The MIP Landscape

Progress: MIP

Example 1: LP still can be HARD

SGM: Schedule Generation Model

157323 rows, 182812 columns, 6348437 nzs

- **LP relaxation at root node:**
  - **18 hours**
- **Branch-and-bound**
  - **1710 nodes, first feasible**
  - **3.7% gap**
  - **Time: 92 days!!**
- **MIP does not appear to be difficult: *LP is a roadblock* (but 1000x LP improvement would make "solvable" in 2 hours!)**

Progress: MIP

Example 2: MIP really is HARD

A customer model: 44 cons, 51 vars, 167 nzs, maxi
51 general integers vars (*no bou*

Branch-and-cut: Initial upper bound -1379.
Initial integer solution -2186.0
…after 1.2 days, 970,000,000 B&C nodes, 79 Gig tr
Integer solution and bound: UNCHANGED

What's wrong?

Example 2: Here's what's wrong

```
Maximize
  x + y + z
Subject To
  2 x + 2 y ≤ 1
  z = 0
  x free, y free
  x,y integer
```

Note: This problem can be solved in several ways

- Removing z=0, objective is integral [*Presolve*]
- Euclidean reduction on the constraint [*Presolve*]

However: Branch-and-bound cannot solve!

Progress: MIP

Changing the rules of business™

Example 3: Supply-chain scheduling

- **Model description:**
  - Weekly model, daily buckets: Objective to minimize end-of-day inventory.
  - Production (single facility), inventory, shipping (trucks), wholesalers (demand known)
- **Initial modeling phase**
  - Simplified prototype + complicating constraints (production run grouping req't, min truck constraints)
  - RESULT: Couldn't get good feasible solutions.
- **Decomposition approach**
  - Talk to current scheduling team: They first decide on "producibles" schedule. Simulate using Constraint Programming.
  - Fixed model: Fix variables and run MIP

Progress: MIP

Example: Supply-chain scheduling (cont.)

CPLEX 5.0:

```
Integer optimal solution (0.0001/0):  Objective =    1.5091900536e+05
Current MIP best bound =    1.5090391809e+05 (gap = 15.0873)
Solution time = 3465.73 sec.  Iterations = 7885711  Nodes = 489870 (2268)
```

CPLEX 9.0:

```
Implied bound cuts applied:  55
Flow cuts applied:  200

Integer optimal solution (0.0001/1e-06):  Objective =    1.5091904146e+05
Current MIP best bound =    1.5090843265e+05 (gap = 10.6088, 0.01%)
Solution time =    0.90 sec.  Iterations = 2418  Nodes = 16 (3)
```

Original model: Now solves to optimality in 40 minutes (20% improvement in solution quality)

Progress: MIP

Computational history: 1950 – 1998

- 1954 Dantzig, Fulkerson, S. Johnson: 42-city TSP
  - Solved to optimality using cutting planes and LP
- 1957 Gomory
  - Cutting plane algorithm: A complete solution
- 1960 Land, Doig; 1965 Dakin
  - Branch-and-bound (B&B)
- 1971 MPSX/370, Benichou et al.
- 1972 UMPIRE, Forrest, Hirst, Tomlin (and Beale)

- 1972 – 1998 Good B&B remained the state-of-the-art in commercial codes, in spite of
  - 1973 Padberg
  - 1974 Balas (disjunctive programming)
  - 1983 Crowder, Johnson, Padberg: PIPX, pure 0/1 MIP
  - 1987 Van Roy and Wolsey: MPSARX, mixed 0/1 MIP
  - Grötschel, Padberg, Rinaldi …TSP (120, 666, 2392 city models solved)

Progress: MIP

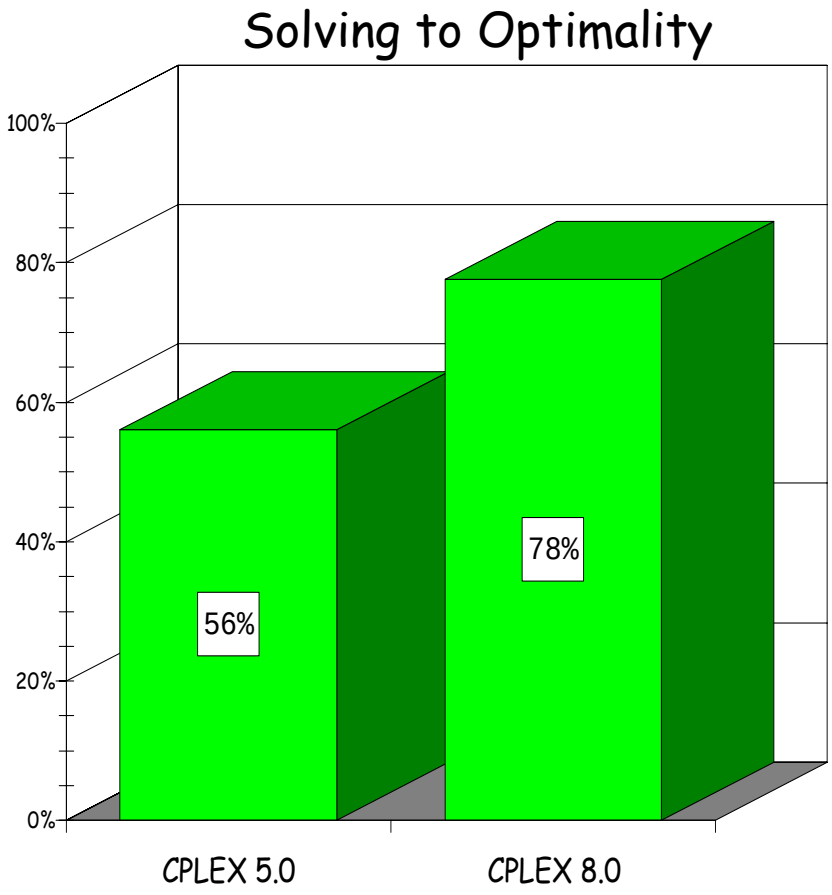
1998 … A New Generation of MIP Codes

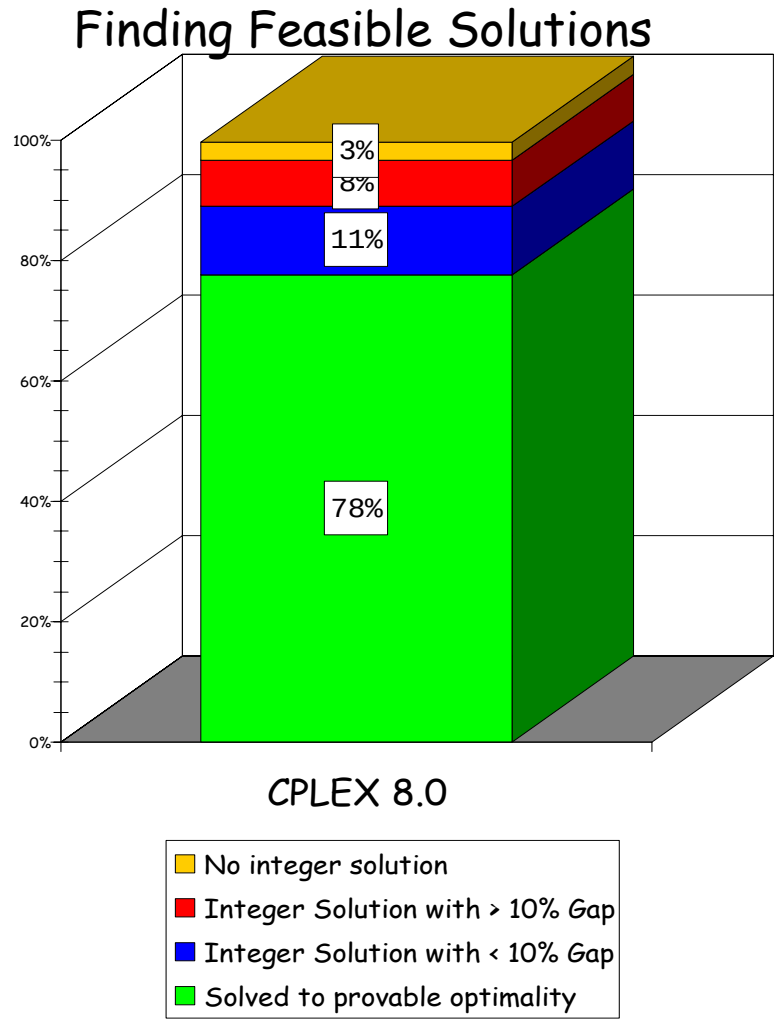
- Linear programming
  - Stable, robust dual simplex
- Variable/node selection
  - Influenced by traveling salesman problem
- Heuristics
  - 11 different tried at root
  - Retried based upon success
- Presolve
  - Numerous small ideas
- Cutting planes
  - **Gomory,** knapsack covers, flow covers, mix-integer rounding, cliques, GUB covers, implied bounds, path cuts

Difficulty of Solving a MIP

Changing the rules of business™

978 models, 30 hour time limit





Progress: MIP

Computational Results II: 758 Models

(all solvable to optimality)

- **Ran for 30 hours using defaults**
- **Relative speedups (versus CPLEX 8.0)**
  - **All models (758):** 12x
  - **CPLEX 5.0 > 1 second (551):** 33x
  - **CPLEX 5.0 > 10 seconds (463):** 59x
  - **CPLEX 5.0 > 100 seconds (375):** 97x
  - **CPLEX 5.0 > 1000 seconds(294):** 191x
- *CPLEX 8.0 versus CPLEX 9.0: 1.5x*