05M2 Lecture
Telecommunication Network Design

Martin Grötschel
Block Course at TU Berlin
"Combinatorial Optimization at Work"
October 4 - 15, 2005
Contents

1. Telecommunication: The General Problem
2. Newspaper Reports
3. Survivability
4. Integrated Topology, Capacity, and Routing Optimization as well as Survivability Planning
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What is the Telecom Problem?

Design excellent technical devices and a robust network that survives all kinds of failures and organize the traffic such that high quality telecommunication between very many individual units at many locations is feasible at low cost!
What is the Telecom Problem?

Design excellent technical devices and a robust network that survives all kinds of failures and organize the traffic such that high quality telecommunication between very many individual units at many locations is feasible at low cost!

Approach in Practice:
- Decompose whenever possible.
- Look at a hierarchy of problems.
- Address the individual problems one by one.
- Recompose to find a good global solution.

This problem is too general to be solved in one step.
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Clyde Monma

- Cornell & Cayuga Lake 1987
Martin Grötschel
USA 1987-1988

THE NIGHTMARE ON LINCOLN ST.

Severed line snags calls long-distance

Illinois Bell experiences its worst service disaster in history as an extra-alarm fire silences phones for nearly two weeks.

The Star Ledger

Date: September 22, 1987

Damage to fiber cable hinders phone service

By TED SHERMAN

Telephone service was disrupted throughout the Northeast yesterday, after a major fiber optic cable was severed north of Trenton.

The problem briefly knocked out all voice and data circuits on American Telephone & Telegraph Co.'s main East Coast fiber cable, which runs from Cambridge, Mass., to Arlington, Va.
Cable snaps, snags area phone calls

American Telephone & Telegraph Co. (AT&T) service along the East Coast was disrupted yesterday when a telephone cable snapped about 13 miles southwest of Newark.

Phone snafu isolates New Jersey
Long-distance cable severed

By J.D. Solomon, Darryl Romano and Robin Siegel
Courier-News Staff Writers

American Telephone & Telegraph Co.'s long-distance telephone service throughout the East Coast was disrupted for about 10 hours yesterday when a major transmission cable was severed by a construction crew working in a Sayreville train yard, AT&T officials said.

Problems were especially severe in portions of central Jersey, and customers closest to Sayreville were expected to be among the last to have their service fully restored, an AT&T spokesman said.

The break in the 1-inch fiber optic cable occurred at 12:18 p.m. Service was restored gradually as computers rerouted calls through other points. Almost all service was restored by about 7 p.m., AT&T said.

“It's almost like a highway in that you have to go along it to get from, say, New York to Florida. This is a major blockage affecting the whole East Coast.”

Jim Nelson, AT&T district manager

In addition to affecting phone service, private customers whose computer networks use AT&T phone line transmissions experienced service problems, Nelson said.

Harry Baumgartner, a spokesman for AT&T in the Basking Ridge section of Bernards, said calls between area codes on the East Coast
Keeping the phone lines open

The telephone network’s moment-by-moment reconfigurations to meet emergencies real and simulated add up to de facto risk management

necessary, Joe as the repeater to get the for done for pure multiple repeaters mine a multir
Sometime after 4:00 p.m. on Sunday, May 8, 1988, on the first floor of a telephone switching center in the Chicago suburb of Hinsdale, a metal cable sheath came into contact with a damaged, energized power cable and touched off an electrical fire. Thus began one of the worst disasters in the history of U.S. telephony.

By the time the smoke had cleared, 35,000 residential and business customers had no service at all, and others served by some 120,000 trunk lines lacked long-distance service. A facility that had relayed 3.5 million telephone calls a day was a messy mix of destroyed and damaged equipment, much of it fast corroding from the caustic combination of water and vapors released by burning paneling.

The community soon found out just how much it depended on telephony. Chicago’s busy O’Hare Airport came to a standstill while technicians jury-rigged some telephone lines for the Federal Aviation Administration to use for air-traffic control. 

Emergency 911 service was no more. Cellular telephones were also out because Hinsdale had housed a key installation in the local system. Automatic teller machines in the Chicago area, which transmit transaction details over telephone lines, were down. Pizza makers, florists, real estate agents, stockbrokers, “mom-and-pop” proprietors, boyfriends and girlfriends—all lost a vital link.

Some areas had no service for a month, and dollar estimates of lost business ranged from the hundreds of millions to the tens of billions.
Graue Mattscheiben und stille Telefone

Totalausfall in Charlottenburg und Spandau bringt Tausende in Rage / Panne bei Bauarbeiten

24. 12. 94
von Bernhard Koch


Die Höhe der Reparaturkosten, mögliche Regressforderungen von Geschädigten sowie die Summe der Gebührenausfälle aufgrund des gestörten Telefonverkehrs seien noch nicht abzusehen, sagte Krüger. Privatkunden, so der Telekom-Sprecher, bekamen grundsätzlich nur dann Vergütungen, wenn das Telefon länger als fünf Tage ausfalle.
High-Tech Terrorism 1995

Anschläge

Stummer Rebellen

Erstmals in Deutschland schlugen in Frankfurt High-Tech-Terroristen gegen die Kommunikationsgesellschaft zu.

Die Täter kamen in der Nacht, im Januar nach dem New Year’s Eve, in die Stadt und begannen mit ihren Anschlägen. Sie benutzten einen Computer und eine ferngesteuerte Bombe, um ihre Abrechnungsziele zu erreichen.

Buchungsschalter im Frankfurter Flughafen: Chaos durch Kabel-Geiz


Die gigantischen Datenmengen der Wirtschaft lassen sich nicht angenehm der
Glasfaserkabel beschädigt
tausende ohne Anschluß


Zwei Drittel der Kapazität stand still

Telefon nach Tirol war unterbrochen


Türkei: Ericsson baut GSM-Netz

WIEN (Red.). Der türkische

ST. PÖLTEN (APA). Es ist ein schwerer Kabelschaden im Bereich Prinzendorf zwischen St. Pölten und Melk.
Bagger kappte Telefon-Hauptkabel

Bei Bauarbeiten an der Westbahnhofstrecke wurde am Donnerstag ein Hauptverkehrsleitungs-Kabel der Telekom beschädigt. Die Verbindungen nach Westösterreich waren bis zum Nachmittag zu zwei Dritteln gestört.

WIAZ (organ.): Durch einen schweren Kabelschnur der Telekom Austria im Bereich von Prinzendorf im Niederösterreicher war am Donnerstag die Telefondienste nach West-Österreich weitgehend unterbrochen. Ais Auslandsanschluss konnte nur eingeschränkt durchgeführt werden.


Die Unfähigkeit liegt zwischen St. Pölten und Markt. Der Neubau von der 300-Meilen-AG (HL-AG) die Westbahnhofstrecke eingebaut. Der Bagger ertappte das Kabel während Bauarbeiten an einer Stelle, an die ein Telekom-Kabel in das Gebäude durch die Glasfasersaile. 38

Bagger legte Telefonverbindung in den Westen lähm


Die Verbindungen nach Salzburg, Tirol und Vorarlberg waren erheblich gestört, teilweise sogar ausgeschaltet. Das Problem: Der durchgeschnittene Hochleistungsnetz bestand aus ei-
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Some Examples

- Locating the sites for antennas (TRXs) and base transceiver stations (BTSs)
- Assignment of frequencies to antennas
- Cryptography and error correcting encoding for wireless communication
- Clustering BTSs
- Locating base station controllers (BSCs)
- Connecting BTSs to BSCs
Network Design: Tasks to be solved

Some Examples (continued)

- Locating Mobile Switching Centers (MSCs)
- Clustering BSCs and Connecting BSCs to MSCs
- Designing the BSC network (BSS) and the MSC network (NSS or core network)
  - Topology of the network
  - Capacity of the links and components
  - Routing of the demand
  - Survivability in failure situations

Most of these problems turn out to be Combinatorial Optimization or Mixed Integer Programming Problems
Chapter 10

Design of Survivable Networks

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I. Overview

This chapter focuses on the important practical and theoretical problem of designing survivable communication networks, i.e., communication networks that are still functional after the failure of certain network components. We motivate this topic in Section 2 by using the example of fiber optic communication network design for telephone companies. A very general model (for undirected networks) is presented in Section 3 which includes practical, as well as theoretical, problems, including the well-studied minimum spanning tree, Steiner tree, and minimum cost k-connected network design problems.
Hence, it is vital to take into account such failure scenarios and their potential negative consequence when designing fiber communication networks. Recall that one of the major functions of a communication network is to provide connectivity between users in order to provide a desired service. We use the term ‘survivability’ to mean the ability to restore network service in the event of a catastrophic failure, such as the complete loss of a transmission link or a facility switching node. Service could be restored by means of routing traffic around the damage through other existing facilities and switches, if this contingency is provided for in the network architecture. This requires additional connectivity in the network topology and a means to automatically reroute traffic after the detection of a failure.

A network topology could provide protection against a single link failure if it remains connected after the failure of any single link. Such a network is called ‘two-edge connected’ since at least two edges have to be removed in order to disconnect the network. However, if there is a node in the network whose removal does disconnect the network, such a network would not protect against a single node failure. Protection against a single node failure can be provided in an analogous manner by ‘two-node connected’ networks.

In the case of fiber communication networks for telephone companies, two-connected topologies provide an adequate level of survivability since most failures usually can be repaired relatively quickly and, as statistical studies have revealed, it is unlikely that a second failure will occur in their duration. However, for other applications it may be necessary to provide higher levels of connectivity.
The Data

The survivability conditions require that the network satisfy certain edge and node connectivity requirements. To specify these, three nonnegative integers $r_{st}$, $k_{st}$ and $d_{st}$ are given for each pair of distinct nodes $s, t \in V$. The numbers $r_{st}$ represent the edge survivability requirements, and the numbers $k_{st}$ and $d_{st}$ represent the node survivability requirements; this means that the network $N = (V, F)$ to be designed has to have the property that, for each pair $s, t \in V$ of distinct nodes, $N$ must contain at least $r_{st}$ edge-disjoint $[s, t]$-paths, and that the removal of at most $k_{st}$ nodes (different from $s$ and $t$) from $N$ must leave at least $d_{st}$ edge-disjoint $[s, t]$-paths. (Clearly, we may assume that $k_{st} \leq |V| - 2$ for all $s, t \in V$, and we will do this throughout this chapter). These conditions ensure that some communication path between $s$ and $t$ will survive a prespecified level of combined failures of both nodes and links. The levels of survivability specified depend on the relative importance placed on maintaining connectivity between different pairs of offices.

Given $G = (V, E)$ and $r, k, d \in \mathbb{Z}^+_{\text{in}}$, extend the functions $r$ and $d$ to functions operating on sets by setting

$$
\text{con}(W) := \max\{r_{st} \mid s \in W, \ t \in V \setminus W\} \quad (1)
$$

and

$$
d(Z, W) := \max\{d_{st} \mid s \in W \setminus Z, \ t \in V \setminus (Z \cup W)\} \text{ for } Z, W \subseteq V. \quad (2)
$$

We call a pair $(Z, W)$ with $Z, W \subseteq V$ eligible (with respect to $k$) if $Z \cap W = \emptyset$ and $|Z| = k_{st}$ for at least one pair of nodes $s, t$ with $s \in W$ and $t \in V \setminus (Z \cup W)$. 
Let us now introduce a variable \( x_e \) for each edge \( e \in E \), and consider the vector space \( \mathbb{R}^E \). Every subset \( F \subseteq E \) induces an incidence vector \( \chi^F = (\chi^F_e)_{e \in E} \in \mathbb{R}^E \) by setting \( \chi^F_e := 1 \) if \( e \in F \), \( \chi^F_e := 0 \) otherwise; and vice versa, each 0/1-vector \( x \in \mathbb{R}^E \) induces a subset \( F^x := \{ e \in E \mid x_e = 1 \} \) of the edge set \( E \) of \( G \). If we speak of the incidence vector of a path in the sequel we mean the incidence vector of the edges of the path. We can now formulate the network design problem introduced above as an integer linear program with the following constraints.

1. \( \sum_{i \in W} \sum_{j \in V \setminus W} x_{ij} \geq \text{con}(W) \) for all \( W \subseteq V, \emptyset \neq W \neq V \),
2. \( \sum_{i \in W} \sum_{j \in V \setminus (Z \cup W)} x_{ij} \geq d(Z, W) \) for all eligible \((Z, W)\) of subsets of \( V \),
3. \( 0 \leq x_{ij} \leq 1 \) for all \( ij \in E \),
4. \( x_{ij} \) integral for all \( ij \in E \).

Note that if \( N - Z \) contains at least \( d_{st} \) edge-disjoint \([s, t]\)-paths for each pair \( s, t \) of distinct nodes in \( V \) and for each set \( Z \subseteq V \setminus \{s, t\} \) with \( |Z| = k_{st} \), and if \( r_{st} = k_{st} + d_{st} \), then all node survivability requirements are satisfied, i.e., inequalities of type (3ii) need not be considered for node sets \( Z \subseteq V \setminus \{s, t\} \) with \( |Z| < k_{st} \). It follows from Menger’s theorem (see [Frank, 1995]) that, for every feasible solution \( x \) of (3), the subgraph \( N = (V, F^x) \) of \( G \) defines a network that satisfies the given edge and node survivability requirements.
To obtain a better LP-relaxation of (3) than the one arising from dropping the integrality constraints (3iv), we define the following polytope. Let $G = (V, E)$ be a graph, let $E_V := \{ st \mid s, t \in V, s \neq t \}$, and let $r, k, d \in \mathbb{Z}_+^E$ be given. Then

$$\text{CON}(G; r, k, d) := \text{conv}\{x \in \mathbb{R}^E \mid x \text{ satisfies (3i)--(3iv)}\}$$ (4)

is the polytope associated with the network design problem given by the graph $G$ and the edge and node survivability requirements $r$, $k$, and $d$. (Above ‘conv’ denotes the convex hull operator.) In the sequel, we will study $\text{CON}(G; r, k, d)$ for various special choices of $r$, $k$ and $d$. Let us mention here a few general properties of $\text{CON}(G; r, k, d)$ that are easy to derive.

Let $G = (V, E)$ be a graph and $r, k, d \in \mathbb{Z}_+^E$ be given as above. We say that $e \in E$ is essential with respect to $(G; r, k, d)$ (short: $(G; r, k, d)$-essential) if $\text{CON}(G - e; r, k, d) = \emptyset$. In other words, $e$ is essential with respect to $(G; r, k, d)$ if its deletion from $G$ results in a graph such that at least one of the survivability requirements cannot be satisfied. We denote the set of edges in $E$ that are essential with respect to $(G; r, k, d)$ by $\text{ES}(G; r, k, d)$. Clearly, for all subsets $F \subseteq E \setminus \text{ES}(G; r, k, d)$, $\text{ES}(G; r, k, d) \subseteq \text{ES}(G - F; r, k, d)$ holds. Let $\dim(S)$ denote the dimension of a set $S \subseteq \mathbb{R}^n$, i.e., the maximum number of affinely independent elements in $S$ minus 1. Then one can easily prove the following two results [see Grötschel & Monma, 1990].

**Theorem 1.** Let $G = (V, E)$ be a graph and $r, k, d \in \mathbb{Z}_+^E$ such that $\text{CON}(G; r, k, d) \neq \emptyset$. Then

$$\text{CON}(G; r, k, d) \subseteq \{x \in \mathbb{R}^E \mid x_e = 1 \text{ for all } e \in \text{ES}(G; r, k, d)\}, \text{ and}$$

$$\dim(\text{CON}(G; r, k, d)) = |E| - |\text{ES}(G; r, k, d)|.$$
**Theorem 2.** Let $G = (V, E)$ be a graph and $r, k, d \in \mathbb{Z}_+^E$ such that $\text{CON}(G; r, k, d) \neq \emptyset$. Then

(a) $x_e \leq 1$ defines a facet of $\text{CON}(G; r, k, d)$ if and only if $e \in E \setminus \text{ES}(G; r, k, d)$;

(b) $x_e \geq 0$ defines a facet of $\text{CON}(G; r, k, d)$ if and only if $e \in E \setminus \text{ES}(G; r, k, d)$ and $\text{ES}(G; r, k, d) = \text{ES}(G - e; r, k, d)$.

Theorems 1 and 2 solve the dimension problem and characterize the trivial facets. But these characterizations are (in a certain sense) algorithmically intractable as the next observation shows, which follows from results of Ling & Kameda [1987].

**Remark 1.** The following three problems are NP-hard.

**Instance:** A graph $G = (V, E)$ and vectors $r, k, d \in \mathbb{Z}_+^E$.

**Question 1:** Is $\text{CON}(G; r, k, d)$ nonempty?

**Question 2:** Is $e \in E$ $(G; r, k, d)$-essential?

**Question 3:** What is the dimension of $\text{CON}(G; r, k, d)$?

However, for most cases of practical interest in the design of survivable networks, the sets $\text{ES}(G; r, k, d)$ of essential edges can be determined easily, and thus the trivial LP-relaxation of (3) can be set up without difficulties by removing the redundant inequalities identified by Theorem 2.
For any subset of edges $F \subseteq E$, we let $x(F)$ stand for the sum $\sum_{e \in F} x_e$. Consider the following integer linear program for a graph $G = (V, E)$ with edge costs $c_e$ for all $e \in E$ and node types $r_s$ for all $s \in V$ [using (5) in the definition of $\operatorname{con}(W)$ in (1)]:

To tie this notation with the previously introduced more general concept, note that

$$k\operatorname{ECON}(G; r) = \operatorname{CON}(G; r', 0, 0),$$

where $r' \in \mathbb{R}^{V \times V}$ with $r'_{st} := \min\{r_s, r_t\}$ for all $s, t \in V$. Also, if there are no parallel edges then

$$k\operatorname{NCON}(G; r) = \operatorname{CON}(G; r', k', d'),$$

where $k'_{st} := \max\{0, r'_{st} - 1\}$ for all $s, t \in V$ and $d' := r' - k'$.

It follows from Menger’s theorem that the feasible solutions of (6) are the incidence vectors of edge sets $F$ such that $N = (V, F)$ satisfies all node survivability conditions; i.e., (6) is an integer programming formulation of the $k\operatorname{NCON}$ problem. Deleting inequalities (6ii) we obtain, again from Menger’s theorem, an integer programming formulation for the $k\operatorname{ECON}$ problem. The inequalities of type (6i) will be called cut inequalities and those of type (6ii) will be called node cut inequalities.

The polyhedral approach to the solution of the $k\operatorname{NCON}$ (and similarly the $k\operatorname{ECON}$) problem consists of studying the polyhedron obtained by taking the convex hull of the feasible solutions of (6). We set

$$k\operatorname{NCON}(G; r) := \operatorname{conv}\{ x \in \mathbb{R}^E \mid x \text{ satisfies (6i)--(6iv)}\},$$

$$k\operatorname{ECON}(G; r) := \operatorname{conv}\{ x \in \mathbb{R}^E \mid x \text{ satisfies (6i), (6iii) and (6iv)}\}.$$
How does one find further classes of valid inequalities? One approach is to infer inequalities from structural investigations. For instance, the cut inequalities ensure that every cut separating two nodes contains at least $r_{st}$ edges. These correspond to partitioning the node set into two parts and guaranteeing that there are enough edges linking them. We can generalize this idea as follows. Let us call a system $W_1, \ldots, W_p$ of nonempty subsets of $V$ with $W_i \cap W_j = \emptyset$ for $1 \leq i < j \leq p$, and $W_1 \cup \ldots \cup W_p = V$ a partition of $V$ and let us call

$$
\delta(W_1, \ldots, W_p) := \{uv \in E \mid \exists i, j, 1 \leq i, j \leq p, i \neq j \text{ with } u \in W_i, v \in W_j\}
$$

a multicut or $p$-cut (if we want to specify the number $p$ of shores $W_1, \ldots, W_p$ of the multicut). Depending on the numbers $\text{con}(W_1), \ldots, \text{con}(W_p)$, any survivable network $(V, F)$ will have to contain at least a certain number of edges of the multicut $\delta(W_1, \ldots, W_p)$. For every partition it is possible to compute a lower bound of this number, and thus to derive a valid inequality for every node partition (resp. multicut). This goes as follows.

Suppose $W_1, \ldots, W_p$ is a partition of $V$ such that $\text{con}(W_i) \geq 1$ for $i = 1, \ldots, p$. Let $I_1 := \{i \in \{1, \ldots, p\} \mid \text{con}(W_i) = 1\}$, and $I_2 := \{i \in \{1, \ldots, p\} \mid \text{con}(W_i) \geq 2\}$. Then the partition inequality (or multicut inequality) induced by $W_1, \ldots, W_p$ is defined as

$$
x(\delta(W_1, \ldots, W_p)) = \frac{1}{2} \sum_{i=1}^{p} x(\delta(W_i)) \geq \begin{cases} \left\lfloor \frac{1}{2} \sum_{i \in I_2} \text{con}(W_i) \right\rfloor + |I_1| & \text{if } I_2 \neq \emptyset, \\ p - 1 & \text{if } I_2 = \emptyset. \end{cases} \tag{10}
$$

Every partition inequality is valid for $k\text{ECON}(G; r)$ and thus for $k\text{NCON}(G; r)$. 
b-matchings and r-covers

on b-matching, see Edmonds [1965]. Edmonds proved that, for any vector \( b \in \mathbb{Z}_+^V \), the vertices of the polyhedron defined by

\[
\begin{align*}
(\text{i}) & \quad y(\delta(v)) \leq b_v \quad \text{for all } v \in V, \\
(\text{ii}) & \quad y(E(H)) + y(\mathcal{T}) \leq \left[ \frac{1}{2} \sum_{v \in H} (b_v + |\mathcal{T}|) \right] \quad \text{for all } W \subseteq V \\
(\text{iii}) & \quad 0 \leq y_e \leq 1
\end{align*}
\]

are precisely the incidence vectors of all (1-capacitated) b-matchings of \( G \), i.e., of edge sets \( M \) such that no node \( v \in V \) is contained in more than \( b_v \) edges of \( M \). For the case \( b_v := |\delta(v)| - r_v \), the b-matchings \( M \) are nothing but the complements \( M = E \setminus F \) of r-covers \( F \) of \( G \). Using the transformation \( x := 1 - y \) and \( T := \delta(H) \setminus \mathcal{T} \) we obtain the system

\[
\begin{align*}
(\text{i}) & \quad x(\delta(v)) \geq r_v \quad \text{for all } v \in V, \\
(\text{ii}) & \quad x(E(H)) + x(\delta(H) \setminus T) \geq \left[ \frac{1}{2} \sum_{v \in H} (r_v - |T|) \right] \quad \text{for all } H \subseteq V \\
(\text{iii}) & \quad 0 \leq x_e \leq 1
\end{align*}
\]

(14) gives a complete description of the convex hull of the incidence vectors of all r-covers of \( G \). We call the inequalities (14ii) r-cover inequalities. Since every solution of the kECON problem for \( G \) and \( r \) is an r-cover, all inequalities (14ii) are valid for kECON\((G; r)\). It is a trivial matter to observe that those inequalities (14ii) where \( \sum_{v \in H} r_v - |T| \) is even are redundant. For the case \( r_v = 2 \) for all
Generalizations of r-cover inequalities

Based on these observations one can extend inequalities (14ii) to more general classes of inequalities valid for \( k\text{ECON}(G; r) \) (but possibly not valid for the \( r \)-cover polytope). We present here one such generalization.

Let \( H \) be a subset of \( V \) called the handle, and \( T \subseteq \delta(H) \) with \( |T| \) odd and \( |T| \geq 3 \). For each \( e \in T \), let \( T_e \) denote the set of the two end nodes of \( e \). The sets \( T_e, e \in T \), are called teeth. Let \( H_1, \ldots, H_p \) be a partition of \( H \) into nonempty pairwise disjoint subsets such that \( r(H_i) \geq 1 \) for \( i = 1, \ldots, p \), and \( |H_i \cap T_e| \leq r(H_i) - 1 \) for all \( i \in \{1, \ldots, p\} \) and all \( e \in T \). Let \( I_1 := \{ i \in \{1, \ldots, p\} \mid r(H_i) = 1 \} \) and \( I_2 := \{ i \in \{1, \ldots, p\} \mid r(H_i) \geq 2 \} \). We call

\[
x(E(H)) - \sum_{i=1}^{p} x(E(H_i)) + x(\delta(H) \setminus T) \geq \left\lfloor \frac{1}{2} \sum_{i \in I_2} (r(H_i) - |T|) \right\rfloor + |I_1|
\]

(15)

the lifted \( r \)-cover inequality (induced by \( H_1, \ldots, H_p, T \)). All inequalities of type (15) are valid for \( k\text{ECON}(G; r) \).
**Theorem 2.** Let $G = (V, E)$ be a graph and $r \in \mathbb{Z}_+^V$ such that $k\text{ECON}(G; r)$ (respectively, $k\text{NCON}(G; r)$) is full-dimensional. Then

a. $x_e \leq 1$ defines a facet of $k\text{ECON}(G; r)$ (respectively, $k\text{NCON}(G; r)$) for all $e$;

b. $x_e \geq 0$ defines a facet of $k\text{ECON}(G; r)$ (respectively, $k\text{NCON}(G; r)$) if and only if for every edge $f \neq e$ the polytope $k\text{ECON}(G - \{e, f\}; r)$ (respectively, $k\text{NCON}(G - \{e, f\}; r)$) is nonempty.
Facets: another example

**Theorem 3.** Let $G = (V, E)$ be a $(k + 1)$-edge connected graph, let $r_v = k$ for all nodes $v \in V$, and let $W \neq V$ be a nonempty node set. Define for each $W_i \subseteq W$ with $\emptyset \neq W_i \neq W$ the deficit of $W_i$ as

$$\text{def}_G(W_i) := \max \{0, k - |\delta_{G[W]}(W_i)|\}.$$

Define similarly for $U_i \subseteq V \setminus W$ with $\emptyset \neq U_i \neq V \setminus W$

$$\text{def}_G(U_i) := \max \{0, k - |\delta_{G[V \setminus W]}(U_i)|\}.$$

**The cut inequality**

$$x(\delta(W)) \geq k$$

defines a facet of the polytope $k\text{ECON}(G; r)$ of $k$-edge connected graphs if and only if

a. $G[W]$ and $G[V \setminus W]$ are connected, and

b. for all edges $e \in E(W) \cup E(V \setminus W)$, for all pairwise disjoint node sets $W_1, \ldots, W_p (p \geq 0)$ of $W$ with $W_i \neq W$ for all $i$, and for all pairwise disjoint node sets $U_1, \ldots, U_q (q \geq 0)$ of $V \setminus W$ with $U_i \neq V \setminus W$ for all $i$, the following inequality holds:

$$\sum_{i=1}^{p} \text{def}_{G-e}(W_i) + \sum_{i=1}^{q} \text{def}_{G-e}(U_i)$$

$$\left[ - \left[ \bigcup_{i=1}^{p} W_i : \bigcup_{i=1}^{q} U_i \right] \right] \leq k.$$
<table>
<thead>
<tr>
<th>Problem</th>
<th>Original graphs</th>
<th>Reduced graphs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0   1  2 Nodes</td>
<td>0   1  2 Nodes</td>
</tr>
<tr>
<td>LATADMA</td>
<td>0   12 24 36 65/0</td>
<td>0   6 15 21 46/4</td>
</tr>
<tr>
<td>LATA1</td>
<td>8   65 14 77 112/0</td>
<td>0   10 14 24 48/2</td>
</tr>
<tr>
<td>LATA5S</td>
<td>0   31 8 39 71/0</td>
<td>0   15 8 23 50/0</td>
</tr>
<tr>
<td>LATA5L</td>
<td>0   36 10 46 98/0</td>
<td>0   20 9 29 77/1</td>
</tr>
<tr>
<td>LATADSF</td>
<td>0  108 8 116 173/40</td>
<td>0   28 11 39 86/26</td>
</tr>
<tr>
<td>LATADS</td>
<td>0  108 8 116 173/0</td>
<td>0   28 11 39 86/3</td>
</tr>
<tr>
<td>LATADL</td>
<td>0  84 32 116 173/0</td>
<td>0   11 28 39 86/6</td>
</tr>
</tbody>
</table>
Reduction

Fig. 4. Original graph of LATADL-problem.

Fig. 5. Reduced graph of LATADL-problem.
## Computational Results

### Table 2
Performance of branch & cut on LATA problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>IT</th>
<th>P</th>
<th>NP</th>
<th>RC</th>
<th>C</th>
<th>COPT</th>
<th>GAP</th>
<th>T</th>
<th>BN</th>
<th>BD</th>
<th>BT</th>
</tr>
</thead>
<tbody>
<tr>
<td>LATADMA</td>
<td>12</td>
<td>65</td>
<td>3</td>
<td>7</td>
<td>1489</td>
<td>1489</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LATA1</td>
<td>4</td>
<td>73</td>
<td>0</td>
<td>1</td>
<td>4296</td>
<td>4296</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LATA5S</td>
<td>4</td>
<td>76</td>
<td>0</td>
<td>0</td>
<td>4739</td>
<td>4739</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LATA5LE</td>
<td>7</td>
<td>120</td>
<td>0</td>
<td>0</td>
<td>4574</td>
<td>4574</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LATA5L</td>
<td>19</td>
<td>155</td>
<td>12</td>
<td>0</td>
<td>4679</td>
<td>4726</td>
<td>0.99</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>LATADS</td>
<td>7</td>
<td>43</td>
<td>0</td>
<td>0</td>
<td>7647</td>
<td>7647</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LATADS</td>
<td>17</td>
<td>250</td>
<td>0</td>
<td>4</td>
<td>7303.60</td>
<td>7320</td>
<td>0.22</td>
<td>4</td>
<td>28</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>LATADL</td>
<td>14</td>
<td>182</td>
<td>0</td>
<td>28</td>
<td>7385.25</td>
<td>7400</td>
<td>0.20</td>
<td>3</td>
<td>32</td>
<td>10</td>
<td>21</td>
</tr>
</tbody>
</table>

- **IT** = number of iterations (= calls to the LP-solver) used in the cutting plane phase; **NP** = number of nodes in the cutting plane phase; **RC** = number of lifted r-cover cuts; **C** = value of the optimum solution after terming the cutting plane phase; **GAP** = 100 \times \frac{(COPT - C)}{COPT} (not including branch & cut); **T** = total running time including the cutting plane phase (not including branch & cut), in minutes; **BN** = number of branch nodes generated; **BD** = maximum depth of the branch & cut algorithm including the cutting plane phase.

### Table 4
Comparison of heuristic values with optimal values

<table>
<thead>
<tr>
<th>Problem</th>
<th>COPT</th>
<th>CHEUR</th>
<th>GAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LATADMA</td>
<td>1489</td>
<td>1494</td>
<td>0.34</td>
</tr>
<tr>
<td>LATA1</td>
<td>4296</td>
<td>4296</td>
<td>0</td>
</tr>
<tr>
<td>LATA5S</td>
<td>4739</td>
<td>4739</td>
<td>0</td>
</tr>
<tr>
<td>LATA5LE</td>
<td>4574</td>
<td>4574</td>
<td>0</td>
</tr>
<tr>
<td>LATA5L</td>
<td>4726</td>
<td>4794</td>
<td>1.44</td>
</tr>
<tr>
<td>LATADS</td>
<td>7647</td>
<td>7727</td>
<td>1.05</td>
</tr>
<tr>
<td>LATADL</td>
<td>7400</td>
<td>7460</td>
<td>0.81</td>
</tr>
</tbody>
</table>
LATA DL: optimum solution

Fig. 4. Original graph of LATADL-problem.

Fig. 6. Solution of LATADL-problem.
The Ship Problem: higher connectivity

Fig. 7. Grid graph of the ship problem.

Fig. 8. Reduced grid graph of the ‘ship13’ problem.
The Ship Problem: higher connectivity

Fig. 8. Reduced grid graph of the ‘ship13’ problem.

Fig. 9. Optimum solution of reduced ‘ship23’ problem.
## The Ship Problem: higher connectivity

### Table 6
Performance of cutting plane algorithm on ship problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>VAR</th>
<th>IT</th>
<th>PART</th>
<th>RCOV</th>
<th>LB</th>
<th>UB</th>
<th>GAP (%)</th>
<th>Time (mins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ship13</td>
<td>1088</td>
<td>3252</td>
<td>777261</td>
<td>0</td>
<td>211957.1</td>
<td>217428</td>
<td>2.58</td>
<td>10122:35</td>
</tr>
<tr>
<td>ship23</td>
<td>1088</td>
<td>15</td>
<td>4090</td>
<td>0</td>
<td>286274</td>
<td>286274</td>
<td>0</td>
<td>27:20</td>
</tr>
<tr>
<td>ship33</td>
<td>1082</td>
<td>42</td>
<td>10718</td>
<td>1</td>
<td>461590.6</td>
<td>483052</td>
<td>4.64</td>
<td>55:26</td>
</tr>
<tr>
<td>ship13red</td>
<td>322</td>
<td>775</td>
<td>200570</td>
<td>0</td>
<td>217428</td>
<td>217428</td>
<td>0</td>
<td>426:47</td>
</tr>
<tr>
<td>ship23red</td>
<td>604</td>
<td>12</td>
<td>2372</td>
<td>0</td>
<td>286274</td>
<td>286274</td>
<td>0</td>
<td>1:54</td>
</tr>
<tr>
<td>ship33red</td>
<td>710</td>
<td>40</td>
<td>9817</td>
<td>0</td>
<td>462099.3</td>
<td>483052</td>
<td>4.53</td>
<td>34:52</td>
</tr>
</tbody>
</table>

Problem = problem name, where ‘red’ means reduced; VAR = number of edges minus number of forced edges; IT = number of LPs solved; PART = number of partition inequalities added; RCOV = number of $r$-cover inequalities added; LB = lower bound (= optimal LP value); UB = upper bound (= heuristic value); GAP = $(UB - LB)/LB$.

### Table 7
Relative running times on ship problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>PT (%)</th>
<th>LPT (%)</th>
<th>CT (%)</th>
<th>MT (%)</th>
<th>Time (mins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ship13</td>
<td>0.0</td>
<td>75.6</td>
<td>23.9</td>
<td>0.5</td>
<td>10122:35</td>
</tr>
<tr>
<td>ship23</td>
<td>0.0</td>
<td>13.1</td>
<td>86.4</td>
<td>0.4</td>
<td>27:20</td>
</tr>
<tr>
<td>ship33</td>
<td>0.0</td>
<td>31.2</td>
<td>68.2</td>
<td>0.6</td>
<td>55:26</td>
</tr>
<tr>
<td>ship13red</td>
<td>0.0</td>
<td>68.5</td>
<td>30.1</td>
<td>1.4</td>
<td>426:47</td>
</tr>
<tr>
<td>ship23red</td>
<td>0.1</td>
<td>39.2</td>
<td>58.6</td>
<td>1.9</td>
<td>1:54</td>
</tr>
<tr>
<td>ship33red</td>
<td>0.0</td>
<td>41.1</td>
<td>58.4</td>
<td>0.5</td>
<td>34:52</td>
</tr>
</tbody>
</table>

Problem = problem name where ‘red’ means reduced; PT = time spent for reduction of problem; LPT = time spent for LP solving; CT = time spent for separation; MT = time on miscellaneous items, input, output, etc.
1. Telecommunication: The General Problem
2. Newspaper Reports
3. Survivability
4. Integrated Topology, Capacity, and Routing Optimization as well as Survivability Planning
Network Design: Tasks to be solved
Some Examples (continued)

- Locating Mobile Switching Centers (MSCs)
- Clustering BSCs and Connecting BSCs to MSCs
- Designing the BSC network (BSS) and the MSC network (NSS or core network)
  - Topology of the network
  - Capacity of the links and components
  - Routing of the demand
  - Survivability in failure situations
Network Optimization

Capacities

Requirements

Cost

Networks
What needs to be planned?

- Topology
- Capacities
- Routing
- Failure Handling (Survivability)

- IP Routing
- Node Equipment Planning
- Optimizing Optical Links and Switches

**DISCNET**: A Network Planning Tool
(Dimensioning Survivable Capacitated NETworks)

Atesio ZIB Spin-Off
The Network Design Problem

Communication Demands

Potential topology & Capacities
## Capacities

(P)SDH = (poly)synchronous digital hierarchy

<table>
<thead>
<tr>
<th>PDH</th>
<th>SDH</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Mbit/s</td>
<td>155 Mbit/s</td>
</tr>
<tr>
<td>34 Mbit/s</td>
<td>622 Mbit/s</td>
</tr>
<tr>
<td>140 Mbit/s</td>
<td>2,4 Gbit/s</td>
</tr>
</tbody>
</table>

... WDM (n x STM-N)

Two capacity models: **Discrete Finite Capacities**  
**Divisible Capacities**

- **WDM** = Wavelength Division Multiplexer  
- **STM-N** = Synchronous Transport Modul with N STM-1 Frames
**Survivability**

**Diversification**

„route node-disjoint“

**Reservation**

„reroute all demands“
(or p% of all demands)

**Path restoration**

„reroute affected demands“
(or p% of all affected demands)
Model: Capacities

Capacity variables: \( e \in E, t = 1, \ldots, T_e \)
\[ x^t_e \in \{0, 1\} \]

Cost function:
\[ \min \sum_{e \in E} \sum_{t=1}^{T_e} k^t_e x^t_e \]

Capacity constraints: \( e \in E \)
\[ 1 = x^0_e \geq x^1_e \geq L \geq x^{T_e}_e \geq 0 \]
\[ y_e = \sum_{t=0}^{T_e} c^t_e x^t_e \]
Model: Routings

Path variables: \( s \in S, uv \in D_s, P \in P^s_{uv} \)
\[
f^s_{uv}(P) \geq 0
\]

Capacity constraints: \( e \in E \)
\[
y_e \geq \sum_{uv \in D} \sum_{P \in P^0_{uv}, e \in P} f^0_{uv}(P)
\]

Demand constraints: \( uv \in D \)
\[
d_{uv} = \sum_{P \in P^0_{uv}} f^0_{uv}(P)
\]
**Model: Survivability (one example)**

Path restoration „reroute affected demands“

\[
\begin{align*}
\sum_{P \in P_{uv}^s \cap P_{uv}^0} f^0(P) + \sum_{P \in P_{uv}^s : e \in P} f^s(P) & \geq \sigma_{uv} d_{uv} \\
\sum_{uv \in D_s} \left( \sum_{P \in P_{uv}^s \cap P_{uv}^0} f^0(P) + \sum_{P \in P_{uv}^s : e \in P} f^s(P) \right) & \leq y_e
\end{align*}
\]
**Mathematical Model**

\[ \min \sum_{e \in E} \sum_{t=1}^{T_e} k_e^t x_e^t \]

- topology decision
- capacity decisions
- normal operation routing
- component failure routing

\[ x_e^t \in \{0,1\} \quad e \in E, t = 1,K, T_e \]

\[ x_e^{t-1} \geq x_e^t \quad e \in E, t = 1,K, T_e \]

\[ y_e = \sum_{t=0}^{T_e} c_e^t x_e^t \quad e \in E \]

\[ y_e \geq \sum_{uv \in D} \sum_{P \in P_{uv}^0} f_{uv}^0 (P) \quad e \in E \]

\[ d_{uv} = \sum_{P \in P_{uv}^0} f_{uv}^0 (P) \quad uv \in D \]

\[ f_{uv}^s (P) \geq 0 \quad s \in S, uv \in D_s, P \in P_{uv}^s \]

\[ \sum_{P \in P_{uv}^0} f_{uv}^0 (P) + \sum_{P \in P_{uv}} f_{uv}^0 (P) \geq \sigma_{uv} d_{uv} \quad s \in S, uv \in D_s \]

\[ \sum_{P \in P_{uv}} f_{uv}^0 (P) + \sum_{P \in P_{uv}} f_{uv}^s (P) \leq y_e \quad \text{s.e.E}_{s}, e \in E_{S} \]
Polyhedral combinatorics
Valid inequalities (facets)
Separation algorithms
Heuristics
Feasibility of a capacity vector

LP-based approach:

- Initialize LP-relaxation
- Solve LP-relaxation
- Separation algorithms
- Augment LP-relaxation
- Separation algorithms
- Solve feasibility problem
- Feasible routings?
- x variables integer?
- Yes
  - Optimal solution
- No
  - No
  - Run heuristics
  - Yes
  - Optimal solution

Flow chart
Finding a Feasible Solution?

Heuristics
- Local search
- Simulated Annealing
- Genetic algorithms
- ...

Manipulation of
- Routings
- Topology
- Capacities

Problem Sizes

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Edges</th>
<th>Demands</th>
<th>Routing-Paths</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>46</td>
<td>78</td>
<td>&gt; 150 x 10^6</td>
</tr>
<tr>
<td>36</td>
<td>107</td>
<td>79</td>
<td>&gt; 500 x 10^9</td>
</tr>
<tr>
<td>36</td>
<td>123</td>
<td>123</td>
<td>&gt; 2 x 10^12</td>
</tr>
</tbody>
</table>
How much to save?

Real scenario
- 163 nodes
- 227 edges
- 561 demands

34% potential savings!

==

> hundred million dollars

PhD Thesis:
http://www.zib.de/wessaely
wessaely@atesio.de
05M2 Lecture
Telecommunication
Network Design

The End