European Graduate Program
Berlin - Zürich
CGC
Combinatorics

03M2 Lecture
Printed Circuit Board Production: Some Issues

Martin Grötschel

Block Course at TU Berlin
"Combinatorial Optimization at Work“
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Martin Grötschel

- Institut für Mathematik, Technische Universität Berlin (TUB)
- DFG-Forschungszentrum “Mathematik für Schlüsseltechnologien” (MATHEON)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

groetschel@zib.de
http://www.zib.de/groetschel

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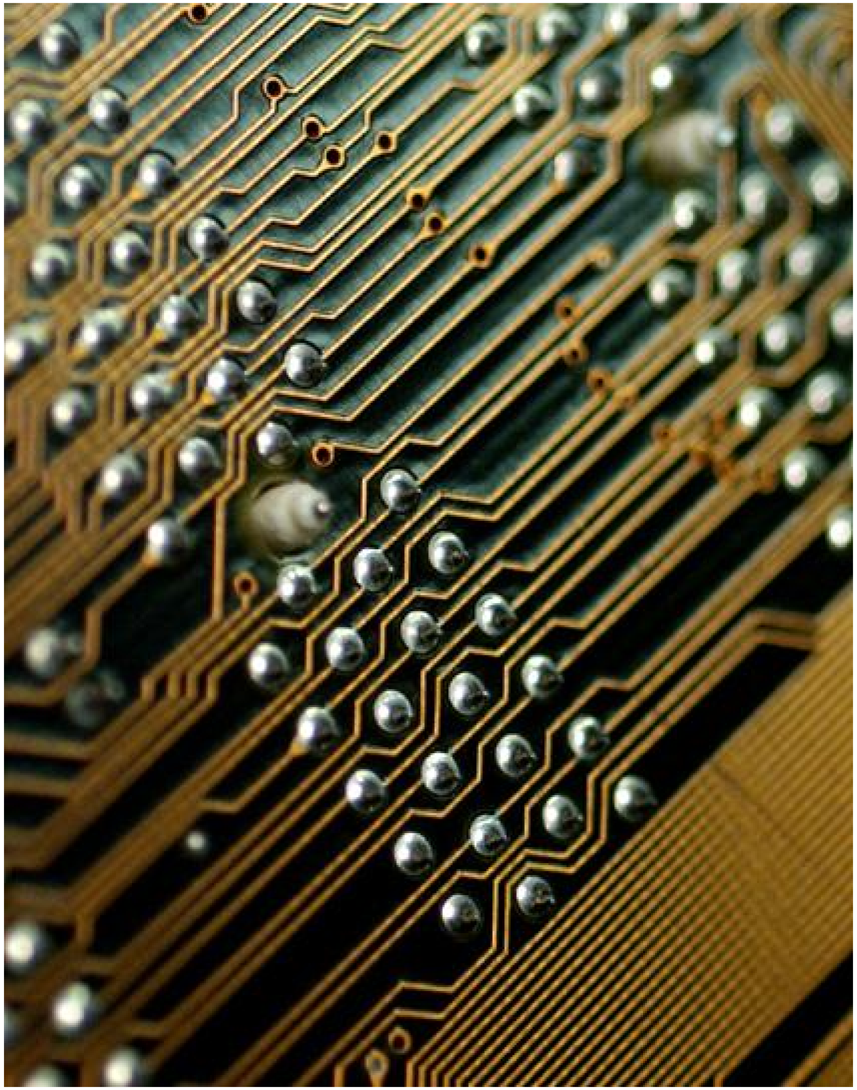
Printed Circuit Boards

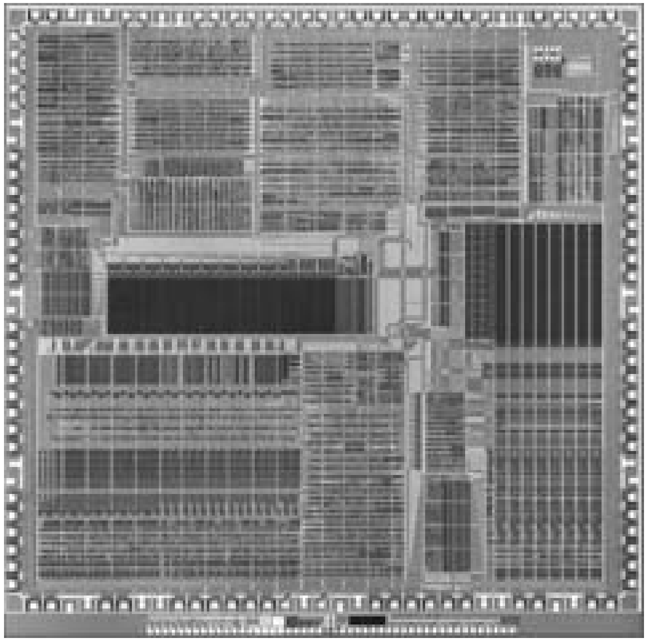
- A printed circuit board consists of "printed wires" attached to a sheet of insulator. The conductive "printed wires" are called "traces" or "tracks". The insulator is called the substrate.
- The inventor of the printed circuit was probably the Austrian engineer Paul Eisler (1907 - 1995) who, while working in England, made one in about 1936 as part of a radio set. In about 1943 the USA began to use the technology on a large scale to make rugged radios for use in World War II. After the war, in 1948, the USA released the invention for commercial use. Printed circuits did not become commonplace in consumer electronics until the mid-1950s.

Printed Circuit Boards: A Picture

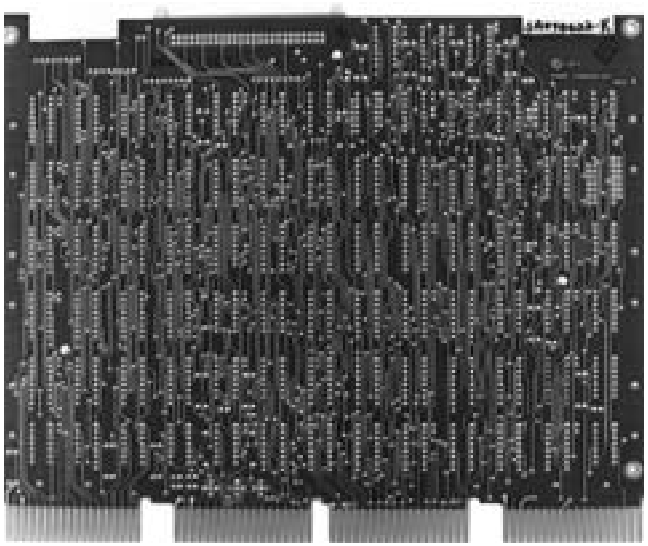


Closeup photo of one side of a motherboard PCB, showing conductive traces and solder points for through-hole components on the opposite side.

Production of ICs and PCBs



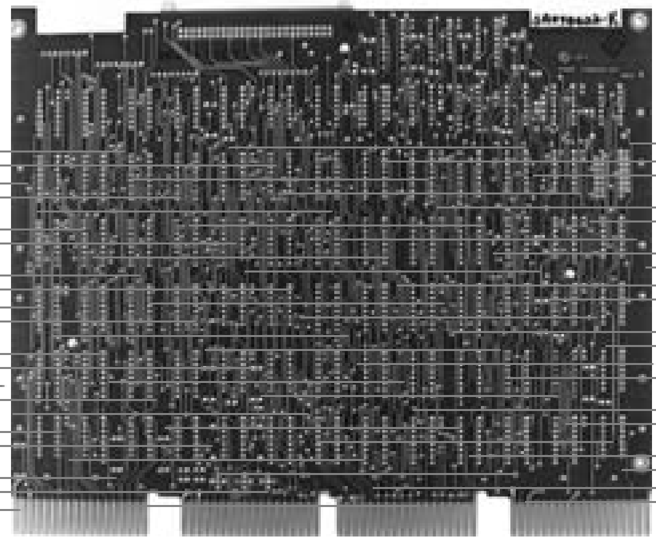
Integrated Circuit (IC)

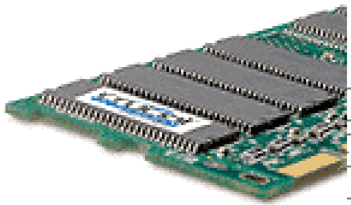


Printed Circuit Board (PCB)

Problems: Logical Design, Physical Design Correctness, Simulation, Placement of Components, Routing, Drilling,...

ICs and PCBs





Integrated Circuit (IC)

Printed Circuit Board (PCB)

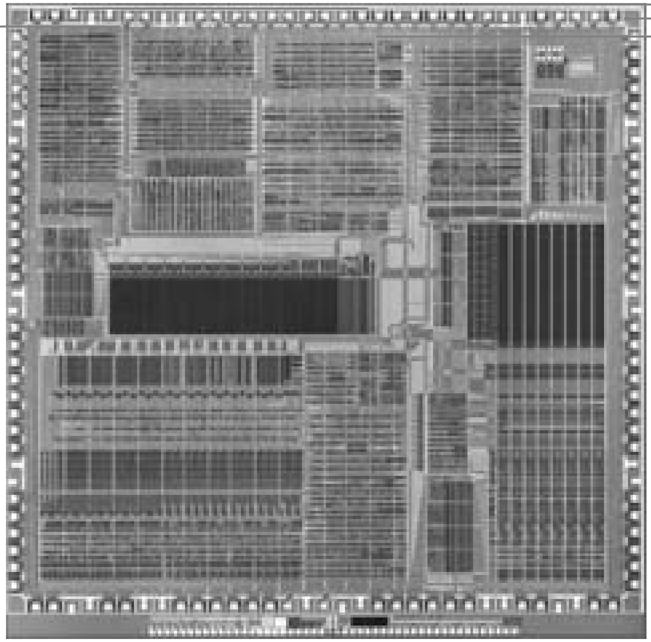


Foto of a “real” PCB (front)

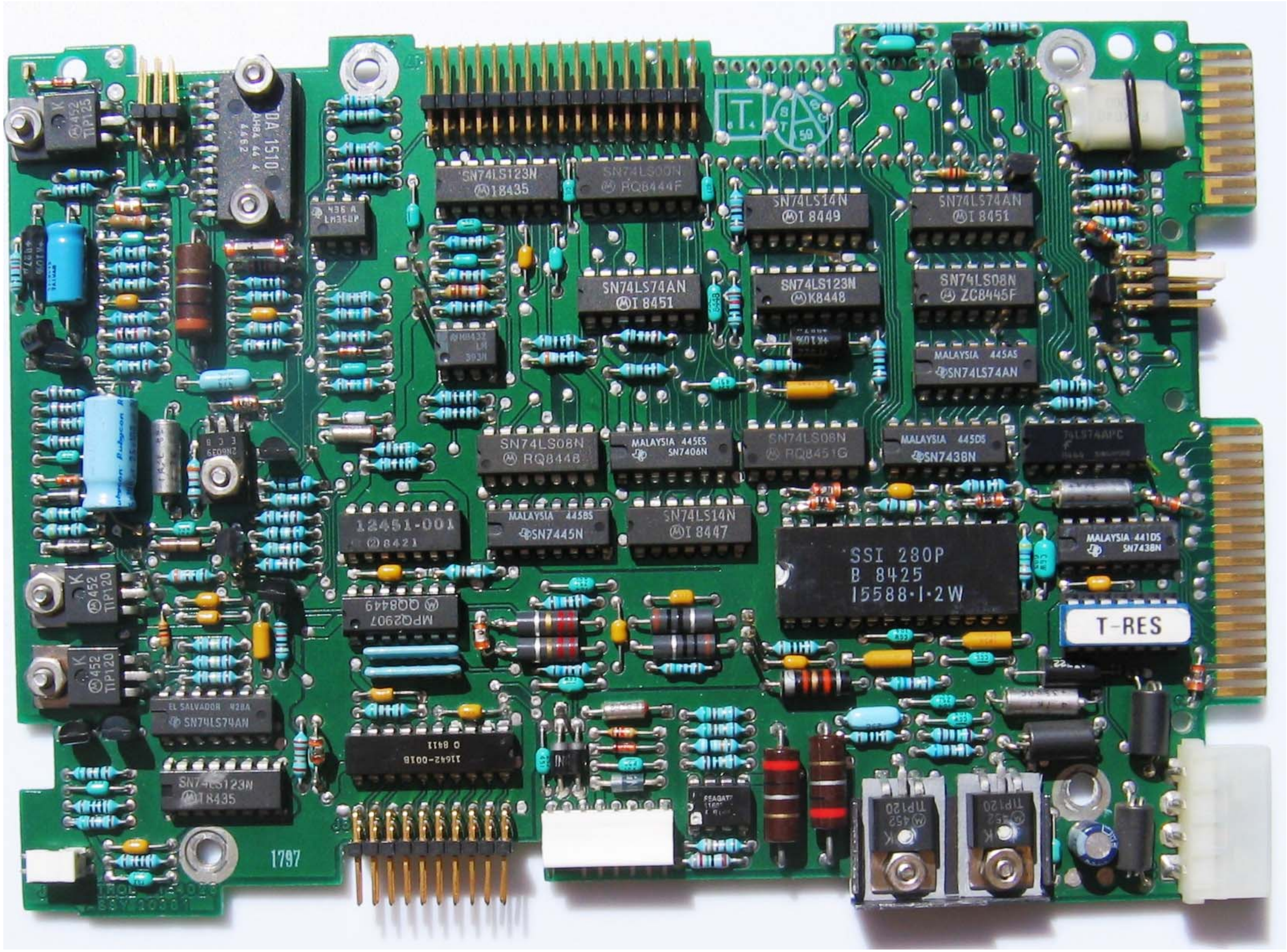
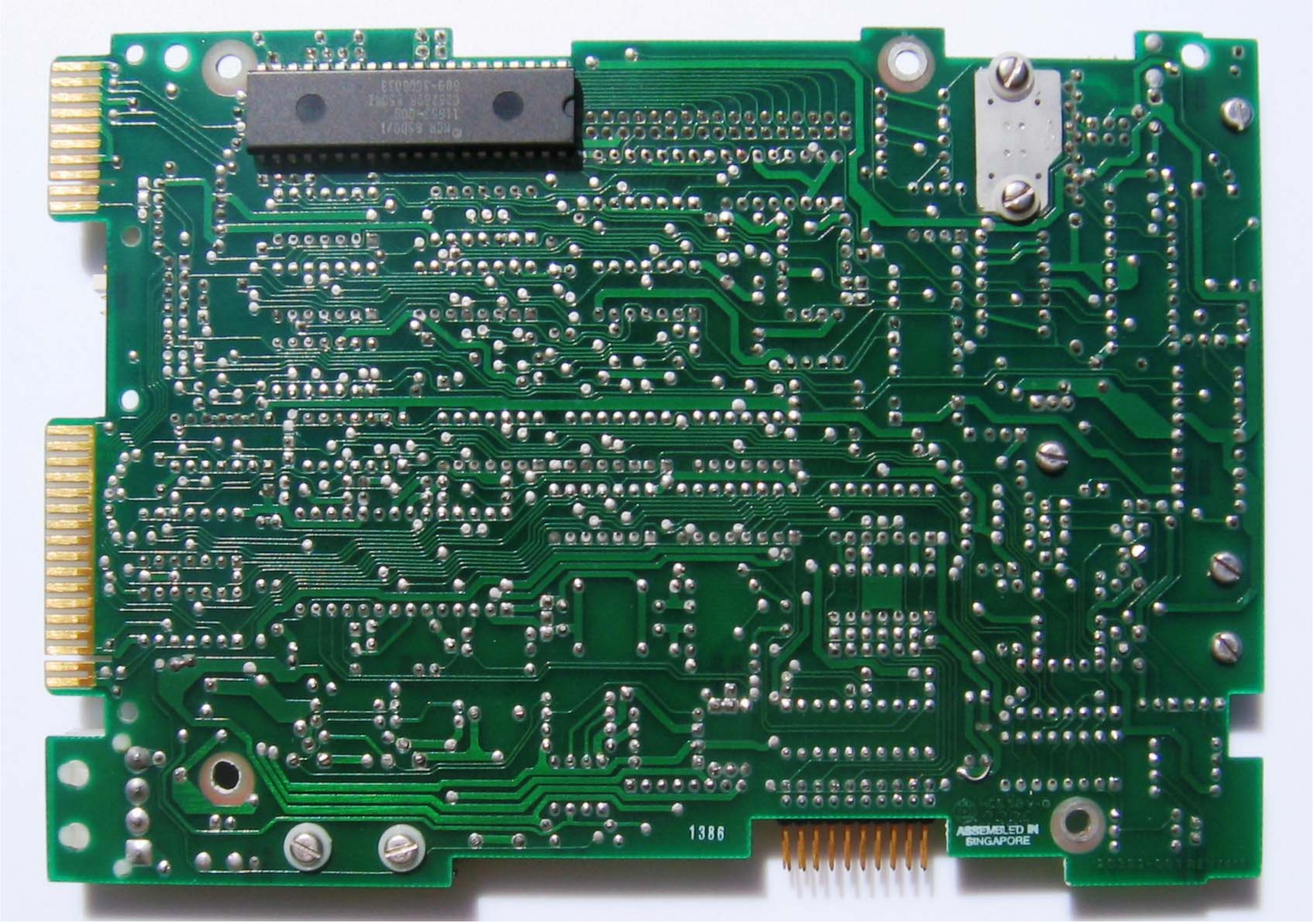


Foto of a “real” PCB (back)



FALKE PCB Assembly Line (Siemens Augsburg)

Picture deleted

Flachbaugruppen-Produktion (Petra Bauer)

We developed and implemented:

- a detailed simulation model
- Optimization heuristics to determine the PCB production seqence
- methods to compute lower bound for the flow time

Expectation: Improvement from 65% to 100% of the projected capacity of the production line

Results

(1) Span between best and worst heuristic solution	~ 20 %
(2) Provable Span	~ 30 %
(3) Solutions used in the factory	~ in the middle of the span
(4) random solution	~ in the middle of the span
(5) best heuristic solution	~ 7 % better as factory solution, but unstable

The whole 9 month effort resulted in no success. We could prove that system design and layout did not match the tasks to be performed well: bottleneck creation, etc.

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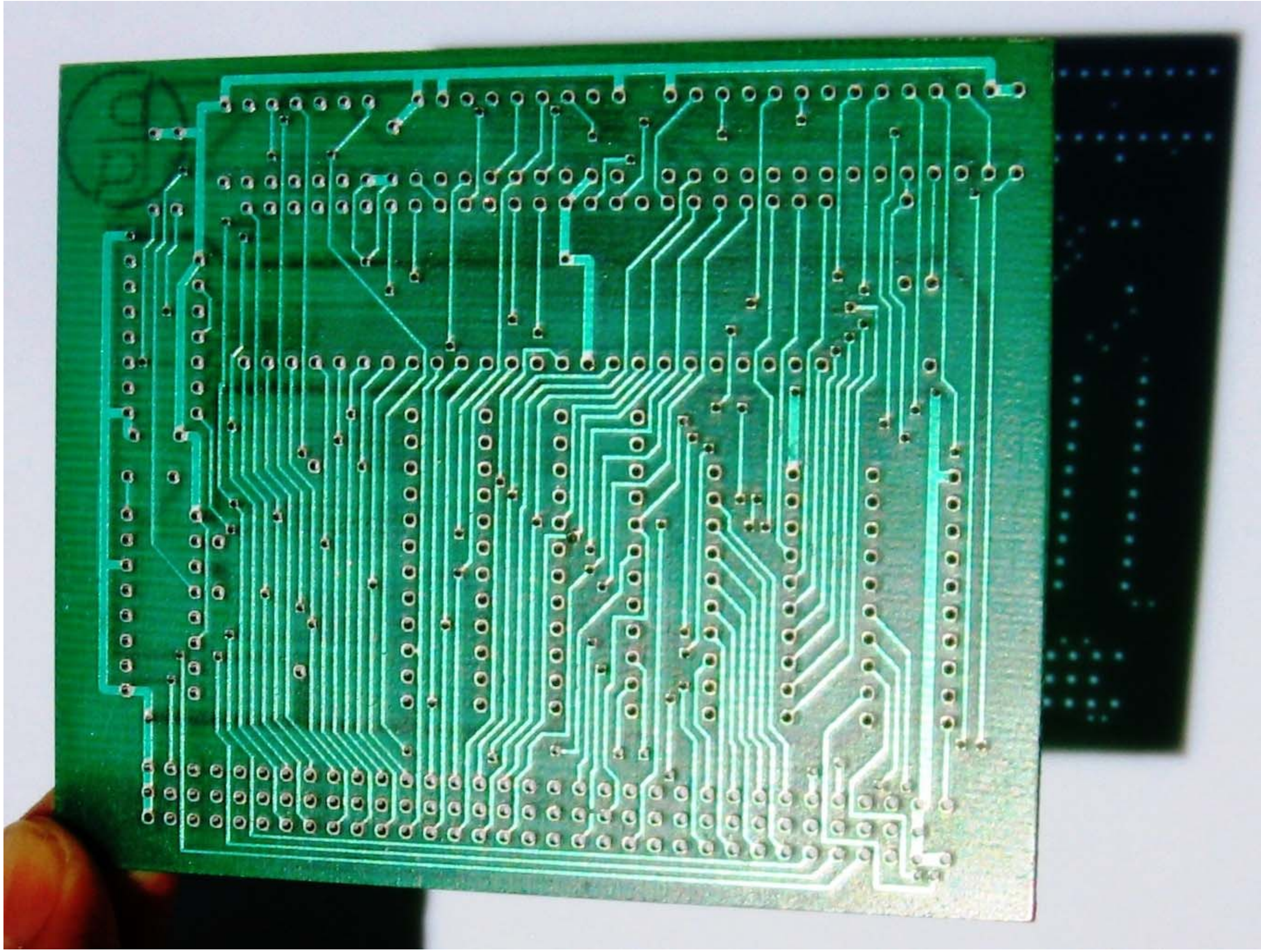
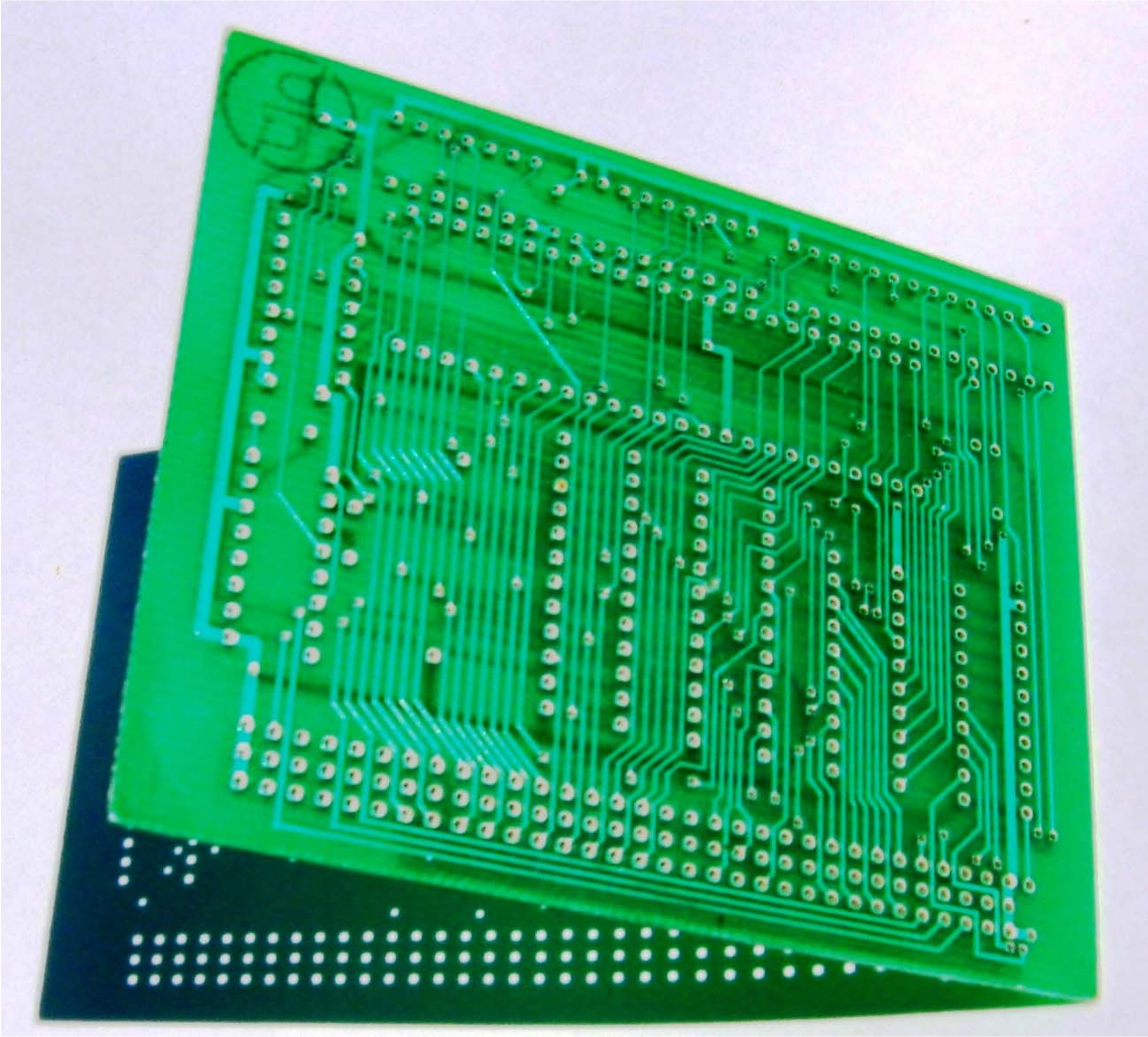
The 442-Holes Problem



The 442-Holes Problem



Drilling/Plotting Laser

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Mechanical PCB Drilling Machine

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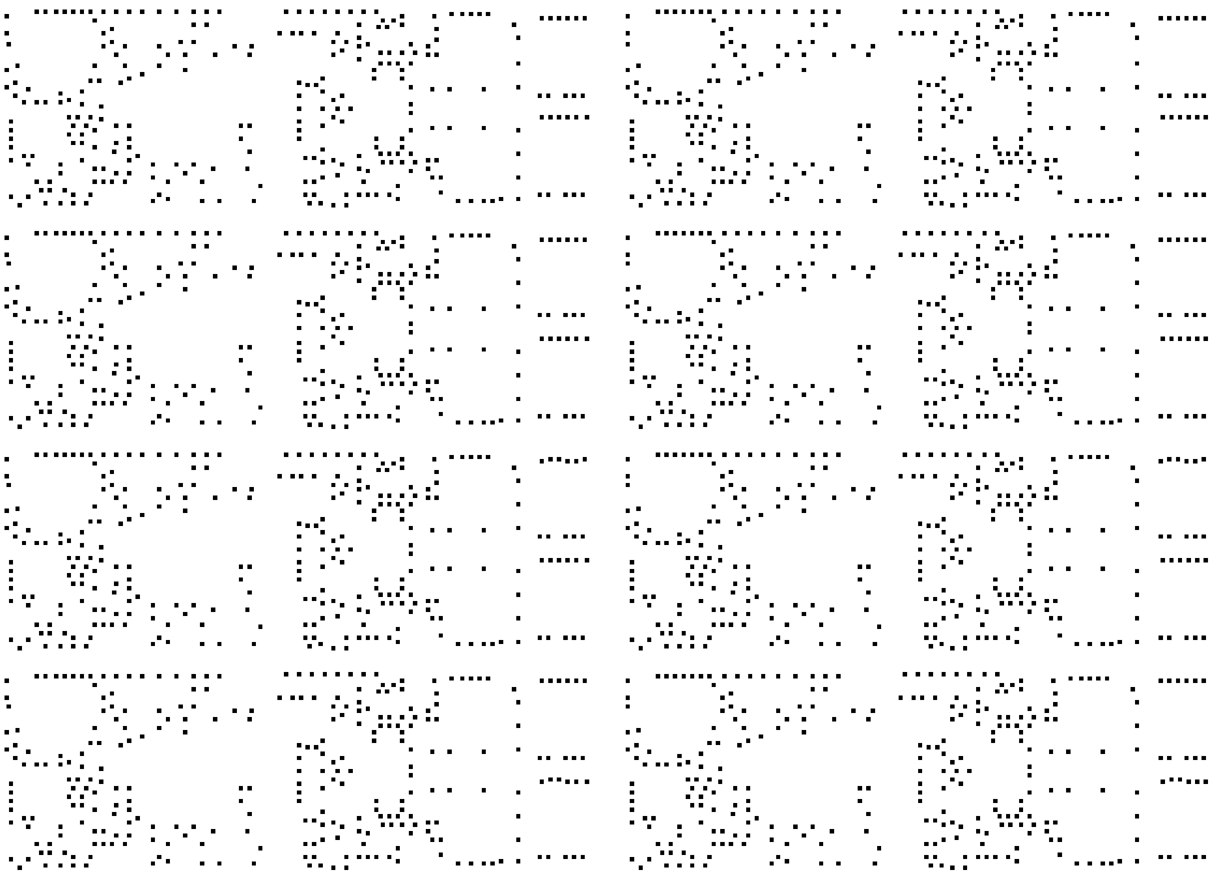
CO at Work

Correct modelling of a printed circuit board drilling problem



length of a move of the drilling head:

Euclidean norm,

Max norm,

Manhattan norm?

2103 holes to be drilled

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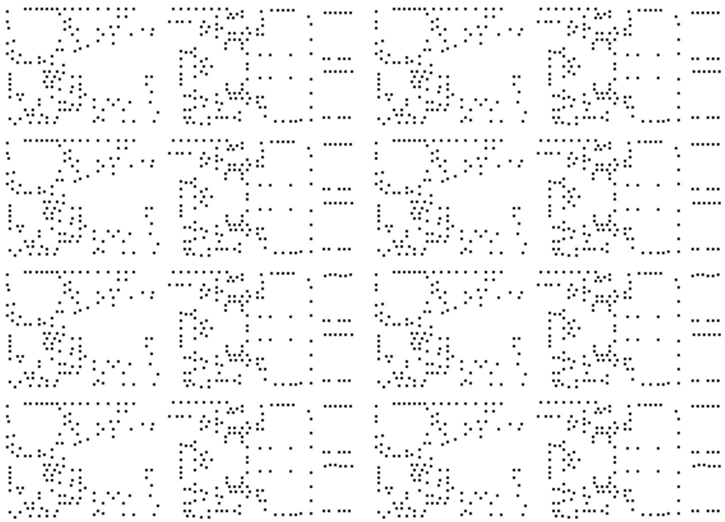
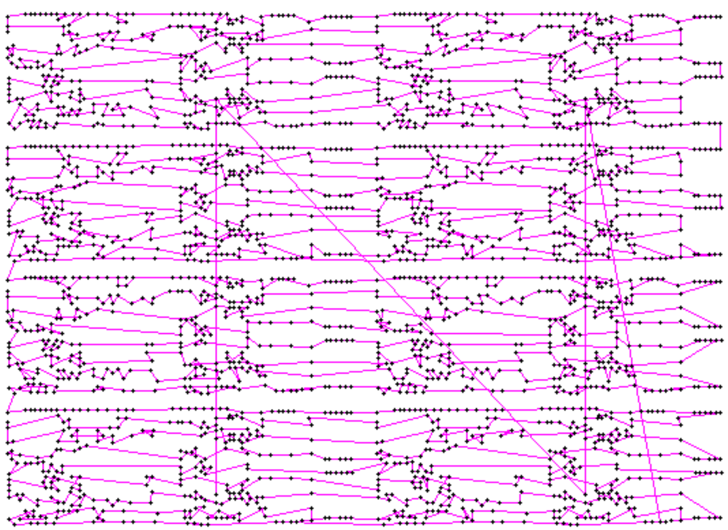
Drilling 2103 holes into a PCB

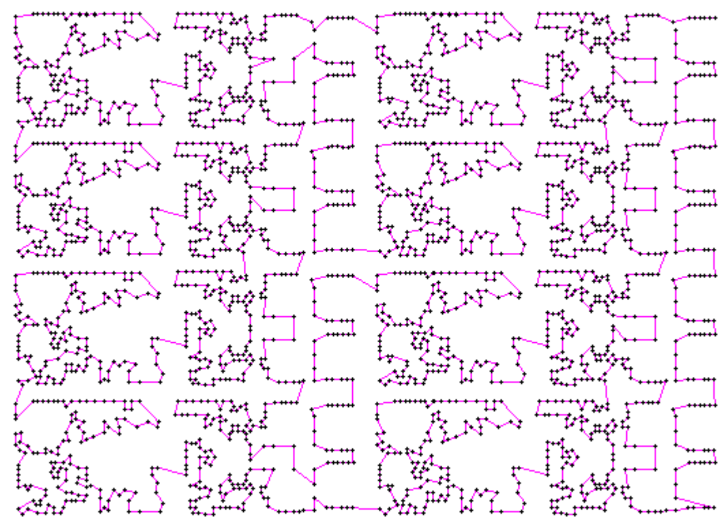


Significant Improvements via TSP

(Padberg & Rinaldi)



industry solution



optimal solution

Siemens-Problem PCB da1

Grötschel, Jünger, Reinelt

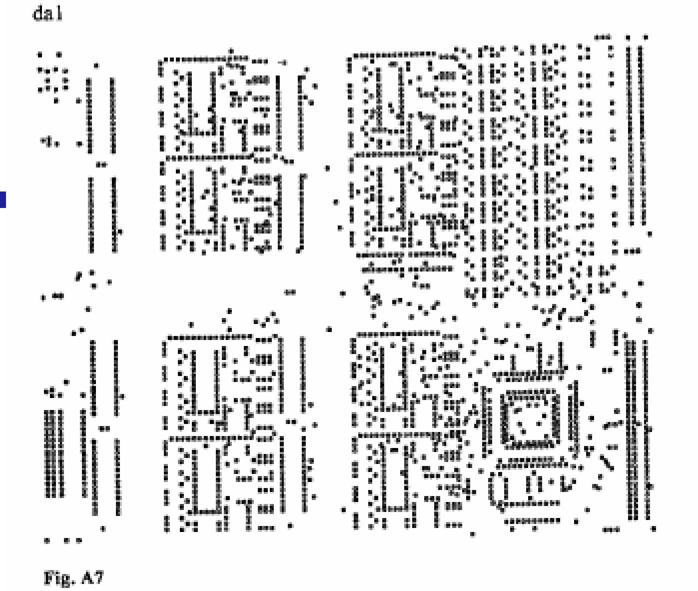


Fig. A7

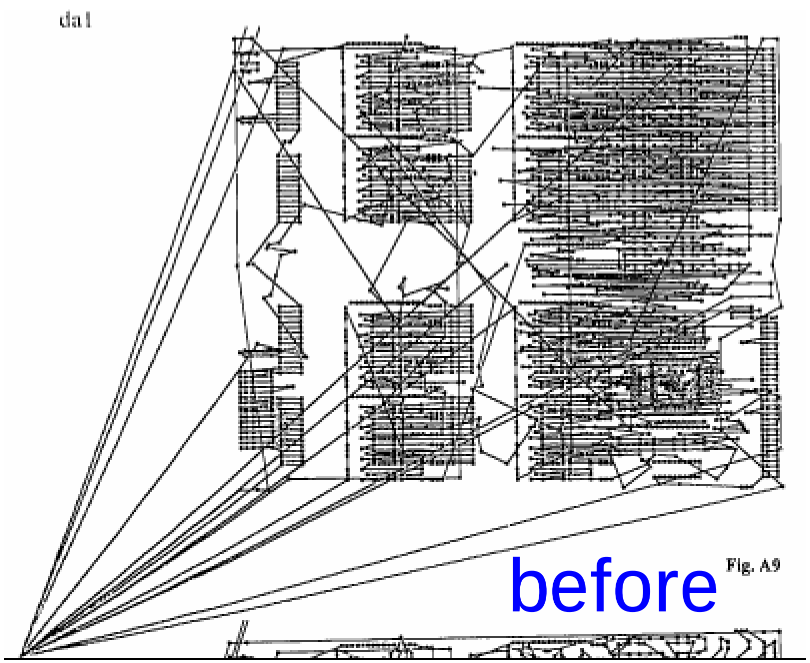


Fig. A9

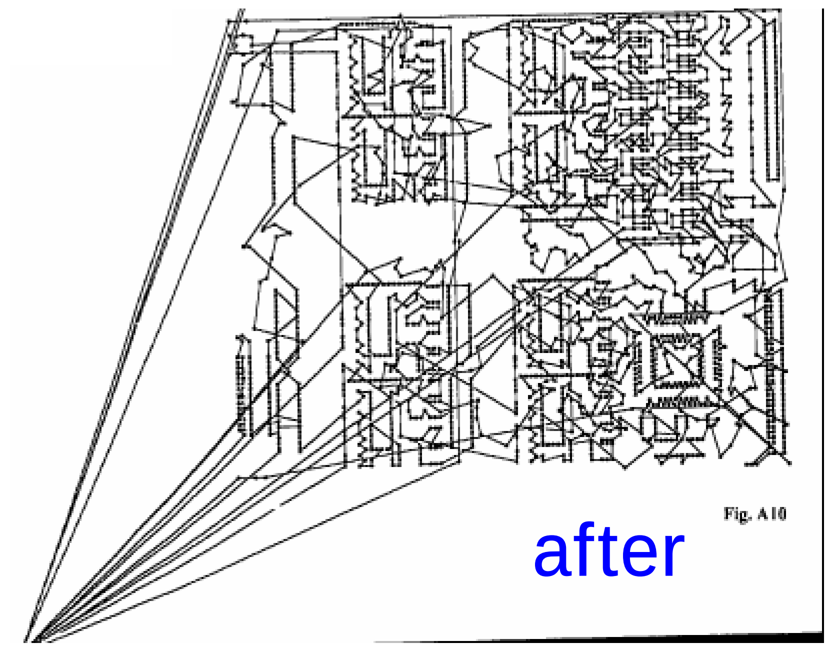


Fig. A10

Siemens-Problem PCB da4

Fig. A8

Grötschel, Jünger, Reinelt

da4

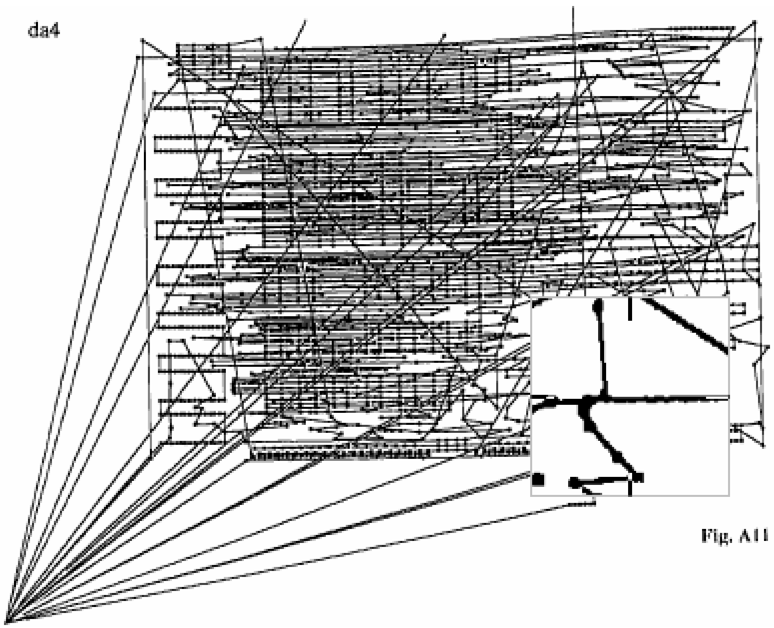


Fig. A11

before

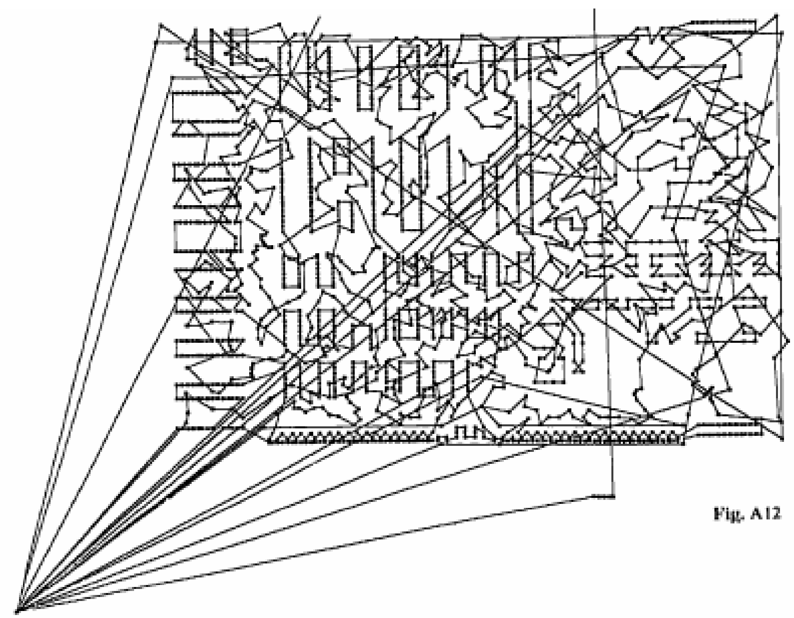


Fig. A12

after

Typical PCB drilling problems at Siemens

	da1	da2	da3	da4
Number of holes	2457	423	2203	2104
Number of drills	7	7	6	10
Tour length	3518728	1049956	1958161	4347902

Table 4

Fast heuristics

	da1	da2	da3	da4
CPU time (min:sec)	1:58	0:05	1:43	1:43
Tour length	1695042	984636	1642027	1928371
Improvement in %	56.87	14.60	26.94	58.38

Table 5

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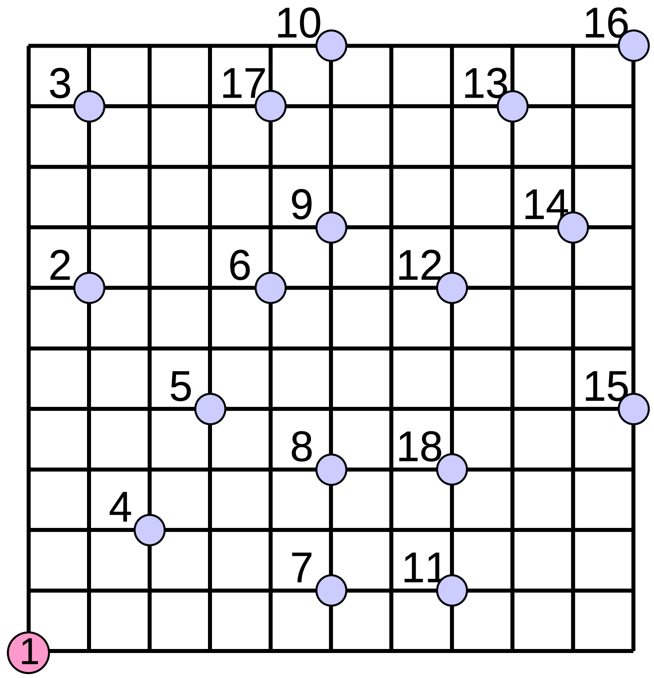
18 Holes, a didactical problem



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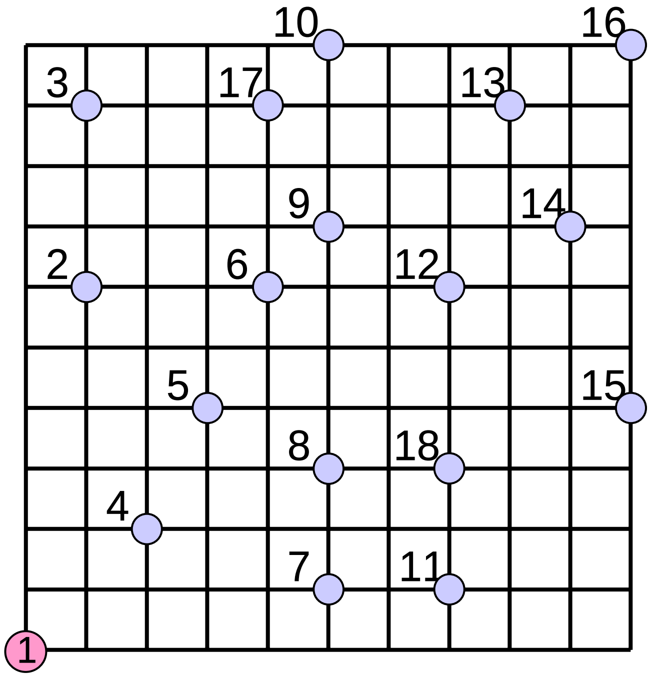
18 Holes: which objectives?



$$c_2(i,j) := \sqrt{|x_i - x_j|^2 + |y_i - y_j|^2}$$

Definition

Der *euklidische Abstand* zweier Punkte (x_i, y_i), (x_j, y_j):

$$c_2(i,j) := \sqrt{|x_i - x_j|^2 + |y_i - y_j|^2}$$

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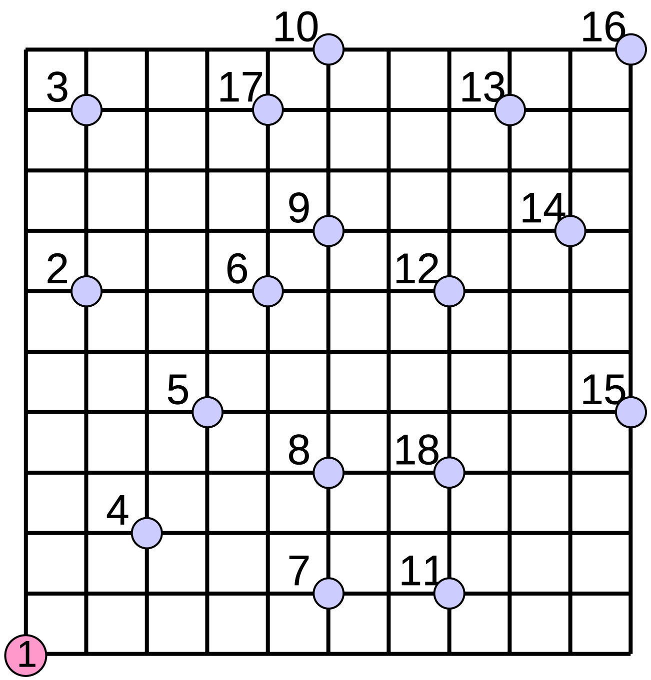
18 Holes: which objectives?



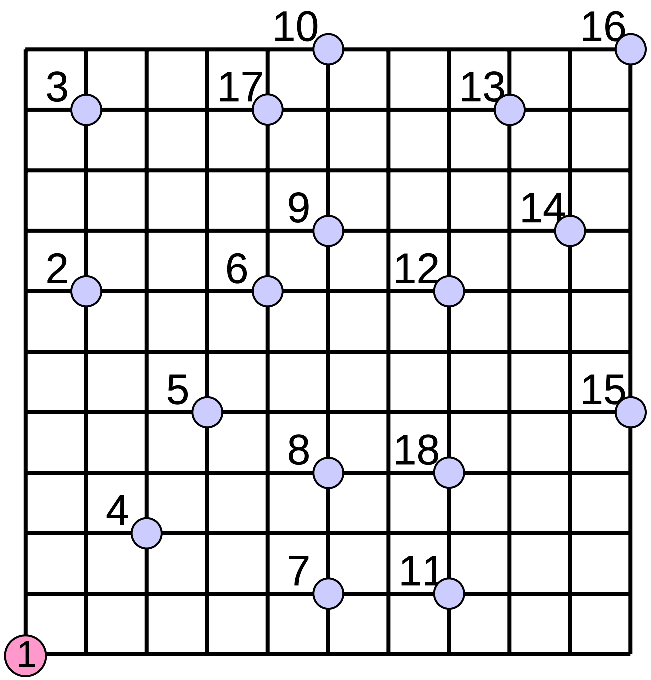
$$c_1(i,j) := |x_i - x_j| + |y_i - y_j|$$

Definition

Der *Manhattan-Abstand* zweier Punkte (x_i, y_i), (x_j, y_j):

$$c_1(i,j) := |x_i - x_j| + |y_i - y_j|$$

18 Holes: which objectives?



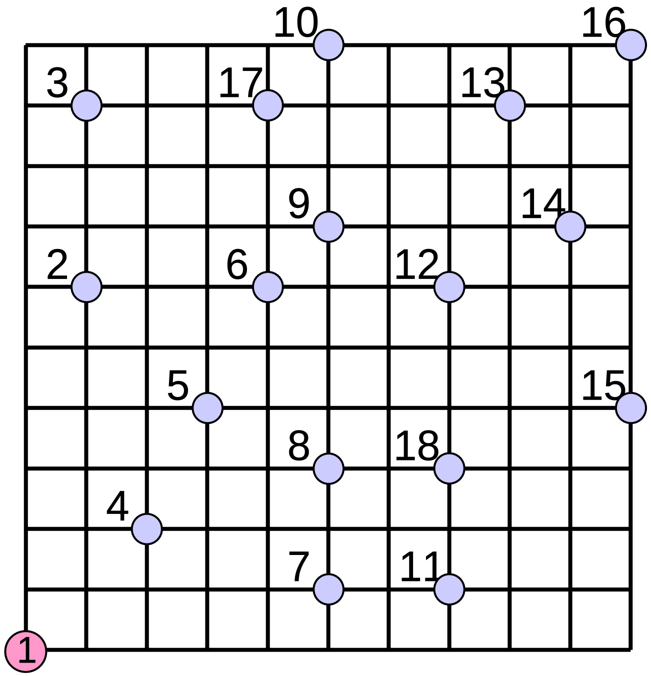
$$c_{\max}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\}$$

Definition

Der max-*Abstand* zweier Punkte $(x_i, y_i), (x_j, y_j)$:

$$c_{\max}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\}$$

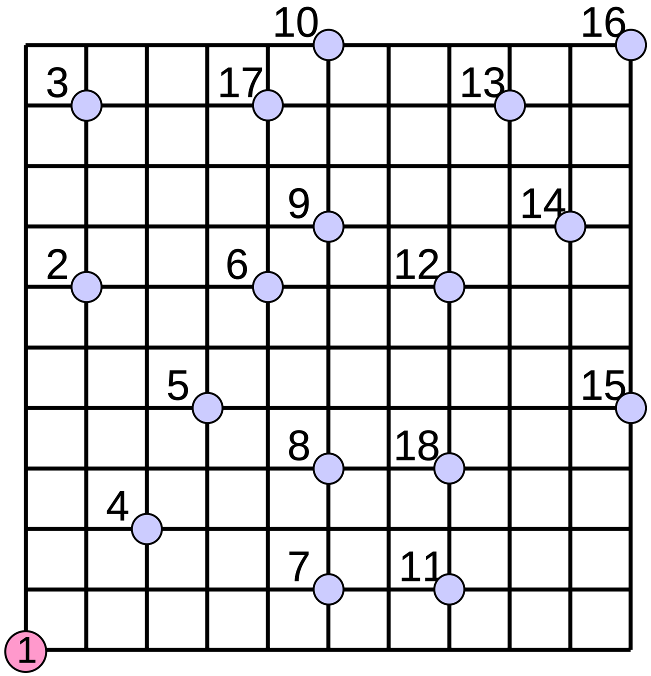
18 Holes: which objectives?



$$c_{\max\min}(i,j) := \frac{199}{200}\, c_{\max}(i,j) + \frac{1}{200}\, c_1(i,j).$$

$$c_{\max\min}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\} + \frac{1}{200} \min\{|x_i - x_j|, |y_i - y_j|\}.$$

18 Holes: which objectives?



$$c_{\max 2x}(i,j) := \max\{2\,|x_i - x_j|, |y_i - y_j|\}.$$

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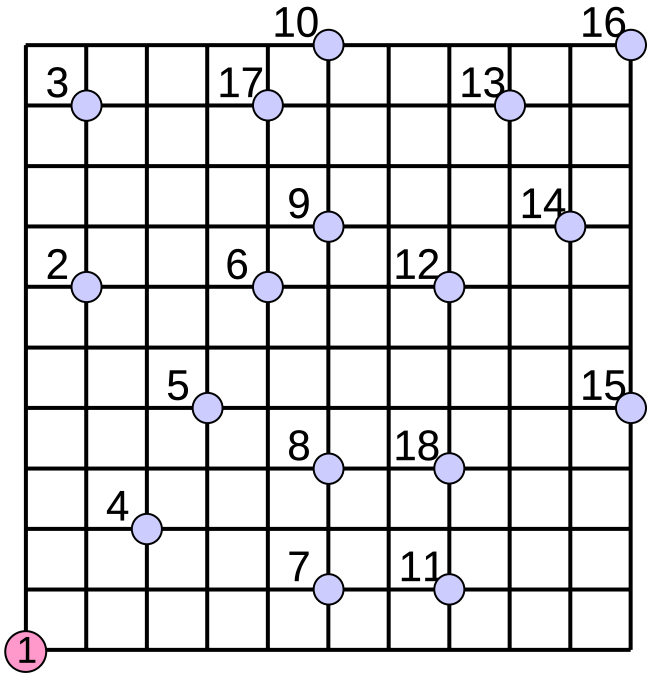
18 Holes: 5 objectives Which are suitable?



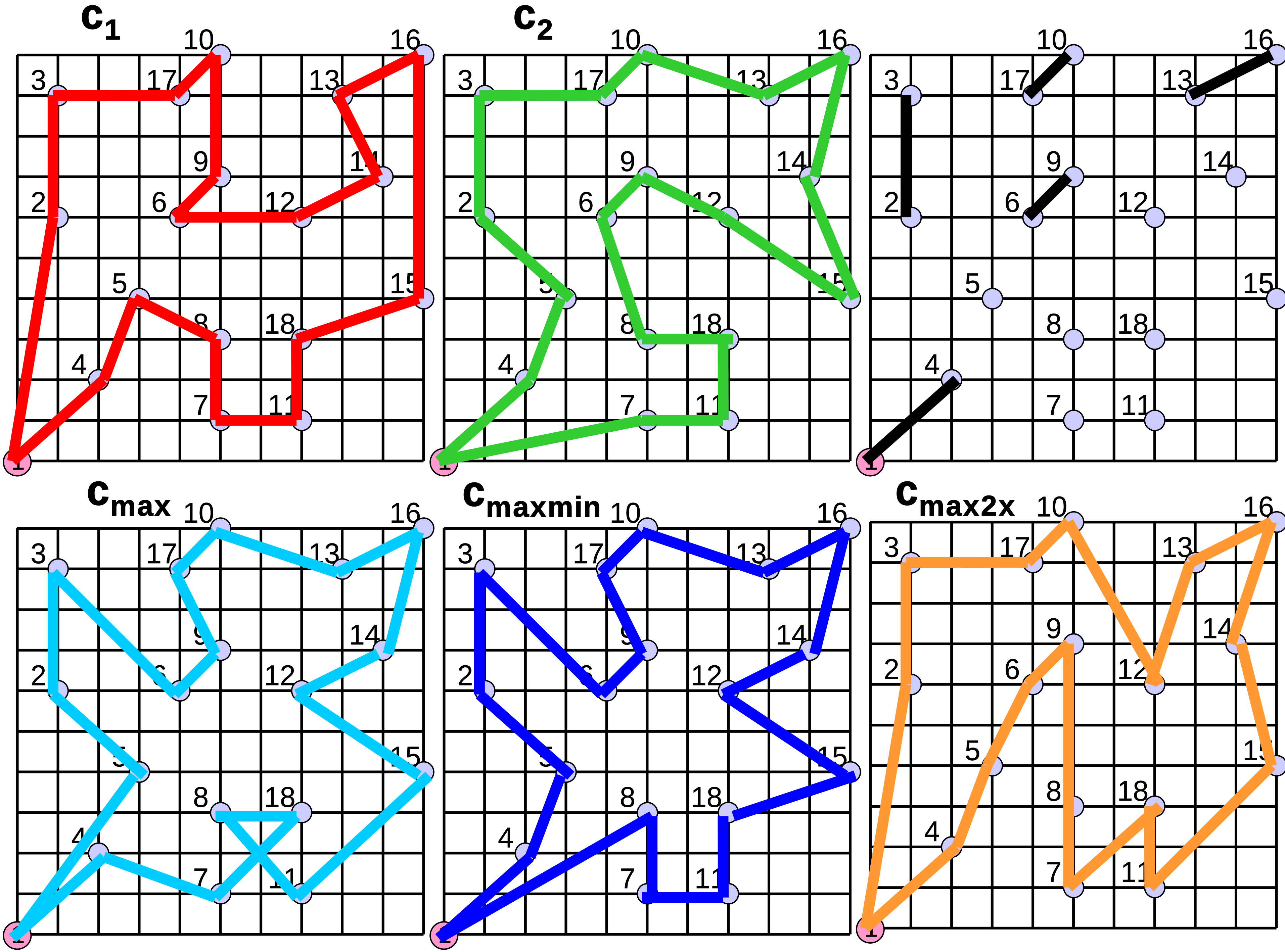
$$c_1(i,j) := |x_i - x_j| + |y_i - y_j|$$

$$c_2(i,j) := \sqrt{|x_i - x_j|^2 + |y_i - y_j|^2}$$

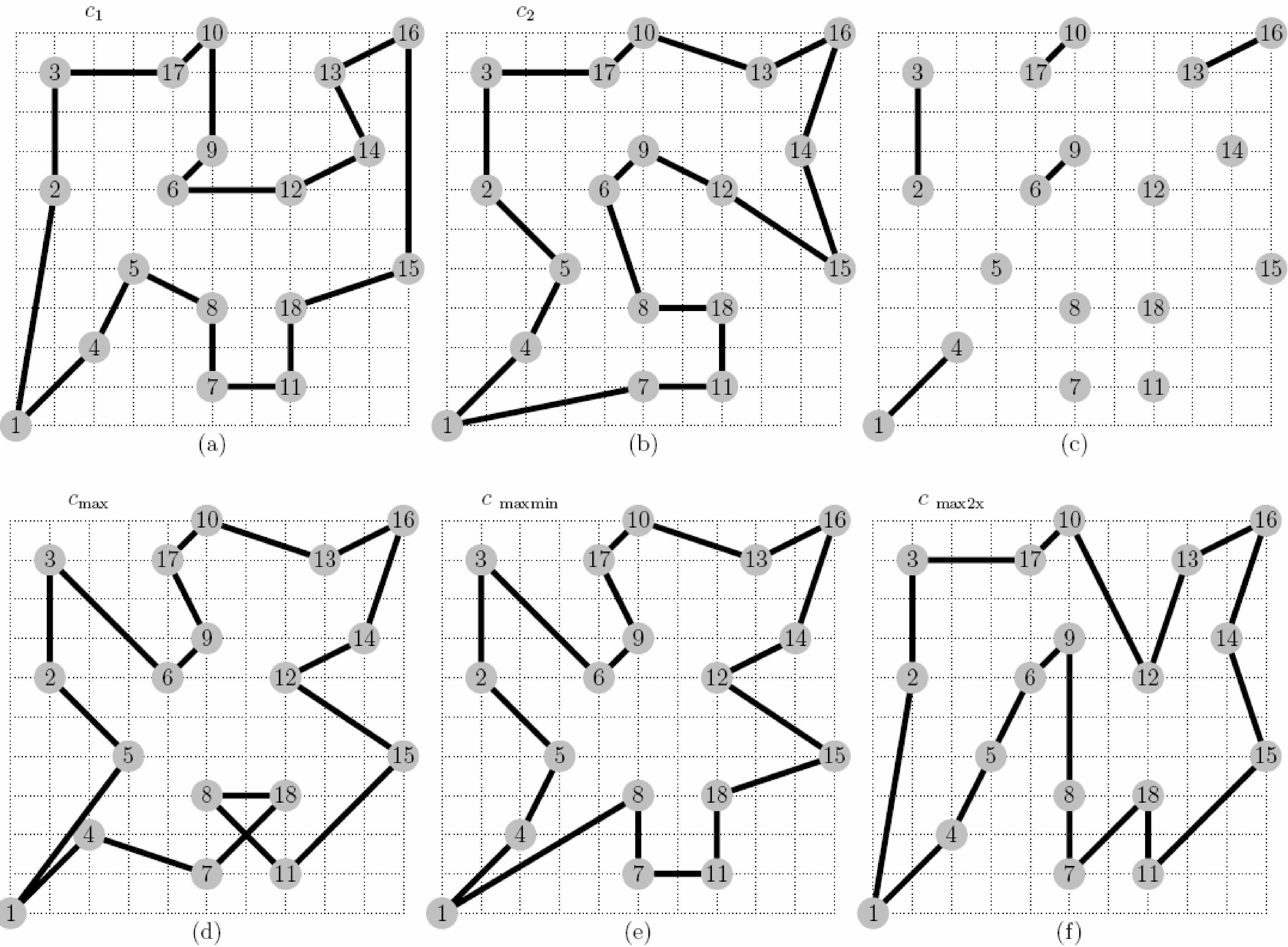
$$c_{max}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\}$$

$$c_{max\,min}(i,j) := \frac{199}{200} c_{max}(i,j) + \frac{1}{200} c_1(i,j).$$

$$c_{max\,2x}(i,j) := \max\{2\,|x_i - x_j|, |y_i - y_j|\}.$$



c1
c2
cmax
cmaxmin
cmax2x


c_1
(a)
c_2
(b)
(c)
$c_{\max}$
(d)
$c_{\max\min}$
(e)
$c_{\max 2x}$
(f)

Optimal Tours: objective values

	c_1-Wert	c_2-Wert	c_{max}-Wert	$c_{max\,min}$-Wert	$c_{max\,2x}$-Wert
Tour (a)	58	50,0822	47	47,0550	68
Tour (b)	60	48,5472	44	44,0800	72
Tour (d)	70	52,4280	43	43,1350	76
Tour (e)	64	49,5955	43	43,1050	72
Tour (f)	66	53,4779	48	48,0900	62

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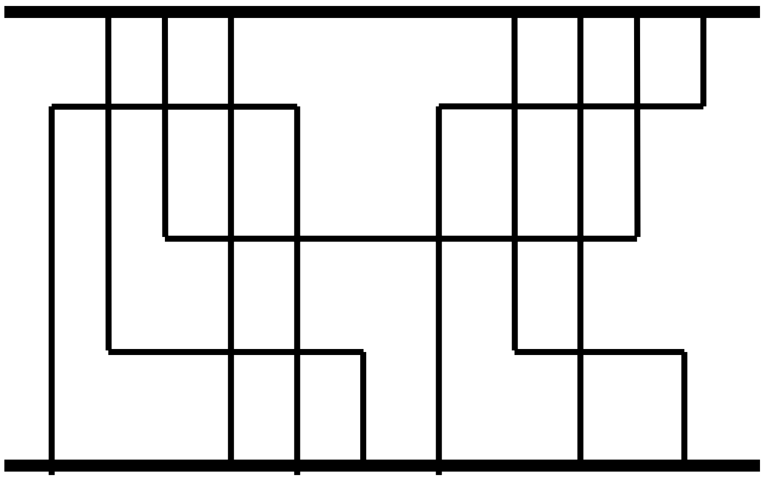
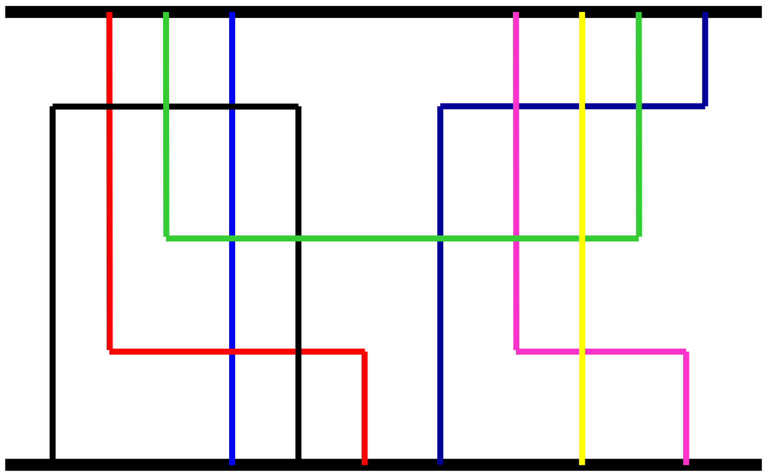
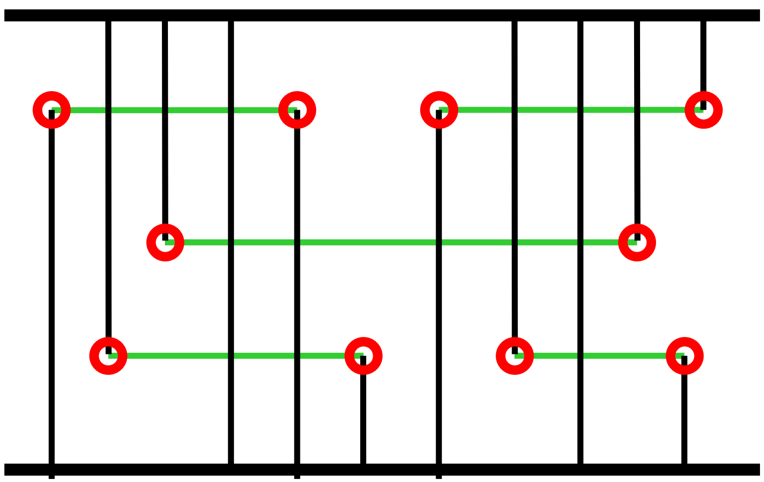
Via Minimization (2 layers)



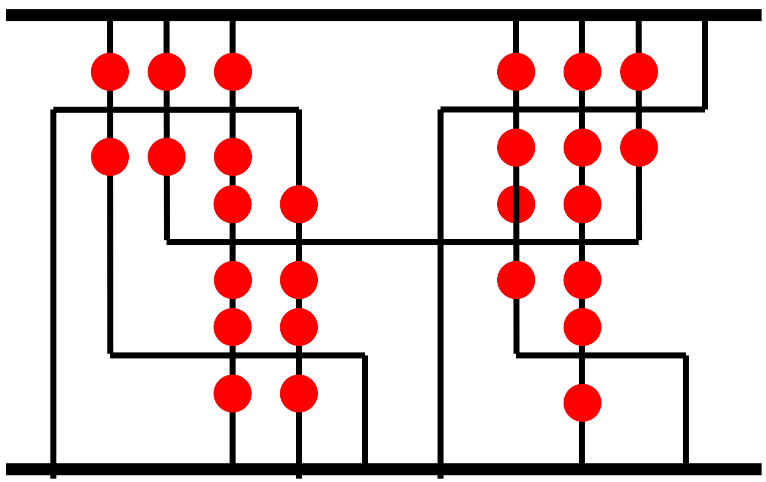
transient routing



7 nets

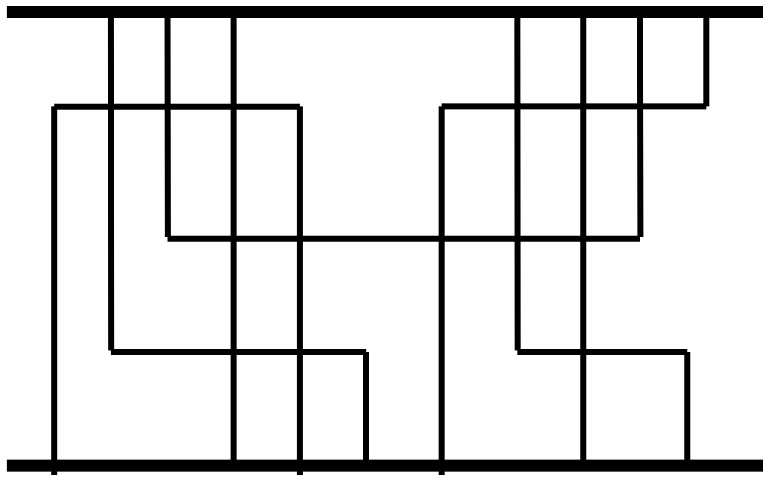


standard solution 10 vias

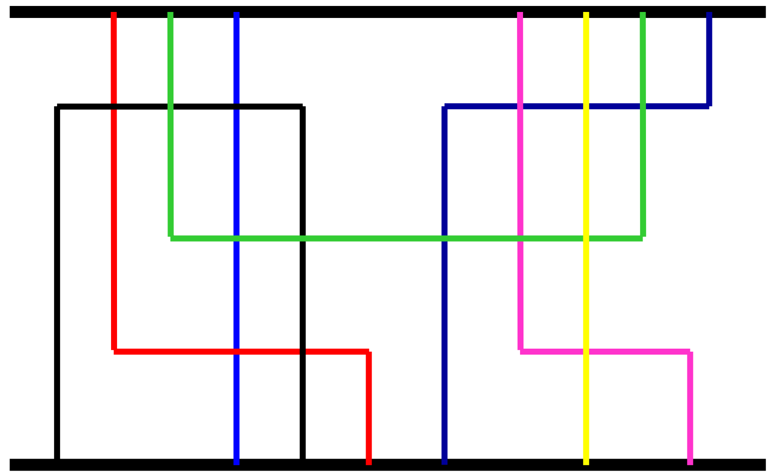


trivial solution 28 vias

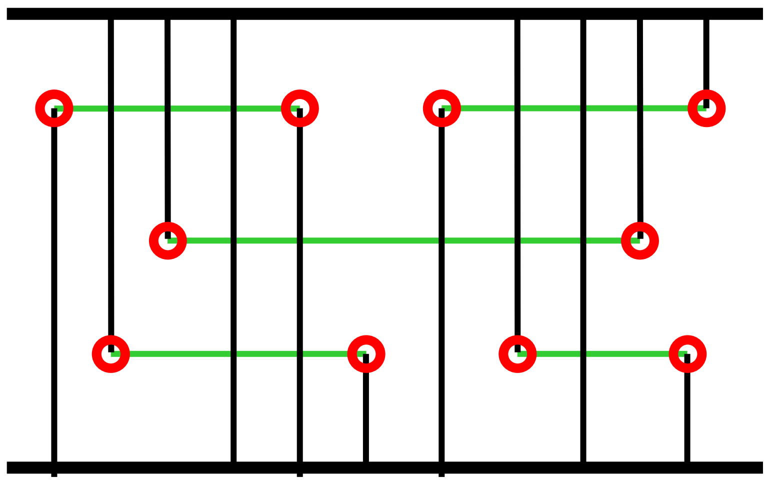
Via Minimization



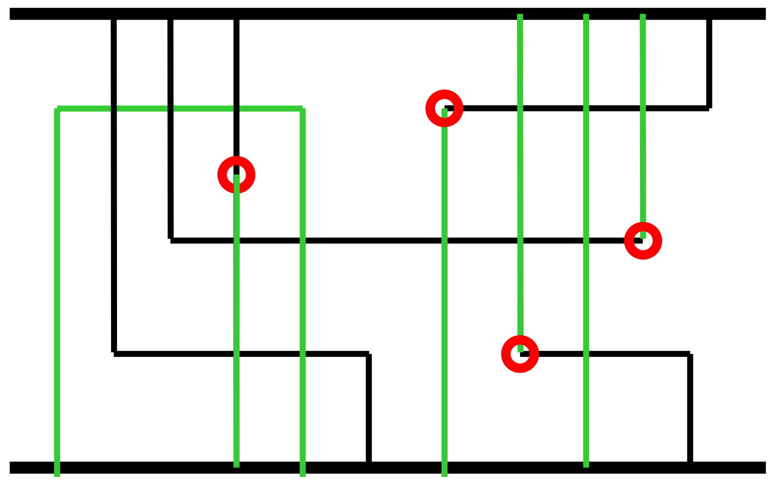
transient routing



7 nets

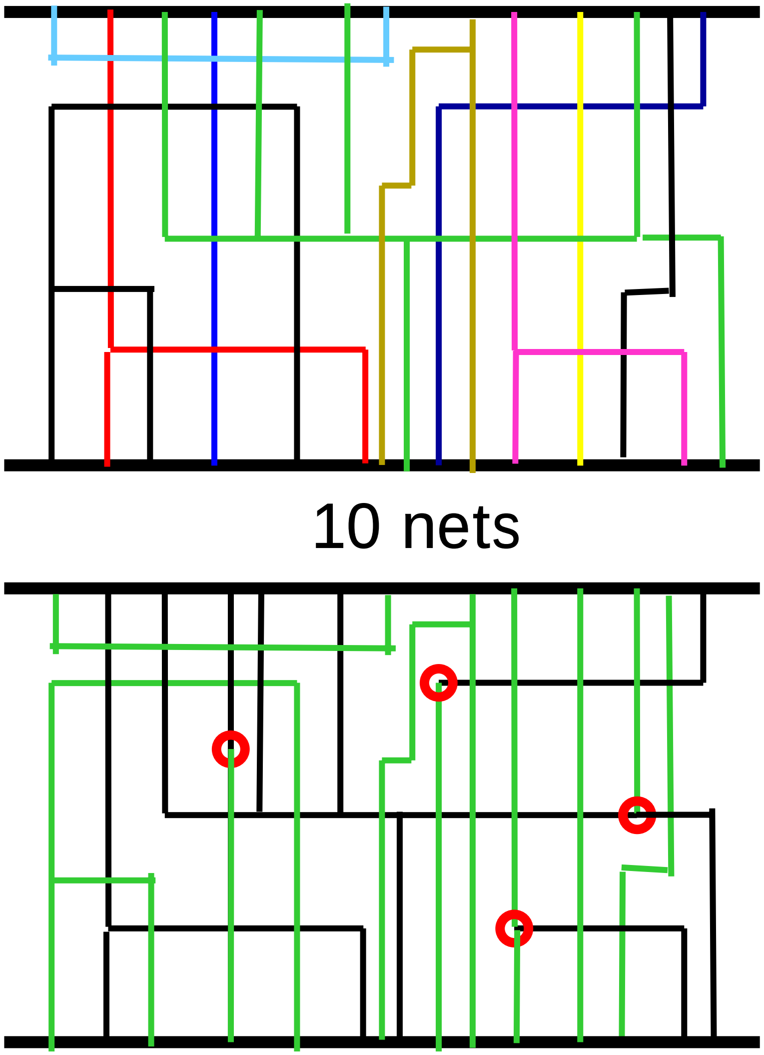


standard solution 10 vias



optimal solution 4 vias

Another Example



10 nets

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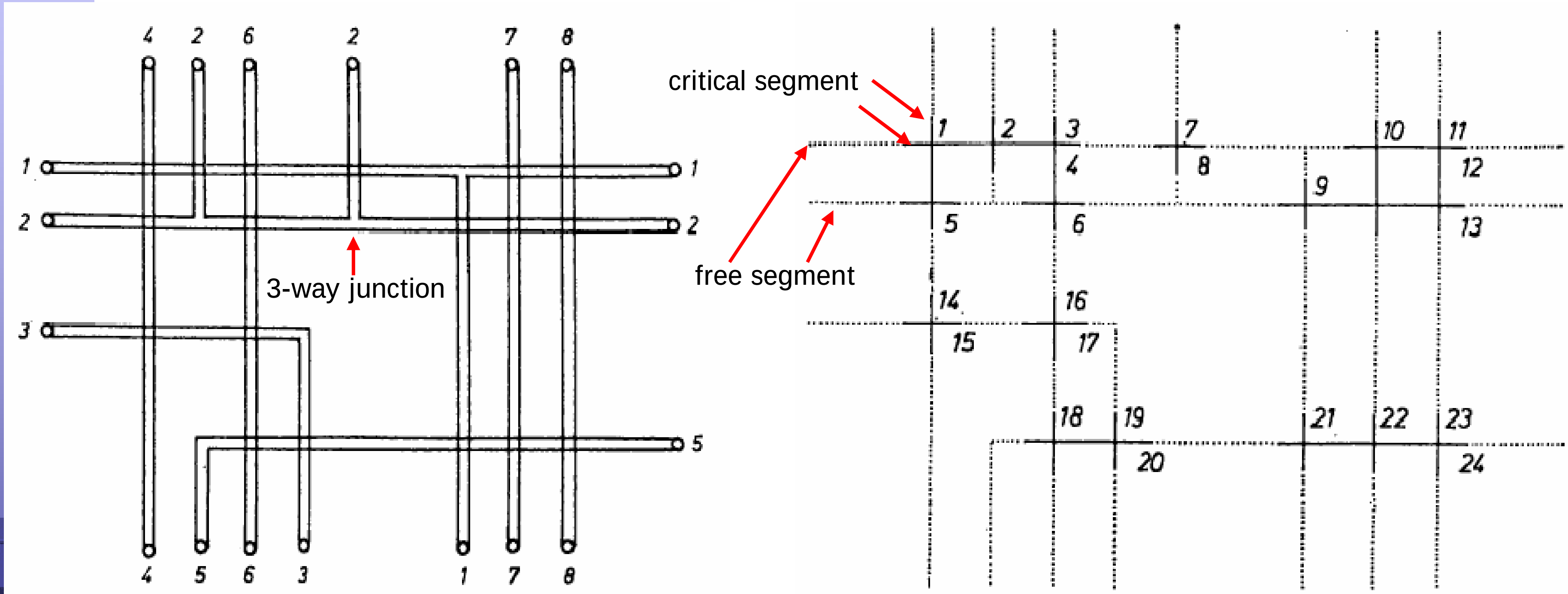
Via minimization

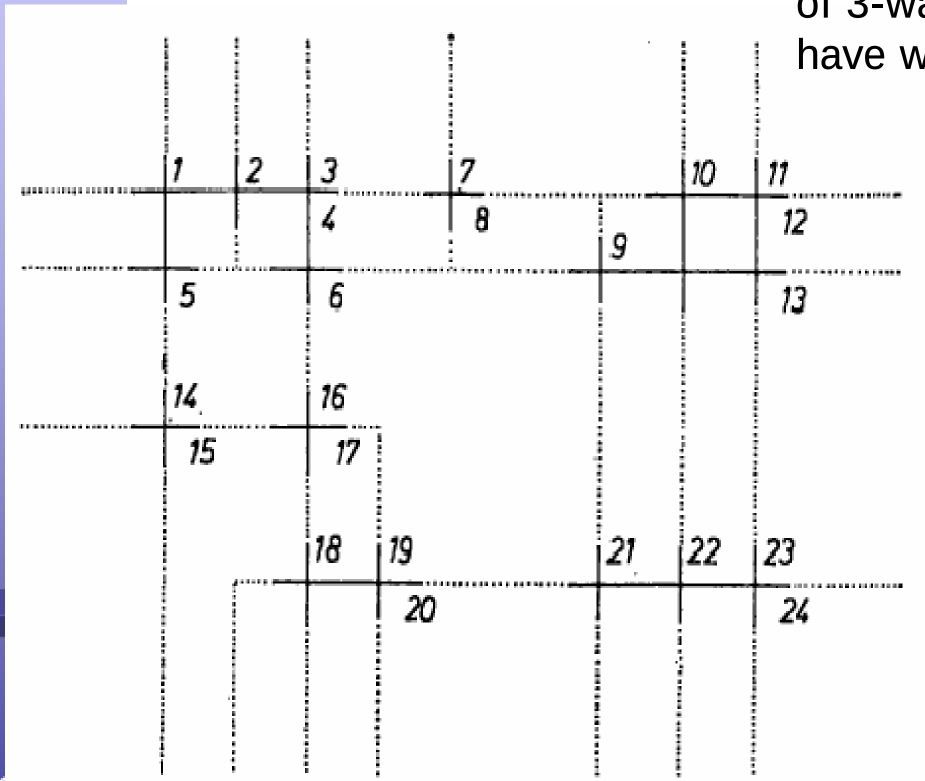


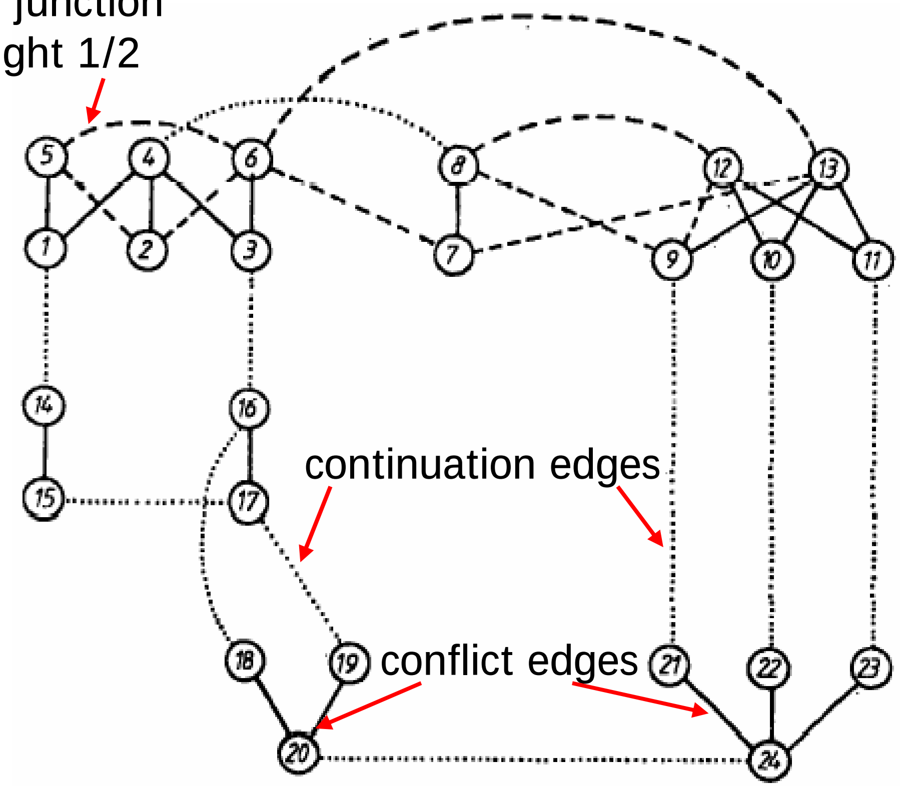
transient routing of 8 nets,
one net is a 4-pin net (2),
one a 3-pin net (1),
all others are 2-pin nets

24 critical and some free segments of the transient routing

Grötschel, Jünger, Reinelt

Constructing the layout graph





24 critical segments
of the transient routing

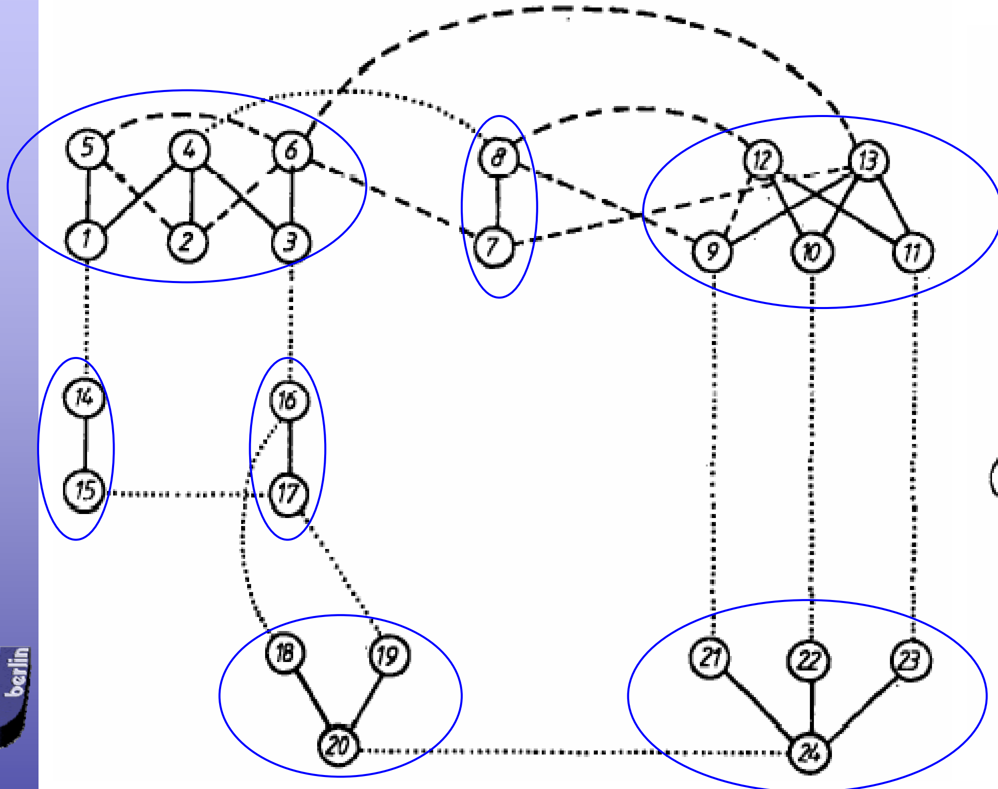
layout graph G = (V, E), E = A I B
A = conflict edges
B = continuation edges

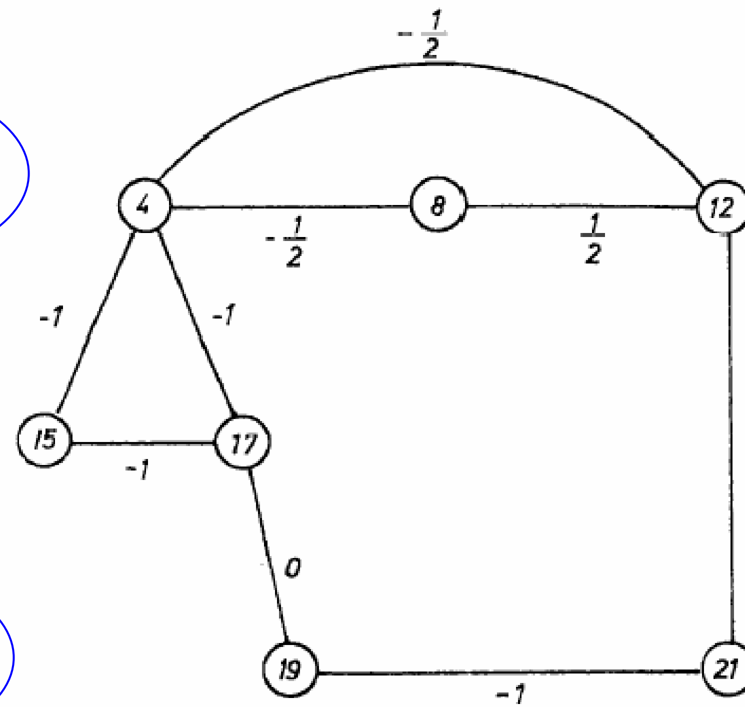
Via Minimization and Max Cut

Result of this construction:

Finding, among all cuts in the layout graph G that contain all conflict edges, a cut C such that the sum of the weights of its continuation edges is as small as possible, yields the smallest number of vias with which a transient layout can be assigned to two layers.

Another reduction step





layout graph G = (V, E), E = A I B

A = conflict edges

B = continuation edges

reduced layout graph R = (W, F)

Observation

- Any cut of maximum weight in the reduced layout graph R yields a cut in the layout graph G that contains all conflict edges such that the sum of the weights of its continuation edges is as small as possible.
- This construction produces a minimal number of vias.

The Max-Cut Problem

Given a graph with edge weights, find a partition of the nodes into two parts so that the sum of the weights of the edges linking the two parts is as large as possible.

Compare to min-cut problem!

$$\max \{c(\delta(W)) \mid W \text{ subset of } V\},\ G=(V,E) \text{ a graph}$$

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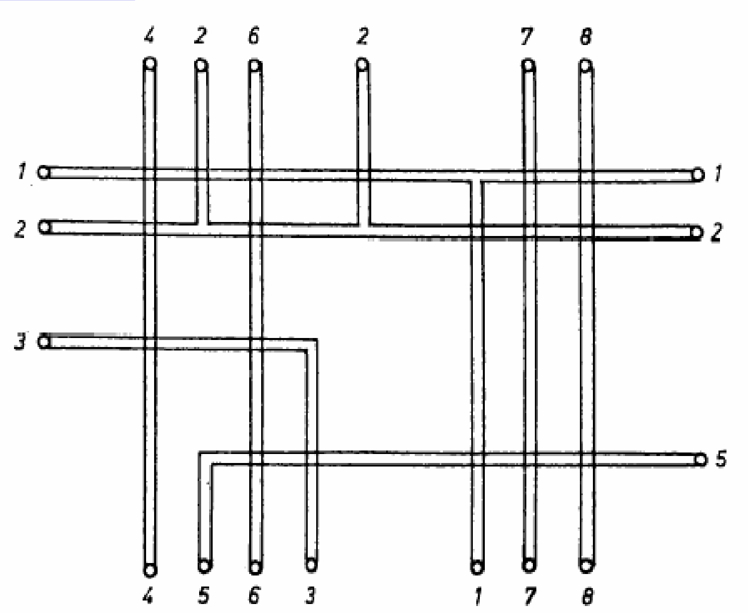
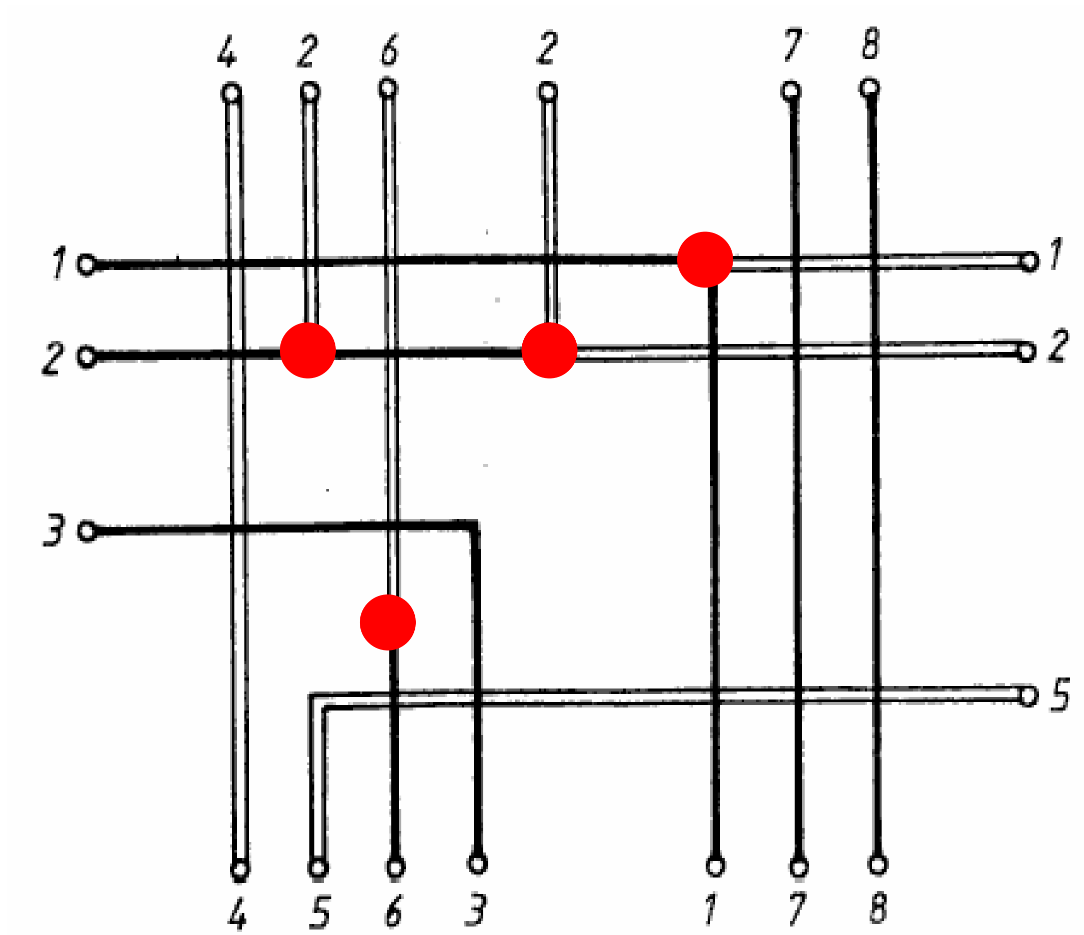
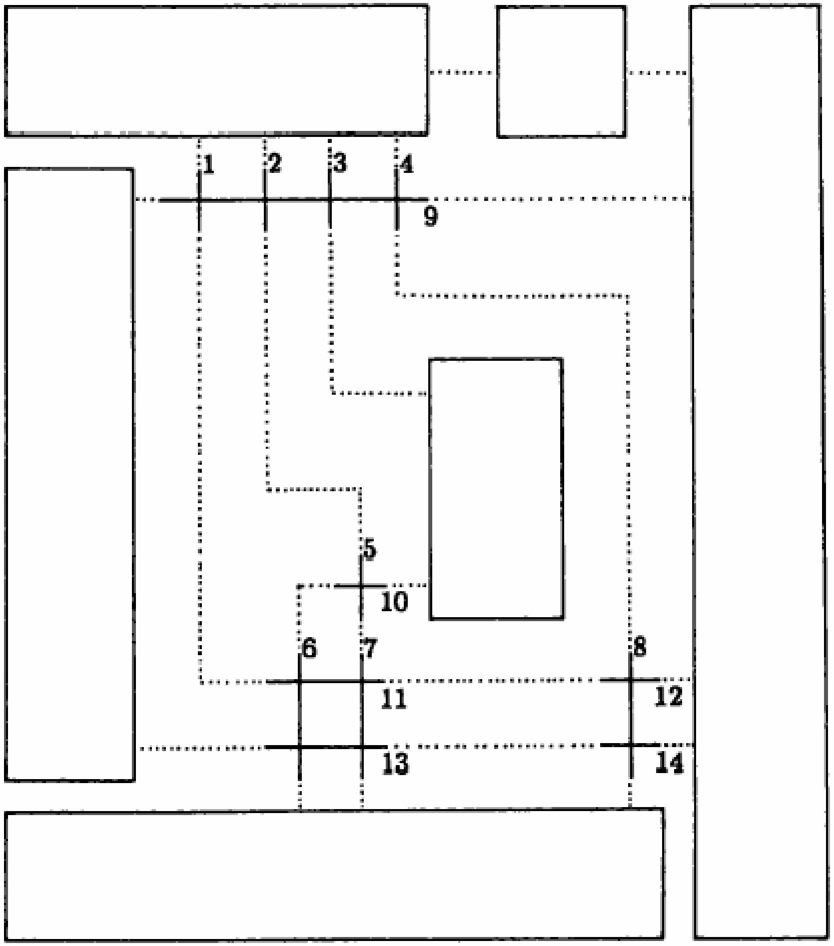
Optimal Layout has 4 vias



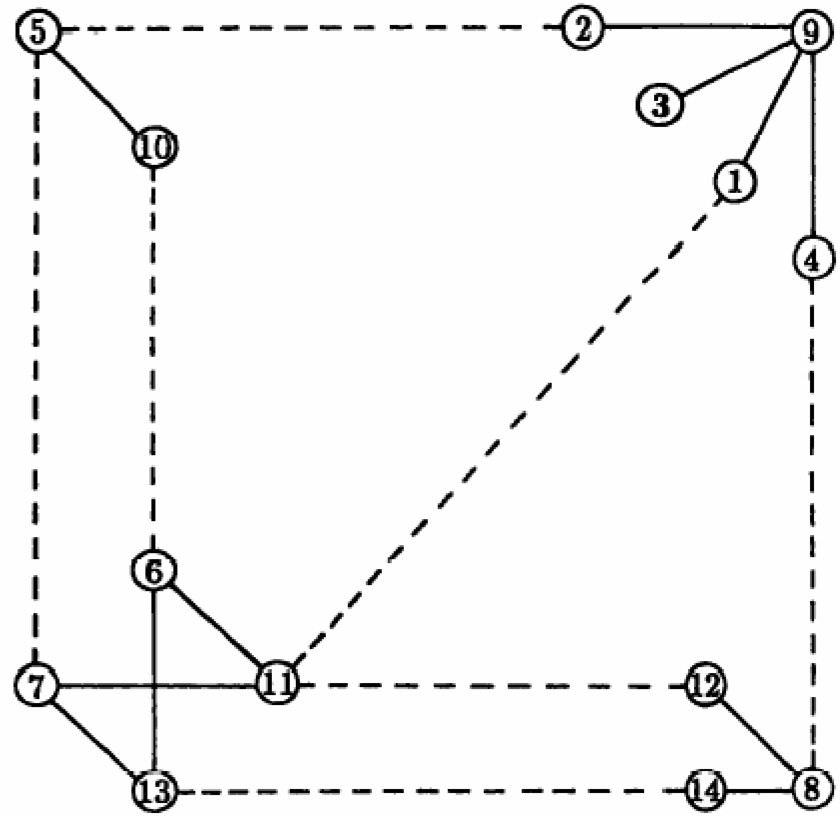


Another example



Printed Circuit Board transient routing with critical segments

Barahona, Grötschel, Jünger, Reinelt



layout graph G = (V, E), E = A I B

A = conflict edges = solid lines

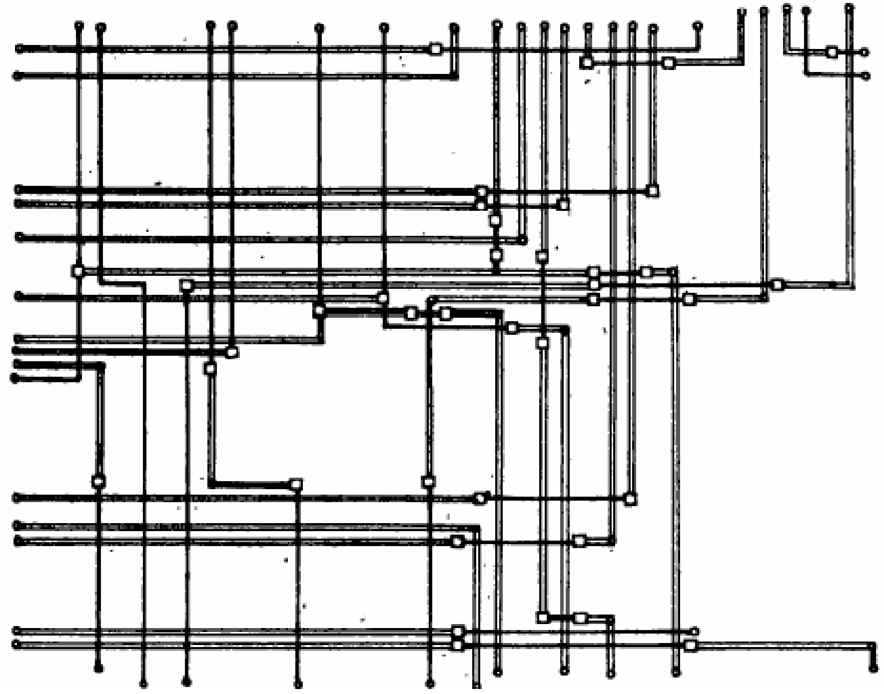
B = continuation edges = dashed lines

Some examples: Siemens printed circuit boards

Table 1

	C1	C2	C3	C4	C5
nodes in reduced layout graph	828	980	1327	1202	1366
edges in reduced layout graph	1445	1775	2480	2234	2606
vias in original design	421	434	683	650	782
via minimization with preassignments	302	376	563	504	645
via minimization without preassignments	272	347	513	475	610

A small cut-out of the optimal solution of a real Siemens PCB

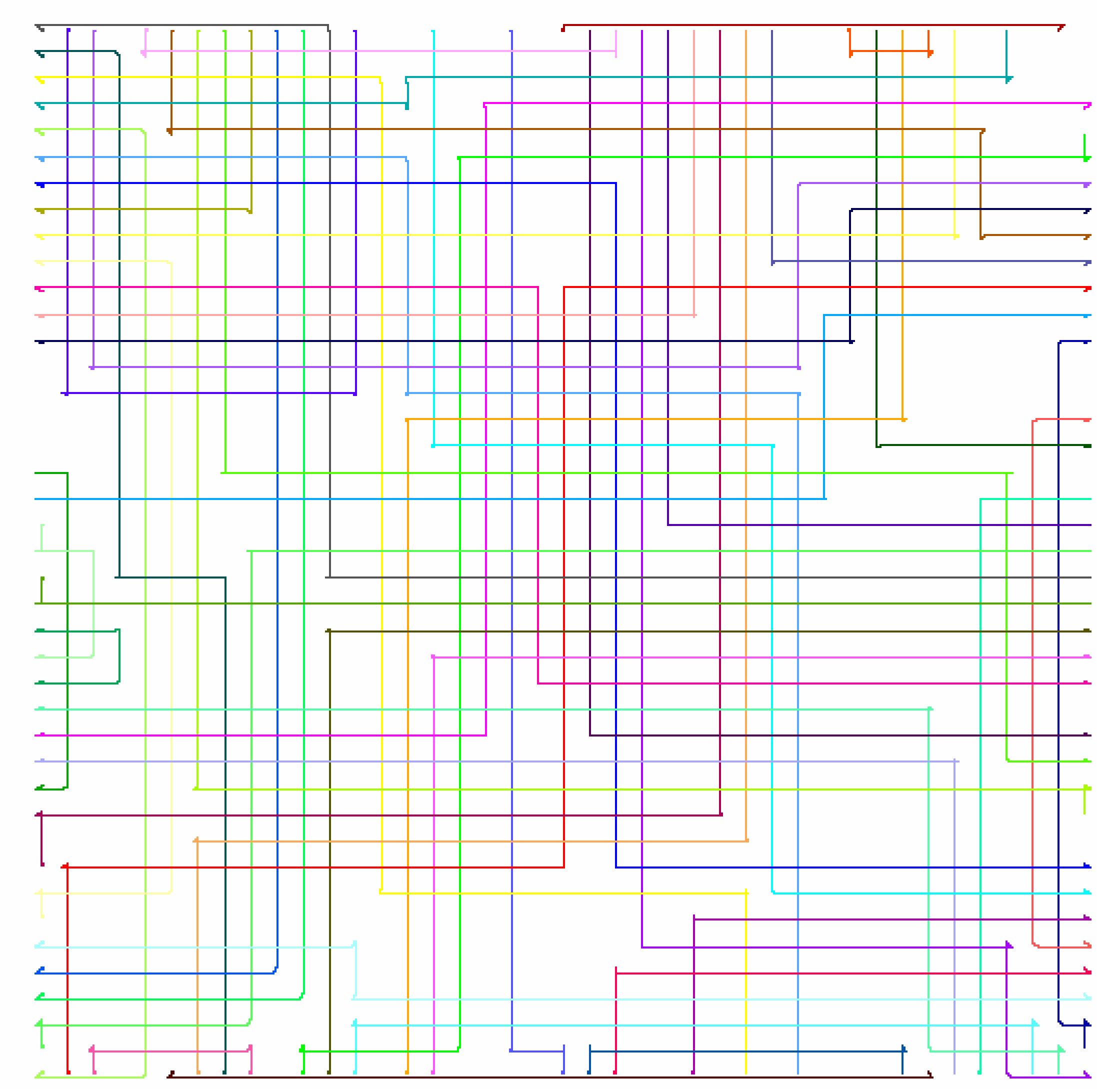


Grötschel, Jünger, Reinelt

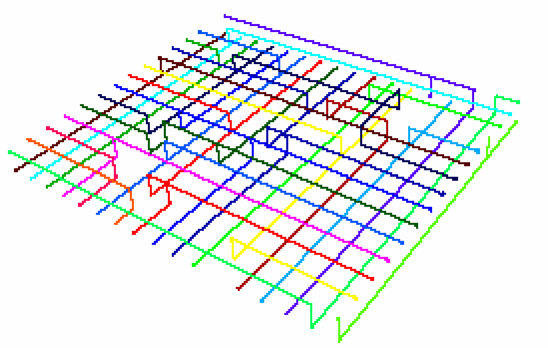
Thorsten Koch PhD Thesis

- new model
- any number of layers
- combines two objectives:
  wiring length & number of vias

Dissertation Thorsten Koch



optimal solution of a track routing problem with simultaneous via minimization



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The Max-Cut problem

Given a graph with edge weights, find a partition of the nodes into two parts so that the sum of the weights of the edges linking the two parts is as large as possible.

Compare to min-cut problem!

$$\max \{c(\delta(W)) \mid W \text{ subset of } V\},\ G=(V,E) \text{ a graph}$$

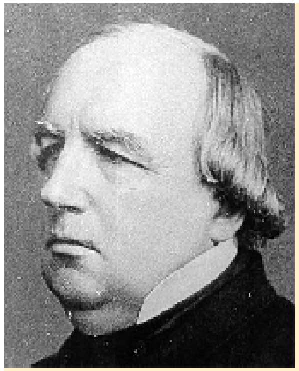
Eulerian Subgraphs and the Chinese Postman Problem

Given a graph $G=(V,E)$ with edge weights $c(e)$, $e \in E$. A subset F of E is called Eulerian if all nodes in $H=(V,F)$ have even degree. (H does not have to be connected.) Find an Eulerian edge set (or subgraph) of maximum weight.

- Kwan, M. K. "Graphic Programming Using Odd or Even Points." *Chinese Math.* **1**, 273-277, 1962.
- Alan Goldman first coined the name 'Chinese Postman Problem' for this problem, as it was originally studied by the Chinese mathematician Mei Ko Kuan.
- Edmonds, J. and Johnson, E. L. "Matching, Euler Tours, and the Chinese Postman." *Math. Programm.* **5**, 88-124, 1973.

Three well known mathematical members of the BBAW

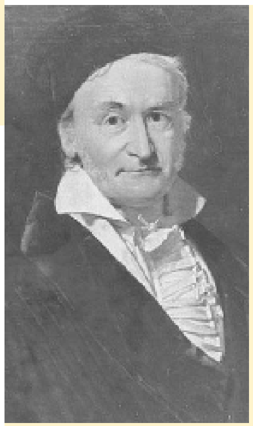
Karl Weierstrass * 31.10.1815 (Ostenfelde/Westf.) - † 19.02.1897 (Berlin)



Mitgliedschaft(en):

19.11.1856 Ordentliches Mitglied

Karl Friedrich Gauss * 30.04.1777 (Braunschweig) - † 23.02.1855 (Göttingen)

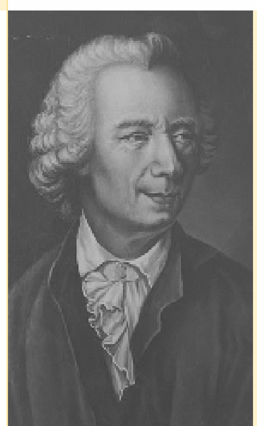


Mitgliedschaft(en):

18.07.1810 Auswärtiges Mitglied

Änderung/Austritt/Ausschluss:
11.11.1824

Leonhard Euler * 15.04.1707 (Basel) - † 18.09.1783 (Petersburg)



Mitgliedschaft(en):

1741 Ordentliches Mitglied

Änderung/Austritt/Ausschluss:
29.05.1766

29.05.1766 Auswärtiges Mitglied

The Origin of Graph Theory

SOLUTIO PROBLEMATIS
AD GEOMETRIAM SITUS PERTINENTIS

Commentatio 53 indicis ENESTROEMIANI
Commentarii academiae scientiarum Petropolitanae 8 (1736), 1741, p. 128—140

Euler

[Link to paper]

This is not Königsberg
but another example
from Euler's article.

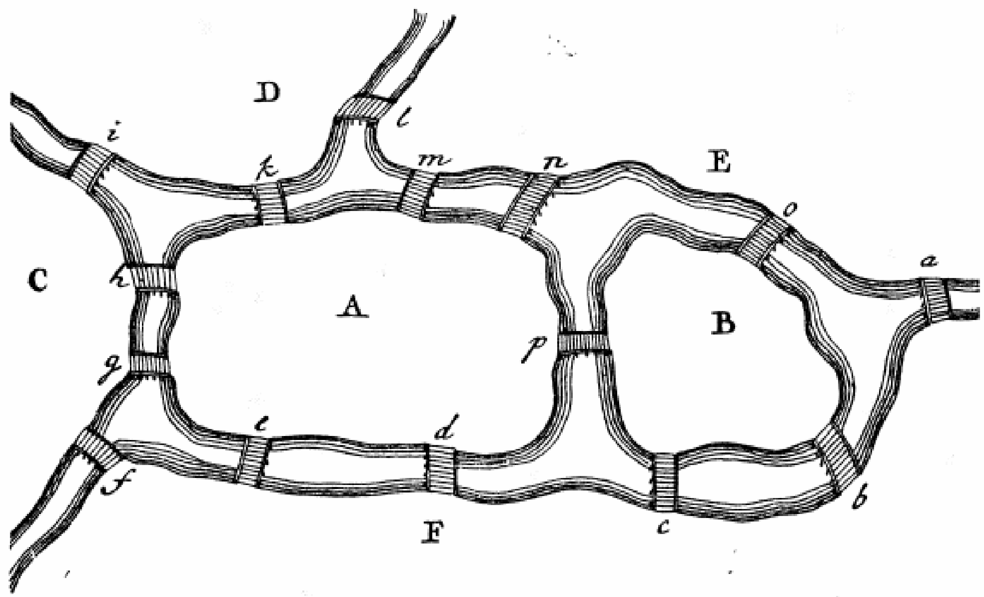


Fig. 3.

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Grötschel

Let us look at the Euler paper

- Euler introduces the symbols A,B,... (for what we call nodes nowadays) and a, b, ... (for edges), and the notation a = AB for edges.
- He introduces the notation ACDBAC for paths and defines path length.
- He discusses notational difficulties with parallel edges.
- ...sequentes observationes in medium protulero...
(...a few preliminary observations...)
- Euler states the First Theorem in Graph Theory:
- Sum of node degrees = twice the number of edges.
- In the second side remark Euler mentions the
- Second Theorem in Graph Theory:
- The number of odd nodes is even.
- Euler's Theorem
Si fuerint plures duabus regiones, ad quas ducentium pontium numerus est impar, tum certo affirmari potest talem transitum non dari

Martin Grötschel

The Chinese Postman and the Max-Cut Problem

Given a graph G=(V,E) with edge weights c(e), eεE. A subset F of E is called Eulerian if all nodes in H=(V,F) have even degree. (H does not have to be connected.) Find an Eulerian edge set (or subgraph) of maximum weight.

Given a graph G=(V,E) with edge weights c(e), eεE. Find a partition of the nodes into two parts so that the sum of the weights of the edges linking the two parts is as large as possible.

2. Problema autem hoc, quod mihi satis notum esse perhibebatur, erat sequens: Regiomonti in Borussia esse insulam *A*, *der Kneiphof* dictam, fluviumque eam cingentem in duos dividi ramos, quemadmodum ex figura (Fig. 1) videre licet; ramos vero huius fluvii septem instructos esse pontibus *a*, *b*, *c*, *d*, *e*, *f* et *g*. Circa hos pontes iam ista proponebatur quaestio, num quis cursum ita instituere queat, ut per singulos pontes semel et non plus quam semel transeat. Hocque fieri posse, mihi dictum est, alios negare alios dubitare; neminem vero affirmare. Ego ex hoc mihi sequens maxime generale formavi problema: quaecunque sit fluvii figura et distributio in ramos atque quicunque fuerit numerus pontium, invenire, utrum per singulos pontes semel tantum transiri queat an vero secus.

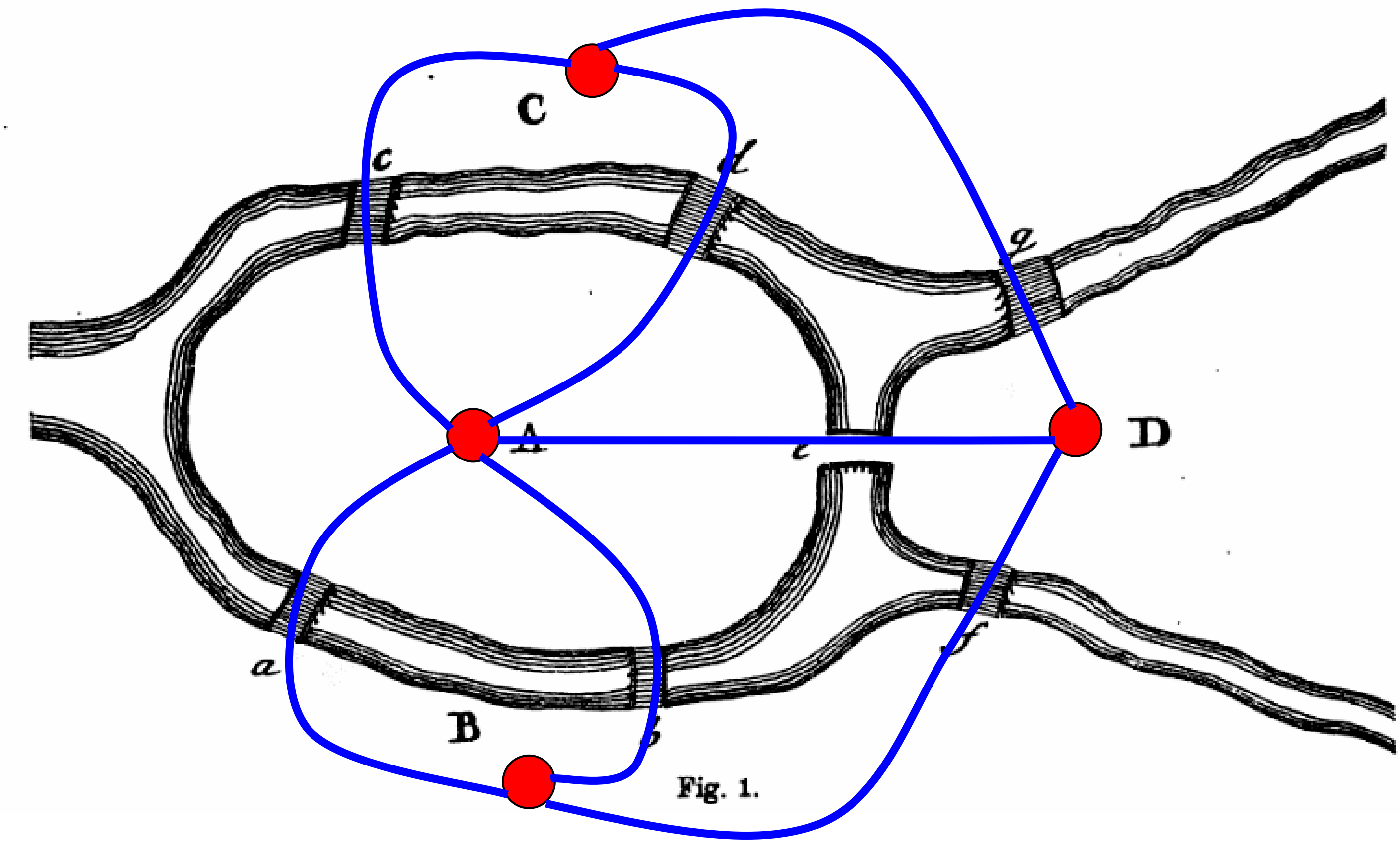
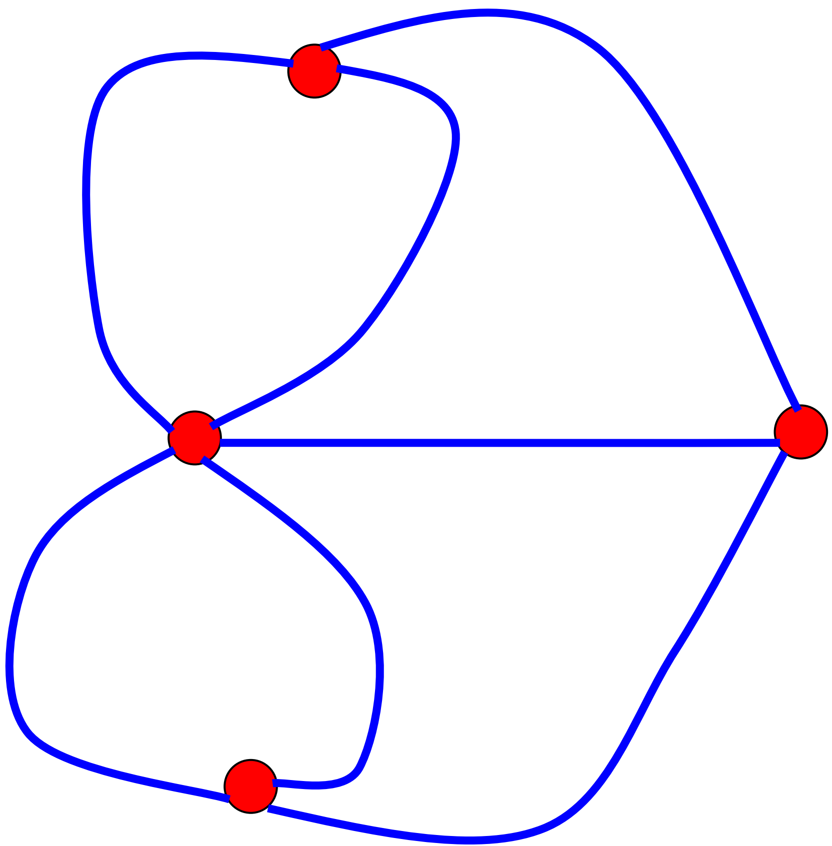


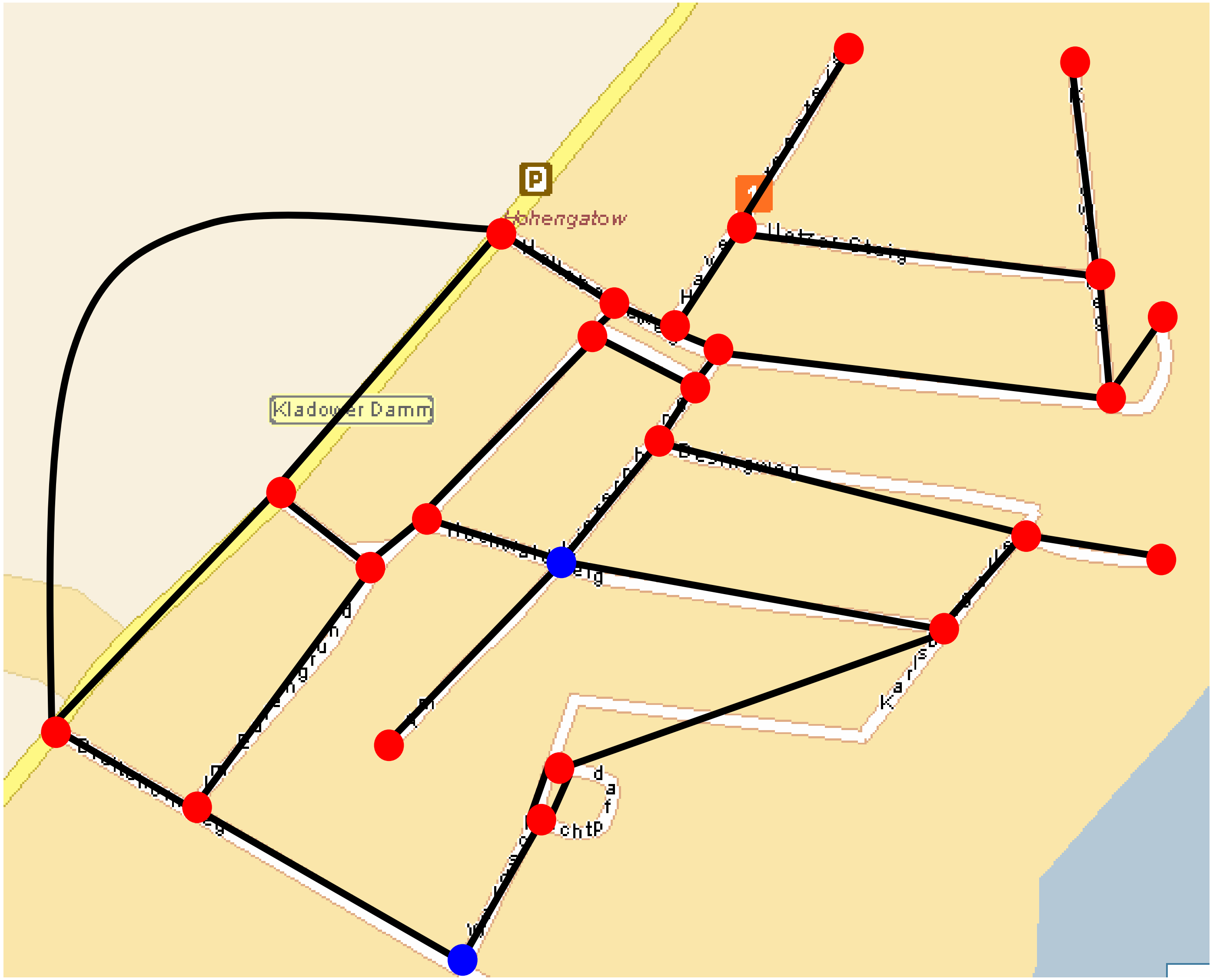
Fig. 1.

Euler’s Theorem (1736)

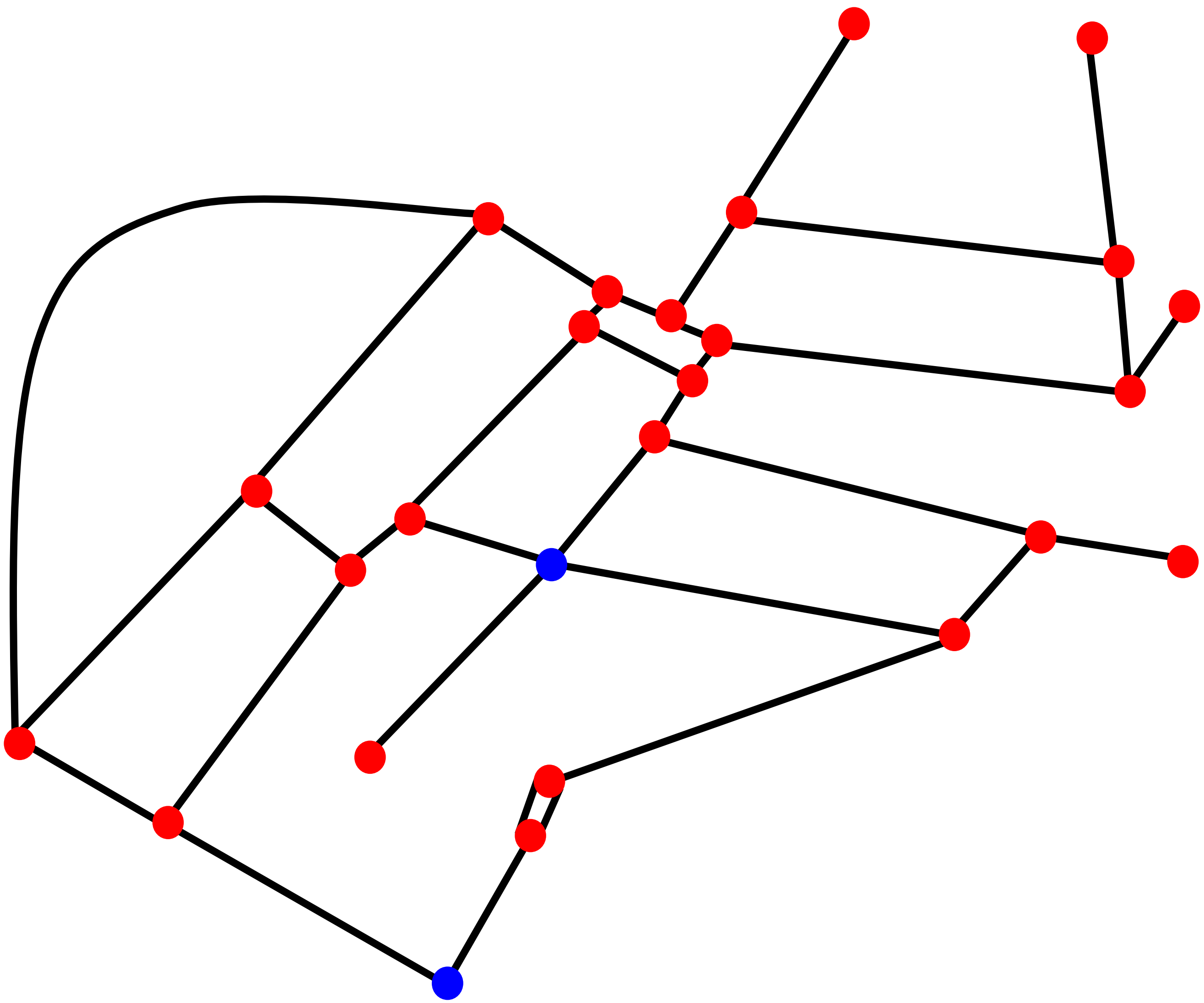
A graph has a (connected) Eulerian Tour if and only if, it is connected and every node has even degree.



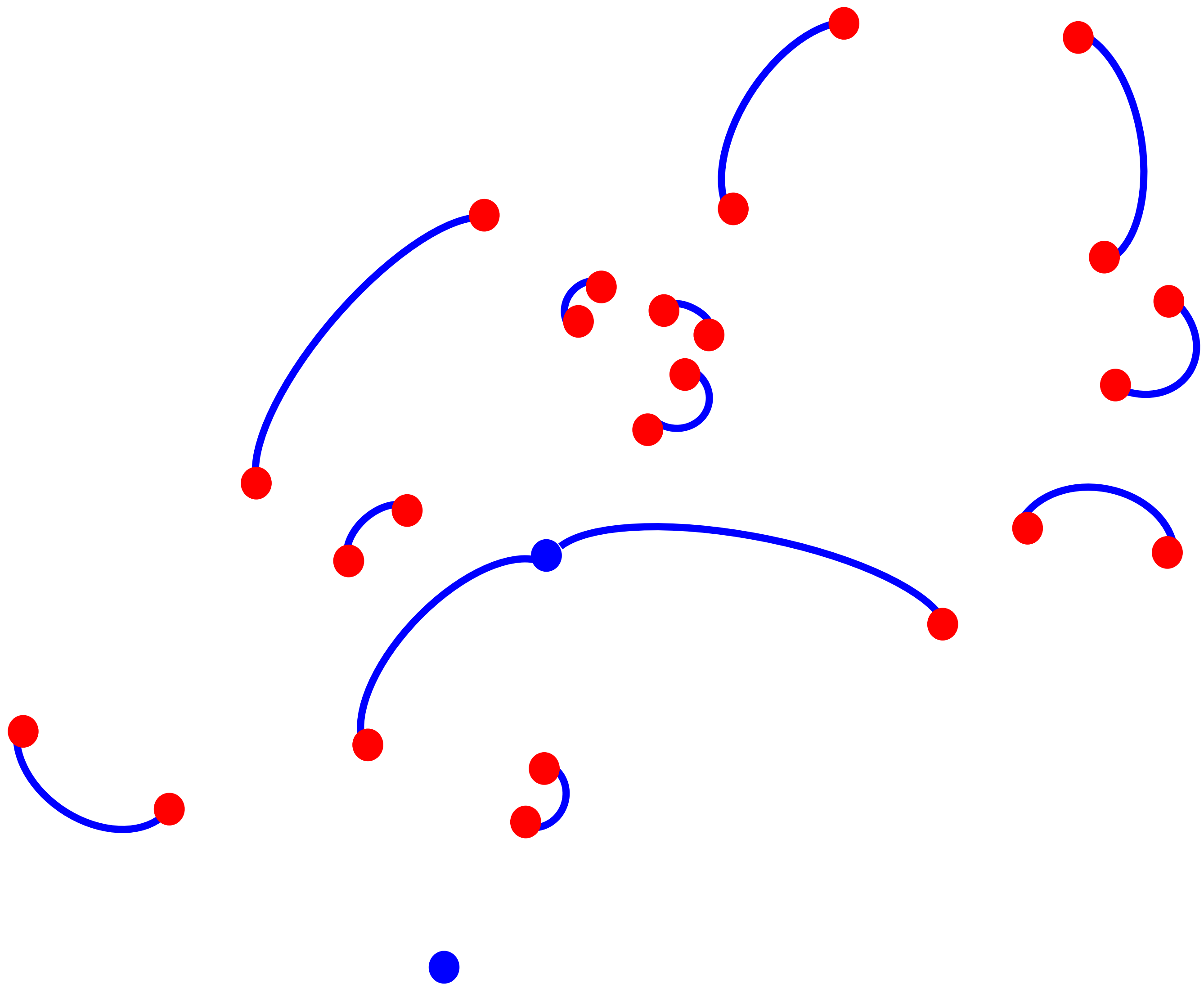
Berlin-Hohengatow



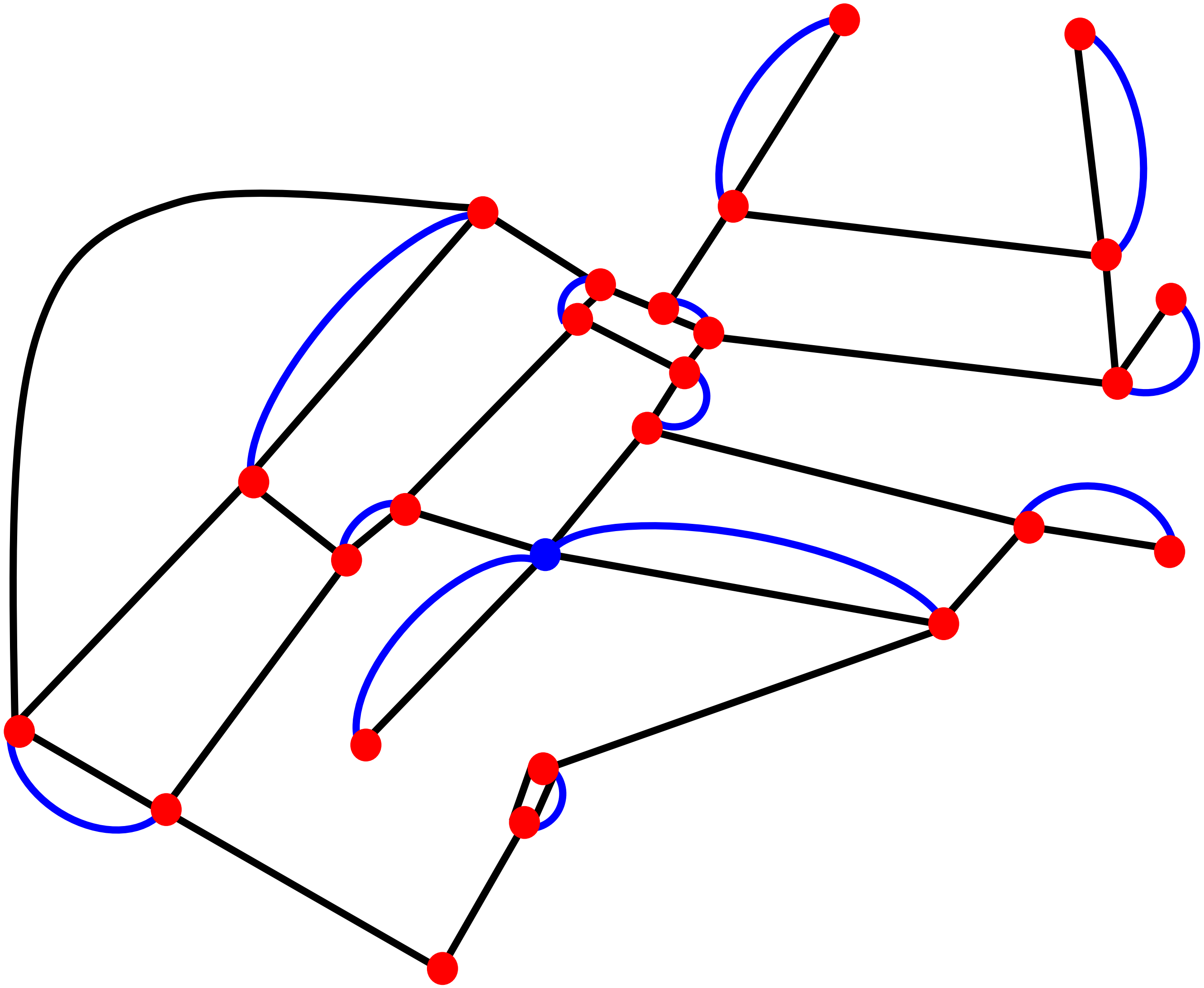
Hohengatow-Graph



Hohengatow-Graph



Hohengatow-Graph



Exercises

- Formulate different versions of the Chinese postman problem.
- Explore the relations between the Chinese postman problem and the max-cut problem.
- Under which additional assumption can one transform one problem into the other and vice versa?
- Provide IP formulations of the two problems.

Contents

Martin Grötschel

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The End

Martin Grötschel
- Institut für Mathematik, Technische Universität Berlin (TUB)
- DFG-Forschungszentrum “Mathematik für Schlüsseltechnologien” (MATHEON)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

groetschel@zib.de http://www.zib.de/groetschel