06M1 Lecture
Frequency Assignment for GSM Mobile Phone Systems

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"Combinatorial Optimization at Work"
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1. Introduction
2. The Telecom Problem & Mobile Communication
3. GSM Frequency/Channel Assignment
4. The UMTS Radio Interface (next talk)
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E-Plus and the Channel Assignment Problem

- How did we get this project?
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Connecting Mobiles
Wireless Communication

Mobile Telecommunication
Generations of Mobile Telecommunications Systems

1G (1980s)
- Analogue
- Voice Only

2G (1990s)
- Digital
- Voice & Data
- GSM mass market
- PCS
- cdmaOne/IS95

3G (2000s)
- UMTS, WiFi/WLAN, cdma2000
- Data Rates ≥ 384 kbit/s
- Various Services

4G (2010 ?)
- more services
- more bandwidth
- fresh spectrum
- new technology
- W-CDMA radio transmissions
Radio Interface: OR & Optimization Challenges

- Location of sites/base stations
  - was investigated in the OR literature („dead subject“)
  - has become „hot“ again
    - UMTS: massive investments around the world
    - GSM: still significant roll-outs
    - special issue: mergers

- Antenna configurations at base stations
  - GSM: coverage based planning
  - UMTS: coverage & capacity considerations

- Radio resource allocation
  - GSM: frequency assignment
  - UMTS: ? (open: real time/online resource management)
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Wireless Communication

GSM: More than 1,000 million users in over 150 countries
Wireless Communication

There are five frequency bands used by GSM mobile phones:

GSM-900, GSM-1800, GSM-850, GSM-1900, GSM-400

GSM-900 and GSM-1800 are used in most of the world.

GSM-900 uses 890 - 915 MHz to send information from the Mobile Station to the Base Transceiver Station (BTS) (This is the “uplink”) and 935 - 960 MHz for the other direction (downlink), providing 124 RF channels spaced at 200 kHz. Duplex spacing of 45 MHz is used. GSM-1800 uses 1710 - 1785 MHz for the uplink and 1805 - 1880 downlink, providing 299 channels. Duplex spacing is 95 MHz.

GSM-850 and GSM-1900 are used in the United States, Canada, and many other countries in the Americas.
Antennas
Initial Idea

- Use graph colouring to assign channels!
Coloring Graphs

Given a graph $G = (V, E)$, color the nodes of the graph such that no two adjacent nodes have the same color.

The smallest number of colors with this property is called chromatic or coloring number and is denoted by $\chi(G)$. 
A typical theoretical question: Given a class $C$ of graphs (e.g., planar or perfect graphs, graphs without certain minors), what can one prove about the chromatic number of all graphs in $C$?

A typical practical question: Given a particular graph $G$ (e.g., arising in some application), how can one determine (or approximate) the chromatic number of $G$?
Coloring Graphs

- Coloring graphs algorithmically
  - NP-hard in theory
  - very hard in practice
  - almost impossible to find optimal colorings
    (symmetry issue)
  - playground for heuristics (e.g., DIMACS challenge)
Coloring in Telecommunication

- Frequency or Channel Assignment for radio-, tv-transmission, etc.
- Our Example: GSM mobile phone systems

Properties of wireless communication

Transmitter
emits electromagnetic oscillations at a frequency

Receiver
detects oscillations

Quality of the received signal:
Signal-to-noise ratio

Poor signal-to-noise ratio:
interference of the signal

Objective: Frequency plan without interference or, second best, with minimum interference
Antennas & Interference

- Cell
- Antenna
- Site
- Backbone network
- Co- & adjacent channel interference
# Cell Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Features</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexagon Cell Model</td>
<td>- sites on regular grid&lt;br&gt;- isotropic propagation conditions&lt;br&gt;- no cell-overlapping</td>
<td>E-Plus Mobilfunk, Germany</td>
</tr>
<tr>
<td>Best Server Model</td>
<td>- realistic propagation conditions&lt;br&gt;- arbitrary cell shapes&lt;br&gt;- no cell-overlapping</td>
<td></td>
</tr>
<tr>
<td>Cell Assignment Probability Model</td>
<td>- realistic propagation conditions&lt;br&gt;- arbitrary cell shapes&lt;br&gt;- cell-overlapping</td>
<td></td>
</tr>
</tbody>
</table>

Source: E-Plus Mobilfunk, Germany
Interference

Level of interference depends on

- distance between transmitters,
- geographical position,
- power of the signals,
- direction in which signals are transmitted,
- weather conditions

- assigned frequencies
  - co-channel interference
  - adjacent-channel interference
Separation/ Blocked Channels

Separation:
Frequencies assigned to the same location (site) have to be separated.

Blocked Channels:
Restricted spectrum at some locations:
- government regulations,
- agreements with operators in neighboring regions,
- requirements of military forces,
- etc.
**Frequency Planning Problem**

Find an assignment of frequencies/channels to transmitters that satisfies

- all separation constraints
- all blocked channels requirements

and either

- avoids interference at all

or

- minimizes the (total/maximum) interference level
Modeling: the interference graph

- **Vertices** represent transmitters (TRXs)
- **Edges** represent separation constraints and co/adjacent-channel interference
  - Separation distance: $d(vw)$
  - Co-channel interference level: $c^{co}(vw)$
  - Adjacent-channel interference level: $c^{ad}(vw)$
There is no way to model interference as some number associated with an edge in some graph.

Modelling is much more complicated, see UMTS talk.
Graph Coloring

Simplifications:

- drop adjacent-channel interference
- drop local blockings
- reduce all separation requirements to 1
- change large co-channel interference into separation distance 1 (inacceptable interference)

Result:

- FAP reduces to coloring the vertices of a graph
- Example
Graph Coloring & Frequency Planning

Unlimited Spectrum

- Vertex Coloring
- T-Coloring
- List T-Coloring
- Minimum Span Frequency Assignment (MS-FAP)

Predefined Spectrum

- k-Colorability
- Min k-Partition
- Set Packing
- Minimum Interference Frequency Assignment (MI-FAP)
- Minimum Blocking Frequency Assignment (MB-FAP)
FAP & Vertex Coloring

• Only co-channel interference
• Separation distance 1
• Minimization of
  – Number of frequencies used (chromatic number)
  – Span of frequencies used
• Objectives are equivalent: span = #colors-1
• FAP is NP-hard
FAP & T-Coloring

Sets of forbidden distances $T_{vw}$

$$|f_v - f_w| \notin T_{vw} \quad T_{vw} = \{0, \ldots, d(vw)-1\}$$

Minimization of number of colors and span are not equivalent!
FAP & List- $\tau$-Coloring

Locally blocked channels:
Sets of forbidden colors $B_v$

No solution with span 3!

Colors: 3
Span: 4
Minimum Span Frequency Assignment

- List-T-Coloring (+ multiplicity)
- Benchmarks: Philadelphia instances

Channel requirements (P1)
Optimal span = 426
Fixed Spectrum

- Is the graph span-$k$-colorable?
- Complete assignment: minimize interference
- Partial assignment without interference

License for frequencies \(\{1,\ldots,4\}\)

No solution with span 3
Hard & Soft constraints

• How to evaluate “infeasible” plans?
  – Hard constraints: separation, local blockings
  – Soft constraints: co- and adjacent-channel interference

• Measure of violation of soft constraints: penalty functions

\[ p_{vw}(f, g) = \begin{cases} 
  c^{co}(vw) & \text{if } f = g \\
  c^{ad}(vw) & \text{if } |f - g| = 1 \\
  0 & \text{otherwise}
\end{cases} \]
Evaluation of infeasible plans

- Minimizing total interference
- Minimizing maximum interference
  - Use of threshold value, binary search

Total penalty: $2 - 2\varepsilon$
Maximum penalty: $1 - \varepsilon$

Total penalty: $1 + \varepsilon$
Maximum penalty: $1 + \varepsilon$
What is a good objective?

Keep interference information!
Use the available spectrum!

Minimize max interference

T-coloring (min span): Hale; Gamst; ...

Minimize sum over interference

Duque-Anton et al.; Plehn; Smith et al.; ...

Minimize max “antenna” interference

Fischetti et al.; Mannino, Sassano
Our Model

Carrier Network:

\[ N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad}) \]

- \((V, E)\) is an undirected graph
- \(C\) is an interval of integers (spectrum)
- \(B_v \subseteq C\) for all \(v \in V\) (blocked channels)
- \(d : E \rightarrow \mathbb{Z}_+\) (separation)
- \(c^{co}, c^{ad} : E \rightarrow [0,1]\) (interference)
Minimum Interference Frequency Assignment

**Integer Linear Program:**

\[
\begin{align*}
\min & \quad \sum_{vw \in E^{co}} c_{vw}^{co} z_{vw}^{co} + \sum_{vw \in E^{ad}} c_{vw}^{ad} z_{vw}^{ad} \\
\text{s.t.} & \quad \sum_{f \in F_v} x_{vf} = 1 & & \forall v \in V \\
& \quad x_{vf} + x_{wg} \leq 1 & & \forall vw \in E^{d}, |f - g| < d(vw) \\
& \quad x_{vf} + x_{wf} \leq 1 + z_{vw}^{co} & & \forall vw \in E^{co}, f \in F_v \cap F_w \\
& \quad x_{vf} + x_{wg} \leq 1 + z_{vw}^{ad} & & \forall vw \in E^{ad}, |f - g| = 1 \\
& \quad x_{vf}, z_{vw}^{co}, z_{vw}^{ad} \in \{0,1\} & & \forall v \in V, f \in C \setminus B_v, \forall vw \in E^{co}, \forall vw \in E^{ad}
\end{align*}
\]
### A Glance at some Instances

| Instance | $|V|$ | density [%] | minimum degree | average degree | maximum degree | diameter | clique number |
|----------|-----|-------------|----------------|----------------|----------------|---------|--------------|
| k        | 267 | 56,8        | 2              | 151,0          | 238            | 3       | 69           |
| B-0-E-20 | 1876| 13,7        | 40             | 257,7          | 779            | 5       | 81           |
| f        | 2786| 4,5         | 3              | 135,0          | 453            | 12      | 69           |
| h        | 4240| 5,9         | 11             | 249,0          | 561            | 10      | 130          |

Expected graph properties: planarity,…
Computational Complexity

Neither high quality nor feasibility are generally achievable within practical running times:

- Testing for feasibility is NP-complete.
- There exists an $\varepsilon > 0$ such that FAP cannot be "approximated" within a factor of $|V|^{\varepsilon}$ unless $P = NP$. 
Heuristic Solution Methods

- Greedy coloring algorithms,
- DSATUR,
- Improvement heuristics,
- Threshold Accepting,
- Simulated Annealing,
- Tabu Search,
- Variable Depth Search,
- Genetic Algorithms,
- Neural networks,
- etc.
Heuristics

- T-coloring
- Dual Greedy
- DSATUR with Costs
- Iterated 1-Opt
- Simulated Annealing
- Tabu-Search
- Variable Depth Search
- MCF
- B&C-based

- construction heuristics
  - o
  - --
  - ++
  - o
  - +

- (randomized) local search
  - +

- other improvement heuristics
  - -
Region with “Optimized Plan”

Instance k, a “toy case” from practice

264 cells
267 TRXs
50 channels

57% density
151 avg. deg.
238 max. deg.
69 clique size

DC5-VDS: Reduction 96.3%
co-channel C/I worst Interferer

Mobile Systems International Plc.

20 km

Commercial software

DC5-IM
Region Berlin - Dresden

2877 carriers

50 channels

Interference reduction: 83.6%
Region Karlsruhe

2877 Carriers

75 channels

Interference Reduction: 83.9%
Guaranteed Quality

Optimal solutions are out of reach!

**Enumeration:** $50^{267} \approx 10^{197}$ combinations
(for trivial instance $k$)

Hardness of approximation

Polyhedral investigation (IP formulation)
  Aardal et al.; Koster et al.; Jaumard et al.; ...
  Used for adapting to local changes in the network

Lower bounds - study of relaxed problems
Lower Bounding Technology

- LP lower bound for coloring
- TSP lower bound for $T$-coloring
- LP lower bound for minimizing interference
- Tree Decomposition approach
- Semidefinite lower bound for minimizing interference
Region with “Optimized Plan”

Instance k, the “toy case” from practice

264 cells
267 TRXs
50 channels
57% density
151 avg. deg.
238 max. deg.
69 clique size

DC5-VDS

Further Reduction: 46.3%
A Simplification of our Model

Simplified Carrier Network:

\[ N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad}) \]

- \((V, E)\) is an undirected graph
- \(C\) is an interval of integers (spectrum)
- \(B_v \subseteq C\) for all \(v \in V\) (blocked channels)
- \(d : E \rightarrow \mathbb{Z}_+ \{0, 1\}\) (separation)
- \(c^{co}, c^{ad} : E \rightarrow [0,1]\) (interference)
MIN $k$-Partition

- No blocked channels
- No separation constraints larger than one
- No adjacent-channel interference

**min $k$-partition (max $k$-cut)**

Chopra & Rao; Deza et al.; Karger et al.; Frieze & Jerrum

IP, LP-based B&C, SDP
MIN k-Partition

Given: an undirected graph $G = (V, E)$ together with real edge weights $w_{ij}$ and an integer $k$. Find a partition of the vertex set into (at most) $k$ sets $V_1, ..., V_k$ such that the sum of the edge weights in the induced subgraphs is minimal!

\[
\min_{V_1, ..., V_k} \sum_{p=1}^{k} \sum_{i,j \in V_p} w_{ij}
\]

NP-hard to approximate optimal solution value.
Integer Linear Programming

\[
\begin{align*}
\text{min} & \quad \sum_{i,j \in V} w_{ij} z_{ij} \\
\sum_{i,j \in V} z_{ih} + z_{hj} - z_{ij} & \leq 1 \quad \forall h, i, j \in V \\
\sum_{i,j \in Q} z_{ij} & \geq 1 \quad \forall Q \subseteq V \text{ with } |Q| = k + 1 \\
z_{ij} & \in \{0, 1\} \\
\end{align*}
\]

**Number of ILP inequalities (facets)**

| Instance*  | |V| | k | Triangle | Clique Inequalities |
|------------|--------|------|----|----------|---------------------|
| cell.k     | 69     | 50   |    | 157182   | 17231414395464984   |
| B-0-E      | 81     | 75   |    | 255960   | 25621596            |
| B-1-E      | 84     | 75   |    | 285852   | 43595145594         |
| B-2-E      | 93     | 75   |    | 389298   | 1724861095493098563 |
| B-4-E      | 120    | 75   |    | 842520   | 1334655509331585084721199905599180 |
| B-10-E     | 174    | 75   |    | 2588772  | 361499854695979558347628887341189586948364637617230 |
Vector Labeling

Lemma: For each $k$, $n$ ($2 \leq k \leq n+1$) there exist $k$ unit vectors $u_1, \ldots, u_k$ in $n$-space, such that their mutual scalar product is $-1/(k-1)$. (This value is least possible.)

Fix $U = \{u_1, \ldots, u_k\}$ with the above property, then the min $k$-partition problem is equivalent to:

$$\min_{\phi: V \to U} \sum_{i \neq j \in E} \left( \frac{k-1}{k} \langle \phi_i, \phi_j \rangle + \frac{1}{k} \right) w_{ij}$$

$X = [\langle \phi_i, \phi_j \rangle]$ is positive semidefinite, has 1’s on the diagonal, and the rest is either $-1/(k-1)$ or 1.
Semidefinite Relaxation

\[
\min \sum_{ij \in E(K_n)} w_{ij} \frac{(k - 1) V_{ij} + 1}{k} \\
V_{ii} = 1 \quad \forall i \in V \\
V_{ij} \geq \frac{-1}{k-1} \quad \forall i, j \in V \\
V \succeq 0
\]

(SDP) is an approximation of (ILP)

Given \( V \), let \( z_{ij} := ((k-1) V_{ij} + 1)/k \), then:

- \( z_{ij} \) in \([0,1]\)
- \( z_{ih} + z_{ih} - z_{ij} < \sqrt{2} \) (\(<=1\))
- \( \sum_{i,j \in Q} z_{ij} > \frac{1}{2} \) (\(>=1\))

Solvable in polynomial time!
Computational Results

S. Burer, R.D.C Monteiro, Y. Zhang; Ch. Helmberg; J. Sturm

<table>
<thead>
<tr>
<th>Instance</th>
<th>clique cover</th>
<th>min k-part.</th>
<th>heuristic</th>
<th>clique cover</th>
<th>min k-part.</th>
<th>heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>cell.k</td>
<td>0.0206</td>
<td>0.0206</td>
<td>0.0211</td>
<td>0.0248</td>
<td>0.1735</td>
<td>0.4023</td>
</tr>
<tr>
<td>B-0-E</td>
<td>0.0016</td>
<td>0.0013</td>
<td>0.0016</td>
<td>0.0018</td>
<td>0.0096</td>
<td>0.8000</td>
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<tr>
<td>B-1-E</td>
<td>0.0063</td>
<td>0.0053</td>
<td>0.0064</td>
<td>0.0063</td>
<td>0.0297</td>
<td>0.8600</td>
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<tr>
<td>B-2-E</td>
<td>0.0290</td>
<td>0.0213</td>
<td>0.0242</td>
<td>0.0378</td>
<td>0.4638</td>
<td>3.1700</td>
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<tr>
<td>B-4-E</td>
<td>0.0932</td>
<td>0.2893</td>
<td>0.3481</td>
<td>0.2640</td>
<td>4.3415</td>
<td>17.7300</td>
</tr>
<tr>
<td>B-10-E</td>
<td>0.2195</td>
<td>2.7503</td>
<td>3.2985</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lower bound on co-channel interference by a factor of 2 to 85 below co- and adjacent-channel interference of best known assignment.
Semidefinite Conclusions

Lower bounding via
Semidefinite Programming works (somewhat),
at least better than LP!

• Challenging computational problems
• Lower bounds too far from cost of solutions to give strong quality guarantees
• How to produce good k-partitions starting from SDP solutions?


FAP web – A website devoted to Frequency Assignment:

http://fap.zib.de
06M1 Lecture
Frequency Assignment for GSM Mobile Phone Systems

The End