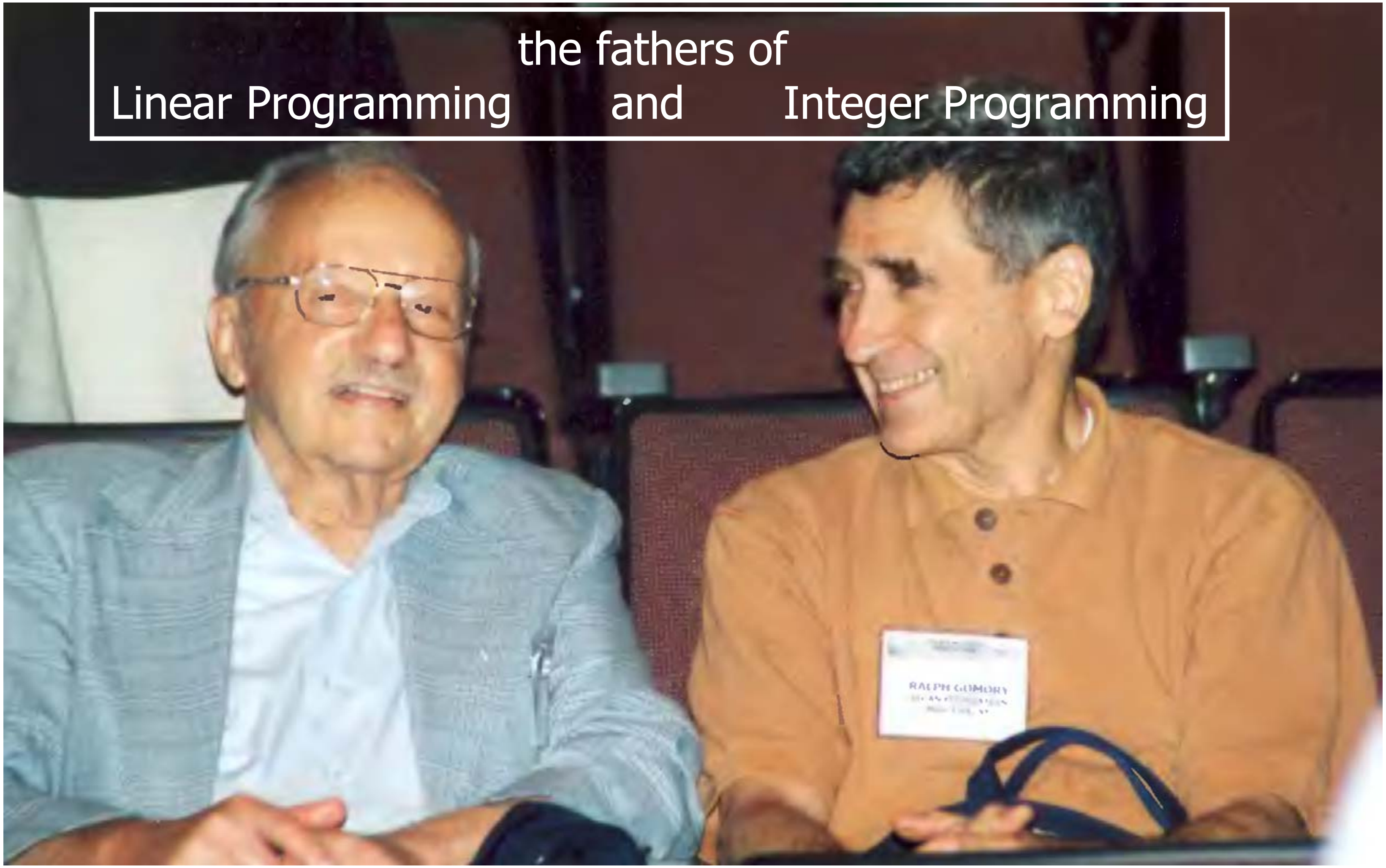
„founding fathers"

~1950
linear programming

~1960
integer programming

George Dantzig and Ralph Gomory

ISMP Atlanta 2000



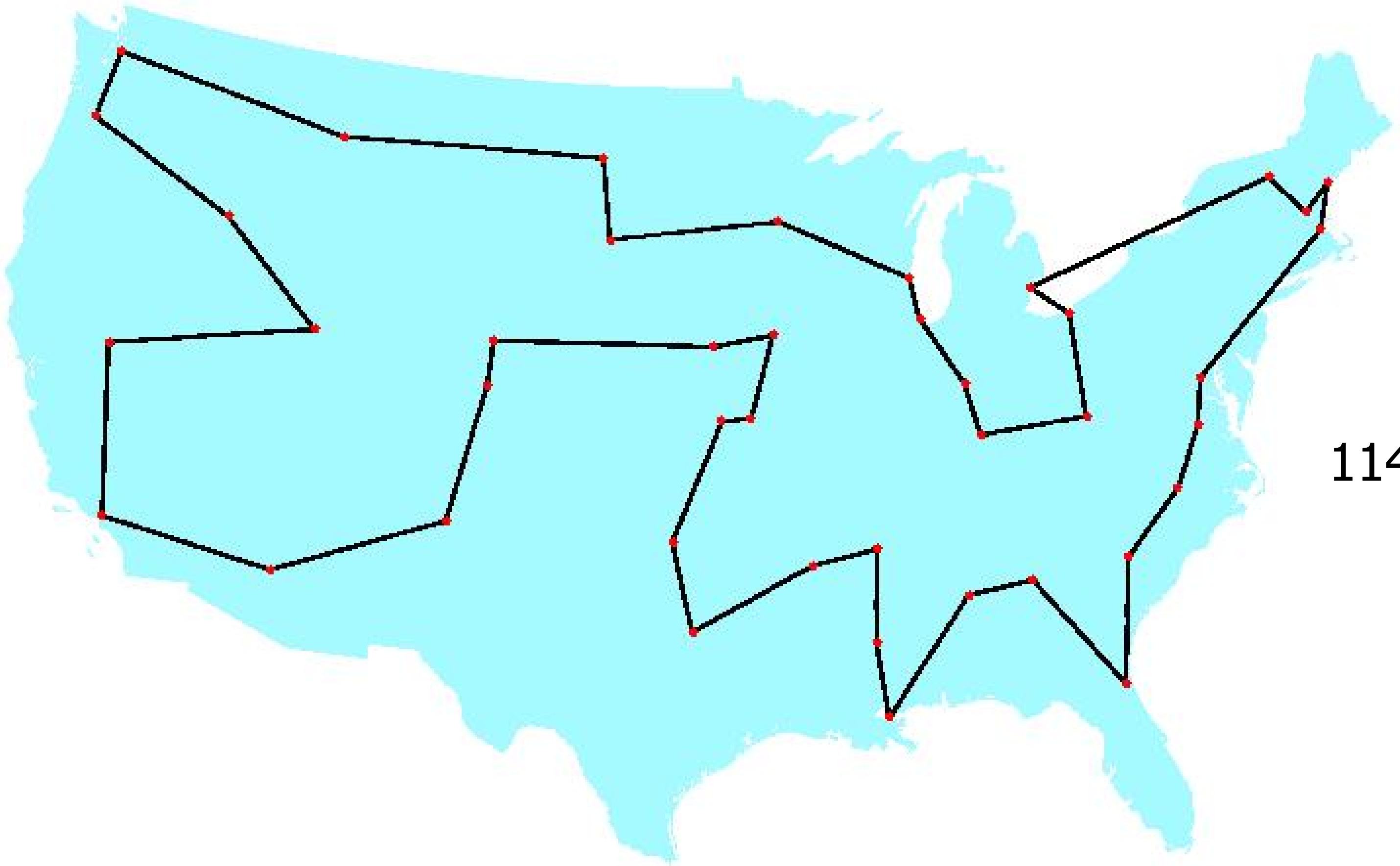
Dantzig and Bixby

George Dantzig and
Bob Bixby

at the International Symposium on Mathematical Programming,

Atlanta, August 2000

1954, the Beginning of IP
G. Dantzig, D.R. Fulkerson, S. Johnson



USA
49 cities
1146 variables

1954

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Commercial software

William Orchard-Hayes (in the period 1953-1954)

The first commercial LP-Code was on the market in 1954 (i.e., ~55 years ago) and available on an IBM CPC (card programmable calculator):

Code: Simplex Algorithm with explicit basis inverse, that was recomputed in each step.

Shortly after, Orchard-Hayes implemented a version with product form of the inverse (idea of A. Orden),
Record: 71 variables, 26 constraints, 8 h running time

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Linear Programming

$$\begin{aligned} \min\ & c^T x \\ & Ax = a \\ & Bx \le b \\ & x \ge 0 \end{aligned}$$

$$\begin{aligned} \min\ & c^T x \\ & Ax = a \\ & Bx \le b \\ & x \ge 0 \end{aligned}$$

$$\begin{aligned} \min\ & c^T x \\ & Ax = a \\ & x \ge 0 \end{aligned}$$

$$\begin{aligned} \min\ & c^T x \\ & Bx \le b \end{aligned}$$

Linear program in various forms.
They are all equivalent!
There are more!

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Optimizers‘ dream: Duality theorems

- Max-Flow Min-Cut Theorem

  The maximal (s,t)-flow in a capacitated network is equal to the minimal capacity of an (s,t)-cut.

- The Duality Theorem of linear programming

$$\begin{aligned} \max\ & c^T x \\ & Ax \le b \\ & x \ge 0 \end{aligned} \quad = \quad \begin{aligned} \min\ & y^T b \\ & y^T A \ge c^T \\ & y \ge 0 \end{aligned}$$

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Optimizers‘ dream: Duality theorems for integer programming

- The Max-Flow Min-Cut Theorem
  does not hold if several source-sink relations are given
  (multicommodity flow).
- The Duality Theorem of linear programming
  does not hold if integrality conditions are added

$$\max c^T x \quad \leq \quad \min y^T b$$

$$Ax \leq b \qquad y^T A \geq c^T$$

$$x \geq 0 \qquad y \geq 0$$

$$x \in \mathbf{Z}^n \qquad y \in \mathbf{Z}^m$$

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Important theorems

- complementary slackness theorems
- redundancy characterizations

LP Solvability

- We assume in this course that the participants are somewhat familiar with complexity theory:
  - Polynomial time solvability, solvability in strongly polynomial time
  - Classes: $\mathcal{P}$ and $\mathcal{NP}$, $\mathcal{NP}$-completeness , $\mathcal{NP}$-hardness , etc.
- Linear programs can be solved in polynomial time with
  - the Ellipsoid Method (Khachiyan, 1979)
  - Interior Points Methods (Karmarkar, 1984, and others)
- Open: is there a strongly polynomial time algorithm for the solution of LPs?
- Certain variants of the Simplex Algorithm run – under certain conditions – in expected polynomial time (Borgwardt, 1977…)
- Open: Is there a polynomial time variant of the Simplex Algorithm?

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LP Solvability: Generalizations

Theorem (GLS 1979, 1988) (modulo technical details) **:**

There exists a polynomial time algorithm to minimize convex functions (e.g., linear functions) over the elements of a class of convex bodies **K** (e. g. polyhedra) if and only if, there exists a polynomial time algorithm that decides, for any given point **x**, whether **x** is in **K**, and that, when **x** is not in **K**, finds a hyperplane that separates **x** from **K**.

Short version: Optimization and Separation are polynomial-time equivalent.

Consequence: Theoretical Foundation of cutting plane algorithms.

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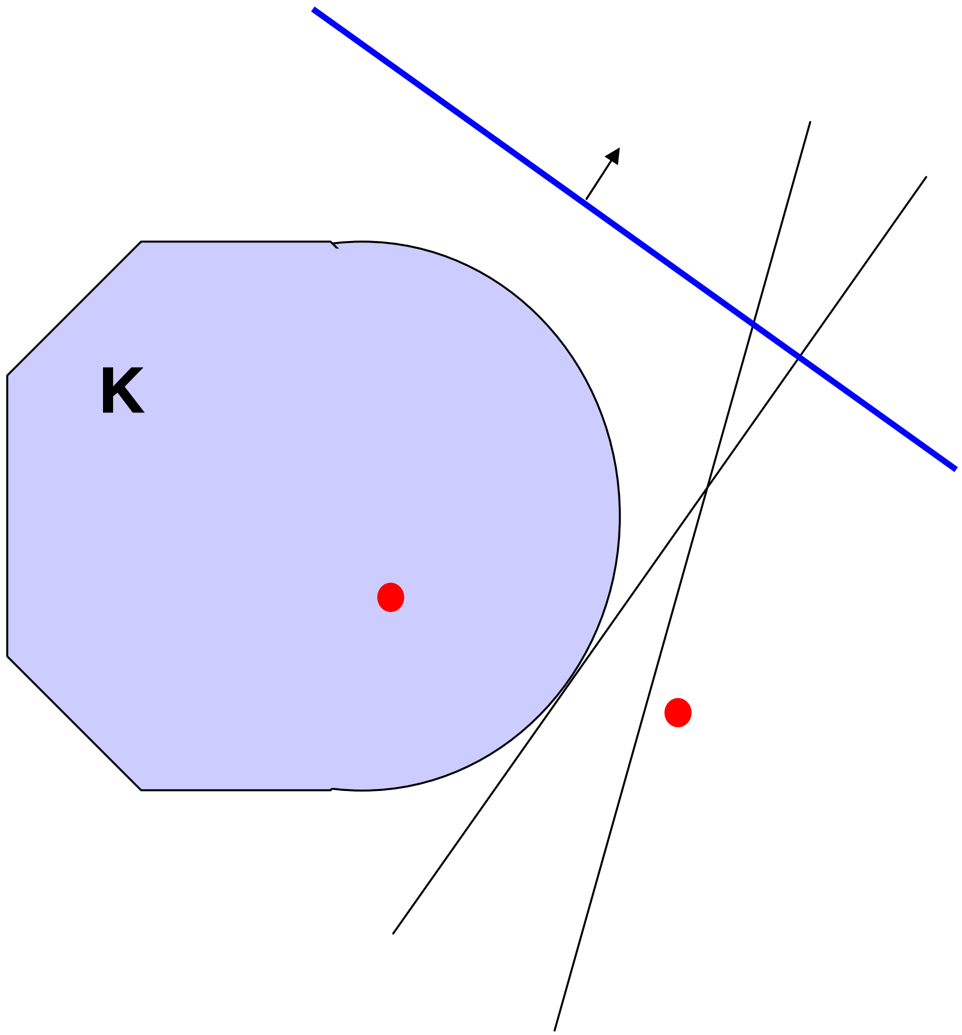
Separation



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IP Solvability

Theorem

Integer, 0/1, and mixed integer programming are NP-hard.

Consequences

- Investigation of special cases
- Exact problem specific special purpose algorithms
- Design of special purpose heuristics

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Algorithms for nonlinear programming

- Iterative methods that solve the equation and inequality sytems representing the necessary local optimality conditions (e.g., KKT).

$$x_{i+1} = x_i + \lambda_i d_i$$
$$d_i \sim \text{"descent direction"}$$
$$\lambda_i \sim \text{"steplength"}$$

$$d_i = -\nabla f(x_i)$$ Steepest descent

$$d_i = -(H(x_i))^{-1} \nabla f(x_i)$$ Newton

(Quasi-Newton, conjugate-gradient-, SQP-, subgradient…methods)

- Sufficient conditions are rarely checked.

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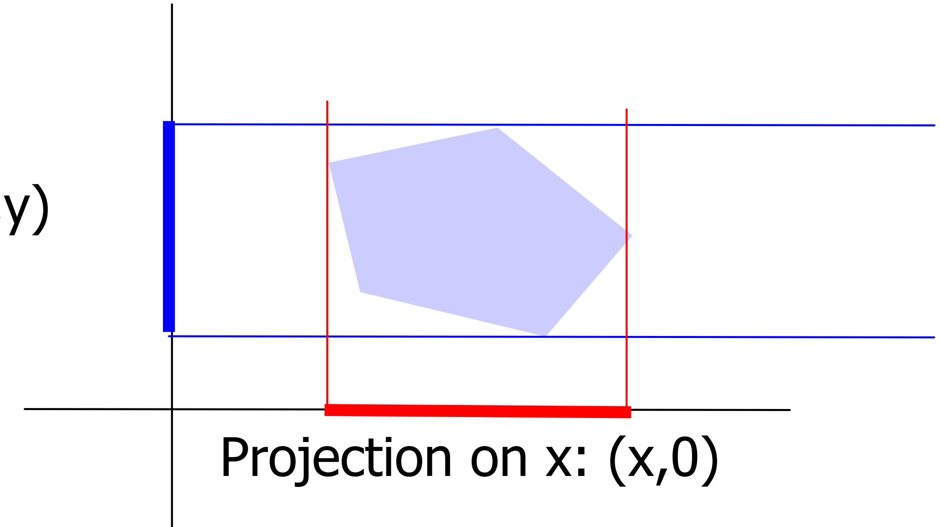
Fourier-Motzkin Elimination

- Fourier, 1847
- Motzkin, 1938
- **Method**: successive projection of a polyhedron in n-dimensional space into a vector space of dimension n-1 by elimination of one variable.



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} will be covered by Bob Bixby on Saturday

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The Simplex Method

- Dantzig, 1947: primal Simplex Method
- Lemke, 1954; Beale, 1954: dual Simplex Method
- Dantzig, 1953: revised Simplex Method
- ....
- **Underlying Idea**: Find a vertex of the set of feasible LP solutions (polyhedron) and move to a better neighbouring vertex, if possible (Fourier‘s idea 1826/27).

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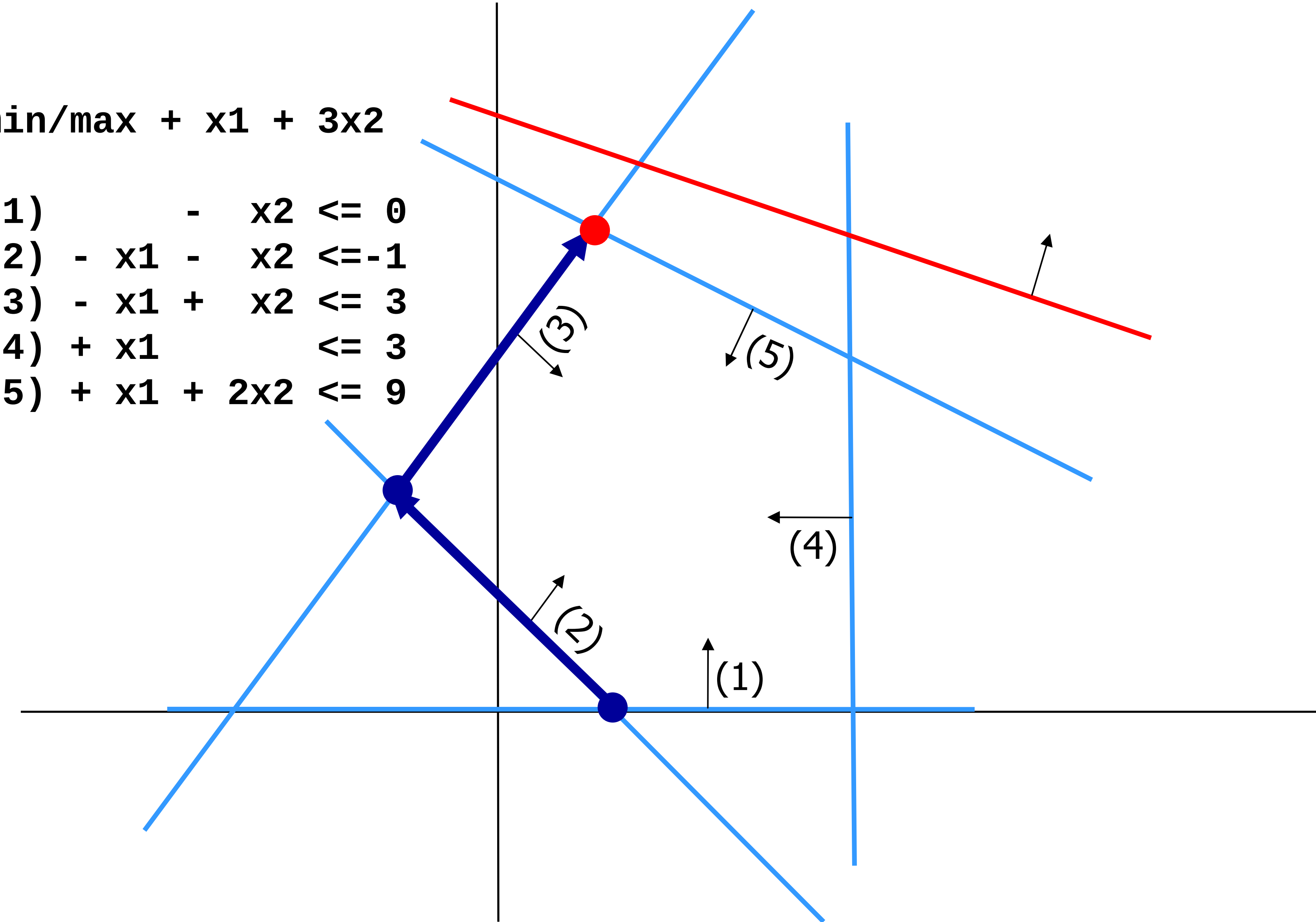
The Simplex Method: an example


```
min/max + x1 + 3x2

(1)       -   x2 <= 0
(2) - x1 -   x2 <=-1
(3) - x1 +   x2 <= 3
(4) + x1        <= 3
(5) + x1 + 2x2 <= 9
```



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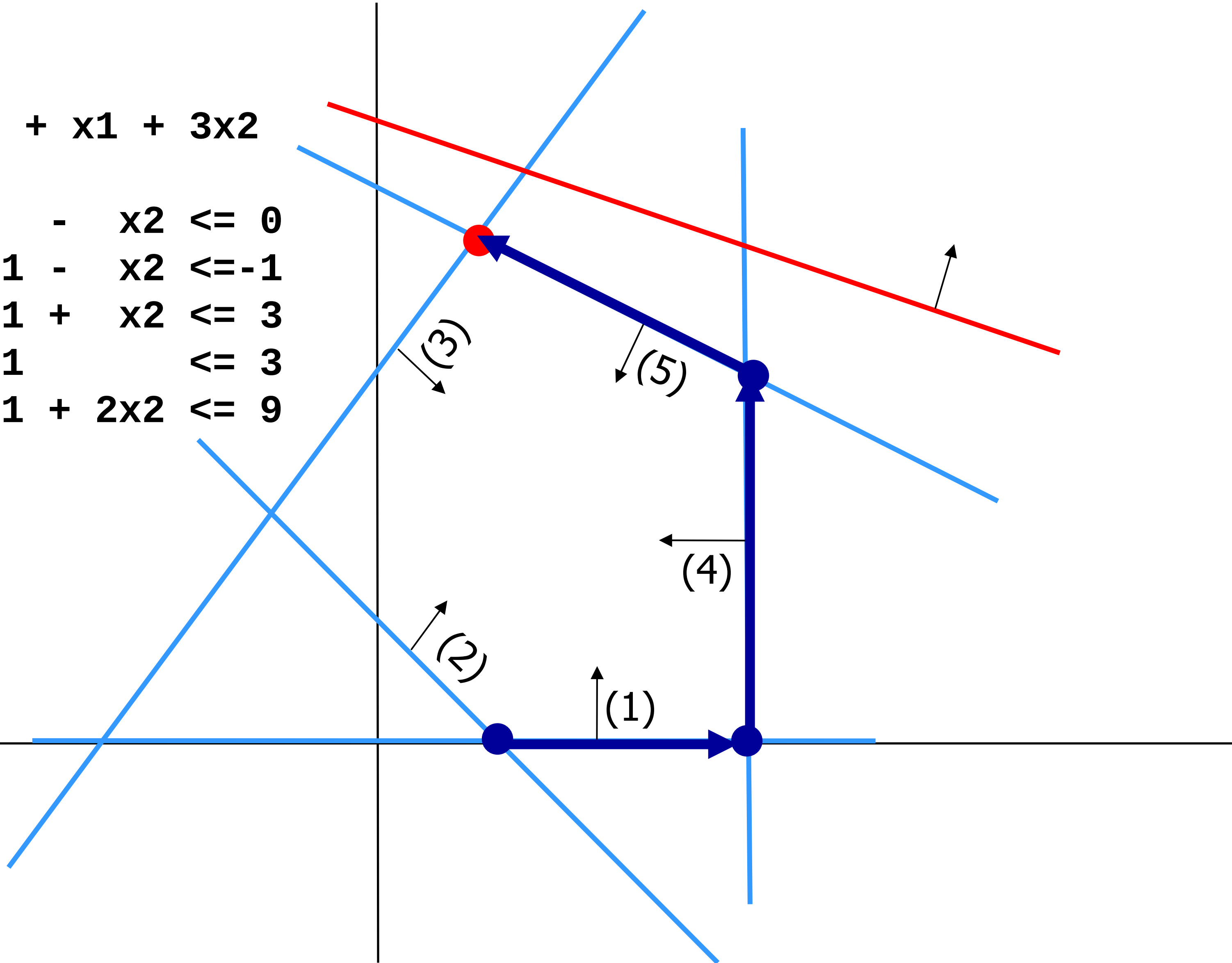

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Hirsch Conjecture

If P is a polytope of dimension n with m facets then every vertex of P can be reached from any other vertex of P on a path of length at most m-n.

In the example before: m=5, n=2 and m-n=3, conjecture is true.

At present, not even a polynomial bound on the path length is known.

Best upper bound: Kalai, Kleitman (1992): The diameter of the graph of an n-dimensional polyhedron with m facets is at most $m^{(\log n+1)}$.

Lower bound: Holt, Klee (1997): at least m-n (m, n large enough).

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Computationally important idea of the Simplex Method

Let a (m,n)-Matrix A with full row rank m, an m-vector b and an n-vector c with m<n be given. For every vertex y of the polyhedron of feasible solutions of the LP,

$$\max c^T x$$
$$Ax = b$$
$$x \geq 0$$

A = | B | N |

there is a non-singular (m,m)-submatrix B (called basis) of A representing the vertex y (basic solution) as follows

$$y_B = B^{-1}b, \quad y_N = 0$$

Many computational consequences:
Update-formulas, reduced cost calculations, number of non-zeros of a vertex,…

You will hear a lot about that in other lectures.

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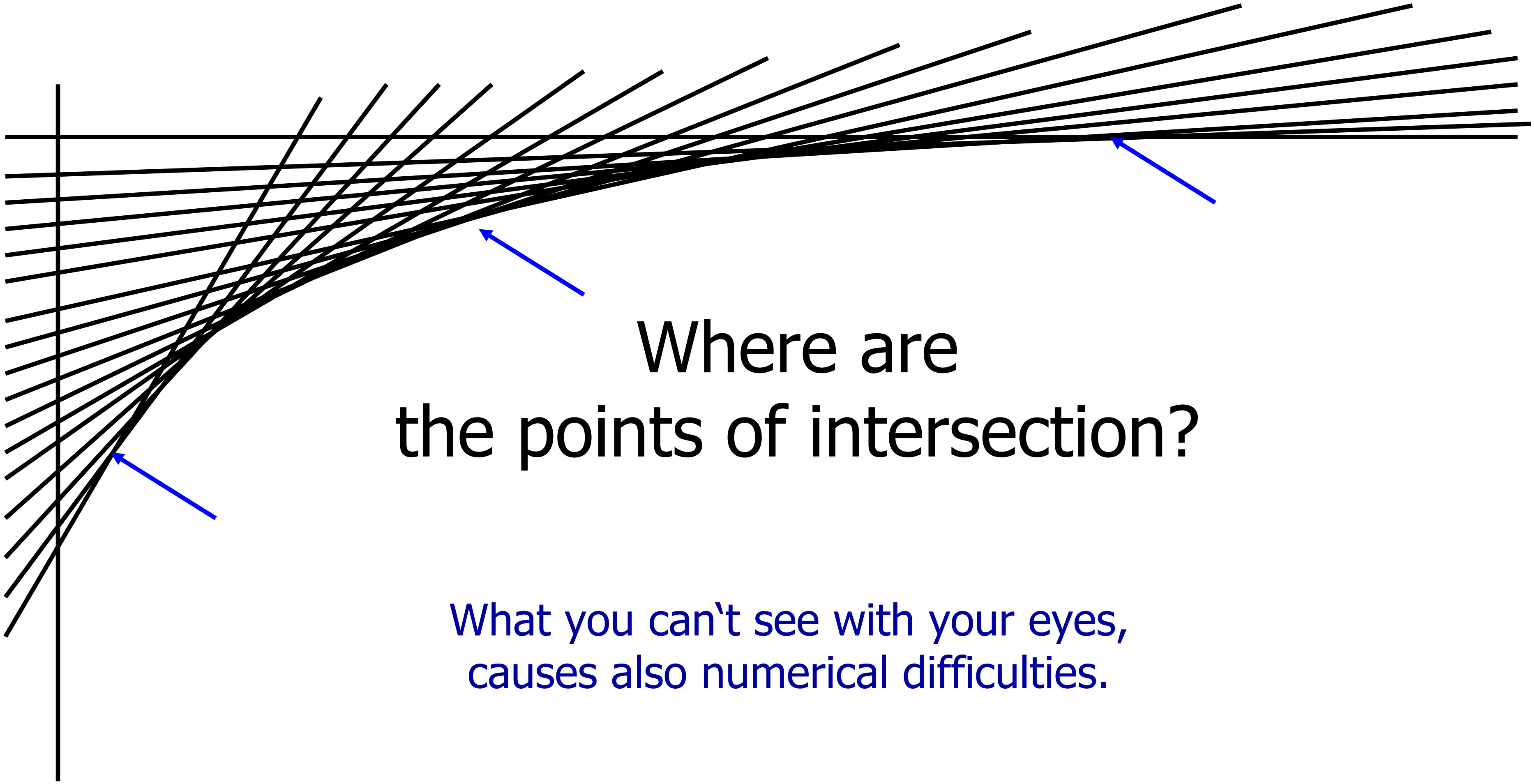
Numerical trouble often has geometric reasons



Where are
the points of intersection?

What you can't see with your eyes,
causes also numerical difficulties.

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The Ellipsoid Method

- Shor, 1970 - 1979
- Yudin & Nemirovskii, 1976
- Khachiyan, 1979
- M. Grötschel, L. Lovász, A. Schrijver,
  Geometric Algorithms and Combinatorial Optimization
  Algorithms and Combinatorics 2, Springer, 1988

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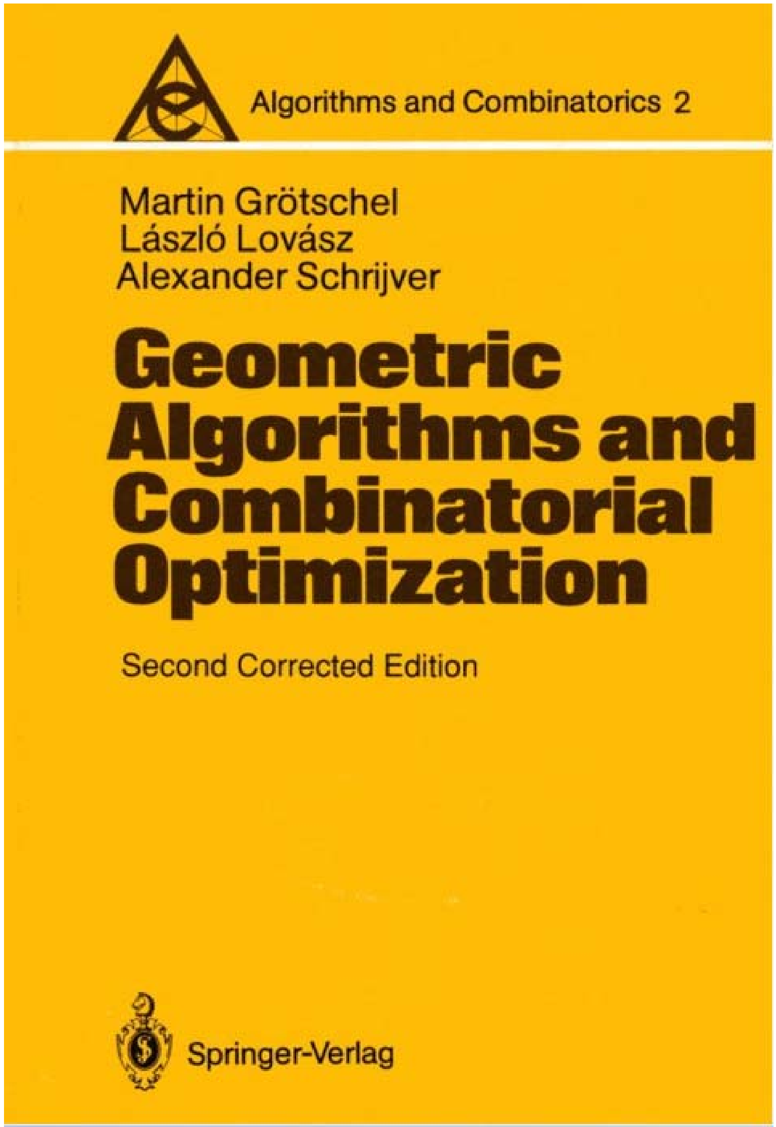
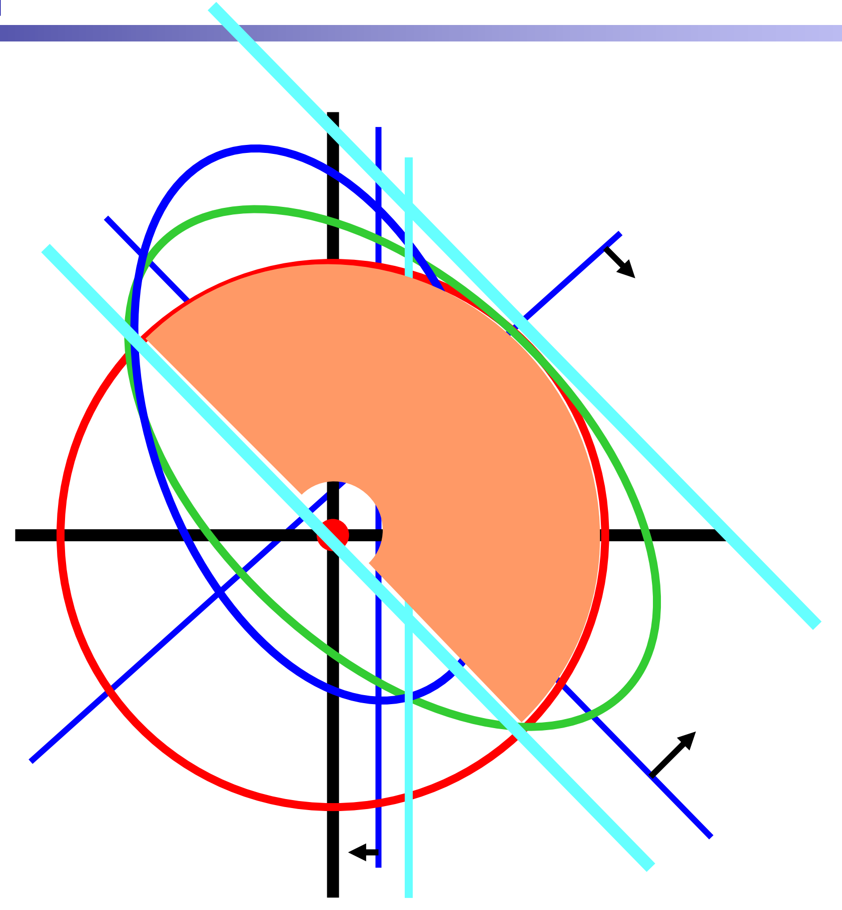
You can download this book from the publications list on my Web page.



http://www.zib.de/groetschel/pubnew/paper/groetschellovaszschrijver1988.pdf

The Ellipsoid Method: an example



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$$k := 0,$$
$$N := 2n((2n+1)\langle C\rangle + n\langle d\rangle - n^3)$$
$$A_0 := R^2 I \quad \text{with } R := \sqrt{n}\, 2^{\langle C,d\rangle - n^2}$$

$$P := \{x \mid Cx \le d\}$$

Initialization

$$a_0 := 0$$

Stopping criterion

If $k = N$, *STOP!* (Declare P empty.)

Feasibility check

If $a_k \in P$, *STOP!* (A feasible solution is found.)

Cutting plane choice

If $a_k \notin P$, *then choose an inequality, say* $c^T x \le \gamma$, *of the system* $Cx \le d$ *that is violated by* a_k.

$$b := \frac{1}{\sqrt{c^T A_k c}} A_k c$$

Update

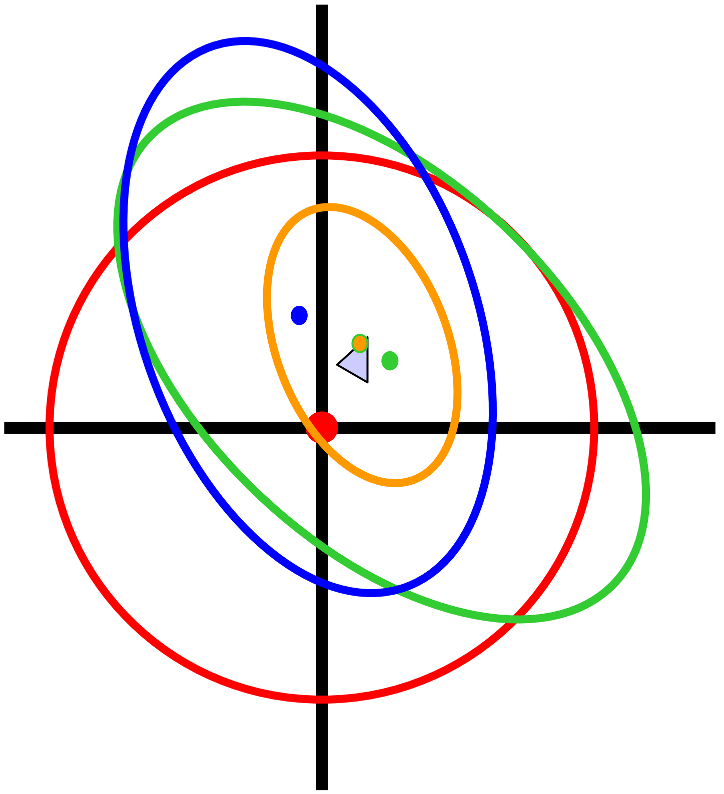
$$a_{k+1} := a_k - \frac{1}{n+1} b$$

$$A_{k+1} := \frac{n^2}{n^2-1}\left(A_k - \frac{2}{n+1} bb^T\right)$$

The Ellipsoid Method

Ellipsoid Method

CO@W



a(0)

a(1)

a(2)

a(7)

feasible solution found

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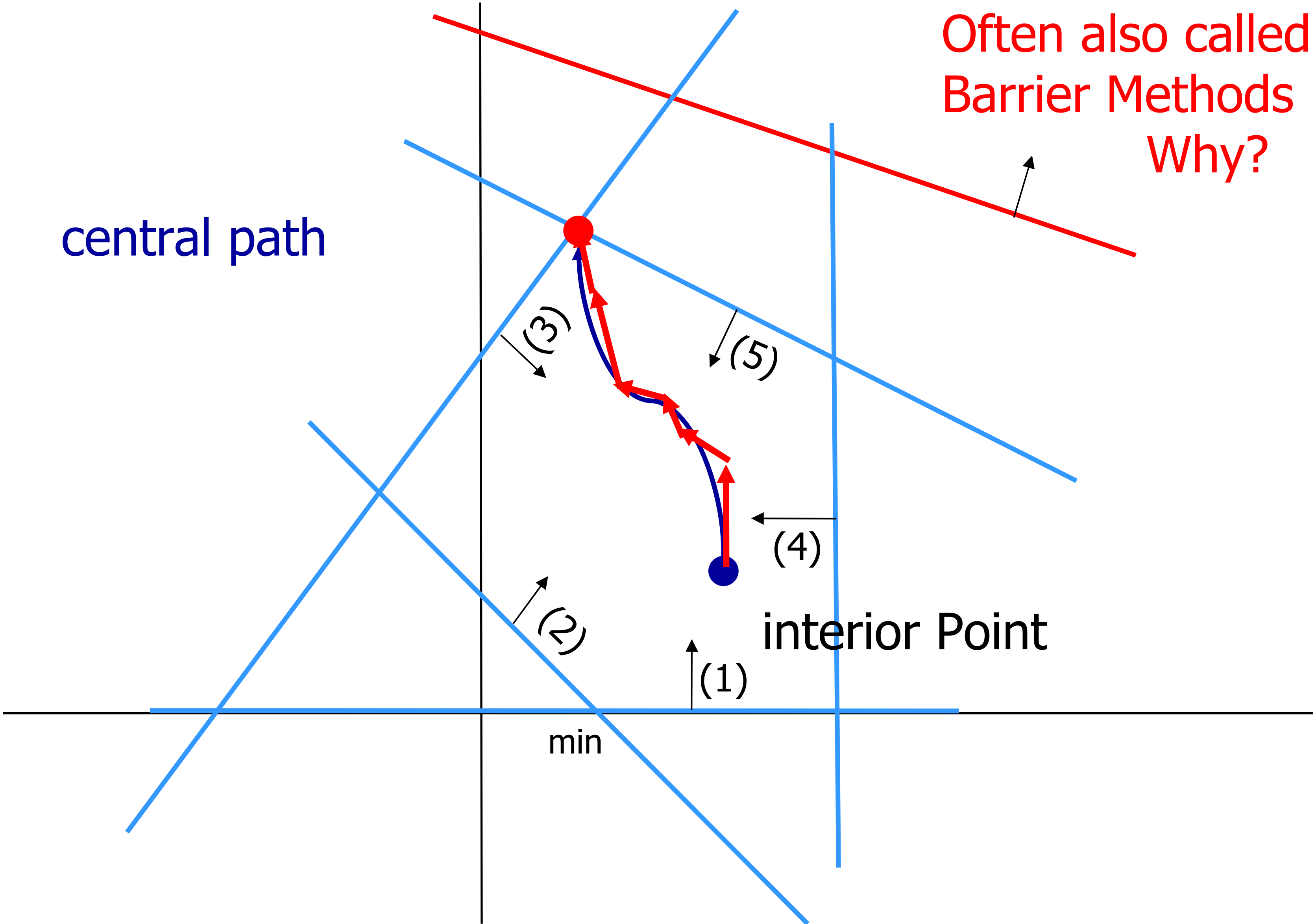
Interior-Point Methods: an example

Often also called
Barrier Methods
Why?



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Other Approaches

- Lagrangean Relaxation
  (for very large scale and structured LPs)
- plus
  - subgradient
  - bundle
  - bundle trust region

  or any other nondifferentiable NLP method that looks promising

Lagrange relaxation, bundle methods, and their use in IP will be explained in detail in other lectures for special IPs.

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special „simple“ combinatorial optimization problems

Finding a

- minimum spanning tree
- shortest path
- maximum matching
- maximal flow through a network
- cost-minimal flow
- …

solvable in polynomial time by special purpose algorithms

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Dijkstra algorithm for shortest paths


```
function Dijkstra(Graph, source):
    for each vertex v in Graph:           // Initializations
        dist[v] := infinity               // Unknown distance function from source to v
        previous[v] := undefined          // Previous node in optimal path from source
    dist[source] := 0                     // Distance from source to source
    Q := the set of all nodes in Graph
    // All nodes in the graph are unoptimized - thus are in Q
    while Q is not empty:                 // The main loop
        u := vertex in Q with smallest dist[]
        if dist[u] = infinity:
            break                         // all remaining vertices are inaccessible from source
        remove u from Q
        for each neighbor v of u:         // where v has not yet been removed from Q.
            alt := dist[u] + dist_between(u, v)
            if alt < dist[v]:             // Relax (u,v,a)
                dist[v] := alt
                previous[v] := u
    return previous[]
```

http://en.wikipedia.org/wiki/Dijkstra's_algorithm

You will learn a lot more about shortest paths with side constraints in this course since such problems have to be solved in subroutines of cutting plane algorithms.

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Special "hard" combinatorial optimization problems

- travelling salesman problem
- location und routing
- set-packing, partitioning, -covering
- max-cut
- linear ordering
- scheduling (with a few exceptions)
- node and edge colouring
- …

NP-hard (in the sense of complexity theory)

The most successful solution techniques employ linear programming.

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4. Algorithms for the solution of linear programs
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   2) The Simplex Method
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The Branch&Bound Technique: An Example

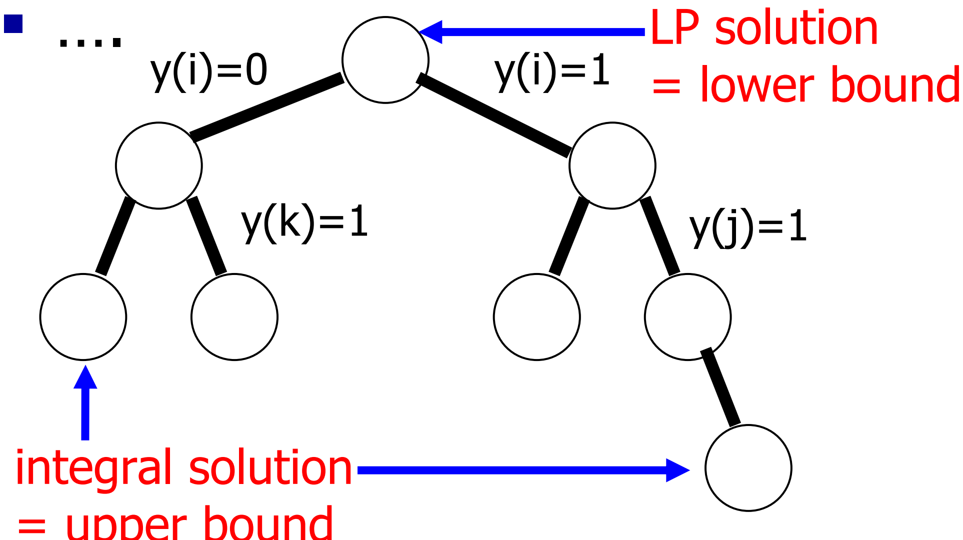
$$\min c^T x$$
$$Ax = a$$
$$Bx \leq b$$
$$x \geq 0$$
$$x \in \{0,1\}^n$$

0/1-program

$$\min c^T x$$
$$Ax = a$$
$$Bx \leq b$$
$$x \geq 0$$
$$\sout{x \in \{0,1\}}^n$$
$$x \leq 1$$

LP-relaxation

- Solve the LP-relaxation and get optimal solution y. (lower bound)
- If y integral, DONE!
- Otherwise pick fractional component y(i).
- Create two new subproblems by adding y(i)=1 and y(i)=0, resp.
- ....



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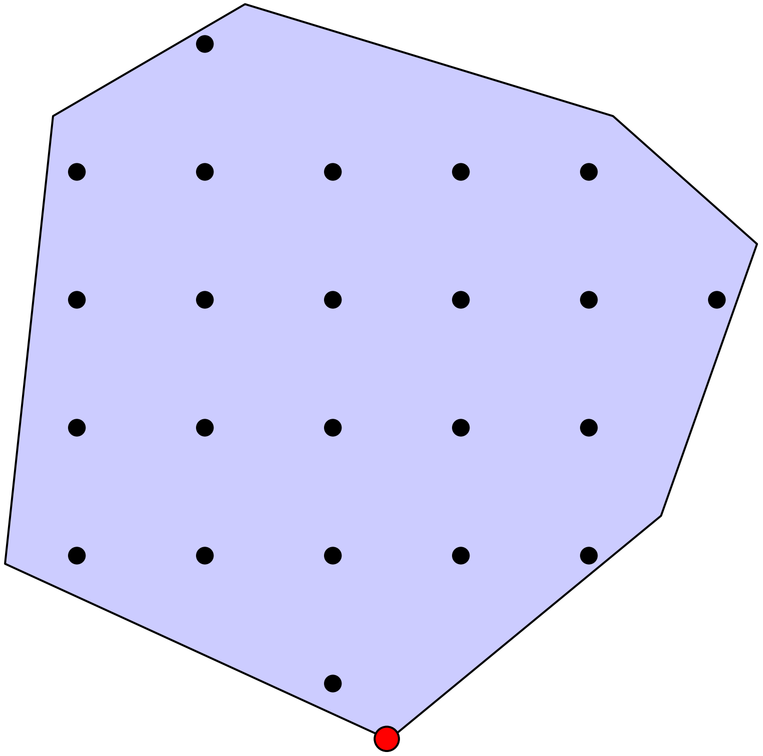
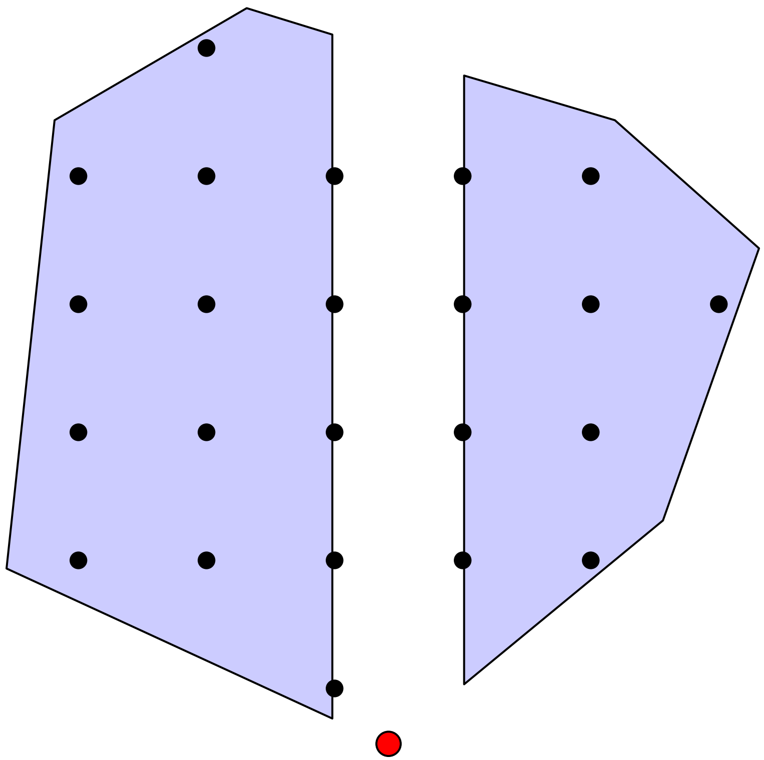
Branching (in general)



- Current solution is infeasible

Branching (in general)

- Rounding a fractional component up and down



- Decomposition into subproblems removes infeasible solution

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A Branching Tree

Applegate

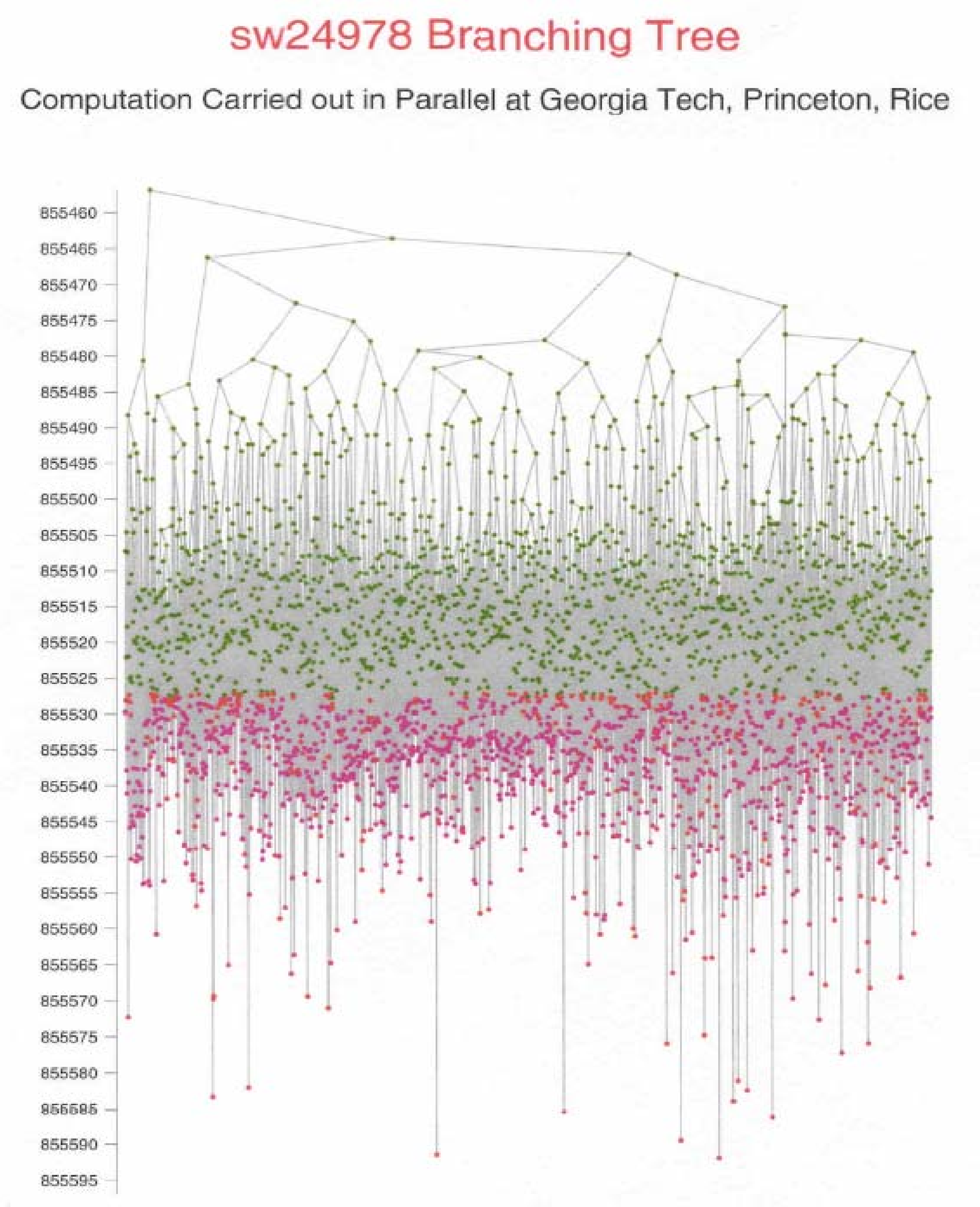
Bixby

Chvátal

Cook

tree copied from

www1.ctt.dtu.dk/ROUTE2003/presentations/cook.pdf



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Cutting plane technique for integer and mixed-integer programming

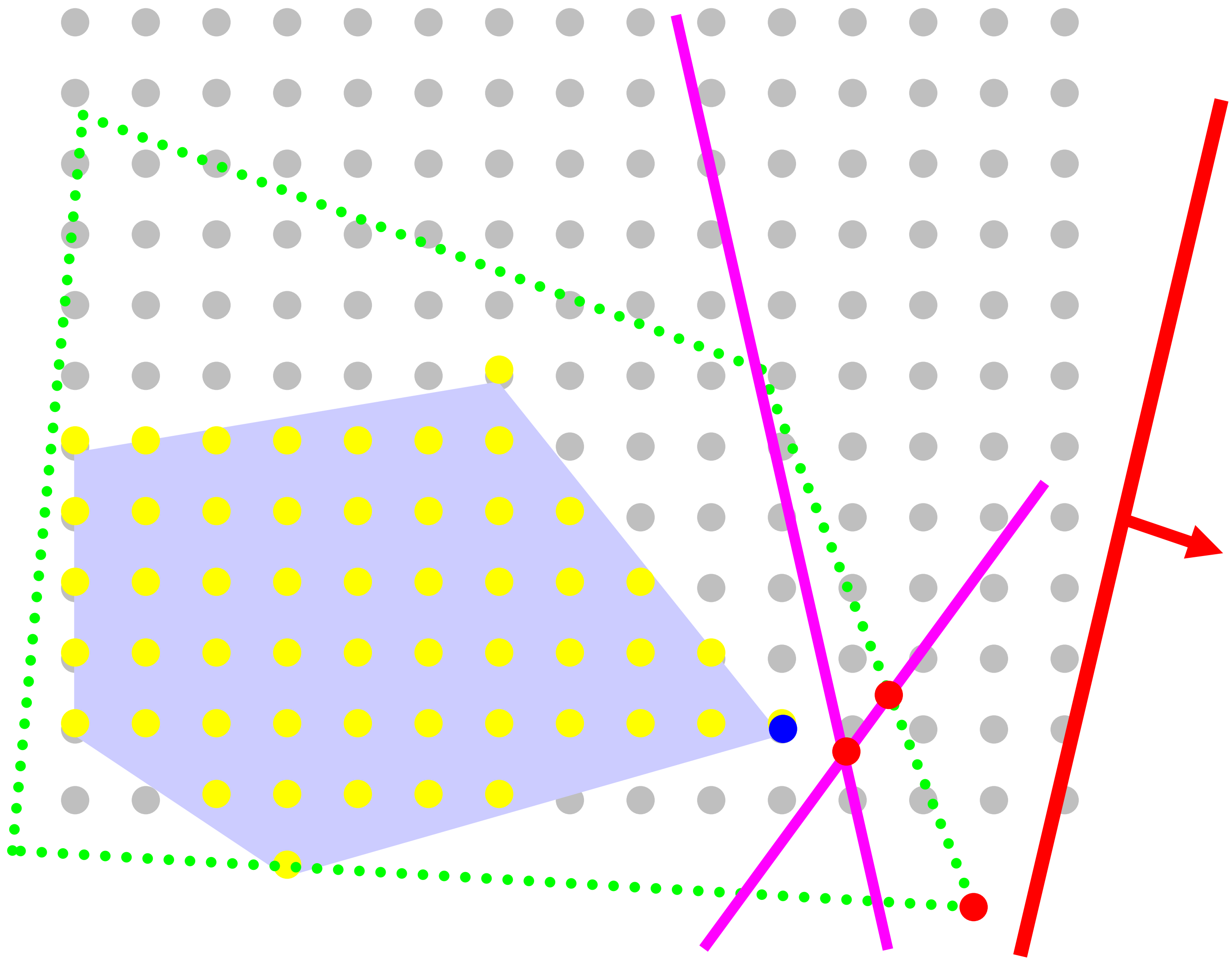
Feasible integer solutions

Objective function

Convex hull

LP-based relaxation

Cutting planes



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Other Names

- Branch & Cut
- Branch & Price
- Branch & Cut & Price
- etc.

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State of the art

- You will learn the state of the art in this course by the world's topmost experts.
- It makes no sense to summarize the state now. Watch out for the lectures on SCIP,

Tu 22.09.		**Introduction to the ZIB Optimization Suite**
09:00-10:30	CR,AW	Porta and Polymake
11:00-12:30	IPG	Basic concepts of SCIP
14:00-15:30	IPG	The ZIB Optimization suite: SCIP, Soplex, Zimpl
16:00-16:30	IPG	Real world data
16:30-17:30	IPG	Refreshing C for SCIP programmers

- Bixby's lectures and the lectures on "commercial solvers day":

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State of the art

Th 24.09.		**Advanced Linear Programming**
09:00-10:30	BB	Solving LPs in practice
11:00-12:30	BB	
14:00-15:30	BB	
16:00-17:30	BB	
Fr 25.09.		**Advanced Mixed Integer Programming**
09:00-10:30	BB	Solving MIPs in practice
11:00-12:30	BB	
14:00-15:30	BB	
16:00-17:30	BB	
Tu 29.09.		**(Commercial) Solvers: state-of-the-art**
09:00-09:15	HM	Introduction
09:15-10:00	BB	Gurobi
10:00-10:45	TA	CPLEX
11:15-12:00	OB	Xpress
12:00-12:45	EA	MOSEK and Interior Point
14:15-15:00	TR	COIN-OR
15:30-16:15	EA	Conic Quadratic Optimization
16:30-17:30	MH	Integer Programming at Siemens

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Linear and Integer Programming: an Introduction

CO@W Berlin

The End

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