08M1 Lecture
Combinatorial Optimization and Transportation, Telebus

Martin Grötschel

Beijing Block Course

"Combinatorial Optimization at Work"

September 25 – October 6, 2006

Martin Grötschel

- Institut für Mathematik, Technische Universität Berlin (TUB)
- DFG-Forschungszentrum "Mathematik für Schlüsseltechnologien" (MATHEON)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

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The “classical” Transportation Problem in Mathematical Programming

$$\min \sum_{(i,j)\in S\times T} c_{ij} x_{ij}$$

subject to

$$\sum_{j\in T} x_{ij} = a_i \quad \forall i \in S$$

$$\sum_{i\in S} x_{ij} = b_j \quad \forall j \in T$$

$$0 \le x_{ij} (\le cap_{ij})$$

- This problem rarely occurs in real life in its pure form.
- It does appear as a subproblem of some much more complex real problems.
- It can be solved very quickly.

S = sources, origins, supply

T = sinks, destinations, demand

High Quality Public Transportation: Mathematical, Social, Political, and Business Aspects

The transport/traffic research group at ZIB has, for more than a decade, worked on various mathematical aspects of public transportation.

We have

- optimized the transport of disabled people in Berlin,
- found the minimal number of busses to run the Berlin and other city or regional bus systems,
- solved driver scheduling problems and
- many other optimization problems of this type.

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High Quality Public Transportation: Mathematical, Social, Political, and Business Aspects

These problems are, in general, of very large scale and represent significant mathematical challenges. I will sketch some of the achievements briefly in my talk.

It turned out, though, that implementing solutions of such problems often creates (unexpected) social or polititical difficulties. I will briefly mention some of these in my presentation. A typical reaction:
"This not good!"

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High Quality Public Transportation: Mathematical, Social, Political, and Business Aspects

But what is a "good" public transportation system?

Can such a system result from deregulation?

How does one deregulate, e.g., the railway system of a country, properly?

We are currently investigating such and related issues which are highly relevant for everybody's everyday life. There are more questions than answers. I will try to give an overview of our approach and will outline some of the aspects that are involved in addressing these issues.

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The ZIB Transportation Team, including former members

Public Transport:

Ralf Borndörfer
Fridolin Klostermeier
Christian Küttner
Andreas Löbel
Sascha Lukac
Marc Pfetsch
Thomas Schlechte
Steffen Weider

Online Transportation:

Norbert Ascheuer
Philipp Friese
Sven O. Krumke
Diana Poensgen
Jörg Rambau
Luis Miguel Torres
Andreas Tuchscherer
Tjark Vredeveld

plus several master's students

The ZIB Transportation Team spin-off companies

Intranetz:

Fridolin Klostermeier

Christian Küttner

Norbert Ascheuer (->atesio)

LBW:

Ralf Borndörfer

Andreas Löbel

Steffen Weider

Application Areas of MATHEON: Key Technologies

A **Life sciences**
(Peter Deuflhard, Hans Jürgen Prömel, Christof Schütte)

B **Traffic and communication networks**
(Martin Grötschel, Rolf Möhring)

C **Production**
(Jürgen Sprekels, Fredi Tröltzsch)

D **Electronic circuits and optical technologies**
(Volker Mehrmann, Frank Schmidt, Caren Tischendorf)

E **Finance**
(Anton Bovier, Hans Föllmer, Peter Imkeller)

F **Visualization**
(John Sullivan, Konrad Polthier, Günter M. Ziegler)

G **Education**
(Ulrich Kortenkamp, Jürg Kramer)

in brackets: **scientists in charge**

Examples of practically important and mathematically challenging problems

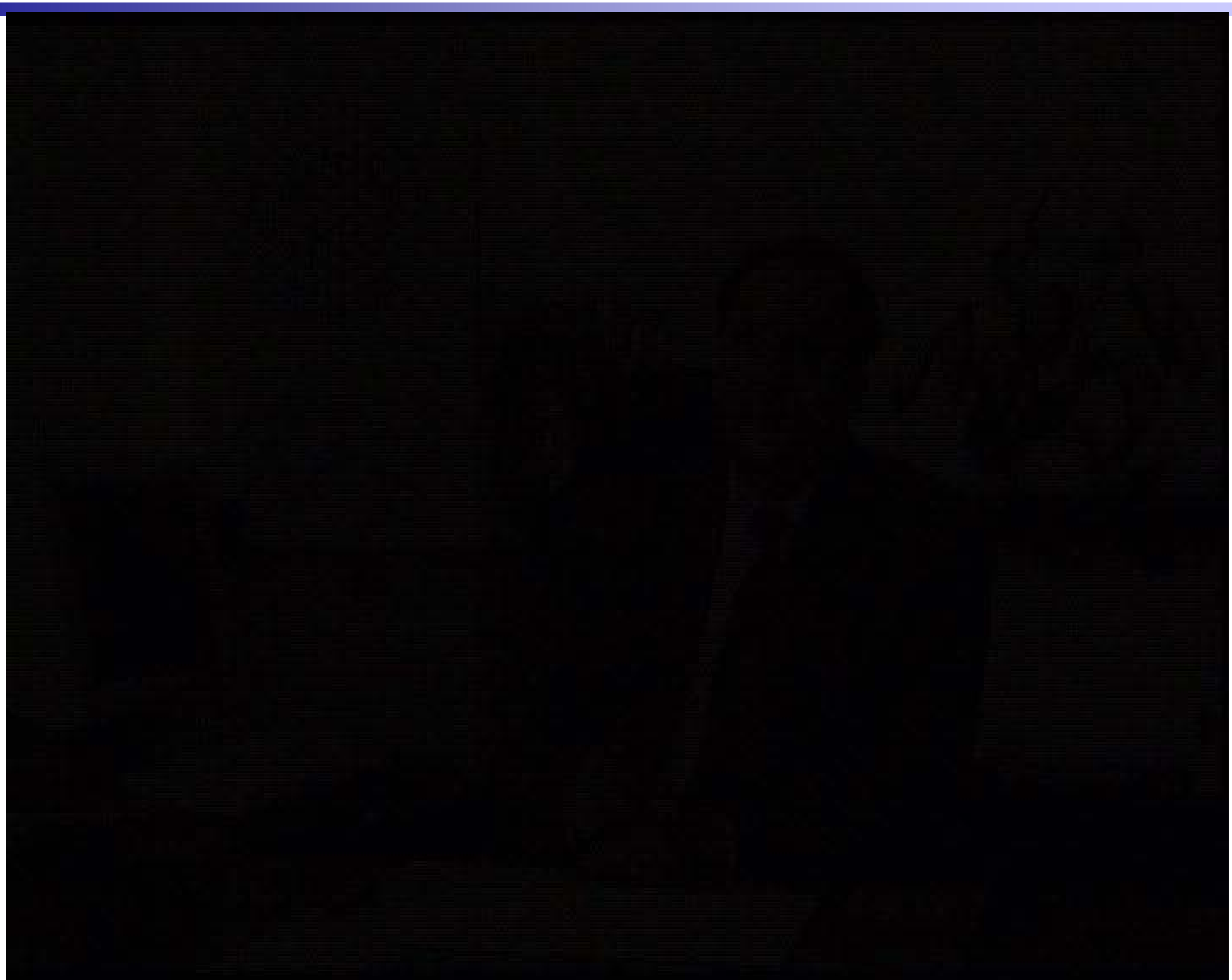
Some of the tasks in public transport to be addressed:

- planning routes
- assigning vehicles
- dispatching drivers
- improving quality
- informing customers
- creating (multi-modal) links
- controlling fleets
- coordinating tours
- keeping track of jobs
- optimizing schedules
- locating vehicles
- failure management/online rescheduling

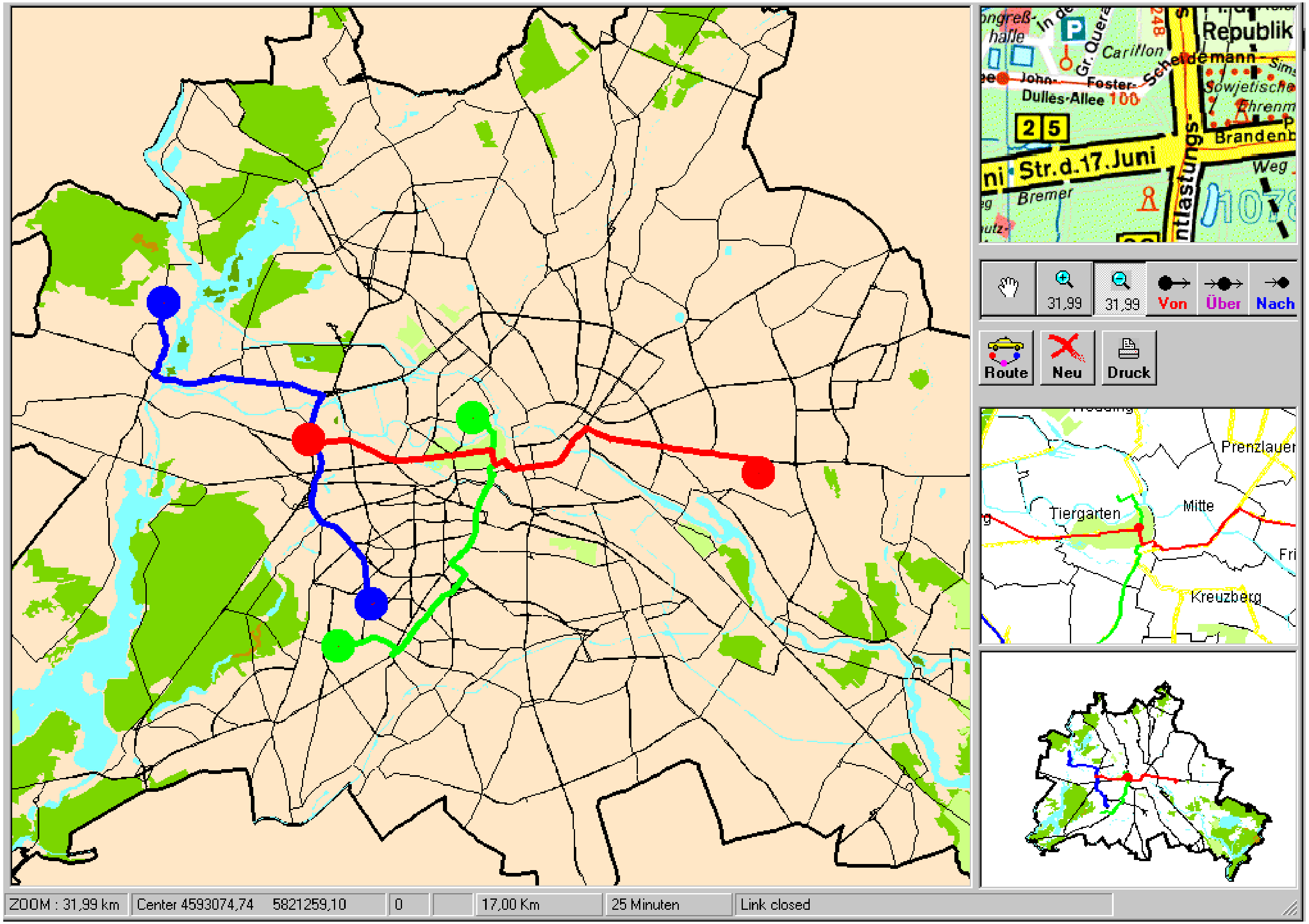
film produced by IVU

Contents

The Telebus Film 1994



Vehicle Routing



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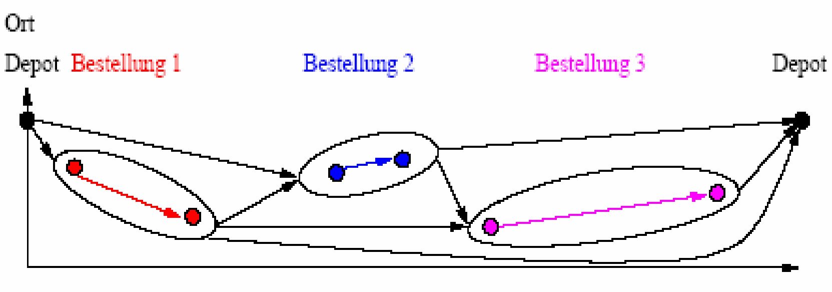
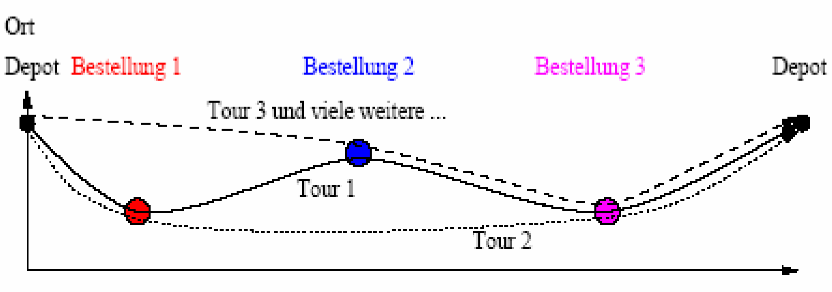
The „Telebus mathematics“

Finding a „Good“ Mathematical Model

Tourengenerierung

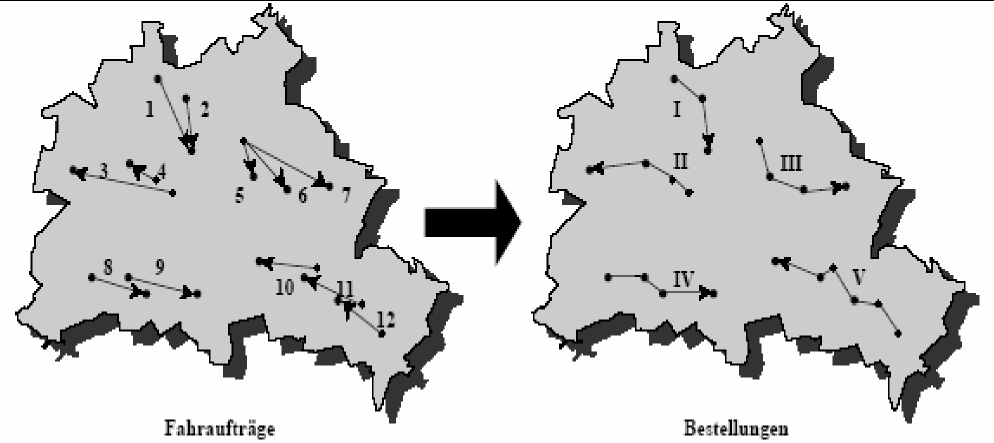


Ermittlung von möglichen Anschlußfahrten



Tourengenerierung

Die Verknüpfungsoptimierung



Vor der eigentlichen Tourenplanung werden Fahraufträge zu Bestellungen verknüpft. Z.B. Zielsammelfahrt (I: 1, 2), Einbindung (II: 3, 4), Startsammelfahrt (III: 5, 6, 7), Anbindung (IV: 8, 9), Mehrfachanbindung (V: 10, 11, 12).

Ziel: Minimierung der Besetztkilometer

Finding a „solvable“ mathematical model: Set Partitioninging

Solution technology

We employ:

- various heuristics:
  clustering
  TSP/Routing
  improvement
- cutting planes
  based on the
  set packing and
  set covering
  polytopes
- column generation
  to dynamically
  generate „good“
  tours/variables
- branch&cut

Set Partitioning

Kosten von Tour 1 = 7, von Tour 2 = 3, ...

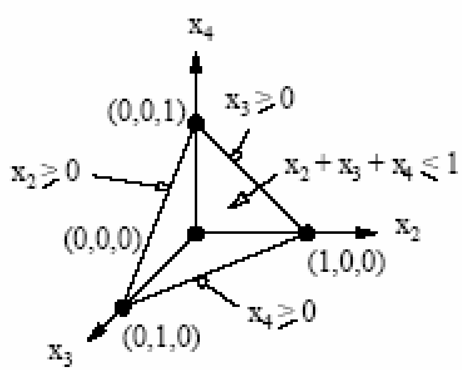
minimiere	$7x_1 +$	$3x_2 +$	$3x_3 +$	$3x_4 +$	$2x_5$		
Fahrtwunsch 1	$1x_1 +$	$1x_2 +$	$0x_3 +$	$1x_4 +$	$0x_5$	$=$	1
Fahrtwunsch 2	$1x_1 +$	$0x_2 +$	$1x_3 +$	$1x_4 +$	$1x_5$	$=$	1
Fahrtwunsch 3	$1x_1 +$	$1x_2 +$	$1x_3 +$	$0x_4 +$	$0x_5$	$=$	1
	Tour 1	Tour 2	Tour 3		x_1,x_2,x_3,x_4,x_5	$\in$	$\{0,1\}$

$$\min c^T x$$

$$Ax = 1$$

$$x \geq 0$$

$$where\ A\ is\ a\ 0/1-matrix$$



$\mathrm{conv}\{(0,0,0),(1,0,0),(0,1,0),(0,0,1)\}$

Polyedrische Kombinatorik: Cliquenungleichung

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Where is the mathematics?

We employ:

- various heuristics:
  clustering
  TSP/Routing
  improvement
- cutting planes
  based on the
  set packing and
  set covering
  polytopes
- column generation
  to dynamically
  generate „good“
  tours/variables
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Where is the mathematics?

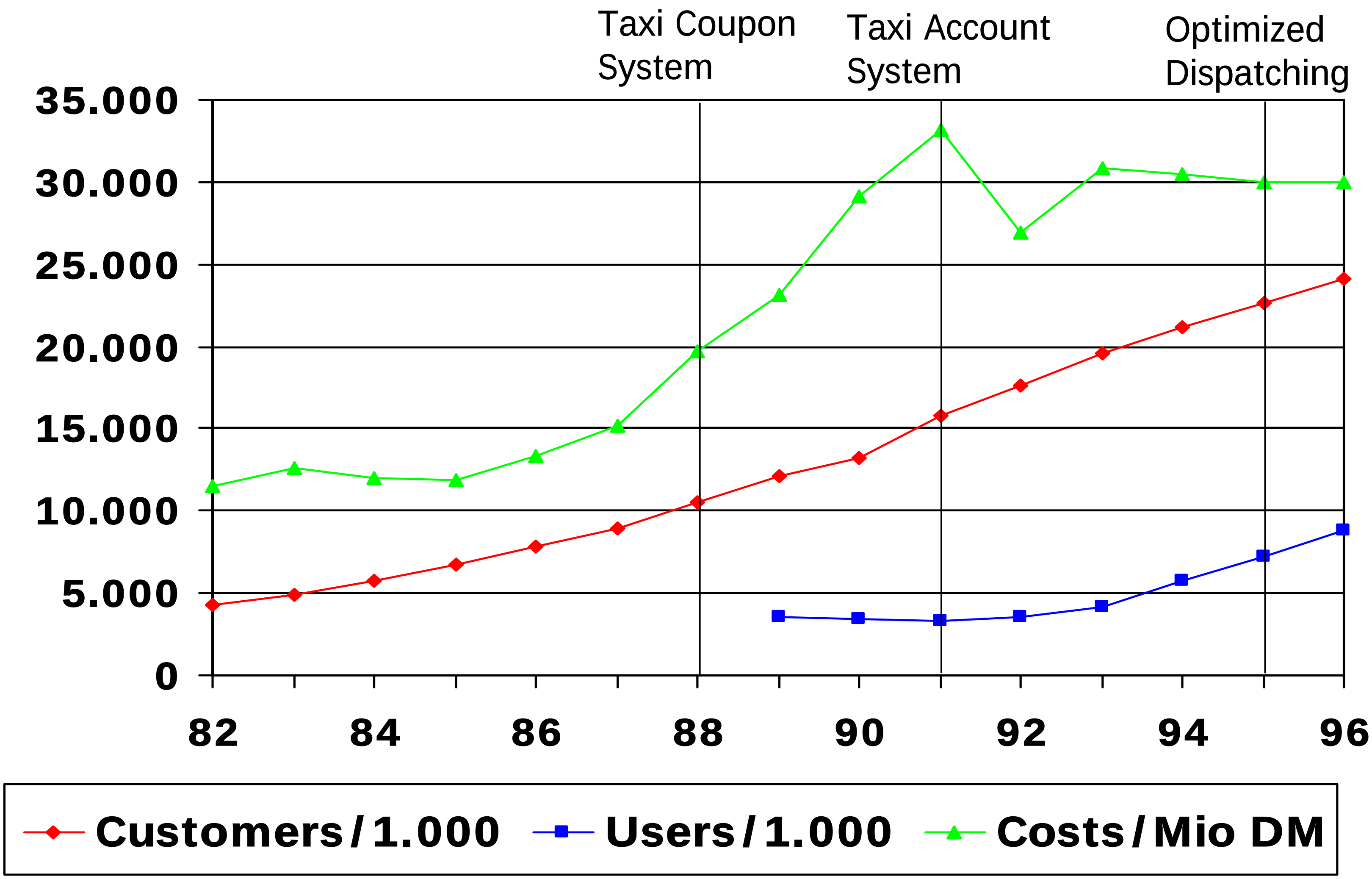
set packing

Study of the stable set problem in graphs

Investigation of the stable set polytope, e.g., finding facets

Designing separation algorithms for valid/facet-defining classes of inequalities

History



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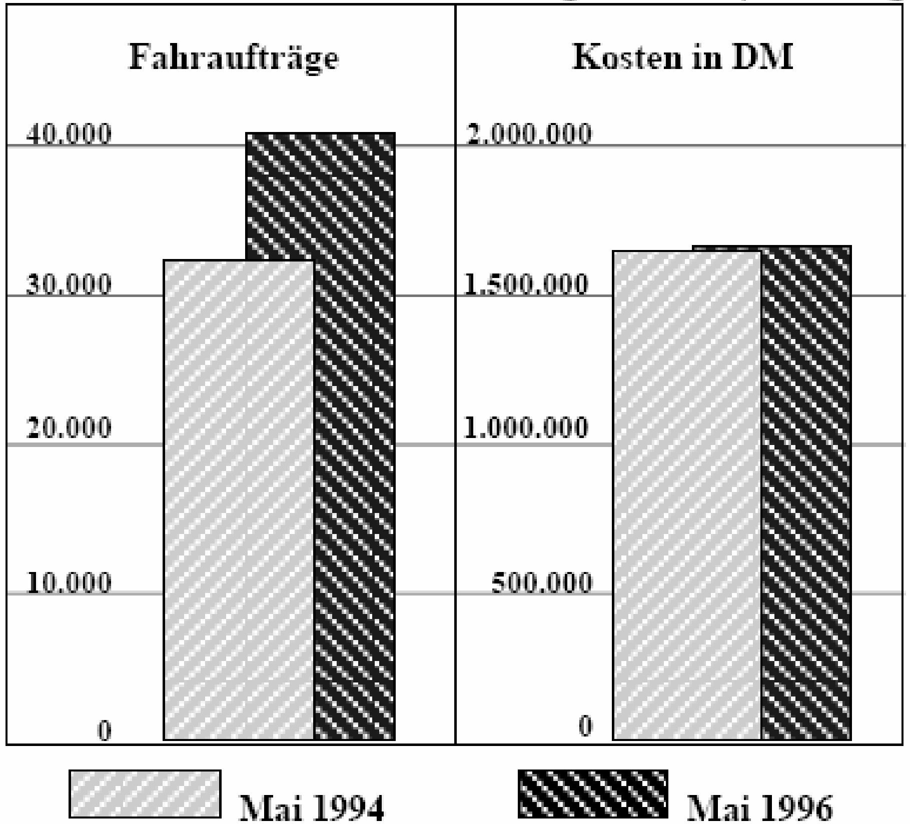
Savings
May 94 – May 96

Historie: Telebus-Projekt

Ziele

- Organisatorische Verbesserungen
- Mathematische Fahrzeugeinsatzplanung



Ergebnisse

- Serviceverbesserung
- Kostenreduktion
- Vereinfachung der Arbeitsabläufe
- Telebus-Computersystem

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Some Telebus stories

- History: The system, newspaper reporter
- Social and political context (Berlin, BMBF)
- Industry interests (subsidies)
- Riding telebusses, psychology of customers
- Taxi and „social“ transport companies
- Union influence
- Psychology of employees and bus companies
- Telebus 2004/05: Another crisis,
  Testimony in Berlin House of Representatives
- Current situation

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Planning Public Transportation

Phase:	Planning	Scheduling	Dispatching
Horizon:	Long Term	Medium term Timetable Period	(very) Short term Day of Operation online planning
Objective:	Service Level	Cost Reduction	Get it done
Steps:	Network Design Line Planning Timetabling	Vehicle Scheduling Duty Scheduling Duty Rostering	Crew Assignment Delay Management Failure Management

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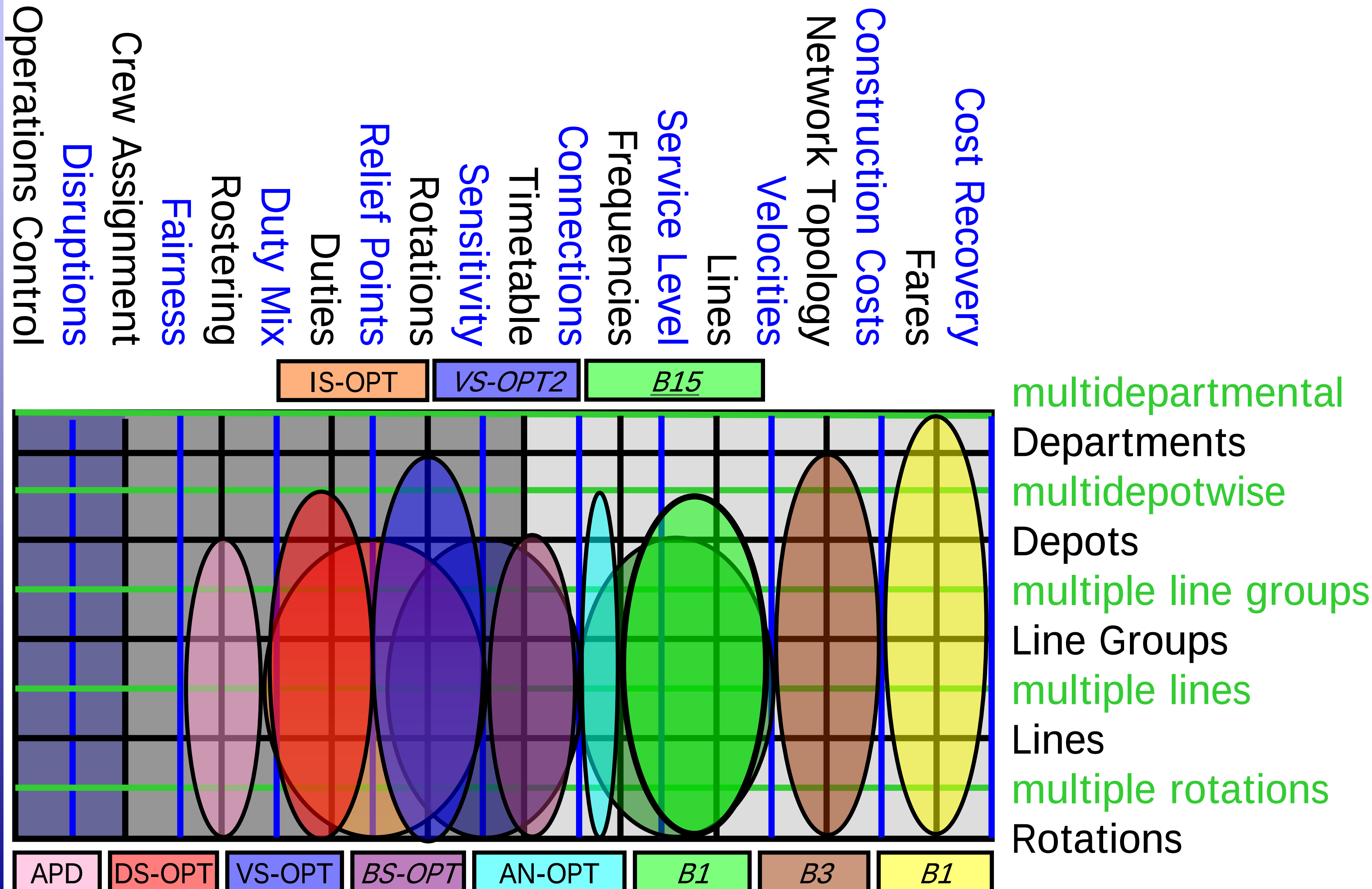
Planning in Public Transport

(Product, *Project*, *Planned*)



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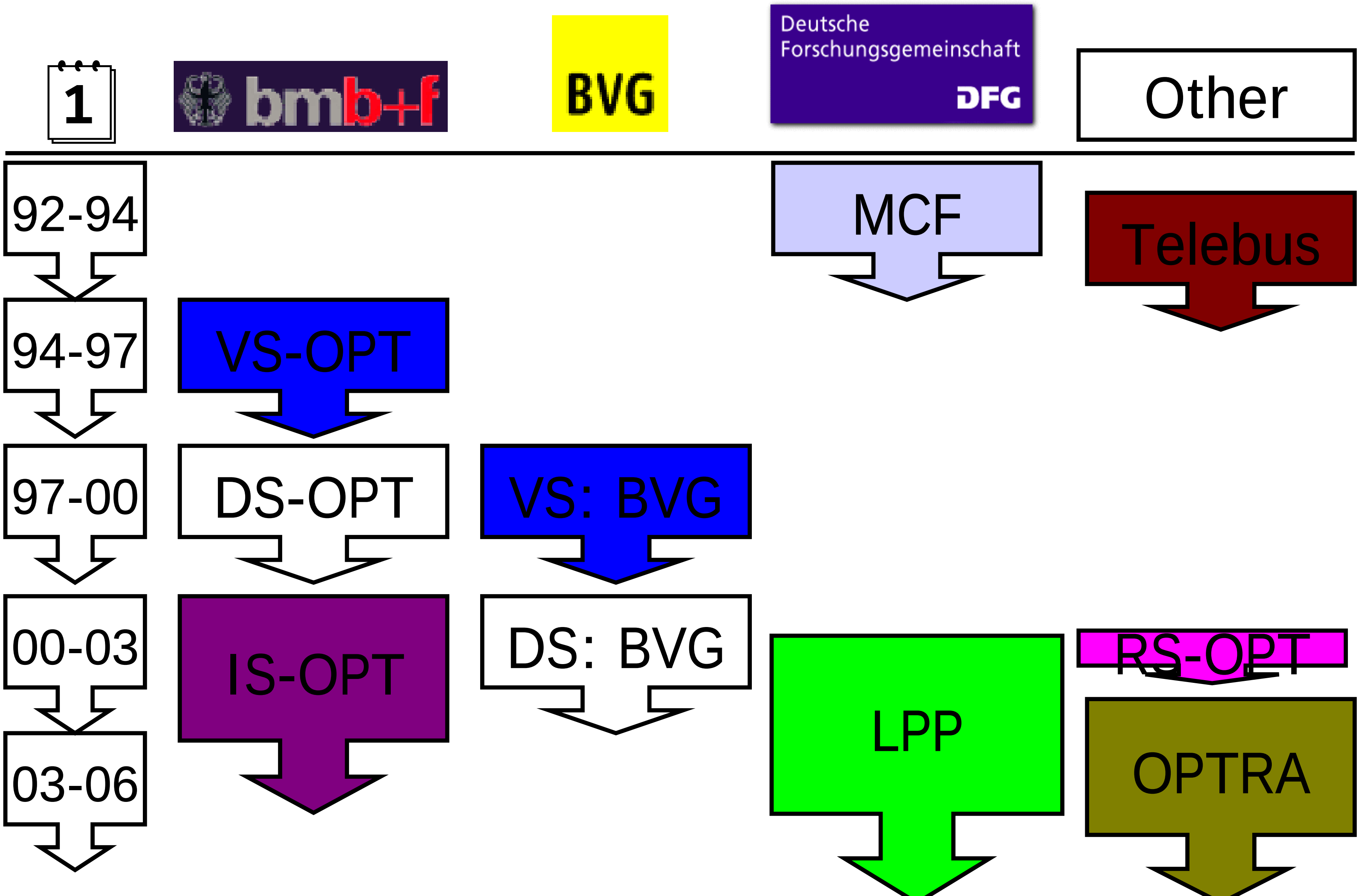
Some of our projects



Partners

Project	Funding	Partner	Product
VS-OPT	bmb+f	ivu BVG SIEMENS	MICROBUS
DS-OPT	bmb+f	ivu BVG	MICROBUS
IS-OPT	bmb+f	ivu RVV mdv	MICROBUS
RS-OPT	LBW	ivu	MICROBUS DIVA
Telebus	Berlin Senatsverwaltung für Wissenschaft, Forschung und Kultur		
LPP	Deutsche Forschungsgemeinschaft DFG	ivu BVG	POTSDAM
OPTRA	bmb+f	TU berlin, TU berlin WIP	

The Main Question

What is a „good“ public transportation network?

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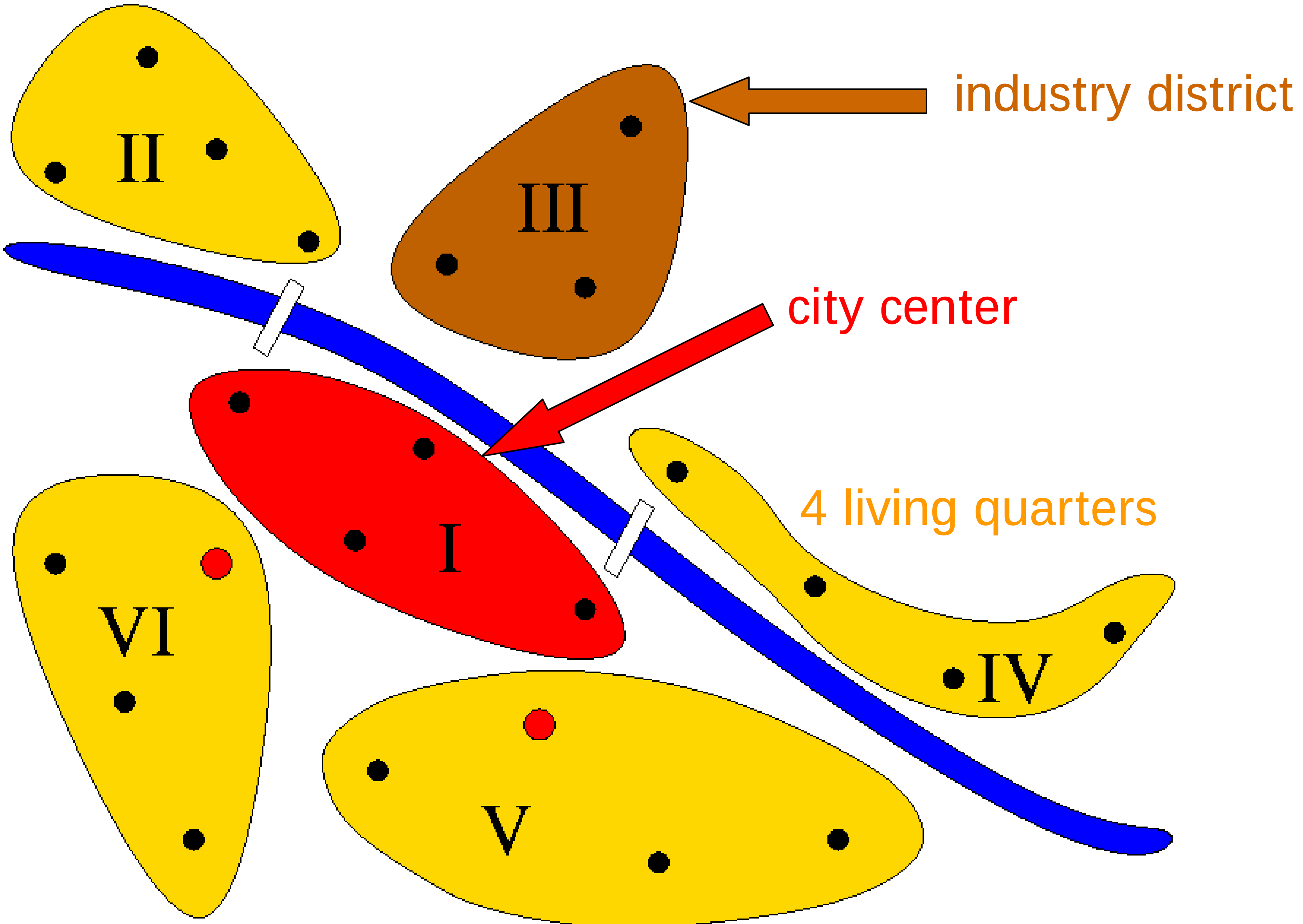
Network Planning: A didactical example city

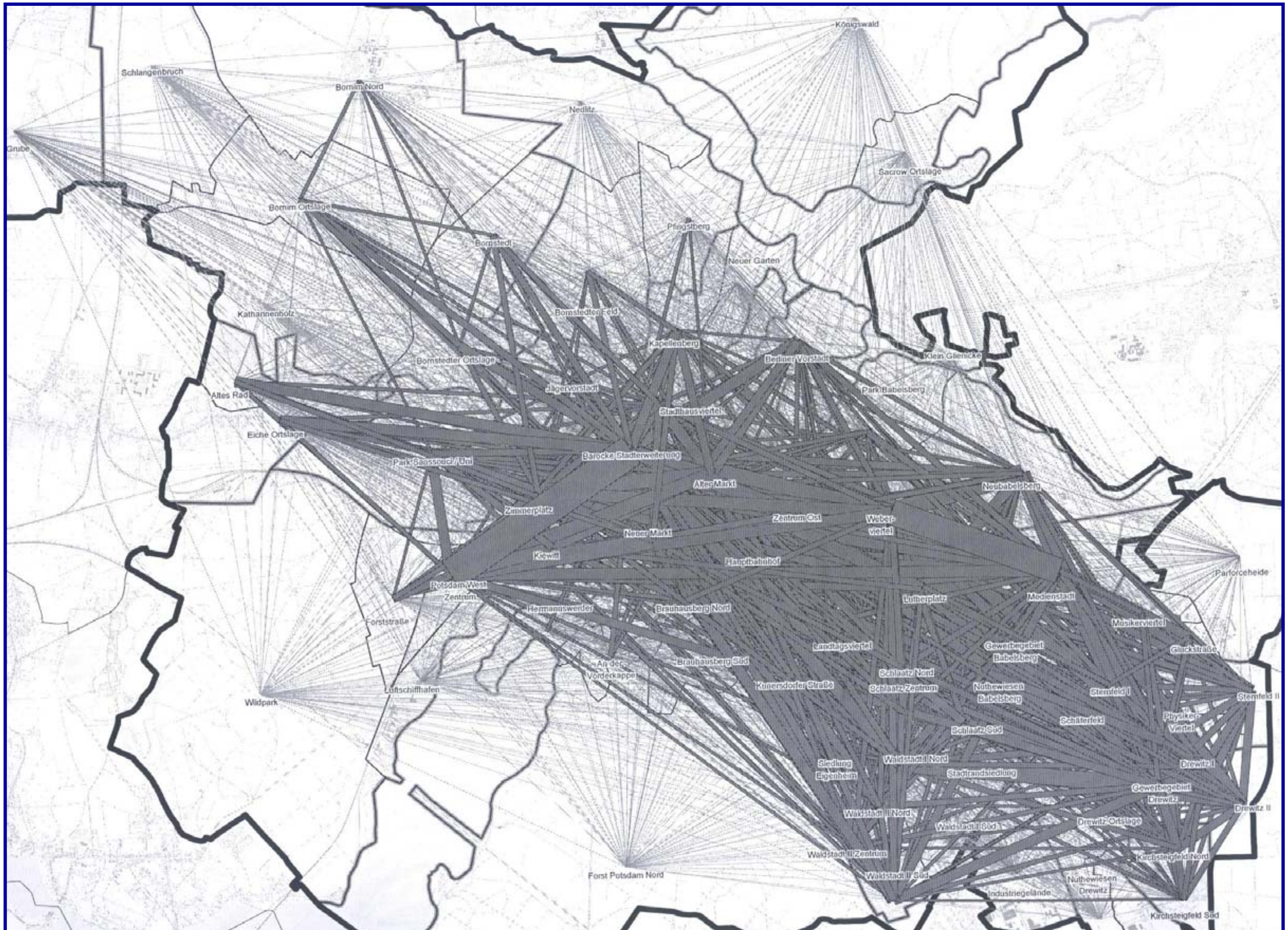


Origin / Destination Matrix for the didactical example

$$
\begin{array}{c|cccccc|}
 & \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} & \text{VI} \\
\hline
\text{I} & 1500 & 0 & 2000 & 0 & 1000 & 500 \\
\text{II} & 500 & 0 & 3000 & 0 & 0 & 500 \\
\text{III} & 500 & 0 & 0 & 0 & 0 & 500 \\
\text{IV} & 1000 & 0 & 3000 & 0 & 500 & 500 \\
\text{V} & 2000 & 0 & 2000 & 0 & 500 & 500 \\
\text{VI} & 1000 & 0 & 1000 & 0 & 500 & 500 \\
\hline
\end{array}
$$

CO at Work

Origin-Destination Matrix of Potsdam



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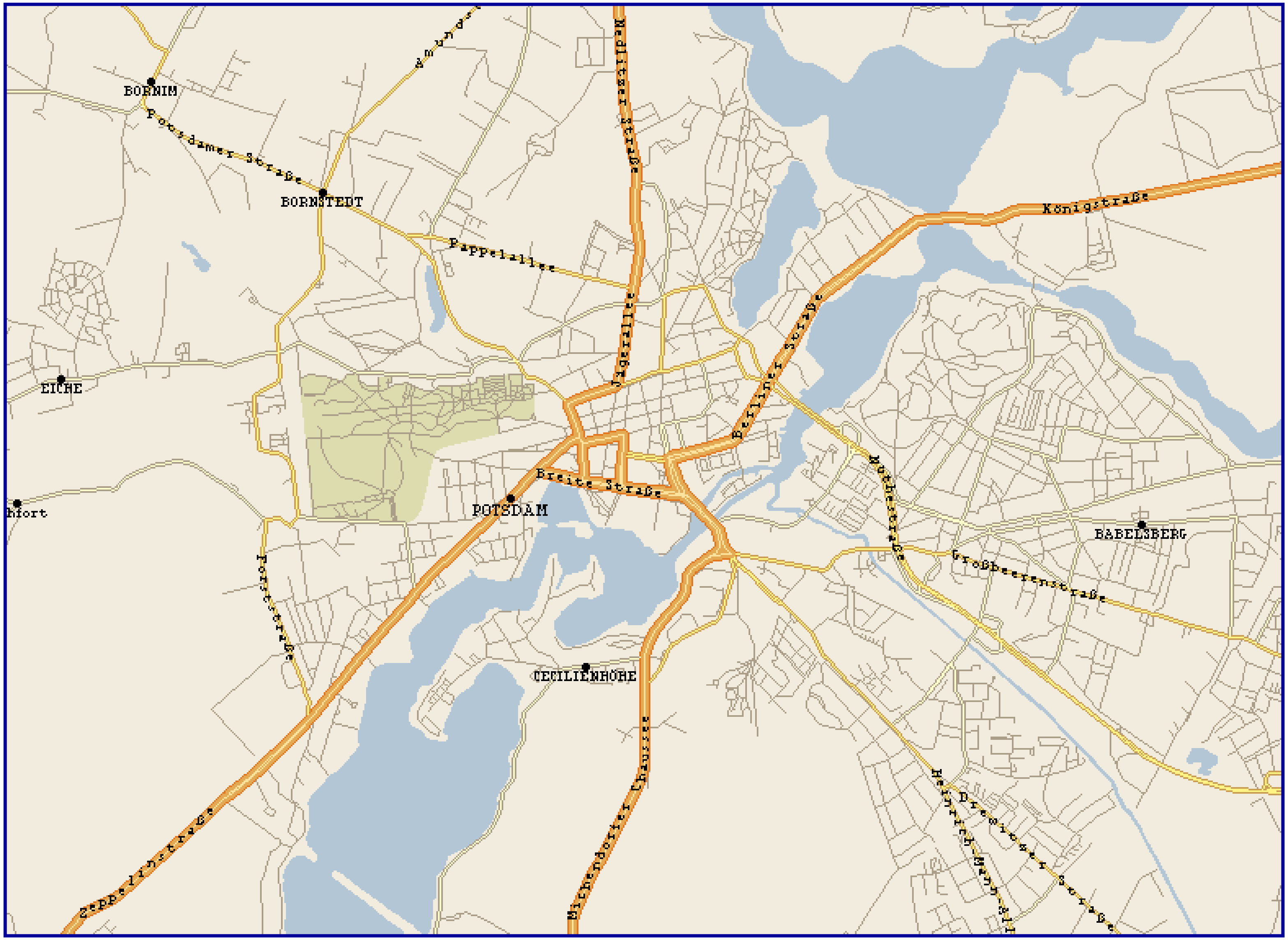
Network



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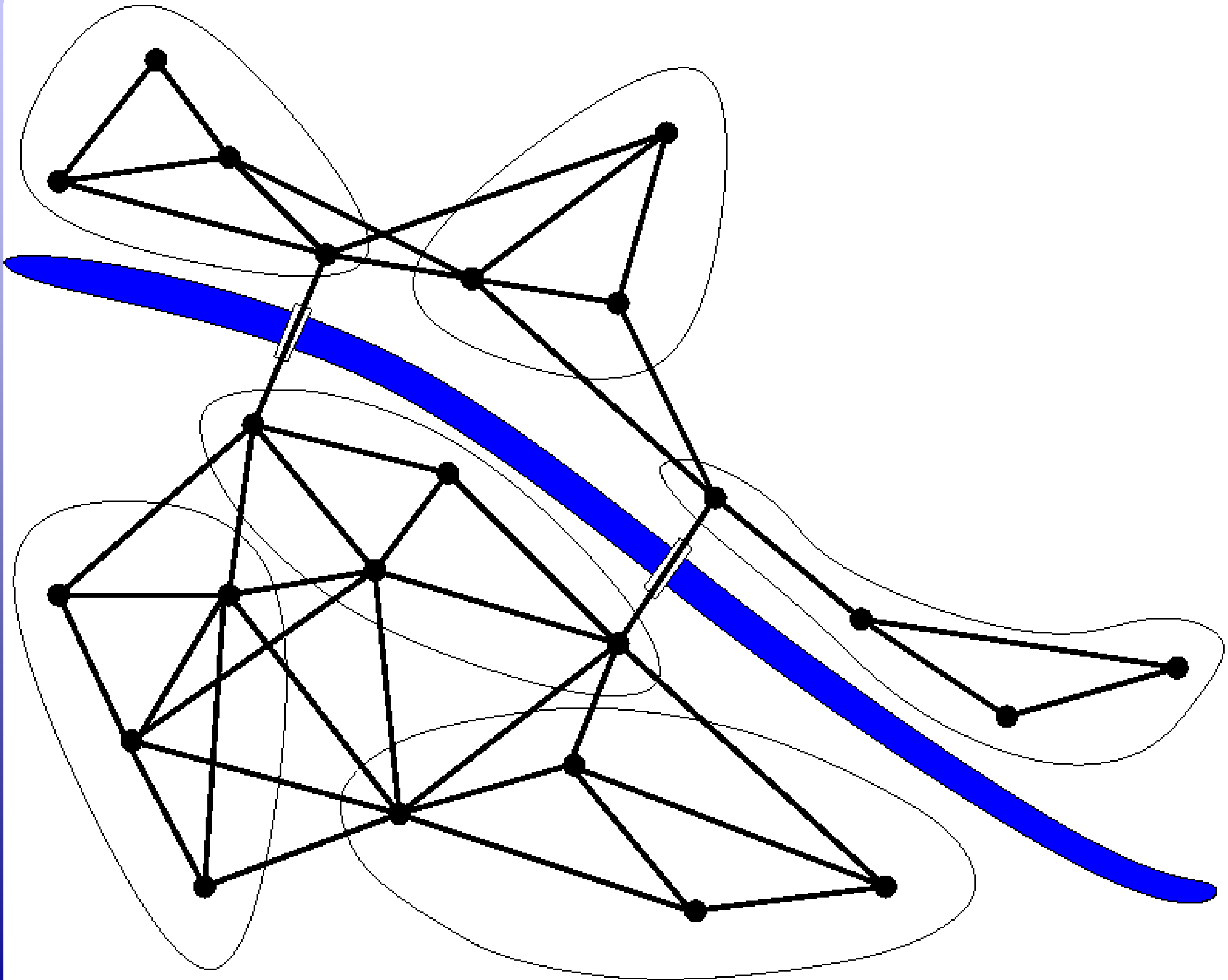
Network Planning (didactical example): available streets/tracks



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MIP Model for Network Planning

Variables: $y_p \in \mathbb{R}_+$ passenger flow on path $p \in P_{st}$

$z_a \in \{0,1\}$ use track/street a

$$\min \sum_a \boxed{\gamma_a} z_a + \sum_p \boxed{\tau_p} y_p$$

getting the data
adequate modelling

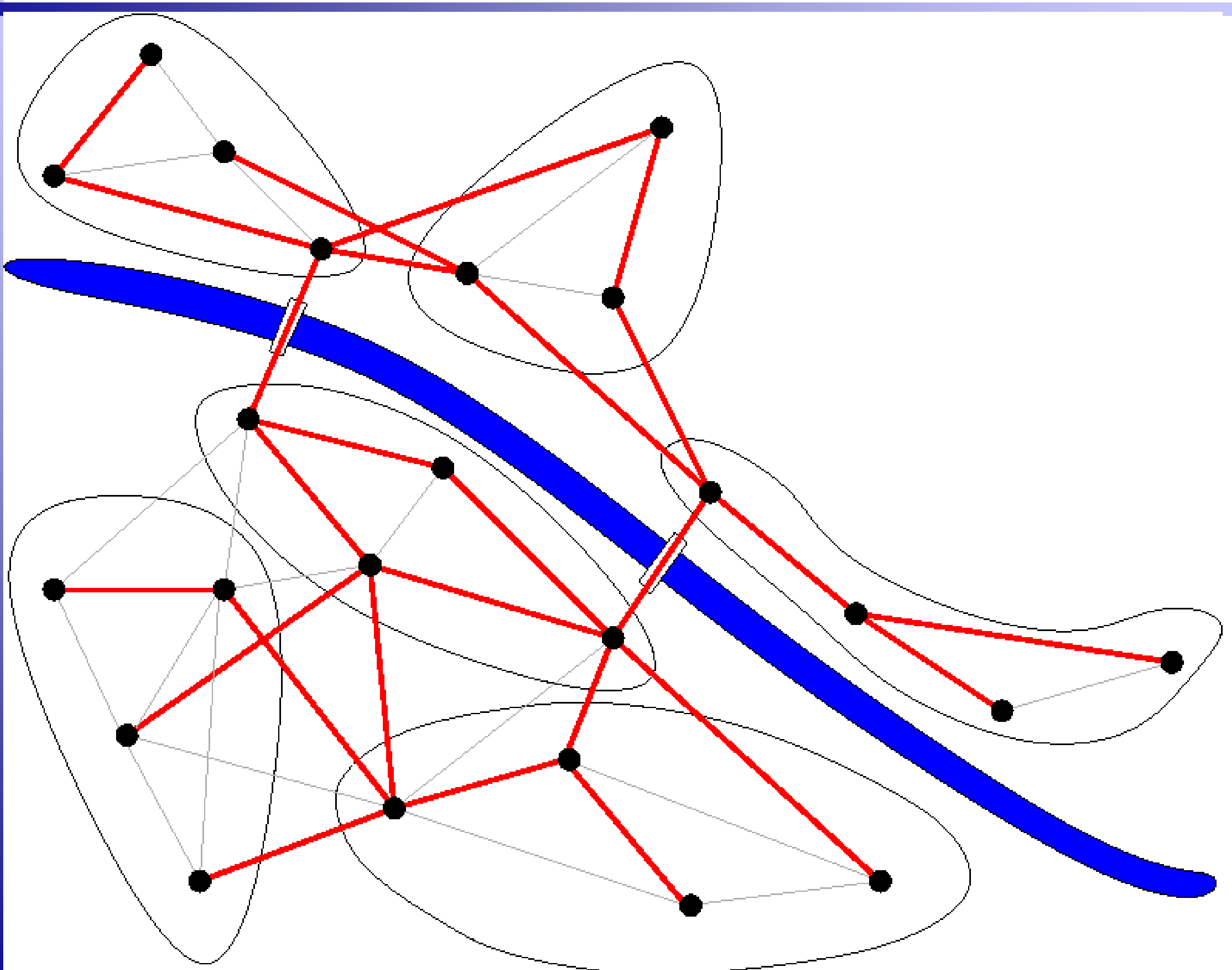
$$s.t. \quad \sum_{p \in P_{st}} y_p = \boxed{d_{st}} \qquad \forall s,t$$

transport all passengers

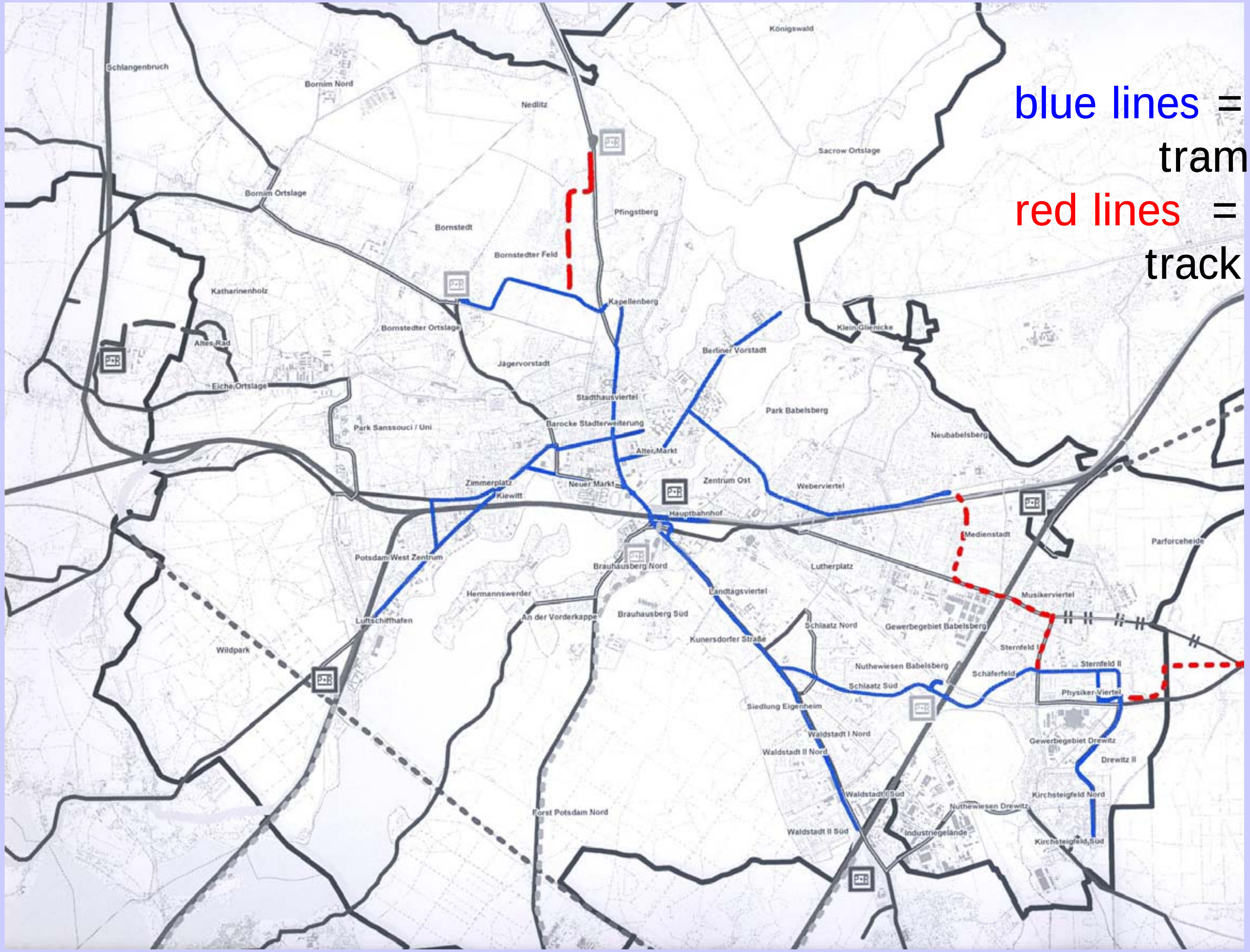
$$\sum_{p:a \in p} y_p \le \boxed{u_a} z_a \qquad \forall a$$

capacity constraints

Network Planning (didactical example): selected network of streets and tracks



Network Planning: The Potsdam Tram Network



blue lines = existing tram tracks

red lines = suggested track extensions

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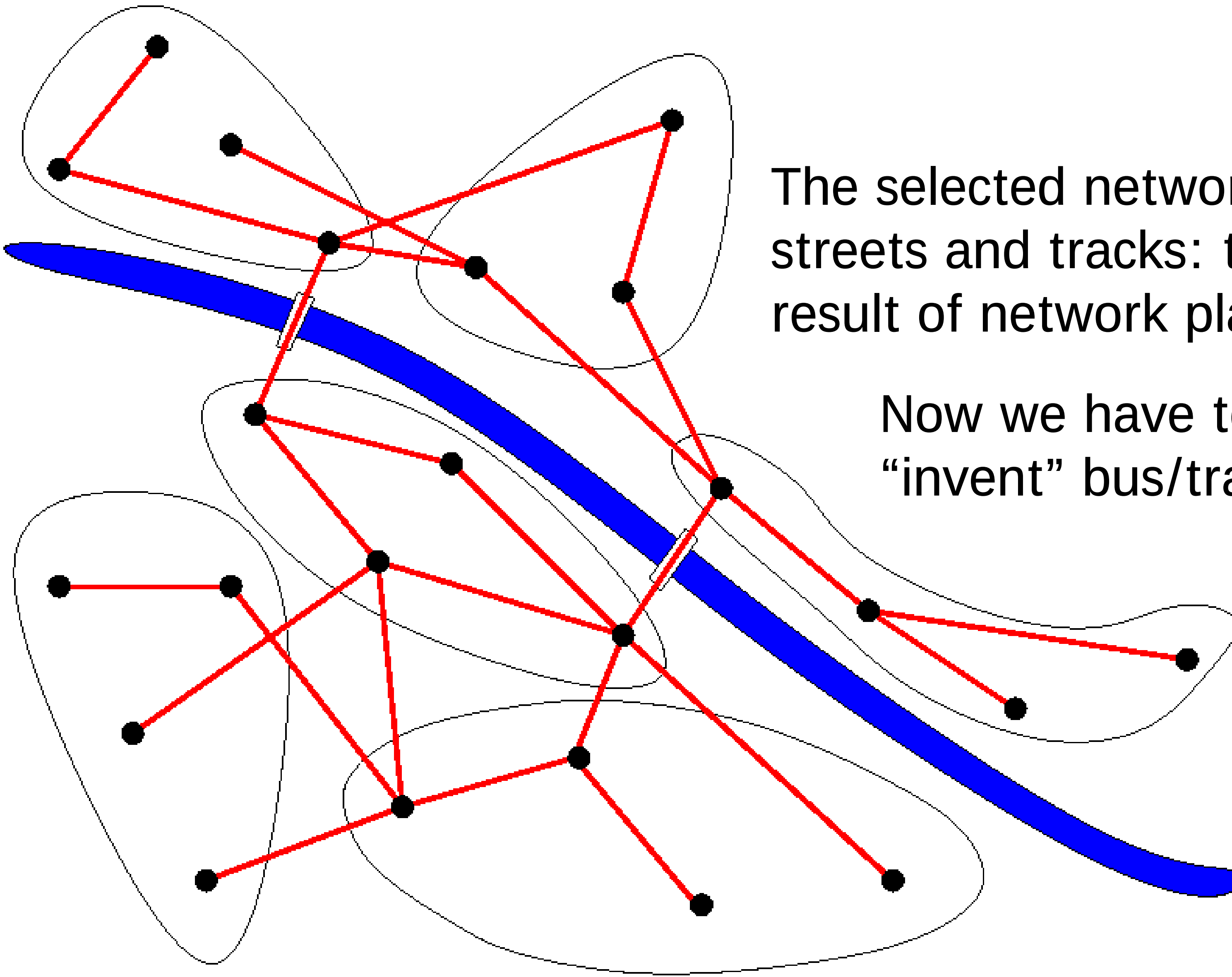
Line Planning: our didactical example city



The selected network of streets and tracks: the result of network planning

Now we have to “invent” bus/tram lines

MIP Model for Line Planning

Variables:

$y_p \in \mathbb{R}_+$ passenger flow on path $p \in P_{st}$

$x_\ell \in \{0,1\}$ choose line ℓ

$f_\ell \in \mathbb{R}_+$ frequency of line ℓ

huge number of variables

$$\min \quad \sum_\ell (C_\ell x_\ell + c_\ell f_\ell) + \sum_p \tau_p y_p$$

$$s.t. \quad \sum_{p \in P_{st}} y_p = d_{st} \qquad \forall s,t$$ transport all passengers

$$\sum_{p: a \in p} y_p \le \sum_{\ell: a \in \ell} \kappa_\ell f_\ell \qquad \forall a$$ capacity constraints

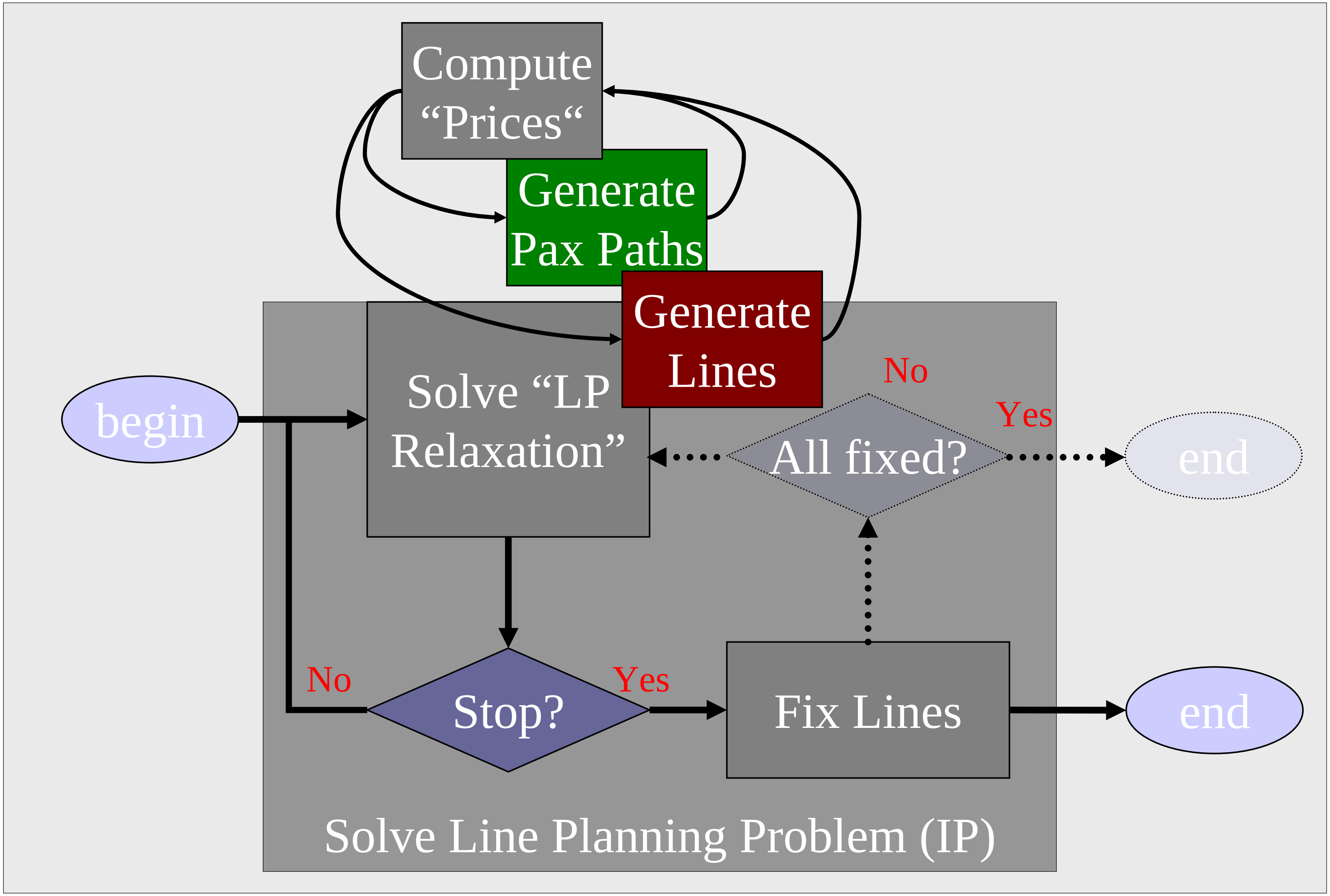
$$f_\ell \le F x_\ell \qquad \forall \ell$$ frequency bounds

$$\ldots$$ other linear constraints

Discussion

- Properties
  - Dynamic line generation (no line pool)
  - Dynamic passenger routing (no system split)
  - Minimize weighted sum of costs and travel times
  - Passenger routes: system optimum = user equilibrium
  - Time horizon: fixed
- Extensions
  - Continuous frequencies $\rightarrow$ discrete frequencies
  - Transfers $\rightarrow$ B11
  - Demand feedback $\rightarrow$ B5, B15

Column Generation Method



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Computational Results

- Cooperation

  ViP Verkehrsbetrieb Potsdam GmbH

  Havelbus Verkehrsgesellschaft mbH

  IVU Traffic Technologies AG

  City of Potsdam

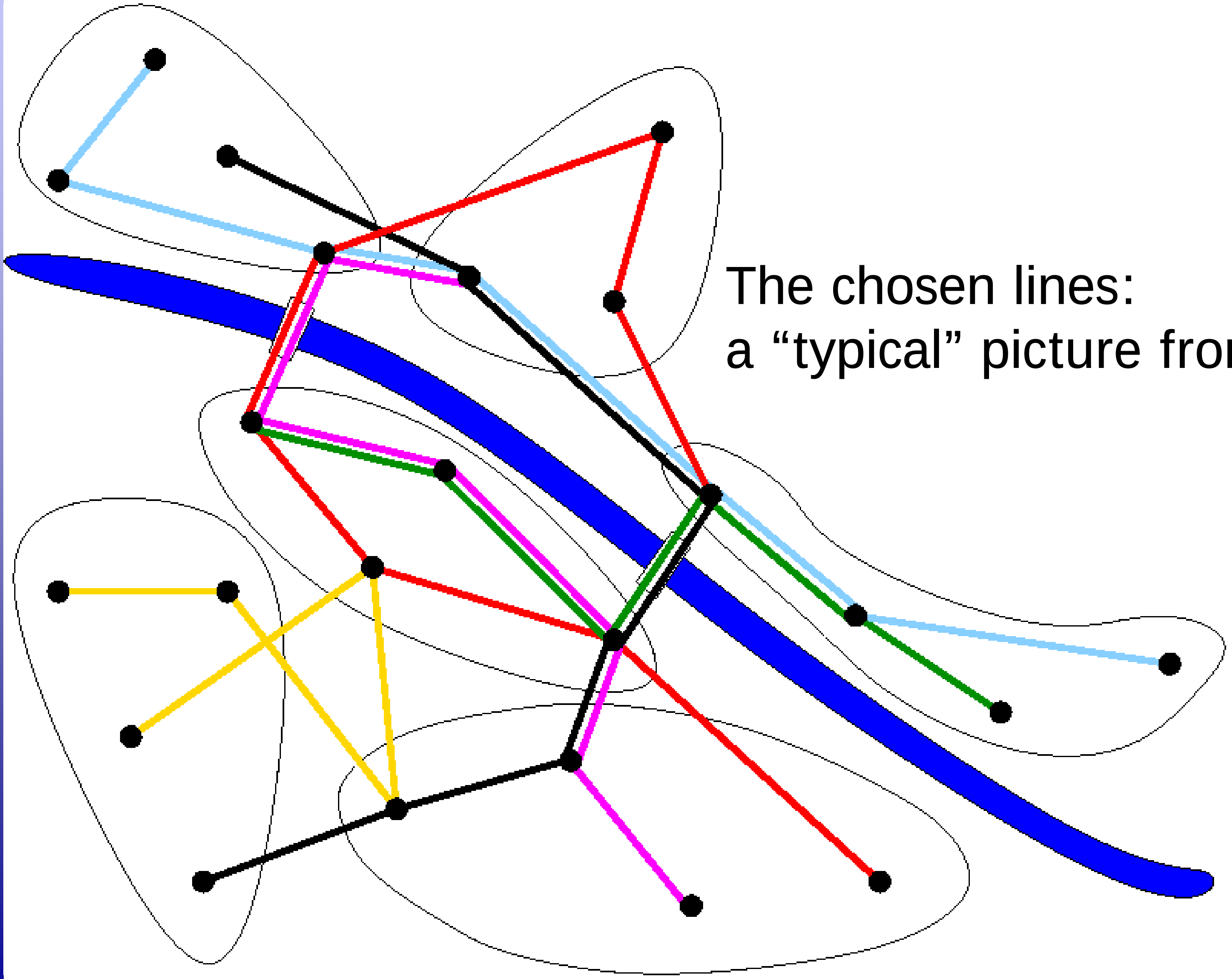
- Potsdam

  130,000 inhabitants, 42,796 trips/afternoon

  27 bus lines, 4 tram lines

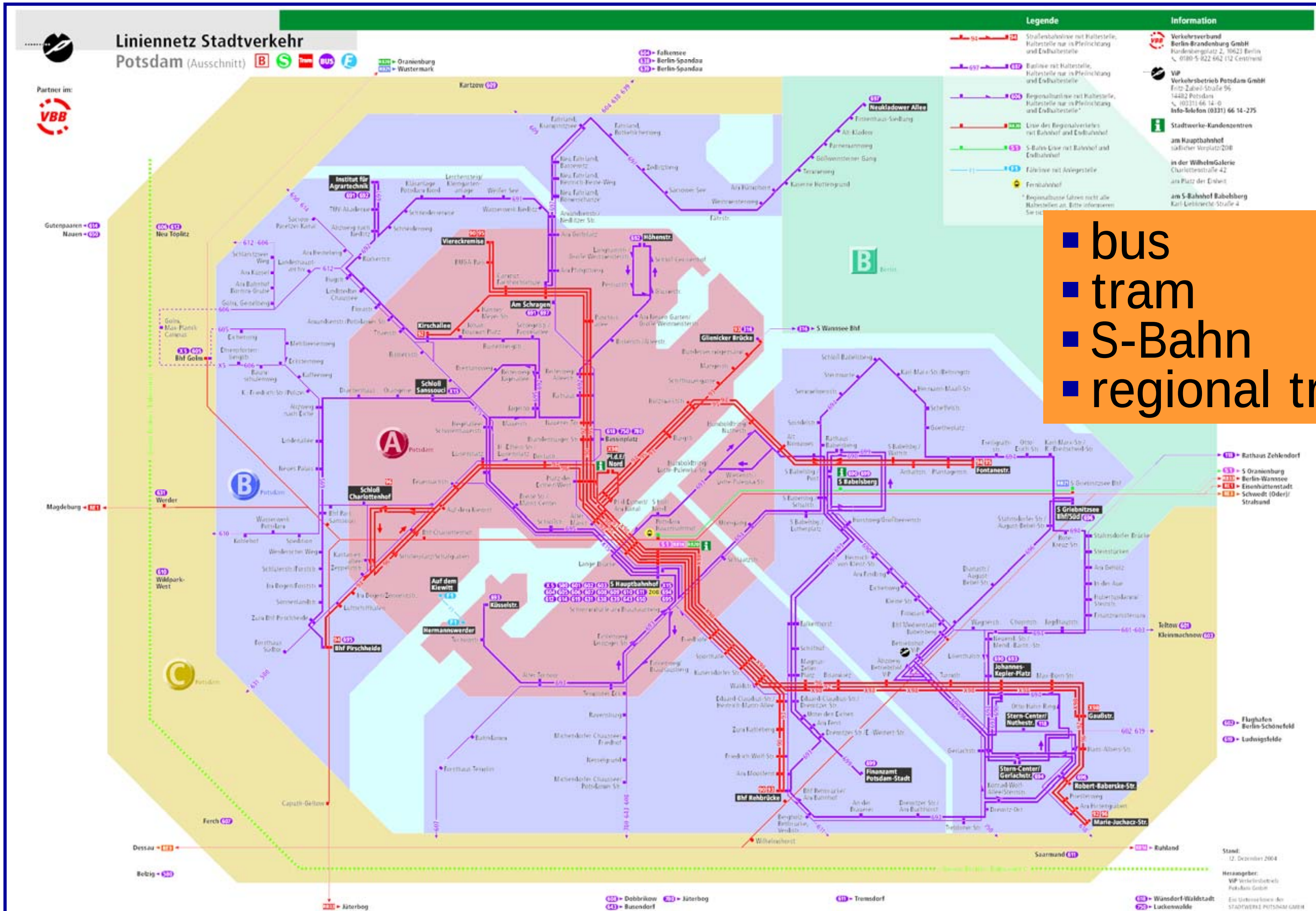
  410 nodes, 106 OD-nodes, 891 edges

Line Planning (didactical example):



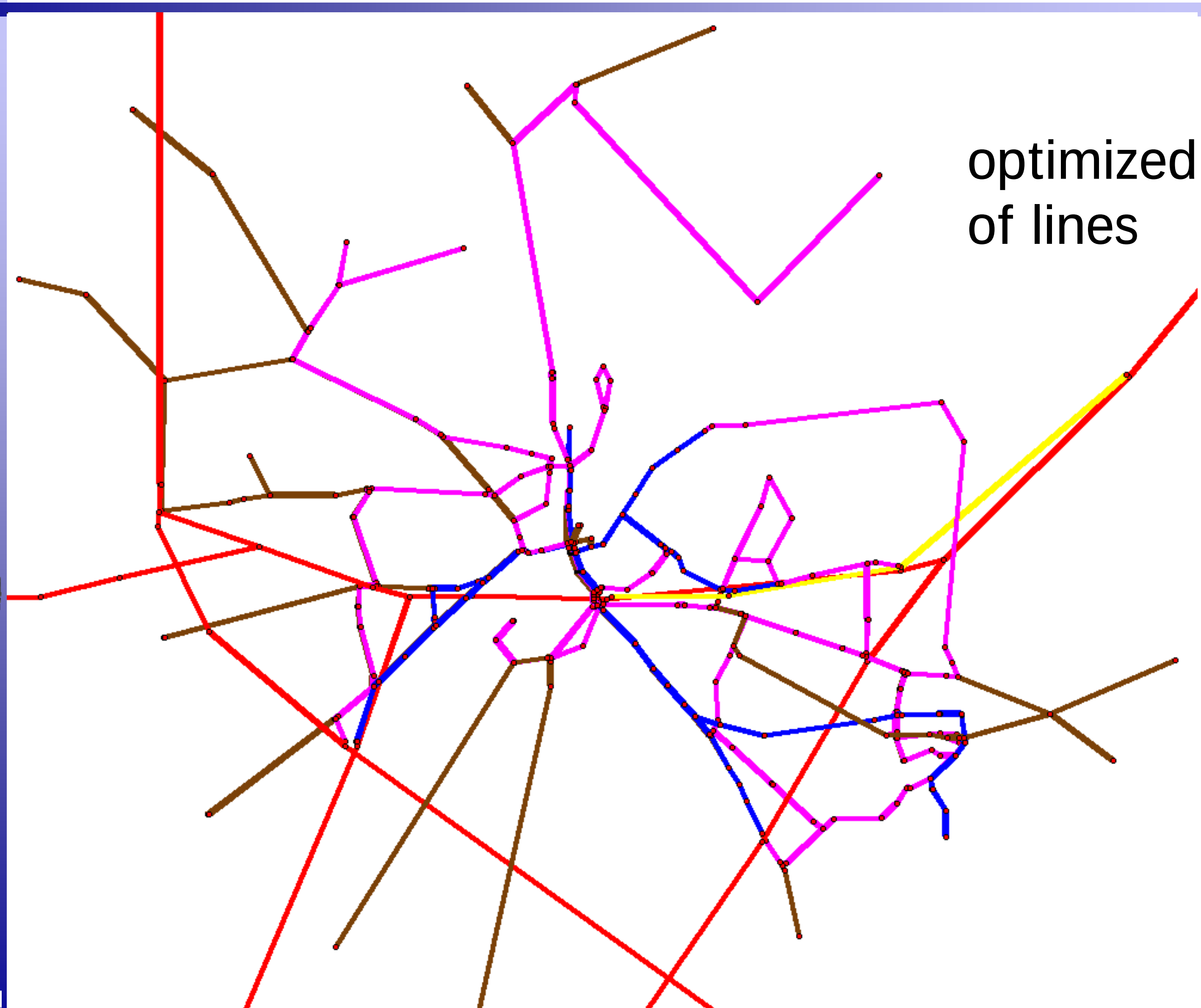
The chosen lines:
a “typical” picture from practice

The currently existing Potsdam Network of Lines



- bus
- tram
- S-Bahn
- regional train

Network of all Public Transportation Lines in Potsdam



optimized system of lines

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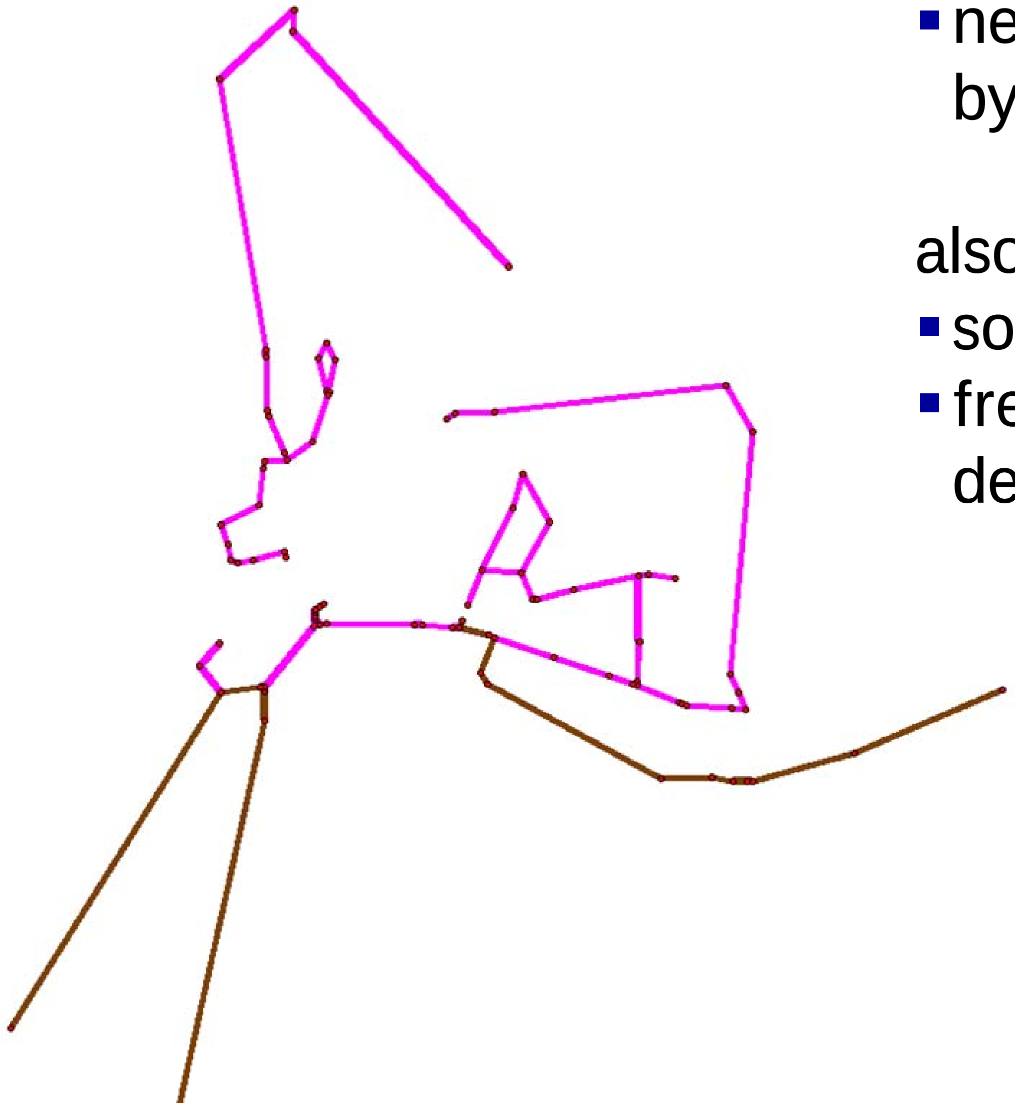
New Lines in Potsdam

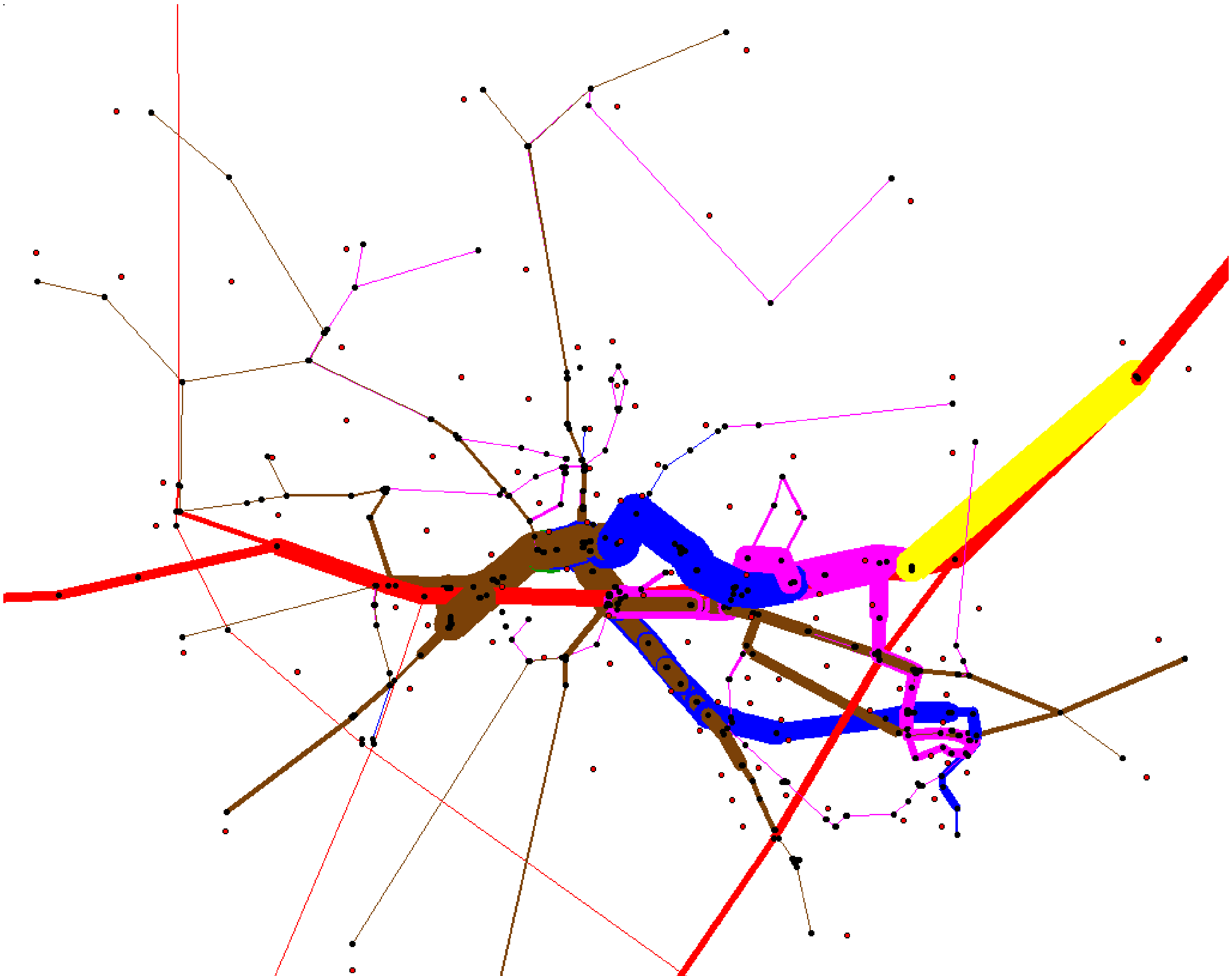


- new lines suggested by optimization

also (but not drawn):

- some lines removed
- frequencies of lines determined

Passenger Flow in the new Potsdam Network of Lines



A Side Remark:

How do we solve these and other MIP problems?

- Commercial LP/IP/MIP solvers, such as CPLEX (ILOG), XPRESS (Dash), ...
  Free solvers Soplex (ZIB),...
- Non-differentiable, non-linear programming techniques (if simplex or interior point algorithms are not applicable or do not work)
- polyhedral combinatorics and cutting planes
- special purpose “tricks”

Contents

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Planning prices and frequencies

- Ansatz
  - „Controlling“ demand via prices and travel times
  - Price system = Individual price + ???
  - Maximize profit
  - Electronic Ticketing
- Status
  - Research project
  - Data?
  - Mathematical models ?
  - Giant amount of literature on topics of questionable value for practice (versions of local elasticities)

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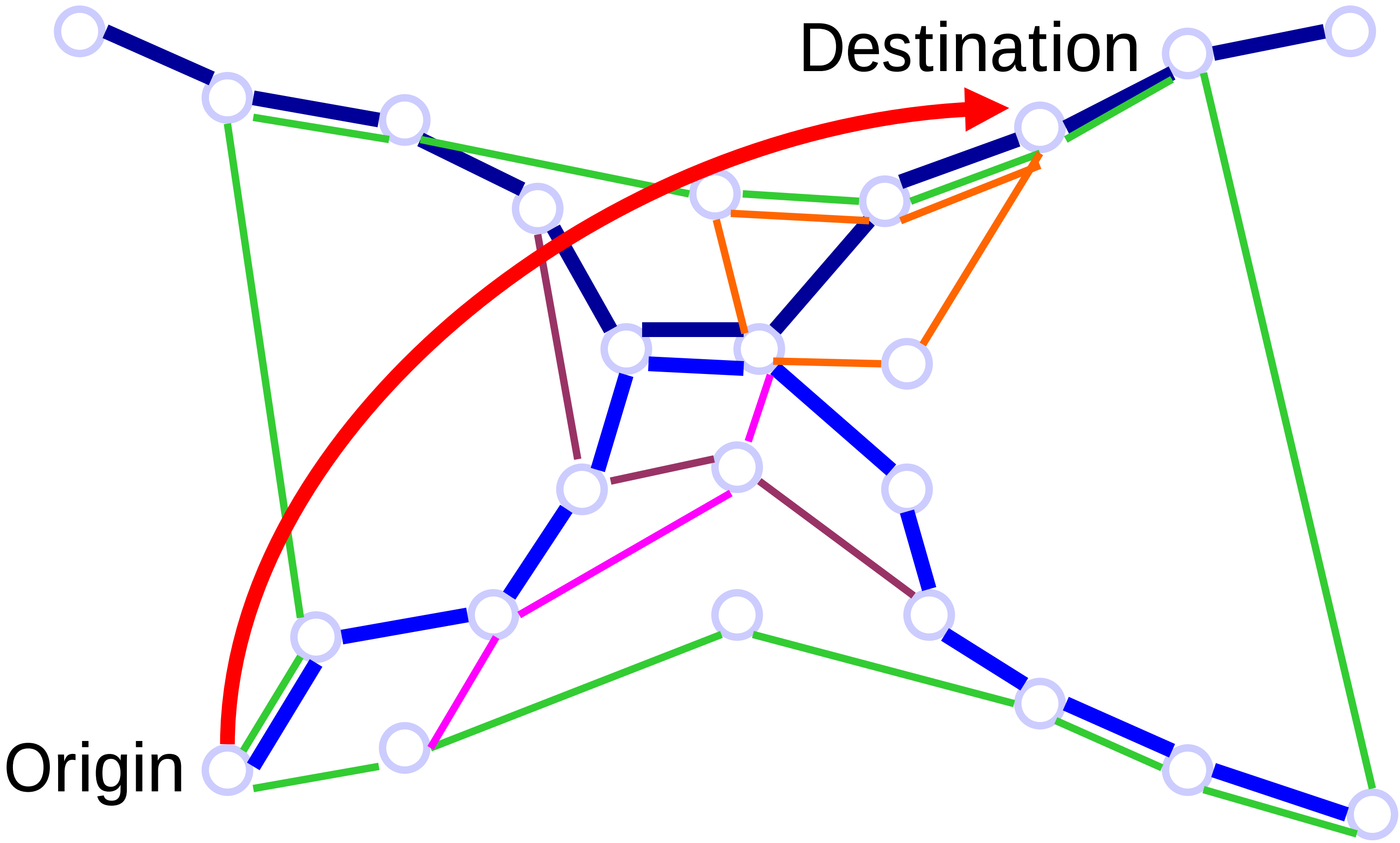
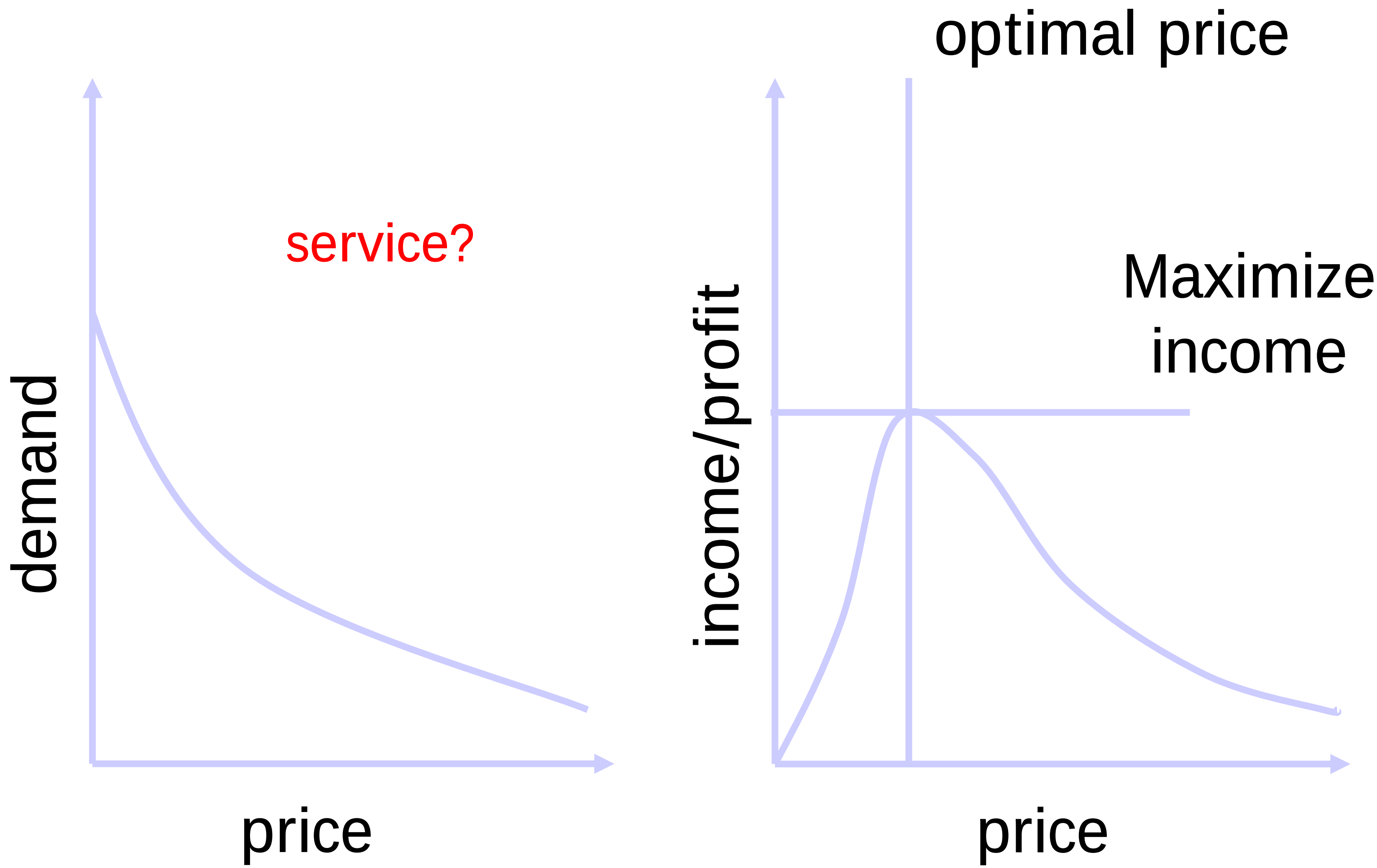
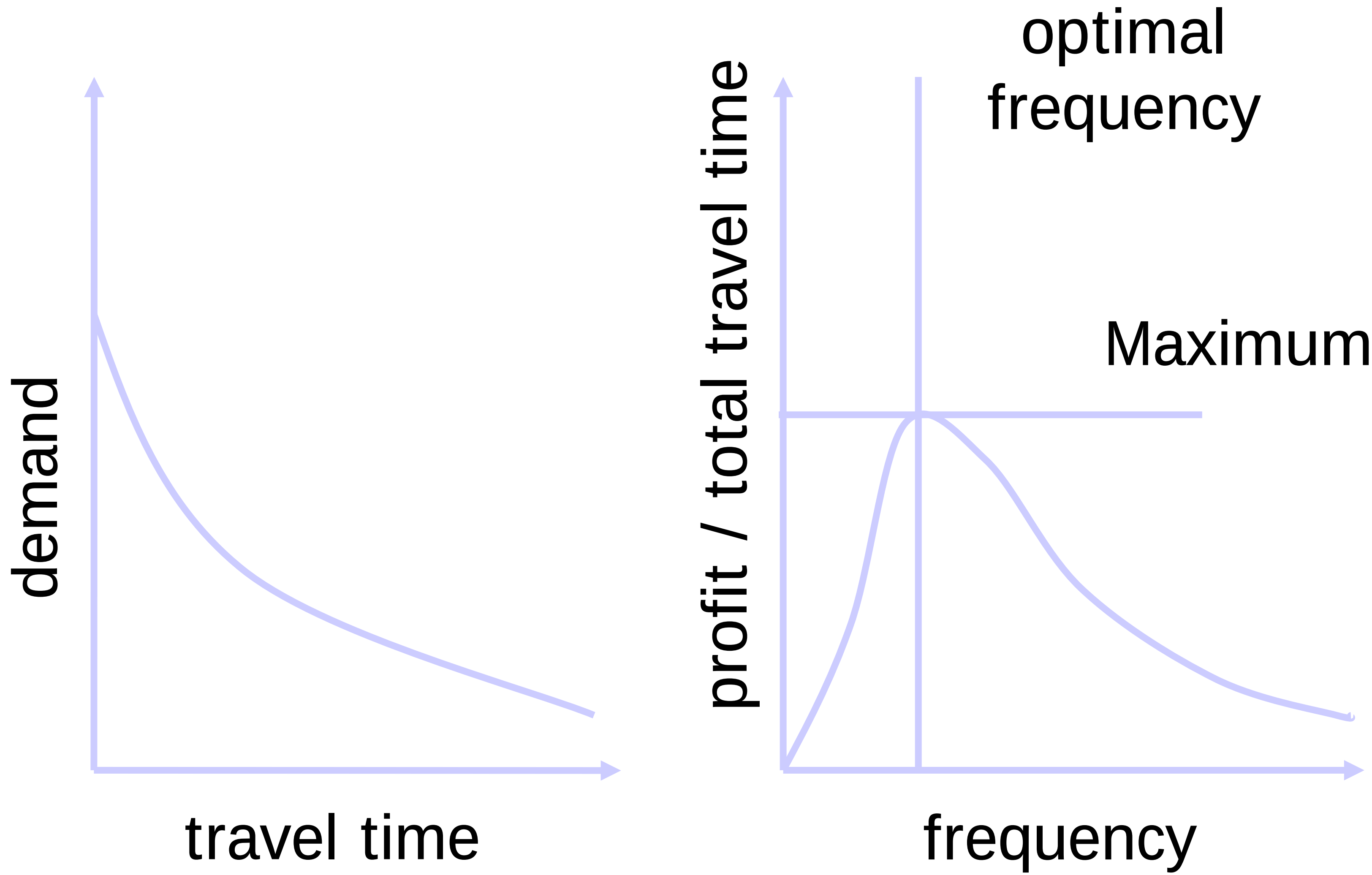
Planning prices



Planning prices



Planning frequencies & synchronized timetables



Current Timetable

BUS 690 S Babelsberg ◂▸ Am Stern, Johannes-Kepler-Platz

ViP

BUS 690

S Babelsberg/Post	S1, Tram 94 95, BUS 694
Rathaus Babelsberg	Tram 94 95, BUS 693 694 699
S Babelsberg/Schulstr.	S1, BUS 693 694 699
Horstweg/Großbeerenstr.	BUS 601 602 603 619 693 699
Heinrich-von-Kleist-Str.	BUS 601 602 603 619
Am Findling	BUS 601 602 603 619
Eichenweg	BUS 601 602 603 619
Kleine Str.	BUS 601 602 603 619
Filmpark	BUS 601 602 603 619
Bhf Medienstadt Babelsberg	BUS 601 602 603 618 619 696
Bhf Medienstadt Babelsberg	B RE3 RB33, BUS 601 602 603, 619 696
Betriebshof ViP	BUS 601 696
Abzweig Betriebshof ViP	Tram 92 96, BUS 601 696
Johannes-Kepler-Platz	Tram X98 92 96, BUS 118 601, 602 602B 618 619 693 694

BUS 690 **täglich**

Verkehrshinweise		8		20	
S Babelsberg/Post	ab	5.27	5.47		21.27
Horstweg/Großbeerenstr.		**5.29**	**5.49**		**21.29**
Heinrich-von-Kleist-Str.		5.31	5.51		21.31
Am Findling		5.32	5.52		21.32
Eichenweg		5.32	5.52		21.32
Kleine Str.		5.33	5.53		21.33
Filmpark		5.35	5.55		21.35
Bhf Medienstadt Babelsberg		**5.36**	**5.56**		**21.36**
Bhf Medienstadt Babelsberg		**5.36**	**5.56**		**21.36**
Betriebshof ViP		5.38	5.58		21.38
Abzweig Betriebshof ViP		5.38	5.58		21.38
Johannes-Kepler-Platz	an	5.42	6.02		21.42

		8		20	
Johannes-Kepler-Platz	ab	5.02	5.22		20.42
Betriebshof ViP		5.05	5.25		20.45
Bhf Medienstadt Babelsberg		**5.08**	**5.28**		**20.48**
Bhf Medienstadt Babelsberg		**5.09**	**5.29**		**20.49**
Filmpark		5.10	5.30		20.50
Kleine Str.		5.11	5.31		20.51
Eichenweg		5.12	5.32		20.52
Am Findling		5.13	5.33		20.53
Heinrich-von-Kleist-Str.		5.14	5.34		20.54
Horstweg/Großbeerenstr.		**5.15**	**5.35**		**20.55**
S Babelsberg/Schulstr.		**5.17**	**5.37**		**20.57**
Rathaus Babelsberg		**5.19**	**5.39**		**20.59**
S Babelsberg/Post	an	5.20	5.40		21.00

8 Montag - Freitag

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Computational Results

- Cooperation

  ViP Verkehrsbetrieb Potsdam GmbH

  Havelbus Verkehrsgesellschaft mbH

  IVU Traffic Technologies AG

  City of Potsdam

- Potsdam

  130,000 inhabitants, 42,796 trips/afternoon

  27 bus lines, 4 tram lines

  410 nodes, 106 OD-nodes, 891 edges

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Fare Planning

- Input
  - Network
  - OD-demand functions
  - Travel times
  - Price system
- Output
  - Prices and passenger flow
- Problem
  - Compute prices and corresponding passenger flow
- Goals
  - Maximize revenue (computing "optimal prices")
  - Optimize modal split (attracting additional passengers)
  - Compare fare systems (decision support)

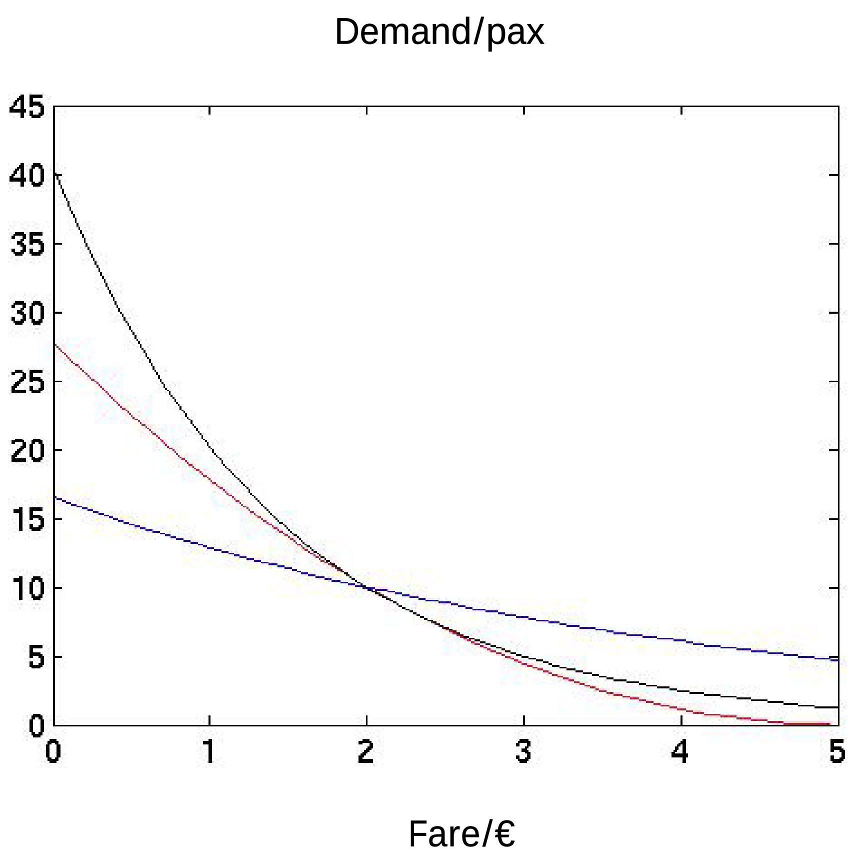
Literature

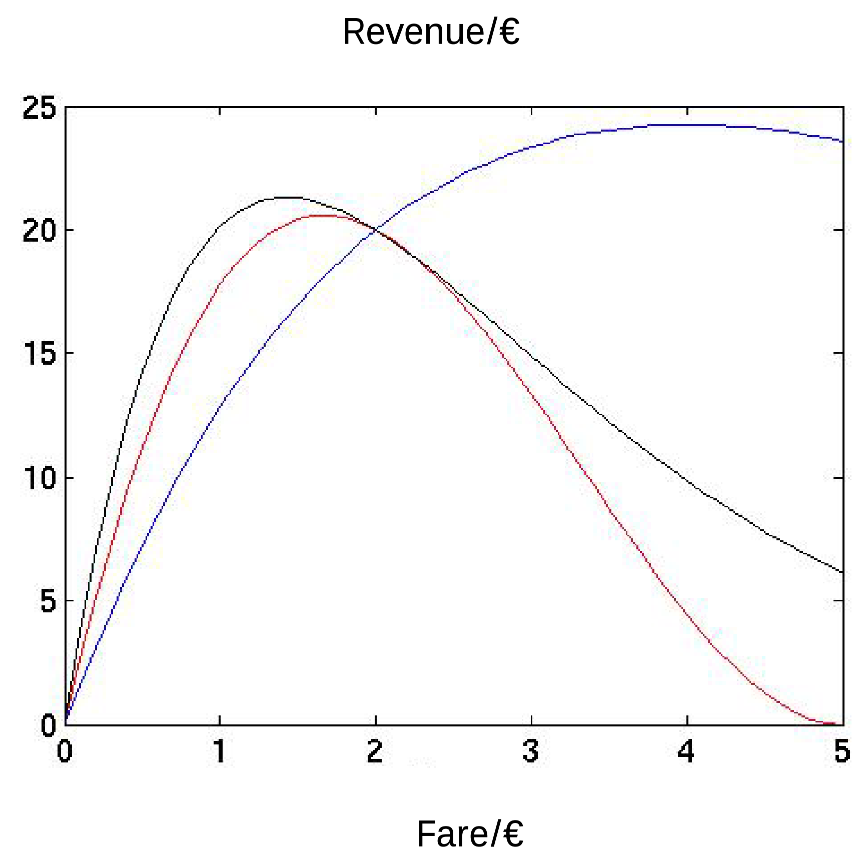
- Price elasticity
  - $\varepsilon(x) := x \cdot d'(x)/d(x) \approx (\Delta d/d)/(\Delta x/x)$
  - Empirical studies: $\varepsilon \approx -0.3$
    Curtin 1968
  - Cobb-Douglas-functions $d(x) = c \cdot x^{\varepsilon}$ with constant elasticity
- Analytical approaches/marginal costs
  Pedersen 2003
- Optimize zone tariffs
  Hamacher & Schöbel 1995, 2004
- Maximize social welfare
  De Borger, Mayeres, Proost, Wouters 1996
- Maximize revenue for bus services
  Kocur & Hendrickson 1982

Literature

- Fare Planning for Public Transport
  Ralf Borndörfer, Marika Neumann, Marc E. Pfetsch
  ZIB-Report 05-20
- Optimal Fares for Public Transport
  Ralf Borndörfer, Marika Neumann, Marc E. Pfetsch
  ZIB-Report 05-35
- Mathematische Preisplanung im ÖPNV
  Marika Neumann
  Diplomarbeit, TU Berlin, 2005

Fare – Demand – Revenue





- Demand $d = d(x)$
- Revenue $r(x) = d(x) \cdot x$

Discrete Choice Demand Model

- A — Set of travel alternatives
- T — Time horizon (here: one month)
- $X_{st} \in \{1,\ldots,n\}$ — Num. of trips per month (rand. variable)
- $C = A \times \{0,1,\ldots,n\}$ — Travel choices (here: no mix)
- $U^a_{st}(x,k) = V^a_{st}(x,k) + v^a_{st}$ — Utility for alt. a, pair st, fares x, k trips

  $V^a_{st}(x,k)$ — Deterministic utility (observable)

  v^a_{st} — Random utility (disturbance term)
- $P^a_{st}(x,k)$ — Prob. that alternative a is chosen

  $P^a_{st}(x,k) = P[V^a_{st}(x,k) + v^a_{st} = \max_{b\in A} V^b_{st}(x,k) + v^b_{st}]$
- $v^a_{st} \sim G(\eta,\mu)$ — Gumbel dist. (here: $\eta = 0$, $\mu = 0.01$)

  $P^a_{st}(x,k) = e^{\mu V^a_{st}(x,k)} / (\Sigma_{b\in A} e^{\mu V^b_{st}(x,k)})$
- $d^{a,k}_{st}(x) = P^a_{st}(x,k) \cdot d_{st} \cdot P[X_{st}=k]$

  $= e^{\mu V^a_{st}(x,k)} / (\Sigma_{b\in A} e^{\mu V^b_{st}(x,k)}) \cdot d_{st} \cdot P[X_{st}=k]$

Nonlinear Programming Model

- $D \subseteq V \times V$ — OD-pairs
- C — Set of travel choices
  $C' \subseteq C$ — Set of public transport travel choices
- $x \in P \subseteq R^n$ — Vector of fare variables
  $P \subseteq R^n$ — Set of feasible fares (polyhedron)
- $p^i_{st}: R^n \to R^+$ — Price function p(x) for all $i \in C$, $st \in D$
- $d^i_{st}: R^n \to R^+$ — Demand function d(x) for all $i \in C$, $st \in D$
- $r: R^n \to R^+$ — Revenue function $r(x) = p(x)^T d(x) = \Sigma_{i,\ st}\, p^i_{st}(x) d^i_{st}(x)$
- Fare planning problem

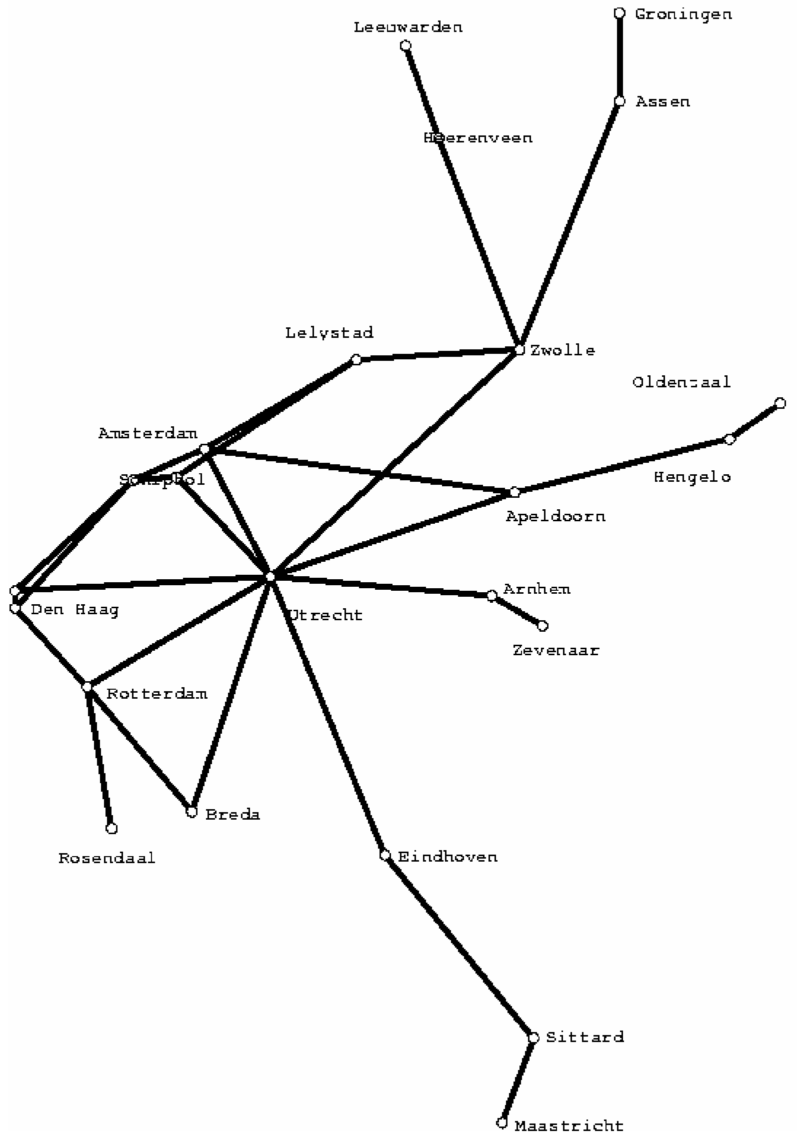
$$
\begin{array}{lll}
\text{(FPP)} & \max & \displaystyle\sum_{i \in C'} \sum_{st \in D} \mathrm{p}^{\mathrm{i}}_{\mathrm{st}}(x) \cdot \mathrm{d}^{\mathrm{i}}_{\mathrm{st}}(x) \\
 & \text{s.t.} & x \in P
\end{array}
$$

Discussion

- Properties
  - Demand extrapolation
  - Revenue maximization
  - Passenger routes: fixed
  - Time horizon: fixed

Dutch Intercity Network

(Bussieck [1998], Bussieck, Kreuzer, Zimmermann [1996], Claessens, van Dijk, Zwaneveld [1998])



Origin-Destination Matrix

(Bussieck [1998], Bussieck, Kreuzer, Zimmermann [1996], Claessens, van Dijk, Zwaneveld [1998])

	Hr	Asn	Zl	Hgl	Ah	Ut	Shl	Asdz	Asd	Gv	Gvc	Rtd	Bd	Ehv	Std	Mt	Lls	Rsdg	Zvg	Odzg
Lw	478		380		13	145	20	21	90	6	26	36	14	9	9	4	77	7	14	
Gn		1720	720			331	48	88	205	12	73	75	34	28	29	13	200	33	14	
Hr			511		11	209	20	16	115	10	48	58	16	11	8	4	77	10	19	
Asn			854		16	502	32	58	235	13	117	125	42	33	28	14	152	48	19	
Zl					56	1112	64	171	400	33	163	182	79	47	46	21	390	100	32	
Apd				468		1160	32	76	917	21	202	143	57	62	10	5		47	83	71
Hgl						422	11	24	287	20	81	52	39	28	20	12		24		75
Ah						4244	60	721	726	109	741	180	136	101			8	320	602	
Ut							278	5826	4919	225	3138	2260	1165	3109	720	359	89	325	996	21
Shl								1456	6469	1339	1503	509	7	99	44	29	103	164		
Asdz										461	207	369	138	542	203	149	819	6	155	
Asd										730	2540	1756	154	437	155	37	2783	2258	489	22
Gv											785	4586	531	35	22	8	29	890		
Gvc												2829	228	335	104	41	31	3	229	7
Rtd													1829	569	179	73	46	1077	157	11
Bd														950	157	79	6	329	14	5
Ehv															936	404	8	75	11	3
Std																863	2	19		
Mt																	1	22		
Lls																		15		

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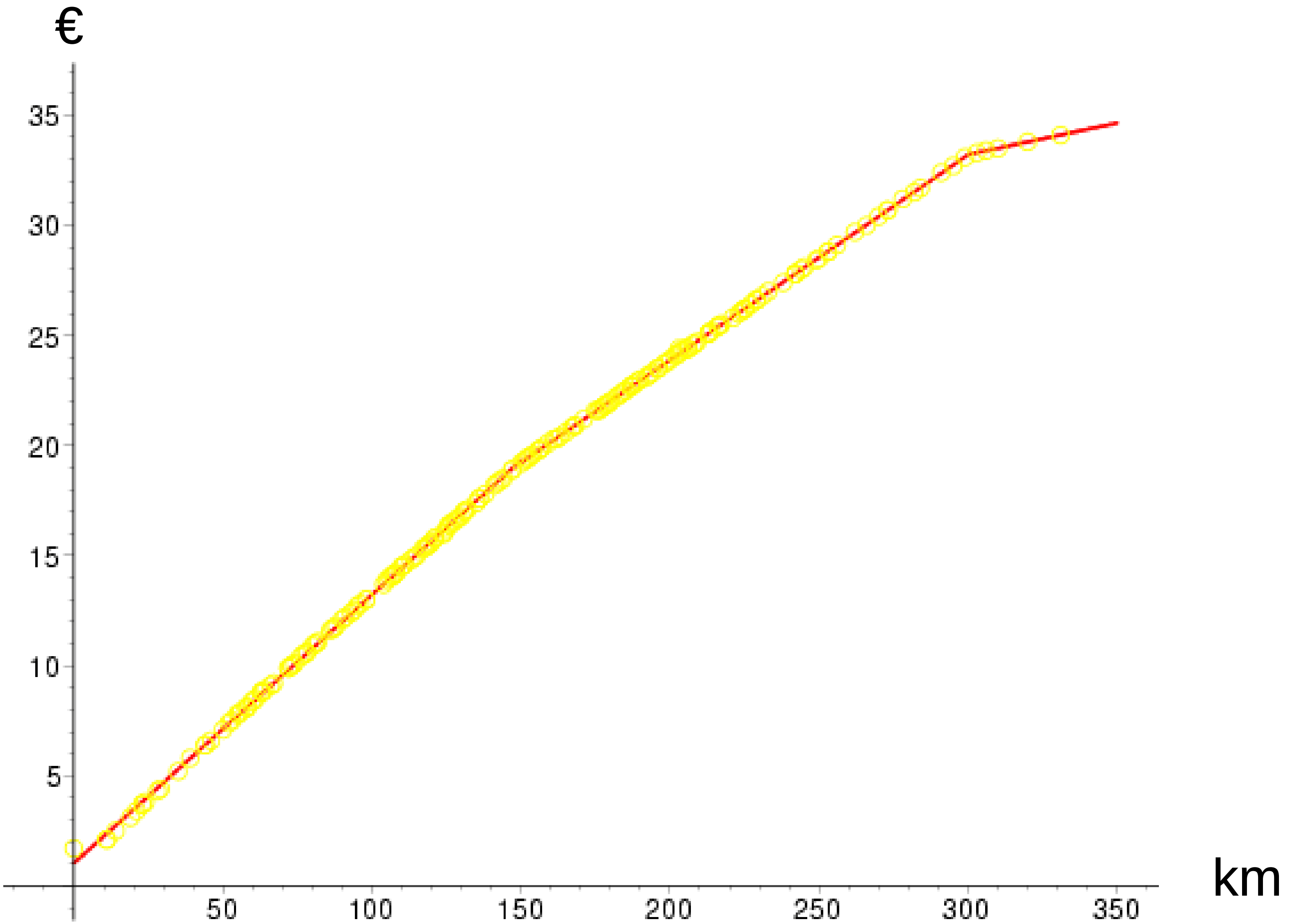
Travel Distances

(Bussieck [1998], Bussieck, Kreuzer, Zimmermann [1996], Claessens, van Dijk, Zwaneveld [1998])

	Hr	Asn	Zl	Hgl	Ut	Shl	Asdz	Asd	Gv	Gvc	Rtd	Bd	Ehv	Std	Mt	Lls	Rsdg	Zvg	Odzg
Lw	29																		
Gn		28																	
Hr			66																
Asn			78																
Zl					85											50			
Apd				69	64			89											
Hgl																			18
Ah					58													19	
Ut							34	39		61	57	92	81						
Shl							9	19	43	43									
Asdz						9										56			
Asd																54			
Gv										1	23								
Rtd												49					18		
Ehv														78					
Std															21				

Fares

(www.ns.nl)



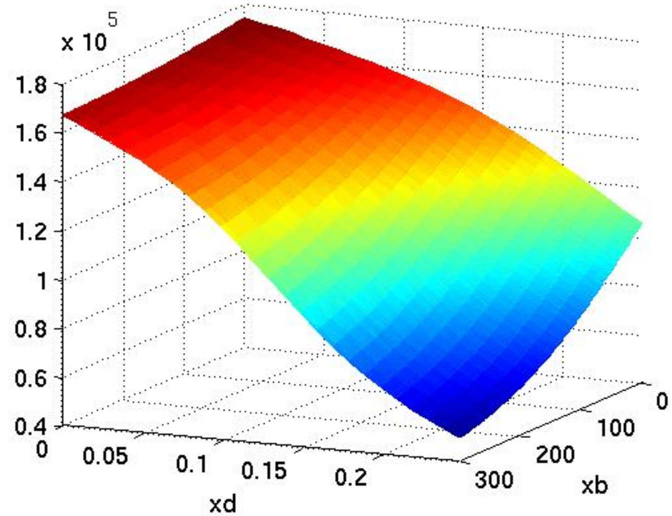
Example 1: Standard and Reduced Ticket

- $A = \{s, r, c\}$ Alternatives standard, reduced, car
- $C = A \times \{1,2,\ldots,60\}$ Travel choices
- $x = (x_b, x_d)$ Reduction entitlement, distance fare
- $p^{S,k}_{st}(x) = x_d \cdot l_{st} \cdot k$ Standard ticket
  $p^{R,k}_{st}(x) = x_b + ½ \cdot x_d \cdot l_{st} \cdot k$ Reduced ticket
  $p^{C,k}_{st}(x) = q_F + q_V \cdot l^C_{st} \cdot k$ Car (here: q_F = 100 €, q_V = 0.1 €/km)
- $V^{S,k}_{st}(x) = - p^{S,k}_{st}(x) - 0.1\ (t_{st} \cdot k)$ Utility for using standard ticket
  $V^{R,k}_{st}(x) = - p^{R,k}_{st}(x) - 0.1\ (t_{st} \cdot k)$ Utility for using reduced ticket
  $V^{C,k}_{st}(x) = - p^{C,k}_{st}(x) - 0.1\ (t^C_{st} \cdot k)$ Utility for using car

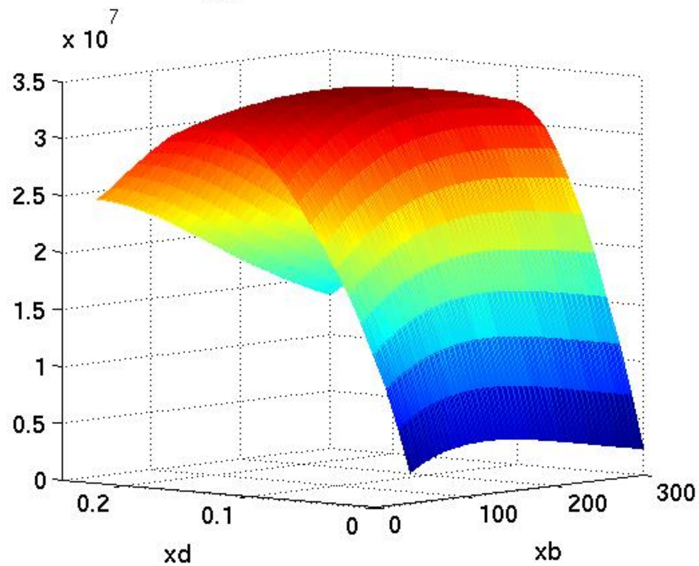
$$\text{(FPP)} \quad \max \quad \sum_{st \in D} \sum_{(a,k) \in C'} p^{a,k}_{st}(x) \cdot \frac{e^{\mu V^{a,k}_{st}(x)}}{\sum_{b \in A} e^{\mu V^{b,k}_{st}(x)}} \cdot d_{st} \cdot \mathrm{P}[X_{st} = k]$$

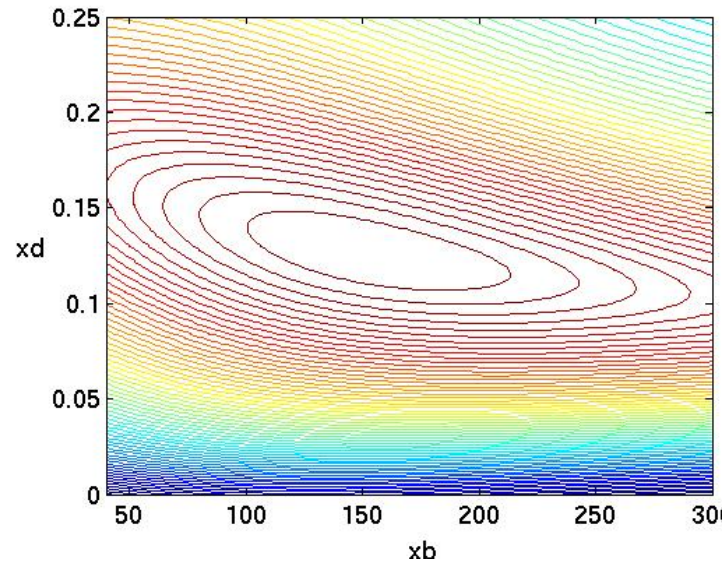
$$\text{s.t.} \qquad x_b, x_d \geq 0$$

Example 1: Standard and Reduced Ticket



- x_b = 153.31 €
- x_d = 0.13 €/km
- r(x) = 34,201,767.8 € (+32.4%)
- d(x) = 126,768 (+38.1%)
- Modal split 68.9% (+18.8%)





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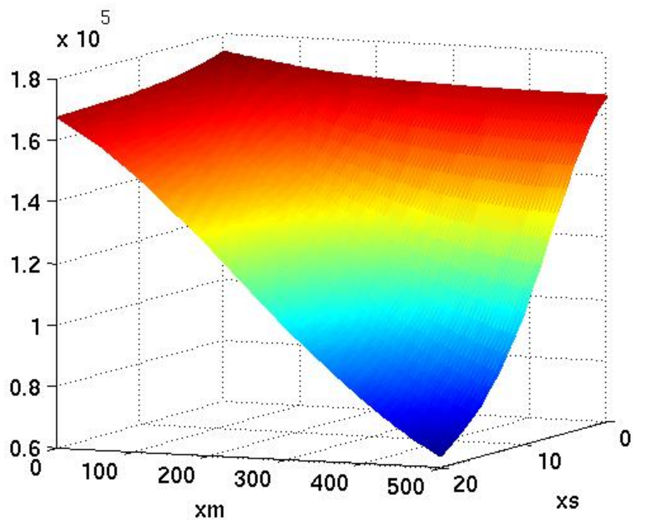
Example 2: Single and Monthly Ticket

- $A = \{s, m, c\}$ Alternatives standard, monthly, car
- $C = A \times \{1,2,\ldots,60\}$ Travel choices
- $x = (x_m, x_s)$ Monthly ticket, single ticket
- $p^{S,k}_{st}(x) = x_s \cdot k$ Single ticket
  $p^{M,k}_{st}(x) = x_m$ Monthly ticket
  $p^{C,k}_{st}(x) = q_F + q_V \cdot l^C_{st} \cdot k$ Car (here: q_F = 100 €, q_V = 0.1 €/km)
- $V^{S,k}_{st}(x) = -\, p^{S,k}_{st}(x) - 0.1\ (t_{st} \cdot k)$ Utility for using standard ticket
  $V^{M,k}_{st}(x) = -\, p^{R,k}_{st}(x) - 0.1\ (t_{st} \cdot k)$ Utility for using reduced ticket
  $V^{C,k}_{st}(x) = -\, p^{C,k}_{st}(x) - 0.1\ (t^C_{st} \cdot k)$ Utility for using car

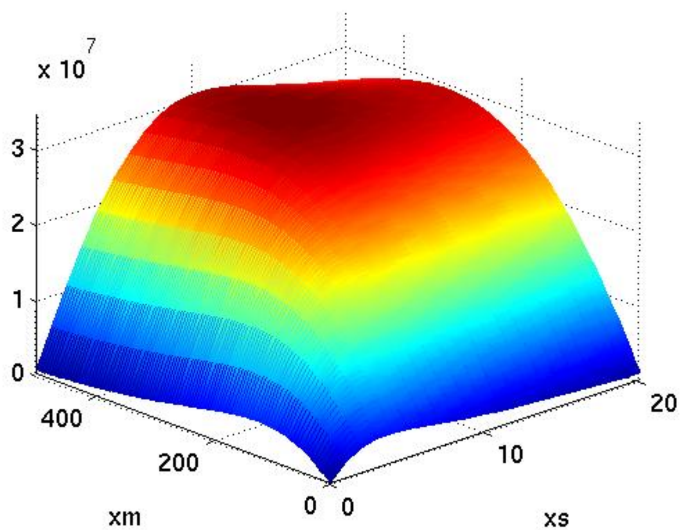
$$\text{(FPP)} \quad \max \quad \sum_{st \in D} \sum_{(a,k) \in C'} p^{a,k}_{st}(x) \cdot \frac{e^{\mu V^{a,k}_{st}(x)}}{\sum_{b \in A} e^{\mu V^{b,k}_{st}(x)}} \cdot d_{st} \cdot P[X_{st} = k]$$

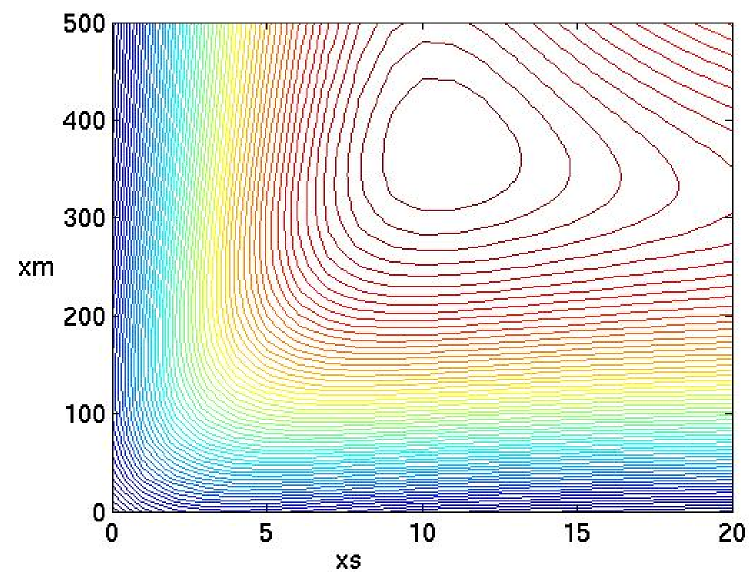
$$\text{s.t.} \qquad x_m, x_s \geq 0$$

Example 2: Single and Monthly Ticket



- x_m = 372.08 €
- x_d = 10.99 €/km
- r(x) = 31,813,156,4 € (+23.1%)
- d(x) = 110,999 (+20.9%)
- Modal split 60.3% (+10.2%)





Comparison

(status quo extrapolated to include car traffic using our model with alternatives nsr, car for each OD-pair)

	revenue	*demand*	*modal split*
status quo	25,928,730	91,791	50.1%
standard / reduced	34,201,767	126,786	68.9%
single / monthly	31,813,156	110,999	60.3%

Contents

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Vehicle Scheduling

VS-OPT

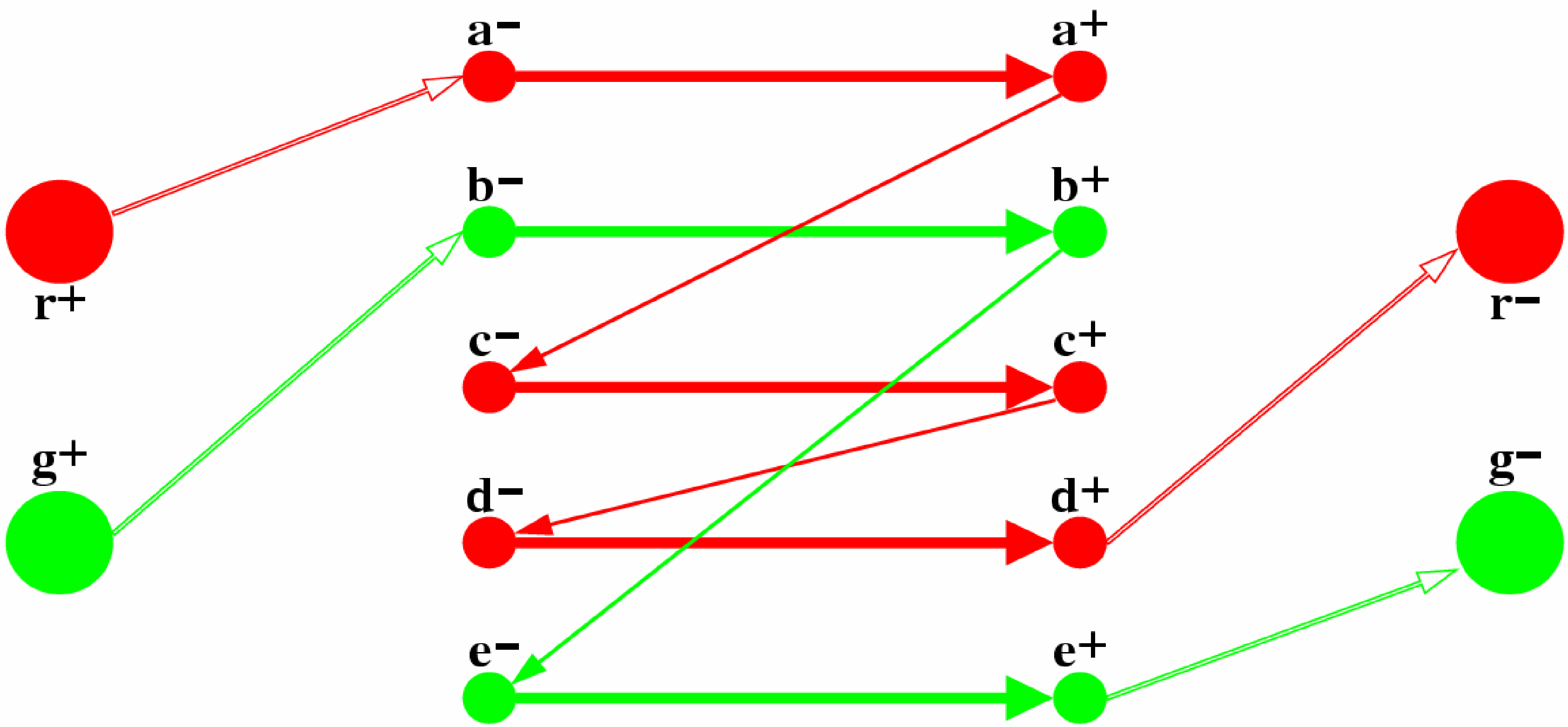
a code developed at ZIB,
maintained and distributed by spin-off company

LBW GbR (Löbel, Borndörfer & Weider)

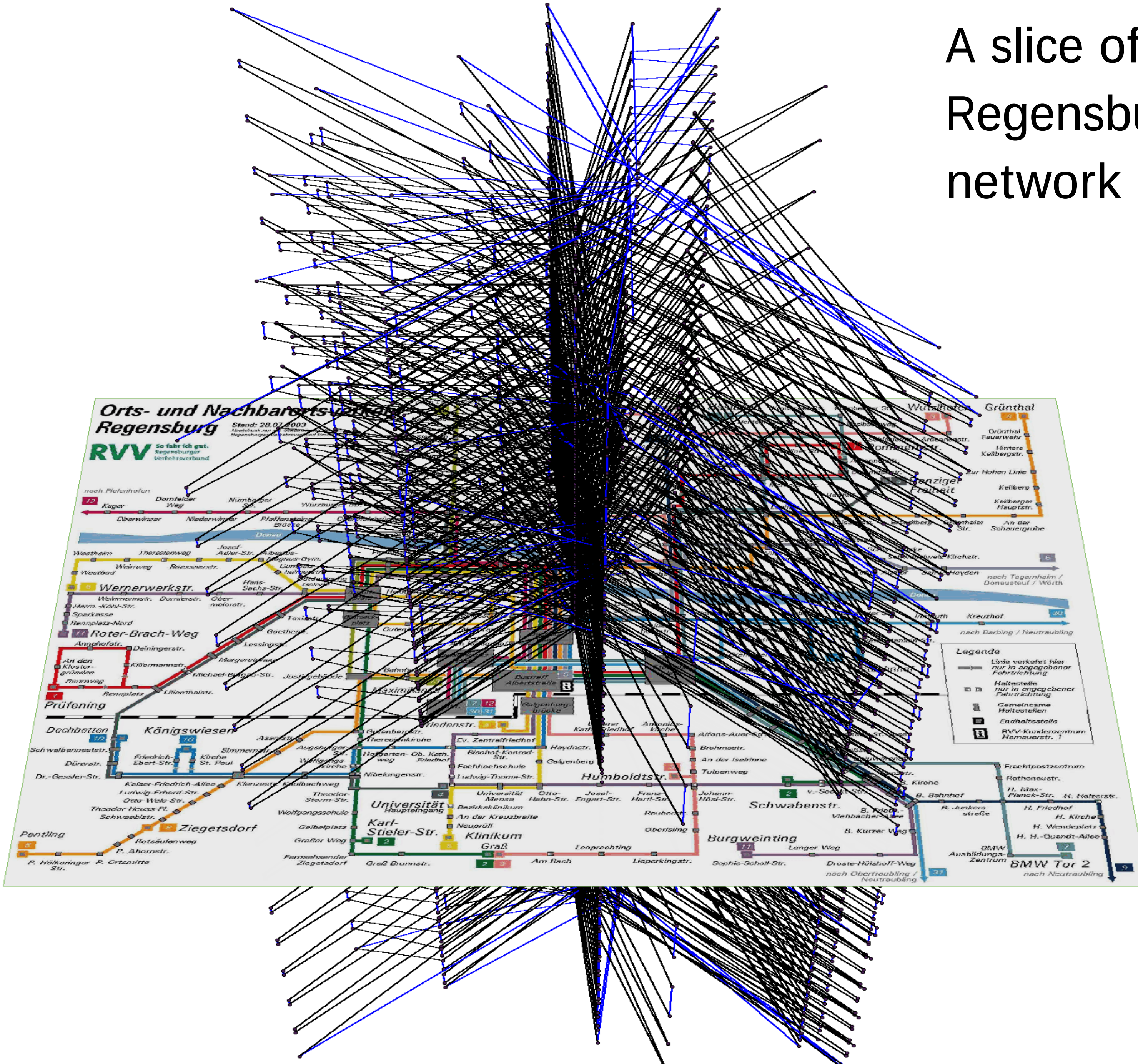
Mathematical Model and Algorithmic Approach:

Multicommodity Flow model solved by
Lagrangean relaxation and dynamic column generation

Graph Theoretic Model



A slice of the Regensburg network



IP Model for Vehicle Scheduling (Multicommodity Flow Problem)

$$
\begin{array}{rcll}
\min \sum_{d \in D} \sum_{ij \in A^d} c_{ij}^d x_{ij}^d & & & \\
\sum_{d \in D} \sum_{tj \in A^d} x_{tj}^d & = & 1 & \forall t \in T \quad \text{(Flow Requ.)} \\
\sum_{d \in D} \sum_{tj \in A^d} x_{tj}^d - \sum_{d \in D} \sum_{it \in A^d} x_{it}^d & = & 0 & \forall t \in T, d \in D \quad \text{(Flow Cons.)} \\
\sum_{dt \in A^d} x_{dt}^d & \leq & \kappa_d & \forall d \in D \quad \text{(Capacities)} \\
x & \in & \mathbb{Z}_+^A & \quad \text{(Integrality)}
\end{array}
$$

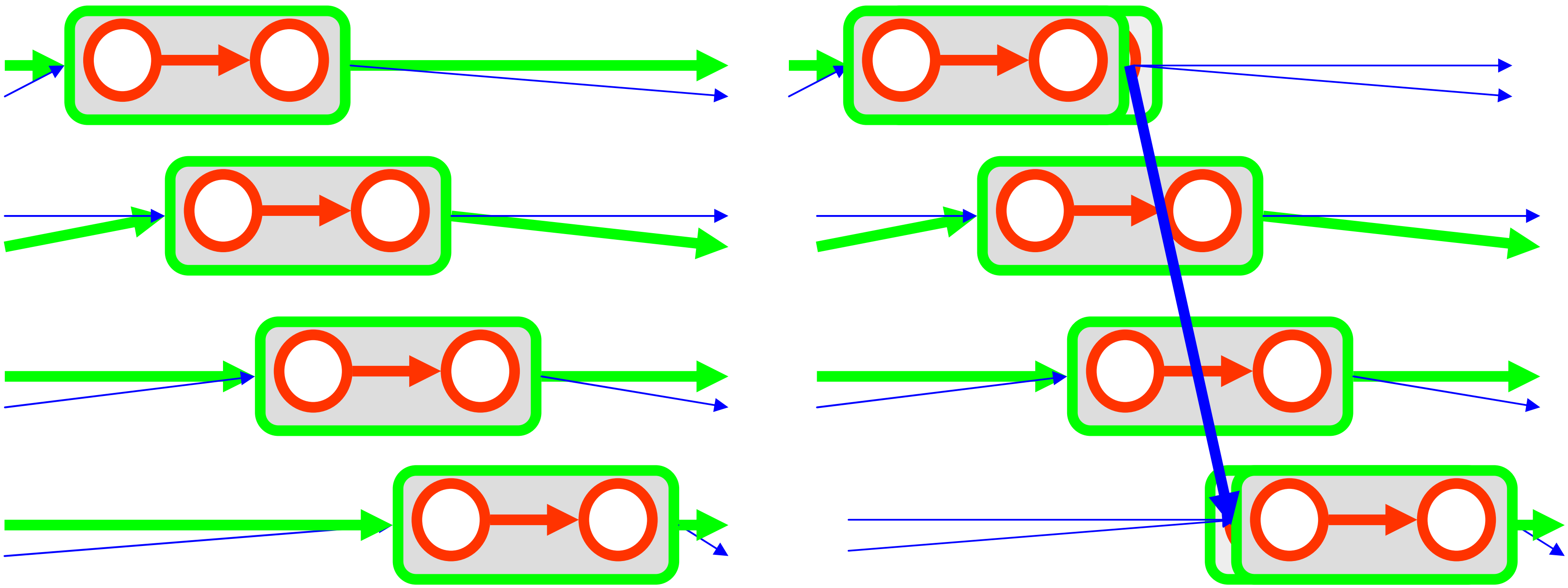
- D – Depots
- T – Timetabled Trips

Discussion

- Properties
  - Exploiting all degrees of freedom
  - Subcontracting constraints → new
- Extensions
  - Trip shifting → current work
  - Multiperiod scheduling
  - Periodic schedules
  - Assimilation
  - Balanced depot exchange
  - Maintenance constraints

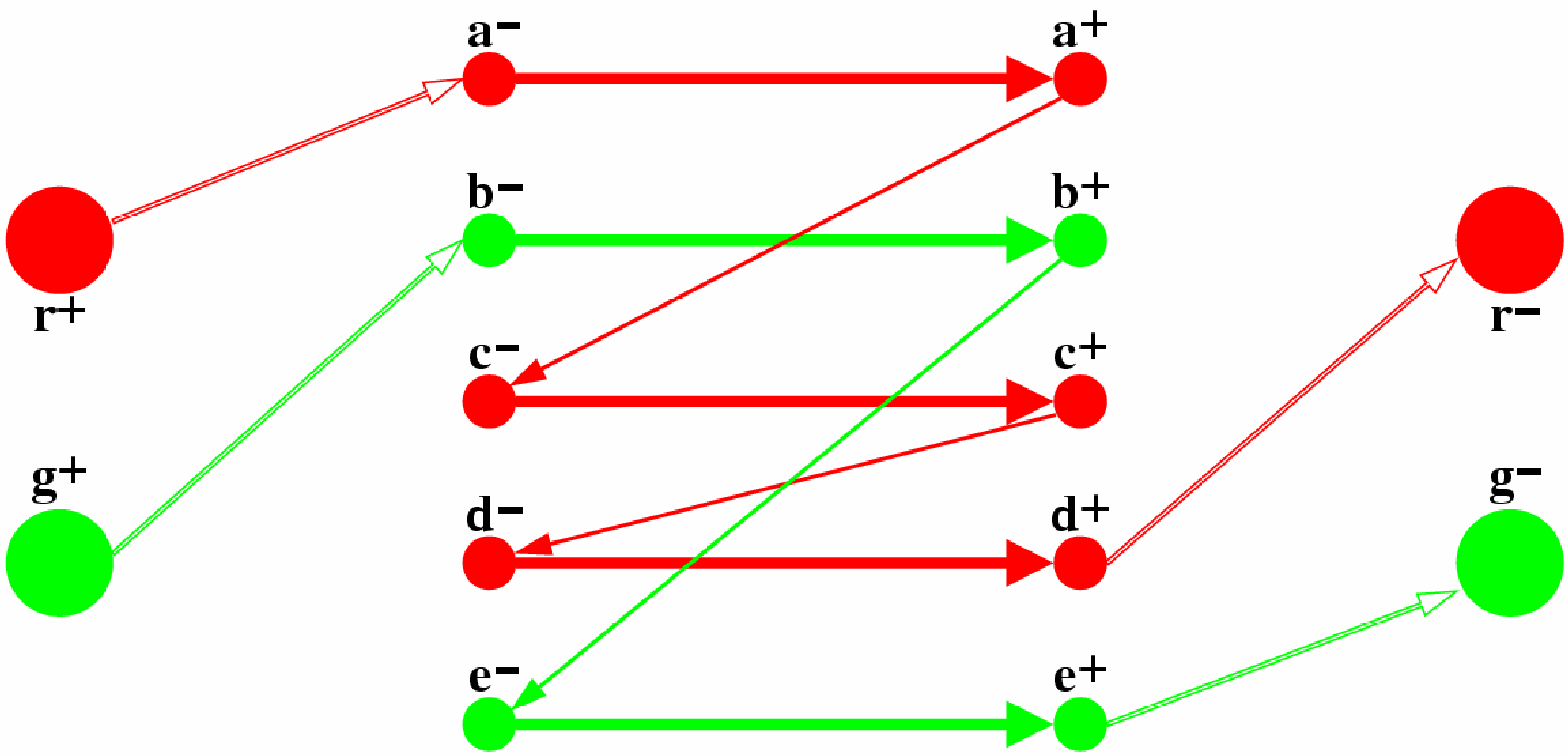
CO at Work

Trip Shifting



Diplomarbeit Cornelia Bönisch

Lagrangean Relaxation Algorithm



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Computational Results

	BVG	*HHA*	*VHH*
depots	10	14	10
vehicle types	44	40	19
timetabled trips	25,000	16,000	5,500
deadheads	70,000,000	15,100,000	10,000,000
cpu mins	200	50	28

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Standard Performance Evaluation Corporation

spec

MCF by Andreas Löbel

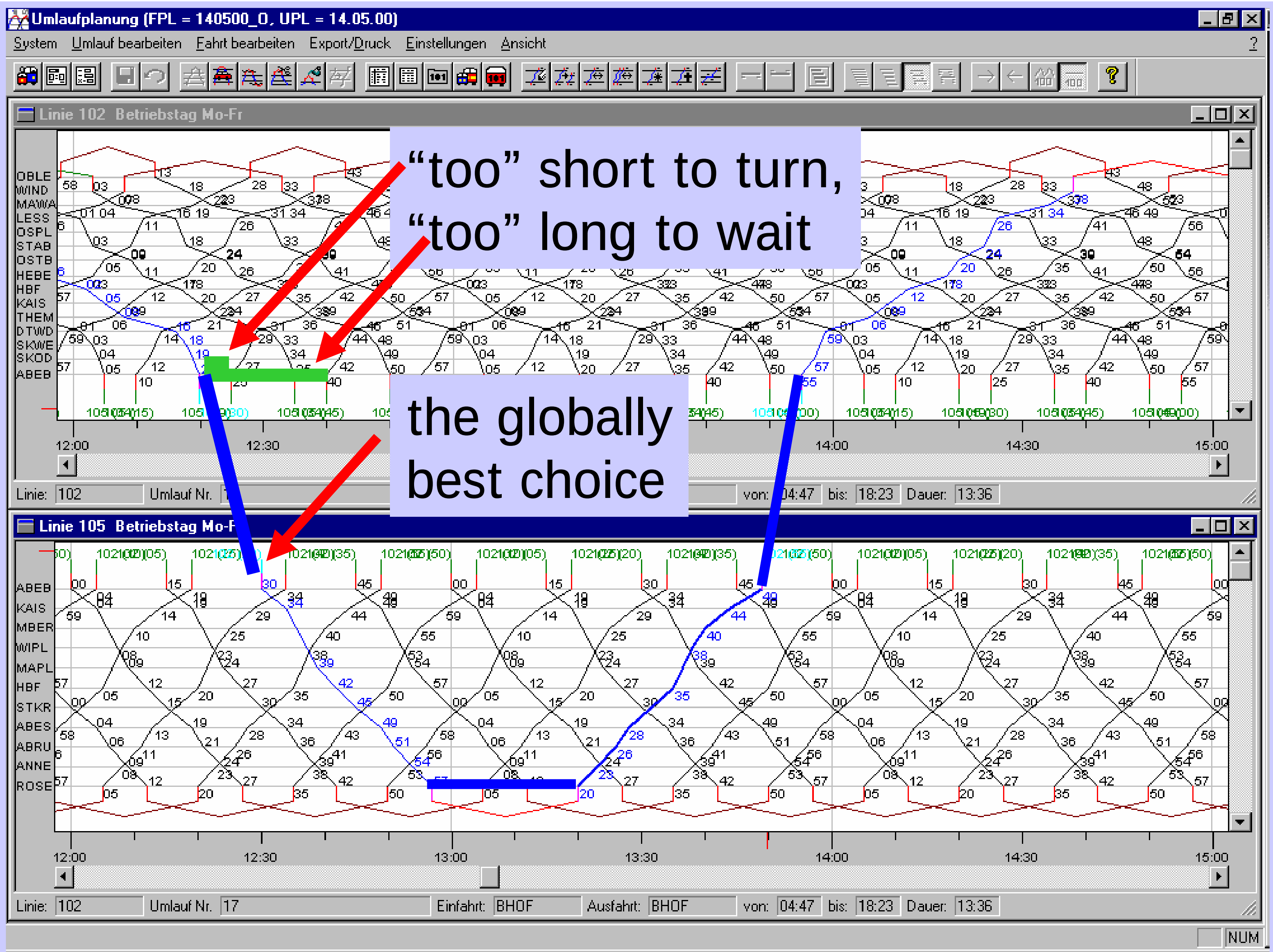
CINT2000 (Integer Component of SPEC CPU2000):

Benchmark	Language	Category	Full Descriptions	
164.gzip	C	Compression	HTML	Text
175.vpr	C	FPGA Circuit Placement and Routing	HTML	Text
176.gcc	C	C Programming Language Compiler	HTML	Text
181.mcf	C	Combinatorial Optimization	HTML	Text
186.crafty	C	Game Playing: Chess	HTML	Text
197.parser	C	Word Processing	HTML	Text
252.eon	C++	Computer Visualization	HTML	Text
253.perlbmk	C	PERL Programming Language	HTML	Text
254.gap	C	Group Theory, Interpreter	HTML	Text
255.vortex	C	Object-oriented Database	HTML	Text
256.bzip2	C	Compression	HTML	Text
300.twolf	C	Place and Route Simulator	HTML	Text

---

webmaster@spec.org
Last updated: *Fri Sep 26 11:10:06 EDT 2003*
Copyright © 1995 - 2004 Standard Performance Evaluation Corporation
URL: *http://www.spec.org/osg/cpu2000/CINT2000/index.html*

Space/Time Chart of a solution



CO at Work

vehicle utilization

Fahrzeugeinsatz

Umlaufversion

Umläufe gültig ab 14.05.2000

Betriebstag

Montag – Freitag

Betriebsbereich

Omnibusverkehr

Linien

V20	V20: Mühlheim -> O
101	101: Markwaldstr.
102	102: A.-Bebel-Ring
103	103: Ffm., Prüflin
104	104: Neusalzer Str
105	105: Rosenhöhe <->
106	106: Buchrainweihe
107	107: Dt. Wetterdie
119	119: OF/Marktplatz
120	120: Dt. Wetterd.
101N	101N: Markwaldstr.
102N	102N: Kaiserlei <-
103N	103N: Ffm., Prüfli
104N	104N: W.-Schramm-S
105N	105N: Rosenhöhe <-
106N	106N: Buchrainweih
120N	120N: Dt. Wetterd.

Zeitraum von 04:00 Uhr

bis 23:00 Uhr

Schrittweite 00:30 [hh:mm]

Auswertung

intervallgenauer Fahrzeugbedarf 68

minutengenauer Fahrzeugbedarf 68

von	bis	SG	SL	TAXI	SLM	SGM	S
4:00	4:30	5	0	0	1	2	
4:30	5:00	16	2	0	8	3	
5:00	5:30	22	4	0	8	3	
5:30	6:00	25	6	0	10	4	
6:00	6:30	26	9	0	10	4	
6:30	7:00	30	13	0	11	4	
7:00	7:30	33	17	0	11	4	
7:30	8:00	33	17	0	11	4	
8:00	8:30	30	11	0	11	4	
8:30	9:00	28	10	0	9	4	
9:00	9:30	27	8	0	9	4	
9:30	10:00	27	7	0	9	4	
10:00	10:30	27	7	0	9	4	
10:30	11:00	27	7	0	9	4	
11:00	11:30	27	7	0	9	4	
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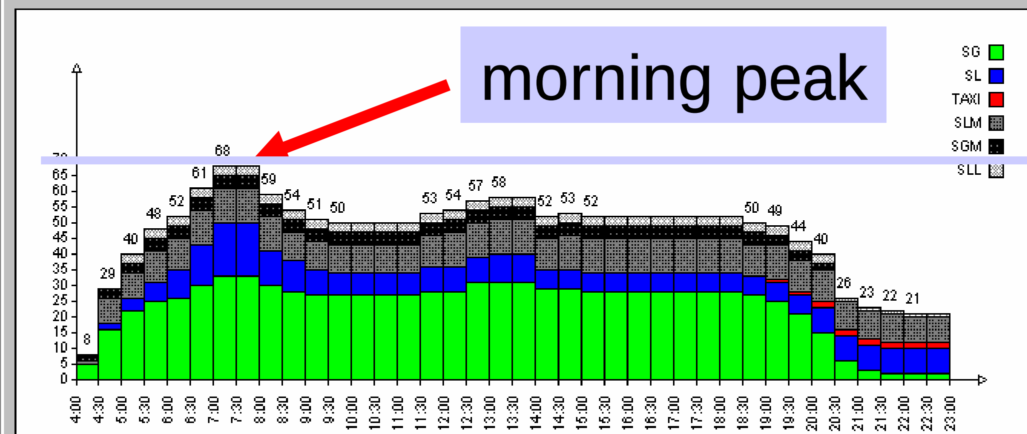
12:00	12:30	28	8	0	11	4	
12:30	13:00	31	8	0	11	4	



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Beenden

Drucken

Office

Microsoft

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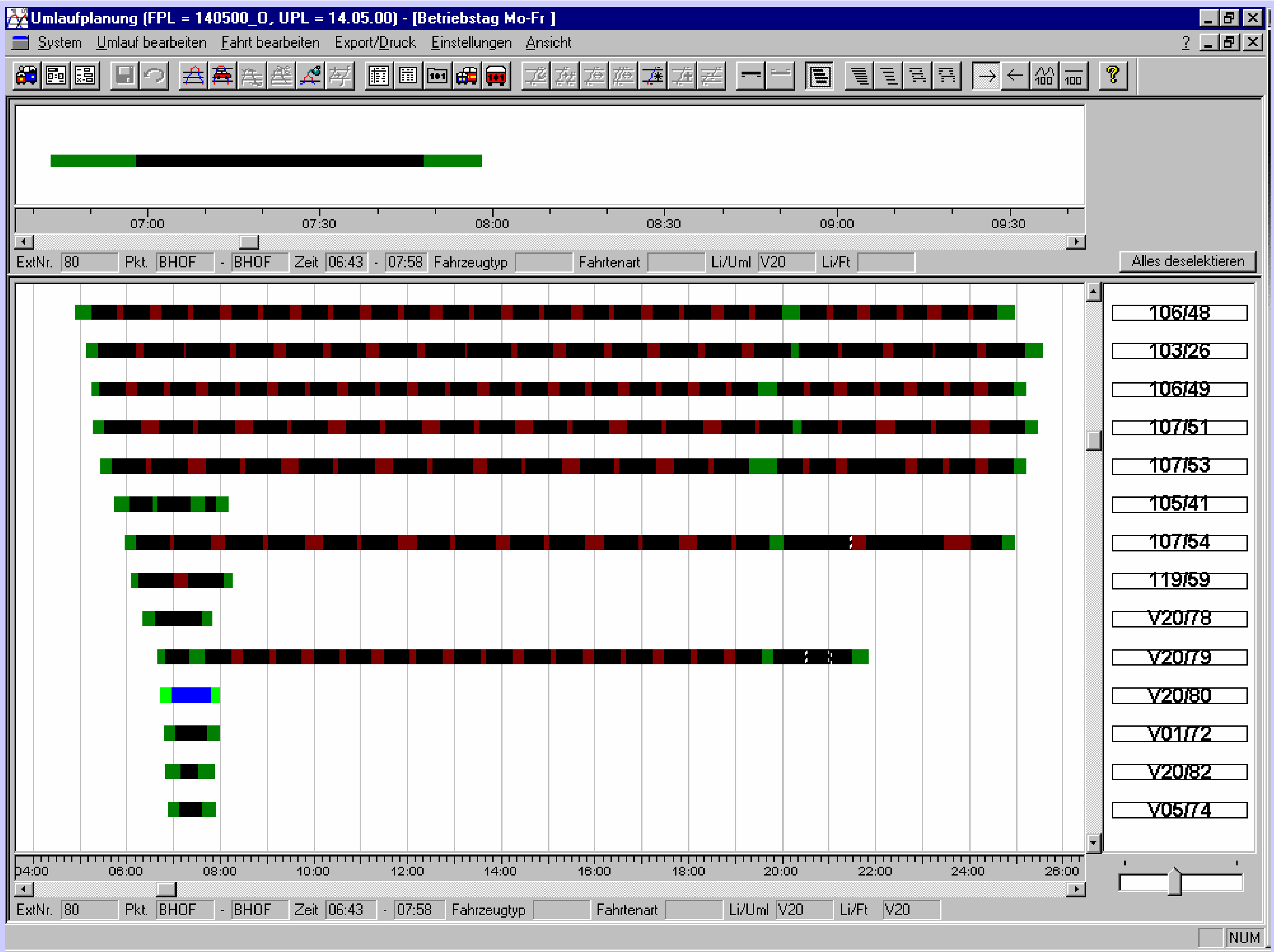
several vehicle schedules



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VS-OPT employed in Berlin (BVG)

- **Real Really Large-Scale Optimization**
  - **28.000 scheduled trips** (worldwide largest known instance)
  - **100 million degrees of freedom** (of 400 mio possible)
  - **optimization of a whole transportation company**
  - **no heuristic simplifications**
  - **„Lagrangean Pricing"-technique**
- **Mathematical quality guarantee**
  - **fleet minimal solution**
  - **at most 1% off minimal cost**
- **Added value**
  - **Scenario analysis**
  - **Sensitivity analysis**
  - **Stability, Fixing, Freezing, Outsourcing, etc.**
- **Running time**
  - **Minutes on standard PCs**

In other words, we have a multicommodity flow problem with 100 million integer variables and can solve it in a few minutes on a laptop

Martin Grötschel

VS-OPT

- Lagrangean Pricing
  - Dynamic Arc Generation
  - Lagrange Relaxation
    - Flow Conservation $\rightarrow$ Min Cost Flow Problem
    - Flow Constraints $\rightarrow$ Circulation Problem
  - Activate Arcs on Paths
- Schedule-Cluster-Reschedule Heuristic
  - Min Cost Flow Solution
  - Assign Paths to Depots
  - Eliminate Conflicts by Local Search

Auf Sparkurs zum Ziel

Systematisierter Einsatz

Die neuen Optimierungsmethoden, die die BVG jetzt nach und nach nutzen will, stammen vom Konrad-Zuse-Zentrum für Informationstechnik und garantieren nach Roß' Angaben Einsparungen von maximal 100 Millionen Mark im Jahr. „Sie sind nötig, um unser Angebot in dieser schweren Lage stabilisieren und dem Einsparungsdruck überhaupt standhalten zu können."

RHEINISCHER MERKUR

POLITISCH.KOMPETENT.ANDERS.

INFORMATIK / *Ein Lehrbeispiel, wie sich Mathematik und Wirtschaft ergänzen*

Auf Sparkurs zum Ziel

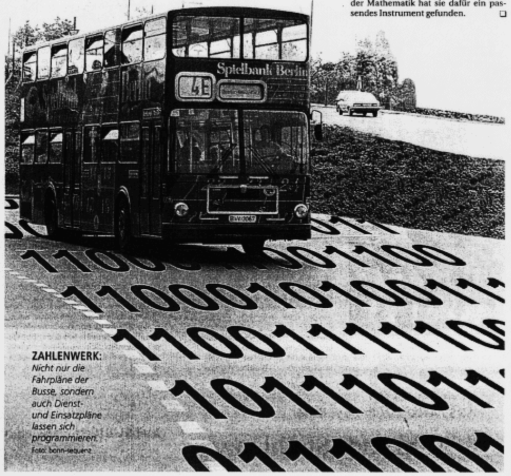
Das Berliner Busnetz kostet jährlich Millionen. Mit Hilfe moderner Software könnte man auf gewaltige Zuschüsse verzichten.

VASCO ALEXANDER SCHMIDT

Komfortable Hilfe

Pfiffige Programme

Systematisierter Einsatz

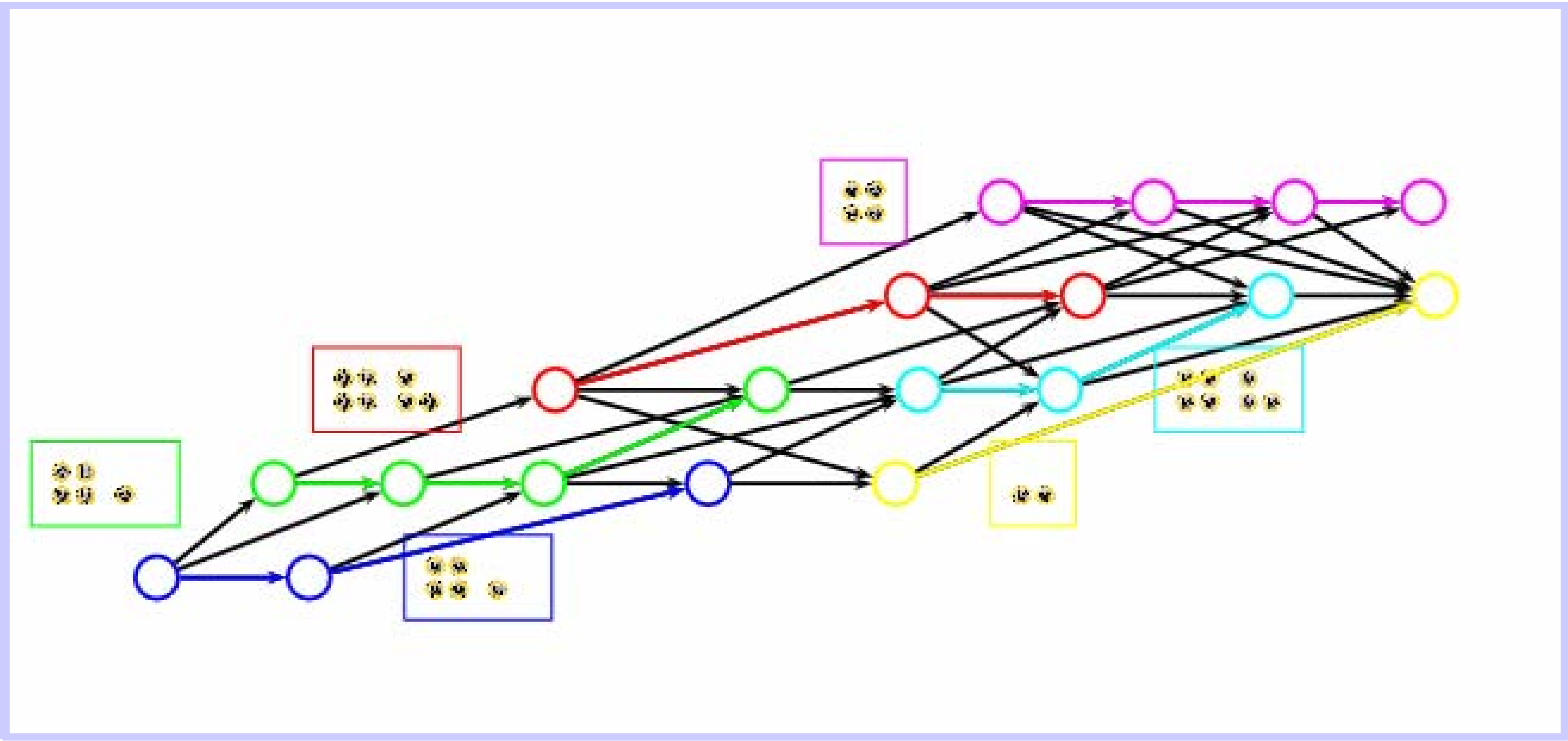


ZAHLENWERK: *Nicht nur die Fahrpläne der Busse, sondern auch Dienst- und Einsatzpläne lassen sich programmieren.*

Artikel erschienen am 26. Sep 1997

Contents

Mathematical duty scheduling DS-OPT



- Types of duties
  - Investigating whole paths
  - supplementary duty elements
- 2.000 duty elements:
  - 50.000 nodes
  - 280.000 links (edges)
  - ∞ duties

IP Model for Duty Scheduling

$$
\begin{array}{rcll}
\min \sum_{j\in J} c_j x_j + \sum_{b\in B} p_b^+ s_b^+ + \sum_{b\in B} p_b^- s_b^- & & & \\
\sum_{j\in J} a_{ij} x_j & = & 1 \quad \forall i \in I & \text{(Tasks)} \\
\sum_{b\in B} d_{bj} x_j + s_b^+ - s_b^- & = & d_b \quad \forall b \in B & \text{(Mix)} \\
x & \in & \{0,1\}^J & \text{(Integrality)}
\end{array}
$$

- I – Tasks
- J – Duties
- B – Mix (Base) Constraints

DS-OPT

- Lagrangean Shortest Path Pricing
  - Linear Resource Constraints
  - Resource Constrained Shortest Path Model
  - Lagrange Relaxation of Linear Constraints
    - Shortest Path Lower Bound
- LP Heuristic
  - Dual Ascent
  - Box Step
- Branch-and-Generate Heuristic
  - Fixing on Links

DS-OPT Computational Results

Name	KS 1	KS 2	KS 3	TR	MI (I)	M (S)	WI	BN
Areas	4	2	2	1	4	1	1	1
Duty Types	3	5	3	3	4	3	3	4
Capacities	5	5	3				1	
Averages				2				
Tasks	4,345	4,345	4,345	2,867	1,317	2,436	4,453	3,014
Nodes	142.261	116.569	91.365	50.979	160.882	42.724	66.602	71.331
Arcs	1,2 Mio	2 Mio	2 Mio	851.160	3 Mio	1,8 Mio	4 Mio	977.701
Duties	114	114	117	103	87	223	252	361
Time	1 d	1 d	1 d	0,5 d	4 h	1 d	0,5 d	0,5 d

Some Users

VS-OPT	DS-OPT
ATC/Terni (I)	**ATC/Terni (I)**
Athen (U) (GR)	
Berlin (D)	**Berlin (D)**
Bonn (D)	**Bonn (D)**
Connex (D)	**Connex (D)**
DB Regio (D)	**DB Regio (D)**
Geilenkirchen (D)	
	Ennepetal (D)
Genua (I)	**Genua (I)**
Mailand (U) (I)	**Mailand (U) (I)**
München (S) (D)	**München (S) (D)**
Norgesbus (N)	**Norgesbus (N)**
Rhein-Neckar (S) (D)	**Rhein-Neckar (S) (D)**
Wiesbaden (D)	**Wiesbaden (D)**

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Optimization Results

	Vehicle Scheduling			Duty Scheduling		
BVG	manual	VS-OPT		manual	DS-OPT	
Bus	195	157	19,5%	n.a.	=	0,0%
SWB	manual	VS-OPT		manual	DS-OPT	
Bus	201	196	2,5%	280	268	4,3%
Tram	n.a.	=	0,0%	120	117	2,5%
VER	manual	VS-OPT		manual	DS-OPT	
Bus	n.a.	-2	n.a.	155	141	9,1%
ESWE	manual	VS-OPT		manual	DS-OPT	
Bus	*		*	280	40	14,3%

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The Psychology of Improvement

- Company goals
- Manager goals
- Dispatcher goals

- The 15% rule

Contents

IP Model for Integrated Scheduling

$$\min \sum_{d\in D} \sum_{ij\in A^d} c_{ij}^d x_{ij}^d + \sum_{j\in J} c_j y_j$$

$$\begin{array}{rclrr}
\sum_{d\in D} \sum_{tj\in A^d} x_{tj}^d & = & 1 & \forall t\in T & \text{(Trips)} \\
\sum_{tj\in A^d} x_{tj}^d - \sum_{it\in A^d} x_{it}^d & = & 0 & \forall t\in T, d\in D & \text{(V-Flow)} \\
\sum_{dt\in A^d} x_{dt}^d & \leq & \kappa_d & \forall d\in D & \text{(V-Cap)} \\
\sum_{j\in J} a_{ij} y_j & = & 1 & \forall i\in t\in T & \text{(Tasks)} \\
\sum_{d\in D} \sum_{tj\in A^d} x_{tj}^d - \sum_{j\in J} a_{ij} y_j & = & 0 & \forall i\in t\in H & \text{(Coupling)} \\
x_{it}^d, y_j & \in & \{0,1\} & & \text{(Integrality)}
\end{array}$$

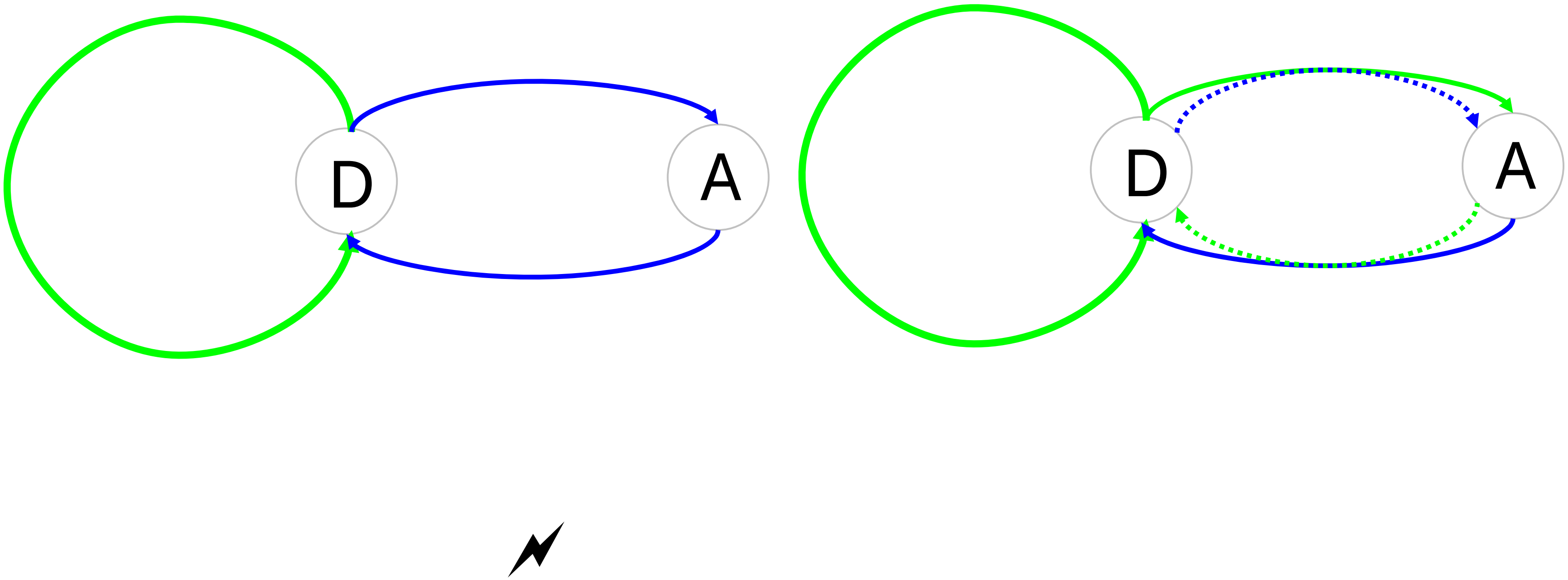
- T – Timetabled Trips
- H – Deadhead Trips

D – Depots
I – Tasks

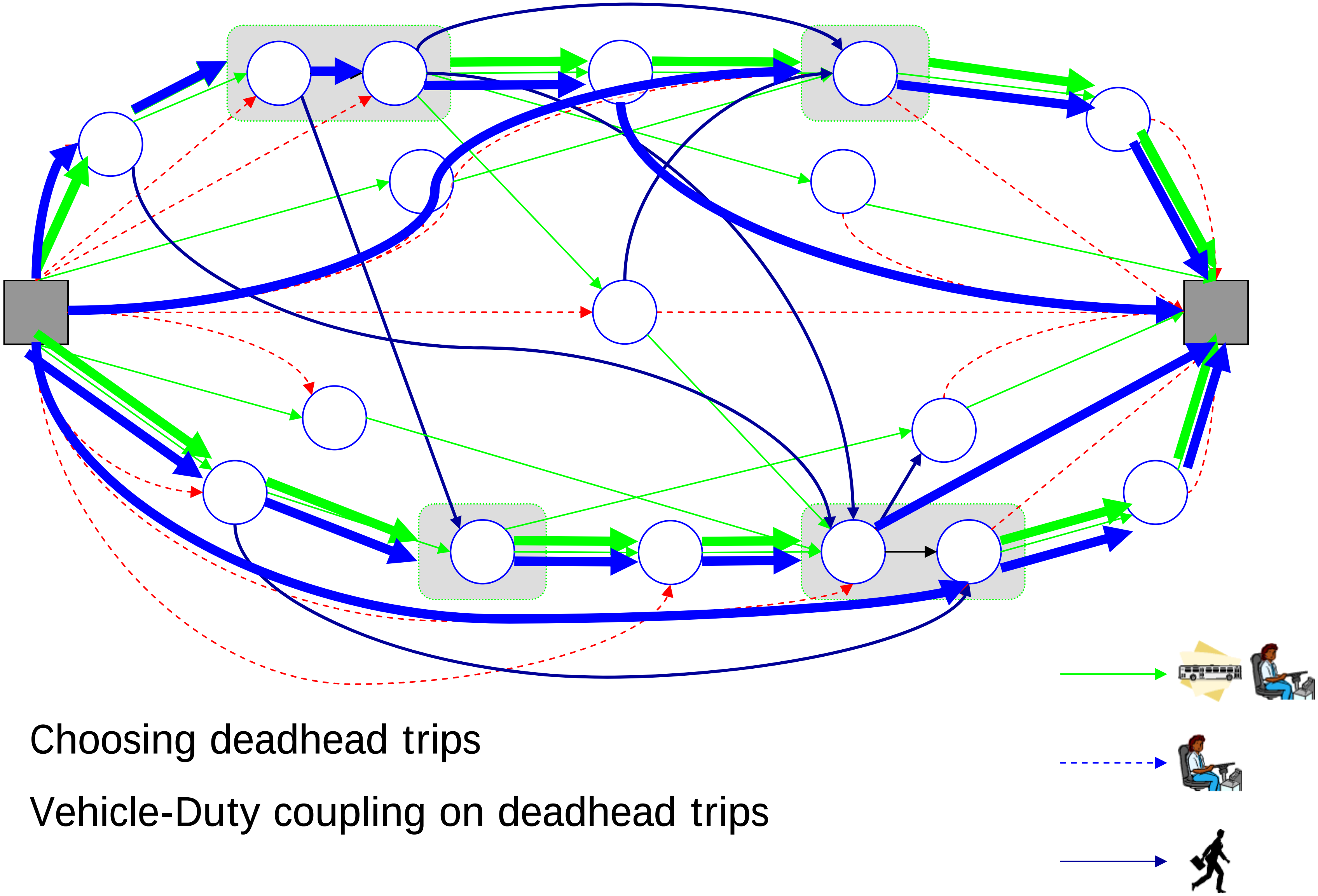
IS-OPT

- Lagrangean Relaxation on Coupling Constraints
  - Proximal Bundle Method
  - Approximate Solution of Vehicle and Duty Scheduling Components
- Branch-and-Generate Heuristic
  - Fixing Deadhead Trips

Regional Public Transport: Relief Problems



Integrated Vehicle and Duty Scheduling



- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips

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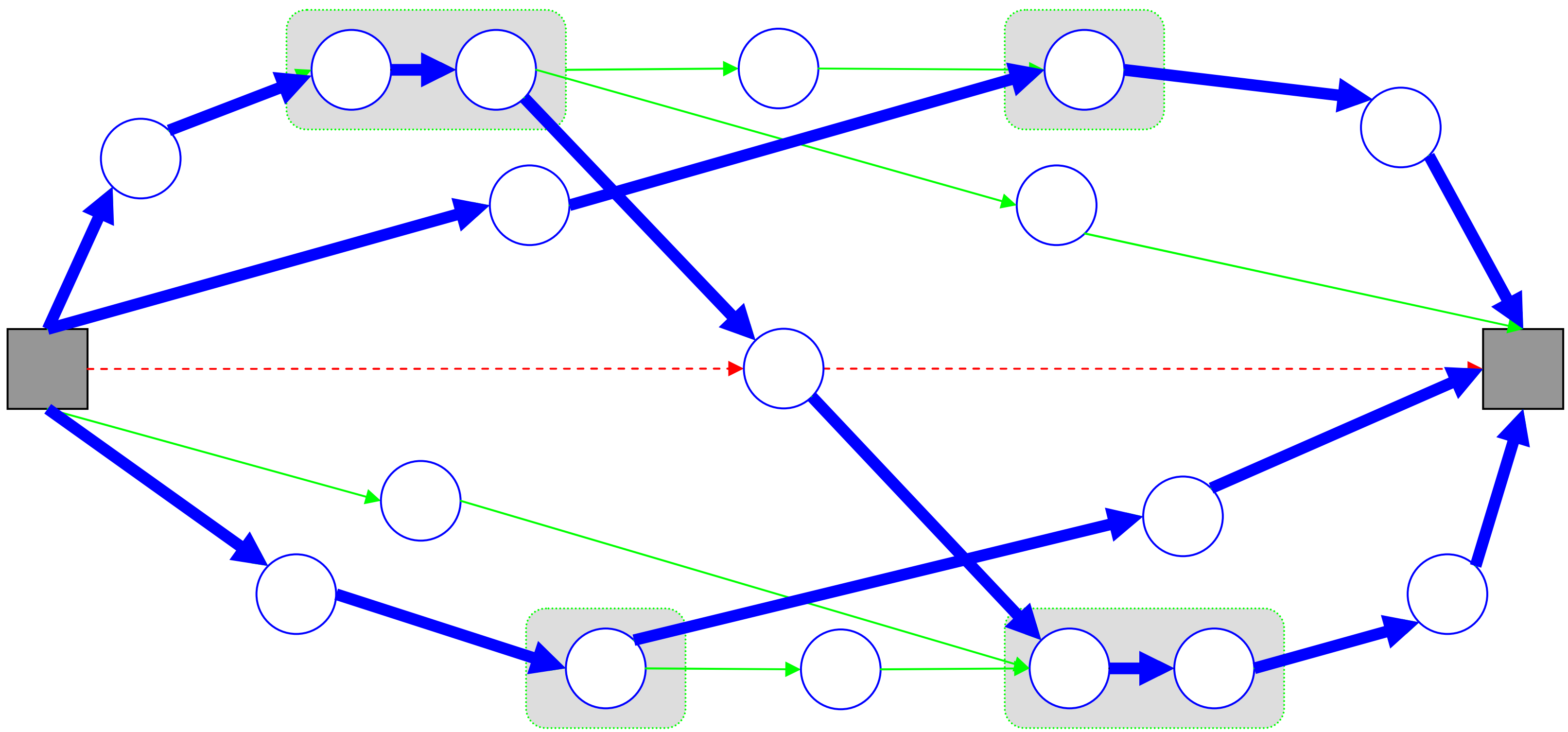
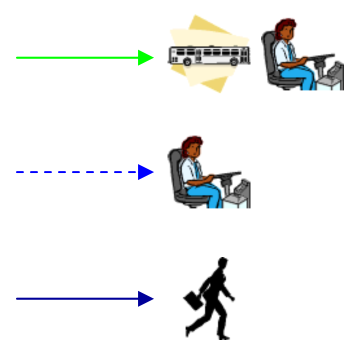
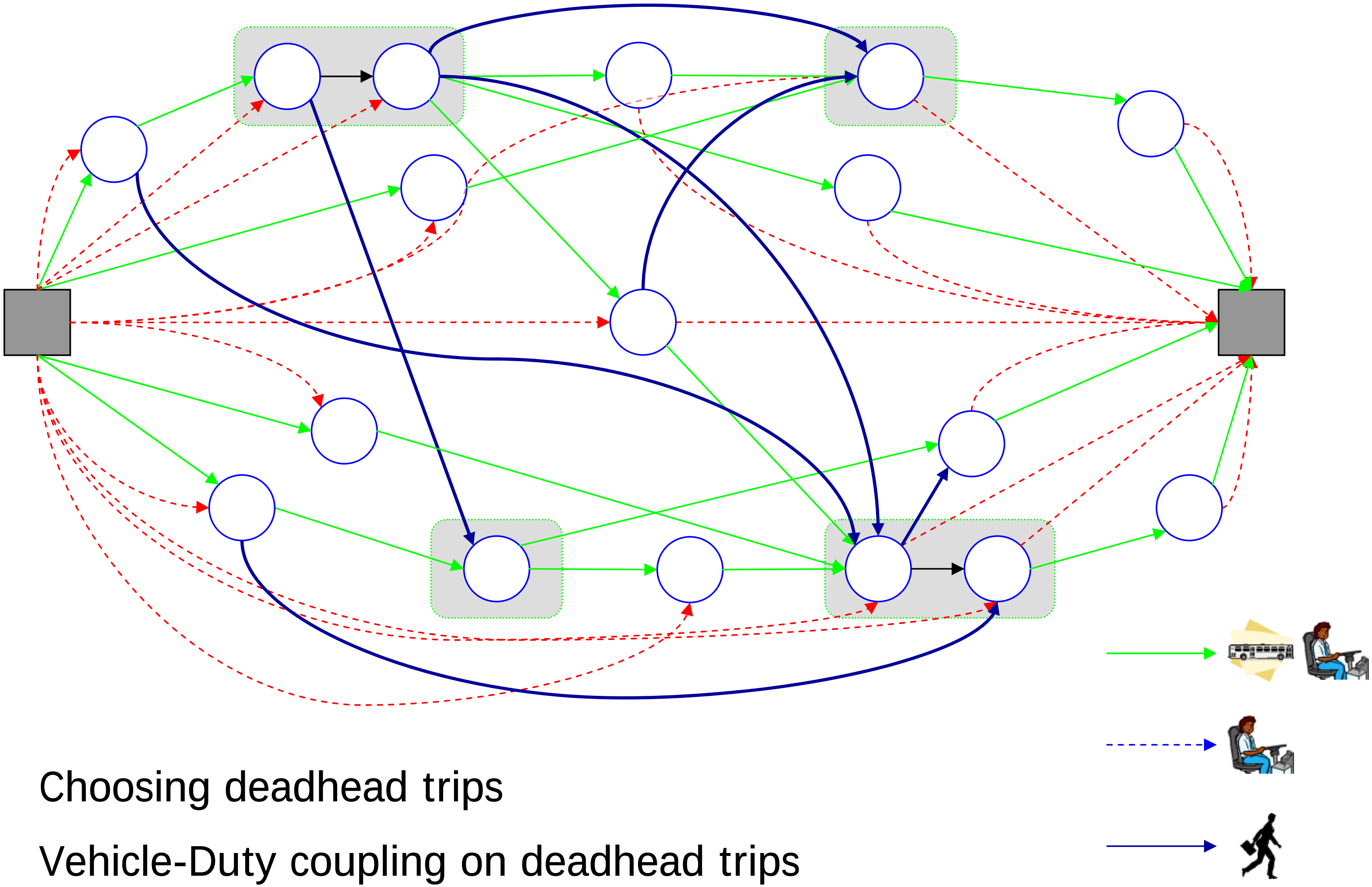
RS-OPT



- Combined Scheduling
  - Rotation = Duty (relief only in the depot)
  - Similar to duty scheduling



Integrated Vehicle and Duty Scheduling



- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips

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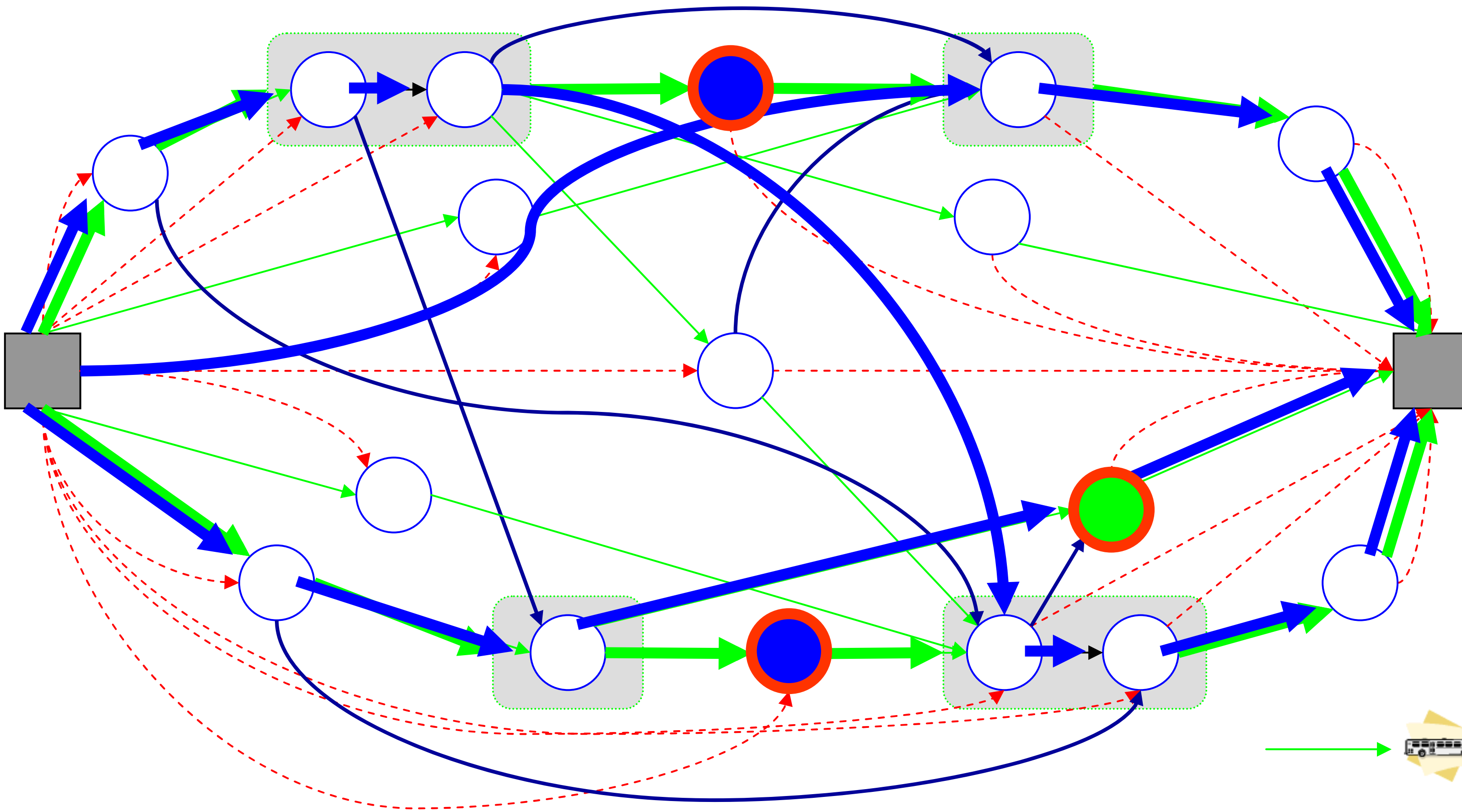
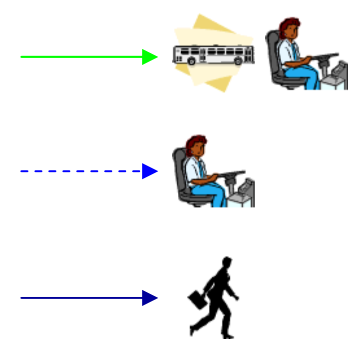
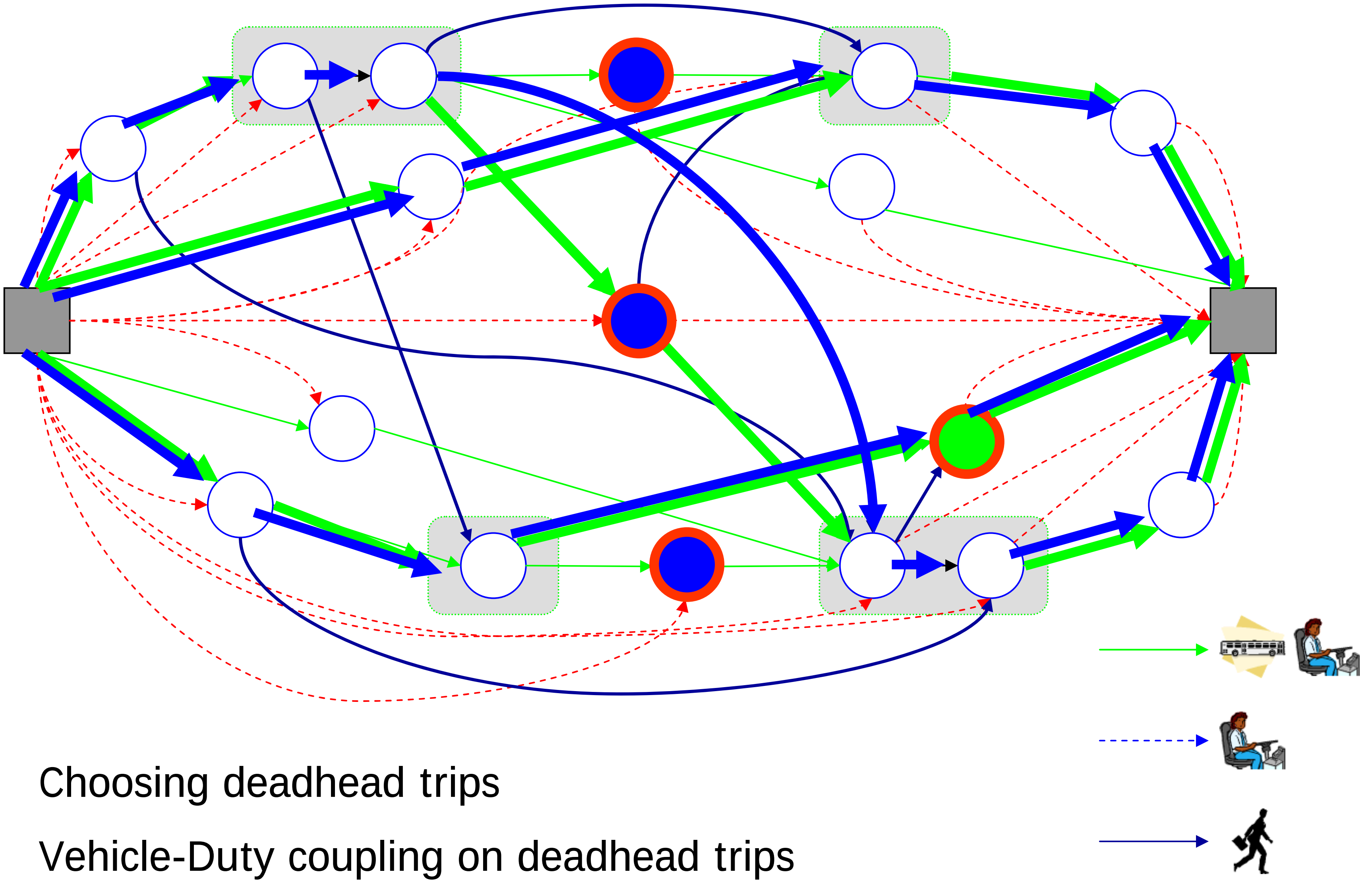
Integrated Vehicle and Duty Scheduling





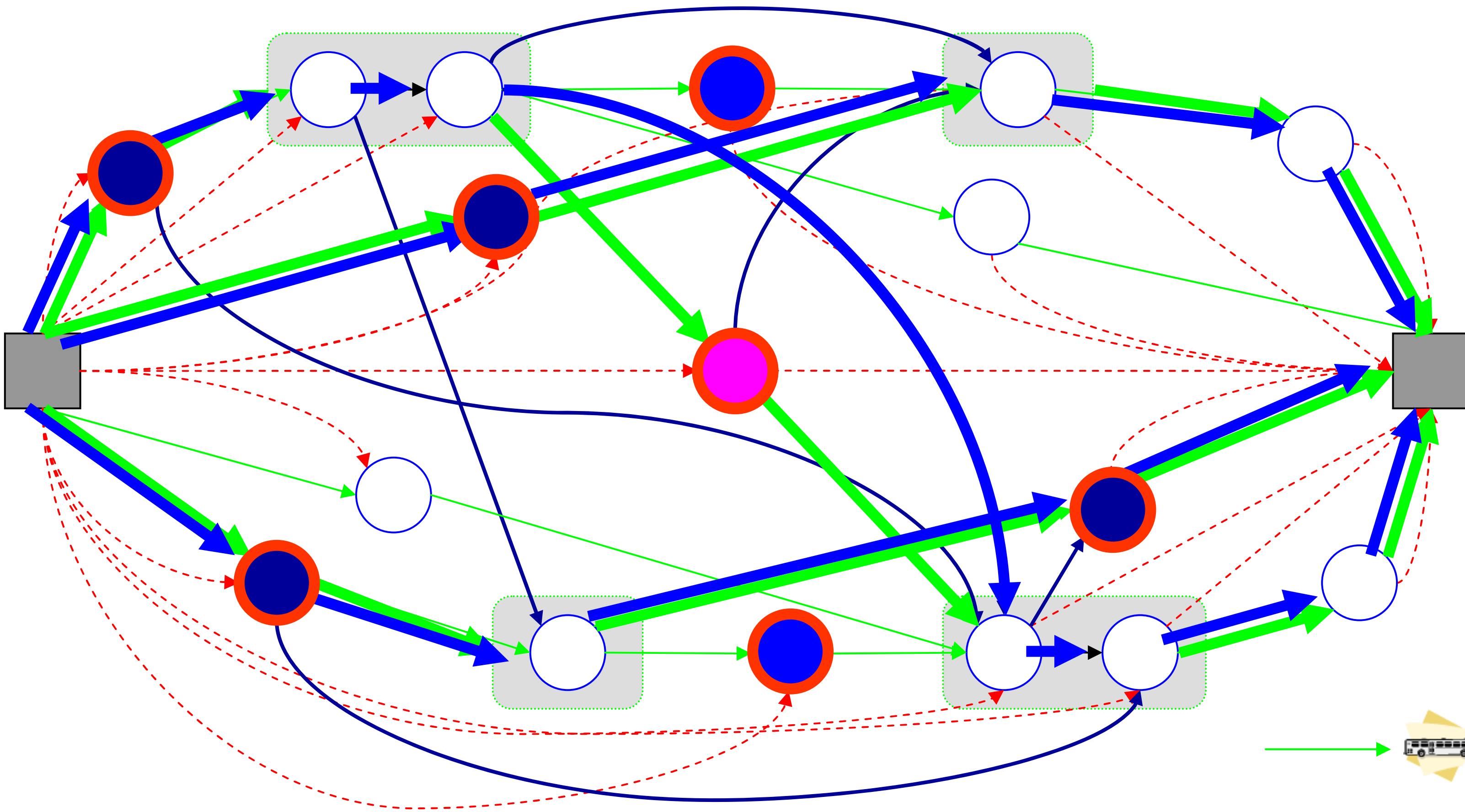
- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips

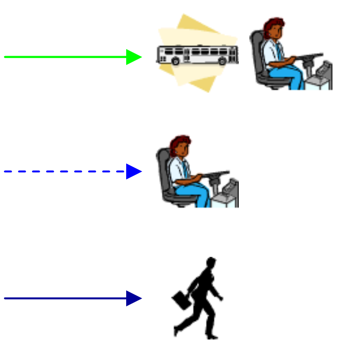
Integrated Vehicle and Duty Scheduling



- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips

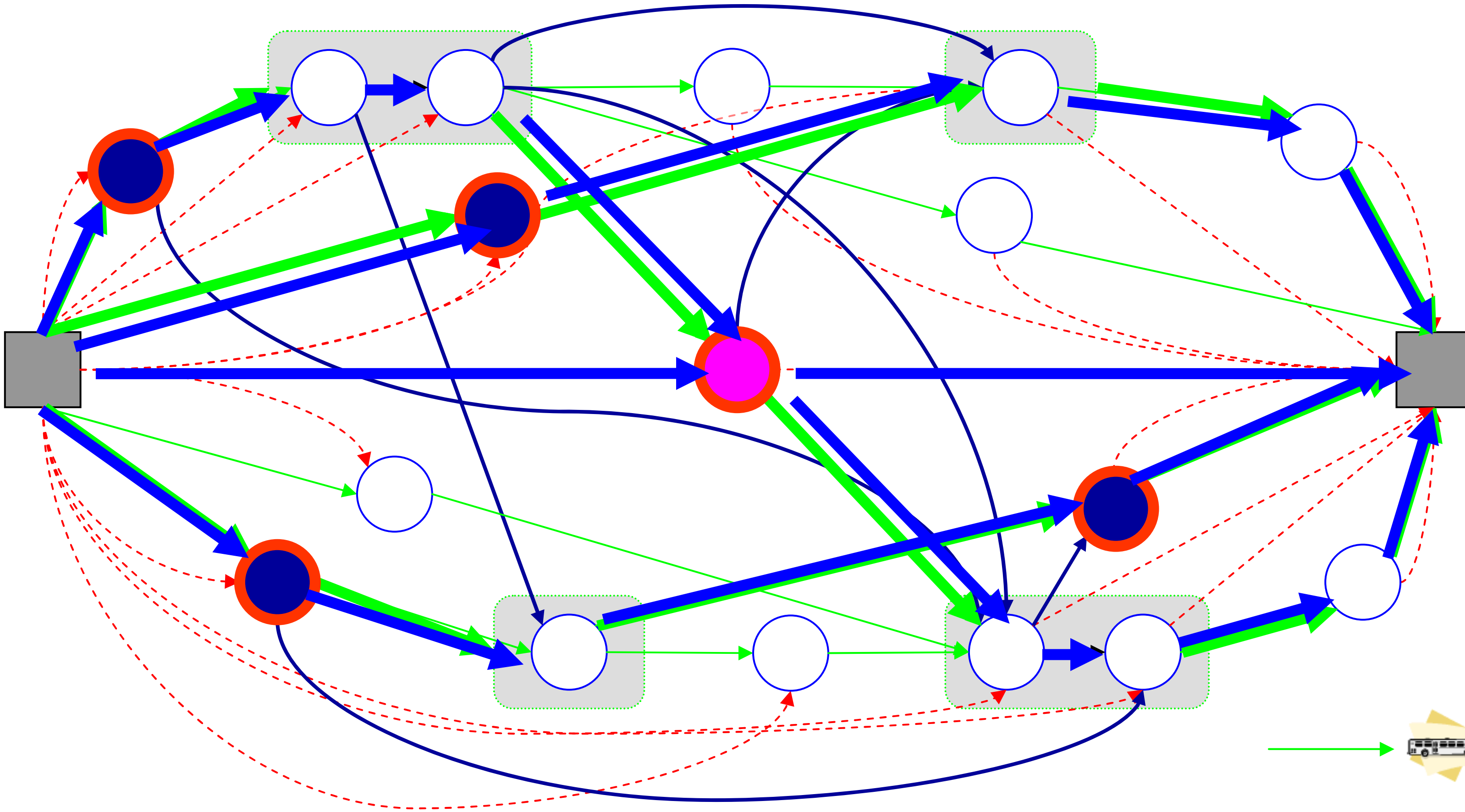
Integrated Vehicle and Duty Scheduling

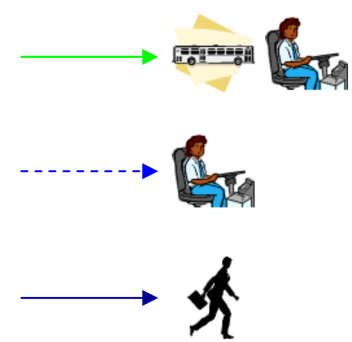




- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips

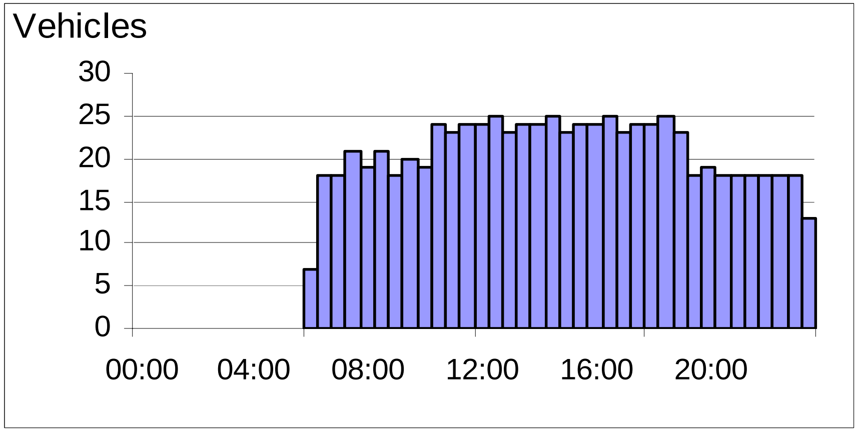
Integrated Vehicle and Duty Scheduling

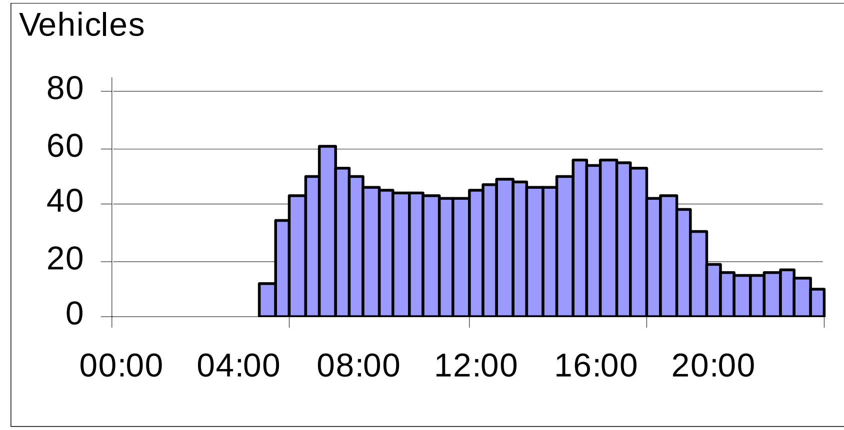




- Choosing deadhead trips
- Vehicle-Duty coupling on deadhead trips

IS-OPT Scenarios (Regensburg)





Day of Week	Su	Mo-Fr
Vehicle Types	**1**	**3**
Timetabled Trips	**794**	**1,414**
Deadhead Trips	**47,523**	**57,646**
Duty Elements on Timetabled Trips	**1,980**	**3,666**
Duty Elements on Deadhead Trips	**47,523**	**57,646**
Extension Elements	**33,224**	**82,308**
Connections	**233,842**	**457,174**
Duty Types	**Early, Middle, Late**	**Early, Middle, Late, Spilt**

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IS-OPT Computational Results: Regensburg on Sunday

Instance	Reference	Variant 1	Variant 2
Vehicle Rotations	350:06 h	335:51 h	346:45 h
Paid Time	545:25 h	514:30 h	526:59 h
Breaks	113:34 h	55:11 h	75:06 h
Unpaid Breaks	1:31 h	0:00 h	0:00 h
Duties	82	78	65
Pieces of Work	87	78	65
Vehicle Rotations	36	33	33
Ø Duty Duration	6:39 h	6:36 h	8:06 h
Comp. Time	—	9:30 h	8:35 h

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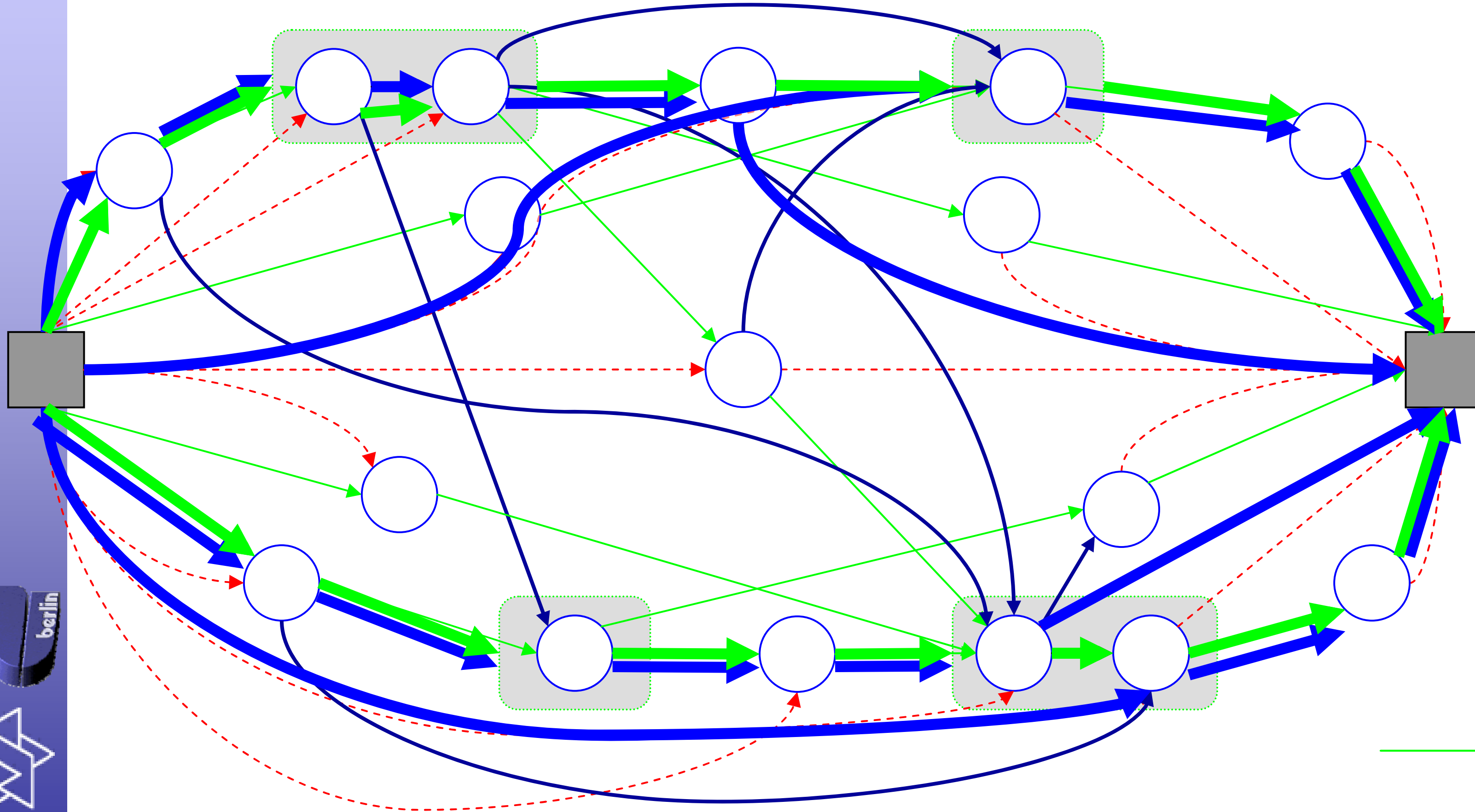
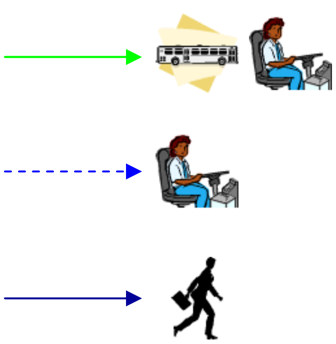
IS-OPT Computational Results: Regensburg Mo-Fr

Instance	Reference	Variant 1	Variant 2
Vehicle Rotations	679:49 h	631:52 h	612:40 h
Paid Time	1103:48 h	1028:10 h	1004:09 h
Breaks	211:53 h	127:06 h	121:21 h
Unpaid Breaks	19:40 h	2:23 h	14:53 h
Duties	139 (1)	136	131
Pieces of Work	217	214	235
Vehicle Rotations	91	80	81
Ø Duty Duration	7:56 h	7:34 h	7:46 h
Comp. Time	—	50:57 h	46:45 h

Integrated Planing (VS-OPT + DS-OPT)



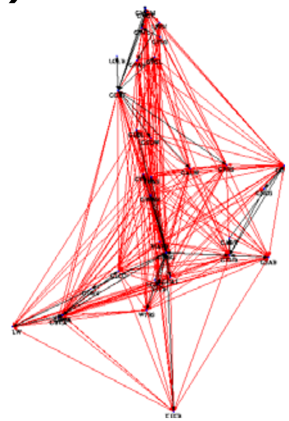


- choice of „unloaded trips“
- coupling of vehicles and duties via unloaded trips

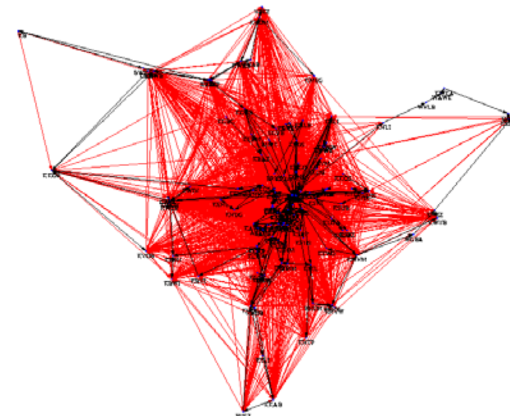
DB Regio

- Das Unternehmen (Busbereich)
  - 12.000 Busse
  - 7.400 Mitarbeiter
  - 17 Regionalgesellschaften

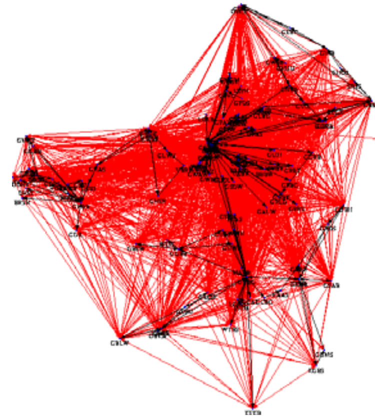
  Quelle: Jahresbericht der DB Regio AG
- Kombinierte Optimierung
  - Studien
- Integrierte Optimierung
  - Forschungsprojekt



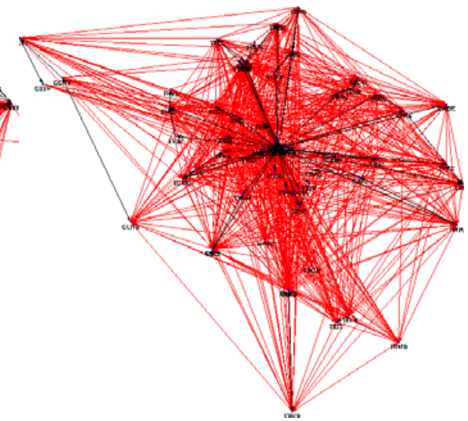
Marb2



Marb4



Marb3



Fulda

Integrated & combined Planning

	Marb1	Marb2	Marb3	Marb4	Fulda
Dep./Fzg.-Typen	1/1	1/2	3/5	1/6	1/1
Fahrten	49	103	1022	705	322
Leerfahrten	733	2.565	142.668	67.287	28.793
IS-OPT	79s 3 Fzg. 4 Dienste	118s 10 Fzg. 10 Dienste	70.984s 51 Fzg. 73 Dienste	16.271s 42 Fzg. 40 Dienste	4.460s 19 Fzg. 19 Dienste
RS-OPT	15s 3 Fzg. 4 Dienste	39s 10 Fzg. 10 Dienste	16.271s 67 Fzg. 74 Dienste	10.548s 44 Fzg. 46 Dienste	2.219s 19 Fzg. 19 Dienste

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Contents

Trassenbörse = track auctioning

European Union:

- Establish a rail traffic market
- open the market to competition
- Deregulate/Regulate this market

History of our project

Martin Grötschel

Contents

Martin Grötschel

Summary

- Possible savings in public transport

- Can public transport break even?
- Where are the bottlenecks?
- What can mathematics do?

08M1 Lecture
Combinatorial Optimization and Transportation, Telebus

The End

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