# 09M2 Lecture Combinatorial Auctions and Rail Track Scheduling

### **Martin Grötschel**

Beijing Block Course at TU Berlin "Combinatorial Optimization at Work" September 25 – October 6, 2006





- DFG-Forschungszentrum MATHEON "Mathematics for key technologies"
  - Konrad-Zuse-Zentrum f
    ür Informationstechnik Berlin (ZIB)
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# Outline

- Auctions
- Rail Track Scheduling
- Rail Track Auctioning
- The Optimal Track Allocation Problem
- Experiments





# **Auctions**

### Commodities/Bids

- Independent commodities (classical autcion)/ commodity bundles (combinatorial auction)
- Combinatorial bids (and/or/xor)

### Bidders

- Cooperation forbidden/ cooperation allowed
- Payment
  - First price/second price (Vickrey) auction

- Information
  - Private Values/Common Values (winner's curse)
  - Sealed Bid/Open Bid
- Mechanism
  - English auction/dutch auction
  - Increment/number of rounds
  - Activity rules/taking bids back
  - Direct bidding/clock/proxy auction



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# **Examples**

### In ancient times ...

- Auctions are known since 500 b.c.
- March 28, 193 a.d.: The pretorians auction the Roman Emperor's throne to Marcus Didius Severus Iulianus, who ruled as Iulianus I. for 66 days



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# The Story of Didius Iulianus

(http://www.roman-emperors.org/didjul.htm)



[193 A.D., March 28] When the emperor **Pertinax** was killed trying to quell a mutiny, no accepted successor was at hand. Pertinax's father-in-law and urban prefect, Flavius Sulpicianus, entered the praetorian camp and tried to get the troops to proclaim him emperor, but he met with little enthusiasm. Other soldiers scoured the city seeking an alternative, but most senators shut themselves in their homes to wait out the crisis. **Didius** Julianus, however, allowed himself to be taken to the camp, where one of the most notorious events in Roman history was about to take place. Didius Julianus was prevented from entering the camp, but he began to make promises to the soldiers from outside the wall. Soon the scene became that of an auction, with Flavius Sulpicianus and Didius Julianus outbidding each other in the size of their donatives to the troops. The Roman empire was for sale to the highest bidder. When Flavius Sulpicianus reached the figure of 20,000 sesterces per soldier, **Didius Julianus** upped the bid by a whopping 5,000 sesterces, displaying his outstretched hand to indicate the amount. The empire was sold, **Didius** Julianus was allowed into the camp and proclaimed emperor.



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# **Examples**

### In ancient times ...

- Auctions are known since 500 b.c.
- March 28, 193 a.d.: The pretorians auction the Roman Emperor's throne to Marcus Didius Severus Iulianus, who ruled as Iulianus I. for 66 days

### In modern times ...

- Traditional auctions (antiques, flowers, stamps, etc.)
- Stock market
- eBay etc.
- Oil drilling rights, energy spot market, etc.
- Procurement
- Sears, Roebuck & Co.
- Frequency auctions in mobile telecommunication
- Regional monopolies (franchising) at British Rail



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# **Arguments for Auctions**

### Auctions can ...

- resolve user conflicts in such a way that the bidder with the highest willigness to pay receives the commodity (efficient allocation, wellfare maximization)
- maximize the auctioneer's earnings
- reveal the bidders' willigness to pay
- reveal bottlenecks and the added value if they are removed

### Economists argue ...

 that a "working auctioning system" is usually superior to alternative methods such as bargaining, fixed prices, etc.



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# Sears, Roebuck & Co.



- 3-year contracts for transports on dedicated routes
- First auction in 1994 with 854 contracts
- Combinatorial auction
  - "And-" and "or-" bids allowed
  - $2^{854}$  ( $\approx 10^{257}$ ) theoretically possible combinations
  - Sequential auction (5 rounds, 1 month between rounds)
- Results
  - 13% cost reduction
  - Extension to 1.400 contracts (14% cost reduction)

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# **Frequency Auctions**

(Cramton 2001, Spectrum Auctions, Handbook of Telecommunications Economics)



- Prices for mobile telecommunication frequencies (2x10 MHz+5MHz)
  - Low earnings are not per se inefficient
  - Only min. prices => insufficient cost recovery

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# **Basic Auctions**

### Revenue Equivalence Theorem (Vickrey):

- Risk neutral bidders i=1,..,n
- Private values  $v_i \in [l, u]$  i.i.d. with strictly monotonously increasing distribution function  $F(v) = P(v_i \le v)$
- Every auction mechanism in which
  - the object is given to the bidder w.t. highest bid
  - a bidder with the lowest possible bid I expects no profit

results in the same revenue.

• Bids 
$$b(v) = v - \int_{l}^{u} F^{n-1}(x) dx / F^{n-1}(v)$$

# **Game Theory**

- Game (N,S,a)
  - N={1,...,n} player
  - S={(s<sub>1</sub>,...,s<sub>n</sub>)} strategies
  - $a:S \rightarrow R^n$  payoff
- Non-cooperative games
  - Dominance
  - (Nash-)Equilibrium ŝ
    - a(ŝ₁,..,s<sub>i</sub>,..,ŝ<sub>n</sub>)≤a(ŝ₁,.., ŝ<sub>n</sub>) ∀i
    - (i.g. no existence/uniqueness)
  - Matrix games: saddle point, minimax

- Theorem (Nash): Every finite non-cooperative n-person game has at least one equilibrium of mixed strategies.
- Theorem (Nikaido, Isoda): Generalization to auction frameworks.
- Cooperative games
  - Imputation (payoff to coalition)
  - Concepts such as core, stable set, bargaining set, kernel, nucleolus, etc.



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# **Combinatorial Auction**

- Combinatorial Auction Problem (CAP)
  - M objects, N bidders,  $b^{j}(S)$  bid by j for S $\subseteq$ M
  - y(S,j) 0/1-variable for giving S to j

$$\max \sum_{\substack{S \subseteq M \\ S \ni i}} \sum_{j \in N} b^{j}(S) y(S, j) \\ \sum_{\substack{S \ni i \\ j \in N}} \sum_{j \in N} y(S, j) \\ y(S, j) \\ \leq 1 \quad \forall i \in M \\ \in \{0, 1\} \quad \forall S \subseteq M, j \in N$$





- Set Packing Problem
- Auction framework

# **Simultaneous Ascending Auction**

Rules

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- Multiple heterogeneous objects
- Combinatorial auction, but only individual bids
- First price sealed bid
- N rounds, minimum increment
- Activity rule #objects
- Fee for taking back
- Empirical efficiency 67%
  - High revenues, partly due to losses for bidders

Equilibrium

	Α	В	AB
1	4	6	9
2	4	5	7
Р	4	6	

Exposure problem



• Efficiency  $\sum v_i(\overline{y}_i) / \sum v_i(y_i^*)$ 



# **Exposure Problem**

# The Chopstick Auction: A Study of the Exposure Problem in Multi-Unit Auctions

Florian Englmaier, Pablo Guillen, Loreto Llorente, Sander Onderstal and Rupert Sausgruber

### JANUARY 2004

Multi-unit auctions are sometimes plagued by what is called the 'exposure problem'. We speak of an exposure problem when bidders aim at winning several objects in a multi-object auction but are *exposed* to the risk of buying too few as competition on some of these objects turns out to be tougher than expected.<sup>1</sup> Several economists have argued that the exposure problem in auction should be prevented as it leads (1) to an inefficient outcome of the auction and (2) to low revenue. In this paper, we will investigate whether these claims are true, both using a game theoretical model and a laboratory experiment.

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# **Simultaneous Ascending Auction**

- Auction #1 (USA 1994)
  - 10 licenses
  - 3 national bandwidths
  - Paging/messaging services
  - ≤3 licenses/bidder
  - Increment 2%
  - 47 rounds (1 week)
  - 617 Mio. USD
     (50 Mio. USD expected)

- Auction #4 (USA 1994)
  - 99 licenses
  - 2 bandwidths, 51 MTAs
  - Mobile telephone services
  - Increment 5%
  - 112 rounds (3 months)
  - 7.000 Mio. USD

# **Adaptive User Selection Mechanism**

- Rules
  - Several heterogeneous objects
  - Combinatorial bids
  - First price open bid
  - Continuos bidding
  - No activity rule
  - Auctioneer determines end
- Empirical efficiency 94%
- Complexity with bidders, lower revenues than SAA

Threshold problem



Proposal: Standbye Q



Free rider problem

# **Threshold problem:**

 Combinatorial auctions present the following strategic impediment to efficient outcomes. Suppose each of two small bidders is bidding on a separate item, but a third bidder is bidding on a package that contains both items. Then the two small bidders must implicitly coordinate through their bidding to ascertain what price each will pay in order for the sum of both bids to exceed the package bid.





# **Free Rider**

In proposing the mechanisms described in this paper we are motivated by public goods in modern communication and computation systems. Consider for example a large distributed database, containing information available to all users, without exclusion. Each user contributes towards the building / maintenance of this database, either in direct monetary terms or through contributed storage resources. Since the information in the database is assumed to be freely available to all users, each user has an incentive to minimize the amount of resources it contributes. However, if every user acts according to these selfish considerations, the net result could be a possibly severe under-provisioning of the resource. This is the classic "free-rider problem:" improper provisioning of a public good – the database – due to selfish behavior. Other examples of public resources are community wireless data access, and file distribution and storage in peer-to-peer networks.



# Vickrey Auction (Nobel price in Economics 1996)

**Combinatorial auction** 

$$E(N,b) \coloneqq \max \sum_{S \subseteq M} \sum_{j \in N} b^{j}(S) y(S,j)$$
$$\sum_{S \ni i} \sum_{j \in N} y(S,j) \le 1 \quad \forall i \in M$$
$$y(S,j) \in \{0,1\} \quad \forall S \subseteq M, j \in N$$

- Private values v<sub>i</sub>
- **Mechanism** 
  - Bids  $b_i = v_i$
  - Payments

$$z_j = E(N \setminus j, v) - E(N, v) | N \setminus j$$



# Vickrey-Clarke-Groves-Mechanism

Combinatorial auction

$$E(N,b) \coloneqq \max \sum_{S \subseteq M} \sum_{j \in N} b^{j}(S) y(S,j)$$
$$\sum_{S \ni i} \sum_{j \in N} y(S,j) \le 1 \quad \forall i \in M$$
$$y(S,j) \in \{0,1\} \quad \forall S \subseteq M, j \in N$$

Example

	Α	В	AB
1	10	5	15
2	1	6	12
Р	6	5	

Collusion



Fraud by auctioneer

- Private values v<sub>j</sub>
- Mechanism
  - Bids  $b_j = v_j$
  - Payments

 $z_j = E(N \setminus j, v) - E(N, v) | N \setminus j$ 

# **Proxy-Auction**

- Combinatorial first price sealed bid auction
- Bids by proxy-agent (program)
- Theorem (Ausubel, Milgrom): A proxy-auction, interpreted as a cooperative game, terminates in the core.
- Theorem (Ausubel, Milgrom): A proxy-auction, interpreted as a non-cooperative games, terminates under certain conditions in a Nash-equilibrium, in particular, if a corresponding Vickrey-Clarke-Groves-auction terminates in a Nash-equilibrium.



 Combinations with other auctions, e.g., clock-proxy, to simplify programming of the agent.

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# **Rail Track Scheduling**



\*Besondere Daten, z. B. fahrdynamische Daten von Triebfahrzeugen, müssen 14 Tage vor der Trassenanmeldung abgegeben werden.

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Geschäfte mit der Bah	<b>n</b>   Reisen   Logistik   Unternehmen <sub>Starts</sub>	eite   Kontakt   Sitemap   FAQ   Hilfe
Sucha		Neue Inhalte
Statter	Geschäfte $\rightarrow$ Infrastruktur & Energie $\rightarrow$ Fahrweg $\rightarrow$ Trassen	
→ Einkauf & Verkauf	Trassennutzung für den Personen- und Güterverkehr	
→ Fahrzeuge Straße/Schiene		
→ Immobilien	Hier finden Sie detaillierte Angebote und Preisinformationen zur Nutzung von Trassen der DB Netz AG für den Personen- und Gütertransport. Die	
→ Infrastruktur & Energie	zusätzlich angebotene Software unterstützt Sie bei der Kalkulation der	
-> Energie	Preise für Ihre gewünschte Trasse.	
-> Fahrweg		Desage device the D. Switcher
→ Netzzugang		Desonderneiten & Fristen
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-> Leistungen	bereitstellung von banninn astruktur	Hassen-Anmeldung
→ Trassen Güterverkehr	Aufraha dar DB Natz ist as leistungsfähiga Eisanhahninfrastrukturan	Bei der Anmeldung von Trassen
→ Trassen	sowie technische Anlagen und Einrichtungen marktgerecht zur Verfügung	gibt es Besonderheiten und Existen die Sie unbediest beschten
Personenverkehr	zu stellen. Das Leistungsangebot setzt sich aus den Produktfeldern	müssen. Alle Informationen zu
→ Trassenpreise	Trassen, Anlagen und Infrastrukturanschlüsse zusammen.	diesem Thema finden Sie hier.
→ Trassenpreisauskunft	Traccor Offerwerkehr	mehr <mark>-&gt;</mark>
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→ Internationale Verkehre	Güterverkehrs-Express-Trassen, Güterverkehr-Standard-Trassen,	müssen Sie für Ihre Anmeldung
-> Baustelleninformationen	Güterverkehr-Zubringer-Trassen und Güterverkehrs-LZ-Trassen	Diese Formulare nebst Erläuterung
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→ IT/TK Infrastruktur		
→ Personaldienstleistungen		
→ Weitere Serviceleistungen	Die Personenverkehrs-Trassen lassen sich in vier verschiedene Kategorien einteilen. Als Kunde haben Sie die Wahl zwischen Personenverkehrs-Express-Trassen, Personenverkehrs-Takt-Trassen, Personenverkehrs-Economy-Trassen und Personenverkehrs-LZ-Trassen. mehr <mark>⊅</mark>	
	Trassenpreise	
	Gültige Preise ab dem 12.12.2004 und 11.12.2005	

Hier finden Sie das seit dem 12.12.2004 gültige und ab dem 11.12.2005 gettende Trassenpreissystem mit seinen Anlagen sowie Streckenkategoriekarten als PDE-Dateien zum Download





#### Gewünschtes Trassenprodukt

Express-Trasse	Standard-Trasse		Zubringer-Trasse	Zur Zubringer-Trasse gehörende
	gewünschte Systemtrasse:			Standard-Trasse

#### Verkehrszeitraum

ab Ort	Zuggattung	Verkehrszeitraum	Zusatztage	Ausfalltage	Konstruktionsspielraum

#### Betrieblich-technische Angaben (Zugcharakteristik)

ab Ort	Vmax	Třz 1	Tfz 2	Schiebel.	(jek uppelt	Last	Bremsstellung	BrH	Länge	EBuLa	Besonderheiten, LÜ, KLV, Gefahrgut

#### Trassenzeiten

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# Outline

- Auctions
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- Rail Track Auctioning
- The Optimal Track Allocation Problem
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# **Rail Track Auctioning**

### Goals

- More traffic at lower cost
- Better service
- How do you measure?
  - Possible answer: in terms of willingness to pay
- What is the "commodity" of this market?
  - Possible answer: timetabled track
    - = dedicated, timetabled track section
- How does the market work?
  - Possible answer: by auctioning timetabled tracks
  - Auctions can be in-company auctions

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# **Rail Track Auction**



# **Rail Track Auction Results**



# **Rail Track Auction Results**

(14,439 Variables, 13,408 Constraints, 48 Minutes)



# **Rail Track Auctioning Modules**



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# **Macroscopic Graph Model**





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# **Blocks & Standardized Dynamics**

- State (i,T,t,v)
  - Directed block i
  - Train type T
  - Starting time t, velocity v



# **Standard Train Types**

<i>train type</i>	<i>V max [km/h]</i>	train length [m]	Security technology	
ICE	250	410	LZB	
IC	200	400	LZB	
RE	160	225	signal	
RB	120	100	signal	
SB	140	125	signal	
ICG	100	600	signal	





# **Bids for Timetabled Tracks**

- Train number(s) and type(s)
- Starting station, earliest starting time
- Final station, latest arrival time
- Bid = Basic Bid
  - + Departure/Arrival Bonus
  - + Travel Time Bonus

- Intermediate stops (Station, min. stopping time, arrival interval)
- Connections
- Combinatorial bids (and/or)



# **Bid Generator**



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### **Bid Generator** (Reuter 2005)

Method	Input	Output	Goal
Minimum spanning tree	Distances	Tracks on a tree	Regional coverage
Maximum spanning tree	OD-Matrix	Tracks on a tree	Demand coverage
Greedy	OD-Matrix	Set of tracks	"Good tracks"
Point-To-Point	Stations	Single track	Direct connections



Gesellschaft für Informationslogistik mbH

### PROSA/prosimExpreß : A line-planning tool for Deutsche Bahn

N. Ascheuer, Ch. Küttner, M. Proksch



J. Dupont, R. Firla, A. Huck, K. Kuchenbecker,

M. Sievers, F. Wagner







prosim Expreß

# **Block Conflicts**





# **Optimal Track Allocation Problem**

- OPTRA
- Input
  - Set of bids for timetabled tracks
  - Available infrastructure (space and time)

# Output

- Conflict free track assignments for the chosen bids
- Track assignment that maximizes total earnings





# **Multicommodity Flow Model**

$$\begin{array}{rcl} \max \ \mathbf{c}^{\mathsf{T}} \mathbf{x} \\ \mathbf{x}^{\mathsf{r}} \left( \delta^{+}(\mathbf{z}) \right) - \mathbf{x}^{\mathsf{r}} \left( \delta^{-}(\mathbf{z}) \right) &=& \mathbf{b}_{\mathbf{z}}^{\mathsf{r}} & \forall \mathbf{r}, \mathbf{z} \\ & \mathbf{x}_{a}^{\mathsf{r}} + \mathbf{x}_{b}^{\mathsf{s}} &\leq& 1 & \forall \mathsf{r}, \mathsf{s}, \mathsf{a}, \mathsf{b} \text{ incomp.} \\ & & \mathbf{x}_{a}^{\mathsf{r}} &\in& \{0, 1\} & \forall \mathsf{r}, \mathsf{a} \end{array}$$

 Space-time graph G=(V,A)

- Nodes  $z=(i,T,t,v) \in V$
- Arcs a=(z1,z2) ∈ A
- Block conflicts on arcs

- Timetabled track
   ≅ path in G
- Timetable

   ≅ set of compatible
   timetabled tracks

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# **Test Network**

- Criteria
  - Important characteristics ("Hildesheimer Kurve")
  - Important subnet
  - Used in earlier studies
- Data
  - 45 sections = 1176 km
  - 31 nodes
  - 6 train types



# **Auction Experiments**

(Reuter 2005, Rounds 8 and 9)

Round	Earnings	Round	Earnings	
1	44563	9	46575	
2	44563	10	47051	
3	44598	11	48096	
4	44799	12	48253	
5	44799	13	48337	
6	44972	14	48391	
7	45551	15	48513	
8	46375			



# **Auction Experiments**

(Reuter 2005)

	ICE		IC		RE		RB		5		ICG	#
# Trains/Type	ind	sync	ind									
Timetable	27	0	27	0	38	19	87	23	0	61	28	_
+24 IC/ICE ind	30	0	29	0	38	19	85	23	0	61	25	18
+24 IC/ICE sync	24	9	27	9	36	19	83	19	0	58	26	22
+27 R*/S ind	27	0	25	0	44	19	89	23	5	58	27	20
+27 R*/S sync	27	0	27	0	36	19	83	32	0	62	27	30
+15 ICG	27	0	27	0	38	19	87	23	0	61	42	19
+66 *	28	0	25	3	38	25	85	29	2	55	31	29





# **Auction Experiments**

(Reuter 2005)

	Ι	CE	1	TC	ŀ	RE	F	RB		5	ICG	Σ
€/km	ind	sync	ind	€								
Timetable												
+24 IC/ICE ind	2.04		1.78		1.24	1.07	0.93	0.90		0.98	1.12	34421
+24 IC/ICE sync	1.89	1.94	1.45	3.27	1.14	1.10	0.89	0.83		0.90	1.10	36031
+27 R*/S ind	1.74		1.41		1.23	1.08	0.91	0.90	1.15	1.10	1.14	31180
+27 R*/S sync	2.31		1.34		1.02	1.04	0.88	1.41		1.06	0.98	33663
+15 ICG	1.45		1.44		1.08	1.08	0.87	0.90		0.88	1.03	32994
+66 *	2.21		1.88	2.87	1.03	1.10	0.89	1.11	1.53	1.47	1.60	41263

# **Tripling Experiment**

variation	cpu time (CPLEX)	earnings (% Status Quo)	trains (% Status Quo)
0 mins	6 secs	52.066 (+ 84%)	420 (+ 47%)
1 mins	8 secs	60.612 (+114%)	496 (+ 74%)
4 mins	1 days	67.069 (+137%)	617 (+117%)
5 mins	3+ days	67.975 (+140%)	737 (+159%)

### Status quo

 284 tracks through 6 hours in the Hannover—Braunschweig— Fulda network, (hypothetical) total income of 28,255 €

### Scenario

 triple requests to 946 bids (~15 minutes alteration, identical willingness to pay)

# 09M2 Lecture Combinatorial Auctions and Rail Track Scheduling

# The End





- DFG-Forschungszentrum MATHEON "Mathematics for key technologies"
  - Konrad-Zuse-Zentrum f
    ür Informationstechnik Berlin (ZIB)
  - Löbel, Borndörfer & Weider GbR (LBW)

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