European Graduate Program
Berlin - Zürich

Combinatorics

# 06M1 Lecture

# Frequency Assignment for GSM Mobile Phone Systems


## Martin Grötschel


**Block Course at TU Berlin**

**"Combinatorial Optimization at Work"**

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# Contents

# Contents

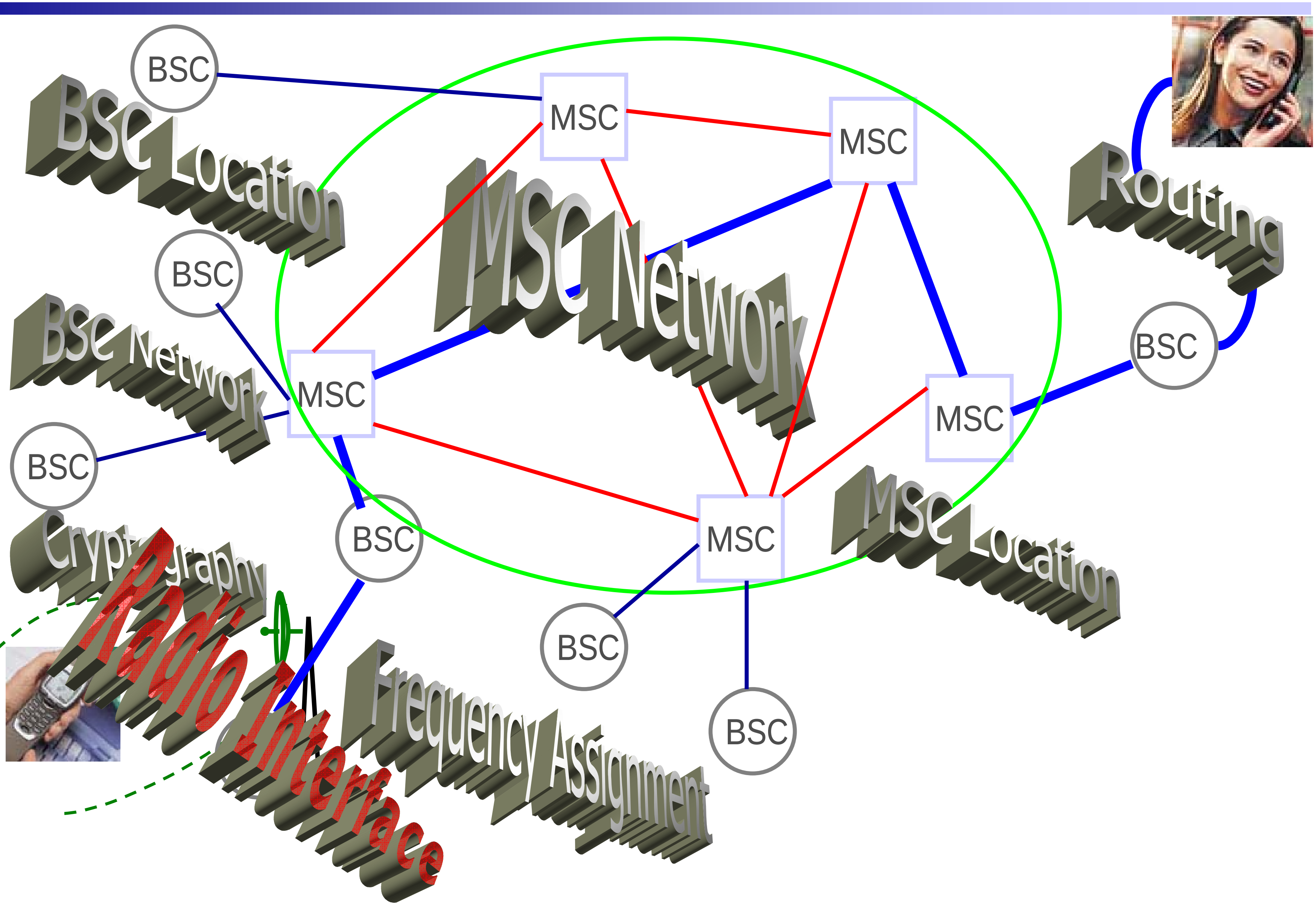
1. Introduction
2. The Telecom Problem & Mobile Communication
3. GSM Frequency/Channel Assignment
4. The UMTS Radio Interface (next talk)

# E-Plus and the Channel Assignment Problem

- How did we get this project?

# Contents

# Connecting Mobiles

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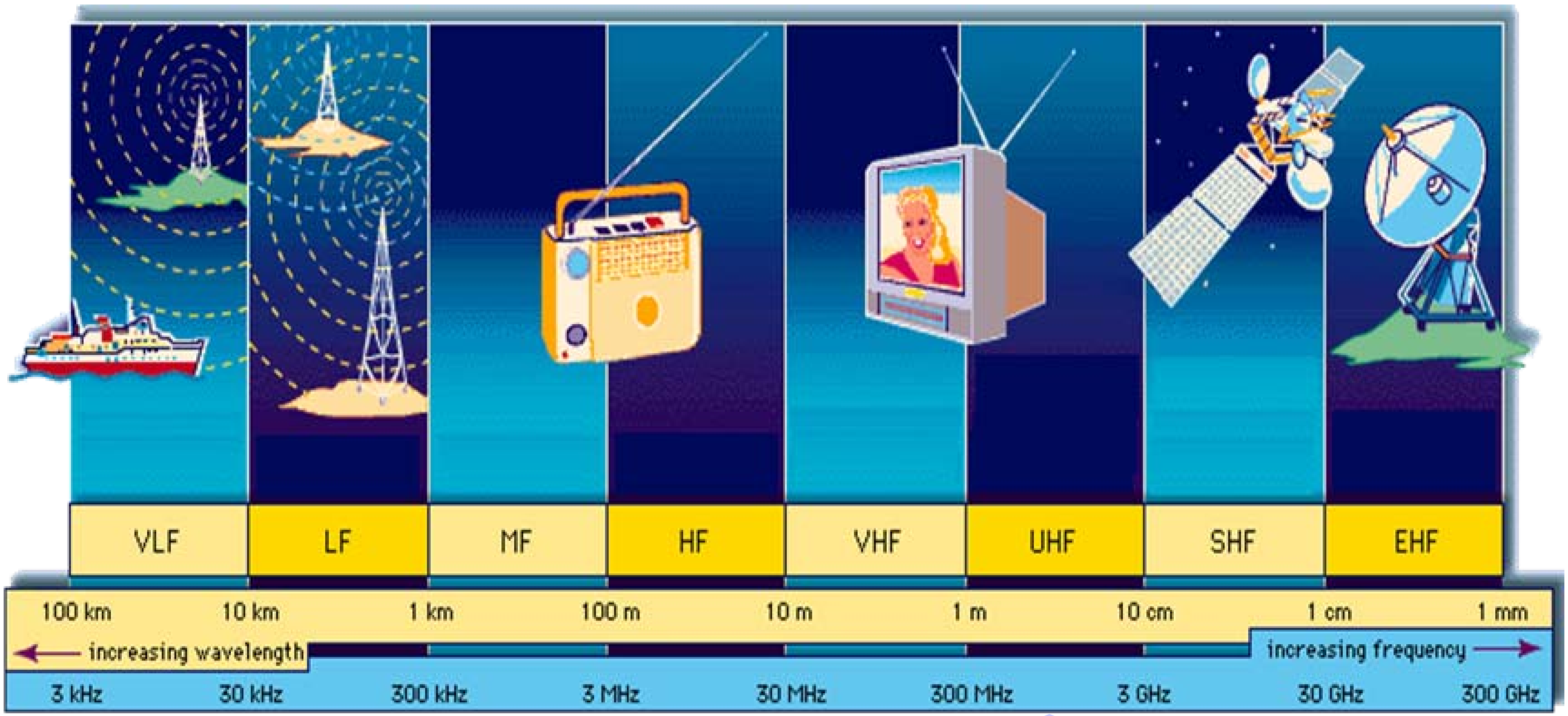
# Wireless Communication






Mobile Telecommunication

# Generations of Mobile Telecommunications Systems

| 1G | 2G | 3G | 4G |
|---|---|---|---|
| 1980s | 1990s | 2000s | 2010 ?? |

**1G (1980s)**

- Analogue
- Voice Only

**2G (1990s)**

- Digital
- Voice & Data
- **GSM** mass market
- PCS
- cdmaOne/IS95

**3G (2000s)**

- **UMTS**, WiFi/WLAN, cdma2000
- Data Rates $\geq$ 384 kbit/s
- Various Services

- more **services**
- more bandwidth
- fresh spectrum
- new technology
- **W-CDMA** radio transmissions

# Radio Interface: OR & Optimization Challenges

- Location of sites/base stations
  - was investgated in the OR literature („dead subject“)
  - has become „hot“ again
    - UMTS: massive investments around the world
    - GSM: still significant roll-outs
    - special issue: mergers
- antenna configurations at base stations
  - GSM: coverage based planning
  - UMTS: coverage & capacity considerations
- radio resource allocation
  - GSM: frequency assignment
  - UMTS: ? (open: real time/online resource management)

# Contents

# Wireless Communication


GSM: More than 1,000 million users in over 150 countries

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# Wireless Communication

There are five frequency bands used by GSM mobile phones:

GSM-900, GSM-1800, GSM-850, GSM-1900, GSM-400

GSM-900 and GSM-1800 are used in most of the world.

GSM-900 uses 890 - 915 MHz to send information from the Mobile Station to the Base Transceiver Station (BTS) (This is the "uplink") and 935 - 960 MHz for the other direction (downlink), providing 124 RF channels spaced at 200 kHz. Duplex spacing of 45 MHz is used. GSM-1800 uses 1710 - 1785 MHz for the uplink and 1805 - 1880 downlink, providing 299 channels. Duplex spacing is 95 MHz.

GSM-850 and GSM-1900 are used in the United States, Canada, and many other countries in the Americas.

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# FAP

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# Antennas

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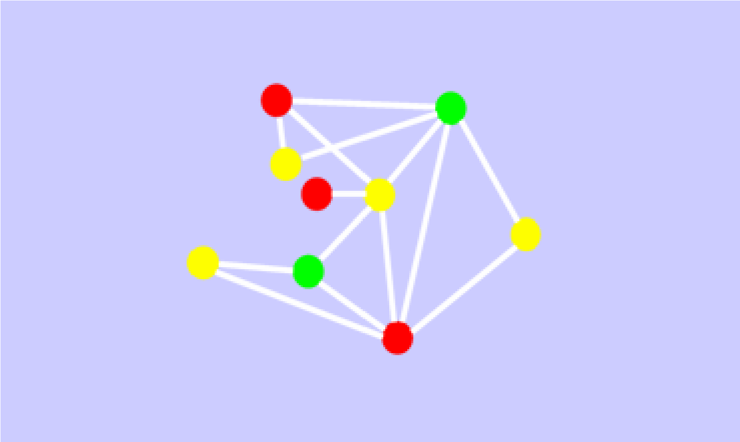
# Initial Idea

- Use graph colouring to assign channels!

# Coloring Graphs

Given a graph G = (V,E), color the nodes of the graph such that no two adjacent nodes have the same color.



The smallest number of colors with this property is called chromatic or coloring number and is denoted by $\chi(G)$.

# Coloring Graphs

A typical theoretical question: Given a

class $C$ of graphs

(e.g., planar or perfect graphs, graphs without certain minors), what can one prove about the chromatic number of all graphs in $C$?

A typical practical question: Given a

particular graph G

(e.g., arising in some application), how can one determine (or approximate) the chromatic number of G?

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# Coloring Graphs

- Coloring graphs algorithmically

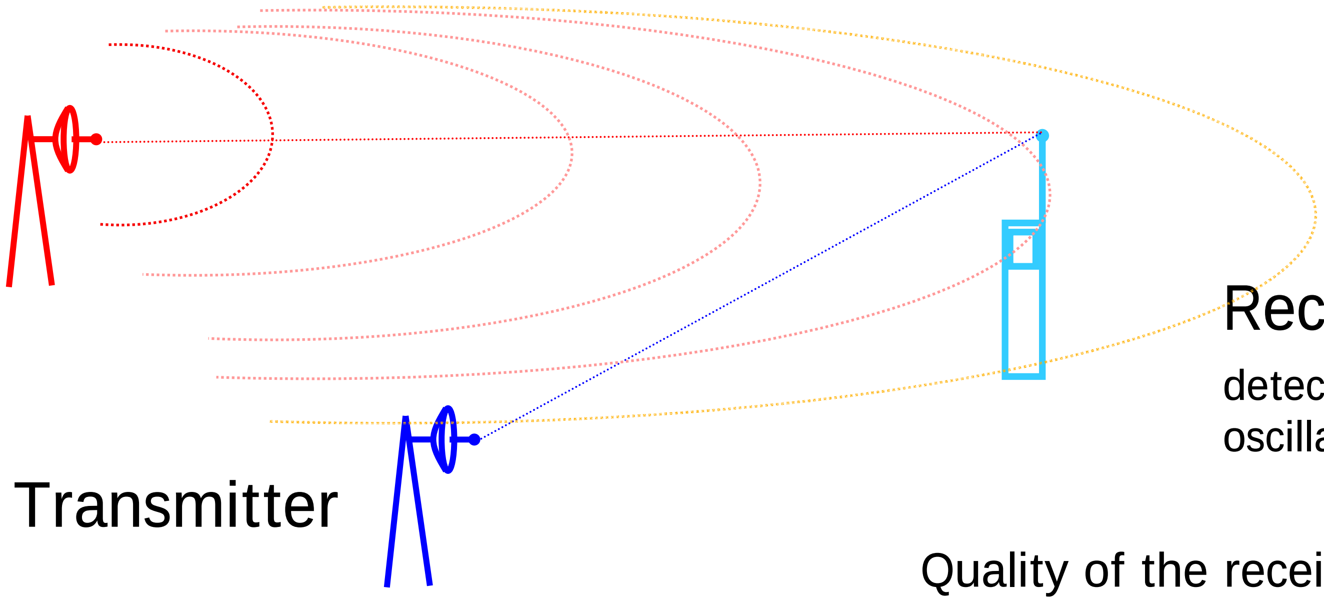
  - NP-hard in theory
  - very hard in practice
  - almost impossible to find optimal colorings (symmetry issue)
  - playground for heuristics (e.g., DIMACS challenge)

# Coloring in Telecommunication

- Frequency or Channel Assignment for radio-, tv-transmission, etc.
- Our Example: GSM mobile phone systems


- Andreas Eseinblätter, Martin Grötschel and Arie M. C. A. Koster, *Frequenzplanung im Mobilfunk*, DMV-Mitteilungen 1(2002)18-25
- Andreas Eisenblätter, Hans-Florian Geerdes, Thorsten Koch, Ulrich Türke: *MOMENTUM Data Scenarios for Radio Network Planning and Simulation* , ZIB-Report 04-07
- Andreas Eisenblätter, Armin Fügenschuh, Hans-Florian Geerdes, Daniel Junglas, Thorsten Koch, Alexander Martin: *Optimization Methods for UMTS Radio Network Planning*, ZIB-Report 03-41

# Properties of wireless communication



Receiver

detects oscillations

Transmitter

emits electromagnetic oscillations at a frequency

Quality of the received signal:
Signal-to-noise ratio

Poor signal-to-noise ratio:
interference of the signal

Objective: Frequency plan without interference or, second best, with minimum interference

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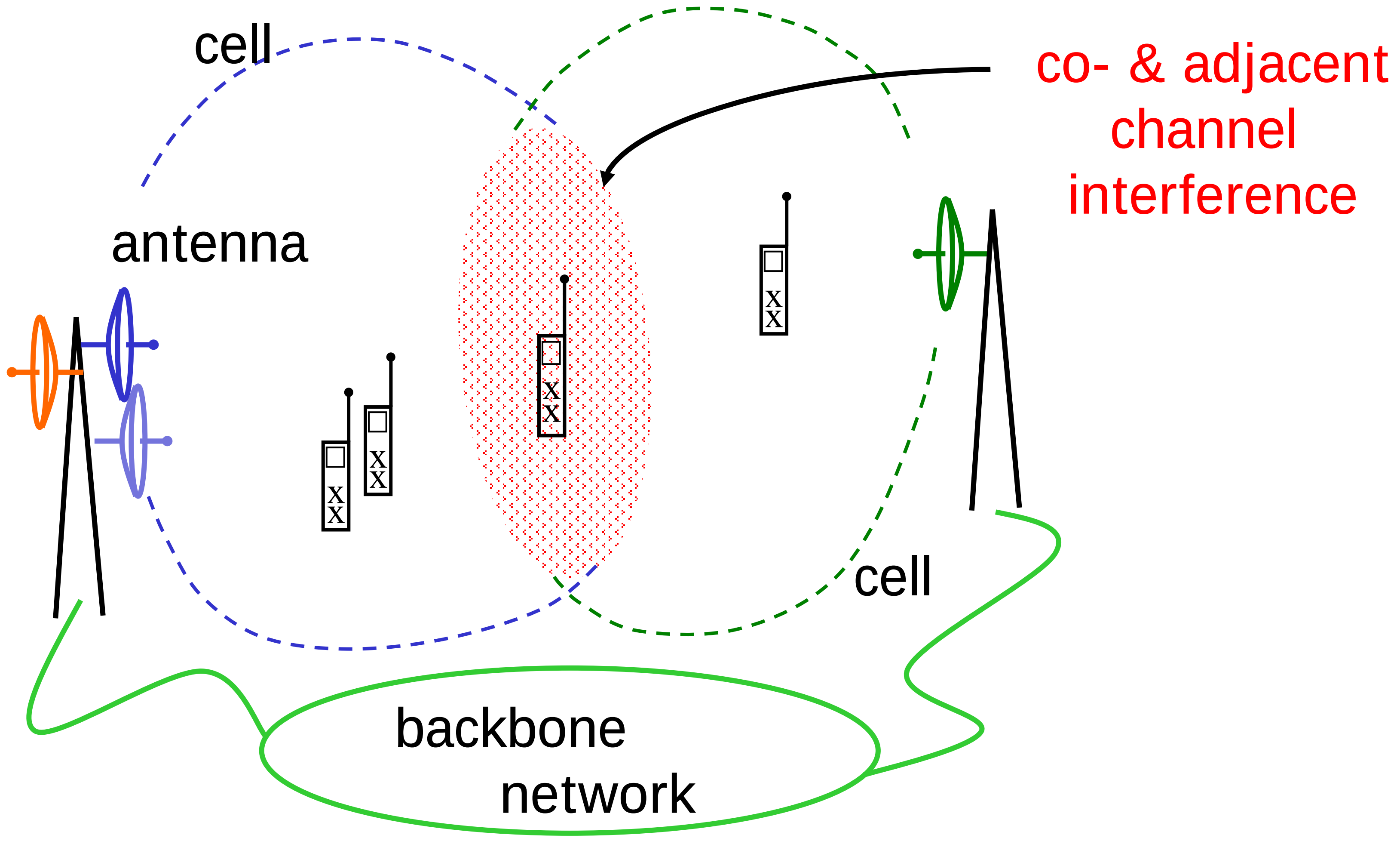
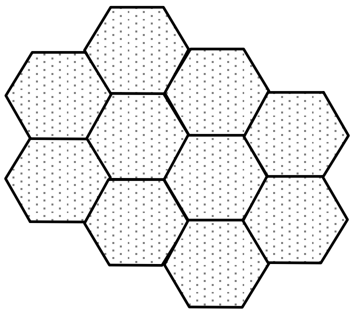
# Antennas & Interference



# Cell Models

### Hexagon Cell Model

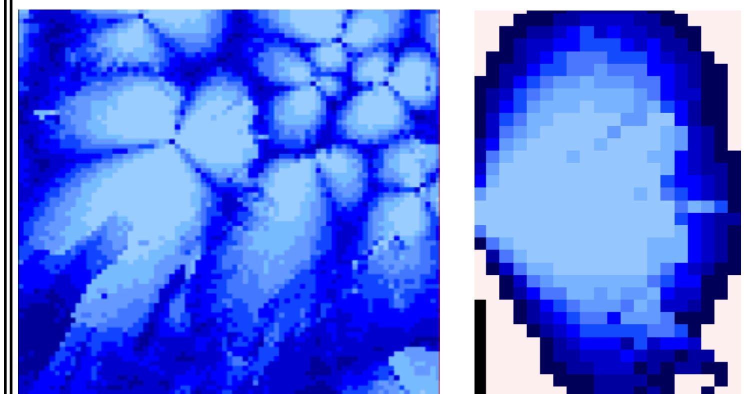


- sites on regular grid
- isotropic propagation conditions
- no cell-overlapping

### Best Server Model

- realistic propagation conditions
- arbitrary cell shapes
- no cell-overlapping

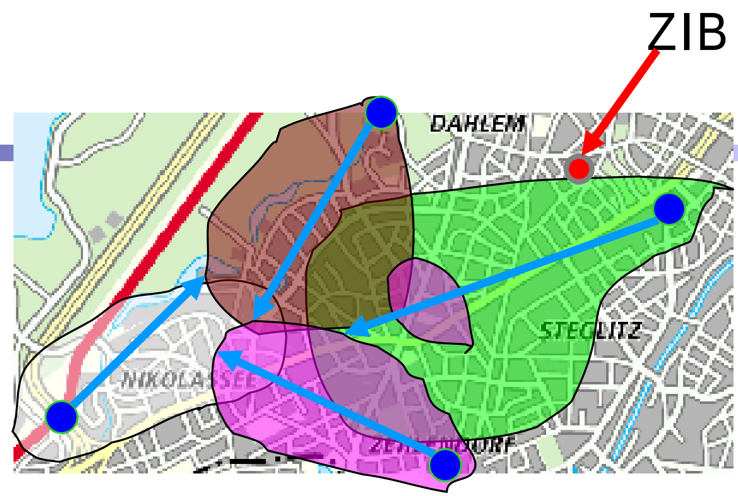
### Cell Assignment Probability Model



- realistic propagation conditions
- arbitrary cell shapes
- cell-overlapping

Source: **E-Plus Mobilfunk, Germany**

# Interference




Level of interference depends on

- distance between transmitters,
- geographical position,
- power of the signals,
- direction in which signals are transmitted,
- weather conditions
- assigned frequencies
  - co-channel interference
  - adjacent-channel interference

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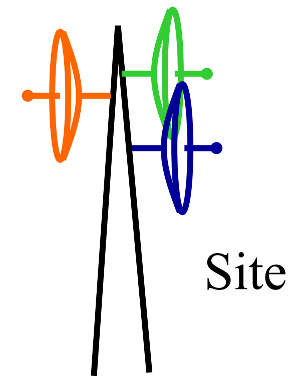
# Separation / Blocked Channels

Separation:

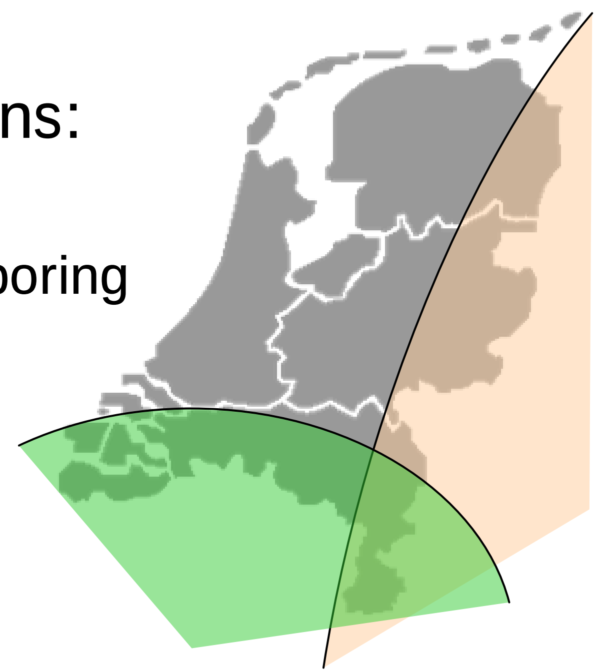
Frequencies assigned to the same location (site) have to be separated




Blocked Channels:

Restricted spectrum at some locations:

- government regulations,
- agreements with operators in neighboring regions,
- requirements of military forces,
- etc.



# Frequency Planning Problem

Find an assignment of frequencies/channels to transmitters that satisfies

- all separation constraints
- all blocked channels requirements

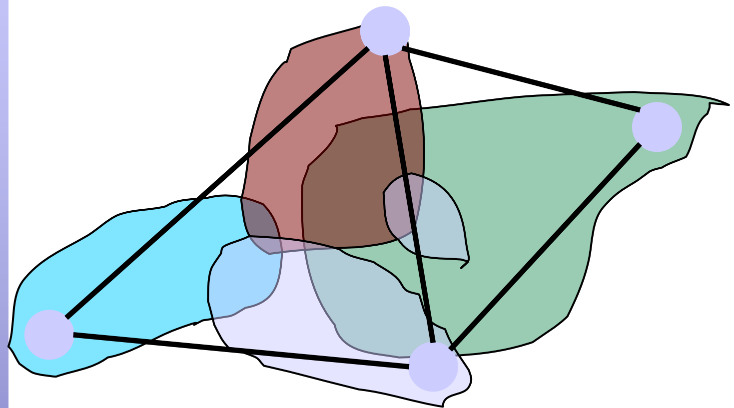
and either

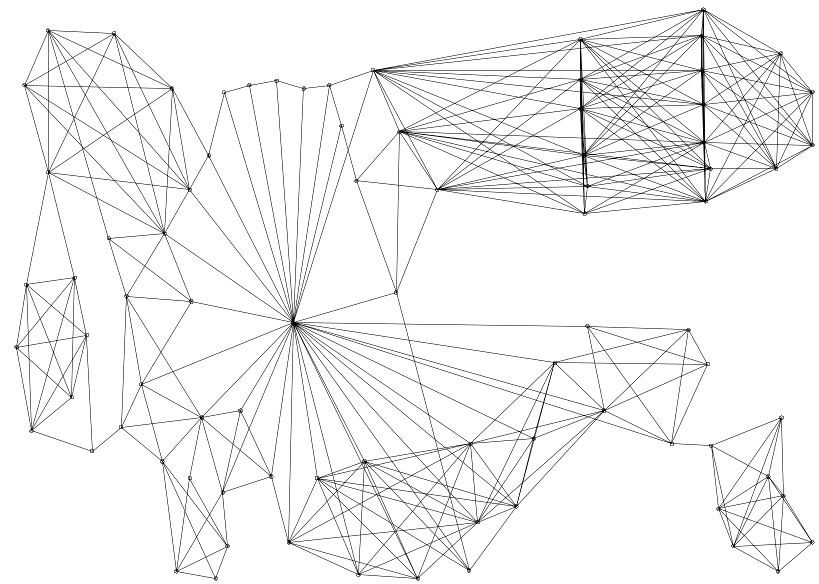
- avoids interference at all

or

- minimizes the (total/maximum) interference level

# Modeling: the interference graph





- Vertices represent transmitters (TRXs)
- Edges represent separation constraints and co/adjacent-channel interference
  - Separation distance: d(vw)
  - Co-channel interference level: $c^{co}(vw)$
  - Adjacent-channel interference level: $c^{ad}(vw)$

# Remark about UMTS

- There is no way to model interference as some number associated with an edge in some graph.
- Modelling is much more complicated, see talk by Hans-Florian Geerdes

# Graph Coloring

## Simplifications:

- drop adjacent-channel interference
- drop local blockings
- reduce all separation requirements to 1
- change large co-channel interference into separation distance 1 (inacceptable interference)

## Result:

- FAP reduces to coloring the vertices of a graph
- Example

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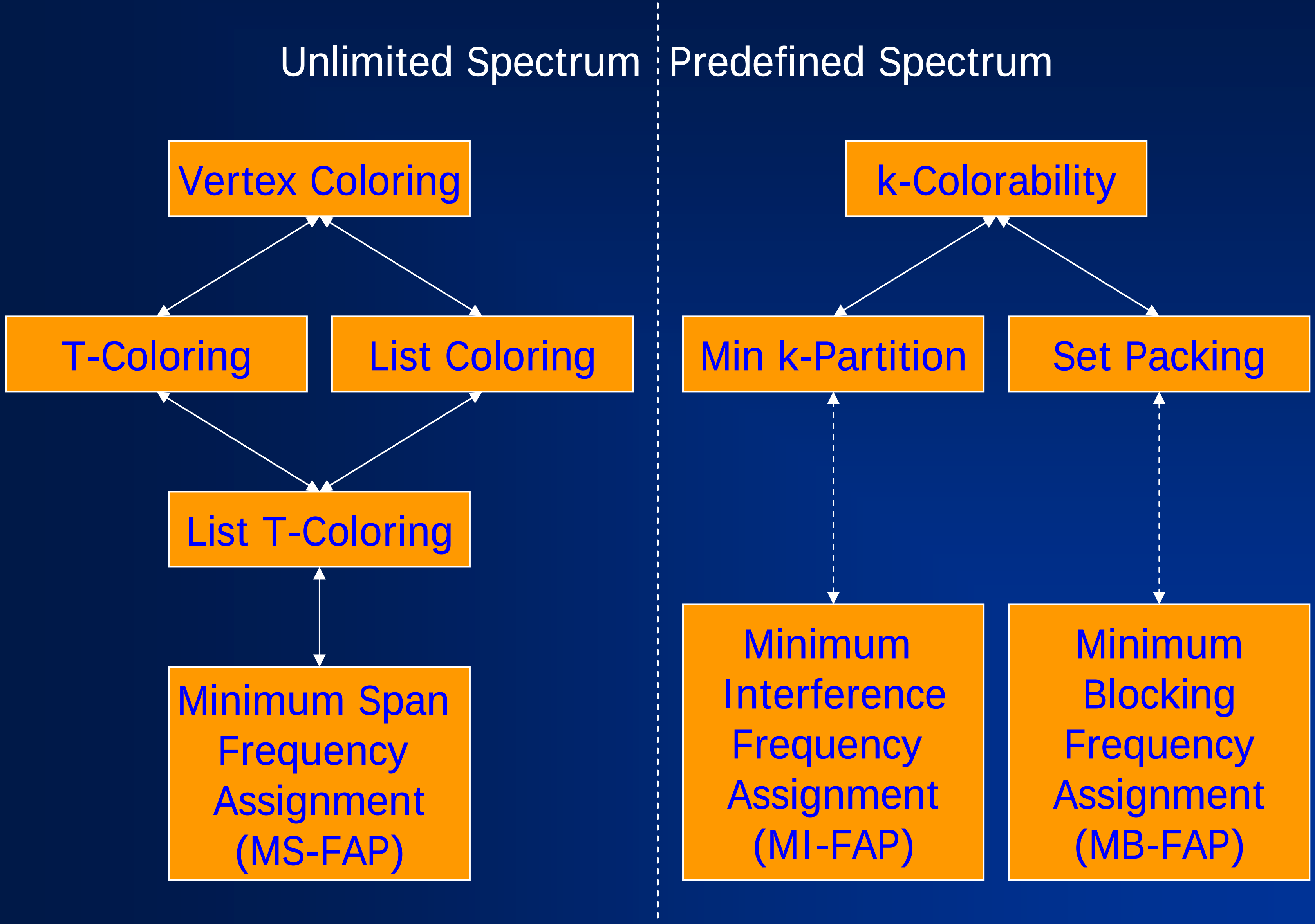
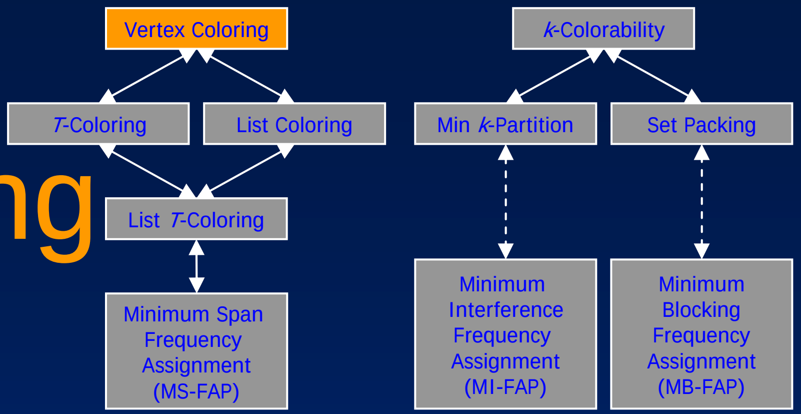
# Graph Coloring & Frequency Planning

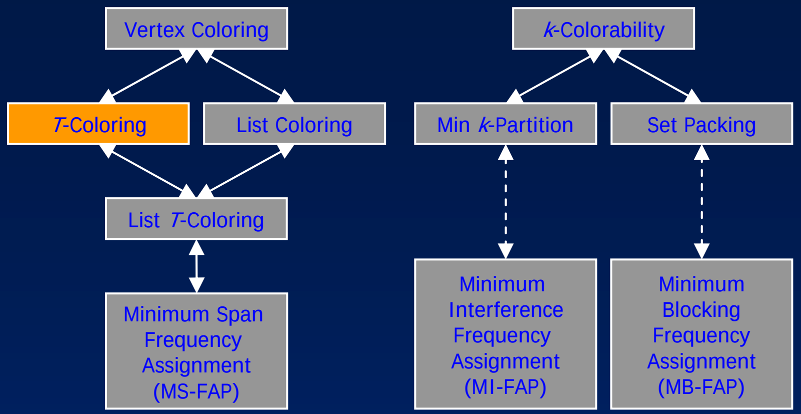


# FAP & Vertex Coloring



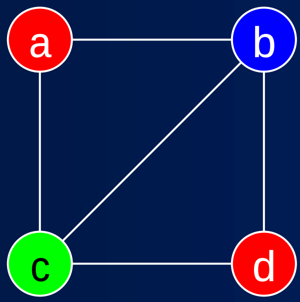

- Only co-channel interference
- Separation distance 1
- Minimization of
  - Number of frequencies used (chromatic number)
  - Span of frequencies used
- Objectives are equivalent: span = #colors-1
- FAP is **NP**-hard

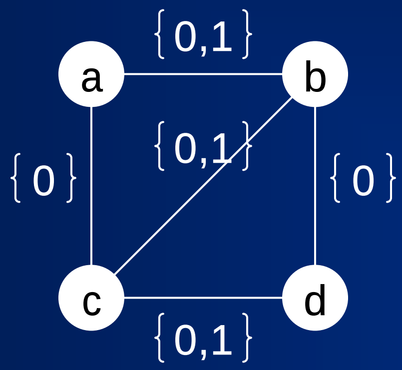
# FAP & T-Coloring

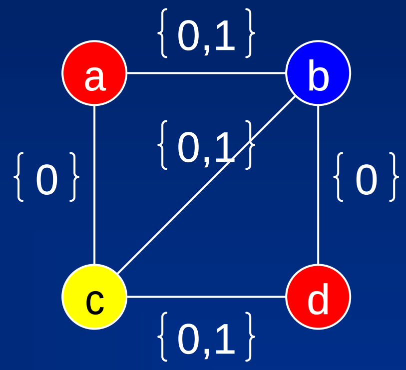



Sets of forbidden distances $T_{vw}$

$$\left|f_v - f_w\right| \notin T_{vw} \qquad T_{vw} = \{0,\ldots,d(vw)\text{-}1\}$$

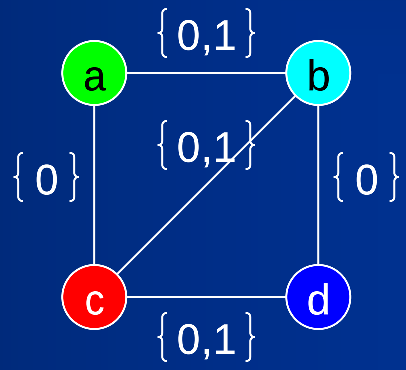


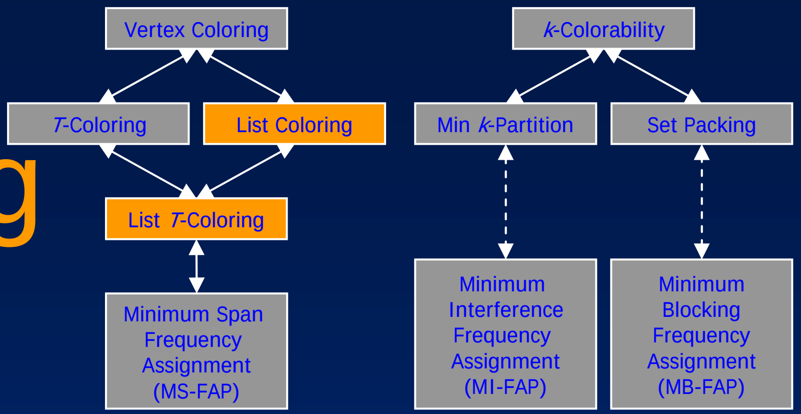

Colors: 3
Span: 4


Minimization of
number of colors and span
are not equivalent!

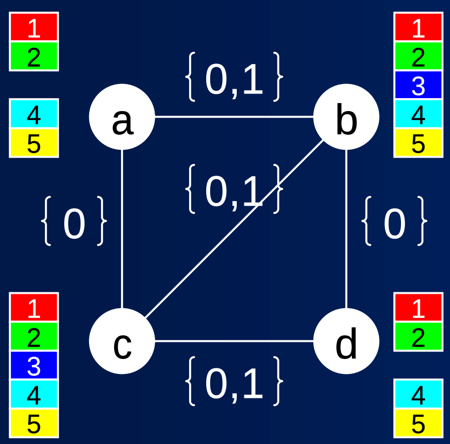


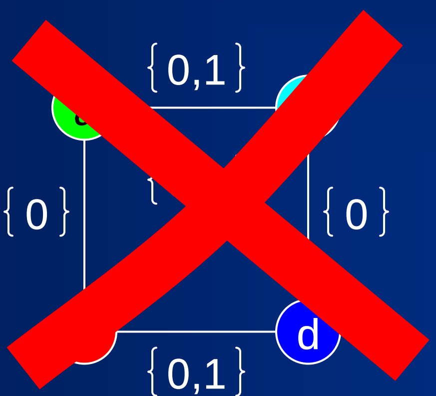

Colors: 4
Span: 3

# FAP & List-*T*-Coloring

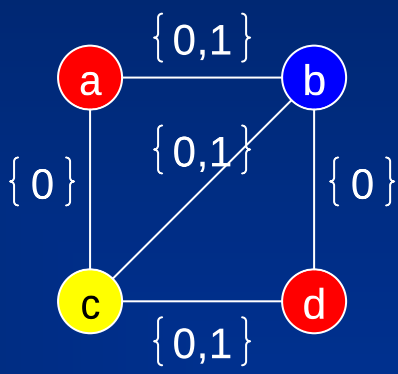



Locally blocked channels:
Sets of forbidden colors $B_v$



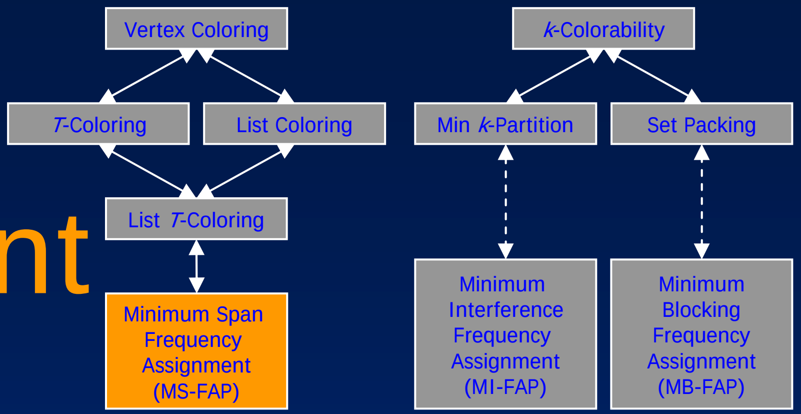




No solution with span 3 !



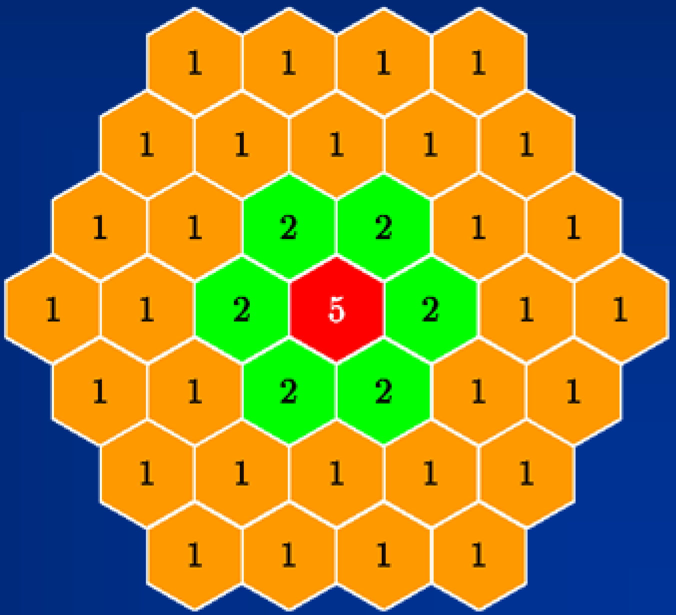

Colors: 3
Span: 4

# Minimum Span Frequency Assignment



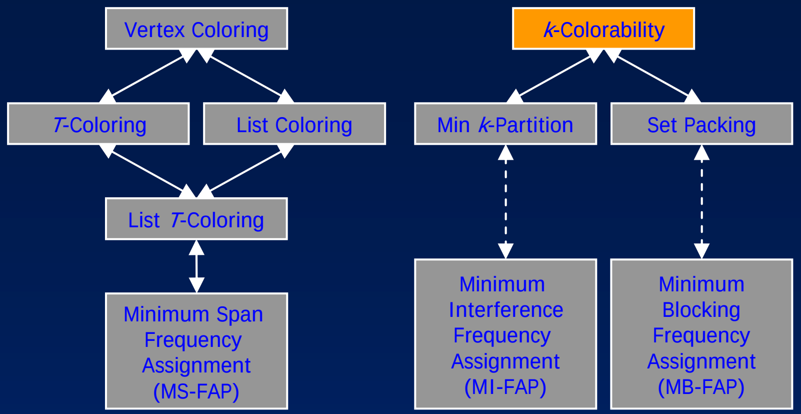

- List-T-Coloring (+ multiplicity)
- Benchmarks: Philadelphia instances

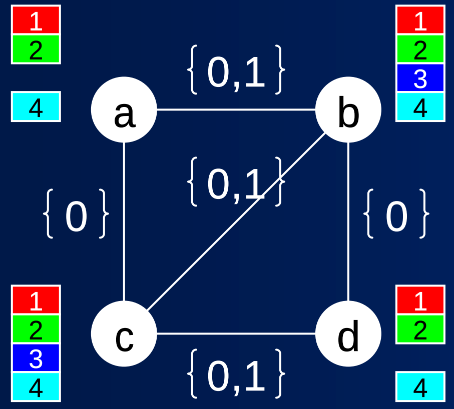

Channel requirements (P1)
Optimal span = 426




Separation distances

# Fixed Spectrum




License for frequencies {1,...,4}

No solution with span 3

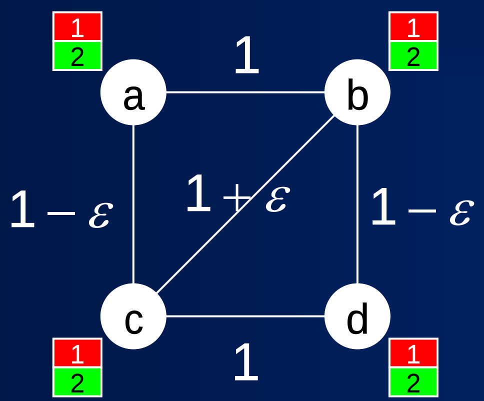
- Is the graph span-*k*-colorable?
- Complete assignment: minimize interference
- Partial assignment without interference

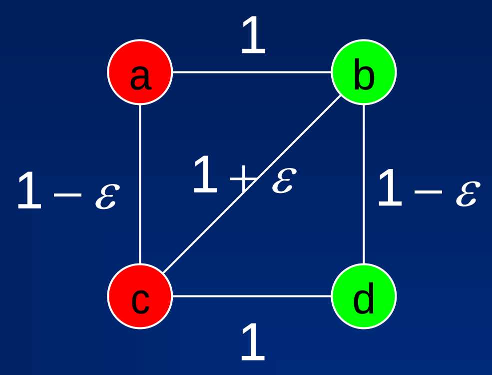
# Hard & Soft constraints

- How to evaluate "infeasible" plans?
  - Hard constraints: separation, local blockings
  - Soft constraints: co- and adjacent-channel interference
- Measure of violation of soft constraints: penalty functions

$$p_{vw}(f,g) = \begin{cases} c^{co}(vw) & \text{if } f = g \\ c^{ad}(vw) & \text{if } |f-g| = 1 \\ 0 & \text{otherwise} \end{cases}$$

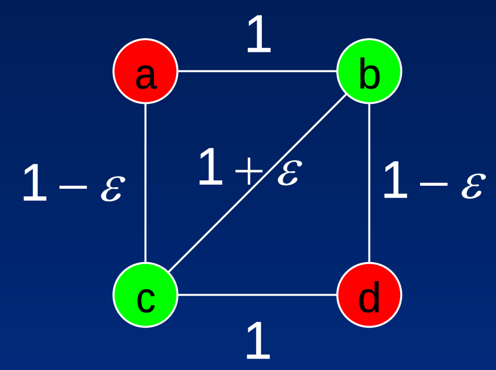
# Evaluation of infeasible plans




Total penalty: $2-2\varepsilon$

Maximum penalty: $1-\varepsilon$




Total penalty : $1+\varepsilon$

Maximum penalty : $1+\varepsilon$

- Minimizing total interference
- Minimizing maximum interference
  - Use of threshold value, binary search

# What is a good objective?

Keep interference information!

Use the available spectrum!

Minimize max interference

T-coloring (min span): Hale; Gamst; …

Minimize sum over interference

Duque-Anton et al.; Plehn; Smith et al.; …

Minimize max “antenna” interference

Fischetti et al.; Mannino, Sassano

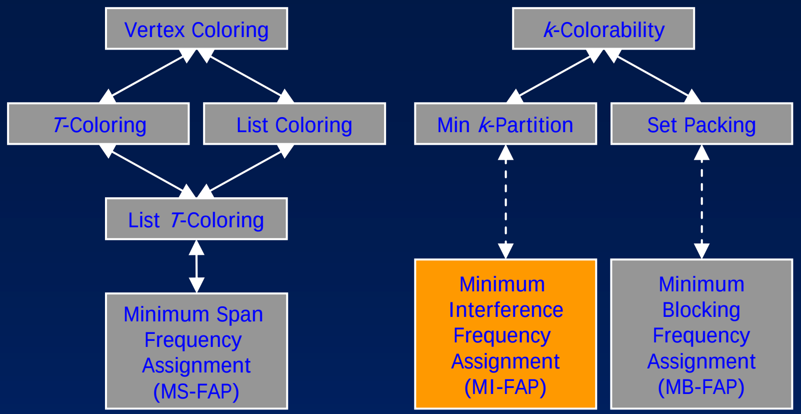
# Our Model

Carrier Network:

$$N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad})$$

- (V,E) is an undirected graph
- $C$ is an interval of integers (spectrum)
- $B_v \subseteq C$ for all $v \in V$ (blocked channels)
- $d : E \to Z_+$ (separation)
- $c^{co}, c^{ad} : E \to [0,1]$ (interference)

# Minimum Interference Frequency Assignment




## Integer Linear Program:

$$\min \quad \sum_{vw \in E^{co}} c_{vw}^{co} z_{vw}^{co} + \sum_{vw \in E^{ad}} c_{vw}^{ad} z_{vw}^{ad}$$

$$s.t. \quad \sum_{f \in F_v} x_{vf} = 1 \qquad \forall v \in V$$

$$x_{vf} + x_{wg} \leq 1 \qquad \forall vw \in E^d, |f - g| < d(vw)$$

$$x_{vf} + x_{wf} \leq 1 + z_{vw}^{co} \qquad \forall vw \in E^{co}, f \in F_v \cap F_w$$

$$x_{vf} + x_{wg} \leq 1 + z_{vw}^{ad} \qquad \forall vw \in E^{ad}, |f - g| = 1$$

$$x_{vf}, z_{vw}^{co}, z_{vw}^{ad} \in \{0,1\} \qquad \forall v \in V, f \in C \setminus B_v, \forall vw \in E^{co}, \forall vw \in E^{ad}$$

# A Glance at some Instances

| Instance | \|V\| | density [%] | minimum degree | average degree | maximum degree | diameter | clique number |
|---|---|---|---|---|---|---|---|
| k | 267 | 56,8 | 2 | 151,0 | 238 | 3 | 69 |
| B-0-E-20 | 1876 | 13,7 | 40 | 257,7 | 779 | 5 | 81 |
| f | 2786 | 4,5 | 3 | 135,0 | 453 | 12 | 69 |
| h | 4240 | 5,9 | 11 | 249,0 | 561 | 10 | 130 |

Expected graph properties: planarity,...

# Computational Complexity

Neither high quality nor feasibility are generally achievable within practical running times:

- Testing for feasibility is NP-complete.
- There exists an $\varepsilon > 0$ such that FAP cannot be “approximated” within a factor of $|V|^{\varepsilon}$ unless P = NP.

# Heuristic Solution Methods

- Greedy coloring algorithms,
- DSATUR,
- Improvement heuristics,
- Threshold Accepting,
- Simulated Annealing,
- Tabu Search,
- Variable Depth Search,
- Genetic Algorithms,
- Neural networks,
- etc.

# Heuristics

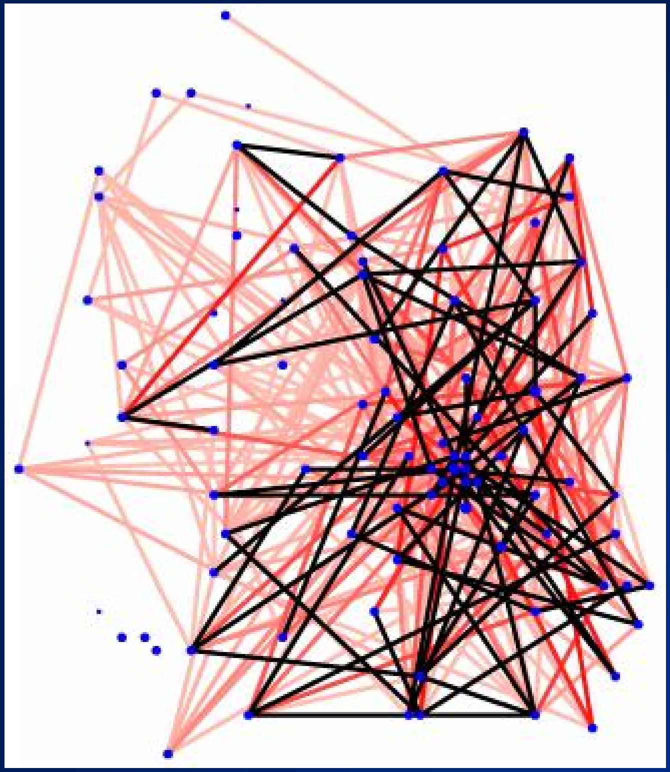
| Heuristic | Rating | Category |
| --- | --- | --- |
| T-coloring | o | construction heuristics |
| Dual Greedy | -- | construction heuristics |
| DSATUR with Costs | ++ | construction heuristics |
| Iterated 1-Opt | o | (randomized) local search |
| Simulated Annealing | + | (randomized) local search |
| Tabu-Search | o | (randomized) local search |
| Variable Depth Search | ++ | (randomized) local search |
| MCF | - | other improvement heuristics |
| B&C-based | + | other improvement heuristics |

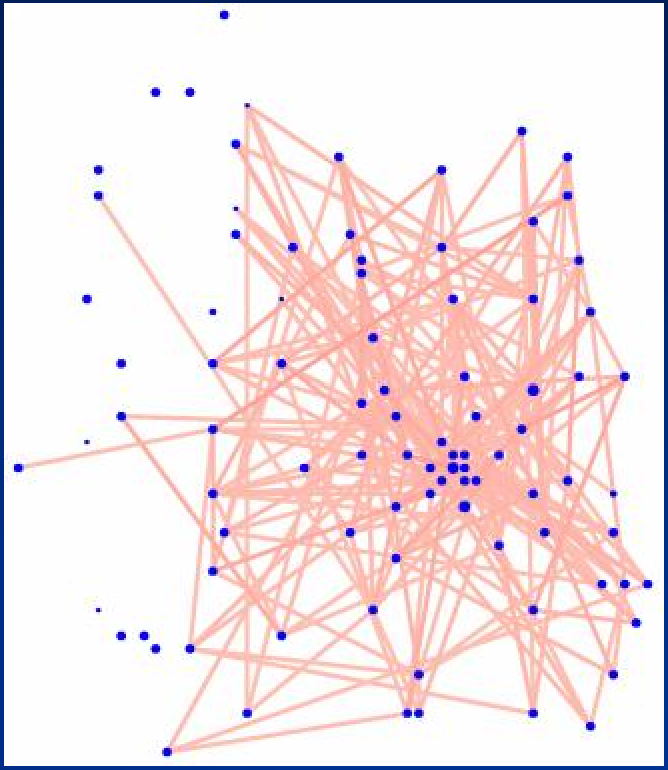
# Region with "Optimized Plan"

## Instance k, a "toy case" from practice

264 cells
267 TRXs
50 channels

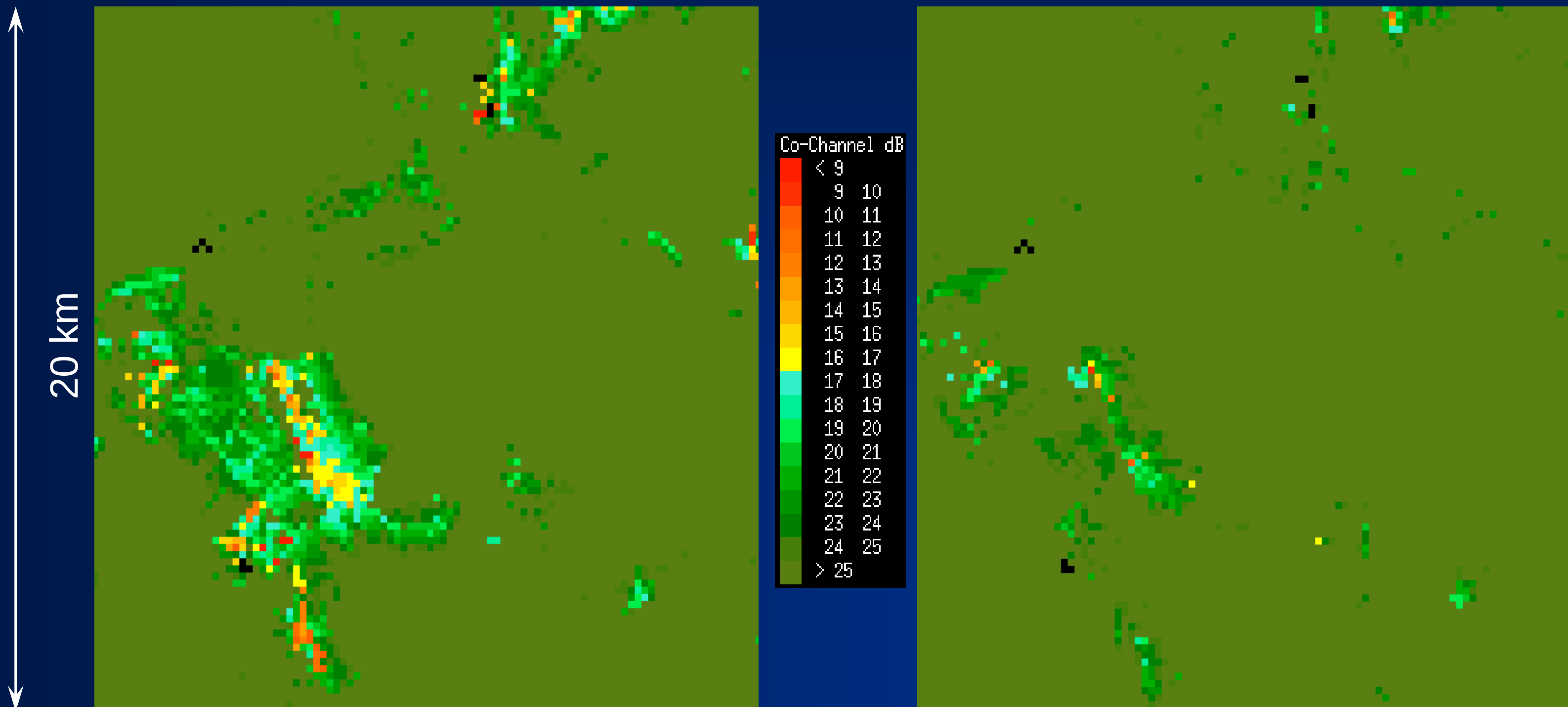
57% density
151 avg.deg.
238 max.deg.
69 clique size





DC5-VDS: Reduction 96,3%

# co-channel C/I worst Interferer



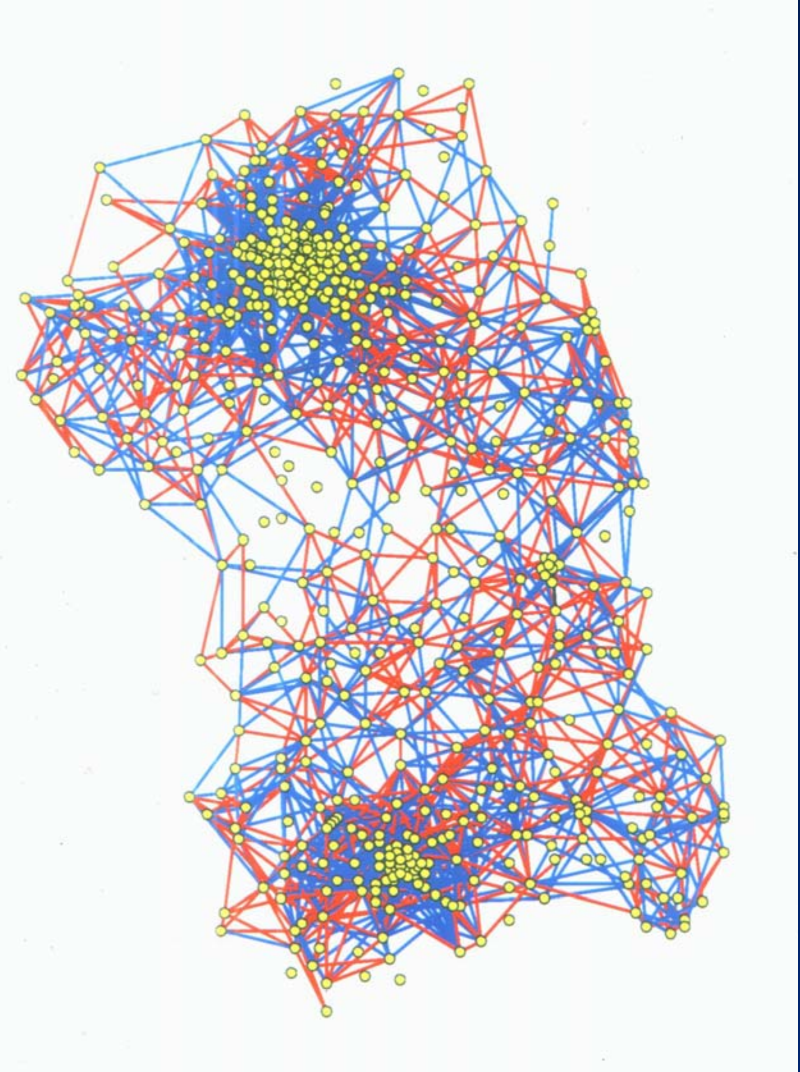

Mobile Systems International Plc.

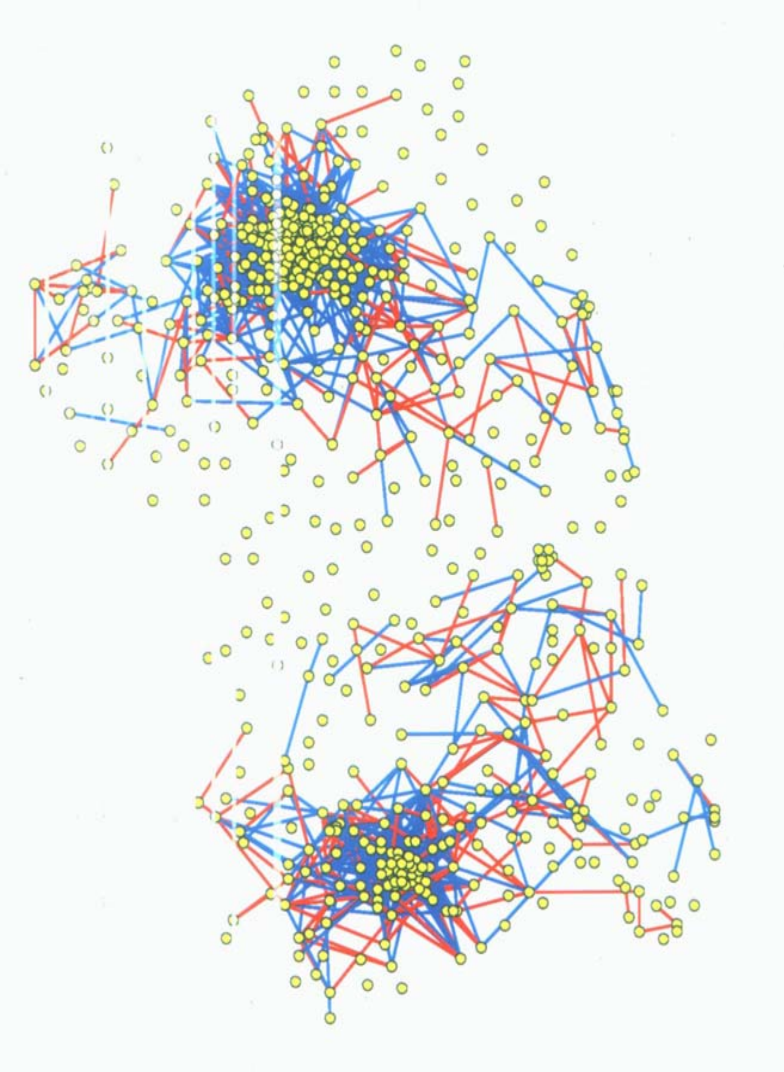
Mobile Systems International Plc.

Commercial software

DC5-IM

# Region Berlin - Dresden



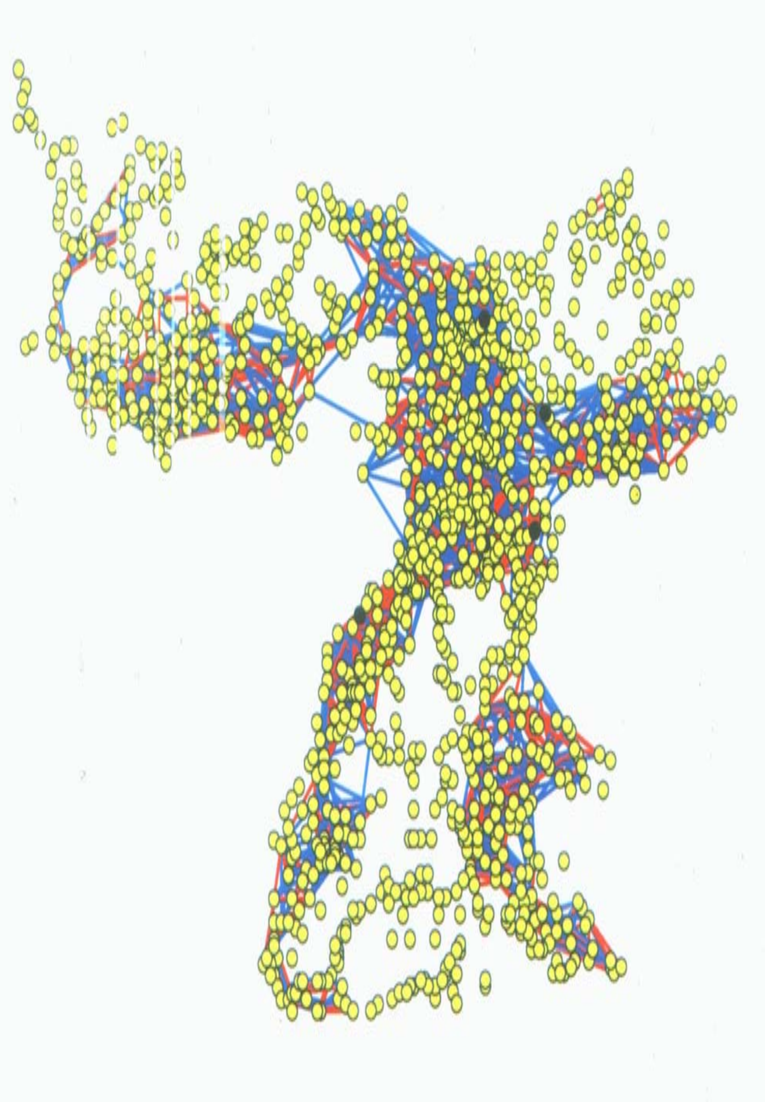


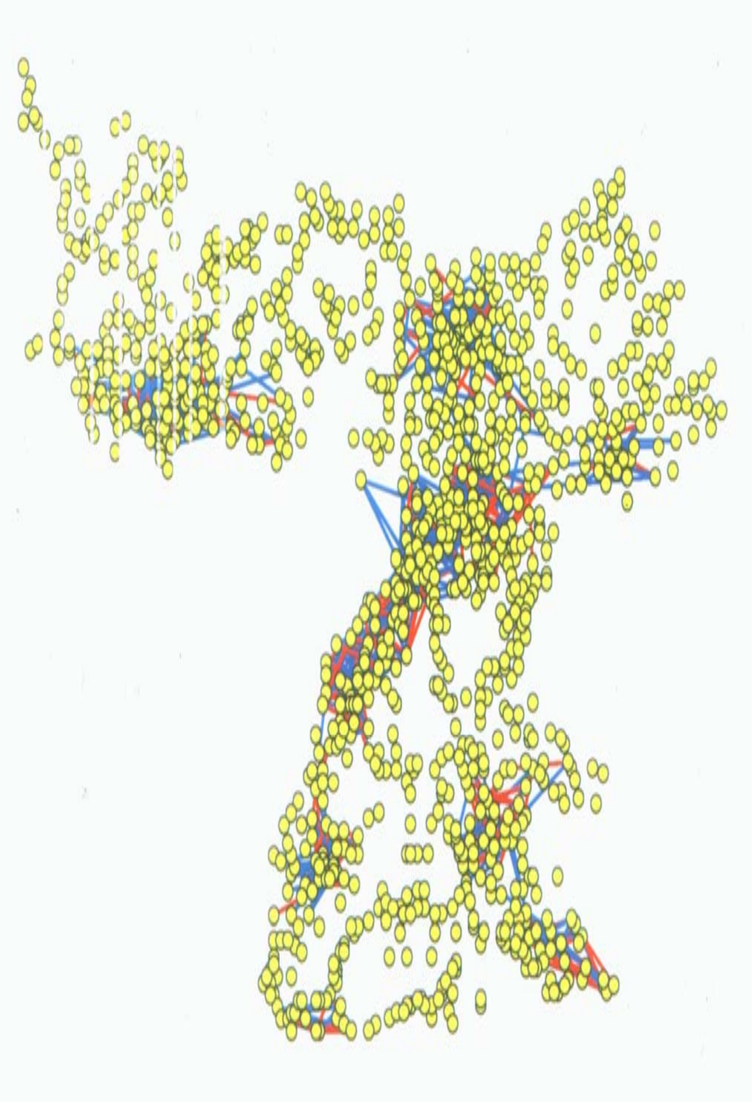
2877 carriers

50 channels

Interference reduction: 83.6%

# Region Karlsruhe





2877
Carriers

75 channels

Interference
Reduction:
83.9 %

# Guaranteed Quality

Optimal solutions are out of reach!

Enumeration: $50^{267} \approx 10^{197}$ combinations (for trivial instance k)

Hardness of approximation

Polyhedral investigation (IP formulation)

Aardal et al.; Koster et al.; Jaumard et al.; ...

Used for adapting to local changes in the network

Lower bounds - study of relaxed problems

# Lower Bounding Technology

- LP lower bound for coloring
- TSP lower bound for *T*-coloring
- LP lower bound for minimizing interference
- Tree Decomposition approach
- Semidefinite lower bound for minimizing interference

# Region with “Optimized Plan”

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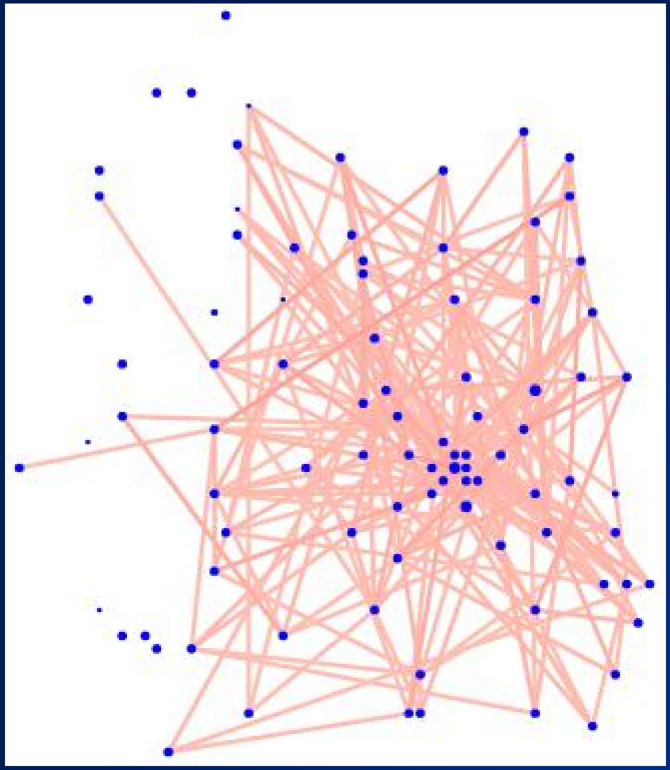
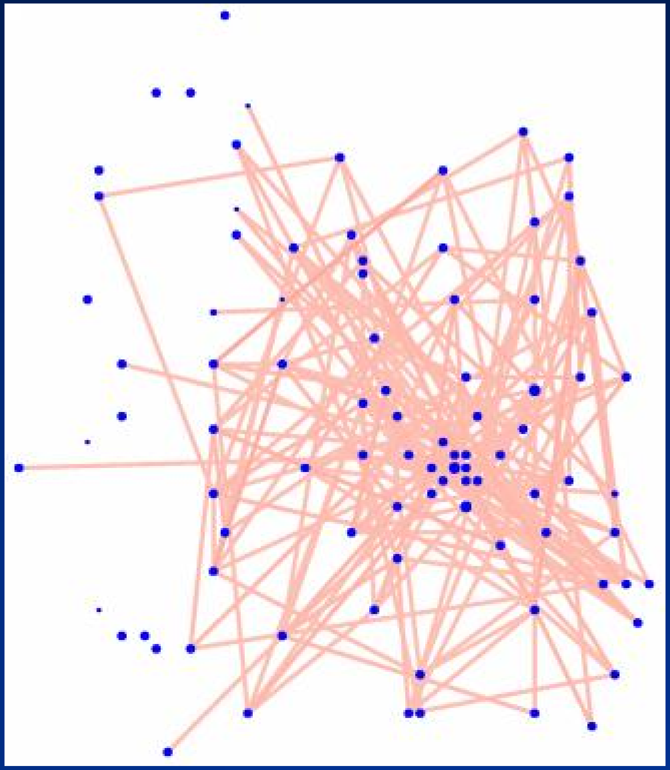


DC5-VDS



Further Reduction: 46.3%

# A Simplification of our Model

Simplified Carrier Network:

$$N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad})$$

- (V,E) is an undirected graph
- $C$ is an interval of integers (spectrum)
- $B_v \subseteq C$ for all $v \in V$ (blocked channels)
- $d : E \to Z_+$ {0, 1} (separation)
- $c^{co}, c^{ad} : E \to [0,1]$ (interference)

# MIN k-Partition

- No blocked channels
- No separation constraints larger than one
- No adjacent-channel interference

**min k-partition (max k-cut)**

Chopra & Rao; Deza et al.; Karger et al.; Frieze & Jerrum

IP, LP-based B&C, SDP

# MIN k-Partition

Given: an undirected graph $G = (V,E)$ together with real edge weights $w_{ij}$ and an integer $k$.

Find a partition of the vertex set into (at most) k sets $V_1, \ldots, V_k$ such that the sum of the edge weights in the induced subgraphs is minimal!

$$\min_{\substack{V_1,\ldots,V_k \\ \text{partition of } V}} \sum_{p=1}^{k} \sum_{i,j \in V_p} w_{ij}$$

NP-hard to approximate optimal solution value.

# Integer Linear Programmming

(ILP)

$$\min \sum_{i,j \in V} w_{ij} z_{ij}$$

$$z_{ih} + z_{hj} - z_{ij} \leq 1 \qquad \forall h, i, j \in V$$ -> partition consistent

$$\sum_{i,j \in Q} z_{ij} \geq 1 \qquad \forall\, Q \subseteq V \text{ with } |Q| = k+1$$ -> use at most k blocks

$$z_{ij} \in \{0, 1\}$$

## Number of ILP inequalities (facets)

| Instance* | \|V\| | k | Triangle | Clique Inequalities |
|---|---|---|---|---|
| **cell.k** | **69** | **50** | **157182** | **17231414395464984** |
| B-0-E | 81 | 75 | 255960 | 25621596 |
| **B-1-E** | **84** | **75** | **285852** | **43595145594** |
| B-2-E | 93 | 75 | 389298 | 1724861095493098563 |
| B-4-E | 120 | 75 | 842520 | 1334655509331585084721199905599180 |
| B-10-E | 174 | 75 | 2588772 | 361499854695979558347628887341189586948364637617230 |

# Vector Labeling

Lemma: For each k, n $(2 \le k \le n+1)$ there exist k unit vectors $u_1, \ldots, u_k$ in n-space, such that their mutual scalar product is $-1/(k-1)$. (This value is least possible.)

Fix $U = \{u_1, \ldots, u_k\}$ with the above property, then the min k-partition problem is equivalent to:

$$\min_{\substack{\phi: V \to U \\ i \mapsto \phi_i}} \sum_{ij \in E} \left( \frac{k-1}{k} \langle \phi_i, \phi_j \rangle + \frac{1}{k} \right) w_{ij}$$

$X = [\langle \phi_i, \phi_j \rangle]$ is positive semidefinite, has 1's on the diagonal, and the rest is either $-1/(k-1)$ or 1.

# Semidefinite Relaxation

(SDP)

$$\min \sum_{ij \in E(K_n)} w_{ij} \frac{(k-1)\,V_{ij}+1}{k}$$

$$V_{ii} = 1 \qquad \forall i \in V$$

$$V_{ij} \geq \frac{-1}{k-1} \qquad \forall i,j \in V$$

$$V \succeq 0$$

Solvable in polynomial time!

Given V, let $z_{ij} := ((k-1)\,V_{ij} + 1)/k$, then:

- $z_{ij}$ in [0,1]
- $z_{ih} + z_{ih} - z_{ij} < \sqrt{2}$ (<=1)
- $\sum_{i,j \text{ in } Q} z_{ij} > ½$ (>=1)

(SDP) is an approximation of (ILP)

# Computational Results

S. Burer, R.D.C Monteiro, Y. Zhang; Ch. Helmberg; J. Sturm

| Instance | clique cover | min k-part. | *heuristic* | clique cover | min k-part. | *heuristic* |
|---|---|---|---|---|---|---|
| **cell.k** | **0,0206** | **0,0206** | ***0,0211*** | **0,0248** | **0,1735** | **0,4023** |
| B-0-E | 0,0016 | 0,0013 | *0,0016* | 0,0018 | 0,0096 | *0,8000* |
| **B-1-E** | **0,0063** | **0,0053** | ***0,0064*** | **0,0063** | **0,0297** | **0,8600** |
| B-2-E | 0,0290 | 0,0213 | *0,0242* | 0,0378 | 0,4638 | *3,1700* |
| B-4-E | 0,0932 | 0,2893 | *0,3481* | 0,2640 | 4,3415 | *17,7300* |
| B-10-E | 0,2195 | 2,7503 | *3,2985* | | | *146,2000* |
| | maximal clique | | | entire scenario | | |

Lower bound on co-channel interference by a factor of 2 to 85 below co- and adjacent-channel interference of best known assignment.

# Semidefinite Conclusions

Lower bounding via
Semidefinite Programming works (somewhat),
at least better than LP!

- Challenging computational problems
- Lower bounds too far from cost of solutions to give strong quality guarantees
- How to produce good k-partitions starting from SDP solutions?

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http://fap.zib.de

European Graduate Program
Berlin - Zürich
Combinatorics

# 01M1 Lecture
# Frequency Assignment for GSM Mobile Phone Systems

# The End

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