08M1 Lecture
Combinatorial Optimization and Transportation, Telebus

Martin Grötschel

Beijing Block Course
"Combinatorial Optimization at Work“
September 25 - October 6, 2006
Contents

1. Introduction
2. Telebus: Transporting disabled people
3. A survey of public transportation tasks
4. Network Planning
5. Line Planning
6. Price and Frequency Planning
7. Vehicle Scheduling
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9. Integrated Vehicle and Duty Scheduling
10. Track Auctioning
11. Summary
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The “classical” Transportation Problem in Mathematical Programming

\[
\min \sum_{(i, j) \in S \times T} c_{ij} x_{ij}
\]

subject to

\[
\sum_{j \in T} x_{ij} = a_i \quad \forall i \in S
\]

\[
\sum_{i \in S} x_{ij} = b_j \quad \forall j \in T
\]

\[
0 \leq x_{ij} (\leq \text{cap}_{ij})
\]

- This problem rarely occurs in real life in its pure form.
- It does appear as a subproblem of some much more complex real problems.
- It can be solved very quickly.

\(S = \text{sources, origins, supply}\)

\(T = \text{sinks, destinations, demand}\)
High Quality Public Transportation: Mathematical, Social, Political, and Business Aspects

The transport/traffic research group at ZIB has, for more than a decade, worked on various mathematical aspects of public transportation.

We have

- optimized the transport of disabled people in Berlin,
- found the minimal number of busses to run the Berlin and other city or regional bus systems,
- solved driver scheduling problems and
- many other optimization problems of this type.
High Quality Public Transportation: Mathematical, Social, Political, and Business Aspects

These problems are, in general, of very large scale and represent significant mathematical challenges. I will sketch some of the achievements briefly in my talk.

It turned out, though, that implementing solutions of such problems often creates (unexpected) social or political difficulties. I will briefly mention some of these in my presentation. A typical reaction:
"This not good!"
High Quality Public Transportation: Mathematical, Social, Political, and Business Aspects

But what is a "good" public transportation system?

Can such a system result from deregulation?

How does one deregulate, e.g., the railway system of a country, properly?

We are currently investigating such and related issues which are highly relevant for everybody’s everyday life. There are more questions than answers. I will try to give an overview of our approach and will outline some of the aspects that are involved in addressing these issues.
The ZIB Transportation Team, including former members

<table>
<thead>
<tr>
<th>Public Transport:</th>
<th>Online Transportation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ralf Borndörfer</td>
<td>Norbert Ascheuer</td>
</tr>
<tr>
<td>Fridolin Klostermeier</td>
<td>Philipp Friese</td>
</tr>
<tr>
<td>Christian Küttner</td>
<td>Sven O. Krumke</td>
</tr>
<tr>
<td>Andreas Löbel</td>
<td>Diana Poensgen</td>
</tr>
<tr>
<td>Sascha Lukac</td>
<td>Jörg Rambau</td>
</tr>
<tr>
<td>Marc Pfetsch</td>
<td>Luis Miguel Torres</td>
</tr>
<tr>
<td>Thomas Schlechte</td>
<td>Andreas Tuchscherer</td>
</tr>
<tr>
<td>Steffen Weider</td>
<td>Tjark Vredeveld</td>
</tr>
</tbody>
</table>

plus several master’s students
# The ZIB Transportation Team

**spin-off companies**

## Intranetz:
- Fridolin Klostermeier
- Christian Küttner
- Norbert Ascheuer (->atesio)

## LBW:
- Ralf Borndörfer
- Andreas Löbel
- Steffen Weider
Application Areas of MATHEON: Key Technologies

A  Life sciences
   (Peter Deuflhard, Hans Jürgen Prömel, Christof Schütte)

B  Traffic and communication networks
   (Martin Grötschel, Rolf Möhring)

C  Production
   (Jürgen Sprekels, Fredi Tröltzsch)

D  Electronic circuits and optical technologies
   (Volker Mehrmann, Frank Schmidt, Caren Tischendorf)

E  Finance
   (Anton Bovier, Hans Föllmer, Peter Imkeller)

F  Visualization
   (John Sullivan, Konrad Polthier, Günter M. Ziegler)

G  Education
   (Ulrich Kortenkamp, Jürg Kramer)

in brackets: scientists in charge
Examples of practically important and mathematically challenging problems

Some of the tasks in public transport to be addressed:

- planning routes
- assigning vehicles
- dispatching drivers
- improving quality
- informing customers
- creating (multi-modal) links
- controlling fleets
- coordinating tours
- keeping track of jobs
- optimizing schedules
- locating vehicles
- failure management/online rescheduling

film produced by IVU
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Vehicle Routing
Finding a „Good“ Mathematical Model

Die Verknüpfungsoptimierung

Vor der eigentlichen Tourenplanung werden Fahraufträge zu Bestellungen verknüpft. Z.B. Zielsammelfahrt (I: 1, 2), Einbindung (II: 3, 4), Startsammmelfahrt (III: 5, 6, 7), Anbindung (IV: 8, 9), Mehrfachanbindung (V: 10, 11, 12).

Ziel: Minimierung der Besetzkilometer

Finding a „solvable“ mathematical model: Set Partitioning
We employ:

- **various heuristics:**
  - clustering
  - TSP/Routing improvement

- **cutting planes**
  - based on the set packing and set covering polytopes

- **column generation**
  - to dynamically generate „good“ tours/variables

- **branch&cut**
Where is the mathematics?

We employ:

- **various heuristics:**
  - clustering
  - TSP/Routing
  - improvement

- **cutting planes**
  - based on the
  - set packing and
  - set covering
  - polytopes

- **column generation**
  - to dynamically
  - generate „good“
  - tours/variables

- **branch&cut**
Where is the mathematics?

Study of the stable set problem in graphs

Investigation of the stable set polytope, e.g., finding facets

Designing separation algorithms for valid/facet-defining classes of inequalities
History

- Taxi Coupon System
- Taxi Account System
- Optimized Dispatching

- Customers/1,000
- Users/1,000
- Costs/Mio DM
Ziele

- Organisatorische Verbesserungen
- Mathematische Fahrzeugeinsatzplanung

Ergebnisse

- Serviceverbesserung
- Kostenreduktion
- Vereinfachung der Arbeitsabläufe
- Telebus-Computersystem
Some Telebus stories

- History: The system, newspaper reporter
- Social and political context (Berlin, BMBF)
- Industry interests (subsidies)
- Riding telebusses, psychology of customers
- Taxi and „social“ transport companies
- Union influence
- Psychology of employees and bus companies
- Telebus 2004/05: Another crisis,
  Testimony in Berlin House of Representatives
- Current situation
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# Planning Public Transportation

<table>
<thead>
<tr>
<th>Phase:</th>
<th>Planning</th>
<th>Scheduling</th>
<th>Dispatching</th>
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</thead>
<tbody>
<tr>
<td>Horizon:</td>
<td>Long Term</td>
<td>Medium term</td>
<td>(very) Short term</td>
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<tr>
<td></td>
<td>Timetable Period</td>
<td></td>
<td>Day of Operation online planning</td>
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<tr>
<td>Objective:</td>
<td>Service Level</td>
<td>Cost Reduction</td>
<td>Get it done</td>
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<td>Steps:</td>
<td>Network Design</td>
<td>Vehicle Scheduling</td>
<td>Crew Assignment</td>
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<td></td>
<td>Line Planning</td>
<td>Duty Scheduling</td>
<td>Delay Management</td>
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<tr>
<td></td>
<td>Timetabling</td>
<td>Duty Rostering</td>
<td>Failure Management</td>
</tr>
</tbody>
</table>
Planning in Public Transport
(Product, Project, Planned)

- Operations Control
- Crew Assignment
- Disruptions
- Fairness
- Rostering
- Duty Mix
- Duties
- Relief Points
- Rotations
- Sensitivity
- Service Level
- Frequencies
- Timetable
- Connections
- Velocities
- Lines
- Relief Points
- Timetables
- Duties
- Disruptions
- Construction Costs
- Fares
- Cost Recovery

- IS-OPT
- VS-OPT
- B15

- multidepartmental
- Departments
- multidepotwise
- Depots
- multiple line groups
- Line Groups
- multiple lines
- Lines
- multiple rotations
- Rotations
Some of our projects

1. **bmb+f**
   - VS-OPT
     - DS-OPT
     - IS-OPT
     - DS: BVG
     - VS: BVG
   - LPP
     - RS-OPT
     - OPTRA

2. BVG

3. Other
   - MCF
   - Telebus

Projects:
- VS-OPT: 92-94
- DS-OPT: 94-97
- IS-OPT: 97-00
- LPP: 00-03
- OPTRA: 03-06

Deutsche Forschungsgemeinschaft (DFG)
## Partners

<table>
<thead>
<tr>
<th>Project</th>
<th>Funding</th>
<th>Partner</th>
<th>Product</th>
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</thead>
<tbody>
<tr>
<td>VS-OPT</td>
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<td>ivu</td>
<td>BVG</td>
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<td>MICROBUS</td>
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<td>DS-OPT</td>
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<td>OPTRA</td>
<td>bmb+f</td>
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The Main Question

What is a „good“ public transportation network?
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Network Planning: A didactical example city

II
III
I
VI
IV
V

industry district
city center
4 living quarters
### Origin/ Destination Matrix for the didactical example

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
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<tr>
<td>III</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>IV</td>
<td>1000</td>
<td>0</td>
<td>3000</td>
<td>0</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>V</td>
<td>2000</td>
<td>0</td>
<td>2000</td>
<td>0</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>VI</td>
<td>1000</td>
<td>0</td>
<td>1000</td>
<td>0</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>
Origin-Destination Matrix of Potsdam
Network
Network Planning (didactical example): available streets/ tracks
MIP Model for Network Planning

Variables: $y_p \in \mathbb{R}_+$ passenger flow on path $p \in P_{st}$
$z_a \in \{0,1\}$ use track/street $a$

$$\min \quad \sum_{a} \gamma_a z_a + \sum_{p} \tau_p y_p$$

getting the data
adequate modelling

s.t. $\sum_{p \in P_{st}} y_p = d_{st}$ $\forall s,t$ transport all passengers

$\sum_{p:a \in p} y_p \leq u_a z_a$ $\forall a$ capacity constraints
Network Planning (didactical example): selected network of streets and tracks
Network Planning:
The Potsdam Tram Network

blue lines = existing tram tracks
red lines = suggested track extensions
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Line Planning: our didactical example city

The selected network of streets and tracks: the result of network planning

Now we have to “invent” bus/tram lines
### MIP Model for Line Planning

**Variables:**
- \( y_p \in \mathbb{R}_+ \) passenger flow on path \( p \in P_{st} \)
- \( x_\ell \in \{0,1\} \) choose line \( \ell \)
- \( f_\ell \in \mathbb{R}_+ \) frequency of line \( \ell \)

\[
\begin{align*}
\text{min} & \quad \sum_\ell (C_\ell x_\ell + c_\ell f_\ell) + \sum_p \tau_p y_p \\
\text{s.t.} & \quad \sum_{p \in P_{st}} y_p = d_{st} \quad \forall s,t \quad \text{transport all passengers} \\
& \quad \sum_{p:a \in p} y_p \leq \sum_{\ell:a \in \ell} \kappa_\ell f_\ell \quad \forall a \quad \text{capacity constraints} \\
& \quad f_\ell \leq Fx_\ell \quad \forall \ell \quad \text{frequency bounds} \\
& \quad \text{...} \quad \text{other linear constraints}
\end{align*}
\]
Discussion

- Properties
  - Dynamic line generation (no line pool)
  - Dynamic passenger routing (no system split)
  - Minimize weighted sum of costs and travel times
  - Passenger routes: system optimum = user equilibrium
  - Time horizon: fixed

- Extensions
  - Continuous frequencies → discrete frequencies
  - Transfers → B11
  - Demand feedback → B5, B15
Column Generation Method

begin

Solve Line Planning Problem (IP)

Solve “LP Relaxation”

Fix Lines

All fixed?

Generate Lines

Generate Pax Paths

Compute “Prices“

Stop?

No

Yes

end

end
Computational Results

- Cooperation
  ViP Verkehrsbetrieb Potsdam GmbH
  Havelbus Verkehrsgesellschaft mbH
  IVU Traffic Technologies AG
  City of Potsdam

- Potsdam
  130,000 inhabitants, 42,796 trips/afternoon
  27 bus lines, 4 tram lines
  410 nodes, 106 OD-nodes, 891 edges
Line Planning (didactical example):

The chosen lines: a “typical” picture from practice
The currently existing Potsdam Network of Lines

- bus
- tram
- S-Bahn
- regional train
Network of all Public Transportation Lines in Potsdam

optimized system of lines
New Lines in Potsdam

- new lines suggested by optimization

also (but not drawn):
- some lines removed
- frequencies of lines determined
Passenger Flow in the new Potsdam Network of Lines
A Side Remark:
How do we solve these and other MIP problems?

- Commercial LP/IP/MIP solvers, such as CPLEX (ILOG), XPRESS (Dash), …
  Free solvers Soplex (ZIB),…
- Non-differentiable, non-linear programming techniques (if simplex or interior point algorithms are not applicable or do not work)
- Polyhedral combinatorics and cutting planes
- Special purpose “tricks”
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Planning prices and frequencies

- Ansatz
  - „Controlling“ demand via prices and travel times
  - Price system = Individual price + ???
  - Maximize profit
  - Electronic Ticketing

- Status
  - Research project
  - Data?
  - Mathematical models?
  - Giant amount of literature on topics of questionable value for practice (versions of local elasticities)
Planning prices
Planning prices

Maximize income

optimal price

service?
Planning frequencies & synchronized timetables

- Demand vs. travel time
- Profit vs. total travel time
- Optimal frequency

Maximum
## Current Timetable

**BUS 690** S Babelsberg ↔ Am Stern, Johannes-Kepler-Platz

### Stundenplan

<table>
<thead>
<tr>
<th>Verkehrshinweise</th>
<th>täglich</th>
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<tbody>
<tr>
<td><strong>S Babelsberg/Post</strong> ab</td>
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<tr>
<td>Horstweg/Großbeerstr.</td>
<td>5.29</td>
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<tr>
<td>Heinrich-von-Kleist-St.</td>
<td>5.31</td>
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<tr>
<td>Am Findling</td>
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<tr>
<td>Eichenweg</td>
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<tr>
<td>Kleine St.</td>
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<tr>
<td>Filmpark</td>
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<tr>
<td>Bhf Medienstadt Babelsberg</td>
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<td>Bhf Medienstadt Babelsberg</td>
<td>5.36</td>
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<tr>
<td>Betriebshof VIP</td>
<td>5.38</td>
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<tr>
<td>Abzweig Betriebshof VIP</td>
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<td>Johannes-Kepler-Platz an</td>
<td>5.42</td>
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</tbody>
</table>

*Montag - Freitag*
Computational Results

- Cooperation
  ViP Verkehrsbetrieb Potsdam GmbH
  Havelbus Verkehrsgesellschaft mbH
  IVU Traffic Technologies AG
  City of Potsdam

- Potsdam
  130,000 inhabitants, 42,796 trips/afternoon
  27 bus lines, 4 tram lines
  410 nodes, 106 OD-nodes, 891 edges
Fare Planning

- **Input**
  - Network
  - OD-demand functions
  - Travel times
  - Price system

- **Output**
  - Prices and passenger flow

- **Problem**
  - Compute prices and corresponding passenger flow

- **Goals**
  - Maximize revenue (computing "optimal prices")
  - Optimize modal split (attracting additional passengers)
  - Compare fare systems (decision support)
Price elasticity

\[ \varepsilon(x) := \frac{x \cdot d'(x)}{d(x)} \approx \frac{(\Delta d/d)}{(\Delta x/x)} \]

Empirical studies: \( \varepsilon \approx -0.3 \)

*Curtin 1968*

Cobb-Douglas-functions \( d(x) = c \cdot x^{\varepsilon} \) with constant elasticity

Analytical approaches/marginal costs

*Pedersen 2003*

Optimize zone tariffs

*Hamacher & Schöbel 1995, 2004*

Maximize social welfare

*De Borger, Mayeres, Proost, Wouters 1996*

Maximize revenue for bus services

*Kocur & Hendrickson 1982*
Literature

- Fare Planning for Public Transport
  *Ralf Borndörfer, Marika Neumann, Marc E. Pfetsch*
  ZIB-Report 05-20

- Optimal Fares for Public Transport
  *Ralf Borndörfer, Marika Neumann, Marc E. Pfetsch*
  ZIB-Report 05-35

- Mathematische Preisplanung im ÖPNV
  *Marika Neumann*
  Diplomarbeit, TU Berlin, 2005
Fare - Demand - Revenue

- **Demand** \( d = d(x) \)
- **Revenue** \( r(x) = d(x) \cdot x \)
Discrete Choice Demand Model

- **A**
  - Set of travel alternatives
- **T**
  - Time horizon (here: one month)
- **X_{st} \in \{1, \ldots, n\}**
  - Num. of trips per month (rand. variable)
- **C = A \times \{0, 1, \ldots, n\}**
  - Travel choices (here: no mix)
- **U_{st}^a(x,k) = V_{st}^a(x,k) + v_{st}^a**
  - Utility for alt. a, pair st, fares x, k trips
- **V_{st}^a(x,k)**
  - Deterministic utility (observable)
- **v_{st}^a**
  - Random utility (disturbance term)
- **P_{st}^a(x,k)**
  - Prob. that alternative a is chosen
  
  \[ P_{st}^a(x,k) = P[V_{st}^a(x,k) + v_{st}^a = \max_{b \in A} V_{st}^b(x,k) + v_{st}^b] \]
- **v_{st}^a \sim G(\eta, \mu)**
  - Gumbel dist. (here: \( \eta = 0, \mu = 0.01 \))
- **d_{st}^{a,k}(x) = P_{st}^a(x,k) \cdot d_{st} \cdot P[X_{st} = k]**
  
  \[ = e^{\mu V_{st}^a(x,k)} / (\sum_{b \in A} e^{\mu V_{st}^b(x,k)}) \cdot d_{st} \cdot P[X_{st} = k] \]
Nonlinear Programming Model

- $D \subseteq V \times V$: OD-pairs
- $C$: Set of travel choices
  - $C' \subseteq C$: Set of public transport travel choices
- $x \in P \subseteq \mathbb{R}^n$: Vector of fare variables
- $P \subseteq \mathbb{R}^n$: Set of feasible fares (polyhedron)
- $p^i_{st}: \mathbb{R}^n \to \mathbb{R}^+$: Price function $p(x)$ for all $i \in C$, $st \in D$
- $d^i_{st}: \mathbb{R}^n \to \mathbb{R}^+$: Demand function $d(x)$ for all $i \in C$, $st \in D$
- $r: \mathbb{R}^n \to \mathbb{R}^+$: Revenue function $r(x) = p(x)^T d(x) = \sum_{i, st} p^i_{st}(x) d^i_{st}(x)$
- Fare planning problem

\[
(FPP) \quad \max \sum_{i \in C'} \sum_{st \in D} p^i_{st}(x) \cdot d^i_{st}(x)
\]
\[\text{s.t.} \quad x \in P \]
Discussion

- Properties
  - Demand extrapolation
  - Revenue maximization
  - Passenger routes: fixed
  - Time horizon: fixed
Dutch Intercity Network
(Bussieck [1998], Bussieck, Kreuzer, Zimmermann [1996], Claessens, van Dijk, Zwaneveld [1998])
# Origin-Destination Matrix


|     | Lw   | Gn   | Hr   | Asn  | Zl   | Apd  | Hgl  | Ah   | Ut   | Shl  | Asdz | Asd  | Gv   | Gvc  | Rtd  | Bd   | Ehv  | Std  | Mt   | Lls  | Rsdg | Zvg | Odzg |
|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|     |
| Lw  | 478  | 380  | 13   | 145  | 20   | 21   | 90   | 6    | 26   | 36   | 14   | 9    | 9    | 4    | 77   | 7    | 14   |      |      |      |      |      |
| Gn  | 1720 | 720  | 331  | 48   | 88   | 205  | 12   | 73   | 75   | 34   | 28   | 29   | 13   | 200  | 33   | 14   |      |      |      |      |      |      |
| Hr  | 511  | 11   | 209  | 20   | 16   | 115  | 10   | 48   | 58   | 16   | 11   | 8    | 4    | 77   | 10   | 19   |      |      |      |      |      |      |
| Asn | 854  | 16   | 502  | 32   | 58   | 235  | 13   | 117  | 125  | 42   | 33   | 28   | 14   | 152  | 48   | 19   |      |      |      |      |      |      |
| Zl  | 56   | 1112 | 64   | 171  | 400  | 33   | 163  | 182  | 79   | 47   | 46   | 21   | 390  | 100  | 32   |      |      |      |      |      |      |      |
| Apd | 468  | 1160 | 32   | 76   | 917  | 21   | 202  | 143  | 57   | 62   | 10   | 5    | 47   | 83   | 71   |      |      |      |      |      |      |      |
| Hgl | 422  | 11   | 24   | 287  | 20   | 81   | 52   | 39   | 28   | 20   | 12   | 24   | 75   |      |      |      |      |      |      |      |      |      |      |      |
| Ah  | 4244 | 60   | 721  | 726  | 109  | 741  | 180  | 101  |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Ut  | 278  | 5826 | 4919 | 225  | 3138 | 2260 | 1165 | 3109 | 720  | 359  | 89   | 325  | 996  | 21   |      |      |      |      |      |      |      |      |      |
| Shl | 1456 | 6469 | 1339 | 1503 | 509  | 7    | 99   | 44   | 29   | 103  | 164  |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Asdz | 461 | 207 | 369 | 138 | 542 | 203 | 149 | 819 | 6 | 155 | | | | | | | | | | | | |
| Asd | 730 | 2540 | 1756 | 154 | 437 | 155 | 37 | 2783 | 2258 | 489 | 22 | | | | | | | | | | | | |
| Gv  | 785 | 4586 | 531 | 35 | 22 | 8 | 29 | 890 | | | | | | | | | | | | | | |
| Gvc | 2829 | 228 | 335 | 104 | 41 | 31 | 3229 | 7 | | | | | | | | | | | | | | |
| Rtd | 1829 | 569 | 179 | 73 | 46 | 1077 | 157 | 11 | | | | | | | | | | | | | | |
| Bd  | 950 | 157 | 79 | 6 | 329 | 14 | 5 | | | | | | | | | | | | | | |
| Ehv | 936 | 404 | 8 | 75 | 11 | 3 | | | | | | | | | | | | | | |
| Std | 863 | 2 | 19 | | | | | | | | | | | | | | | | | | |
| Mt  | 1 | 22 | | | | | | | | | | | | | | | | | | | |
| Lls | | | | | | | | | | | | | | | | | | | | | | 15 |
## Travel Distances

(Bussieck [1998], Bussieck, Kreuzer, Zimmermann [1996], Claessens, van Dijk, Zwaneveld [1998])

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Fares
(www.ns.nl)
Example 1: Standard and Reduced Ticket

- $A = \{s, r, c\}$ Alternatives standard, reduced, car
- $C = A \times \{1, 2, \ldots, 60\}$ Travel choices
- $x = (x_b, x_d)$ Reduction entitlement, distance fare
- $p_{S,k}^{st}(x) = x_d \cdot l_{st} \cdot k$ Standard ticket
- $p_{R,k}^{st}(x) = x_b + \frac{1}{2} \cdot x_d \cdot l_{st} \cdot k$ Reduced ticket
- $p_{C,k}^{st}(x) = q_F + q_V \cdot l_{C}^{st} \cdot k$ Car (here: $q_F = 100 \, \text{€}$, $q_V = 0.1 \, \text{€/km}$)
- $V_{S,k}^{st}(x) = -p_{S,k}^{st}(x) - 0.1 \cdot (t_{st} \cdot k)$ Utility for using standard ticket
- $V_{R,k}^{st}(x) = -p_{R,k}^{st}(x) - 0.1 \cdot (t_{st} \cdot k)$ Utility for using reduced ticket
- $V_{C,k}^{st}(x) = -p_{C,k}^{st}(x) - 0.1 \cdot (t_{C}^{st} \cdot k)$ Utility for using car

\[\text{(FPP)} \quad \max \sum_{st \in D} \sum_{(a,k) \in C'} p_{st}^{a,k}(x) \cdot \frac{e^{\mu V_{st}^{a,k}(x)}}{\sum_{b \in A} e^{\mu V_{st}^{b,k}(x)}} \cdot d_{st} \cdot P[X_{st} = k]\]

s.t. $x_b, x_d \geq 0$
Example 1: Standard and Reduced Ticket

- $x_b = 153.31 \, \text{€}$
- $x_d = 0.13 \, \text{€/km}$
- $r(x) = 34,201,767.8 \, \text{€} \ ( +32.4\% )$
- $d(x) = 126,768 \ ( +38.1\% )$
- Modal split 68.9\% \ ( +18.8\% )
Example 2: Single and Monthly Ticket

- **A = \{s, m, c\}**  Alternatives standard, monthly, car
- **C = A \times \{1,2,\ldots,60\}**  Travel choices
- **x = (x_m, x_s)**  Monthly ticket, single ticket
- **p^{S,k}_{st}(x) = x_s \cdot k**  Single ticket
- **p^{M,k}_{st}(x) = x_m**  Monthly ticket
- **p^{C,k}_{st}(x) = q_f + q_v \cdot l^C_{st} \cdot k**  Car (here: \(q_f = 100 \text{ €}\), \(q_v = 0.1 \text{ €/km}\))

- **\(V^{S,k}_{st}(x) = -p^{S,k}_{st}(x) - 0.1 \cdot (t_{st} \cdot k)\)**  Utility for using standard ticket
- **\(V^{M,k}_{st}(x) = -p^{R,k}_{st}(x) - 0.1 \cdot (t_{st} \cdot k)\)**  Utility for using reduced ticket
- **\(V^{C,k}_{st}(x) = -p^{C,k}_{st}(x) - 0.1 \cdot (t^C_{st} \cdot k)\)**  Utility for using car

\[
\text{(FPP)} \quad \max \sum_{st \in D} \sum_{(a,k) \in C'} p^{a,k}_{st}(x) \cdot \frac{e^{\mu V^{a,k}_{st}(x)}}{\sum_{b \in A} e^{\mu V^{b,k}_{st}(x)}} \cdot d_{st} \cdot P[X_{st} = k] \\
\text{s.t.} \quad x_m, x_s \geq 0
\]
Example 2: Single and Monthly Ticket

- \( x_m = 372.08 \text{ €} \)
- \( x_d = 10.99 \text{ €/km} \)
- \( r(x) = 31,813,156,4 \text{ €} \ (+23.1\%) \)
- \( d(x) = 110,999 \ (+20.9\%) \)
- Modal split 60.3\% (+10.2\%)
## Comparison

(status quo extrapolated to include car traffic using our model with alternatives nsr, car for each OD-pair)

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Vehicle Scheduling
a code developed at ZIB, maintained and distributed by spin-off company LBW GbR (Löbel, Borndörfer & Weider)

Mathematical Model and Algorithmic Approach:

Multicommodity Flow model solved by Lagrangean relaxation and dynamic column generation
Graph Theoretic Model
A slice of the Regensburg network
IP Model for Vehicle Scheduling (Multicommodity Flow Problem)

\[
\begin{align*}
\text{min} & \quad \sum_{d \in D} \sum_{ij \in A^d} C_{ij} x_{ij}^d \\
\sum_{d \in D} \sum_{tj \in A^d} x_{tj}^d &= 1 \quad \forall t \in T \quad \text{(Flow Requ.)} \\
\sum_{d \in D} \sum_{tj \in A^d} x_{tj}^d - \sum_{d \in D} \sum_{it \in A^d} x_{it}^d &= 0 \quad \forall t \in T, d \in D \quad \text{(Flow Cons.)} \\
\sum_{dt \in A^d} x_{dt}^d &\leq \kappa_d \quad \forall d \in D \quad \text{(Capacities)} \\
x &\in \mathbb{Z}_+^A \quad \text{(Integrality)}
\end{align*}
\]

- D – Depots
- T – Timetabled Trips
Discussion

- Properties
  - Exploiting all degrees of freedom
  - Subcontracting constraints $\rightarrow$ new

- Extensions
  - Trip shifting $\rightarrow$ current work
  - Multiperiod scheduling
  - Periodic schedules
  - Assimilation
  - Balanced depot exchange
  - Maintenance constraints
Trip Shifting

Diplomarbeit Cornelia Bönisch
Lagrangean Relaxation Algorithm
## Computational Results

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## CINT2000 (Integer Component of SPEC CPU2000):

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webmaster@spec.org

Last updated: Fri Sep 26 11:10:06 EDT 2003

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“too” short to turn, 
“too” long to wait
the globally best choice
vehicle utilization

**Morning Peak**

The chart and table illustrate the vehicle utilization during the morning peak. The data shows the distribution of vehicles across different routes and time slots. The chart highlights the intensity of vehicle use, with peak times marked by higher values in the table. The page also includes a graphical representation along with a tabular summary, providing a comprehensive view of vehicle activity.
several vehicle schedules
VS-OPT employed in Berlin (BVG)

- Real Really Large-Scale Optimization
  - 28,000 scheduled trips (worldwide largest known instance)
  - 100 million degrees of freedom (of 400 mio possible)
  - optimization of a whole transportation company
  - no heuristic simplifications
  - „Lagrangean Pricing“-technique

- Mathematical quality guarantee
  - fleet minimal solution
  - at most 1% off minimal cost

- Added value
  - Scenario analysis
  - Sensitivity analysis
  - Stability, Fixing, Freezing, Outsourcing, etc.

- Running time
  - Minutes on standard PCs

In other words, we have a multicommodity flow problem with 100 million integer variables and can solve it in a few minutes on a laptop.
VS-OPT

- Lagrangean Pricing
  - Dynamic Arc Generation
  - Lagrange Relaxation
    - Flow Conservation $\rightarrow$ Min Cost Flow Problem
    - Flow Constraints $\rightarrow$ Circulation Problem
  - Activate Arcs on Paths

- Schedule-Cluster-Reschedule Heuristic
  - Min Cost Flow Solution
  - Assign Paths to Depots
  - Eliminate Conflicts by Local Search
Auf Sparkurs zum Ziel

Systematisierter Einsatz

Die neuen Optimierungsmethoden, die die BVG jetzt nach und nach nutzen will, stammen vom Konrad-Zuse-Zentrum für Informationstechnik und garantieren nach Roß' Angaben Einsparungen von maximal 100 Millionen Mark im Jahr. "Sie sind nötig, um unser Angebot in dieser schweren Lage stabilisieren und dem Einsparungsdruck überhaupt standhalten zu können."
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Mathematical duty scheduling DS-OPT

- Types of duties
  - Investigating whole paths
  - supplementary duty elements

- 2.000 duty elements:
  - 50.000 nodes
  - 280.000 links (edges)
  - $\infty$ duties
IP Model for Duty Scheduling

\[
\begin{align*}
\min & \sum_{j \in J} c_j x_j + \sum_{b \in B} p^+_b s^+_b + \sum_{b \in B} p^-_b s^-_b \\
\sum_{j \in J} a_{ij} x_j & = 1 \quad \forall i \in I \quad \text{(Tasks)} \\
\sum_{b \in B} d_{bj} x_j + s^+_b - s^-_b & = d_b \quad \forall b \in B \quad \text{(Mix)} \\
x & \in \{0,1\}^J \quad \text{(Integrality)}
\end{align*}
\]

- I - Tasks
- J - Duties
- B - Mix (Base) Constraints
DS-OPT

- Lagrangean Shortest Path Pricing
  - Linear Resource Constraints
  - Resource Constrained Shortest Path Model
  - Lagrange Relaxation of Linear Constraints
    - Shortest Path Lower Bound

- LP Heuristic
  - Dual Ascent
  - Box Step

- Branch-and-Generate Heuristic
  - Fixing on Links
# DS-OPT Computational Results

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### Some Users

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<td>ATC/ Terni (I)</td>
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<tr>
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<td>Connex (D)</td>
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### Optimization Results

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<td>120 117 2,5%</td>
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<td><strong>VER</strong></td>
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<tr>
<td>Bus</td>
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<td>manual</td>
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<td>n.a.</td>
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<td>Bus</td>
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<tr>
<td>*</td>
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<td>280 40 14,3%</td>
</tr>
</tbody>
</table>
The Psychology of Improvement

- Company goals
- Manager goals
- Dispatcher goals
- The 15% rule
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11. Summary
IP Model for Integrated Scheduling

\[
\begin{align*}
\min & \sum_{d \in D} \sum_{ij \in A^d} c_{ij}^d x_{ij}^d + \sum_{j \in J} c_j y_j \\
\sum_{d \in D} \sum_{tj \in A^d} x_{tj}^d &= 1 \quad \forall t \in T \quad \text{(Trips)} \\
\sum_{tj \in A^d} x_{tj}^d - \sum_{it \in A^d} x_{it}^d &= 0 \quad \forall t \in T, d \in D \quad \text{(V-Flow)} \\
\sum_{dt \in A^d} x_{dt}^d &\leq \kappa_d \quad \forall d \in D \quad \text{(V-Cap)} \\
\sum_{j \in J} a_{ij} y_j &= 1 \quad \forall i \in t \in T \quad \text{(Tasks)} \\
\sum_{d \in D} \sum_{tj \in A^d} x_{tj}^d - \sum_{j \in J} a_{ij} y_j &= 0 \quad \forall i \in t \in H \quad \text{(Coupling)} \\
x_{it}^d, y_j &\in \{0,1\} \quad \text{(Integrality)}
\end{align*}
\]

- T - Timetabled Trips
- D - Depots
- H - Deadhead Trips
- I - Tasks
CO at Work

IS-OPT

- Lagrangean Relaxation on Coupling Constraints
  - Proximal Bundle Method
  - Approximate Solution of Vehicle and Duty Scheduling Components
- Branch-and-Generate Heuristic
  - Fixing Deadhead Trips
Regional Public Transport: Relief Problems
Integrated Vehicle and Duty Scheduling

- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips
RS-OPT

- Combined Scheduling
  - Rotation = Duty (relief only in the depot)
  - Similar to duty scheduling
Integrated Vehicle and Duty Scheduling

- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips
Integrated Vehicle and Duty Scheduling

- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips
Integrating Vehicle and Duty Scheduling

- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips
Integrated Vehicle and Duty Scheduling

- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips
Integrated Vehicle and Duty Scheduling

- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips
IS-OPT Scenarios (Regensburg)

Day of Week | Su | Mo-Fr
--- | --- | ---
Vehicle Types | 1 | 3
Timetabled Trips | 794 | 1,414
Deadhead Trips | 47,523 | 57,646
Duty Elements on Timetabled Trips | 1,980 | 3,666
Duty Elements on Deadhead Trips | 47,523 | 57,646
Extension Elements | 33,224 | 82,308
Connections | 233,842 | 457,174
Duty Types | Early, Middle, Late | Early, Middle, Late, Spilt
## IS-OPT Computational Results: Regensburg on Sunday

<table>
<thead>
<tr>
<th>Instance</th>
<th>Reference</th>
<th>Variant 1</th>
<th>Variant 2</th>
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<tbody>
<tr>
<td>Vehicle Rotations</td>
<td>350:06 h</td>
<td>335:51 h</td>
<td>346:45 h</td>
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<tr>
<td>Paid Time</td>
<td>545:25 h</td>
<td>514:30 h</td>
<td>526:59 h</td>
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<tr>
<td>Breaks</td>
<td>113:34 h</td>
<td>55:11 h</td>
<td>75:06 h</td>
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<td>Unpaid Breaks</td>
<td>1:31 h</td>
<td>0:00 h</td>
<td>0:00 h</td>
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<tr>
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<td>65</td>
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<tr>
<td>Pieces of Work</td>
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<td>65</td>
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<tr>
<td>Vehicle Rotations</td>
<td>36</td>
<td>33</td>
<td>33</td>
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<tr>
<td>Ø Duty Duration</td>
<td>6:39 h</td>
<td>6:36 h</td>
<td>8:06 h</td>
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<tr>
<td>Comp. Time</td>
<td>—</td>
<td>9:30 h</td>
<td>8:35 h</td>
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### IS-OPT Computational Results: Regensburg Mo-Fr

<table>
<thead>
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<th>Instance</th>
<th>Reference</th>
<th>Variant 1</th>
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<td>Unpaid Breaks</td>
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<td>50:57 h</td>
<td>46:45 h</td>
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Integrated Planing (VS-OPT + DS-OPT)

- choice of "unloaded trips"
- coupling of vehicles and duties via unloaded trips
Das Unternehmen (Busbereich)
- 12.000 Busse
- 7.400 Mitarbeiter
- 17 Regionalgesellschaften
Quelle: Jahresbericht der DB Regio AG

Kombinierte Optimierung
- Studien

Integrierte Optimierung
- Forschungsprojekt
### Integrated & combined Planning

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<td>46 Dienste</td>
<td>19 Dienste</td>
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Trassenbörse = track auctioning

European Union:
- Establish a rail traffic market
- Open the market to competition
- Deregulate/Regulate this market

History of our project
Contents

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11. Summary
Summary

- Possible savings in public transport
- Can public transport break even?
- Where are the bottlenecks?
- What can mathematics do?
08M1 Lecture
Combinatorial Optimization and Transportation, Telebus

The End