MIP Heuristics
Why not wait for branching?

• Produce feasible solutions as quickly as possible
  • Often satisfies user demands
  • Avoid exploring unproductive subtrees
  • Better reduced-cost fixing

• Avoid “tree pollution”
  • Good fixings in a heuristic are often not good branches
CPLEX Heuristics

Two classes

- **Plunging heuristics:**
  - Maintain linear feasibility
  - Try to achieve integer feasibility

- **Local improvement heuristics:**
  - Maintain integer feasibility
  - Try to achieve linear feasibility
Plunging Heuristic Structure

• Fix a set of integer infeasible variables
  • Usually by rounding
• Perform bound strengthening to propagate implications
• Solve LP relaxation
• Repeat
• Methods for choosing fixings:
  • Non-basic variables
    • Sorted in order of increasing $d_j$
  • Fractional variables
    • Sorted in order of increasing distance to an integer in relaxation
How many variables to fix per round:

- All of them?
  - Inexpensive; no need to solve LP relaxations
  - But ‘flying blind’ after a few fixings
    - Bound strengthening helps

- A few?
  - More expensive
  - LP relaxation can guide later choices
    - (variable values, reduced costs, etc.)

In what order are variables fixed?

- Variations useful for diversification
Local Improvement Heuristics

High-level structure

- Choose integer values for all integer variables
  - Produces linear infeasibility
- Iterate over integer variables:
  - Does adding/subtracting 1 reduce linear infeasibility?
- Infeasibility metrics:
  - Primary: number of violated constraints
  - Secondary: |b-Ax|
Local Improvement Details

• What initial values to assign to integer variables?
  • Rounded relaxation values
  • Bounds

• What to do when local improvement gets stuck?
  • Reverse infeasibility metrics
General Heuristic Strategies

Apply 11 different variations

- Apply all heuristics before beginning the branch and bound search
- Apply the least expensive heuristics after every round of root cutting planes
- Apply them every 10 nodes in the MIP tree
- Decrease the frequency of a particular heuristic when it is not finding new feasible solutions
Heuristic Results

Effectiveness

• Feasible solution found for most models before branch and bound begins

• Roughly 10% improvement in time to proven optimality (978 model test set)

• Often finds solutions branching does not
Combining Local Search and MIP Heuristics to Solve Very Difficult MIP Models
Local Search for MIP

Relaxation Induced Neighborhood Search (RINS)

• New local search heuristic in CPLEX
• Local search uses *neighborhoods* to improve a given solution
  • Neighborhoods generally based on problem structure
    • Example: Nodes and edges in a graph
  • No high level structural information available in an arbitrary MIP model
Local Search for MIP

Basic Approach

• Form sub-MIP from variables whose values differ in incumbent and relaxation

• Combine desirable properties of two solutions:
  • Incumbent: feasible
  • Relaxation: optimal

• Neighborhood contains both solutions

• Extend promising partial solution
### Local Search for MIP

**Example: RINS = off**

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<tr>
<th>Node</th>
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<th>Objective</th>
<th>IInf</th>
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<th>Best Node</th>
<th>ItCnt</th>
<th>Gap</th>
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## Local Search for MIP

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