05M1 Lecture
Combinatorial Optimization and Telecommunication

Martin Grötschel

Beijing Block Course
"Combinatorial Optimization at Work"

September 25 - October 6, 2006

Martin Grötschel
- Institut für Mathematik, Technische Universität Berlin (TUB)
- DFG-Forschungszentrum "Mathematik für Schlüsseltechnologien" (MATHEON)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

groetschel@zib.de http://www.zib.de/groetschel

The ZIB Telecom Team

The Telecom Group

Andreas Bley

Andreas Eisenblätter

Hans Florian Geerdes

Martin Grötschel

Tobias Harks

Thorsten Koch

Arie Koster

Sebastian Orlowski

Roland Wessäly

Adrian Zymolka

Manfred Brandt

Former Members

Dimitris Alevras (ZIB, IBM)

Christoph Helmberg (ZIB, Chemnitz)

Sven Krumke (ZIB, Kaiserslautern)

Alexander Martin (ZIB, Darmstadt)

Mechthild Opperud (ZIB, Telenor)

Diana Poensgen (ZIB, McKinsey)

Jörg Rambau (ZIB, Bayreuth)

Clyde Monma (BellCore, ...)

plus several MSc students

ZIB Partners from Industry

Bell Communications Research (now Telcordia)

Telenor (Norwegian Telecom)

E-Plus (acquired by KPN in 01/2002)

DFN-Verein

Bosch Telekom (bought by Marconi in 1999)

Siemens

Telekom Austria (Italia Telecom, ..., ÖIAG)

T-Systems Nova (T-Systems, Deutsche Telekom)

KPN

Telecel-Vodafone

Atesio (ZIB spin-off company)

Contents

Martin Grötschel

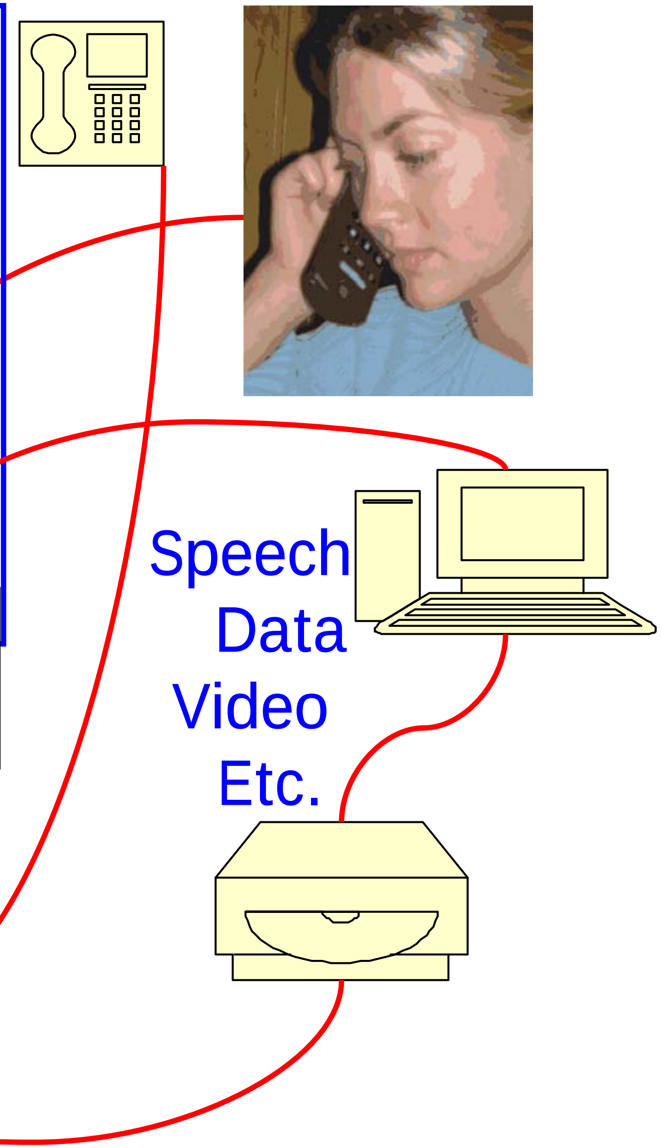
Contents

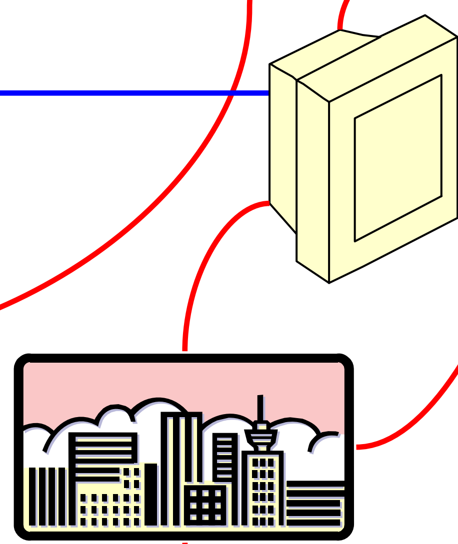
Advertisement

- Modern telecommunication is impossible without mathematics. Cryptography, digital signal encoding, queue management come to your mind immediately.
- But modern mathematics also supports the innovative design and the cost-efficient production of devices and equipment. Mathematics plans low-cost, high-capacity, survivable networks and optimizes their operation.
- Briefly: no efficient use of scarce resources without mathematics – not only in telecommunication.
- Many of these achievements are results of newest research. Their employment in practice is fostered by significant improvements in computing technology.

What is the Telecom Problem?

Design excellent technical devices and a robust network that survives all kinds of failures and organize the traffic such that high quality telecommunication between very many individual units at many locations is feasible at low cost!





What is the Telecom Problem?

Design excellent technical devices and a robust network that survives all kinds of failures and organize the traffic such that high quality telecommunication between very many individual units at many locations is feasible at low cost!

This problem is too general to be solved in one step.

Approach in Practice:

- Decompose whenever possible.
- Look at a hierarchy of problems.
- Address the individual problems one by one.
- Recompose to find a good global solution.

Contents

Martin Grötschel

Cell Phones and Mathematics

- Designing mobile phones
- Task partitioning
- Chip design (VLSI)
- Component design

- Computational logic
- Combinatorial optimization
- Differential algebraic equations

- Producing Mobile Phones
- Production facility layout
- Control of CNC machines
- Control of robots
- Lot sizing
- Scheduling
- Logistics

- Operations research
- Linear and integer programming
- Combinatorial optimization
- Ordinary differential equations

Marketing and Distributing Mobiles

- Financial mathematics
- Transportation optimization

Production and Mathematics: Examples

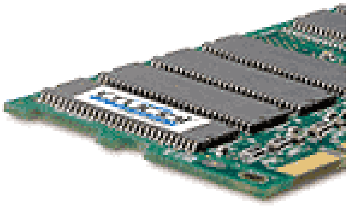
CNC Machine for 2D and 3D cutting and welding (IXION ULM 804)

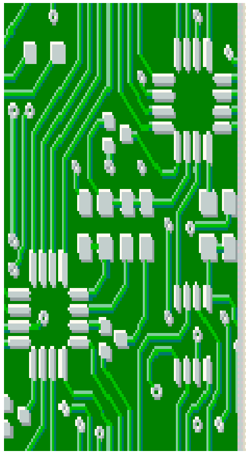
Sequencing of Tasks and Optimization of Moves

SMD

Mounting Devices

Minimizing Production Time via TSP or IP





Printed Circuit Boards

Optimization of Manufacturing

Mobile Phone Production Line

Fujitsu Nasu plant

Contents

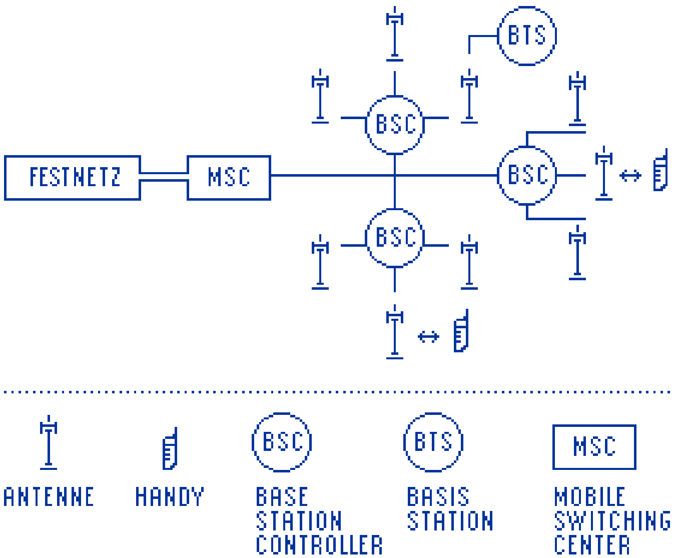
Martin Grötschel

Network Components

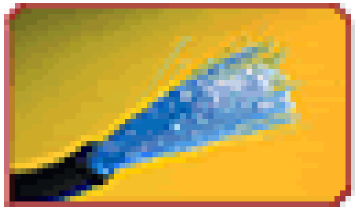
Design, Production, Marketing, Distribution:

Similar math problems as for mobile phones

- Fiber (and other) cables
- Antennas and Transceivers
- Base stations (BTSs)
- Base Station Controllers (BSCs)
- Mobile Switching Centers (MSCs)
- and more...



Component „Cables“



Martin Grötschel

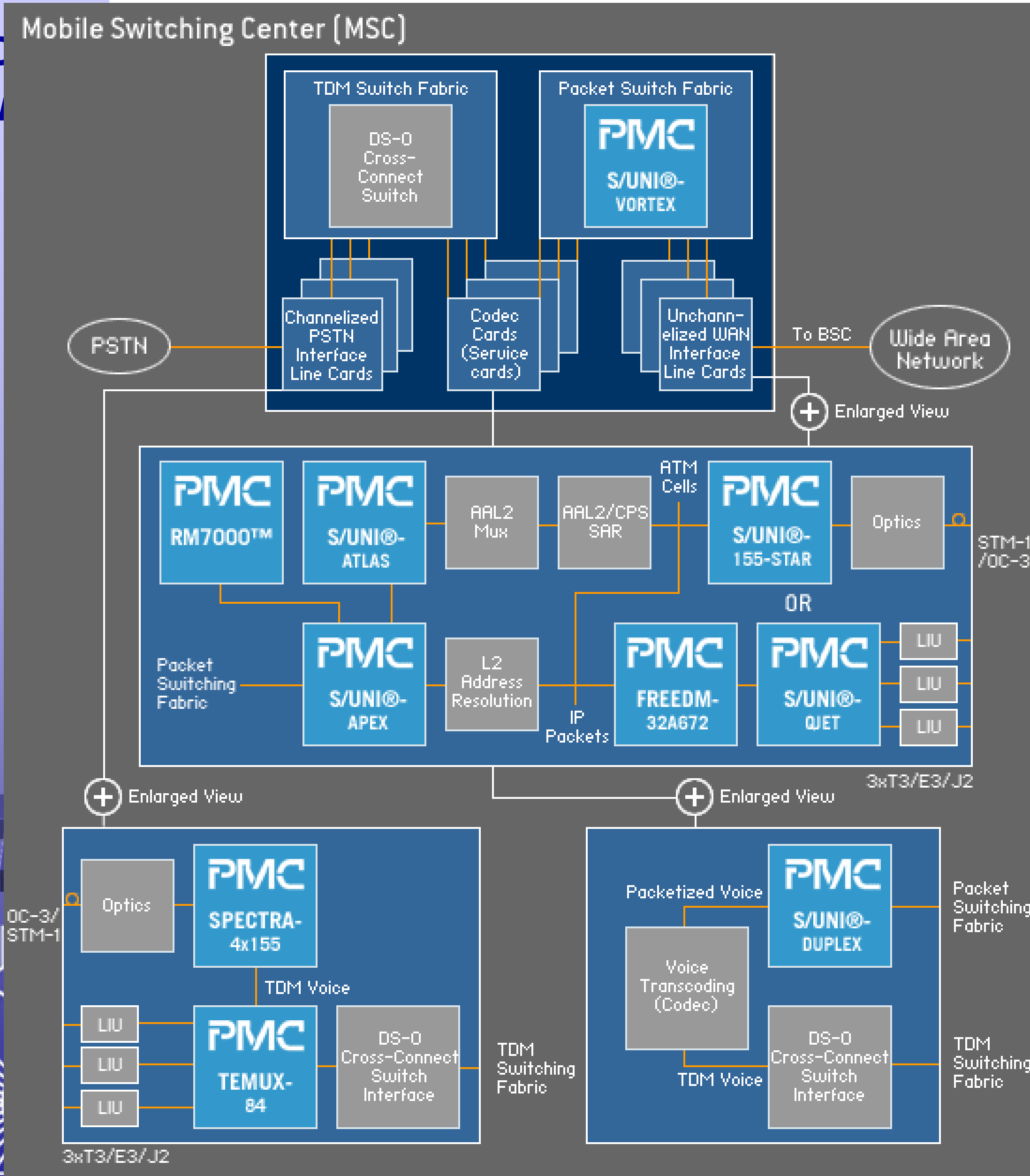
Component „Antennas“

Component „Base Station“

Nokia MetroSite

Nokia UltraSite

Martin Grötschel



Component „Mobile Switching Center":

Example of an MSC Plan

Contents

Network Design: Tasks to be solved Some Examples

- Locating the sites for antennas (TRXs) and base transceiver stations (BTSs)
- Assignment of frequencies to antennas
- Cryptography and error correcting encoding for wireless communication
- Clustering BTSs
- Locating base station controllers (BSCs)
- Connecting BTSs to BSCs

Martin Grötschel

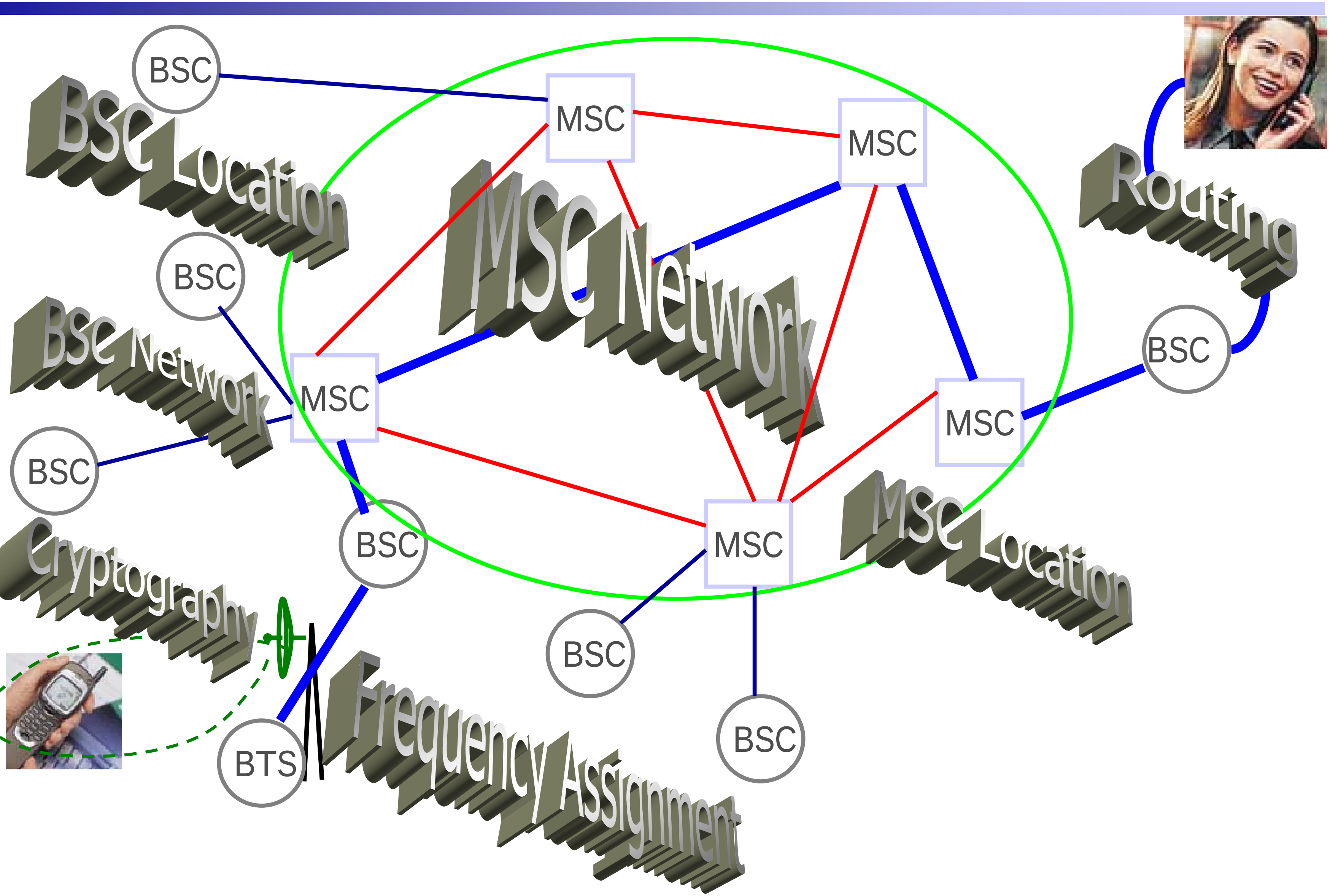
Network Design: Tasks to be solved
Some Examples (continued)

- Locating Mobile Switching Centers (MSCs)
- Clustering BSCs and Connecting BSCs to MSCs
- Designing the BSC network (BSS) and the MSC network (NSS or core network)
  - Topology of the network
  - Capacity of the links and components
  - Routing of the demand
  - Survivability in failure situations

Most of these problems turn out to be Combinatorial Optimization or Mixed Integer Programming Problems

CO at Work

Connecting Mobiles: What´s up?



Martin Grötschel

Contents

Frequency or Channel Assignment

There will be two special lectures on this aspect, one on

- GSM technology

  (GSM = Global System for Mobile Communications)

  and one on

- UMTS

  (UMTS = Universal Mobile Telecommunications System),

  a system that is based on CDMA technology

  (CDMA = Code Division Multiple Access)

  which is currently being deployed.

Martin Grötschel

Contents

G-WiN Data

G-WiN = Gigabit-Wissenschafts-Netz of the DFN-Verein
Internet access of all German universities
and research institutions

- Locations to be connected: 750
- Data volume in summer 2000: 220 Terabytes/month
- Expected data volume in 2004: 10.000 Terabytes/month

Clustering (to design a hierarchical network):

- 10 nodes in Level 1a
- 20 nodes in Level 1b
- All other nodes in Level 2

261 nodes eligible for Level 1

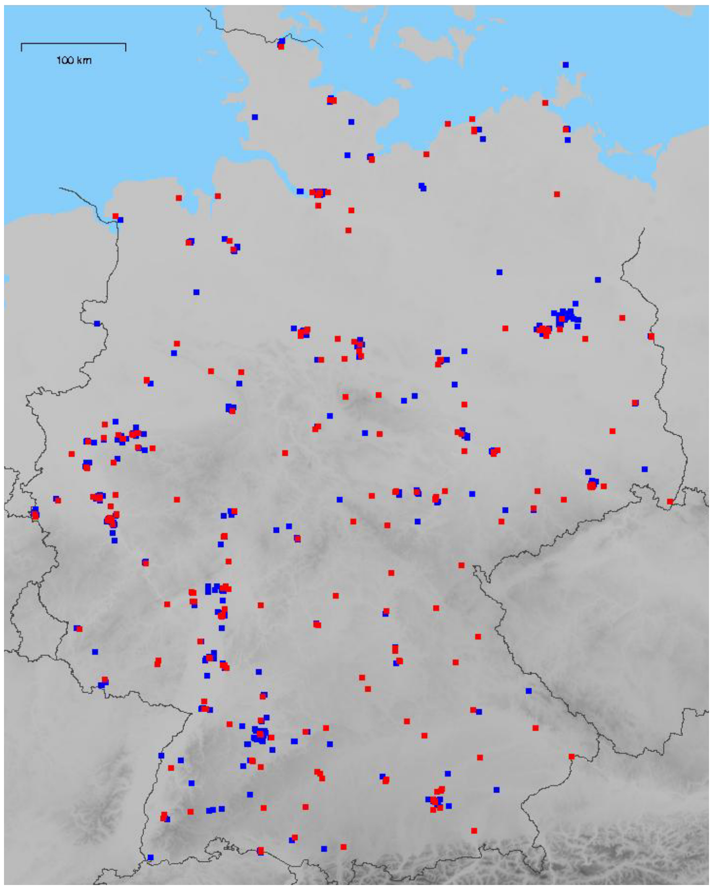
G-WiN Problem

- Select the 10 nodes of Level 1a.
- Select the 20 nodes of Level 1b.

- Each Level 1a node has to be linked to two Level 1b nodes.
- Link every Level 2 node to one Level 1 node.

- Design a Level 1a Network such that
  - Topology is survivable (2-node connected)
  - Edge capacities are sufficient (also in failure situations)
  - Shortest path routing (OSPF) leads to balanced capacity use (objective in network update)
- The whole network should be „stable for the future".
- The overall cost should be as low as possible.

Potential node locations for the 3-Level Network of the G-WIN



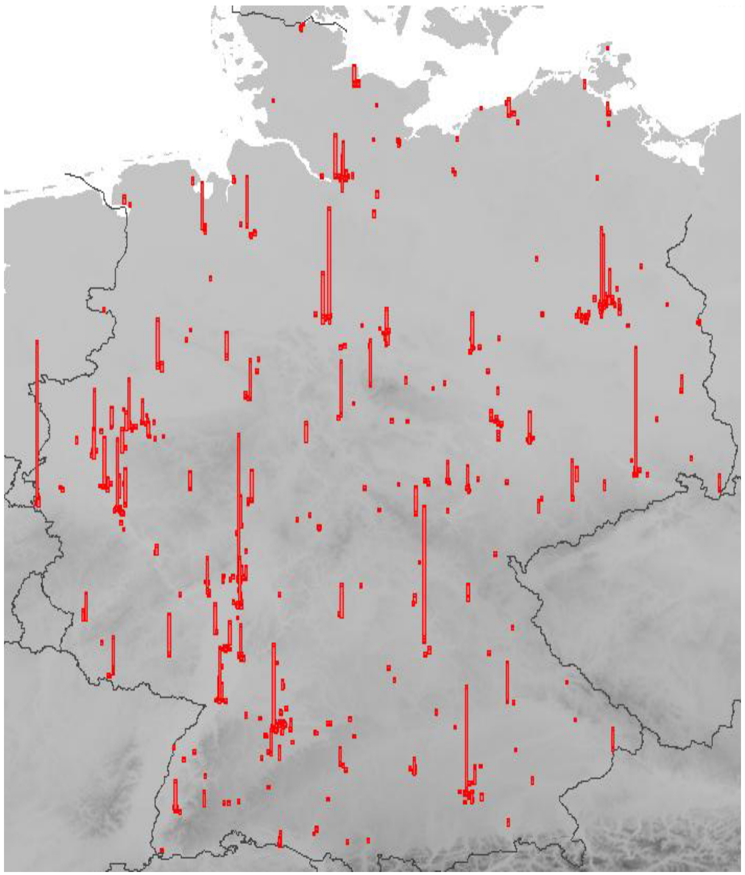
Red nodes are potential level 1 nodes

Blue nodes are all remaining nodes

Cost:
- Connection between nodes
- Capacity of the nodes

berlin
ZIB
Martin Grötschel

Demand distribution



The demand scales with the height of each red line

Aim

Select backbone nodes and connect all non-backbone nodes to a backbone node such that the overall network cost is minimal (access+backbone cost)

Martin Grötschel

G-WiN Location Problem: Data

V = set of locations

Z = set of potential Level 1a locations (subset of V)

K_p = set of possible configurations at location p in Level 1a

For $i \in V$, $p \in Z$ and $k \in K_p$:

w_{ip} = connection costs from i to p

d_i = traffic demand at location i

c_p^k = capacity of location p in configuration k

w_p^k = costs at location p in configuration k

x_{ip} = 1 if location i is connected to p (else 0)

z_p^k = 1 if configuration k is used at location p (else 0)

G-WiN Location/Clustering Problem

$$\min \sum_{p \in Z} \sum_{i \in V} w_{ip} x_{ip} + \sum_{p \in Z} \sum_{k \in K_p} w_p^k z_p^k$$

$$\sum_p x_{ip} = 1$$ Each location i must be connected to a Level 1 node

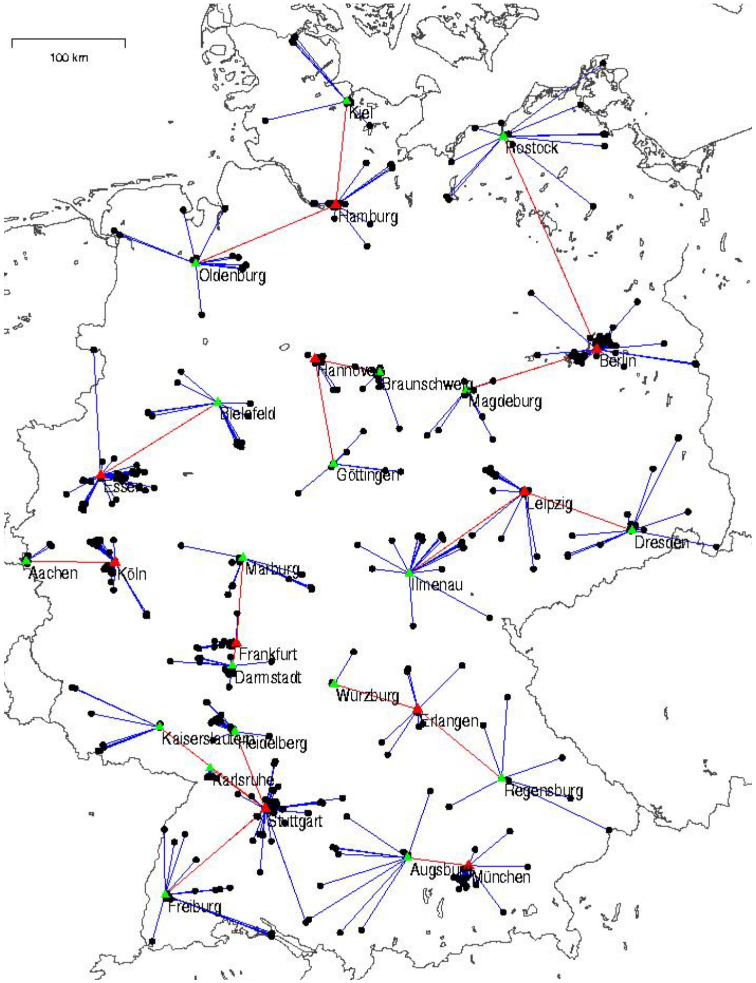
$$\sum_i d_i x_{ip} \leq \sum_k c_p^k z_p^k$$ Capacity at p must be large enough

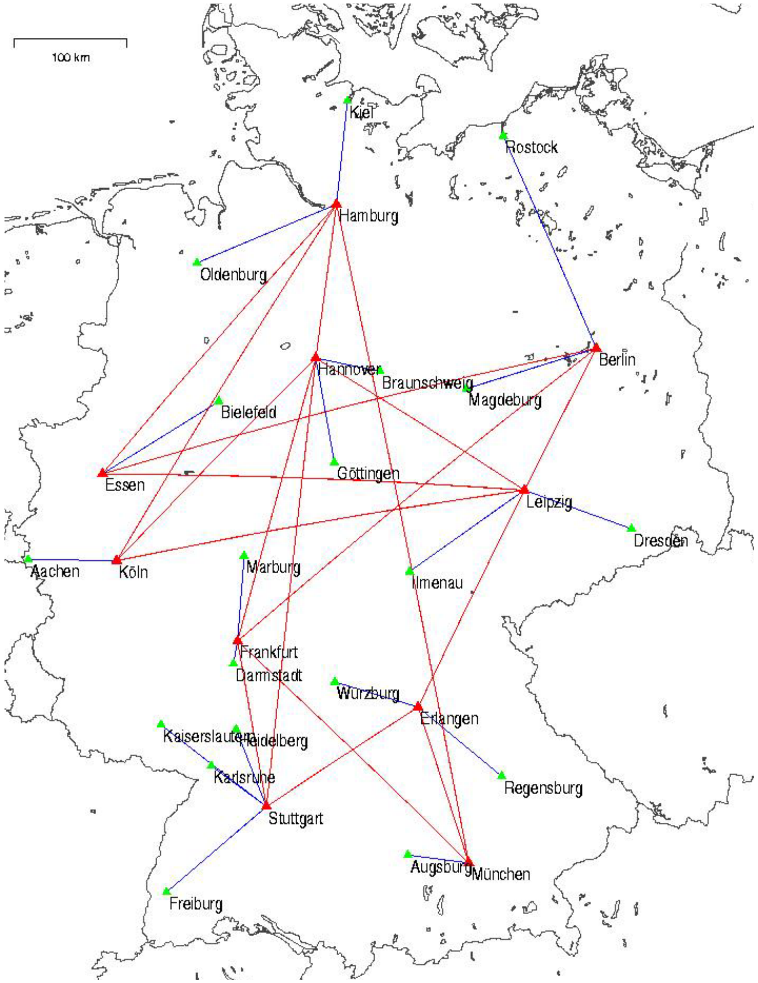
$$\sum_k z_p^k = 1$$ Only one configuration at each Location 1 node

$$\sum_p z_p^k = const$$ # of Level 1a nodes

All variables are 0/1.

Solution: Hierarchy & Backbone





Martin Grötschel

G-WiN Location Problem: Solution Statistics

The DFN problem leads to ~100.000 0/1-variables.
Typical computational experience:
Optimal solution via CPLEX in a few seconds!

A very related problem at Telekom Austria has ~300.000 0/1-variables plus some continuous variables and capacity constraints.
Computational experience (before problem specific fine tuning):

10% gap after 6 h of CPLEX computation,
60% gap after „simplification“
(dropping certain capacities).

Contents

Re-Optimization of Signaling Transfer Points

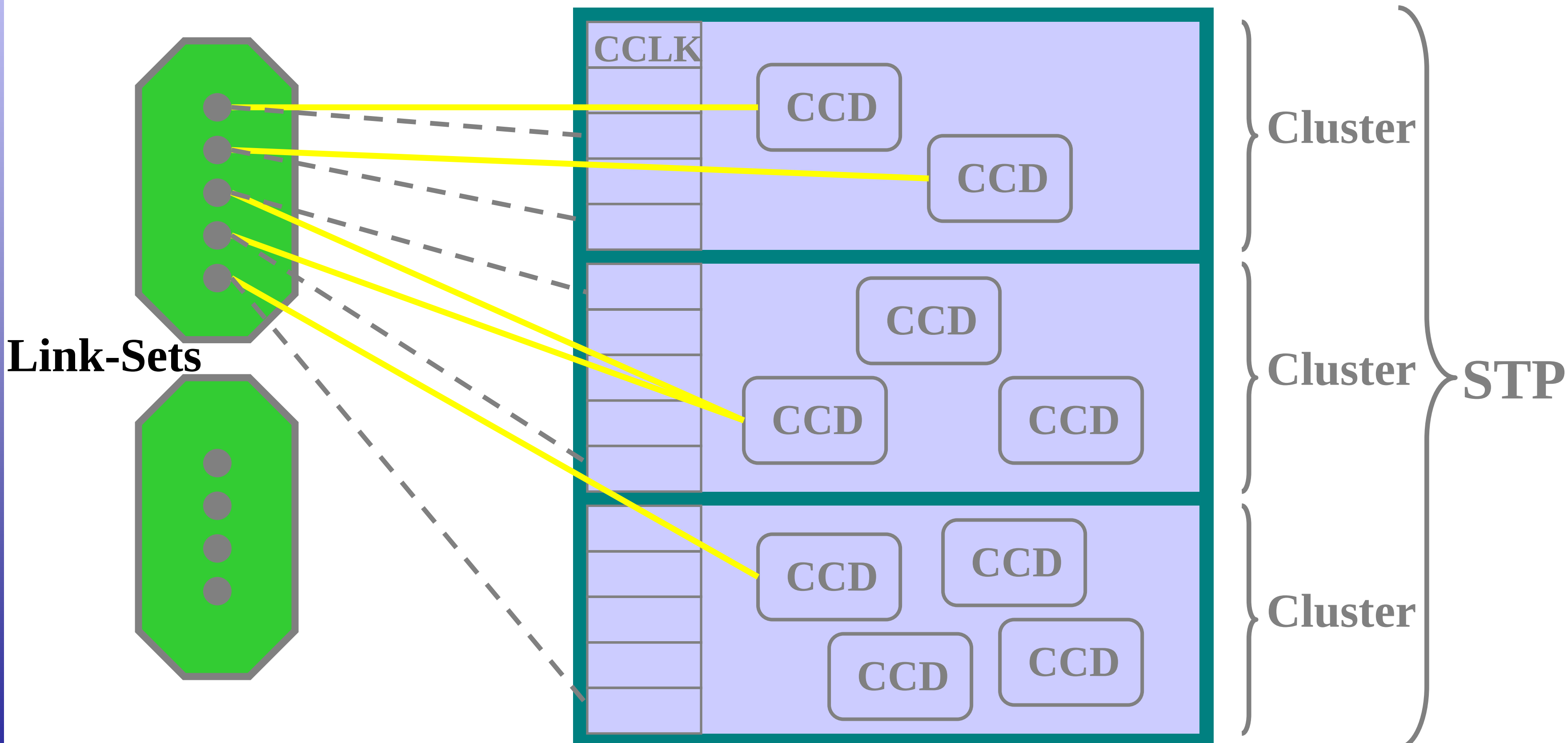
Telecommunication companies maintain a signaling network (in adition to their communication transport network). This is used for management tasks such as:

- Basic call setup or tear down
- Wireless roaming
- Mobile subscriber authentication
- Call forwarding
- Number display
- SMS messages
- Etc.

A. Eisenblätter, A. M. C. A. Koster, R. Wallbaum, R. Wessäly
Load Balancing in Signaling Transfer Points
ZIB-Report 02-50,

Signaling Transfer Point (STP)

CCD=routing unit, CCLK=interface card



CCD=Common Channel Distributors, CCLK=Common Channel Link Controllers

Martin Grötschel

STP – Problem description

Target

Assign each link to a CCD/CCLK

Constraints

At most 50% of the links in a linkset can be assigned to a single cluster

Number of CCLKs in a cluster is restricted

Objective

Balance load of CCDs

STP – Mathematical model

Data

- C set of CCDs j
- L set of links i
- D_i demand of link i
- P set of link-sets
- Q set of clusters
- L_p subset of links in link-set p
- C_q subset of CCDs in cluster q
- c_q #CCLKs in cluster q

Variables

$$x_{ij} \in \{0,1\}, i \in L, j \in C$$

$x_{ij} = 1$ if and only if link i is assigned to CCD j

STP – Mathematical model

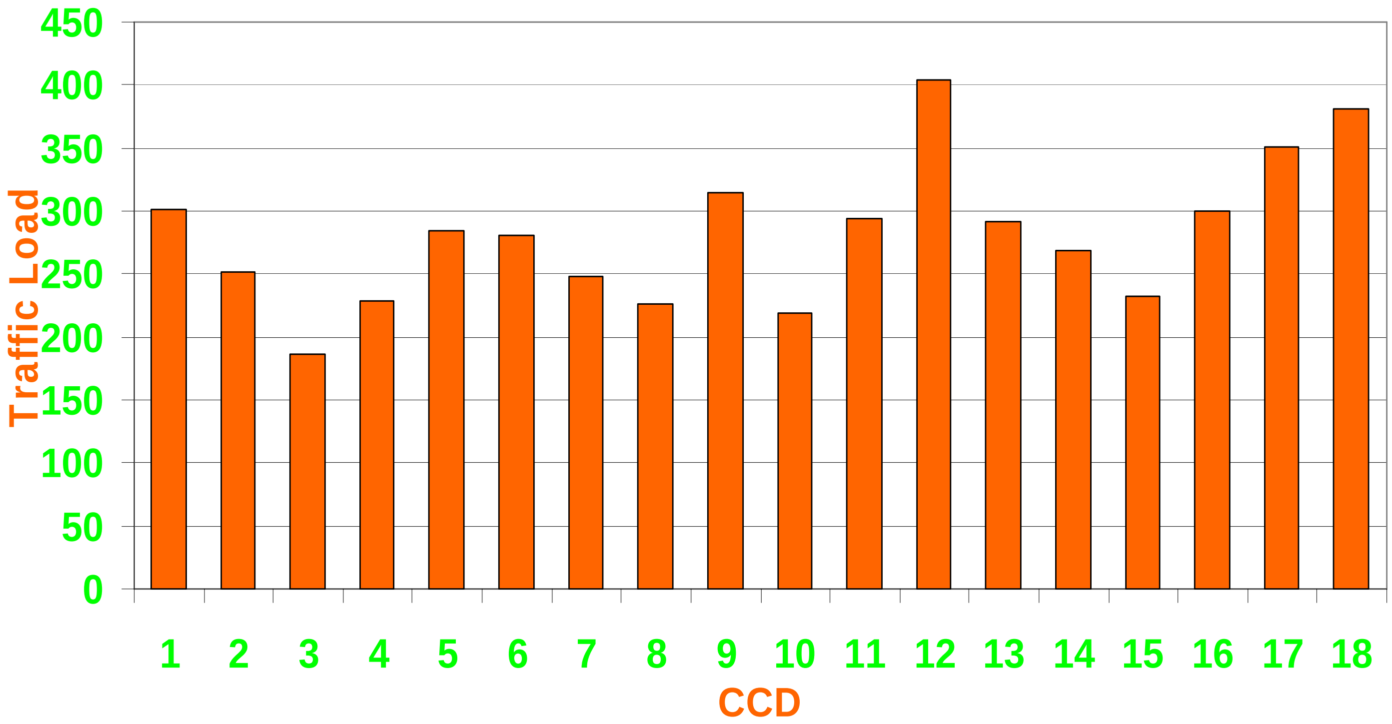
Min load difference

$$\min\ y - z$$

$\sum_{j \in C} x_{ij} = 1$	$i \in L$	Assign each link
$\sum_{i \in L} D_i x_{ij} \leq y$	$j \in C$	Upper bound of CCD-load
$\sum_{i \in L} D_i x_{ij} \geq z$	$j \in C$	Lower bound of CCD-load
$\sum_{i \in L_p} \sum_{j \in C_q} x_{ij} \leq \left\lceil \lvert L_p \rvert / 2 \right\rceil$	$p \in P, q \in Q$	Diversification
$\sum_{i \in L} \sum_{j \in C_q} x_{ij} \leq c_q$	$q \in Q$	CCLK-bound
$x_{ij} \in \{0,1\}$		Integrality

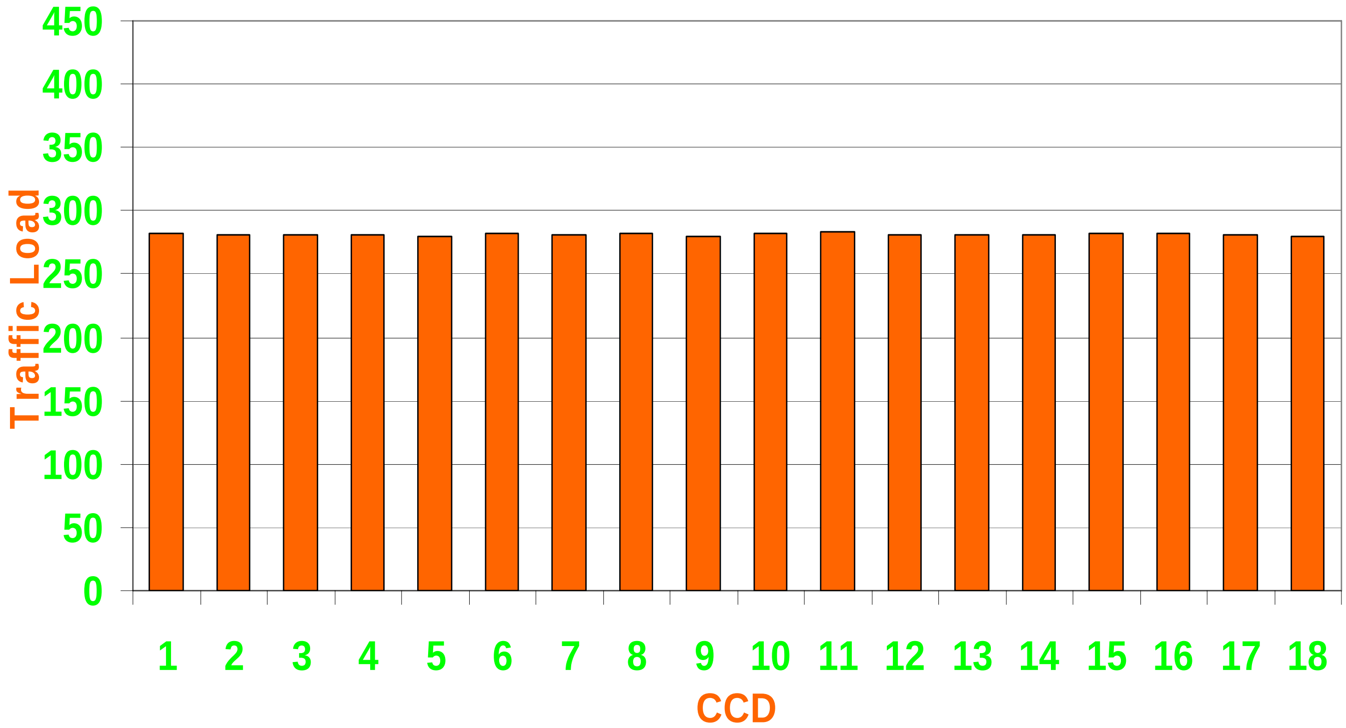
STP – former (unacceptable) solution

Minimum: 186 **Maximum:** 404 **Load difference:** 218



STP – „Optimal solution“

Minimum: 280 Maximum: 283 Load difference: 3



STP – Practical difficulty

Problem: 311 rearrangements are necessary to migrate to the optimal solution

→ **Reformulation with new objective**

Find a best solution with a restricted number of changes

STP – Reformulated Model

Min load difference

$$\min\; y - z$$

$$\sum_{j \in C} x_{ij} = 1 \qquad i \in L$$ Assign each link

$$\sum_{i \in L} D_i x_{ij} \le y \qquad j \in C$$ Upper bound of CCD-load

$$\sum_{i \in L} D_i x_{ij} \ge z \qquad j \in C$$ Lower bound of CCD-load

$$\sum_{i \in L_p} \sum_{j \in C_q} x_{ij} \le \left\lceil |L_p| / 2 \right\rceil \qquad p \in P, q \in Q$$ Diversification

$$\sum_{i \in L} \sum_{j \in C_q} x_{ij} \le c_q \qquad q \in Q$$ CCLK-bound

$$\sum_{i \in L} \sum_{j \in C, j \ne j^*(i)} x_{ij} \le B$$ Restricted number of changes!

$$x_{ij} \in \{0,1\}$$ Integrality

Martin Grötschel

STP – Alternative Model

$$\min \sum_{i \in L} \sum_{j \in C, j \neq j^*(i)} x_{ij}$$ **Min # changes**

$$\sum_{j \in C} x_{ij} = 1 \qquad i \in L$$ Assign each link

$$\sum_{i \in L} D_i x_{ij} \leq y \qquad j \in C$$ Upper bound of CCD-load

$$\sum_{i \in L} D_i x_{ij} \geq z \qquad j \in C$$ Lower bound of CCD-load

$$\sum_{i \in L_p} \sum_{j \in C_q} x_{ij} \leq \left\lceil |L_p| / 2 \right\rceil \qquad p \in P, q \in Q$$ Diversification

$$\sum_{i \in L} \sum_{j \in C_q} x_{ij} \leq c_q \qquad q \in Q$$ CCLK-bound

$$y - z \leq D$$ **Restricted load difference**

$$x_{ij} \in \{0,1\}$$ Integrality

STP – New Solutions

White: D=50, (alternative)

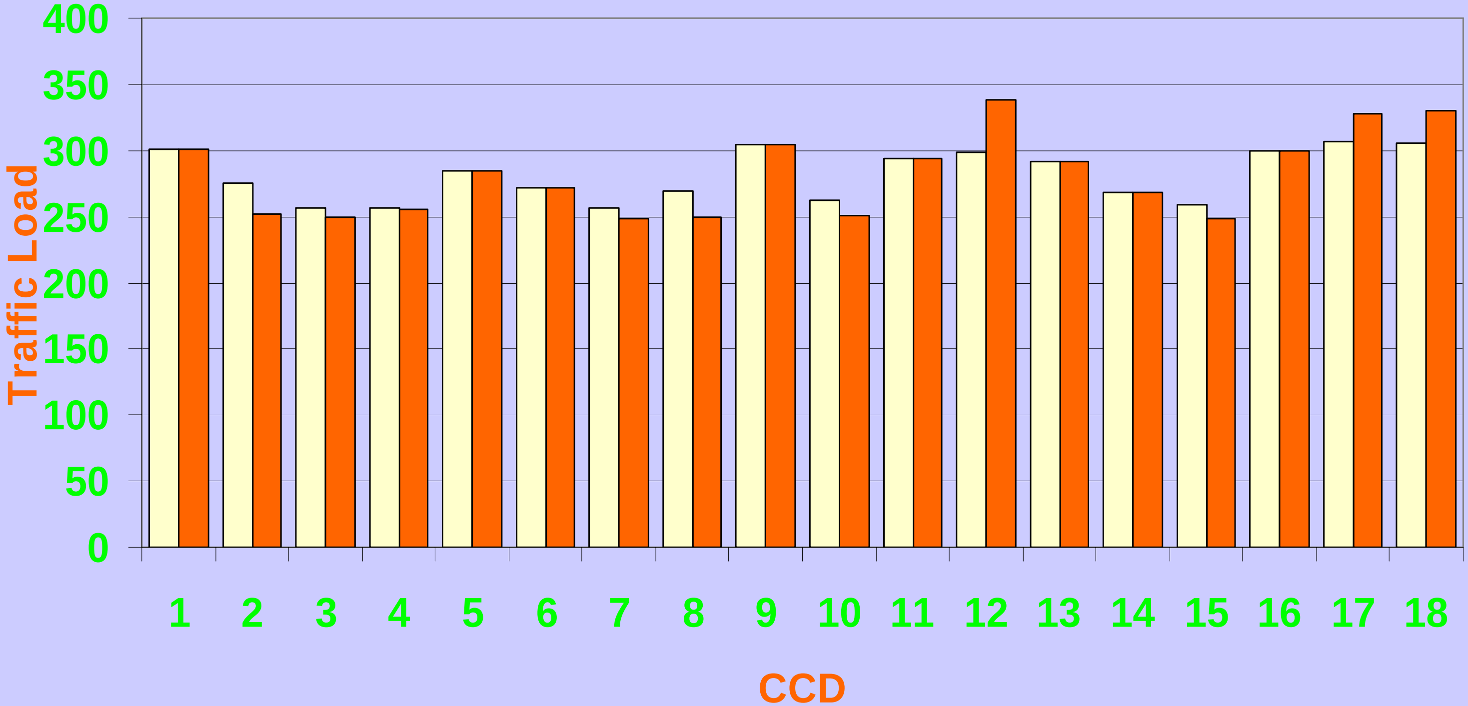
Minimum: 257 Maximum: 307

D=Load difference: 50 Number of changes: 12

Orange: B=8, (reformulated)

Minimum: 249 Maximum: 339

Load difference: 90 Number of changes : 8



STP – Experimental results

Max changes	0	5	10	15	20
Load differences	218	129	71	33	14

1 hour application of CPLEX MIP-Solver for each case

STP - Conclusions

It is possible to achieve

85%

of the optimal improvement with less than

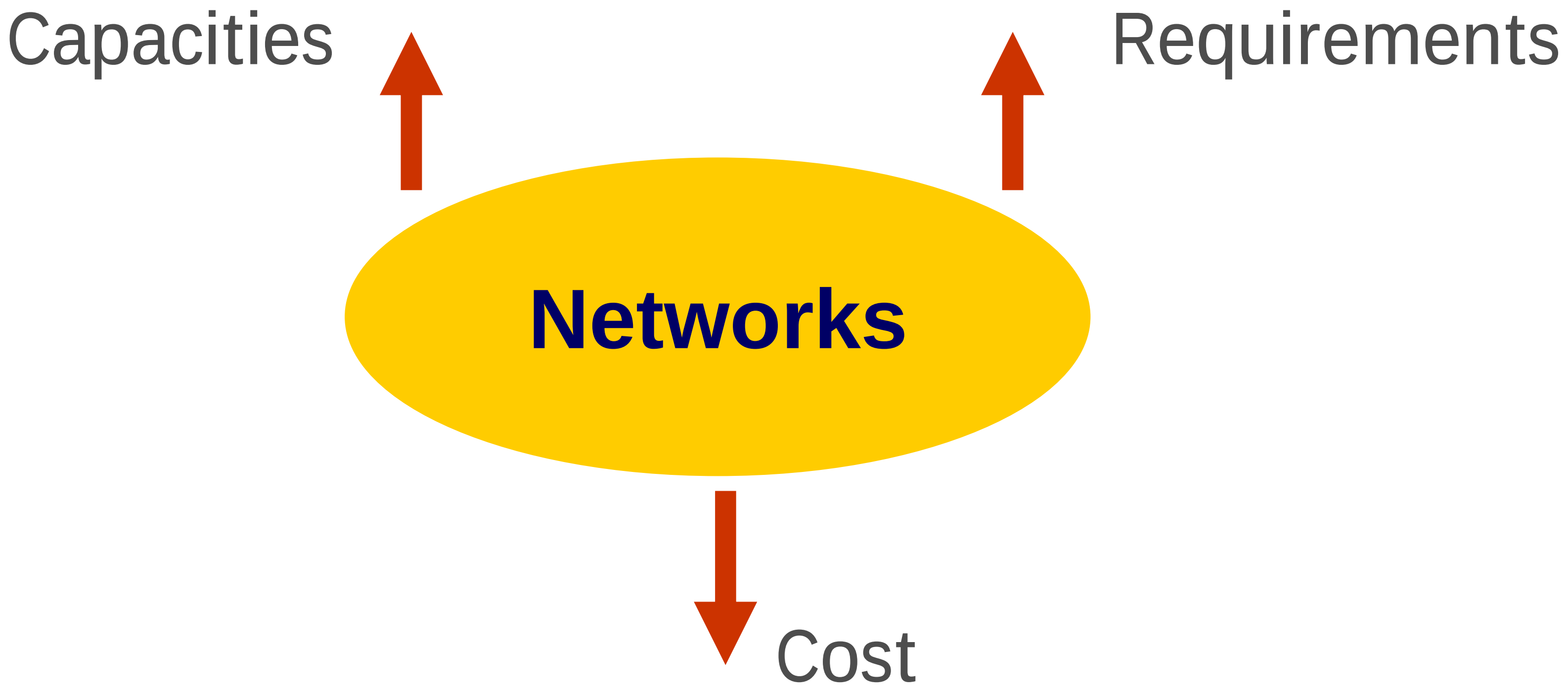
5%

of the changes necessary to obtain a load balance optimal solution !

Martin Grötschel

Contents

Network Optimization



What needs to be planned?

- Topology
- Capacities
- Routing
- Failure Handling (Survivability)

} special lecture

- IP Routing
- Node Equipment Planning
- Optimizing Optical Links and Switches

DISCNET: A Network Planning Tool
(Dimensioning Survivable Capacitated NETworks)

Atesio ZIB Spin-Off

Contents

Martin Grötschel

DFN: German Research Network

Internet for German universities, scientific institutions, museums, libraries, etc.

B-WiN: Breitband WissenschaftsNetz, 1996 – 2000

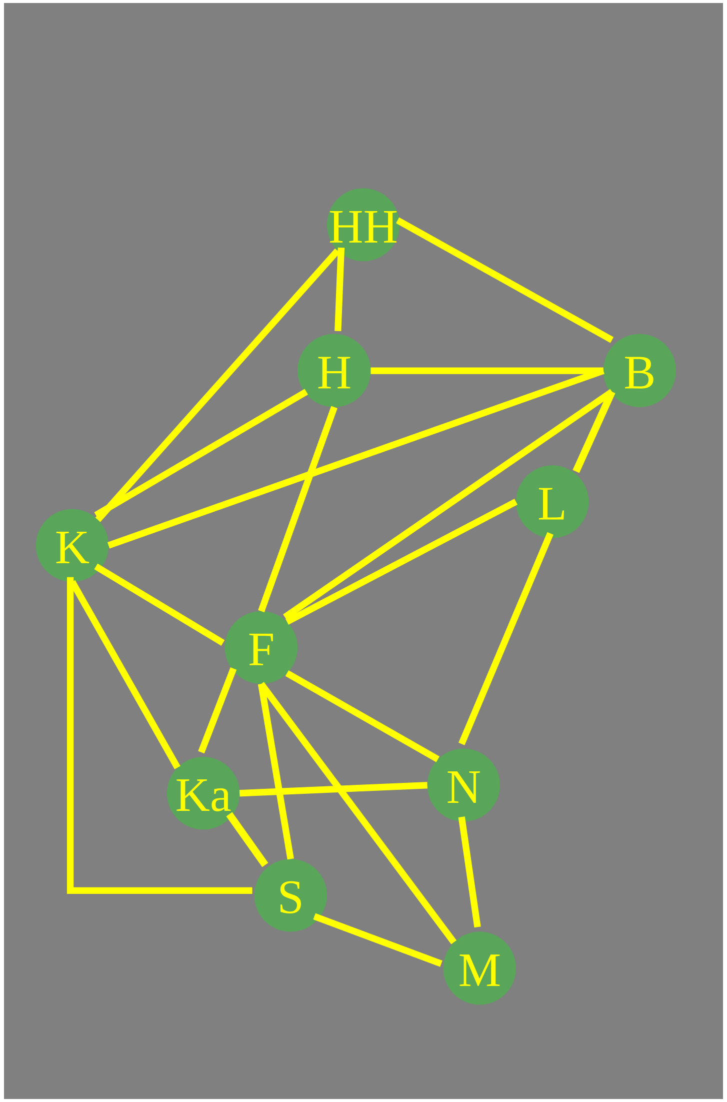
- virtual private network from DeTeSystems
- ~ 400 users
- Backbone links 35 – 155 Mbit/s

G-WiN: Gigabit WissenschaftsNetz, 2000 - ...

- virtual private SDH/WDM network from DeTeSystems
- IP over SDH/WDM
- Backbone links 155 Mbit/s – 10 Gbit/s

X-Win: ...

The network design problem



Supply Graph

Demands

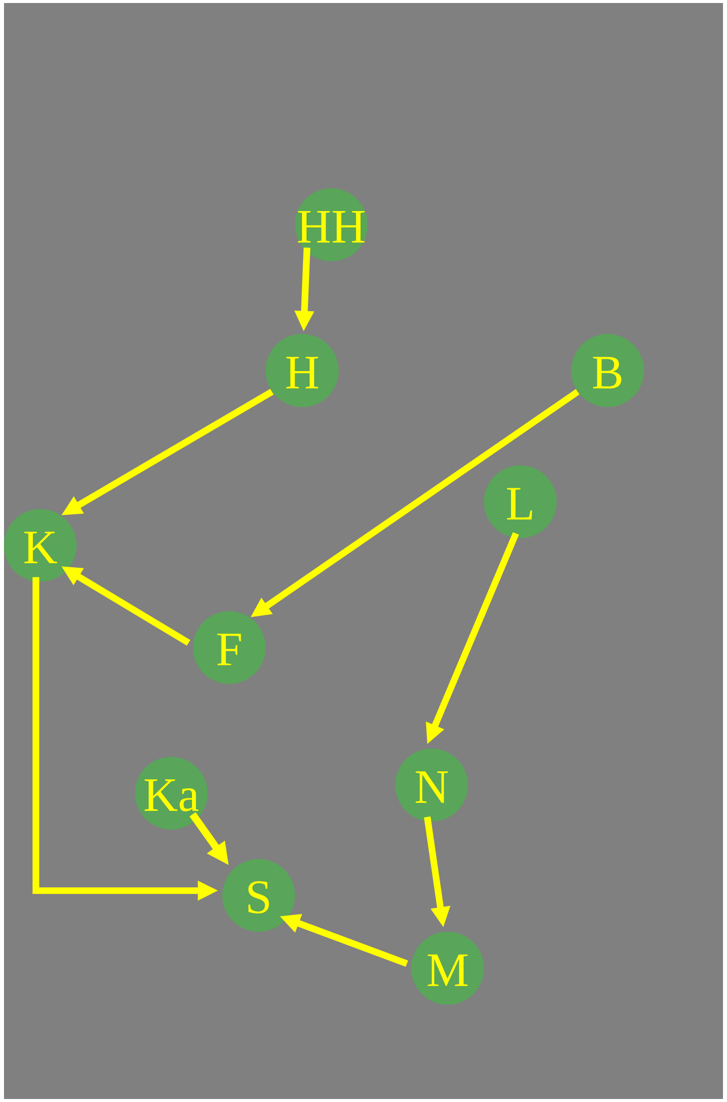
Discrete Capacities & Costs

OSPF-Routing

Survivability

Further technical constraints

OSPF-Routing: Weights

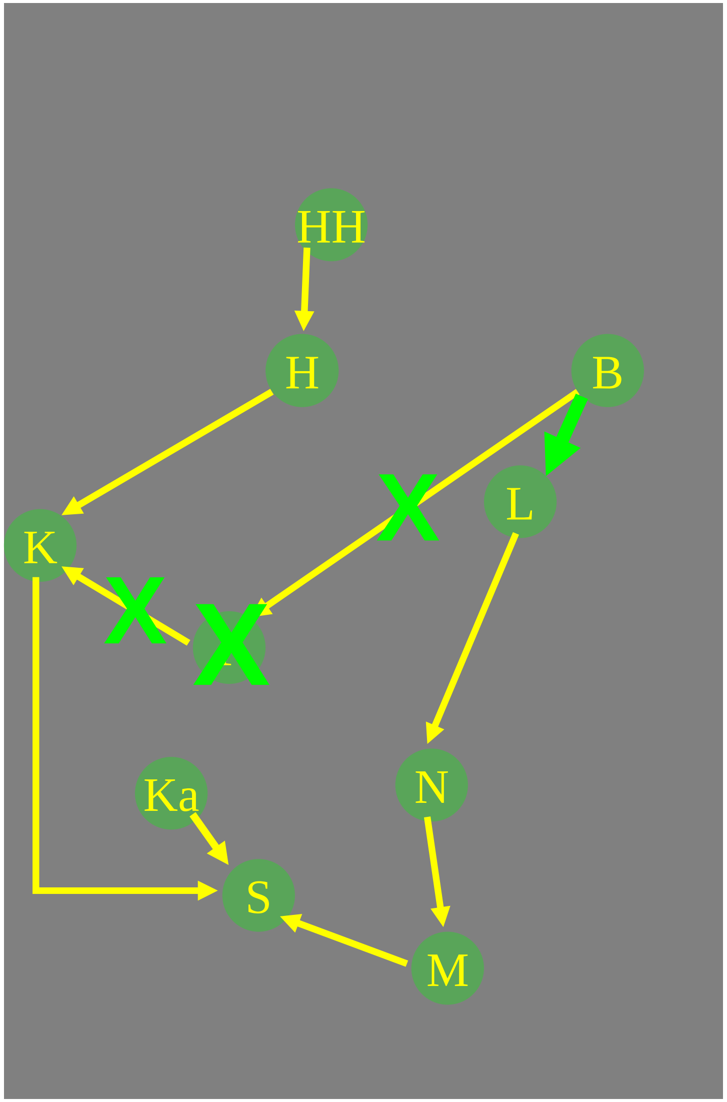


Non-bifurcated routing on shortest paths **w.r.t. non-negative** link weights

Sink-tree for each destination

Unique shortest paths necessary to guarantee feasible routing in practice!

OSPF-Routing: Survivability



Survivability: Capacities must accommodate a feasible OSPF-routing in

- the normal operating state
- single edge and single node failure states

➡ **2-node-connectivity**

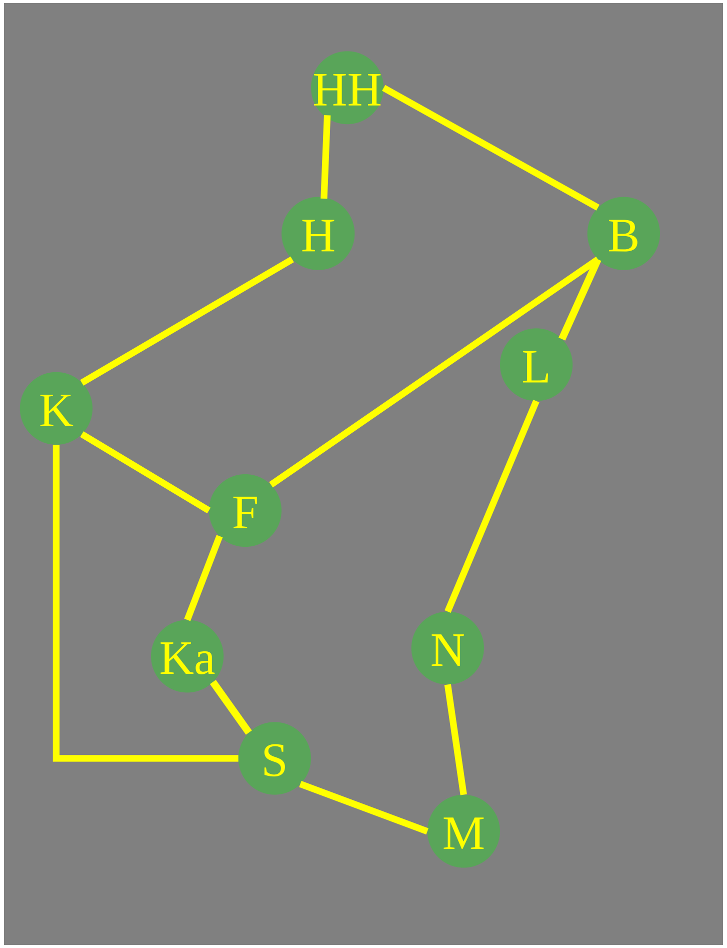
Model & Solution approach

Mixed-integer programming model

Solution approach (Decomposition)

- Network design
  - Cutting plane algorithm
  - Heuristics
- Weight computation
  - Linear programming

Results: Original network



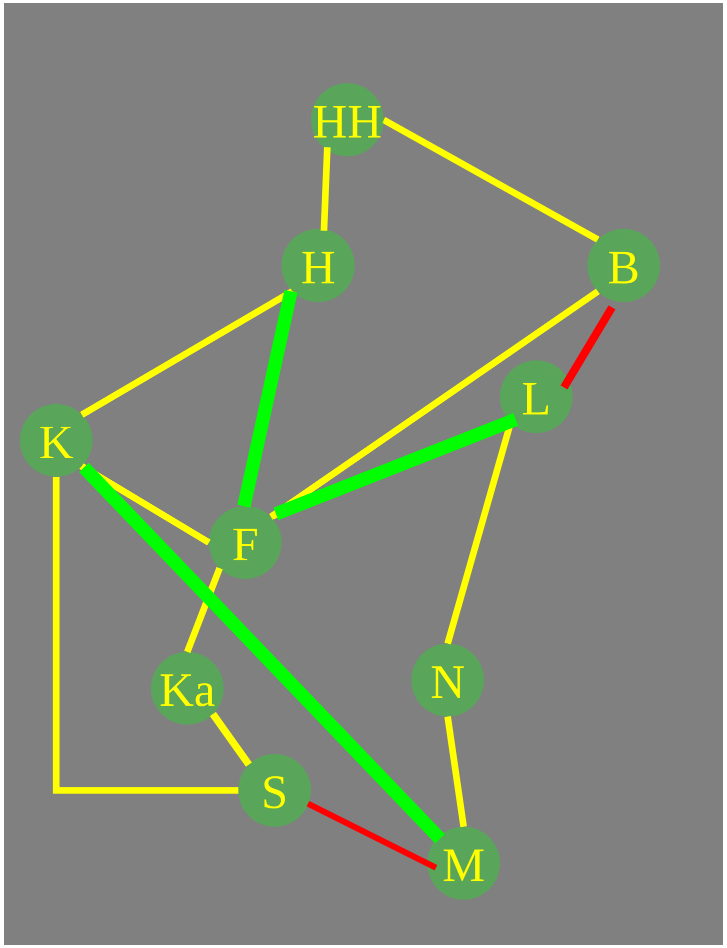
Demands: Nov 1997

Routing with perturbed unit-weights

Original topology

Cost: 12.04

Results: New Network



Demands: Nov 1997

Routing with perturbed unit-weights

Maximal 3 topology changes

Cost: 10.71

10 % improvement on the network that has already been optimized with our algorithm

Martin Grötschel

Conclusion

OSPF-routing (weights)

and

topology & capacities

must be simultaneously optimized !

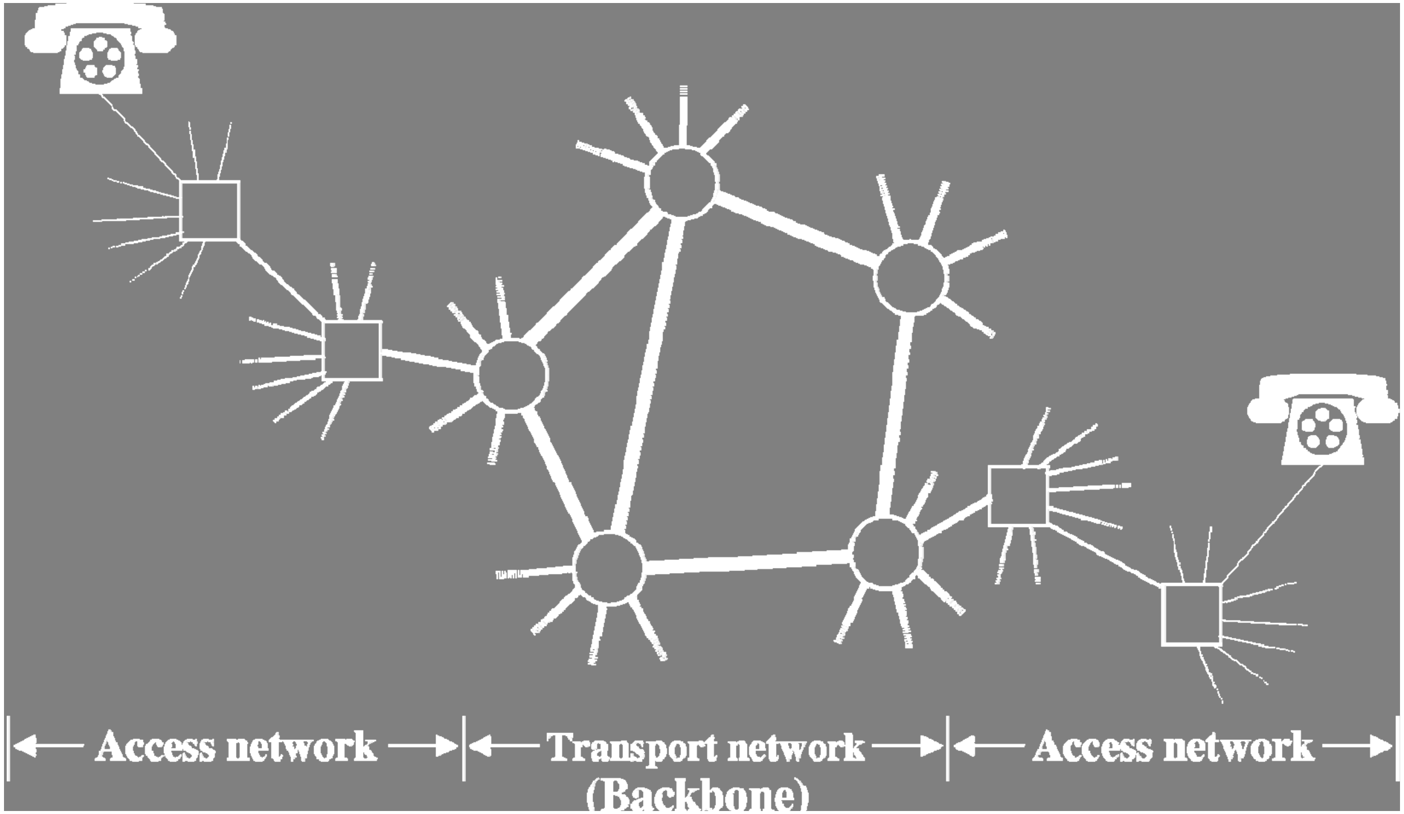
X-Win

- ZIB (Andreas Bley) has been involved in the planning of the X-Win of Deutsches Forschungsnetz (DFN) to be installed in 2006/7:
  - Locations
  - Network
  - Hub and Line Capacities

Contents

Martin Grötschel

Telecommunication networks

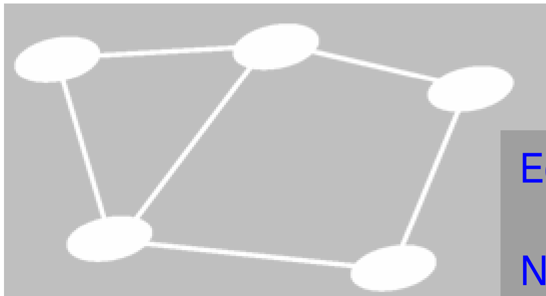


- Focus: Backbone layer
- Planning-objective: Cost-minimal network
- Reason: New technology, new services

Martin Grötschel

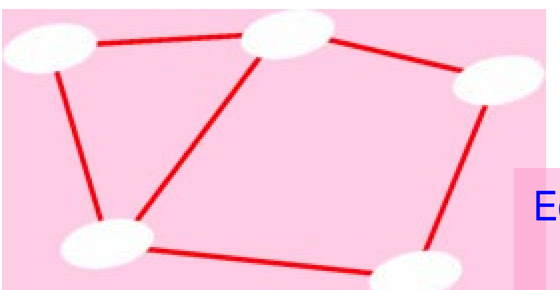
From copper to fiber...

Electronic Networks



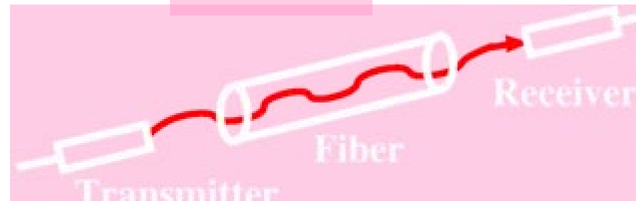
Edges:
electronic
Nodes:
electronic

Fiber-Networks



Edges:
optic
Nodes:
electronic

Fiber

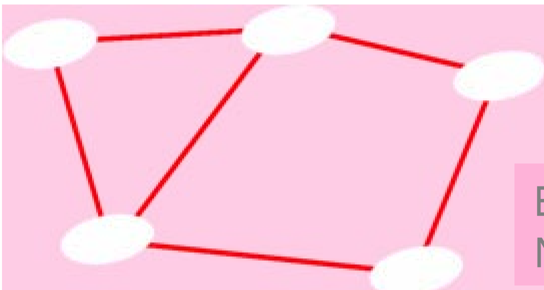


- high transmission-capacity, but restricted mileage
- various types

  (uni- and bi-directional)
- various qualities

...to WDM ...

Fiber-Networks

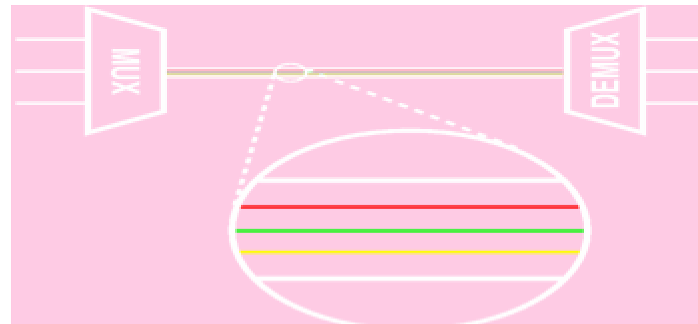


Edges: optic
Nodes: electronic

(Point-to-Point-) WDM-Networks

Edges:
optic (WDM)
Nodes:
electronic

Wavelength Division Multiplexing (WDM)



- Capacity is multiplied
- growing multiplex factors
- different systems (#channels and spectra)

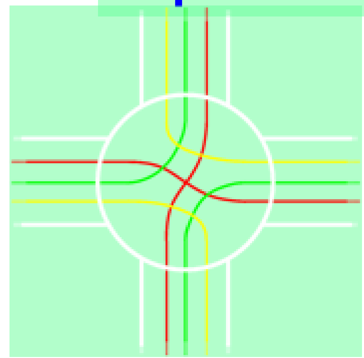
Martin Grötschel

... to all optical networks

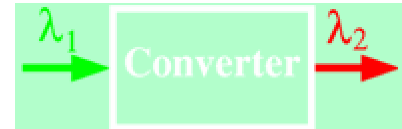
(Point-to-Point-) WDM-Networks

Edges:
optic (WDM)
Nodes:
electronic

Optical devices

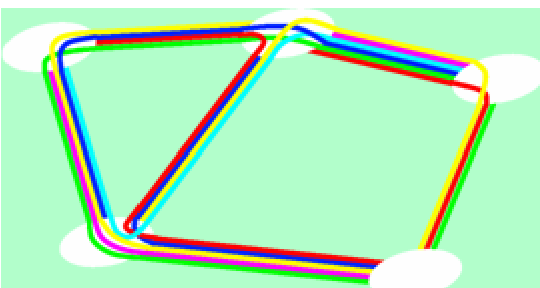


Optical Cross-connect (OXC)



Optical Wavelength Converter

Optical Networks



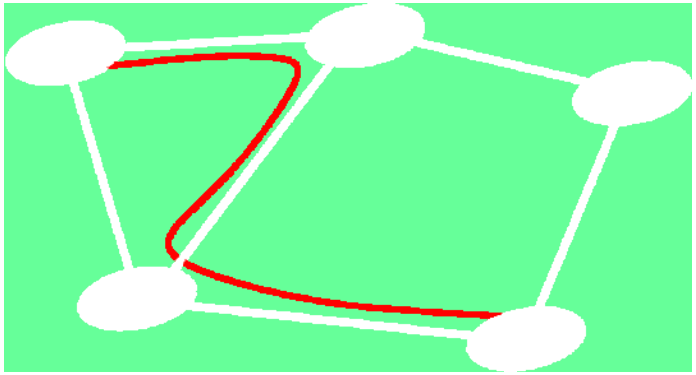
Edges:
optic (WDM)
Nodes:
optic

- **Switching optical channels w/o o-e-o-conversion**
- **Switching of arbitrary wavelengths**

Martin Grötschel

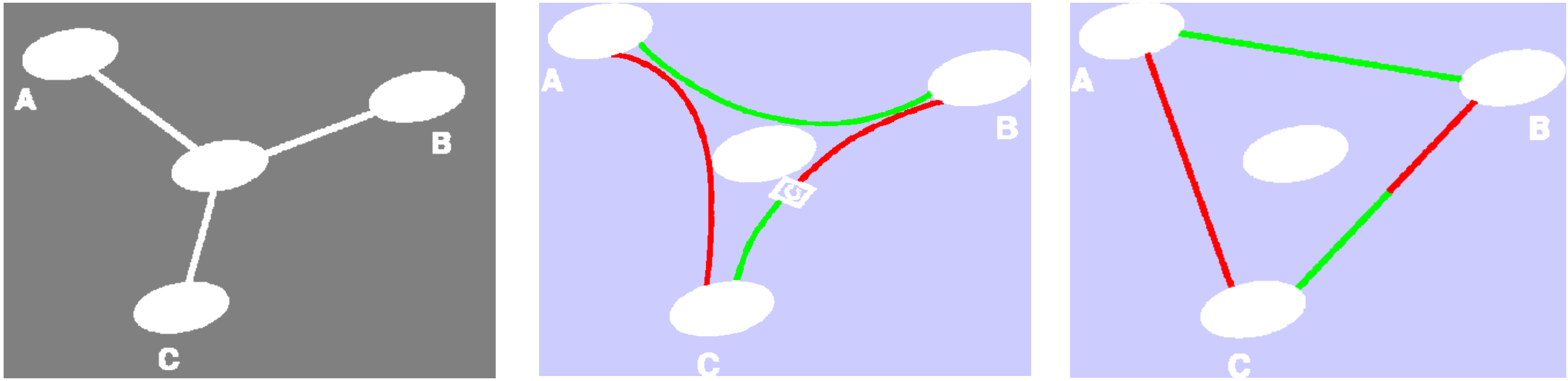
Lightpaths

Lightpath = pure optical connection between two nodes via one or multiple fibers with optical switching in traversed nodes



- length restriction (dispersion and attenuation)
- wavelength assignment

Optical Network Configuration



physical topology

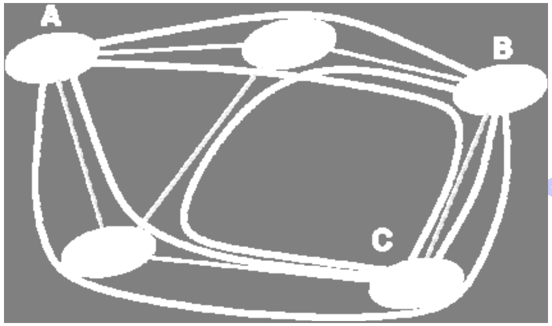
lightpath configuration

virtual topology

Planning Optical Networks

Input: Network topology and demand-matrix

Output: Cost-minimal network configuration with:



Planning present networks

Dimensioning

Edges: Transmission-capacity

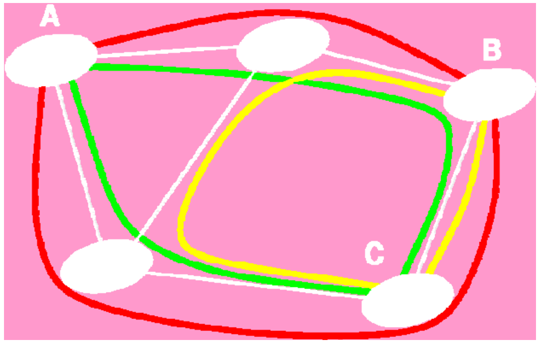
Nodes: Switching-capacity

Routing

Determination of routing (with survivability)

Coloring

Conflict-free wavelength assignment (with converters)



Planning optical networks

Modeling Optical Networks

- Overall problem is too complex
  - extreme large mathematical model (intractable)
- Decomposition into two subproblems:
  - Dimensioning and Routing
    - connection to previous network planning
    - integer routing requirement
  - Wavelength Assignment
    - conflict-free wavelength assignment to lightpaths

Dimensioning and Routing

Present

Network Dimensioning and Routing:

- Capacity planning
  - mainly edge capacities

  ⇒ integer capacity variables
- Routing either bifurcated (splittable)

  ⇒ continuous flow or path variables
- or non-bifurcated (unsplittable)

  ⇒ 0-1 flow or path variables

Optical

Network Dimensioning and Routing:

- Capacity planning
  - both edge and node related

  ⇒ integer capacity variables
- Routing in lightpaths (integer-valued)

  ⇒ general integer routing variables
- Lightpath length restriction

  ⇒ only via path variables

Martin Grötschel

Integer Programming Formulation

$G=(V,E)$	Physical topology
Q	Demand set, (s^q,t^q,d^q) source, target and lightpath demand
P^q	Set of paths from s^q to t^q that are allowed to route lightpaths for commodity q
T_{mn}	Index set of available edge capacity levels (fibers + WDM systems) for edge mn
κ^0_{mn} , κ^t_{mn}	Installed edge capacity (channels), available capacity levels
c^T_{mn}	Cost of installing edge capacity level t at edge mn
Θ_m	Index set of available node capacity levels (OXCs) for node m
κ^0_m, κ^θ_m	Installed node capacity (ports), available capacity levels
c^θ_m	Cost of installing node capacity level θ at node m

Integer Programming Formulation

z_p^q	# of lightpaths of commodity q routed via path p
ℓ_{mn}^t	0-1 variable indicating whether edge capacity level t is used at edge mn
x_m^θ	0-1 variable indicating whether node capacity level θ is used at node m

Integer Programming Formulation

$$\min \sum_{m\in V}\sum_{\theta\in\Theta_m} c_{OXC} x_m^\theta + \sum_{mn\in E}\sum_{t\in T_{mn}} c_{mn}^t \ell_{mn}^t$$

$$s.t. \quad \sum_{p\in P^q} z_p^q = d^q \qquad \forall q\in Q$$

Routing

$$\sum_{q\in Q}\sum_{p\in P^q:mn\in p} z_p^q \le \kappa_{mn}^0 + \sum_{t\in T_{mn}} \kappa_{mn}^t \ell_{mn}^t \qquad \forall mn\in E$$

$$\sum_{t\in T_{mn}} \ell_{mn}^t \le 1 \qquad \forall mn\in E$$

Edge Capacity

$$\sum_{q\in Q}\sum_{p\in P^q:m\in p} z_p^q + \sum_{q\in Q:m=s^q} d^q \le \kappa_m^0 + \sum_{\theta\in\Theta_m} \kappa_m^\theta x_m^\theta \qquad \forall m\in V$$

$$\sum_{\theta\in\Theta_m} x_m^\theta \le 1 \qquad \forall m\in V$$

Node Capacity

$$z_p^q \in Z_0^+;\ \ell_{mn}^t \in \{0,1\};\ x_m^\theta \in \{0,1\}$$

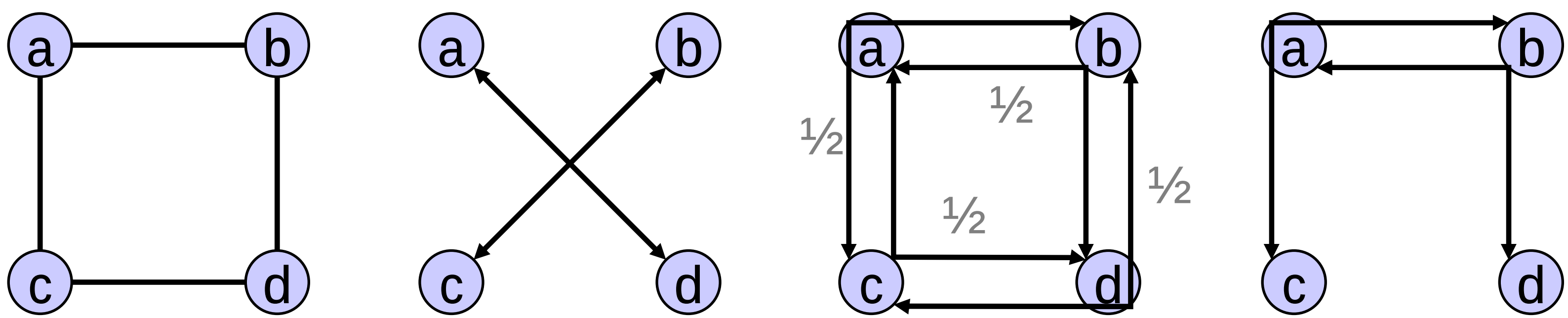
Formulation Alternatives

- Depending on concrete capacity structure other variables can be used
- Every lightpath can be considered as a single commodity:
  - non-bifurcated routing of commodities, all with unit demand
  - number of variables is blown up
  - available inequalities for non-bifurcated routing are less/not effective for unit demands (with integer capacity)

Martin Grötschel

Computational Experiments

- Deleting integrality requirements yields surprisingly few non-integer routings



Remaining free capacity 1 at every edge

Lightpaths to be routed

Continuous routing

Lightpath routing

Martin Grötschel

Concluding Remarks on Optical Networks

- Lightpath routing implies (general) integer routing variables
- Formulation alternative with d^q non-bifurcated commodities unattractive
- IP traffic results in assymmetric demand matrix:
  - symmetric routing not possible
  - asymmetric routing formulation
- Multi-hop networks require 2-layer formulation
- Wavelength assignment introduces a new aspect of optical network design
- Survivability concepts have to be added

Contents

Martin Grötschel

Summary

Telecommunication Problems such as

- Frequency Assignment
- Locating the Nodes of a Network Optimally
- Balancing the Load of Signaling Transfer Points
- Integrated Topology, Capacity, and Routing Optimization as well as Survivability Planning
- Planning IP Networks
- Optical Network Design
- and many others

can be succesfully attacked with optimization techniques.

Summary

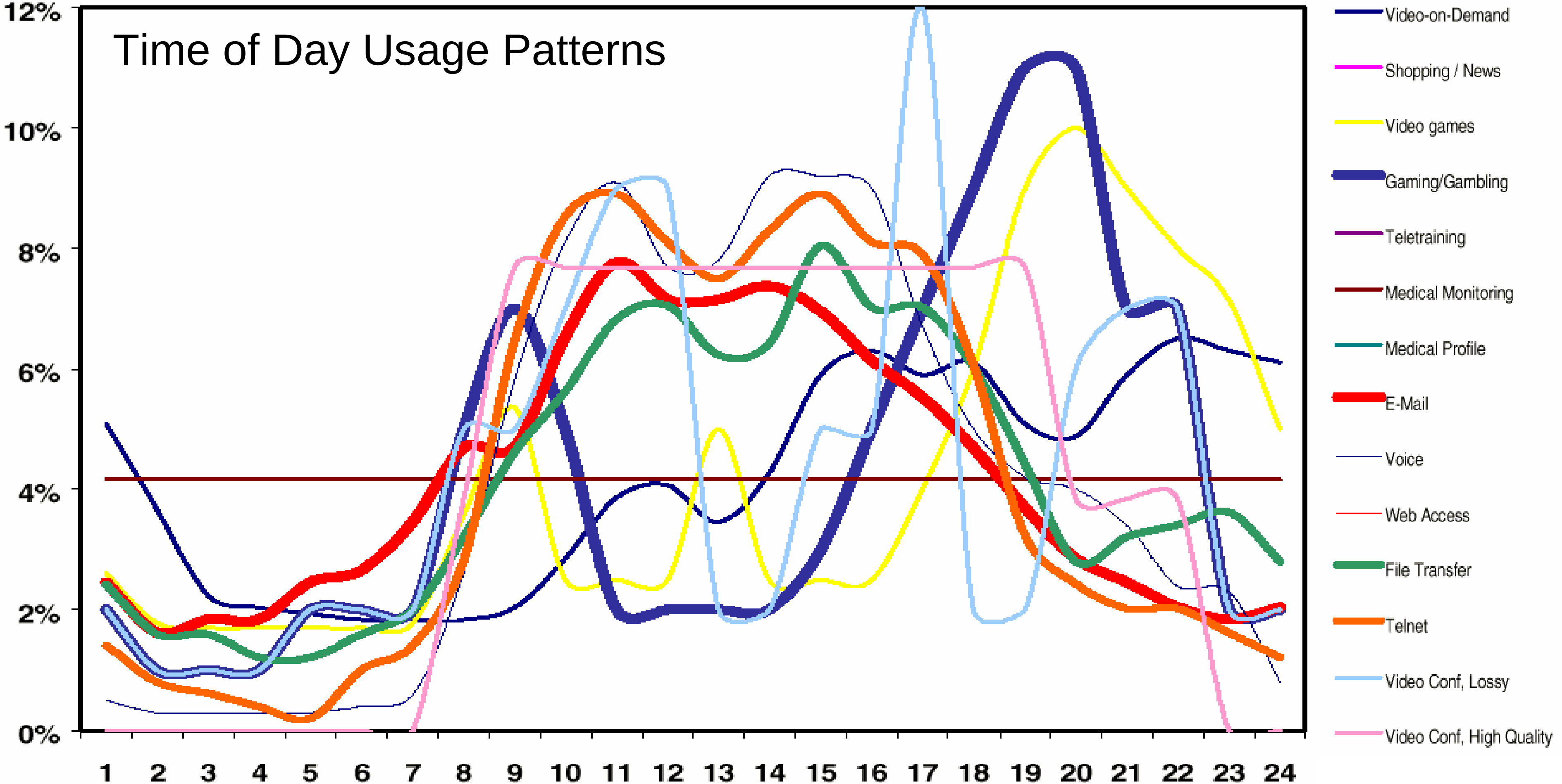
The mathematical programming approach

- Helps understanding the problems arising
- Makes much faster and more reliable planning possible
- Allows considering variations and scenario analysis
- Allows the comparison of different technologies
- Yields feasible solutions
- Produces much cheaper solutions than traditional planning techniques
- Helps evaluating the quality of a network.

There is still a lot to be done, e.g.,
for the really important problems,
optimal solutions are way out of reach!

Integrating Variable Multimedia Services

Courtesy Dr. Winter (E-Plus)



The Mathematical Challenges

- Finding the right ballance between flexibility and controlability of future networks
- Controlling such a flexible network
- Handling the huge complexity
- Integrating new services easily
- Guaranteeing quality

- Finding appropriate Mathematical Models
- Finding appropriate solution techniques (exact, approximate , interactive, quality guaranteed)

05M1 Lecture
Combinatorial Optimization and Telecommunication

The End

Martin Grötschel

- Institut für Mathematik, Technische Universität Berlin (TUB)
- DFG-Forschungszentrum “Mathematik für Schlüsseltechnologien” (MATHEON)
- Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

groetschel@zib.de
http://www.zib.de/groetschel