06M1 Lecture

Frequency Assignment for GSM Mobile Phone Systems

Martin Grötschel

Beijing Block Course

"Combinatorial Optimization at Work“

September 25 – October 6, 2006

Martin Grötschel

- Institut für Mathematik, Technische Universität Berlin (TUB)
- DFG-Forschungszentrum “Mathematik für Schlüsseltechnologien” (MATHEON)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

groetschel@zib.de http://www.zib.de/groetschel

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1. Introduction
2. The Telecom Problem & Mobile Communication
3. GSM Frequency/Channel Assignment
4. The UMTS Radio Interface (next talk)

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E-Plus and the Channel Assignment Problem

- How did we get this project?

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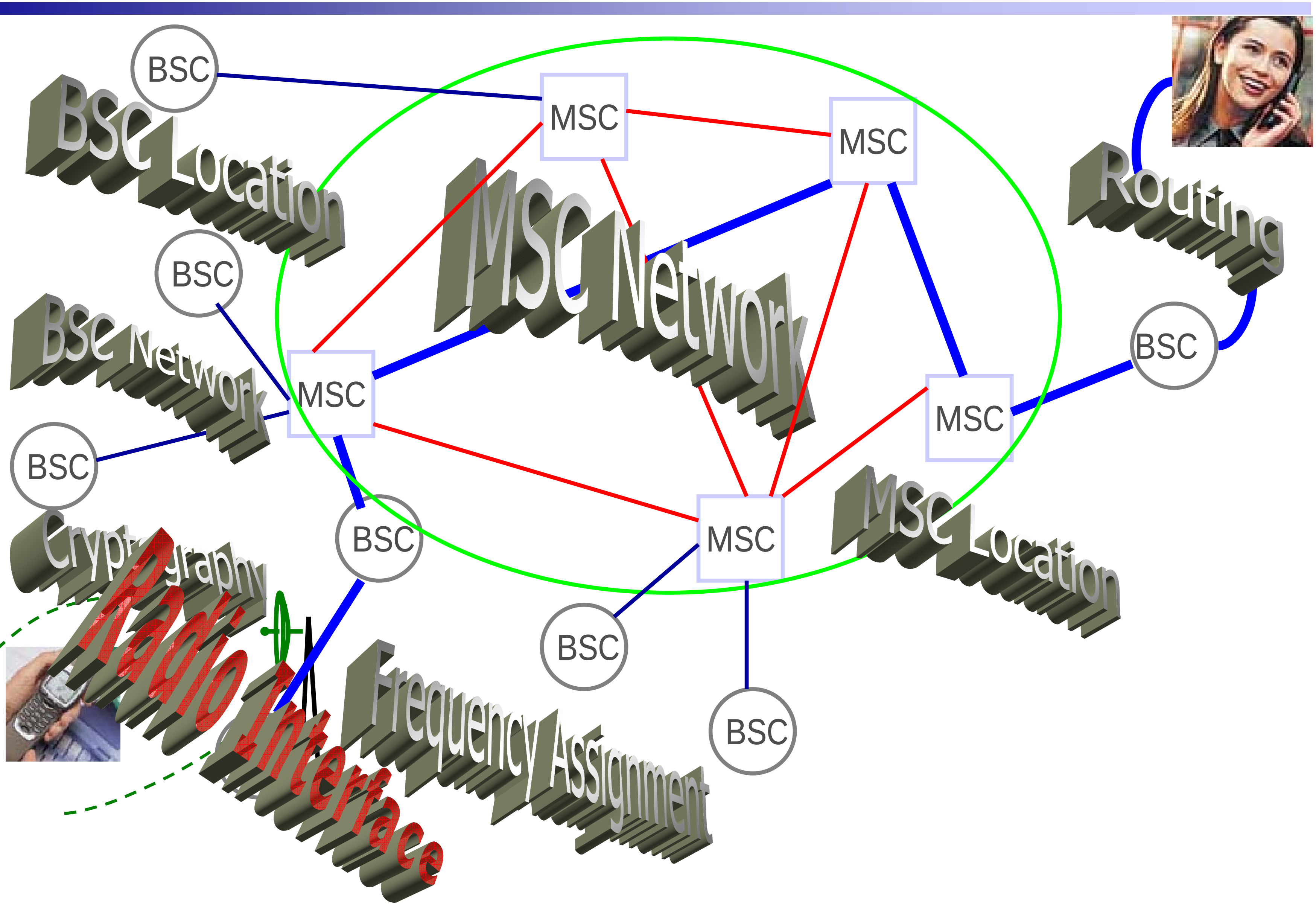
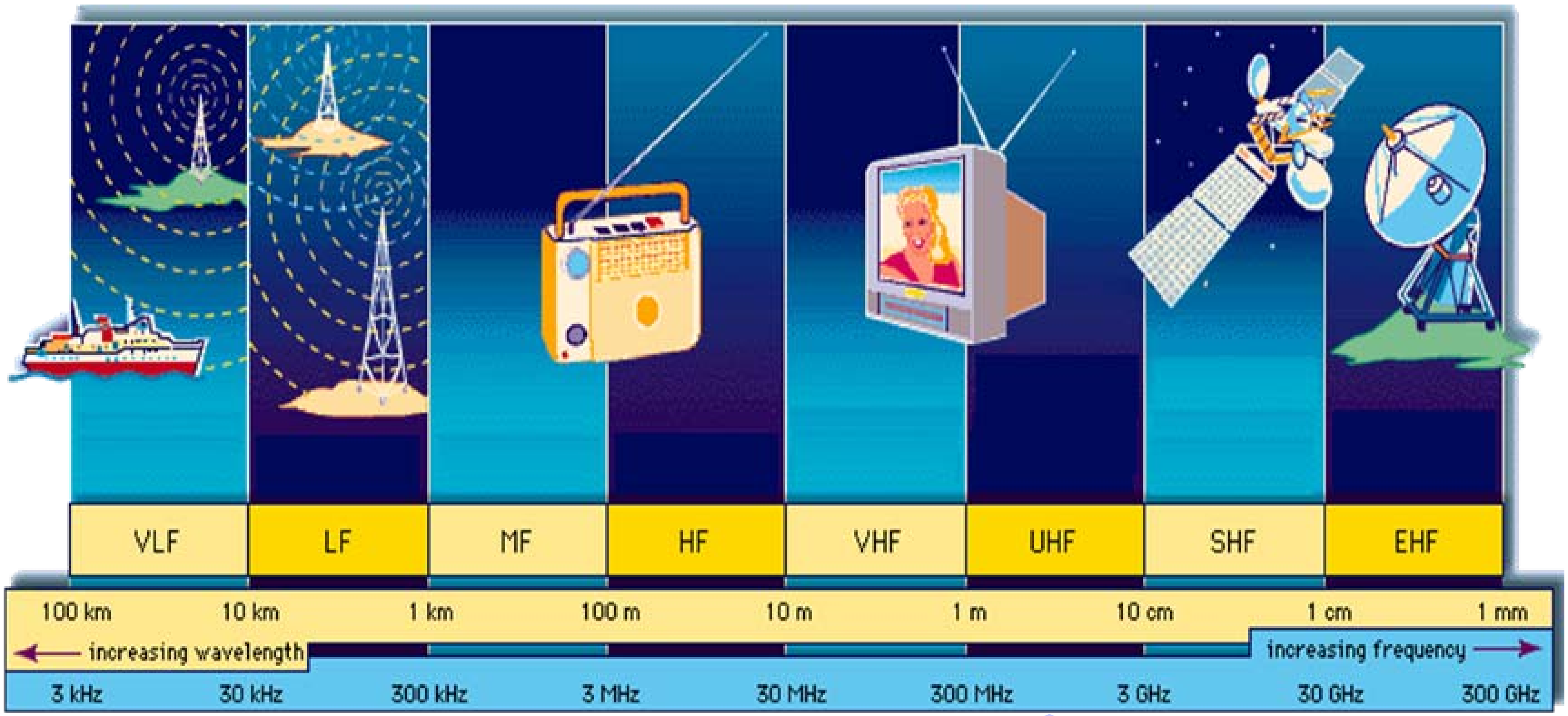
Connecting Mobiles



Wireless Communication



Mobile Telecommunication

Generations of Mobile Telecommunications Systems

1G	2G	3G	4G
1980s	1990s	2000s	2010 ??

1G
- Analogue
- Voice Only

2G
- Digital
- Voice & Data
- **GSM** mass market
- PCS
- cdmaOne/IS95

3G
- **UMTS**, WiFi/WLAN, cdma2000
- Data Rates ≥ 384 kbit/s
- Various Services

4G
- more **services**
- more bandwidth
- fresh spectrum
- new technology
- **W-CDMA** radio transmissions

Radio Interface: OR & Optimization Challenges

- Location of sites/base stations
  - was investgated in the OR literature („dead subject“)
  - has become „hot“ again
    - UMTS: massive investments around the world
    - GSM: still significant roll-outs
    - special issue: mergers
- antenna configurations at base stations
  - GSM: coverage based planning
  - UMTS: coverage & capacity considerations
- radio resource allocation
  - GSM: frequency assignment
  - UMTS: ? (open: real time/online resource management)

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Wireless Communication

GSM: More than 1,000 million users in over 150 countries

Wireless Communication

There are five frequency bands used by GSM mobile phones:

GSM-900, GSM-1800, GSM-850, GSM-1900, GSM-400

GSM-900 and GSM-1800 are used in most of the world.

GSM-900 uses 890 - 915 MHz to send information from the Mobile Station to the Base Transceiver Station (BTS) (This is the “uplink”) and 935 - 960 MHz for the other direction (downlink), providing 124 RF channels spaced at 200 kHz. Duplex spacing of 45 MHz is used. GSM-1800 uses 1710 - 1785 MHz for the uplink and 1805 - 1880 downlink, providing 299 channels. Duplex spacing is 95 MHz.

GSM-850 and GSM-1900 are used in the United States, Canada, and many other countries in the Americas.

FAP

F
i
l
m

Antennas

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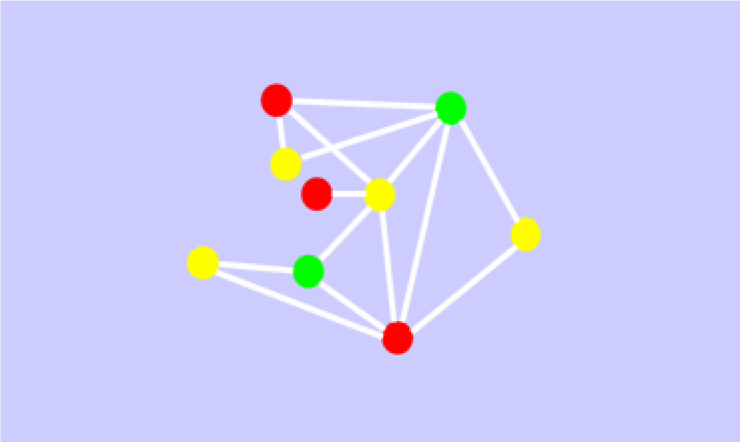
Initial Idea

- Use graph colouring to assign channels!

Coloring Graphs

Given a graph G = (V,E), color the nodes of the graph such that no two adjacent nodes have the same color.



The smallest number of colors with this property is called chromatic or coloring number and is denoted by $\chi(G)$.

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Coloring Graphs

A typical theoretical question: Given a

class *C* of graphs

(e.g., planar or perfect graphs, graphs without certain minors), what can one prove about the chromatic number of all graphs in *C*?

A typical practical question: Given a

particular graph G

(e.g., arising in some application), how can one determine (or approximate) the chromatic number of G?

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Coloring Graphs

- Coloring graphs algorithmically

  - NP-hard in theory
  - very hard in practice
  - almost impossible to find optimal colorings (symmetry issue)
  - playground for heuristics (e.g., DIMACS challenge)

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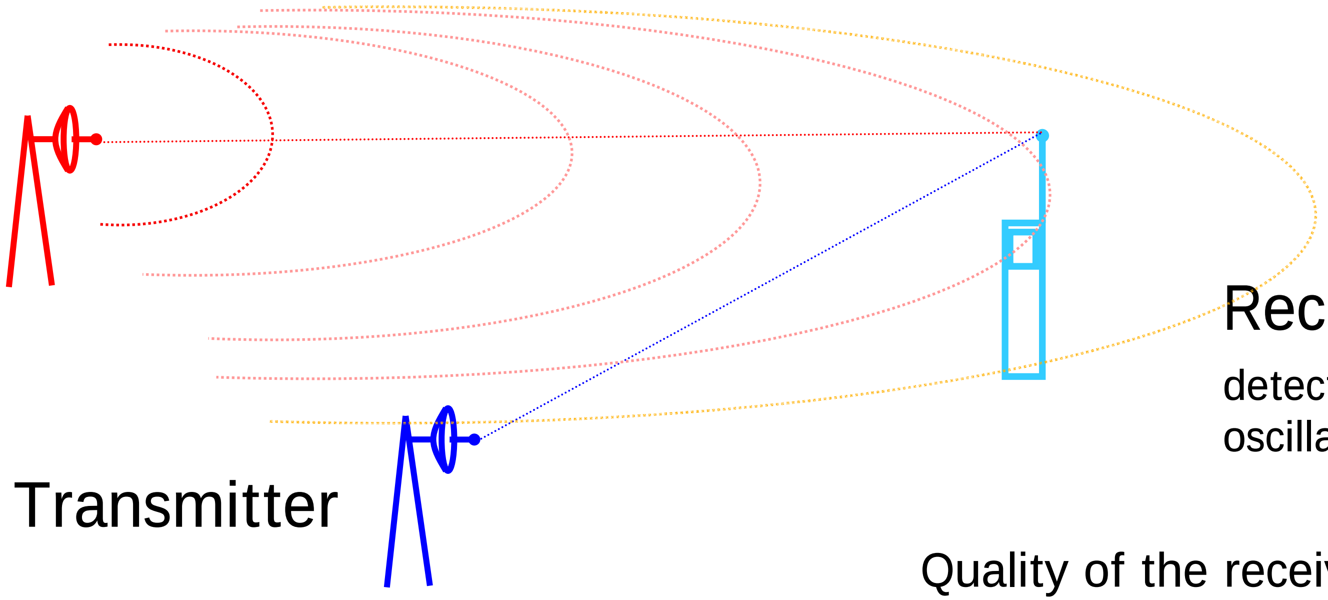
Coloring in Telecommunication

- Frequency or Channel Assignment for radio-, tv-transmission, etc.
- Our Example: GSM mobile phone systems

- Andreas Eseinblätter, Martin Grötschel and Arie M. C. A. Koster, *Frequenzplanung im Mobilfunk*, DMV-Mitteilungen 1(2002)18-25
- Andreas Eisenblätter, Hans-Florian Geerdes, Thorsten Koch, Ulrich Türke: *MOMENTUM Data Scenarios for Radio Network Planning and Simulation* , ZIB-Report 04-07
- Andreas Eisenblätter, Armin Fügenschuh, Hans-Florian Geerdes, Daniel Junglas, Thorsten Koch, Alexander Martin: *Optimization Methods for UMTS Radio Network Planning*, ZIB-Report 03-41

Properties of wireless communication



Transmitter

emits electromagnetic
oscillations at a frequency

Receiver

detects
oscillations

Quality of the received signal:
Signal-to-noise ratio

Poor signal-to-noise ratio:
interference of the signal

Objective: Frequency plan without interference or,
second best, with minimum interference

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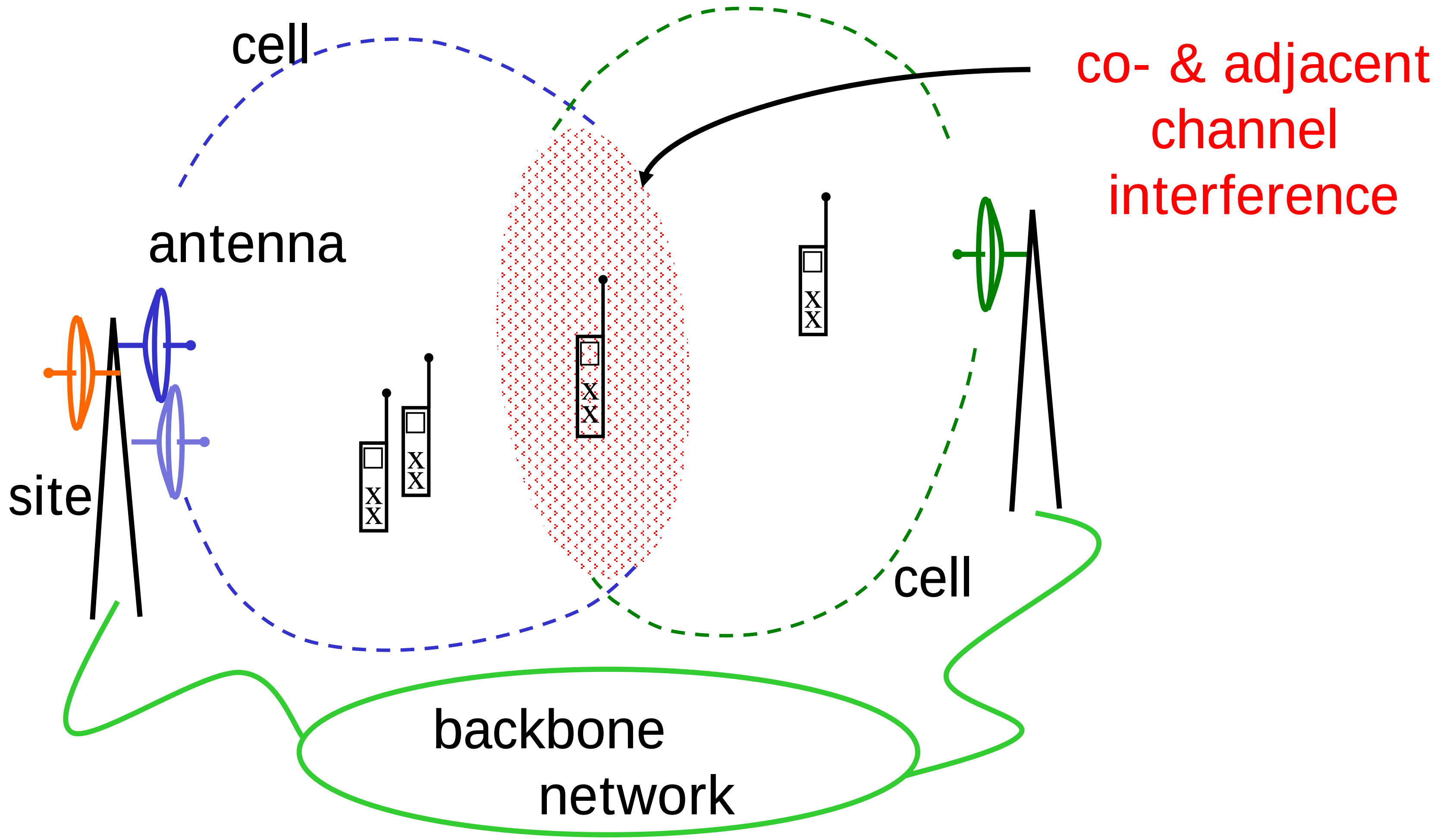
Antennas & Interference



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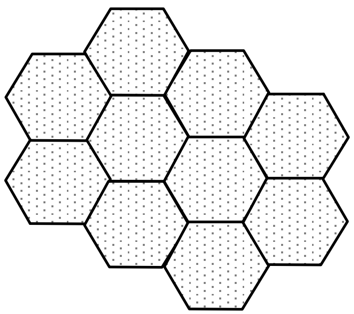
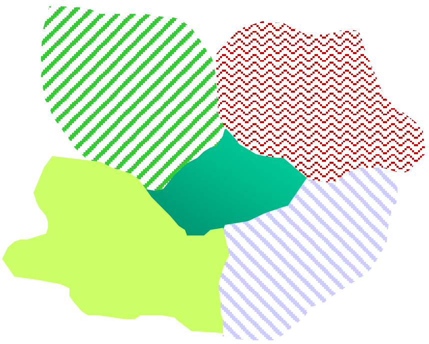
Cell Models

Hexagon Cell Model



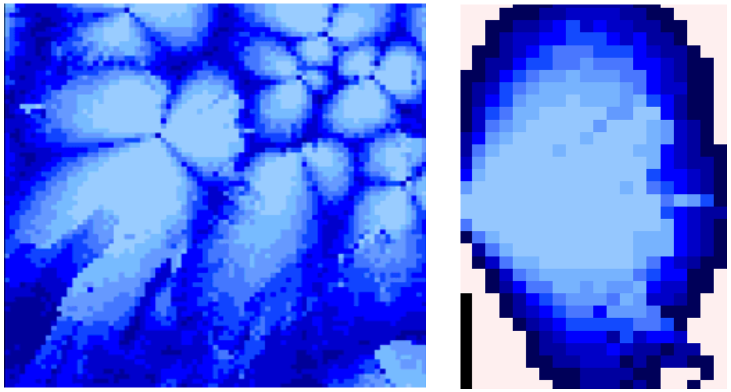
- sites on regular grid
- isotropic propagation conditions
- no cell-overlapping

Best Server Model



- realistic propagation conditions
- arbitrary cell shapes
- no cell-overlapping

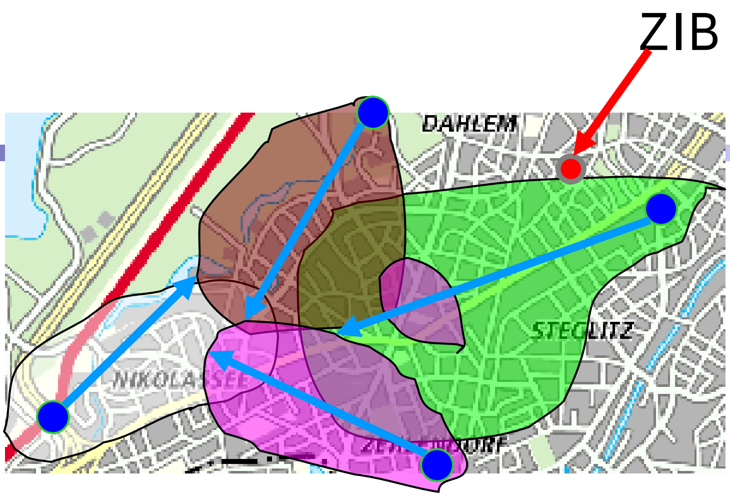
Cell Assignment Probability Model



- realistic propagation conditions
- arbitrary cell shapes
- cell-overlapping

Source: **E-Plus Mobilfunk, Germany**

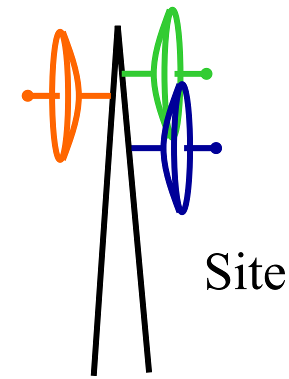
Interference



Level of interference depends on

- distance between transmitters,
- geographical position,
- power of the signals,
- direction in which signals are transmitted,
- weather conditions
- assigned frequencies
  - co-channel interference
  - adjacent-channel interference

Separation / Blocked Channels



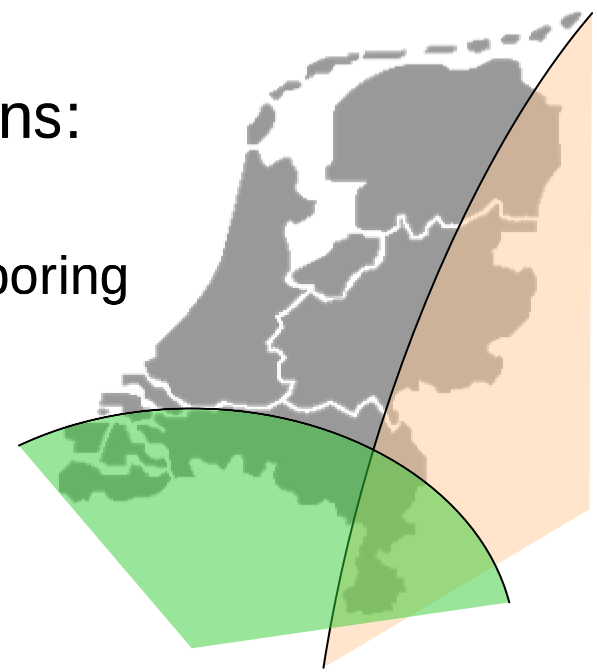
Separation:

Frequencies assigned to the same location (site) have to be separated

Blocked Channels:

Restricted spectrum at some locations:

- government regulations,
- agreements with operators in neighboring regions,
- requirements of military forces,
- etc.



Frequency Planning Problem

Find an assignment of frequencies/channels to transmitters that satisfies

- all separation constraints
- all blocked channels requirements

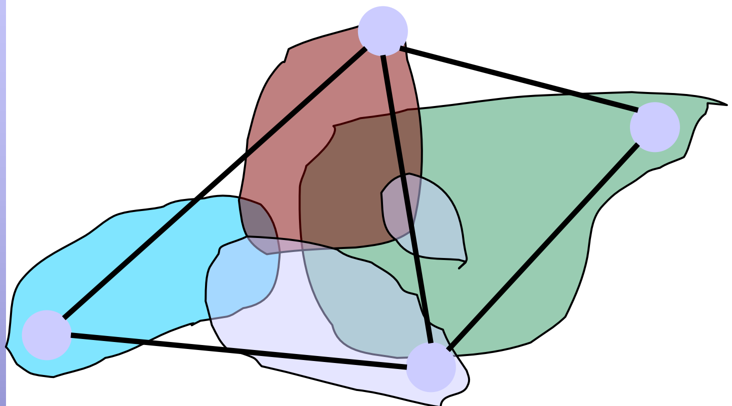
and either

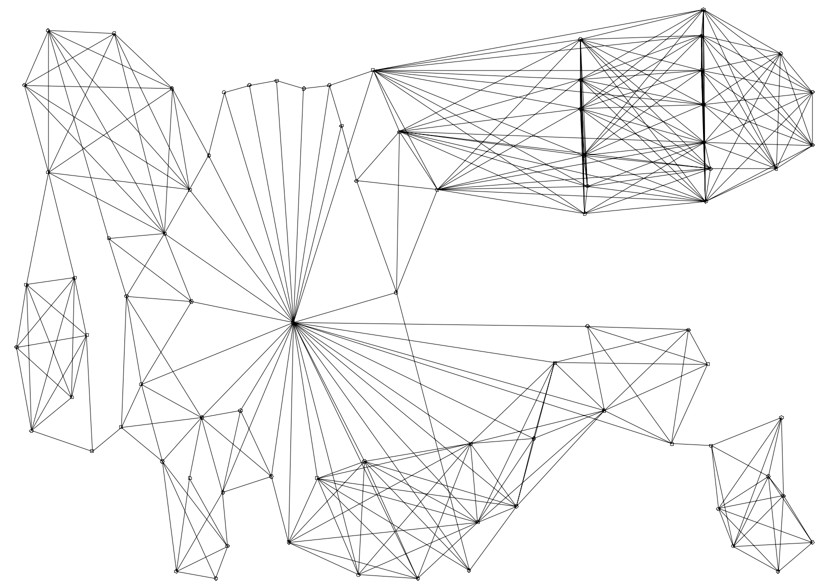
- avoids interference at all

or

- minimizes the (total/maximum) interference level

Modeling: the interference graph





- Vertices represent transmitters (TRXs)
- Edges represent separation constraints and co/adjacent-channel interference
  - Separation distance: $d(vw)$
  - Co-channel interference level: $c^{co}(vw)$
  - Adjacent-channel interference level: $c^{ad}(vw)$

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Remark about UMTS

- There is no way to model interference as some number associated with an edge in some graph.
- Modelling is much more complicated, see UMTS talk

Graph Coloring

Simplifications:

- drop adjacent-channel interference
- drop local blockings
- reduce all separation requirements to 1
- change large co-channel interference into separation distance 1 (inacceptable interference)

Result:

- FAP reduces to coloring the vertices of a graph
- Example

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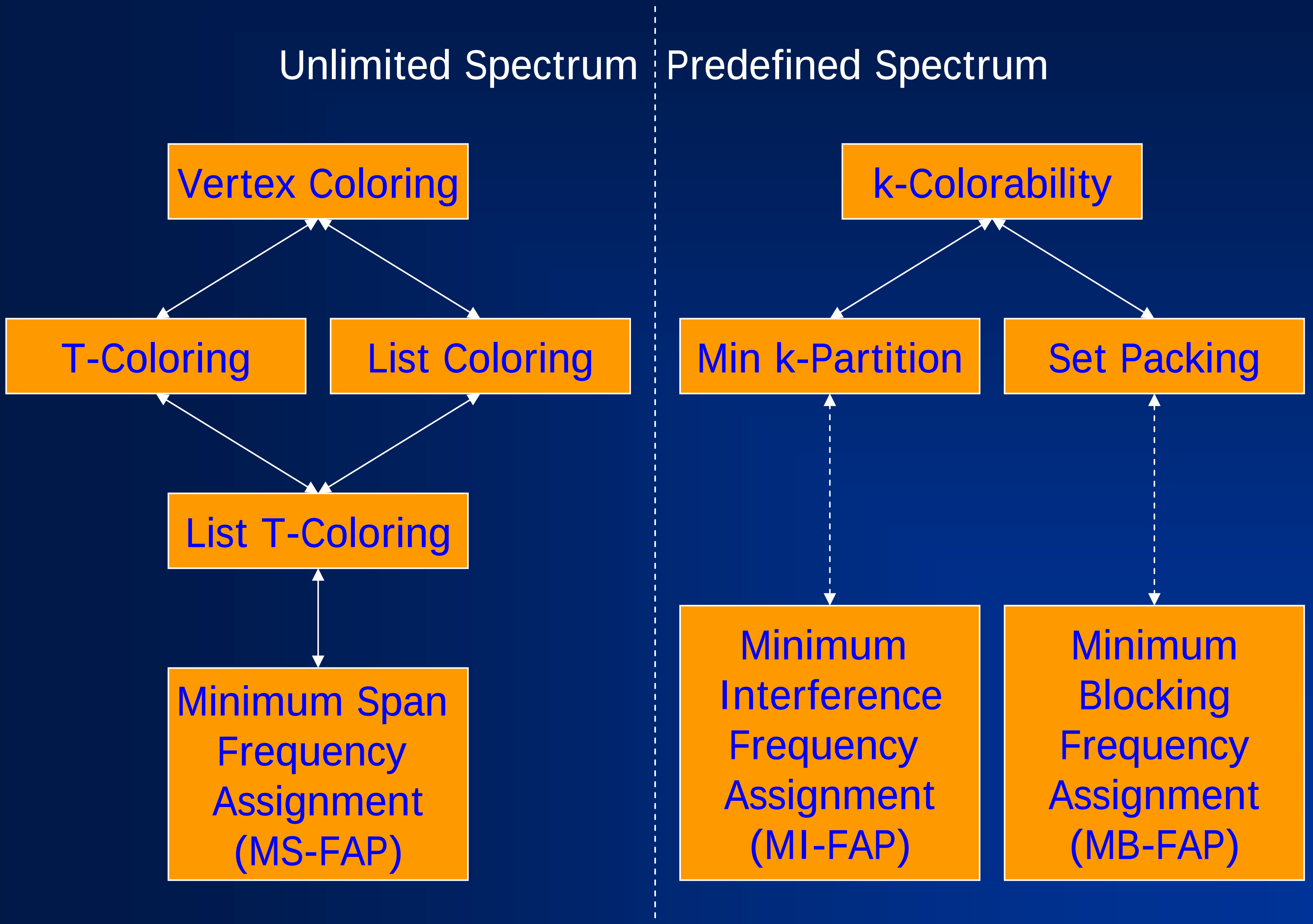
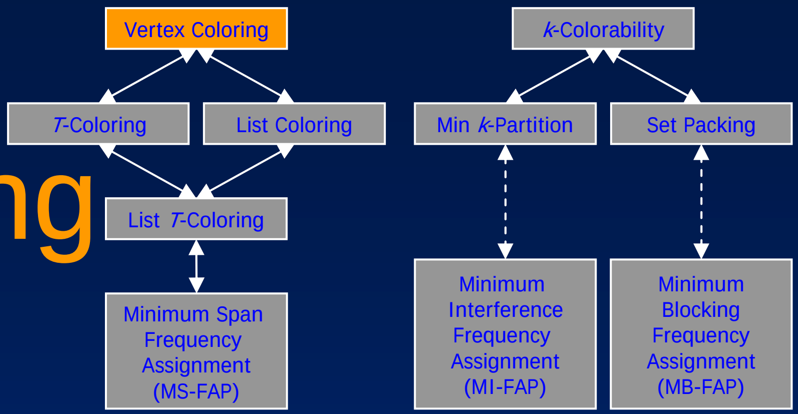
Graph Coloring & Frequency Planning

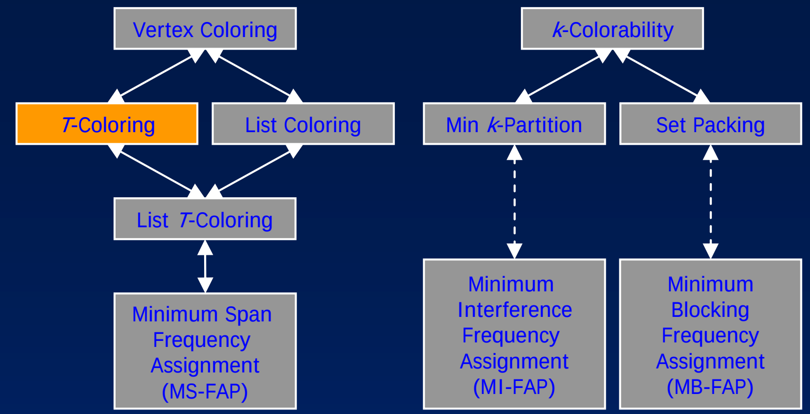


FAP & Vertex Coloring



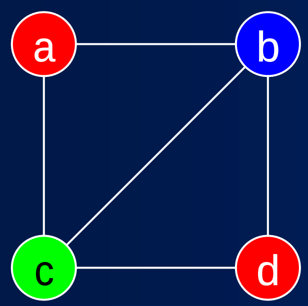
- Only co-channel interference
- Separation distance 1
- Minimization of
  - Number of frequencies used (chromatic number)
  - Span of frequencies used
- Objectives are equivalent: span = #colors-1
- FAP is **NP**-hard

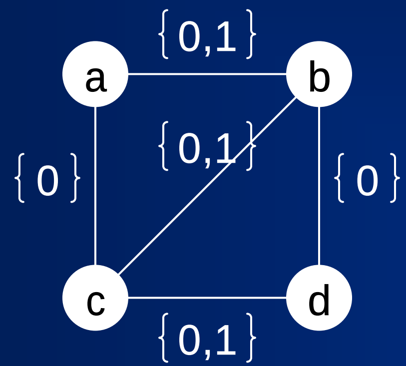
FAP & T-Coloring

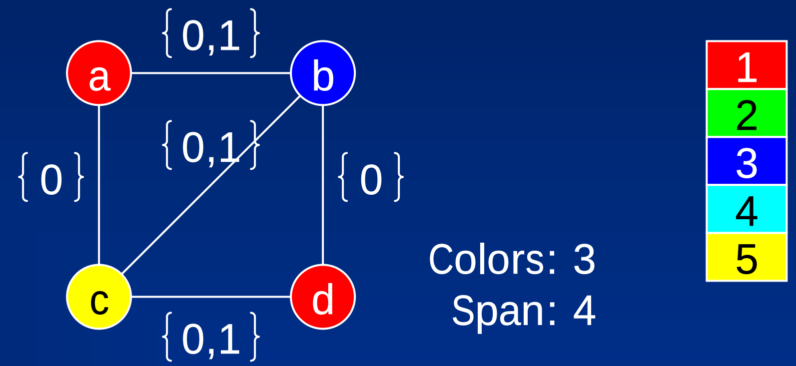


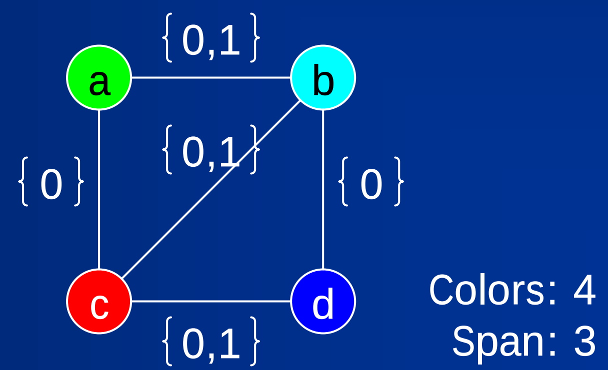
Sets of forbidden distances T_{vw}

$$|f_v - f_w| \notin T_{vw} \qquad T_{vw} = \{0,\dots,d(vw)\text{-}1\}$$



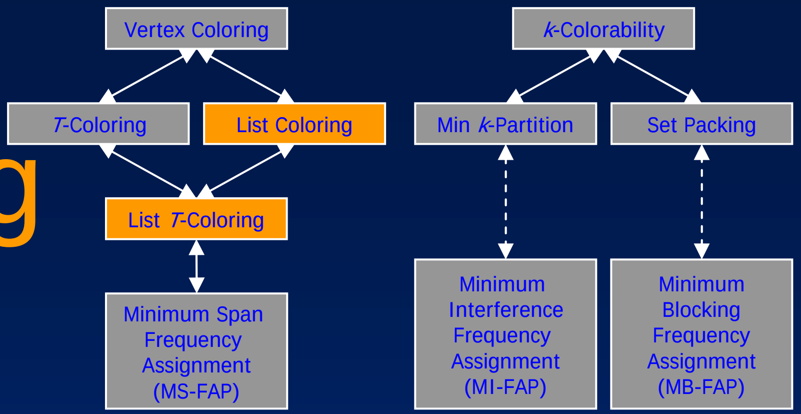




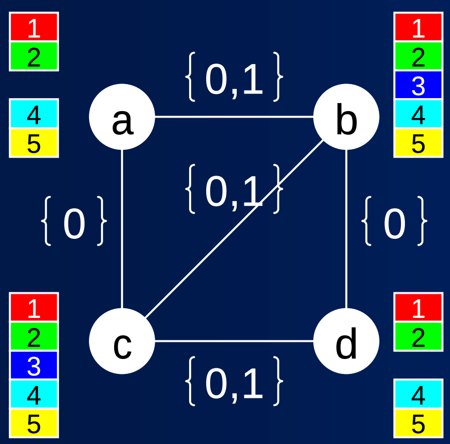


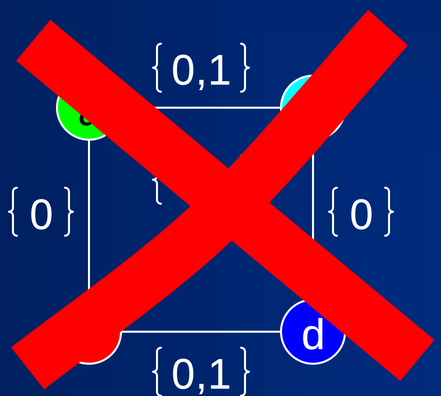
Minimization of
number of colors and span
are not equivalent!

FAP & List- *T*-Coloring

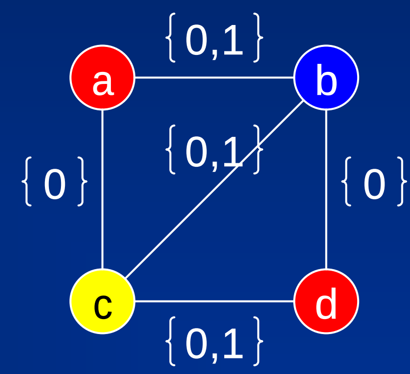


Locally blocked channels:
Sets of forbidden colors B_v



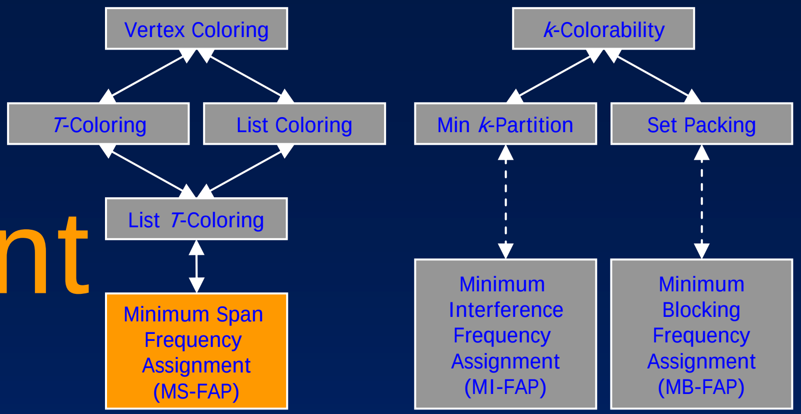


No solution with span 3 !



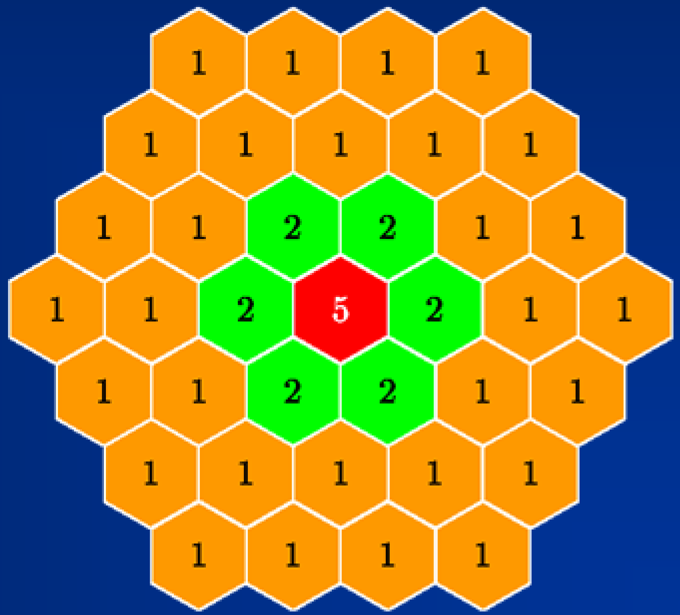
Colors: 3
Span: 4

Minimum Span Frequency Assignment



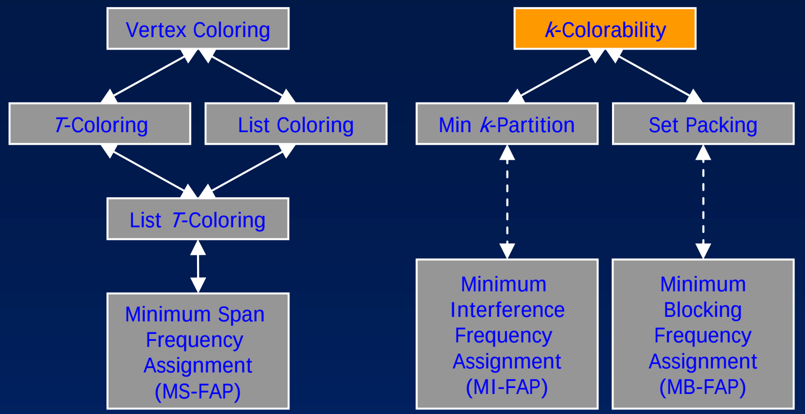
- List-T-Coloring (+ multiplicity)
- Benchmarks: Philadelphia instances

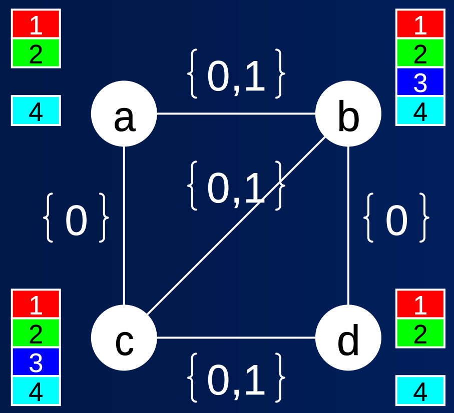
Channel requirements (P1)
Optimal span = 426



Separation distances

Fixed Spectrum





License for frequencies {1,...,4}

No solution with span 3

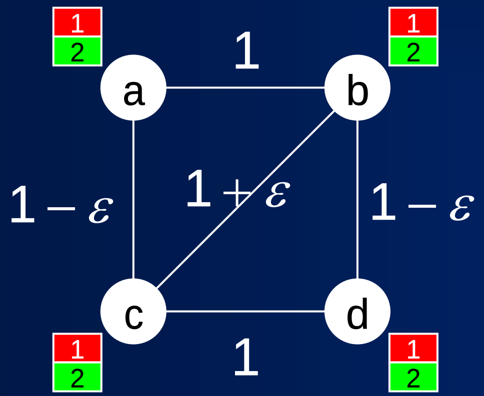
- Is the graph span-*k*-colorable?
- Complete assignment: minimize interference
- Partial assignment without interference

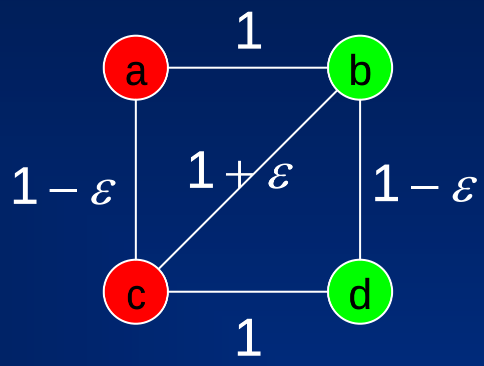
Hard & Soft constraints

- How to evaluate "infeasible" plans?
  - Hard constraints: separation, local blockings
  - Soft constraints: co- and adjacent-channel interference
- Measure of violation of soft constraints: penalty functions

$$p_{vw}(f,g) = \begin{cases} c^{co}(vw) & \text{if } f = g \\ c^{ad}(vw) & \text{if } \left|f-g\right| = 1 \\ 0 & \text{otherwise} \end{cases}$$

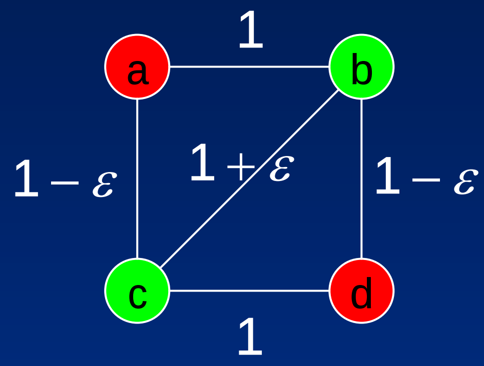
Evaluation of infeasible plans





Total penalty: $2-2\varepsilon$

Maximum penalty: $1-\varepsilon$



Total penalty : $1+\varepsilon$

Maximum penalty : $1+\varepsilon$

- Minimizing total interference
- Minimizing maximum interference
  - Use of threshold value, binary search

What is a good objective?

Keep interference information!

Use the available spectrum!

Minimize max interference

- T-coloring (min span): Hale; Gamst; …

Minimize sum over interference

- Duque-Anton et al.; Plehn; Smith et al.; …

Minimize max "antenna" interference

- Fischetti et al.; Mannino, Sassano

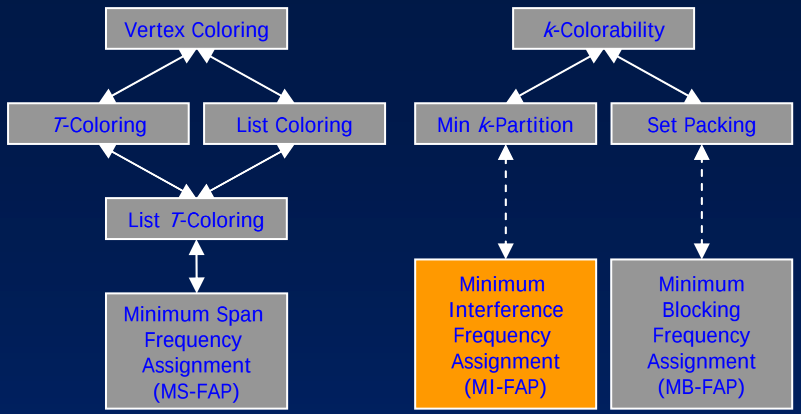
Our Model

Carrier Network:

$$N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad})$$

- (V,E) is an undirected graph
- C is an interval of integers (spectrum)
- $B_v \subseteq C$ for all $v \in V$ (blocked channels)
- $d : E \to Z_+$ (separation)
- $c^{co}, c^{ad} : E \to [0,1]$ (interference)

Minimum Interference Frequency Assignment



Integer Linear Program:

$$
\begin{array}{lll}
\min & \displaystyle\sum_{vw\in E^{co}} c^{co}_{vw} z^{co}_{vw} + \sum_{vw\in E^{ad}} c^{ad}_{vw} z^{ad}_{vw} & \\
s.t. & \displaystyle\sum_{f\in F_v} x_{vf} = 1 & \forall v \in V \\
 & x_{vf} + x_{wg} \le 1 & \forall vw \in E^{d}, |f-g| < d(vw) \\
 & x_{vf} + x_{wf} \le 1 + z^{co}_{vw} & \forall vw \in E^{co}, f \in F_v \cap F_w \\
 & x_{vf} + x_{wg} \le 1 + z^{ad}_{vw} & \forall vw \in E^{ad}, |f-g| = 1 \\
 & x_{vf}, z^{co}_{vw}, z^{ad}_{vw} \in \{0,1\} & \forall v \in V, f \in C \setminus B_v, \forall vw \in E^{co}, \forall vw \in E^{ad}
\end{array}
$$

A Glance at some Instances

Instance	|V|	density [%]	minimum degree	average degree	maximum degree	diameter	clique number
k	267	56,8	2	151,0	238	3	69
B-0-E-20	1876	13,7	40	257,7	779	5	81
f	2786	4,5	3	135,0	453	12	69
h	4240	5,9	11	249,0	561	10	130

Expected graph properties: planarity,...

Computational Complexity

Neither high quality nor feasibility are generally achievable within practical running times:

- Testing for feasibility is NP-complete.
- There exists an $\varepsilon > 0$ such that FAP cannot be "approximated" within a factor of $|V|^{\varepsilon}$ unless P = NP.

Heuristic Solution Methods

- Greedy coloring algorithms,
- DSATUR,
- Improvement heuristics,
- Threshold Accepting,
- Simulated Annealing,
- Tabu Search,
- Variable Depth Search,
- Genetic Algorithms,
- Neural networks,
- etc.

Heuristics

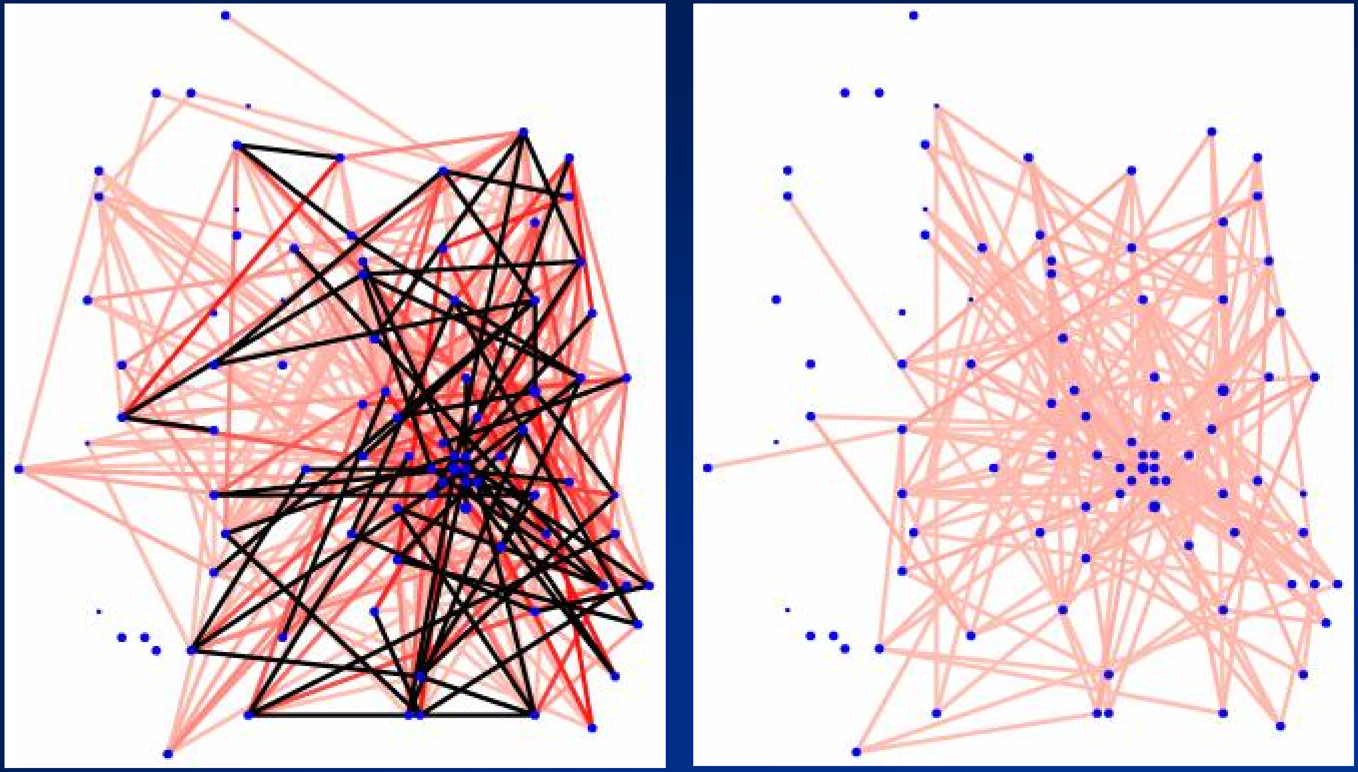
• T-coloring	o	construction heuristics
• Dual Greedy	--	
• DSATUR with Costs	++	
• Iterated 1-Opt	o	(randomized) local search
• Simulated Annealing	+	
• Tabu-Search	o	
• Variable Depth Search	++	
• MCF	-	other improvement heuristics
• B&C-based	+	

Region with “Optimized Plan”

Instance k, a “toy case” from practice

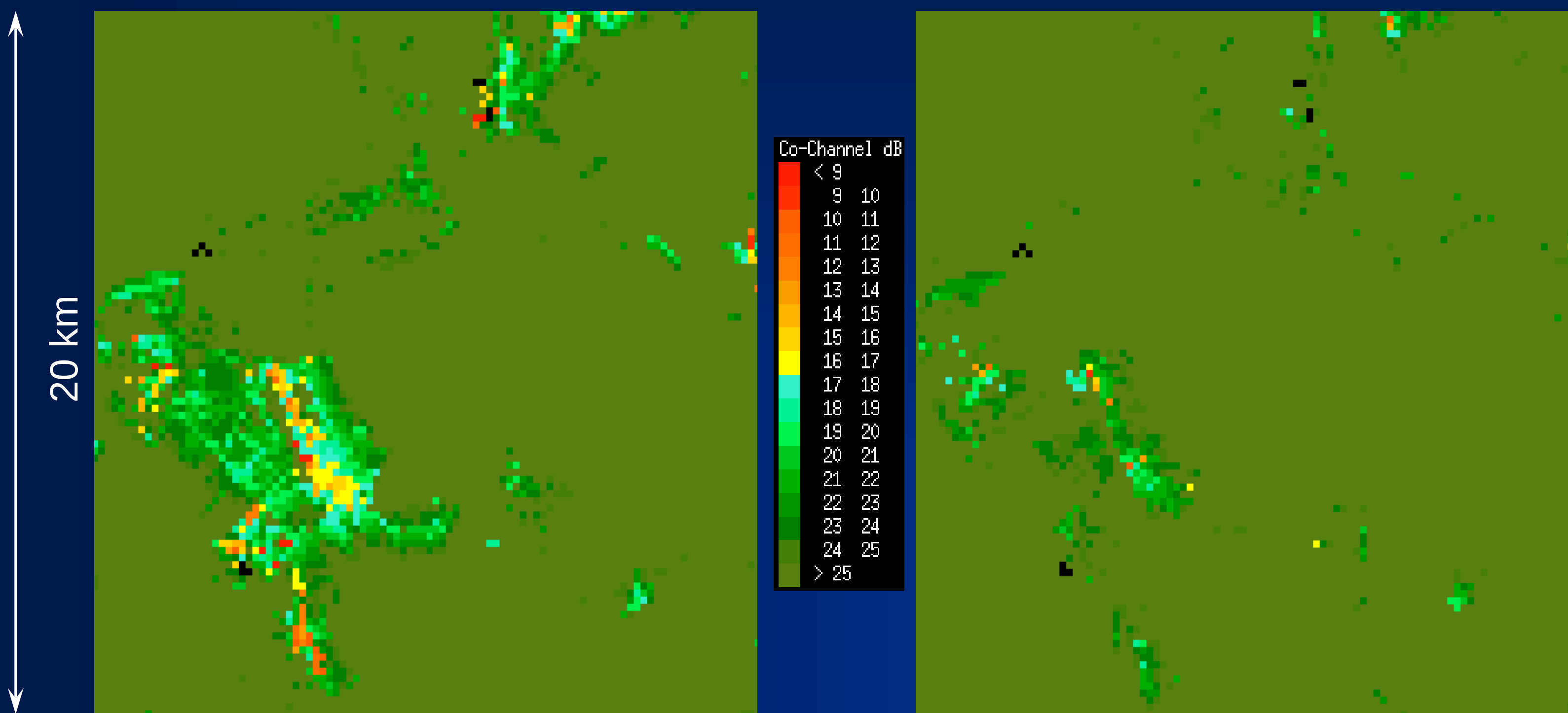
264 cells
267 TRXs
50 channels

57% density
151 avg.deg.
238 max.deg.
69 clique size



DC5-VDS: Reduction 96,3%

co-channel C/I worst Interferer



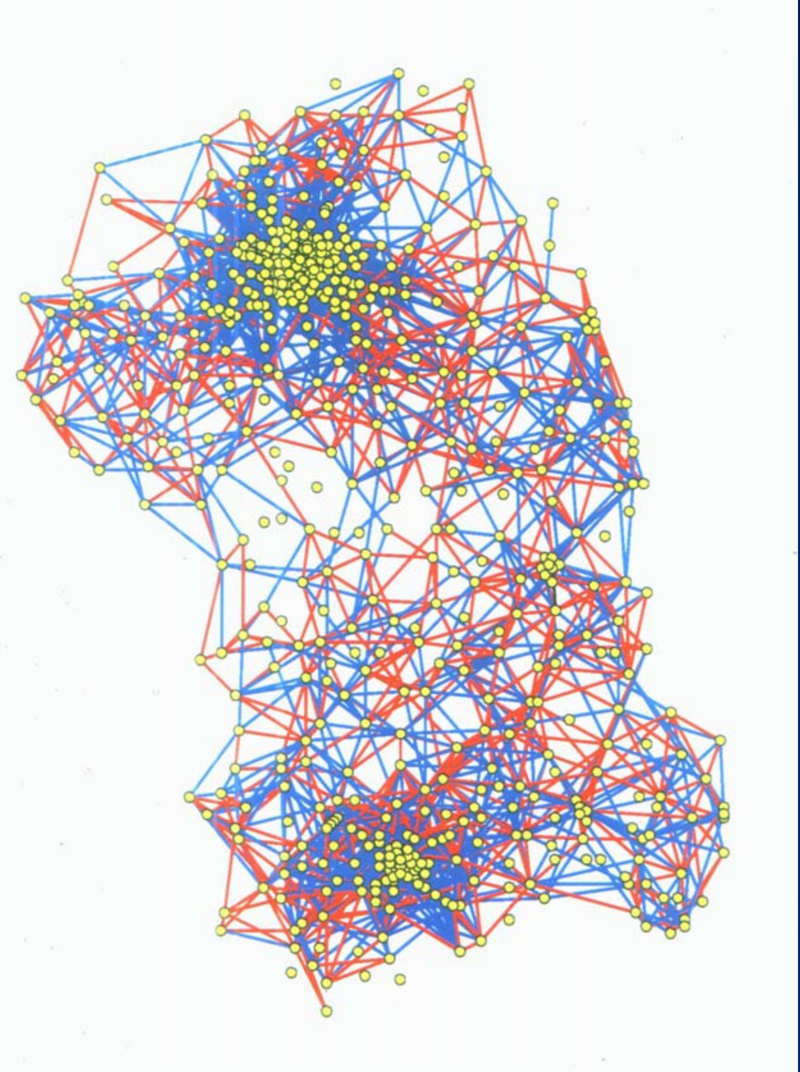
Mobile Systems International Plc.

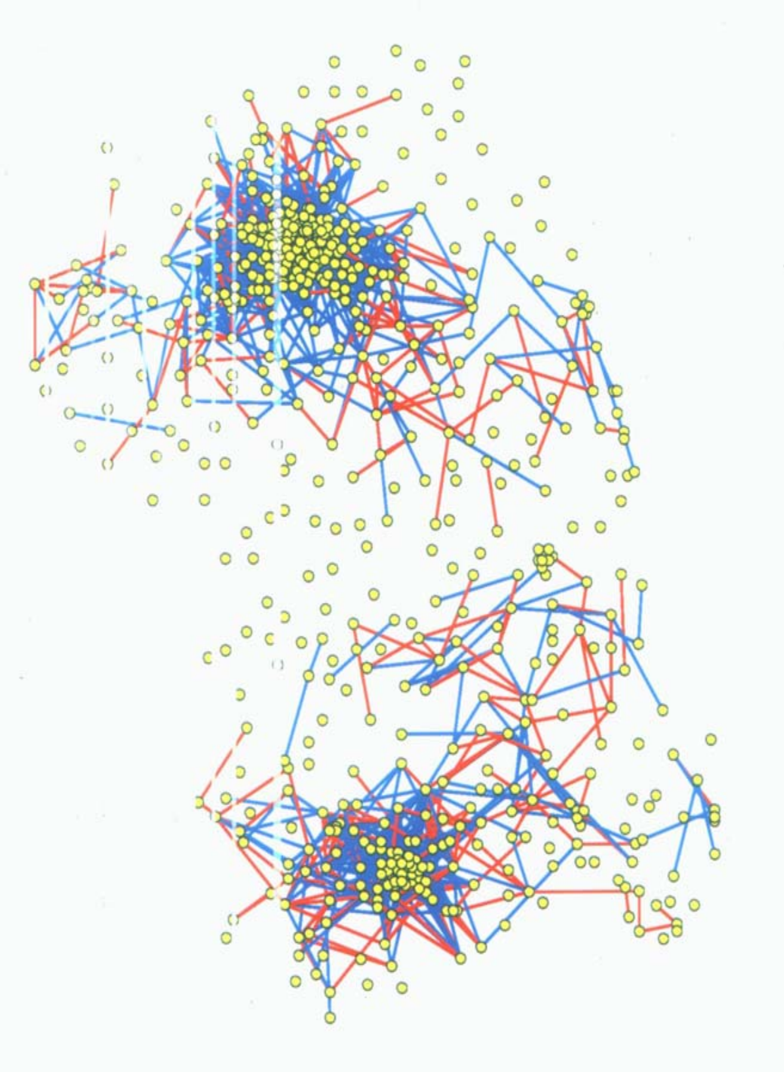
Mobile Systems International Plc.

Commercial software

DC5-IM

Region Berlin - Dresden



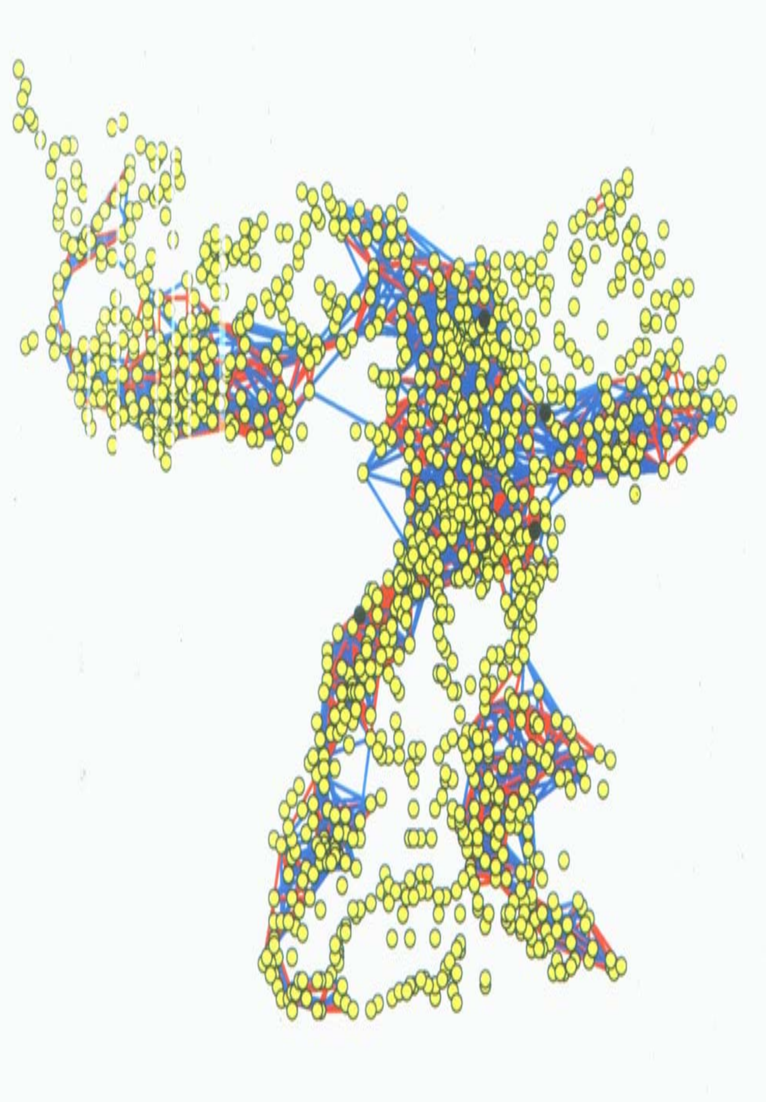


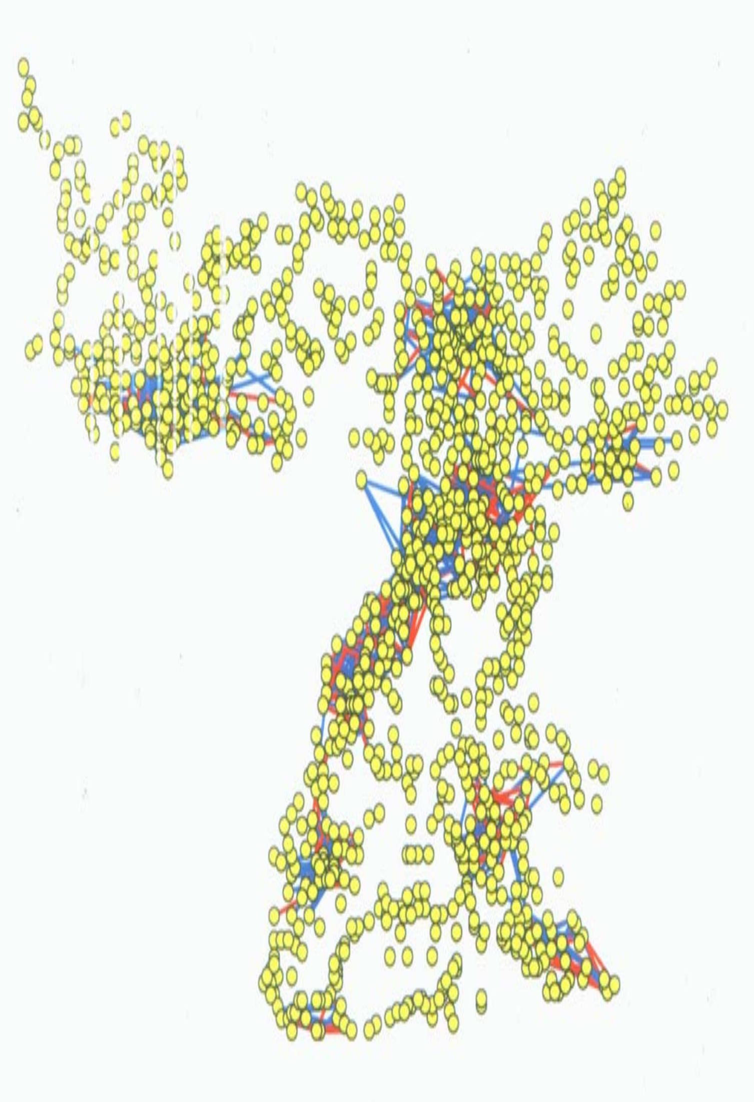
2877
carriers

50 channels

Interference
reduction:
83.6%

Region Karlsruhe





2877
Carriers

75 channels

Interference
Reduction:
83.9 %

Guaranteed Quality

Optimal solutions are out of reach!

Enumeration: $50^{267} \approx 10^{197}$ combinations

(for trivial instance k)

Hardness of approximation

Polyhedral investigation (IP formulation)

Aardal et al.; Koster et al.; Jaumard et al.; ...

Used for adapting to local changes in the network

Lower bounds - study of relaxed problems

Lower Bounding Technology

- LP lower bound for coloring
- TSP lower bound for *T*-coloring
- LP lower bound for minimizing interference
- Tree Decomposition approach
- Semidefinite lower bound for minimizing interference

Region with "Optimized Plan"

Instance k, the "toy case" from practice

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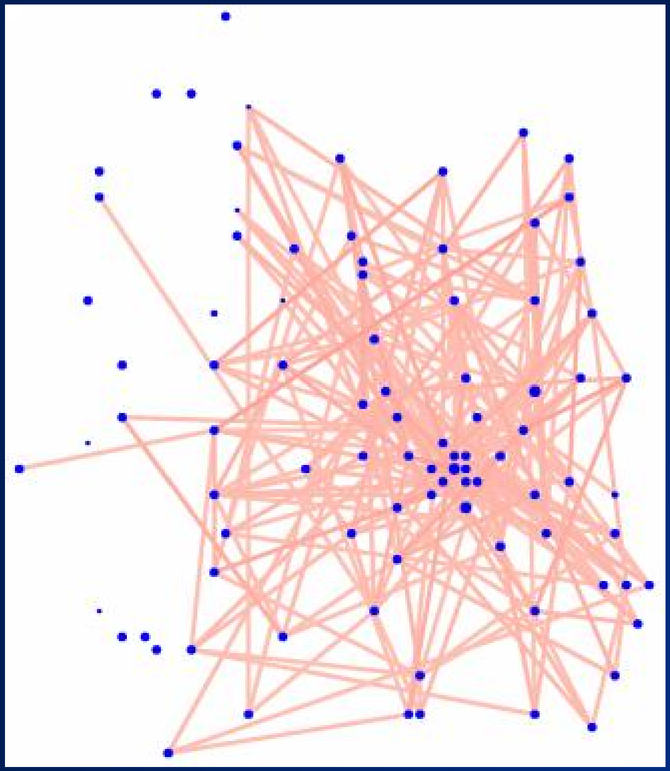
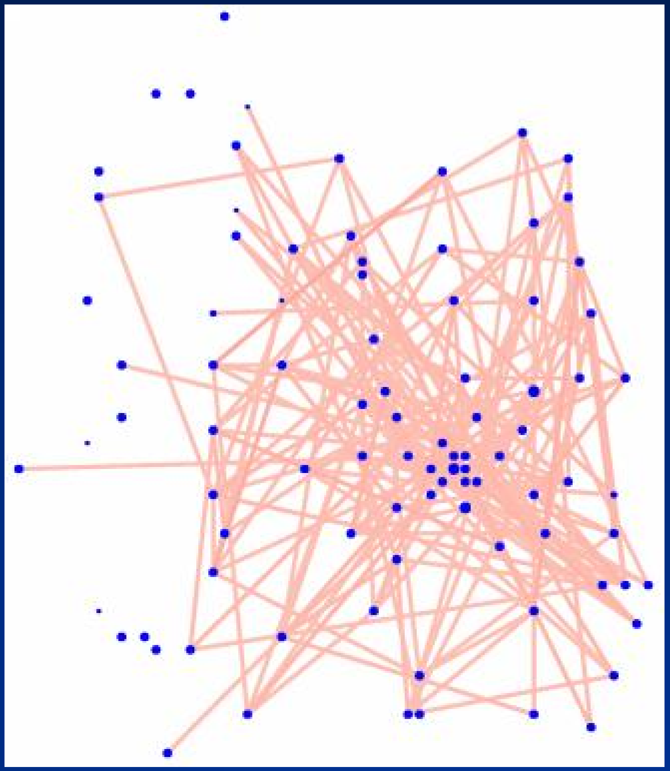
238 max.deg.
69 clique size



DC5-VDS



Further Reduction: 46.3%

A Simplification of our Model

Simplified Carrier Network:

$$N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad})$$

- (V,E) is an undirected graph
- C is an interval of integers (spectrum)
- $B_v \subseteq C$ for all $v \in V$ (blocked channels)
- $d : E \to Z_+$ {0, 1} (separation)
- $c^{co}, c^{ad} : E \to [0,1]$ (interference)

MIN k-Partition

- No blocked channels
- No separation constraints larger than one
- No adjacent-channel interference

min k-partition (max k-cut)

Chopra & Rao; Deza et al.; Karger et al.; Frieze & Jerrum

IP, LP-based B&C, SDP

MIN k-Partition

Given: an undirected graph $G = (V,E)$ together with real edge weights w_{ij} and an integer k.

Find a partition of the vertex set into (at most) k sets $V_1, \ldots, V_k$ such that the sum of the edge weights in the induced subgraphs is minimal!

$$\min_{\substack{V_1,\ldots,V_k \\ \text{partition of } V}} \sum_{p=1}^{k} \sum_{i,j \in V_p} w_{ij}$$

NP-hard to approximate optimal solution value.

Integer Linear Programmming

(ILP)

$$\min \sum_{i,j \in V} w_{ij} z_{ij}$$

$$z_{ih} + z_{hj} - z_{ij} \leq 1 \qquad \forall h, i, j \in V$$ -> partition consistent

$$\sum_{i,j \in Q} z_{ij} \geq 1 \qquad \forall\, Q \subseteq V \text{ with } |Q| = k+1$$ -> use at most k blocks

$$z_{ij} \in \{0, 1\}$$

Number of ILP inequalities (facets)

Instance*	|V|	k	Triangle	Clique Inequalities
cell.k	**69**	**50**	**157182**	**1723141439546484**
B-0-E	81	75	255960	25621596
B-1-E	**84**	**75**	**285852**	**43595145594**
B-2-E	93	75	389298	1724861095493098563
B-4-E	120	75	842520	1334655509331585084721199905599180
B-10-E	174	75	2588772	361499854695979558347628887341189586948364637617230

Vector Labeling

Lemma: For each k, n ($2 \leq k \leq n+1$) there exist k unit vectors $u_1, \ldots, u_k$ in n-space, such that their mutual scalar product is -1/(k-1). (This value is least possible.)

Fix $U = \{u_1, \ldots, u_k\}$ with the above property, then the min k-partition problem is equivalent to:

$$\min_{\substack{\phi: V \to U \\ i \mapsto \phi_i}} \sum_{ij \in E} \left(\frac{k-1}{k} \langle \phi_i, \phi_j \rangle + \frac{1}{k} \right) w_{ij}$$

$X = [\langle \phi_i, \phi_j \rangle]$ is positive semidefinite, has 1's on the diagonal, and the rest is either -1/(k-1) or 1.

Karger et al.; Frieze & Jerrum

Semidefinite Relaxation

(SDP)

$$\min \sum_{ij \in E(K_n)} w_{ij} \frac{(k-1)\, V_{ij} + 1}{k}$$

$$V_{ii} = 1 \qquad \forall i \in V$$

$$V_{ij} \geq \frac{-1}{k-1} \qquad \forall i,j \in V$$

$$V \succeq 0$$

Solvable in polynomial time!

Given V, let $z_{ij} := ((k-1)\, V_{ij} + 1)/k$, then:

- z_{ij} in [0,1]
- $z_{ih} + z_{ih} - z_{ij} < \sqrt{2}$ (<=1)
- $\sum_{i,j \text{ in } Q} z_{ij} > ½$ (>=1)

(SDP) is an approximation of (ILP)

Computational Results

S. Burer, R.D.C Monteiro, Y. Zhang; Ch. Helmberg; J. Sturm

	maximal clique			entire scenario		
Instance	clique cover	min k-part.	*heuristic*	clique cover	min k-part.	*heuristic*
cell.k	**0,0206**	**0,0206**	***0,0211***	**0,0248**	**0,1735**	**0,4023**
B-0-E	0,0016	0,0013	*0,0016*	0,0018	0,0096	*0,8000*
B-1-E	**0,0063**	**0,0053**	***0,0064***	**0,0063**	**0,0297**	**0,8600**
B-2-E	0,0290	0,0213	*0,0242*	0,0378	0,4638	*3,1700*
B-4-E	0,0932	0,2893	*0,3481*	0,2640	4,3415	*17,7300*
B-10-E	0,2195	2,7503	*3,2985*			*146,2000*

Lower bound on co-channel interference by a factor of 2 to 85 below co- and adjacent-channel interference of best known assignment.

Semidefinite Conclusions

Lower bounding via
Semidefinite Programming works (somewhat),
at least better than LP!

- Challenging computational problems
- Lower bounds too far from cost of solutions to give strong quality guarantees
- How to produce good k-partitions starting from SDP solutions?

Literature (ZIB PaperWeb)

K. Aardal, S. van Hoesel, A. Koster, C. Mannino, A. Sassano, “Models and Solution Techniques for the Frequency Assignment Problem”, ZIB-report 01-40, 2001.

A. Eisenblätter, “Frequency Assignment in GSM Networks: Models, Heuristics, and Lower Bounds”, Ph.D. thesis TU Berlin, 2001.

A. Eisenblätter, M. Grötschel, A. Koster, “Frequency Planning and Ramifications of Coloring”, Discussiones Mathematicae, Graph Theory, 22 (2002) 51-88.

A. Eisenblätter, M. Grötschel, A. Koster, "Frequenzplanung im Mobilfunk", DMV-Mitteilungen 1/2002, 18-25

A. Koster, “Frequency Assignment – Models and Algorithms”, Ph.D. thesis Universiteit Maastricht, 1999.

FAP web – A website devoted to Frequency Assignment:

http://fap.zib.de

06M1 Lecture
Frequency Assignment for GSM Mobile Phone Systems

The End

Martin Grötschel

- Institut für Mathematik, Technische Universität Berlin (TUB)
- DFG-Forschungszentrum "Mathematik für Schlüsseltechnologien" (MATHEON)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

groetschel@zib.de
http://www.zib.de/groetschel