09M2

Combinatorial Auctions and Rail Track Scheduling

Ralf Borndörfer

joint work with

Martin Grötschel   Sascha Lukac   Thomas Schlechte

Block Course at TU Berlin
"Combinatorial Optimization at Work"

October 4 – 15, 2005

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- DFG-Forschungszentrum MATHEON "Mathematics for key technologies"
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)
- Löbel, Borndörfer & Weider GbR (LBW)

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Outline

- Auctions
- Rail Track Scheduling
- Rail Track Auctioning
- The Optimal Track Allocation Problem
- Experiments

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Auctions

- Commodities/Bids
  - Independent commodities (classical autcion)/ commodity bundles (combinatorial auction)
  - Combinatorial bids (and/or/xor)
- Bidders
  - Cooperation forbidden/ cooperation allowed
- Payment
  - First price/second price (Vickrey) auction
- Information
  - Private Values/Common Values (winner's curse)
  - Sealed Bid/Open Bid
- Mechanism
  - English auction/dutch auction
  - Increment/number of rounds
  - Activity rules/taking bids back
  - Direct bidding/clock/proxy auction

Examples

- **In ancient times ...**
  - Auctions are known since 500 b.c.
  - March 28, 193 a.d.: The pretorians auction the Roman Emperor`s throne to Marcus Didius Severus Iulianus, who ruled as Iulianus I. for 66 days

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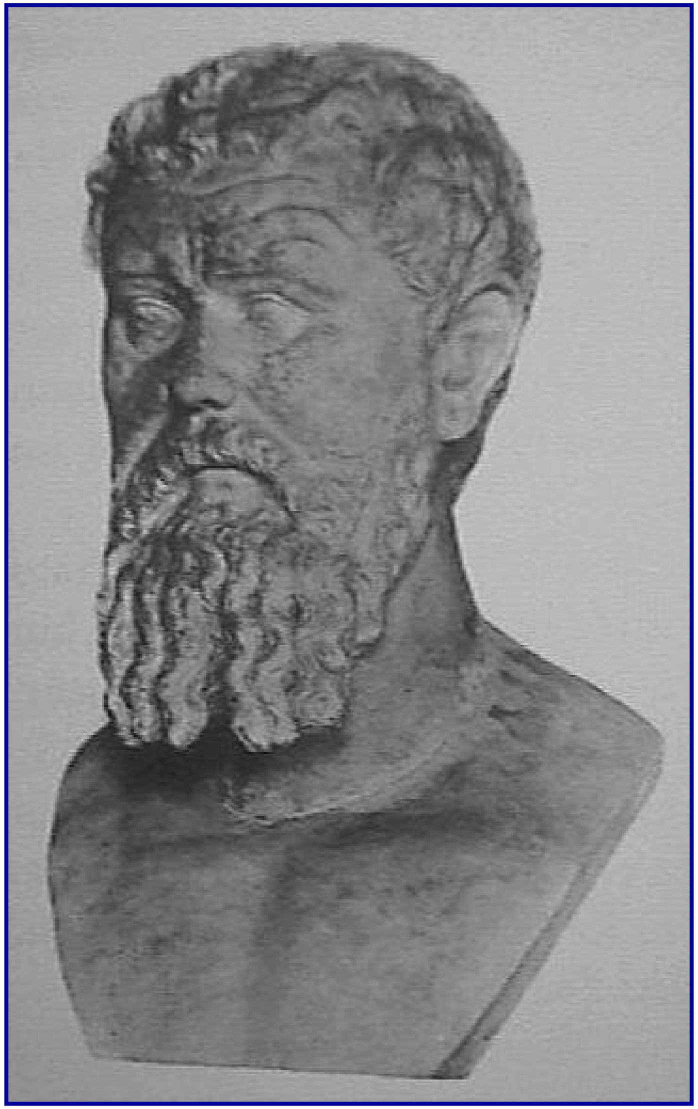
The Story of Didius Iulianus

(http://www.roman-emperors.org/didjul.htm)



[193 A.D., March 28] When the emperor **Pertinax** was killed trying to quell a mutiny, no accepted successor was at hand. **Pertinax's** father-in-law and urban prefect, Flavius Sulpicianus, entered the praetorian camp and tried to get the troops to proclaim him emperor, but he met with little enthusiasm. Other soldiers scoured the city seeking an alternative, but most senators shut themselves in their homes to wait out the crisis. **Didius Julianus**, however, allowed himself to be taken to the camp, where one of the most notorious events in Roman history was about to take place. **Didius Julianus** was prevented from entering the camp, but he began to make promises to the soldiers from outside the wall. Soon the scene became that of an auction, with Flavius Sulpicianus and **Didius Julianus** outbidding each other in the size of their donatives to the troops. The Roman empire was for sale to the highest bidder. When Flavius Sulpicianus reached the figure of 20,000 sesterces per soldier, **Didius Julianus** upped the bid by a whopping 5,000 sesterces, displaying his outstretched hand to indicate the amount. The empire was sold, **Didius Julianus** was allowed into the camp and proclaimed emperor.

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Examples

- **In ancient times ...**
  - Auctions are known since 500 b.c.
  - March 28, 193 a.d.: The pretorians auction the Roman Emperor's throne to Marcus Didius Severus Iulianus, who ruled as Iulianus I. for 66 days
- **In modern times ...**
  - Traditional auctions (antiques, flowers, stamps, etc.)
  - Stock market
  - eBay etc.
  - Oil drilling rights, energy spot market, etc.
  - Procurement
  - Sears, Roebuck & Co.
  - Frequency auctions in mobile telecommunication
  - Regional monopolies (franchising) at British Rail

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Arguments for Auctions

- Auctions can …
  - resolve user conflicts in such a way that the bidder with the highest willigness to pay receives the commodity (efficient allocation, wellfare maximization)
  - maximize the auctioneer's earnings
  - reveal the bidders' willigness to pay
  - reveal bottlenecks and the added value if they are removed
- Economists argue …
  - that a "working auctioning system" is usually superior to alternative methods such as bargaining, fixed prices, etc.

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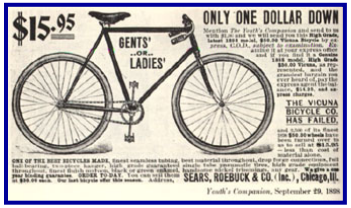
Sears, Roebuck & Co.



- 3-year contracts for transports on dedicated routes
- First auction in 1994 with 854 contracts
- Combinatorial auction
  - „And-“ and „or-“ bids allowed
  - 2^{854} ($\approx 10^{257}$) theoretically possible combinations
  - Sequential auction (5 rounds, 1 month between rounds)
- Results
  - 13% cost reduction
  - Extension to 1.400 contracts (14% cost reduction)

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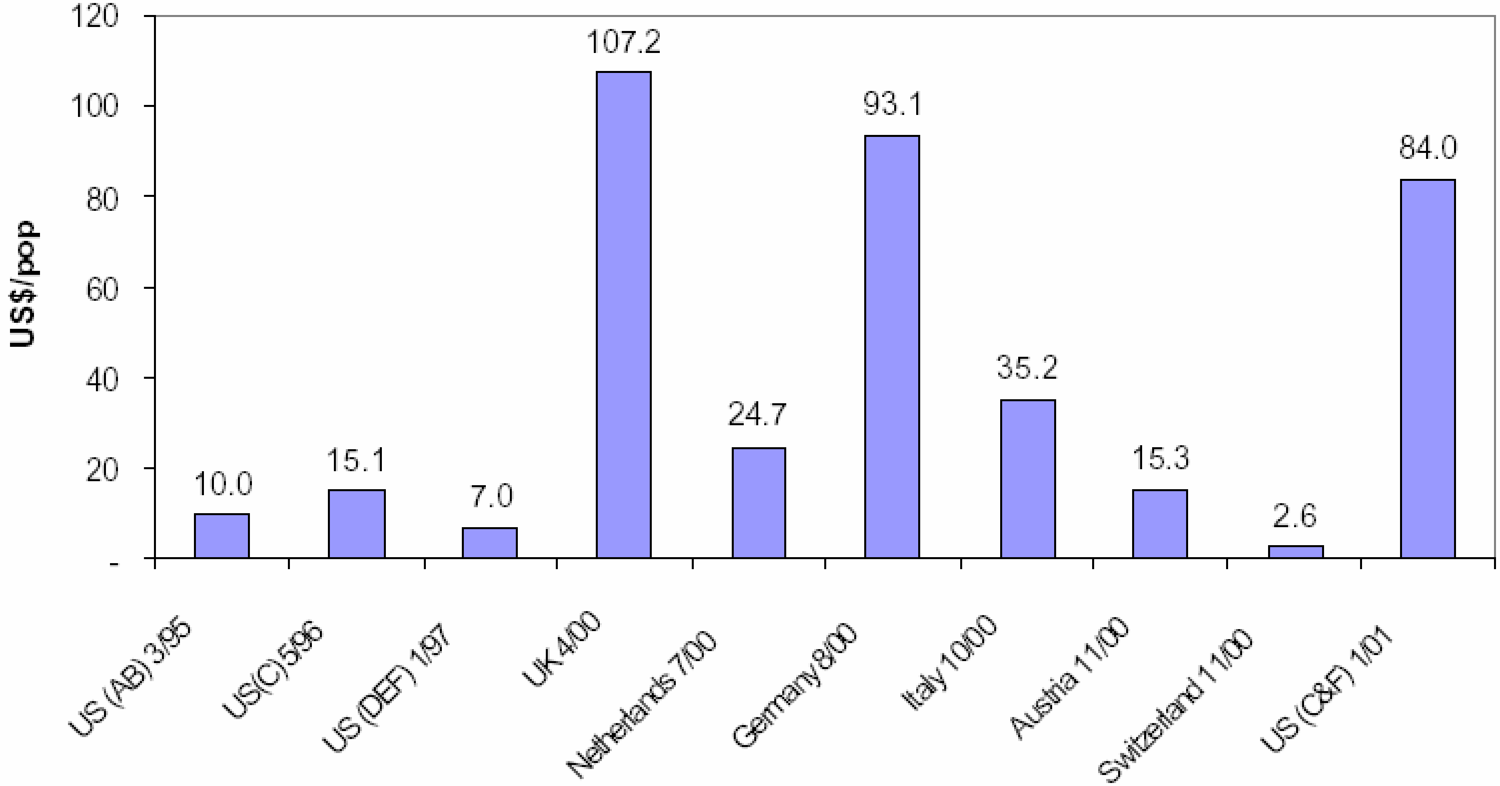
Frequency Auctions

(Cramton 2001, Spectrum Auctions, *Handbook of Telecommunications Economics*)



- Prices for mobile telecommunication frequencies (2x10 MHz+5MHz)
- Low earnings are not per se inefficient
- Only min. prices => insufficient cost recovery

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Basic Auctions

- **Revenue Equivalence Theorem (Vickrey):**
  - Risk neutral bidders i=1,..,n
  - Private values $v_i \in [l,u]$ i.i.d. with strictly monotonously increasing distribution function $F(v) = P(v_i \leq v)$
  - Every auction mechanism in which
    - the object is given to the bidder w.t. highest bid
    - a bidder with the lowest possible bid l expects no profit

    results in the same revenue.
  - Bids $b(v) = v - \int_l^u F^{n-1}(x)dx \Big/ F^{n-1}(v)$

Game Theory

- Game (N,S,a)
  - $N=\{1,\ldots,n\}$ player
  - $S=\{(s_1,\ldots,s_n)\}$ strategies
  - $a:S\rightarrow R^n$ payoff
- Non-cooperative games
  - Dominance
  - (Nash-)Equilibrium $\hat{s}$

    $a_i(\hat{s}_1,..,s_i,..,\hat{s}_n)\leq a_i(\hat{s}_1,..,\hat{s}_n)\ \forall i$

    (i.g. no existence/uniqueness)
  - Matrix games: saddle point, minimax
- **Theorem (Nash):** Every finite non-cooperative n-person game has at least one equilibrium of mixed strategies.
- **Theorem (Nikaido, Isoda):** Generalization to auction frameworks.
- Cooperative games
  - Imputation (payoff to members of a coalition)
  - Concepts such as core, stable set, bargaining set, kernel, nucleolus, etc.

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Combinatorial Auction

- Combinatorial Auction Problem (CAP)
  - M objects, N bidders, $b^j(S)$ bid by j for $S \subseteq M$
  - y(S,j) 0/1-variable for giving S to j

$$\max \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S,j)$$

$$\sum_{S \ni i} \sum_{j \in N} y(S,j) \leq 1 \quad \forall i \in M$$

$$y(S,j) \in \{0,1\} \quad \forall S \subseteq M, j \in N$$

  - Set Packing Problem
- Auction framework

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Simultaneous Ascending Auction

- Rules
  - Multiple heterogeneous objects
  - Combinatorial auction, but only individual bids
  - First price sealed bid
  - N rounds, minimum increment
  - Activity rule #objects
  - Fee for taking back
- Empirical efficiency 67%
- High revenues, partly due to losses for bidders

- Equilibrium

	A	B	AB
1	4	6	9
2	4	5	7
P	4	6	

- Exposure problem

	A	B	AB
1	2	2	7
2	4	4	6
P	?	?	

- Efficiency

$$\sum v_i(\bar{y}_i) \Big/ \sum v_i(y_i^*)$$

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Simultaneous Ascending Auction

- Auction #1 (USA 1994)
  - 10 licenses
  - 3 national bandwidths
  - Paging/messaging services
  - ≤3 licenses/bidder
  - Increment 2%
  - 47 rounds (1 week)
  - 617 Mio. USD

    (50 Mio. USD expected)

- Auction #4 (USA 1994)
  - 99 licenses
  - 2 bandwidths, 51 MTAs
  - Mobile telephone services
  - Increment 5%
  - 112 rounds (3 months)
  - 7.000 Mio. USD

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Adaptive User Selection Mechanism

- Rules
  - Several heterogeneous objects
  - Combinatorial bids
  - First price open bid
  - Continuos bidding
  - No activity rule
  - Auctioneer determines end
- Empirical efficiency 94%
- Complexity with bidders, lower revenues than SAA

- Threshold problem

	A	B	AB
1	7	2	8
2	2	7	8
3	4	4	10
P	?	?	10

- Proposal: Standbye Q

	A	B	AB
1	7	2	8
2	2	7	8
3	4	4	10
P	6	?	10

- Free rider problem

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Vickrey Auction

(Nobel price in Economics 1996)

- Combinatorial auction

$$E(N,b) := \max \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S,j)$$
$$\sum_{S \ni i} \sum_{j \in N} y(S,j) \le 1 \quad \forall i \in M$$
$$y(S,j) \in \{0,1\} \quad \forall S \subseteq M, j \in N$$

- Private values v_j
- Mechanism
  - Bids $b_j = v_j$
  - Payments
    $z_j = E(N \backslash j, v) - E(N,v)|N \backslash j$

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Vickrey-Clarke-Groves-Mechanism

- Combinatorial auction

$$E(N,b) := \max \sum_{S \subseteq M} \sum_{j \in N} b^j(S) y(S,j)$$
$$\sum_{S \ni i} \sum_{j \in N} y(S,j) \leq 1 \quad \forall i \in M$$
$$y(S,j) \in \{0,1\} \quad \forall S \subseteq M, j \in N$$

- Private values v_j
- Mechanism
  - Bids $b_j = v_j$
  - Payments
    $z_j = E(N \setminus j, v) - E(N,v)|N \setminus j$

- Example

	A	B	AB
1	10	5	15
2	1	6	12
P	6	5	

- Collusion

	A	B	C	ABC
1				2
2	1			
3		1		
4			1	
P	0	0	0	

- Fraud by auctioneer

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Proxy-Auction

- Combinatorial first price sealed bid auction
- Bids by proxy-agent (program)
- **Theorem (Ausubel, Milgrom):** A proxy-auction, interpreted as a cooperative game, terminates in the core.
- **Theorem (Ausubel, Milgrom):** A proxy-auction, interpreted as a non-cooperative games, terminates under certain conditions in a Nash-equilibrium, in particular, if a corresponding Vickrey-Clarke-Groves-auction terminates in a Nash-equilibrium.
- Combinations with other auctions, e.g., clock-proxy, to simplifiy programming of the agent.

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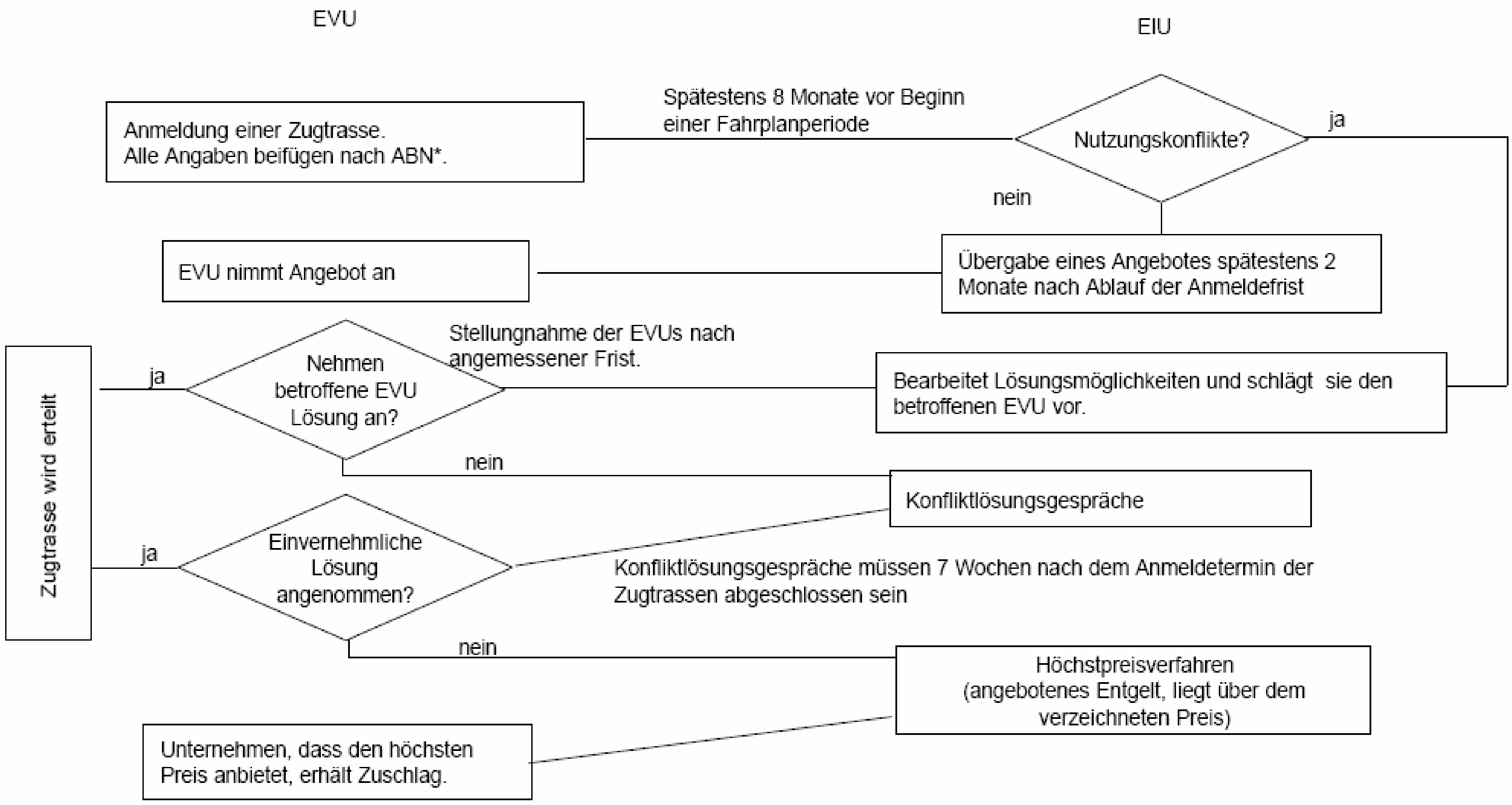
Rail Track Scheduling



*Besondere Daten, z. B. fahrdynamische Daten von Triebfahrzeugen, müssen 14 Tage vor der Trassenanmeldung abgegeben werden.

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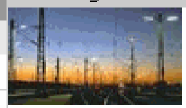
Trassennutzung für den Personen- und Güterverkehr

Hier finden Sie detaillierte Angebote und Preisinformationen zur Nutzung von Trassen der DB Netz AG für den Personen- und Gütertransport. Die zusätzlich angebotene Software unterstützt Sie bei der Kalkulation der Preise für Ihre gewünschte Trasse.

Leistungen

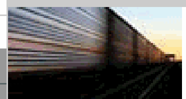


Leistungsangebot der DB Netz AG zur Bereitstellung von Bahninfrastruktur

Aufgabe der DB Netz ist es leistungsfähige Eisenbahninfrastrukturen sowie technische Anlagen und Einrichtungen marktgerecht zur Verfügung zu stellen. Das Leistungsangebot setzt sich aus den Produktfeldern Trassen, Anlagen und Infrastrukturanschlüsse zusammen.

mehr →

Trassen Güterverkehr



Trassen für den Güterverkehr

Als Kunde im Güterverkehr haben Sie die Wahl zwischen mehreren Produkten. Je nach Nutzung wird zwischen Güterverkehrs-Express-Trassen, Güterverkehr-Standard-Trassen, Güterverkehr-Zubringer-Trassen und Güterverkehrs-LZ-Trassen unterschieden.

mehr →

Trassen Personenverkehr

Trassen für den Personenverkehr

Die Personenverkehrs-Trassen lassen sich in vier verschiedene Kategorien einteilen. Als Kunde haben Sie die Wahl zwischen Personenverkehrs-Express-Trassen, Personenverkehrs-Takt-Trassen, Personenverkehrs-Economy-Trassen und Personenverkehrs-LZ-Trassen.

mehr →

Trassenpreise

Gültige Preise ab dem 12.12.2004 und 11.12.2005

Hier finden Sie das seit dem 12.12.2004 gültige und ab dem 11.12.2005 geltende Trassenpreissystem mit seinen Anlagen sowie Streckenkategoriekarten als PDF-Dateien zum Download

Besonderheiten & Fristen

Grundsätzliches zur Trassen-Anmeldung

Bei der Anmeldung von Trassen gibt es Besonderheiten und Fristen, die Sie unbedingt beachten müssen. Alle Informationen zu diesem Thema finden Sie hier.

mehr →

Formulare

Formulare als PDF-Download

Wenn Sie als Eisenbahnverkehrsunternehmen Trassen anmelden möchten, müssen Sie für Ihre Anmeldung bestimmte Formulare verwenden. Diese Formulare nebst Erläuterung finden Sie hier.

mehr →

DB Netz
Deutsche Bahn Gruppe

Trassenanmeldung ☐ / Trassenstudie ☐ / Fahrzeitrechnung ☐ / Preisanfrage ☐ / Fahrplananpassung ☐
(Zutreffendes bitte ankreuzen)

für Güterzugtrassen

Zugart	Zugnummer	Nutzungsdauer	bestehende Vergleichstrasse	Interne Bearbeitungs-ID Kunde	Kunden-Nr

Kunde, Bevollmächtigter laut ABN	Telefon	Fax	E-Mail

Eingang der Anmeldung

Gewünschtes Trassenprodukt

☐ Express-Trasse

☐ Standard-Trasse
gewünschte Systemtrasse: ....

☐ Zubringer-Trasse

Zur Zubringer-Trasse gehörende Standard-Trasse: ....

Verkehrszeitraum

ab Ort	Zuggattung	Verkehrszeitraum	Zusatztage	Ausfalltage	Konstruktionsspielraum

Betrieblich-technische Angaben (Zugcharakteristik)

ab Ort	Vmax	Tfz 1	Tfz 2	Schiebel.	gekuppelt	Last	Bremsstellung	BrH	Länge	EBuLa	Besonderheiten, LÜ, KLV, Gefahrgut

Trassenzeiten

Kundenanmeldung							Konstruktionsergebnisse 1		Konstruktionsergebnisse 2	
Ort	Gleis	Ank	Halt	Art	Abf	Vorgaben// Änderungen der Zugcharakteristik	Ank	Abf	Ank	Abf

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Fahrplan		Bestellung EVU			Angebot GB Netz				
Abk.	Bahnhof	an	Min.-halt	ab	an	ab	Bemerkungen		
	120 60 Min nach Ankunft 84079 in RLB								
	Zwischen Lu BASF und Lu-Oggersheim								
	nächstmögliche Trasse rd 3 Std nach								
	Ankunft 84079. We 14.01.2001								
RLB	Ludwigshafen BASF					16.42			
RLB	Ludwigshafen BASF					14.47			
RLO	Lu-hafen Oggersheim				16.53	17.15	Kopfmachen		
RLO	Lu-hafen Oggersheim				14.58	15.21	Kopfmachen		
RWOR	Worms Hbf					17.29	We 14.01.2002		
RWOR	Worms Hbf					15.35	We 06.02.02		(In Tr. 54050 Sc)
RMZ	Mainz Hbf					18.28			
Biblis					15.45.6	15.47.4	Ra 07.02.02		
FKBWO	Kaiserbrücke Ost					18.33.3	Ra 17.01.02		
FKBWO	Kaiserbrücke Ost								
——	Oberlahnstein Gbf					20.02			
KOL G	Oberlahnstein Gbf								
—	Linz (Rh)					20.35	Ha 17.01.02		
KLI	Linz (Rh)								
FMB	16.56/17.12								
FKOS	Abzw Kostheim					17.20			
KOL G	Oberlahnstein Gbf					18.57			
KLI	Linz (Rh)					19.35	Ha 11.02.02		
KUN	Unkel				+19.42	19.48			
KG G	Gremberg Gnf					20.27	An 19.2.02		
KDO	Dormagen	19:00							

Bemerkungen Kunde

BR 185 mit 5.6 MW

Preisgünstigster Weg gewünscht.

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Outline

- Auctions
- Rail Track Scheduling
- Rail Track Auctioning
- The Optimal Track Allocation Problem
- Experiments

Rail Track Auctioning

- Goals
  - More traffic at lower cost
  - Better service
- How do you measure?
  - Possible answer: in terms of willingness to pay
- What is the "commodity" of this market?
  - Possible answer: timetabled track

    = dedicated, timetabled track section
- How does the market work?
  - Possible answer: by auctioning timetabled tracks
- Auctions can be in-company auctions

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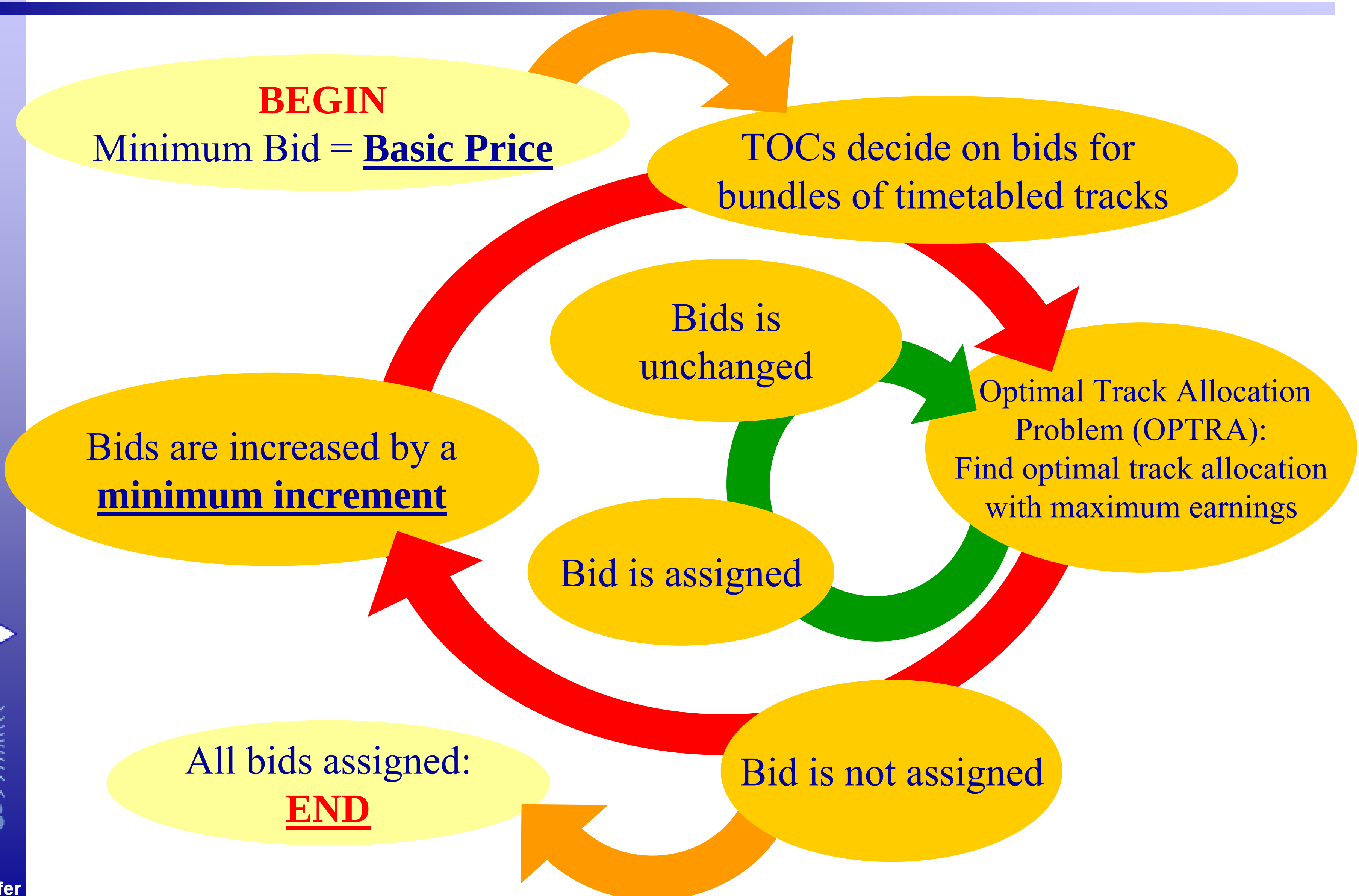
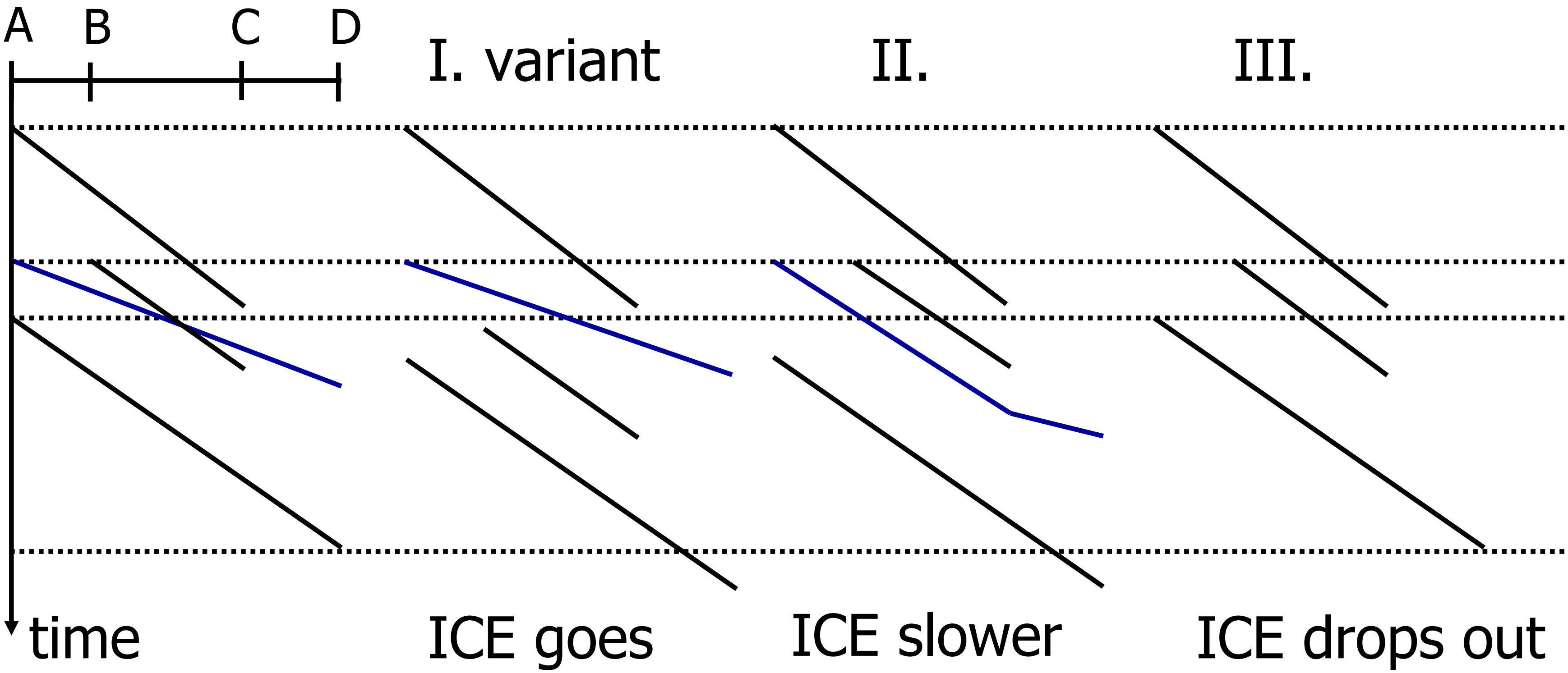
Rail Track Auction



Rail Track Auction Results

3 x  + 1 x  = ???



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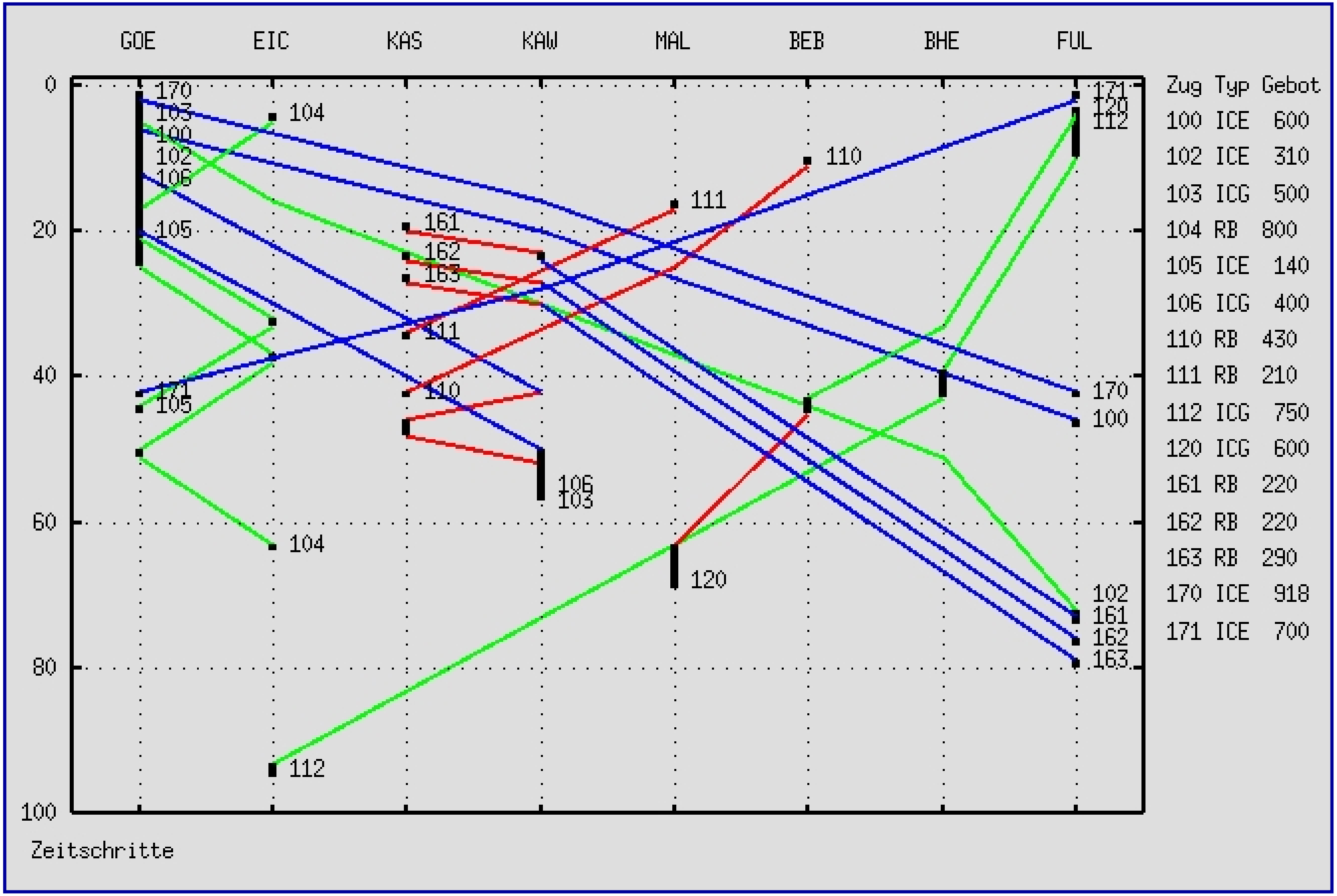
Rail Track Auction Results

(14,439 Variables, 13,408 Constraints, 48 Minutes)



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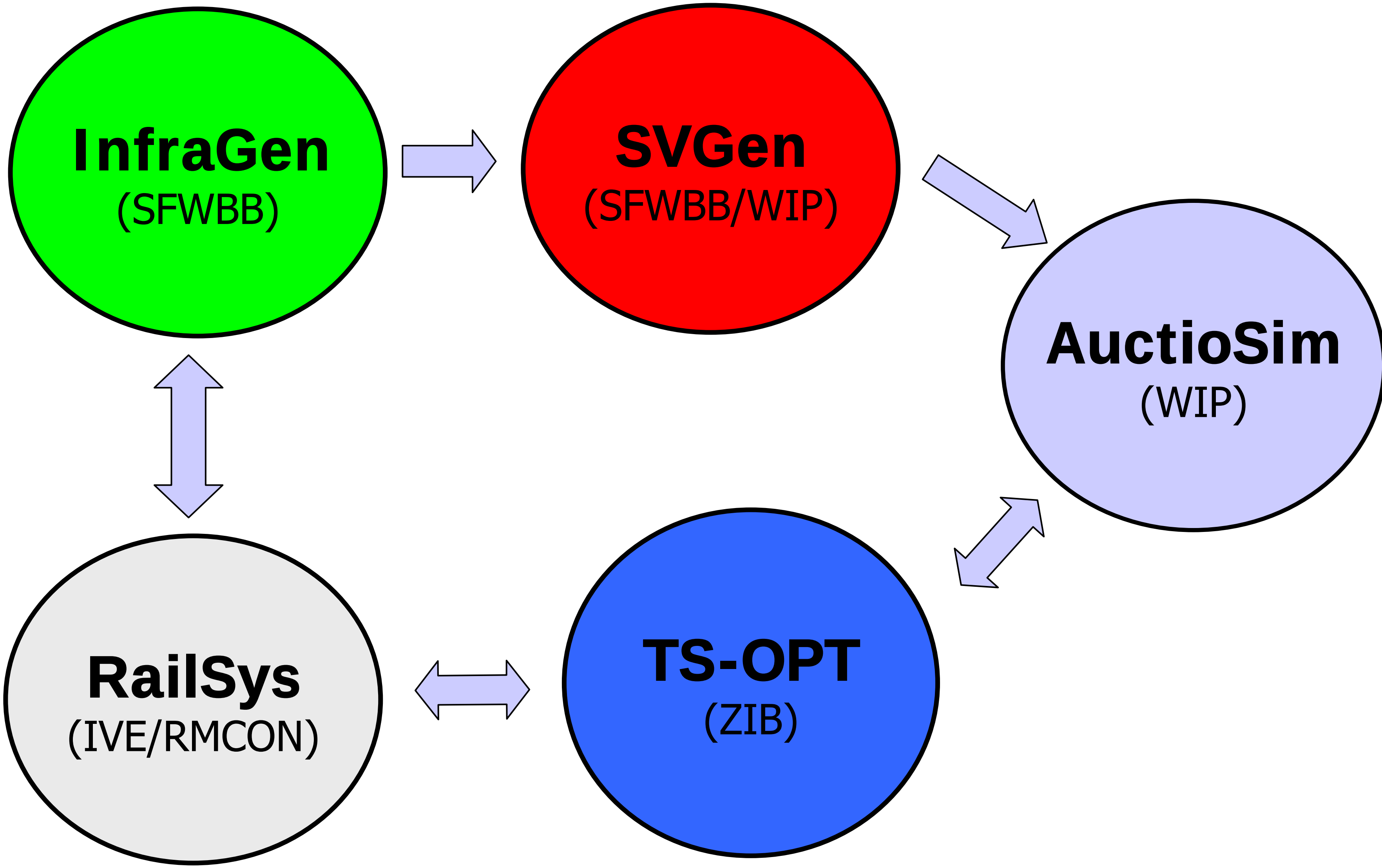
Rail Track Auctioning Modules



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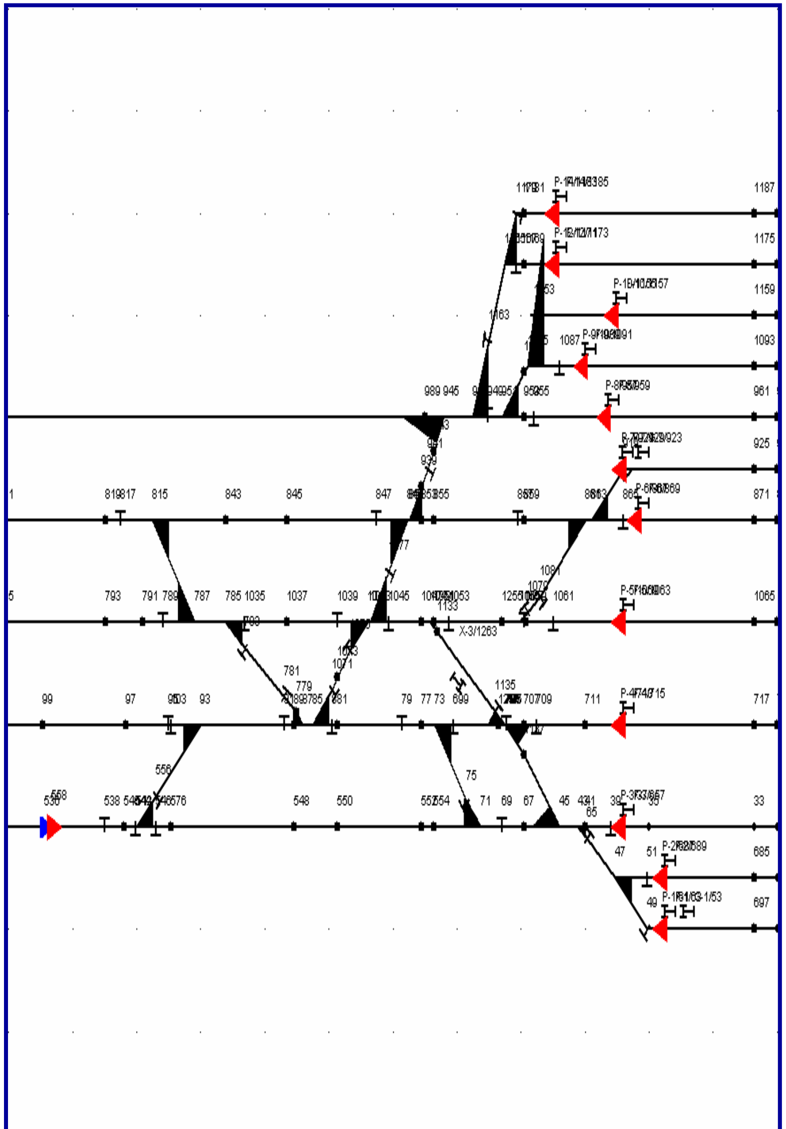
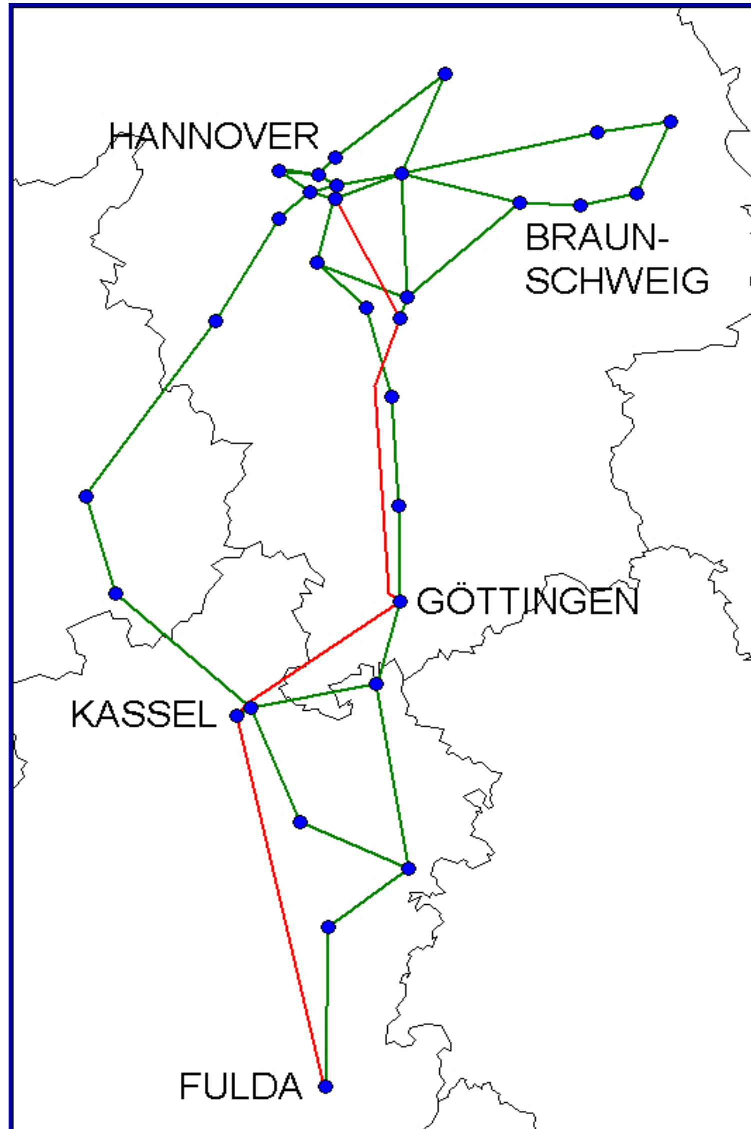
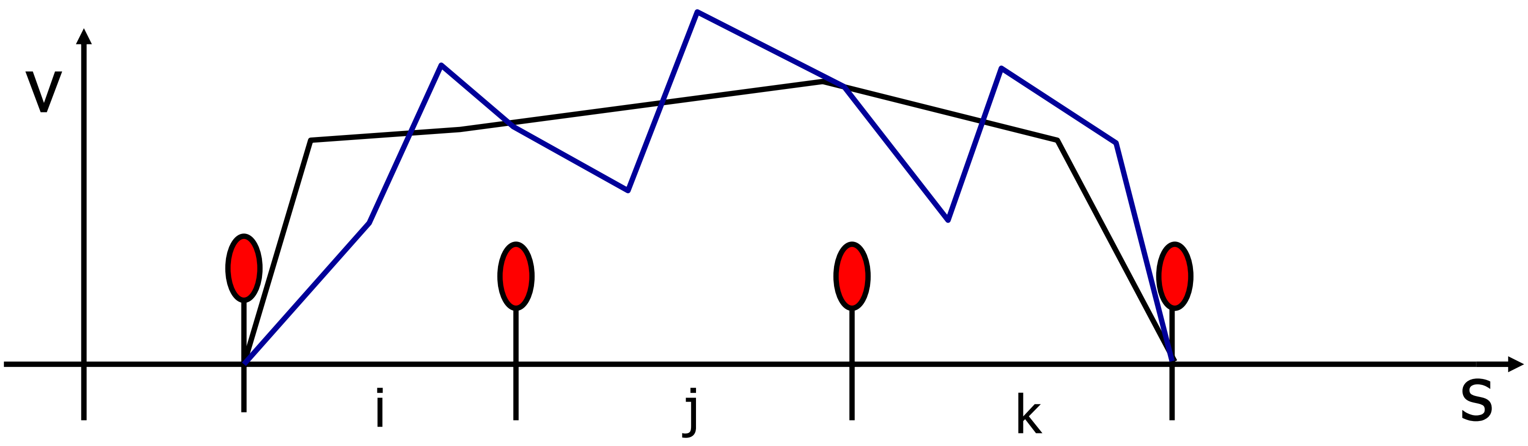
Macroscopic Graph Model





Blocks & Standardized Dynamics

- State (i,T,t,v)
  - Directed block i
  - Train type T
  - Starting time t, velocity v



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Standard Train Types

train type	*V max [km/h]*	*train length [m]*	*Security technology*	*...*
ICE	250	410	LZB	
IC	200	400	LZB	
RE	160	225	signal	
RB	120	100	signal	
SB	140	125	signal	
ICG	100	600	signal	

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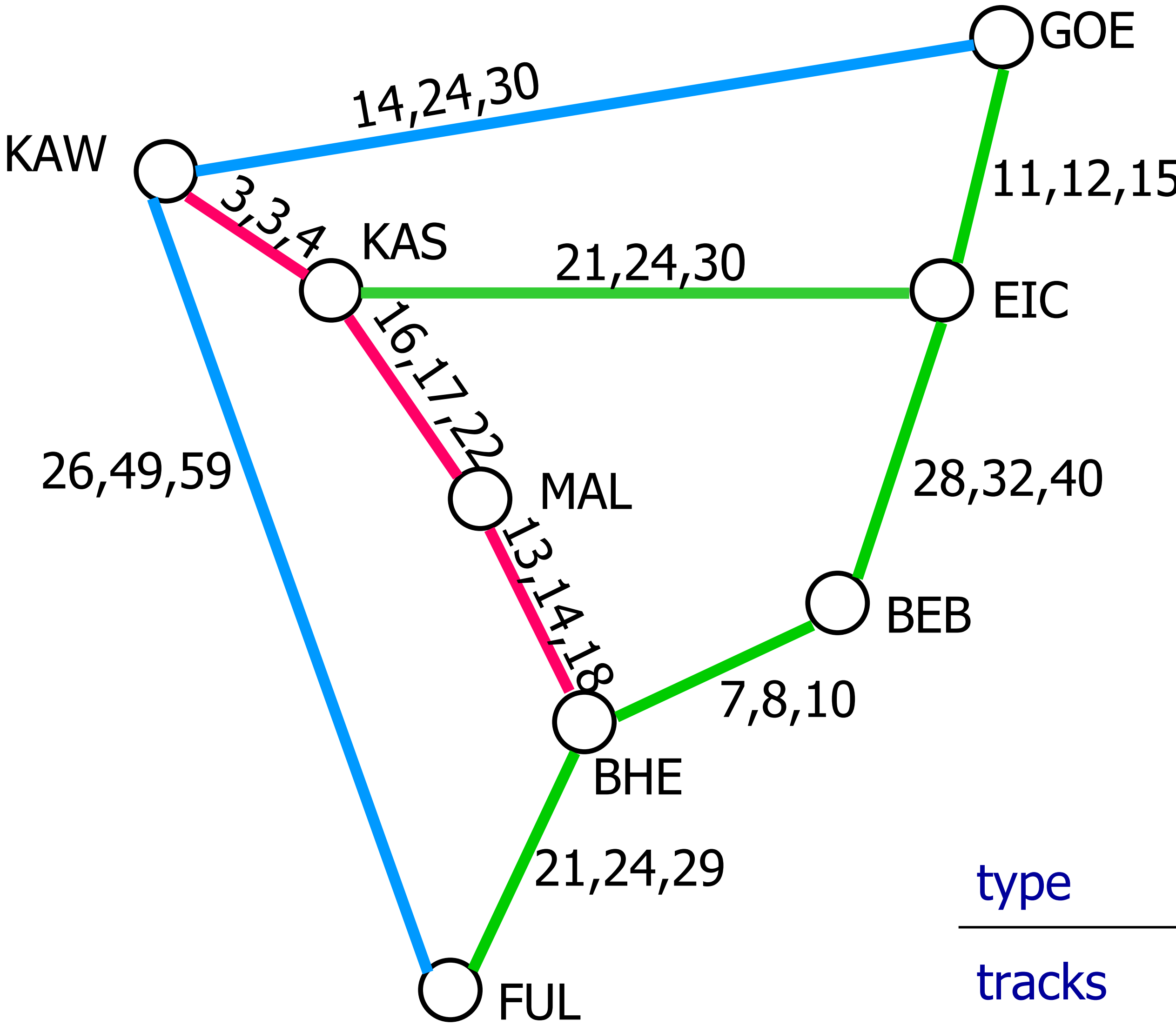
Example

(travel times in mins)



type	ICE, RB, ICG
tracks	766 km

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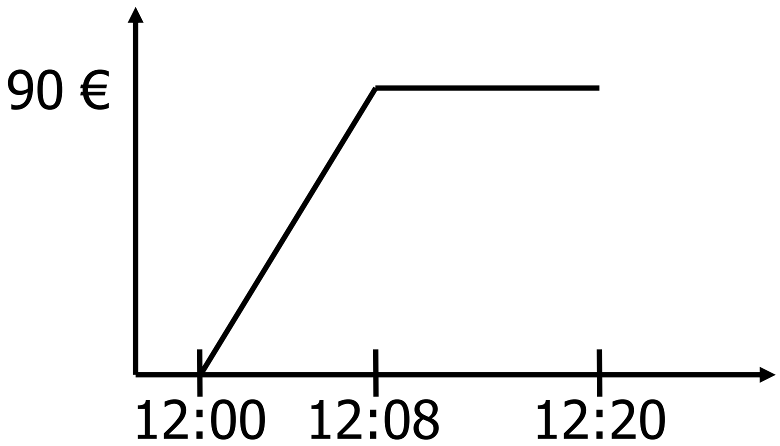
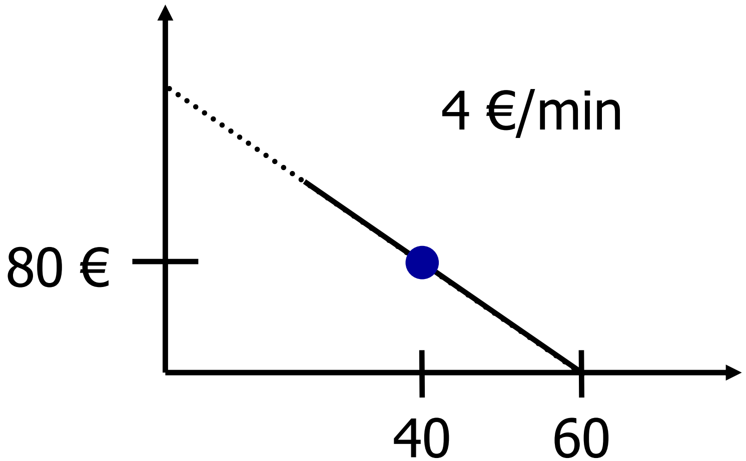
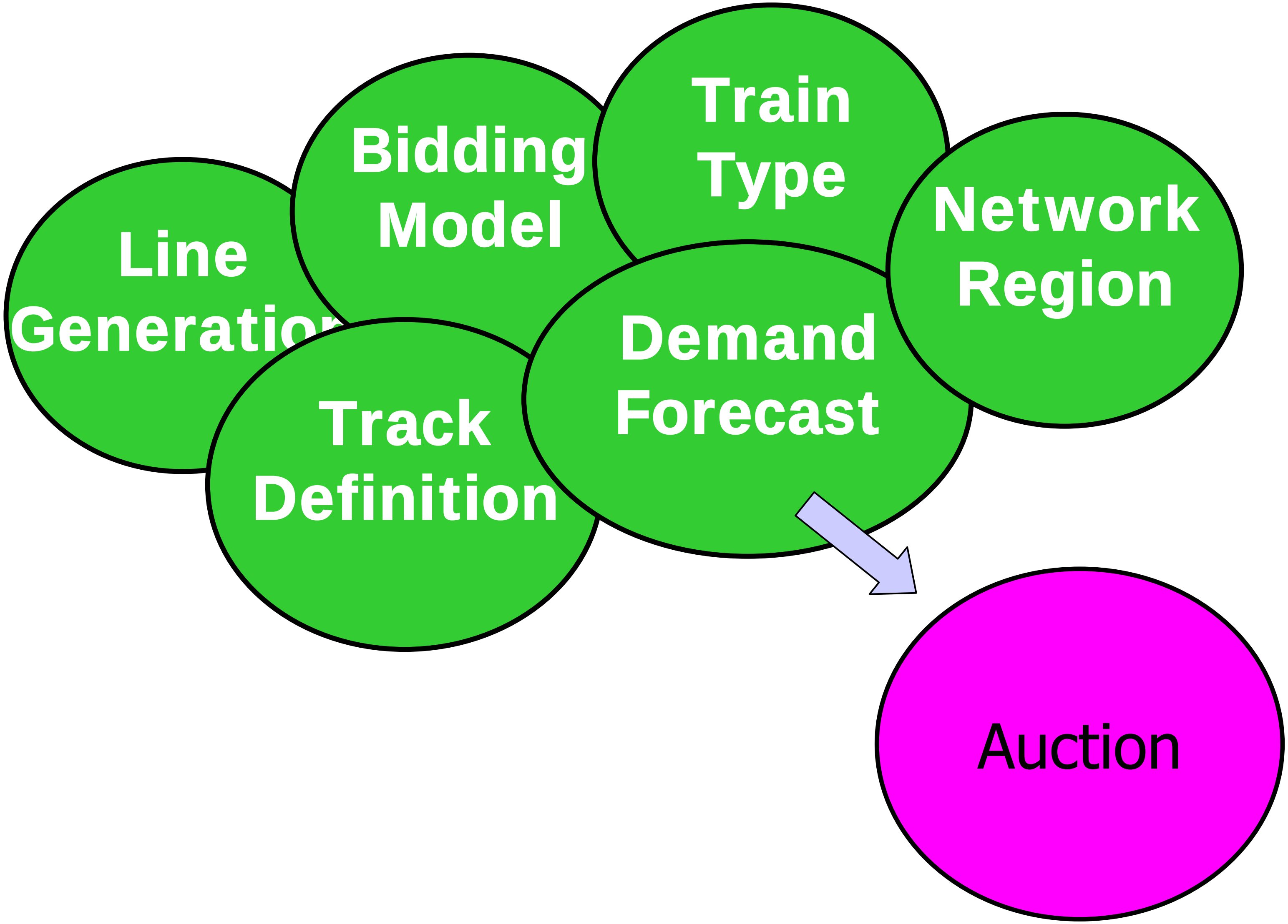
Bids for Timetabled Tracks

- Train number(s) and type(s)
- Starting station, earliest starting time
- Final station, latest arrival time
- Bid = Basic Bid
  + Departure/Arrival Bonus
  + Travel Time Bonus
- Intermediate stops (Station, min. stopping time, arrival interval)
- Connections
- Combinatorial bids (and/or)





Bid Generator



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Bid Generator

(Reuter 2005)

Method	*Input*	*Output*	*Goal*
Minimum spanning tree	Distances	Tracks on a tree	Regional coverage
Maximum spanning tree	OD-Matrix	Tracks on a tree	Demand coverage
Greedy	OD-Matrix	Set of tracks	"Good tracks"
Point-To-Point	Stations	Single track	Direct connections

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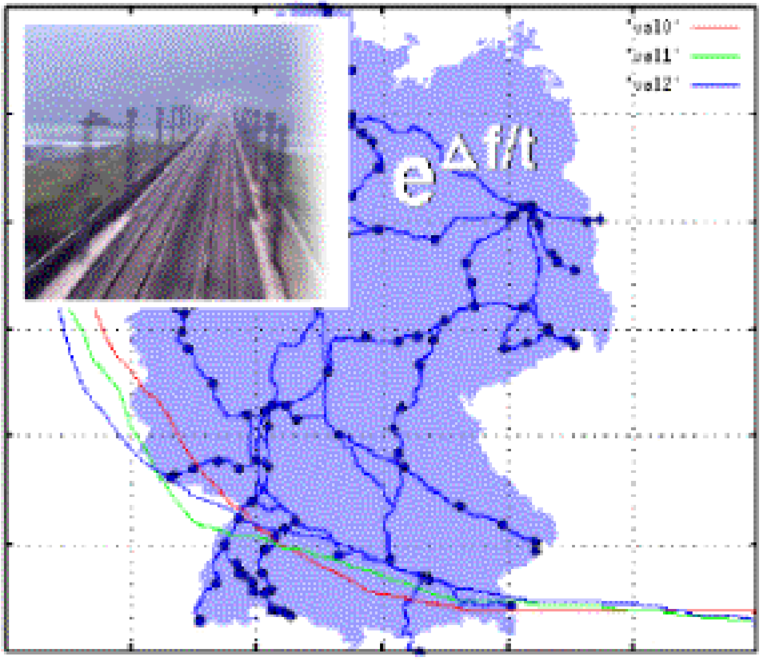
Informations technik
Software logistik

Intranetz
Gesellschaft für Informationslogistik mbH

PROSA/prosimExpreß :
A line-planning tool for Deutsche Bahn



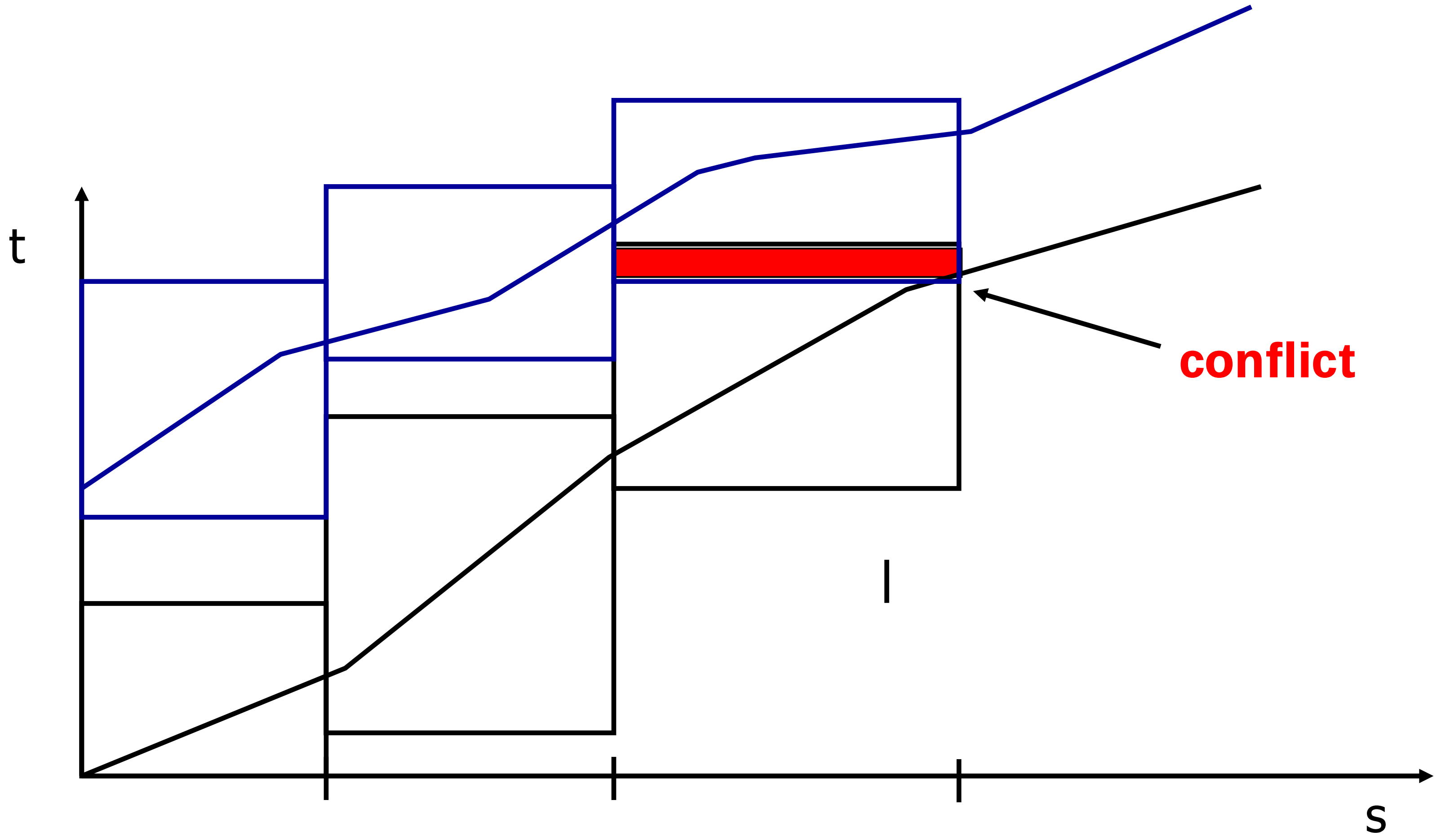
N. Ascheuer, Ch. Küttner, M. Proksch

Intranetz
Gesellschaft für Informationslogistik mbH

J. Dupont, R. Firla, A. Huck, K. Kuchenbecker,
M. Sievers, F. Wagner

DB

Block Conflicts



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Optimal Track Allocation Problem

- OPTRA
- Input
  - Set of bids for timetabled tracks
  - Available infrastructure (space and time)
- Output
  - Conflict free track assignments for the chosen bids
  - Track assignment that maximizes total earnings

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Multicommodity Flow Model

$$
\begin{aligned}
\max\ & c^T x \\
x^r(\delta^+(z)) - x^r(\delta^-(z)) &= b_z^r && \forall r, z \\
x_a^r + x_b^s &\le 1 && \forall r, s, a, b \text{ incomp.} \\
x_a^r &\in \{0,1\} && \forall r, a
\end{aligned}
$$

- Space-time graph G=(V,A)
  - Nodes z=(i,T,t,v) $\in$ V
  - Arcs a=(z1,z2) $\in$ A
- Block conflicts on arcs
- Timetabled track $\cong$ path in G
- Timetable $\cong$ set of compatible timetabled tracks

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- Rail Track Auctioning
- The Optimal Track Allocation Problem
- Experiments

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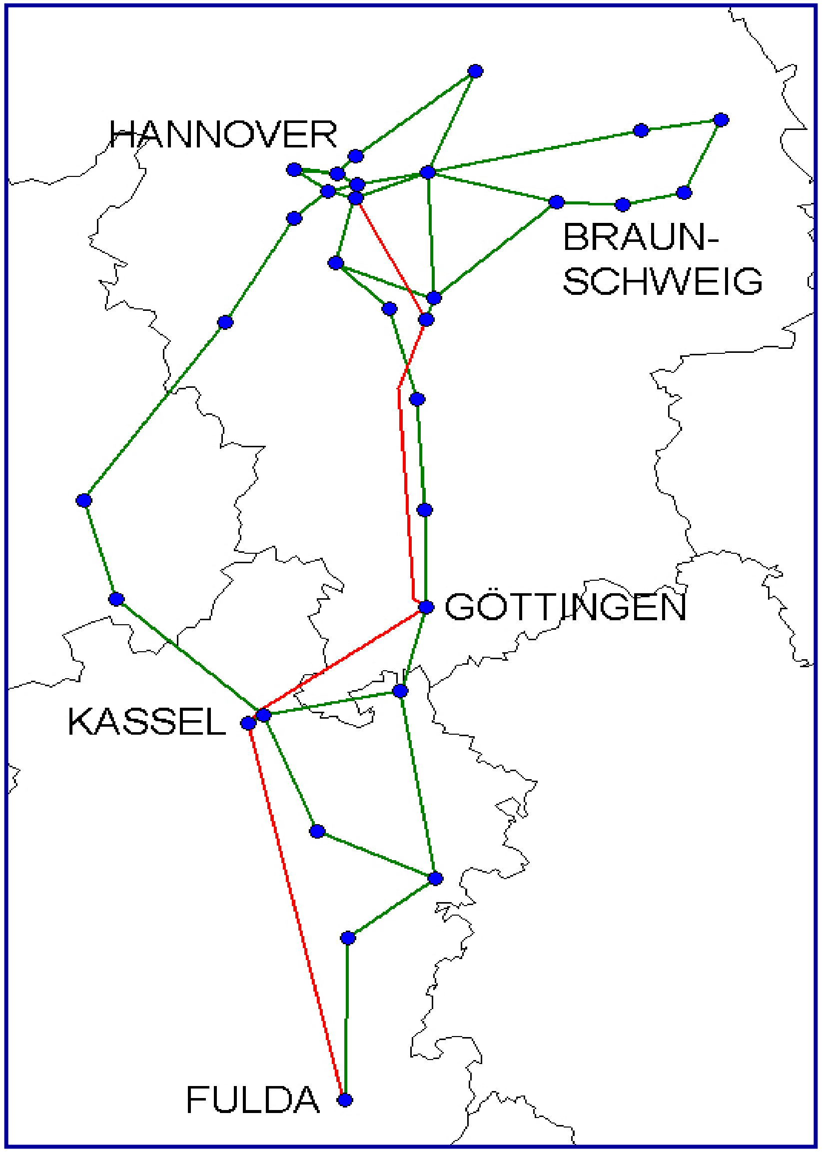
Test Network

- Criteria
  - Important characteristics ("Hildesheimer Kurve")
  - Important subnet
  - Used in earlier studies
- Data
  - 45 sections = 1176 km
  - 31 nodes
  - 6 train types



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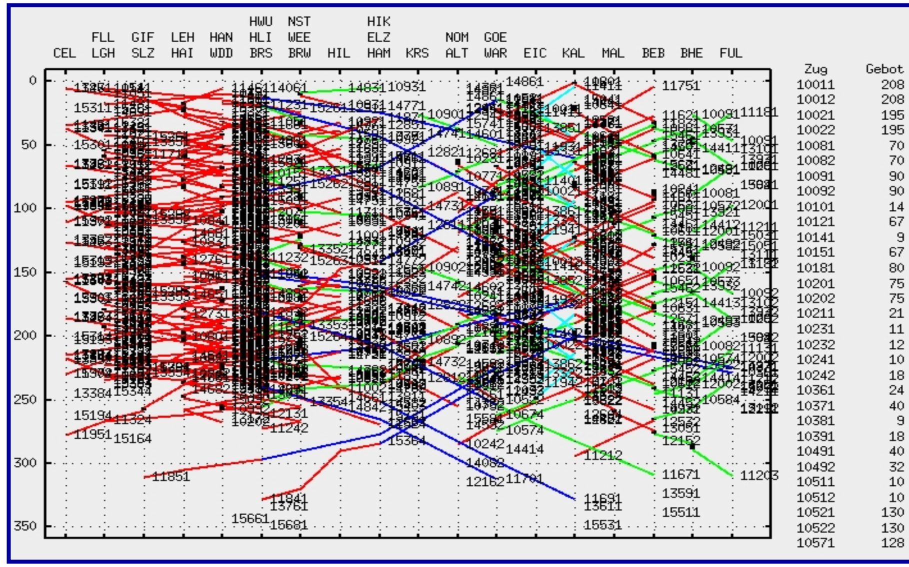
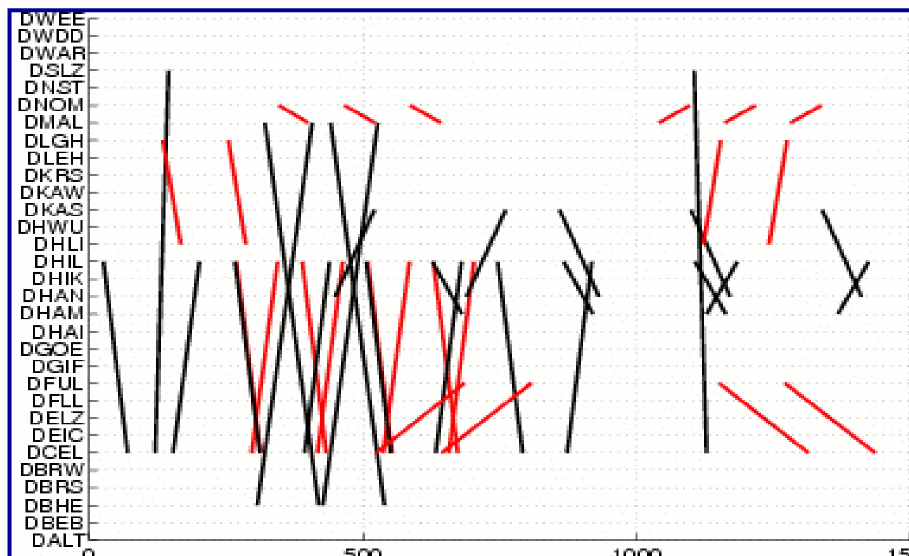
Auction Experiments

(Reuter 2005, Rounds 8 and 9)

Round	*Earnings*	*Round*	*Earnings*
1	44563	9	46575
2	44563	10	47051
3	44598	11	48096
4	44799	12	48253
5	44799	13	48337
6	44972	14	48391
7	45551	15	48513
8	46375		





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Auction Experiments
(Reuter 2005)

	ICE		*IC*		*RE*		*RB*		*S*		*ICG*	*#*
# Trains/Type	*ind*	*sync*	*ind*	*sync*	*ind*	*sync*	*ind*	*sync*	*ind*	*sync*	*ind*	
Timetable	27	0	27	0	38	19	87	23	0	61	28	—
+24 IC/ICE ind	30	0	29	0	38	19	85	23	0	61	25	18
+24 IC/ICE sync	24	9	27	9	36	19	83	19	0	58	26	22
+27 R*/S ind	27	0	25	0	44	19	89	23	5	58	27	20
+27 R*/S sync	27	0	27	0	36	19	83	32	0	62	27	30
+15 ICG	27	0	27	0	38	19	87	23	0	61	42	19
+66 *	28	0	25	3	38	25	85	29	2	55	31	29

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Auction Experiments

(Reuter 2005)

	ICE		*IC*		*RE*		*RB*		*S*		*ICG*	*Σ*
€/km	*ind*	*sync*	*ind*	*sync*	*ind*	*sync*	*ind*	*sync*	*ind*	*sync*	*ind*	*€*
Timetable												
+24 IC/ICE ind	2.04		1.78		1.24	1.07	0.93	0.90		0.98	1.12	34421
+24 IC/ICE sync	1.89	1.94	1.45	3.27	1.14	1.10	0.89	0.83		0.90	1.10	36031
+27 R*/S ind	1.74		1.41		1.23	1.08	0.91	0.90	1.15	1.10	1.14	31180
+27 R*/S sync	2.31		1.34		1.02	1.04	0.88	1.41		1.06	0.98	33663
+15 ICG	1.45		1.44		1.08	1.08	0.87	0.90		0.88	1.03	32994
+66 *	2.21		1.88	2.87	1.03	1.10	0.89	1.11	1.53	1.47	1.60	41263

Tripling Experiment

variation	*cpu time (CPLEX)*	*earnings (% Status Quo)*	*trains (% Status Quo)*
0 mins	6 secs	52.066 (+ 84%)	420 (+ 47%)
1 mins	8 secs	60.612 (+114%)	496 (+ 74%)
4 mins	1 days	67.069 (+137%)	617 (+117%)
5 mins	3+ days	67.975 (+140%)	737 (+159%)

- Status quo
  - 284 tracks through 6 hours in the Hannover—Braunschweig—Fulda network, (hypothetical) total income of 28,255 €
- Scenario
  - triple requests to 946 bids (~15 minutes alteration, identical willingness to pay)

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Thank you for your attention.

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