03M2 Lecture
Printed Circuit Board Production: Some Issues

Martin Grötschel

Beijing Block Course
"Combinatorial Optimization at Work"

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Contents

1. Printed Circuit Board Production: a Brief Overview
2. Drilling Holes into Printed Circuit Boards (PCBs)
3. Via Minimization
4. The Max-Cut and the Chinese Postman Problem
5. Optimization Problems in PCB Assembly (Petra Bauer)

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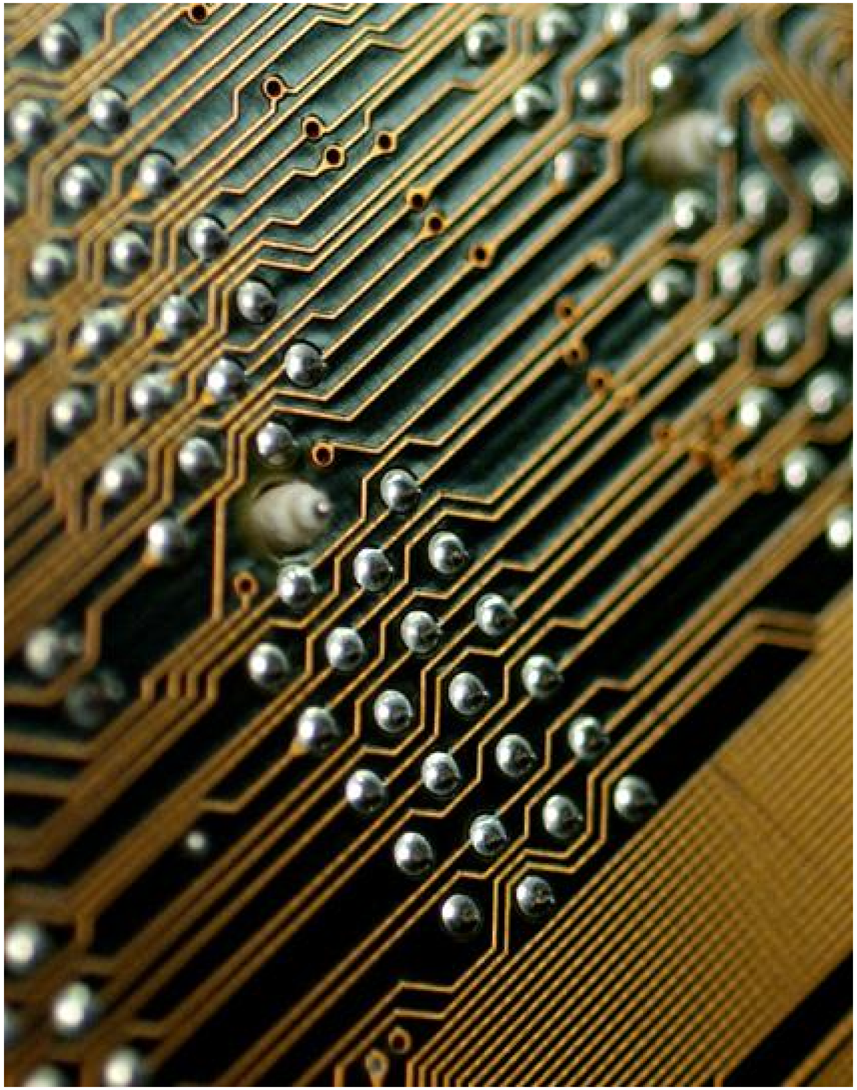
Printed Circuit Boards

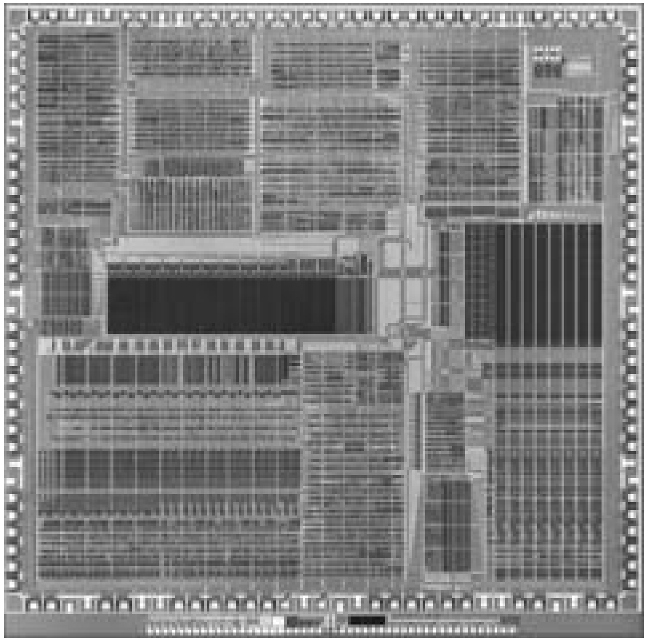
- A printed circuit board consists of "printed wires" attached to a sheet of insulator. The conductive "printed wires" are called "traces" or "tracks". The insulator is called the substrate.
- The inventor of the printed circuit was probably the Austrian engineer Paul Eisler (1907 - 1995) who, while working in England, made one in about 1936 as part of a radio set. In about 1943 the USA began to use the technology on a large scale to make rugged radios for use in World War II. After the war, in 1948, the USA released the invention for commercial use. Printed circuits did not become commonplace in consumer electronics until the mid-1950s.

Printed Circuit Boards: A Picture

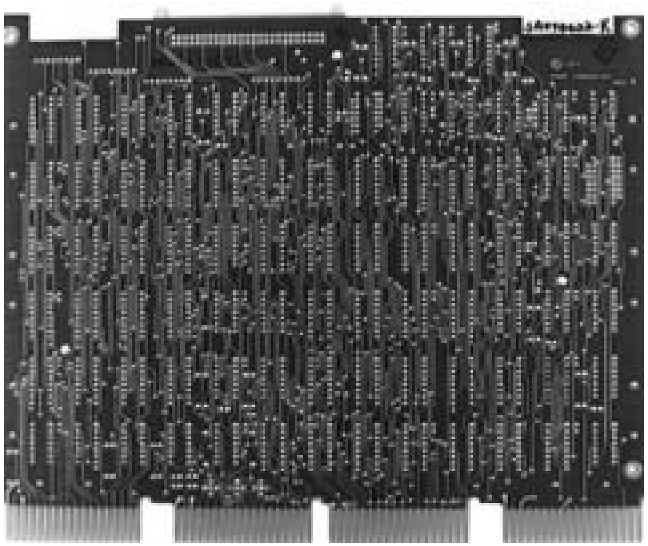


Closeup photo of one side of a motherboard PCB, showing conductive traces and solder points for through-hole components on the opposite side.

Production of ICs and PCBs



Integrated Circuit (IC)



Printed Circuit Board (PCB)

Problems: Logical Design, Physical Design Correctness, Simulation, Placement of Components, Routing, Drilling,...

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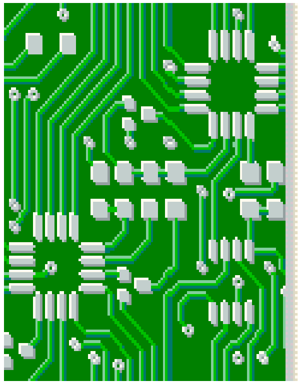
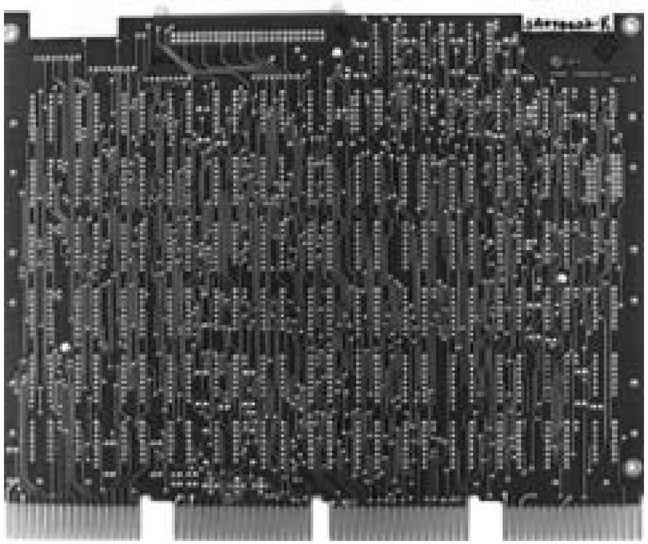
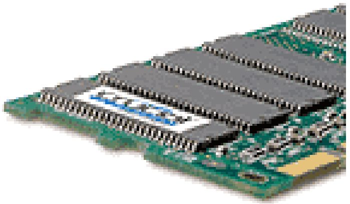
ICs and PCBs







Integrated Circuit (IC)

Printed Circuit Board (PCB)

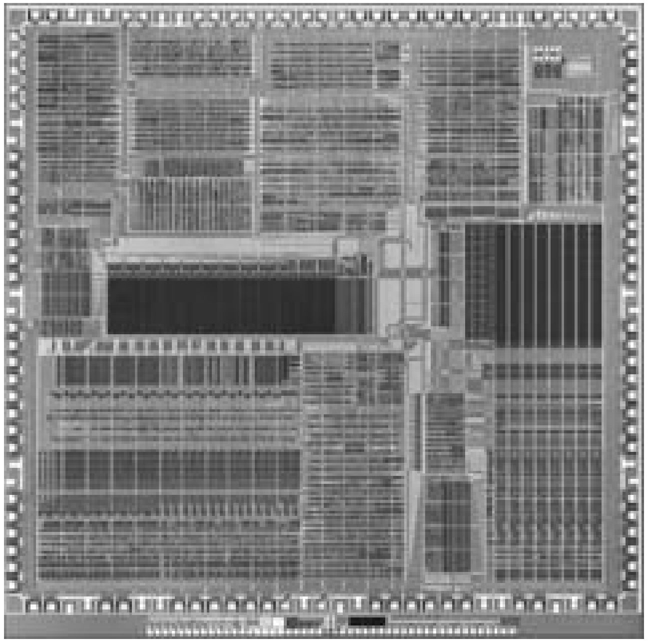


Foto of a “real” PCB (front)

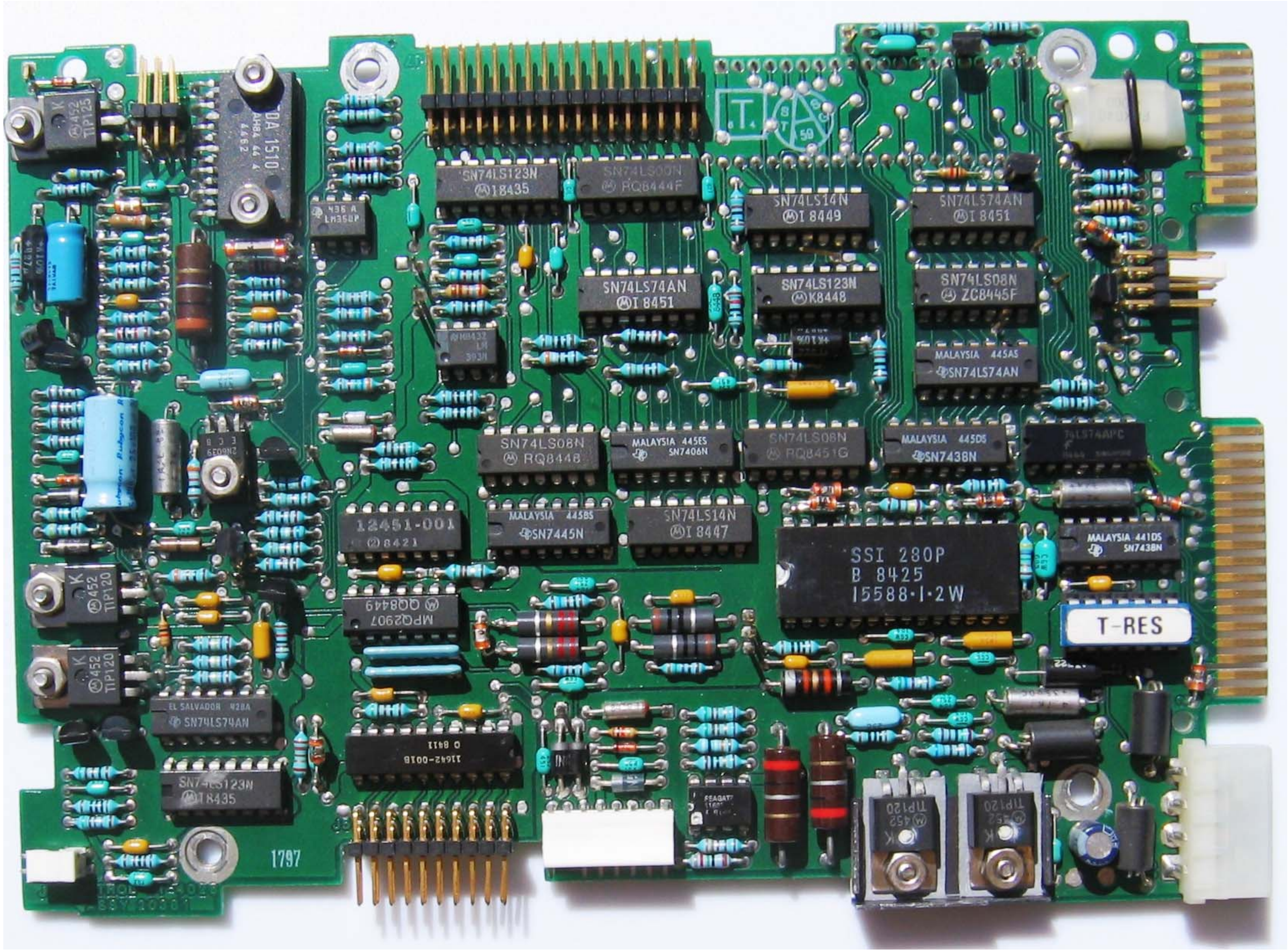
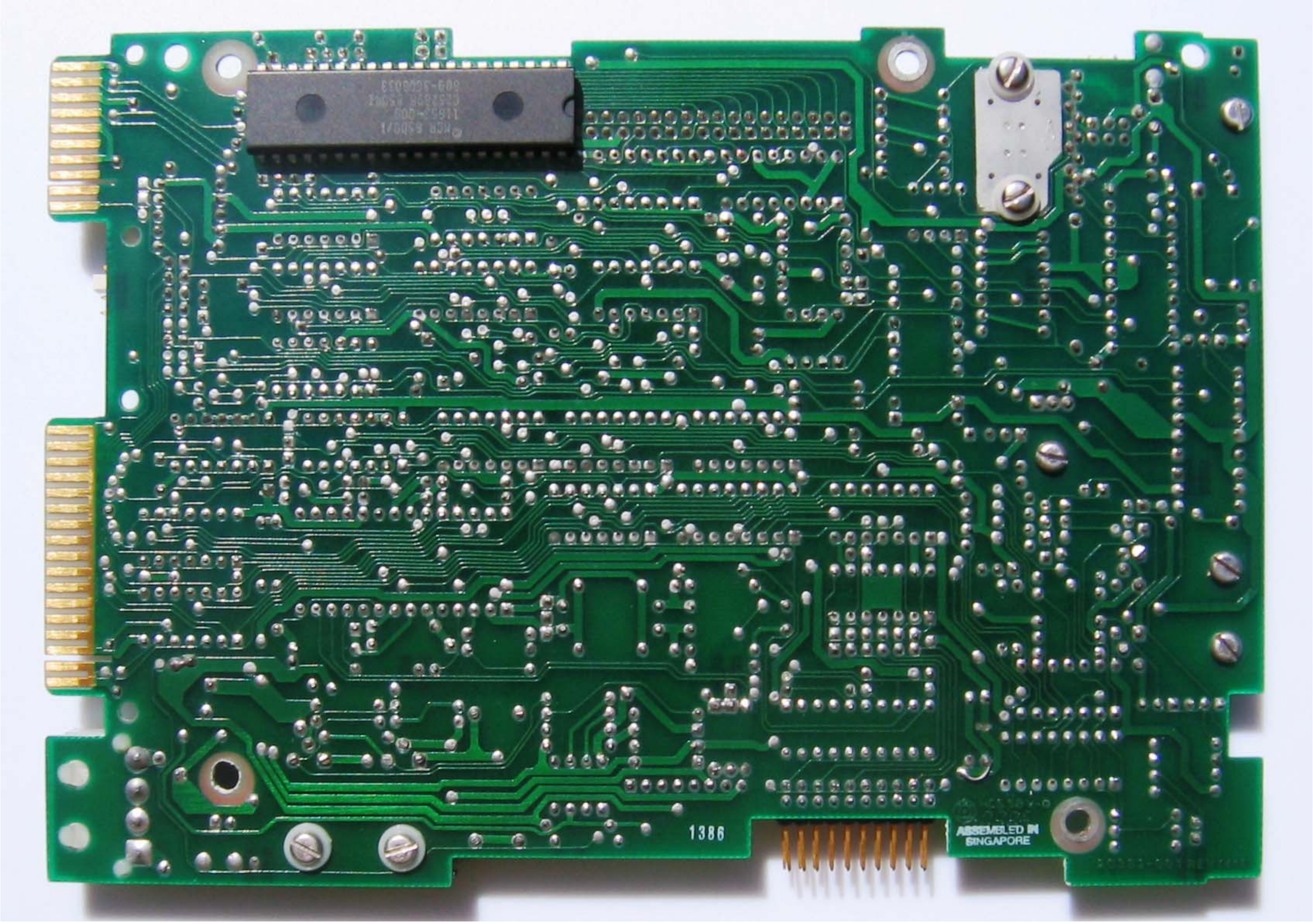
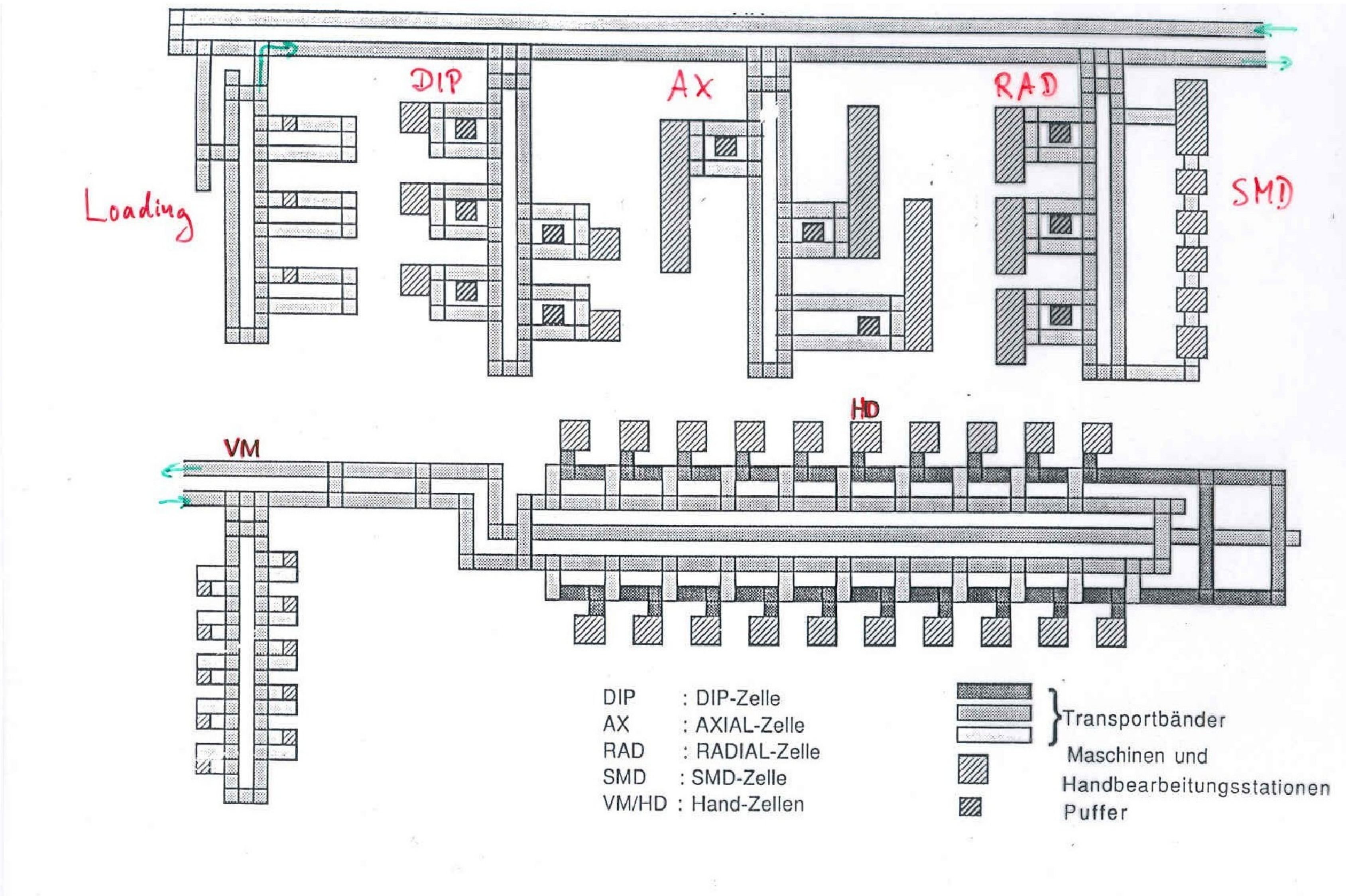


Foto of a “real” PCB (back)



FALKE PCB Assembly Line (Siemens Augsburg)



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Flachbaugruppen-Produktion (Petra Bauer)

We developed and implemented:

- a detailed simulation model
- Optimization heuristics to determine the PCB production seqence
- methods to compute lower bound for the flow time

Expectation: Improvement from 65% to 100% of the projected capacity of the production line

Results

(1) Span between best and worst heuristic solution	~ 20 %
(2) Provable Span	~ 30 %
(3) Solutions used in the factory	~ in the middle of the span
(4) random solution	~ in the middle of the span
(5) best heuristic solution	~ 7 % better as factory solution, but unstable

The whole 9 month effort resulted in no success. We could prove that system design and layout did not match the tasks to be performed well: bottleneck creation, etc.

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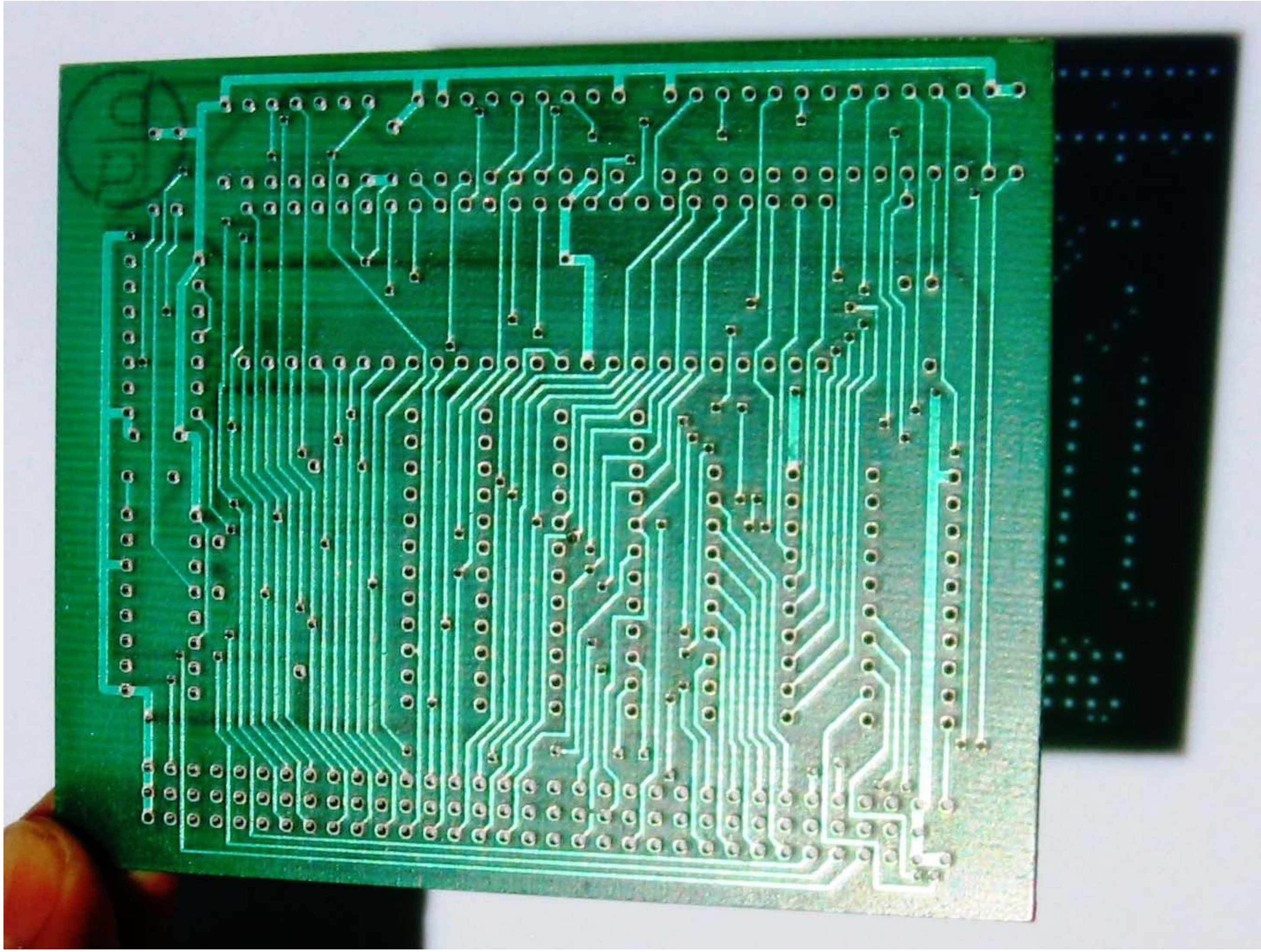
The 442-Holes Problem



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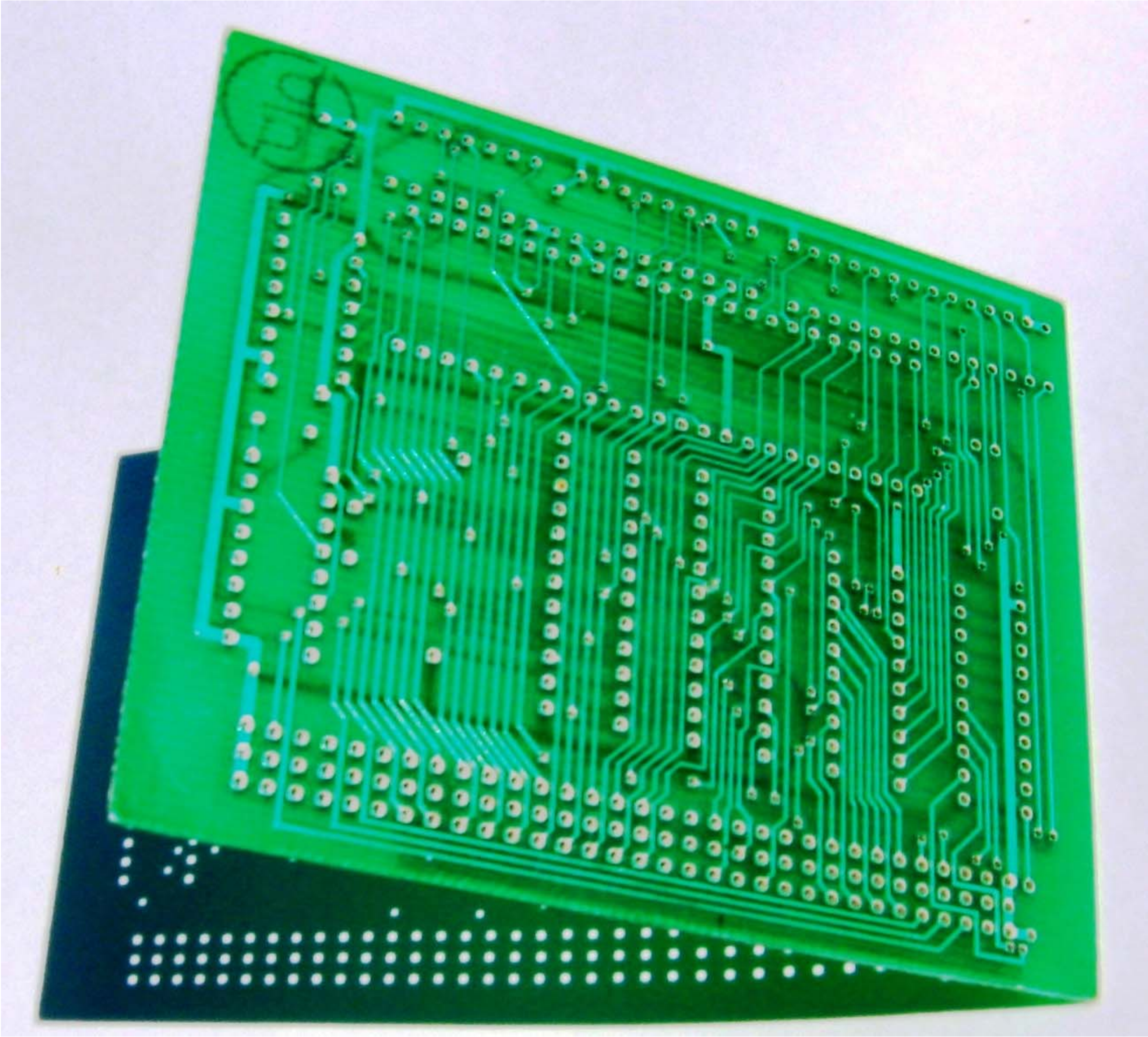
The 442-Holes Problem



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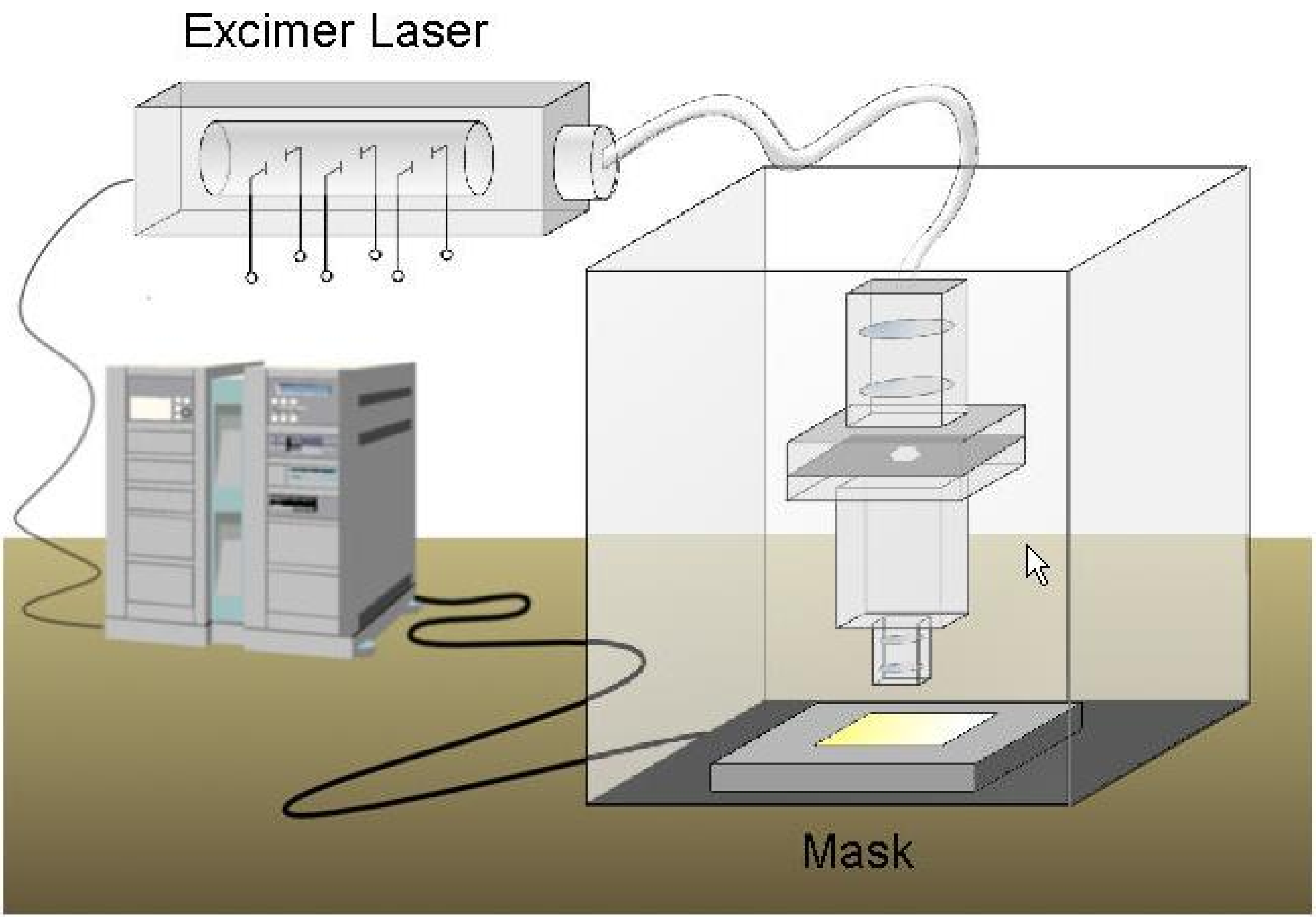
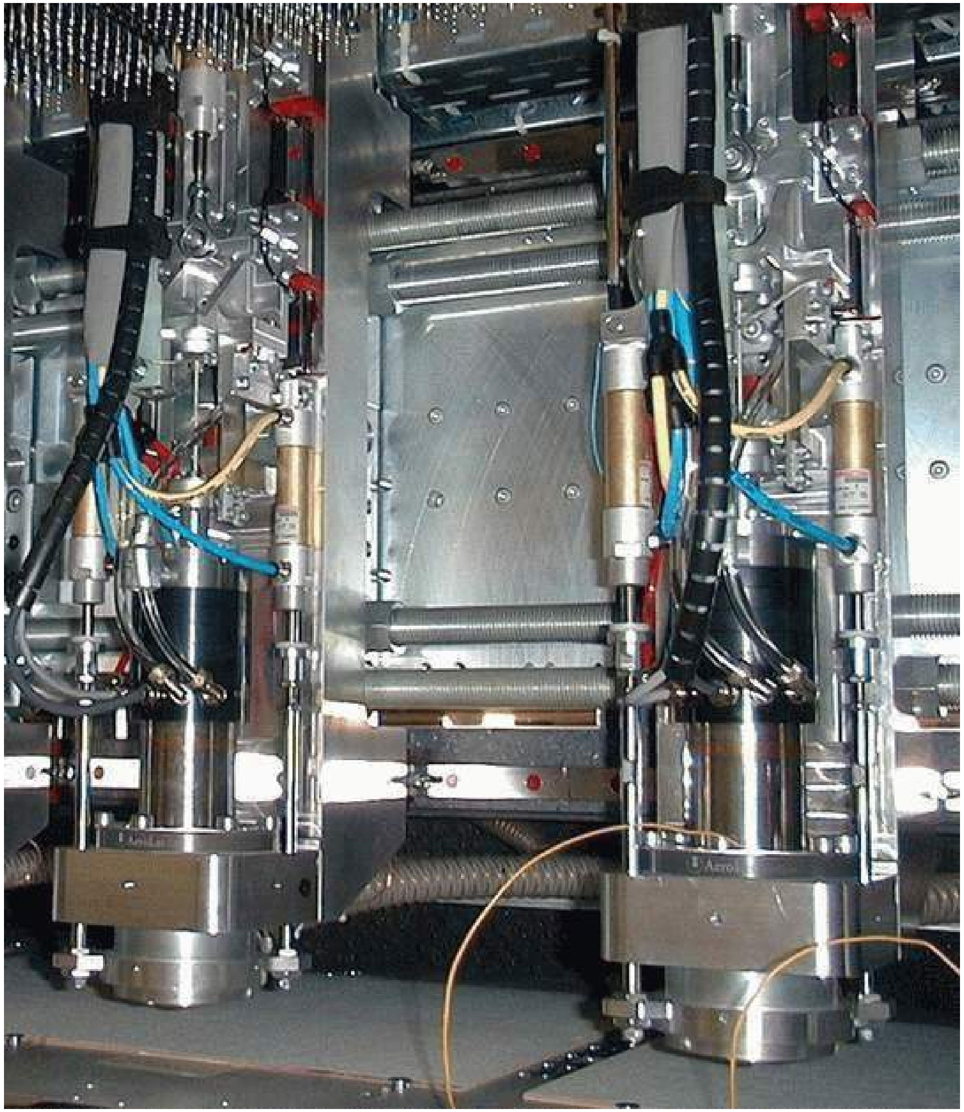
Drilling / Plotting Laser



Mechanical PCB Drilling Machine



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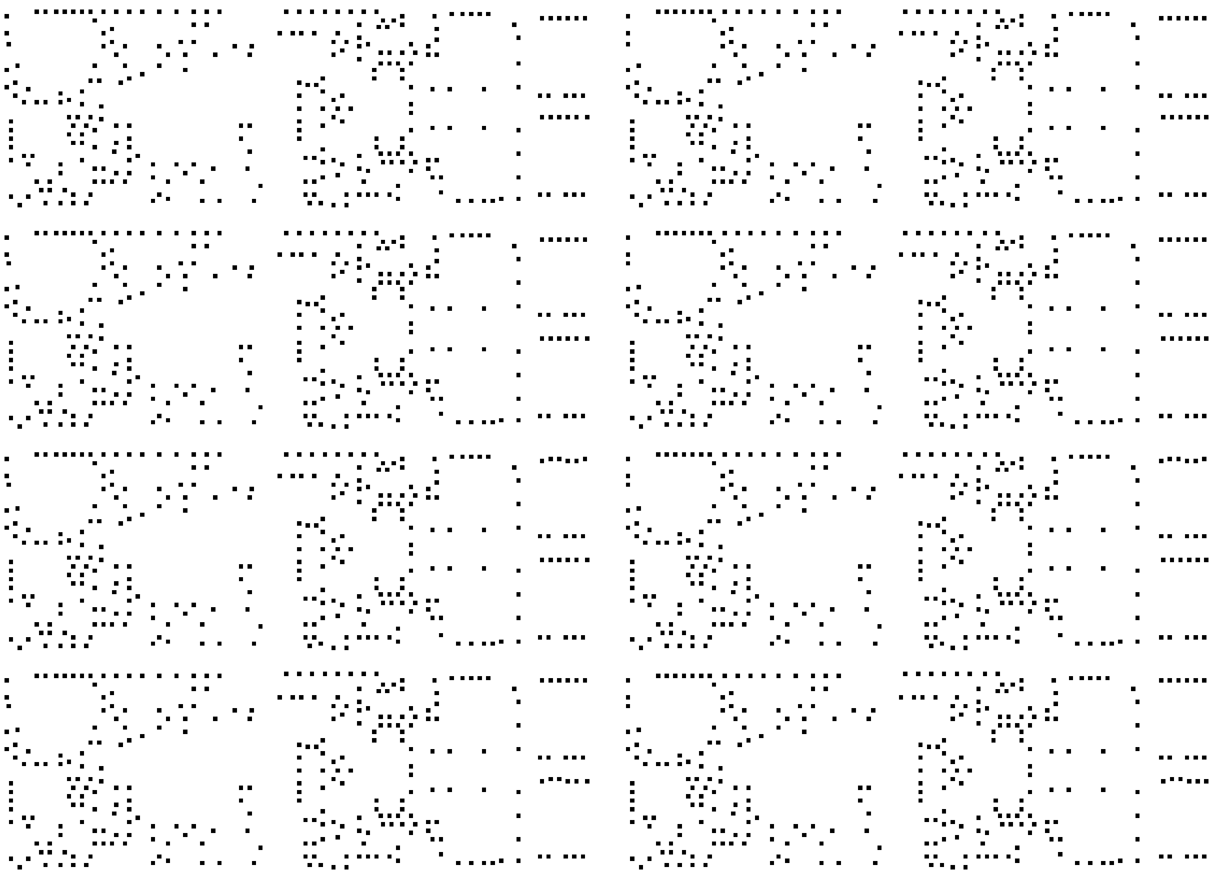
Correct modelling of a printed circuit board drilling problem



2103 holes to be drilled

length of a move of the drilling head:

Euclidean norm,

Max norm,

Manhattan norm?

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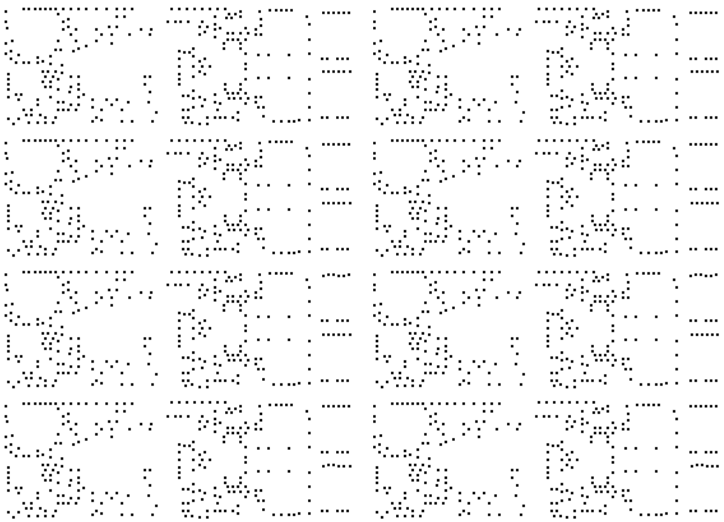
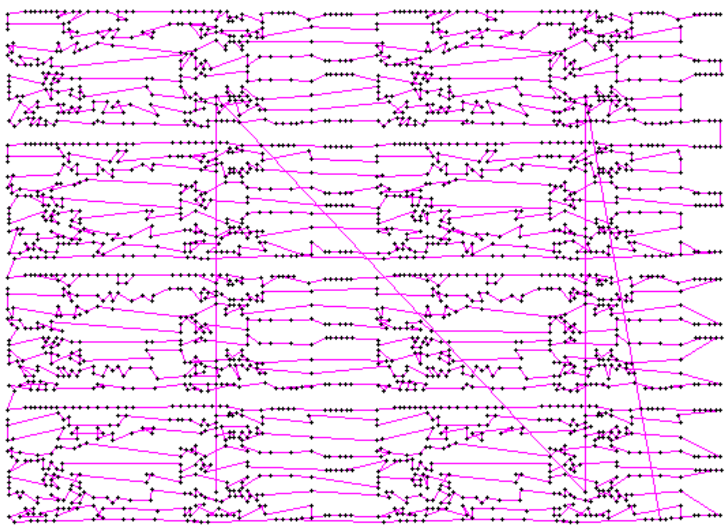
Drilling 2103 holes into a PCB

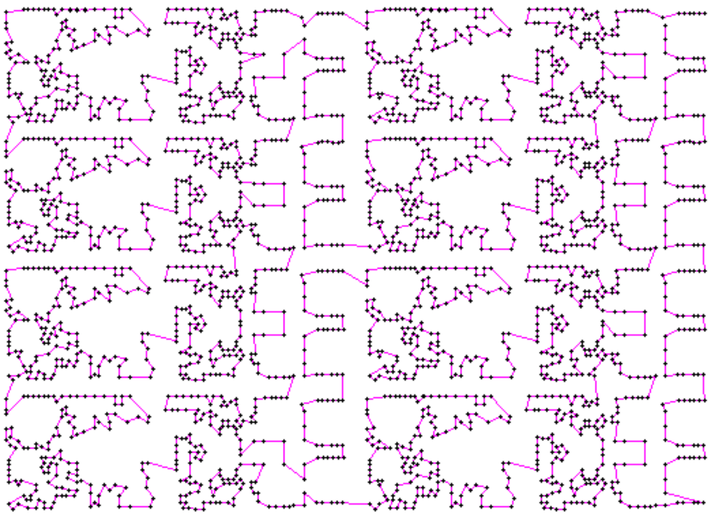


Significant Improvements via TSP

(Padberg & Rinaldi)



industry solution



optimal solution

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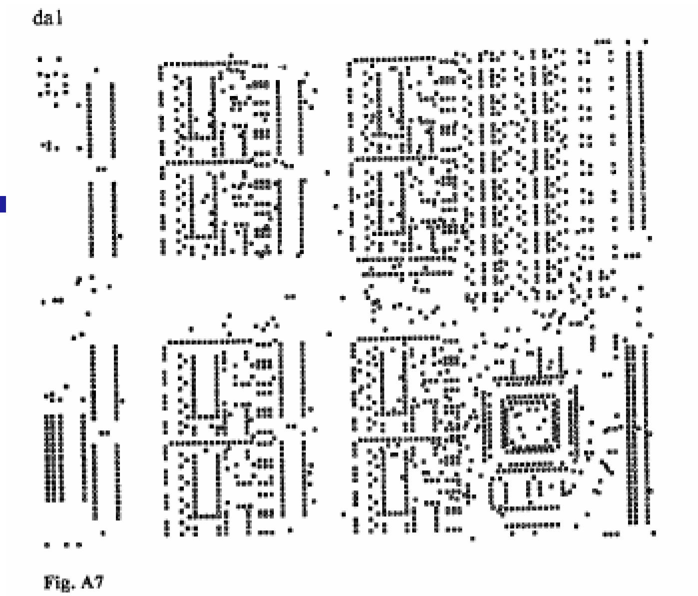
Siemens-Problem PCB da1



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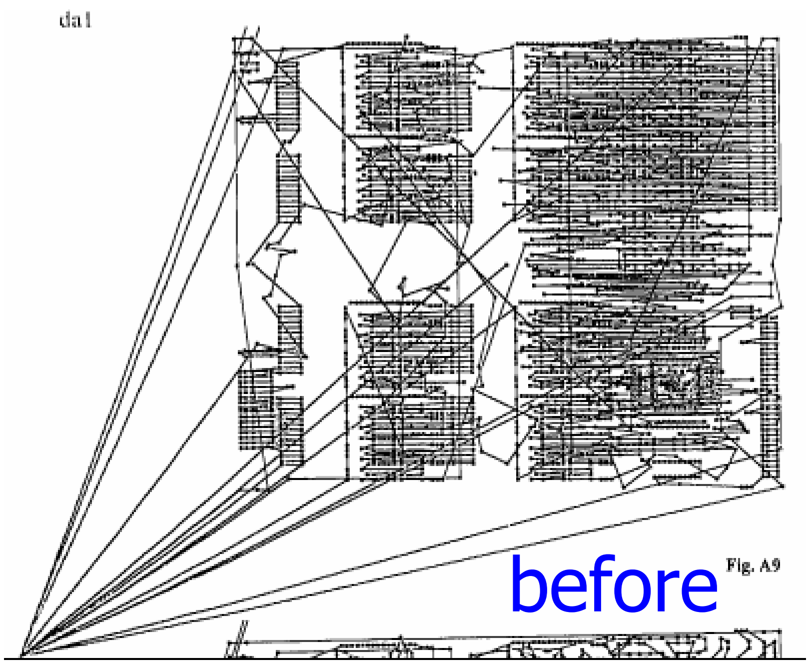

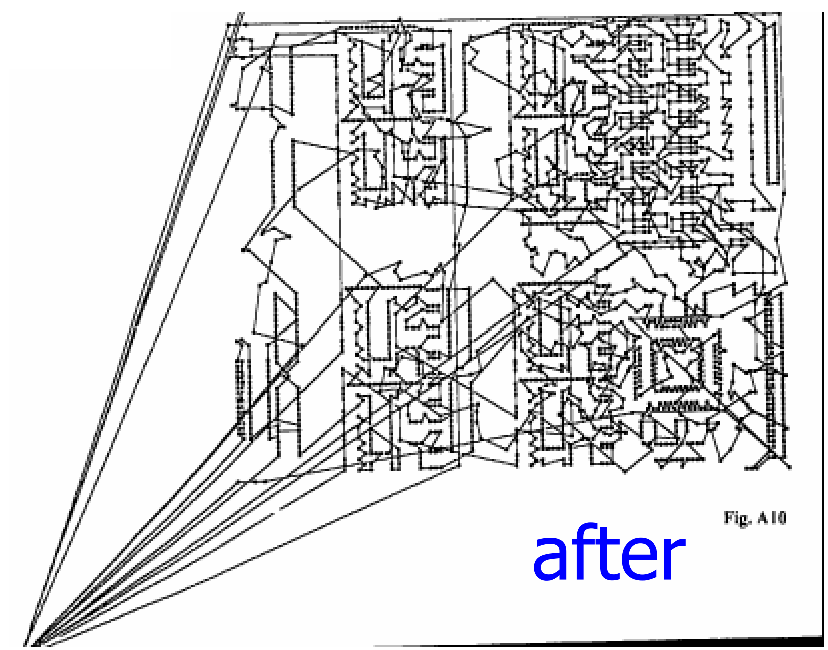

Siemens-Problem PCB da4

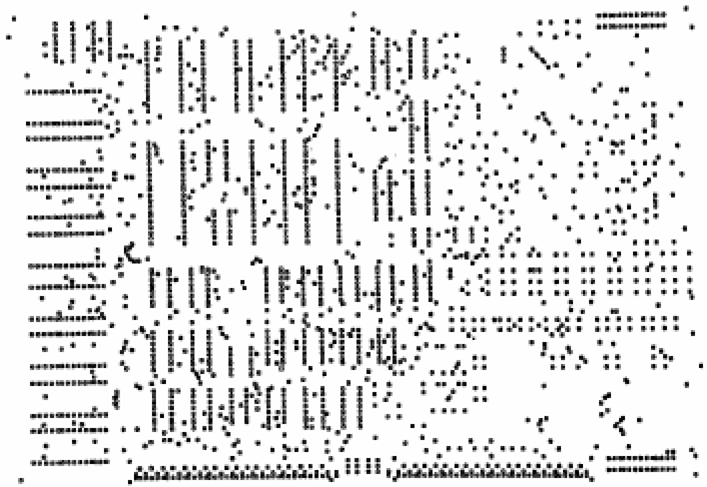

Fig. A8

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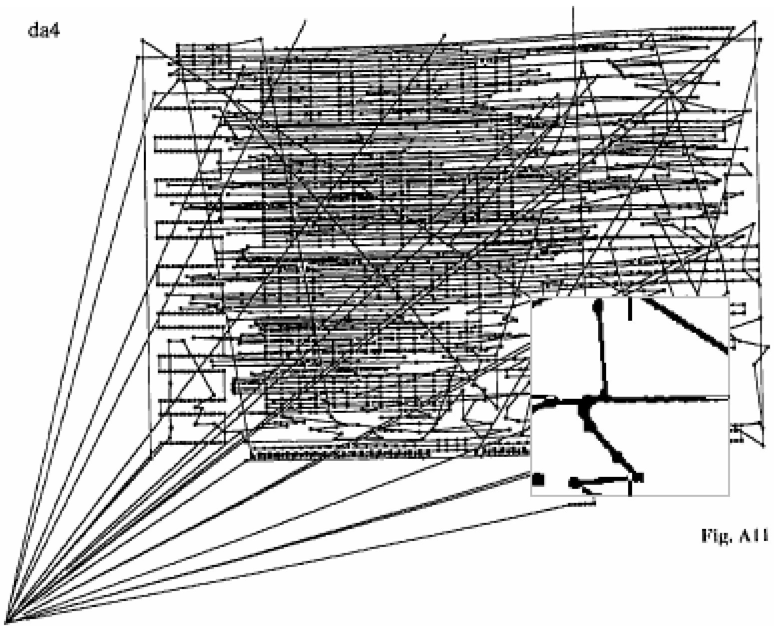

Fig. A11

before

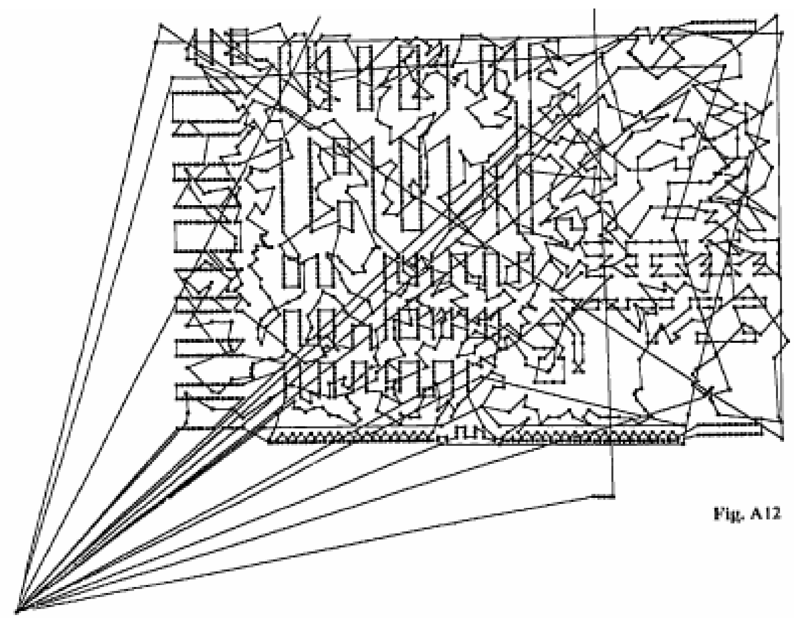

Fig. A12

after

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Typical PCB drilling problems at Siemens

	da1	da2	da3	da4
Number of holes	2457	423	2203	2104
Number of drills	7	7	6	10
Tour length	3518728	1049956	1958161	4347902

Table 4

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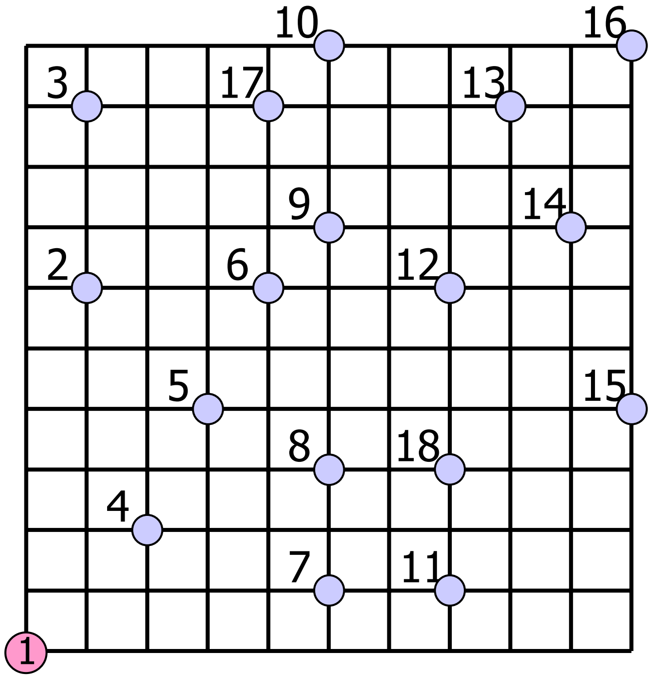
Fast heuristics

	da1	da2	da3	da4
CPU time (min:sec)	1:58	0:05	1:43	1:43
Tour length	1695042	984636	1642027	1928371
Improvement in %	56.87	14.60	26.94	58.38

Table 5

18 Holes, a didactical problem



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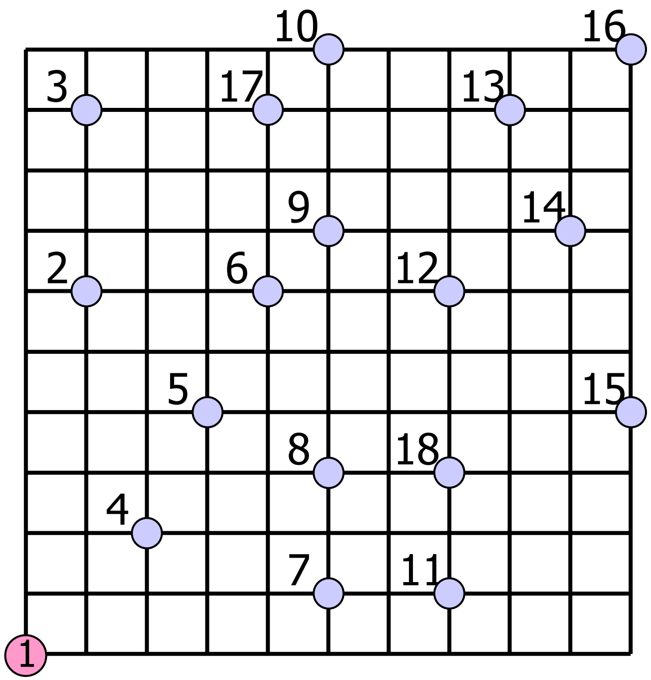
18 Holes: which objectives?



$$c_2(i,j) := \sqrt{|x_i - x_j|^2 + |y_i - y_j|^2}$$

Definition

Der *euklidische Abstand* zweier Punkte (x_i, y_i), (x_j, y_j):

$$c_2(i,j) := \sqrt{|x_i - x_j|^2 + |y_i - y_j|^2}$$

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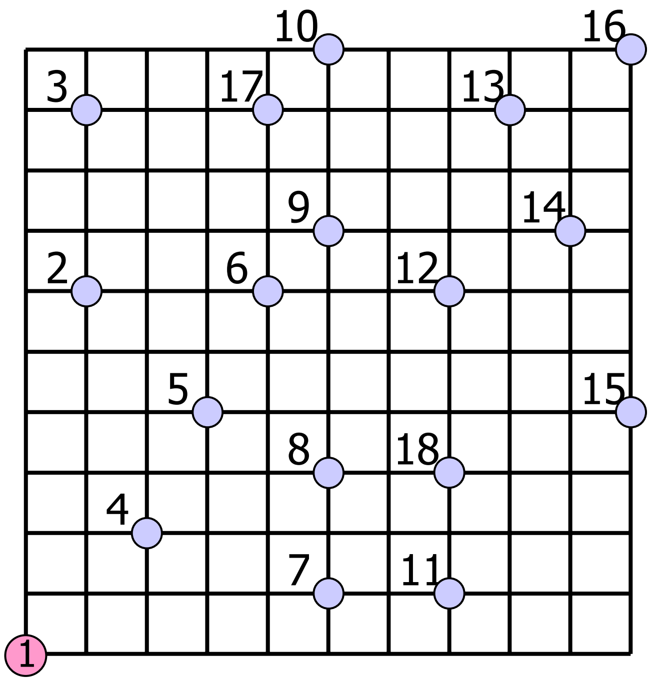
18 Holes: which objectives?



$$c_1(i,j) := |x_i - x_j| + |y_i - y_j|$$

Definition

Der *Manhattan-Abstand* zweier Punkte $(x_i, y_i), (x_j, y_j)$:

$$c_1(i,j) := |x_i - x_j| + |y_i - y_j|$$

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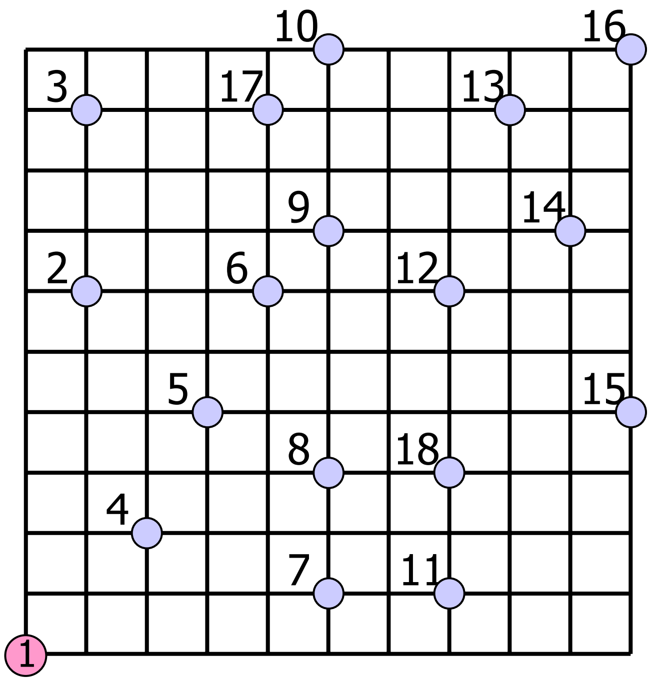
18 Holes: which objectives?



$$c_{max}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\}$$

Definition

Der max-*Abstand* zweier Punkte $(x_i, y_i), (x_j, y_j)$:

$$c_{max}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\}$$

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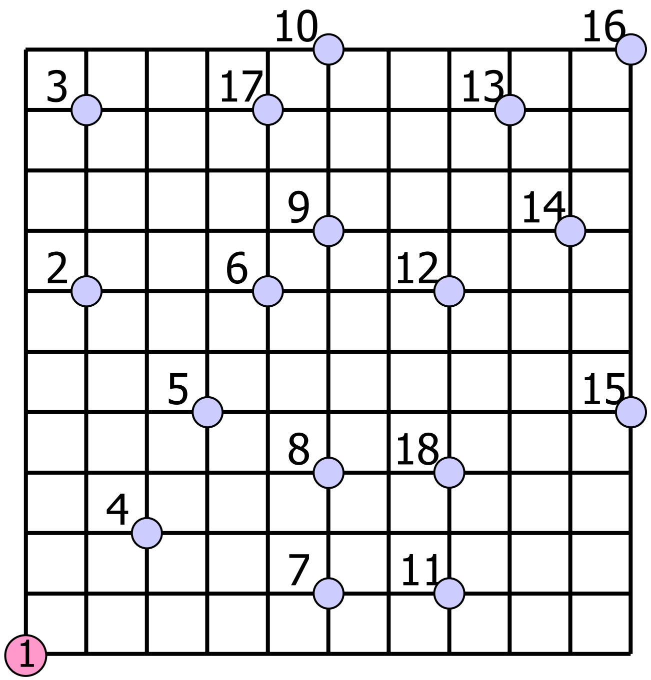
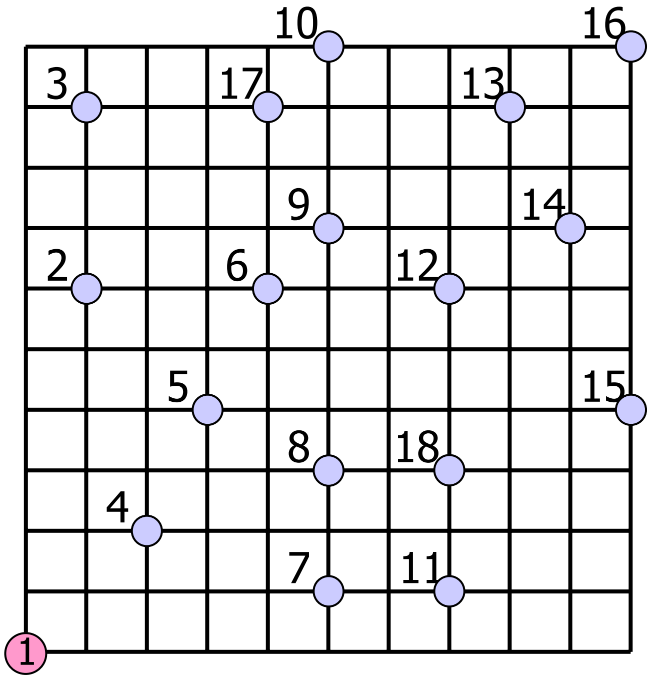
18 Holes: which objectives?



$$c_{\max\min}(i,j) := \frac{199}{200}\, c_{\max}(i,j) + \frac{1}{200}\, c_1(i,j).$$

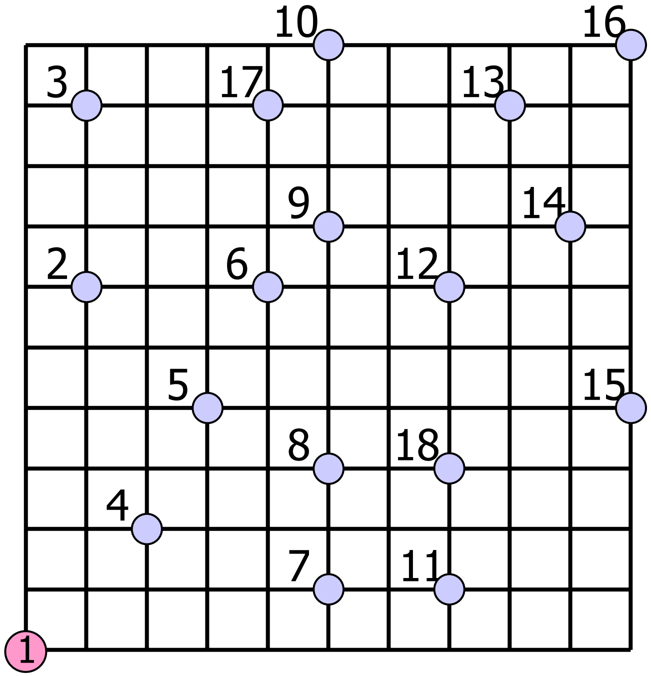
$$c_{\max\min}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\} + \frac{1}{200} \min\{|x_i - x_j|, |y_i - y_j|\}.$$

18 Holes: which objectives?



$$c_{\max 2x}(i,j) := \max\{2\,|x_i - x_j|, |y_i - y_j|\}.$$

18 Holes: 5 objectives Which are suitable?



$$c_1(i,j) := |x_i - x_j| + |y_i - y_j|$$

$$c_2(i,j) := \sqrt{|x_i - x_j|^2 + |y_i - y_j|^2}$$

$$c_{\max}(i,j) := \max\{|x_i - x_j|, |y_i - y_j|\}$$

$$c_{\max\min}(i,j) := \frac{199}{200} c_{\max}(i,j) + \frac{1}{200} c_1(i,j).$$

$$c_{\max 2x}(i,j) := \max\{2\,|x_i - x_j|, |y_i - y_j|\}.$$

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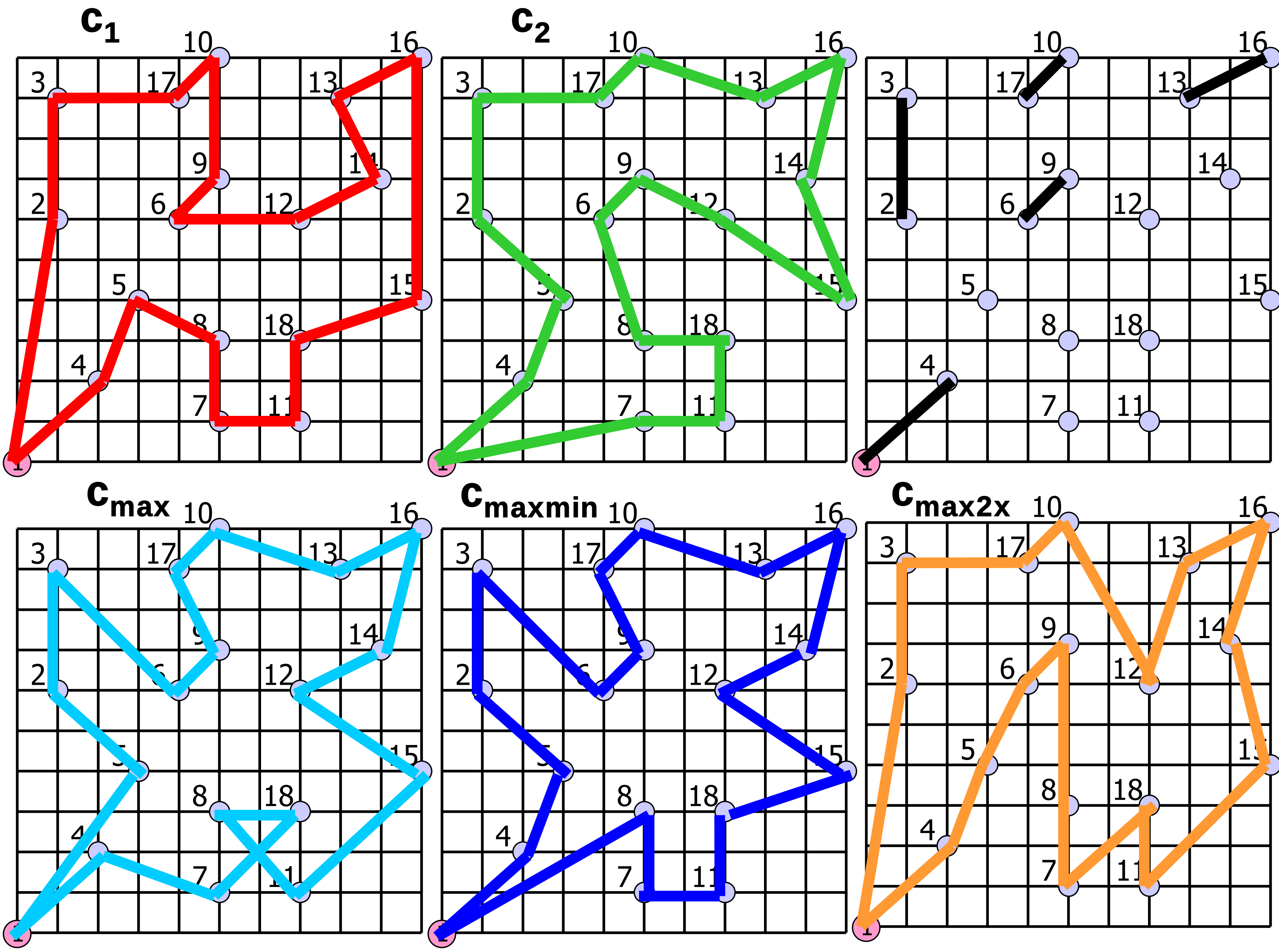
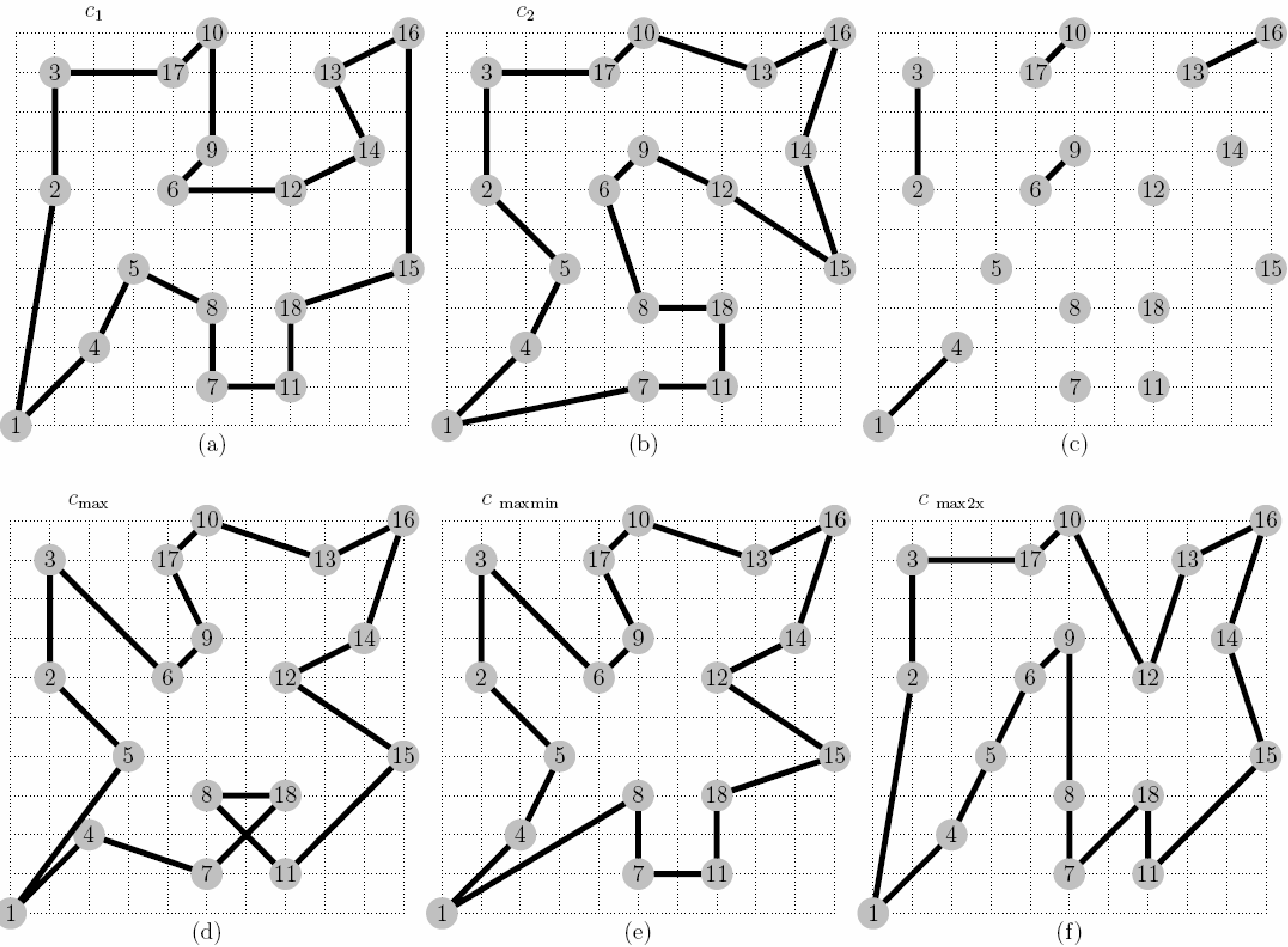

c1
c2
cmax
cmaxmin
cmax2x


c_1
c_2
$c_{\max}$
$c_{\max\min}$
$c_{\max 2x}$
(a)
(b)
(c)
(d)
(e)
(f)

Optimal Tours: objective values

	c_1-Wert	c_2-Wert	c_{max}-Wert	$c_{max\,min}$-Wert	$c_{max\,2x}$-Wert
Tour (a)	58	50,0822	47	47,0550	68
Tour (b)	60	48,5472	44	44,0800	72
Tour (d)	70	52,4280	43	43,1350	76
Tour (e)	64	49,5955	43	43,1050	72
Tour (f)	66	53,4779	48	48,0900	62

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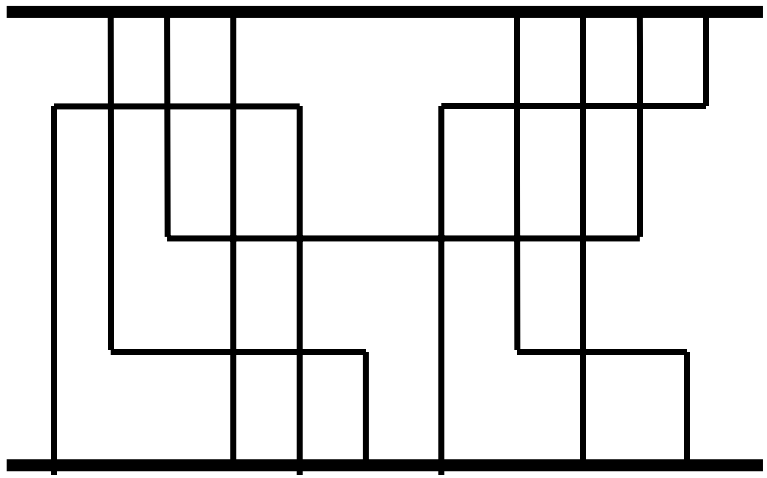
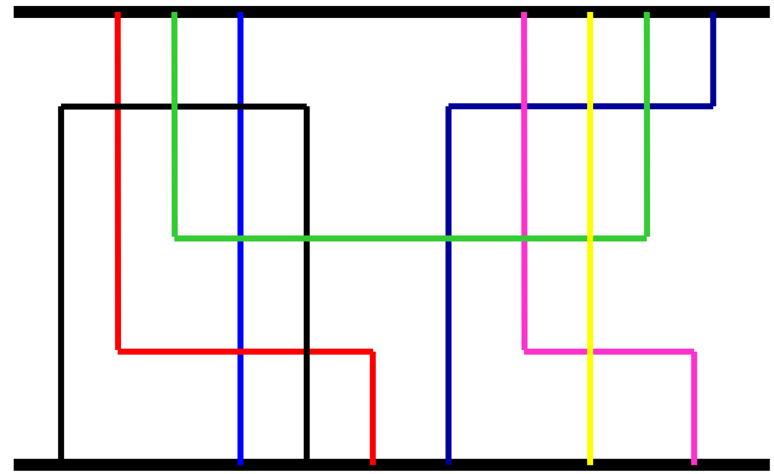
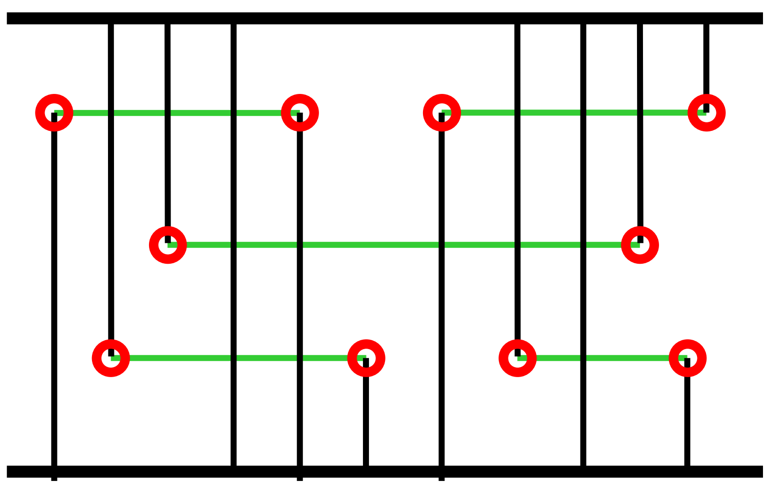
Via Minimization (2 layers)



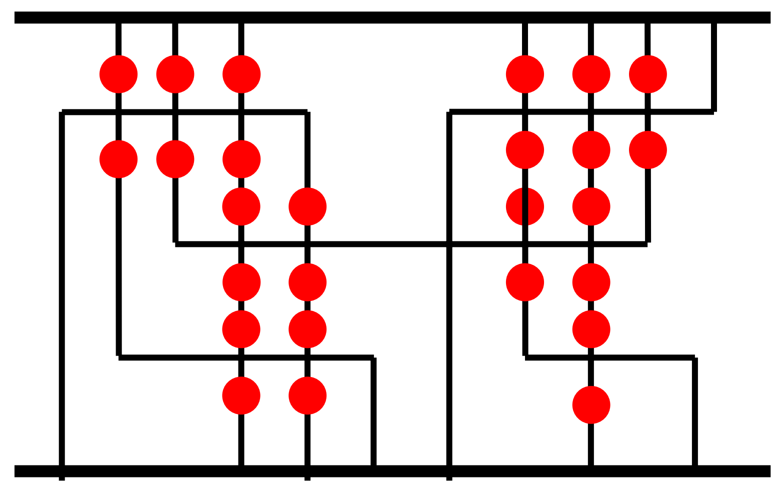
transient routing



7 nets

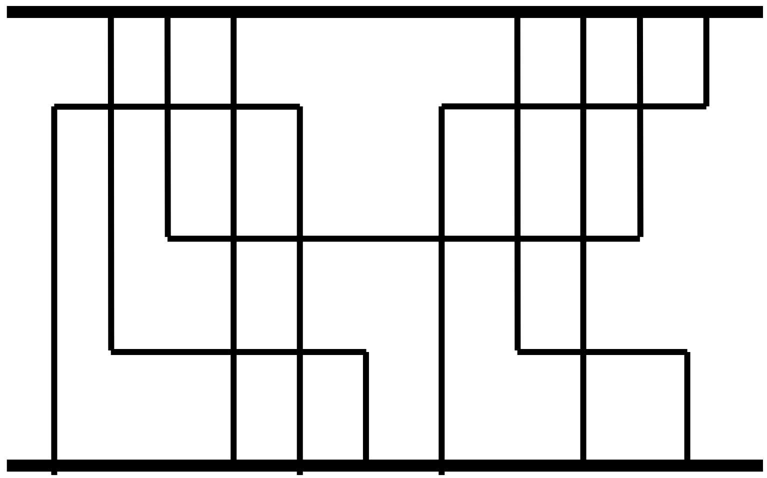


standard solution 10 vias

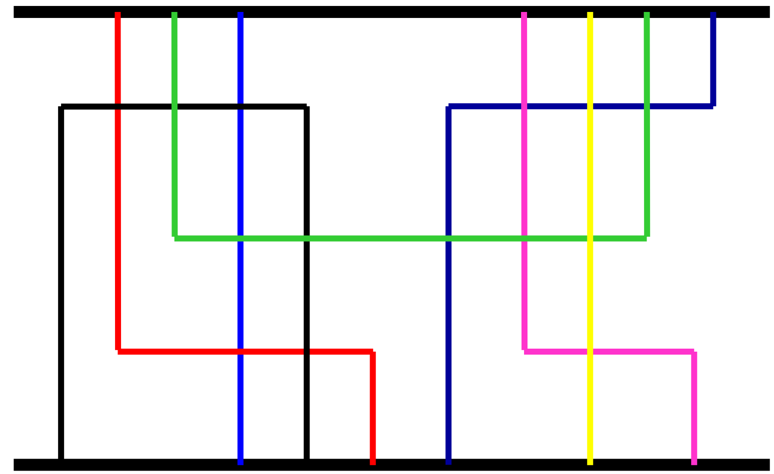


trivial solution 28 vias

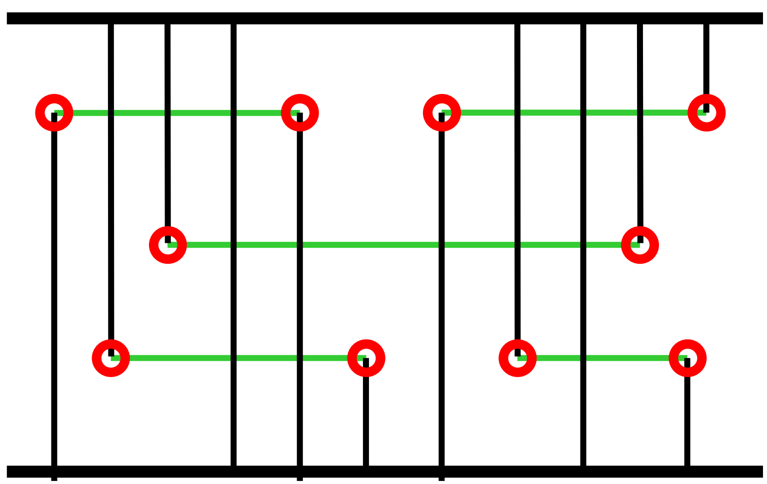
Via Minimization



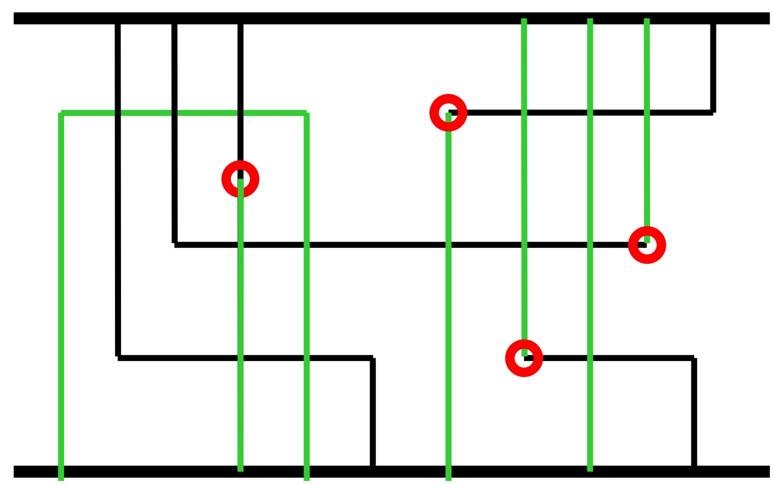
transient routing



7 nets



standard solution 10 vias



optimal solution 4 vias

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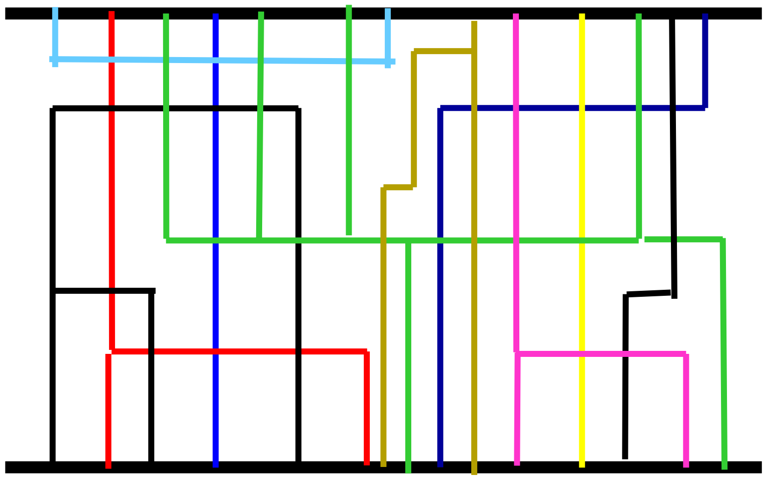
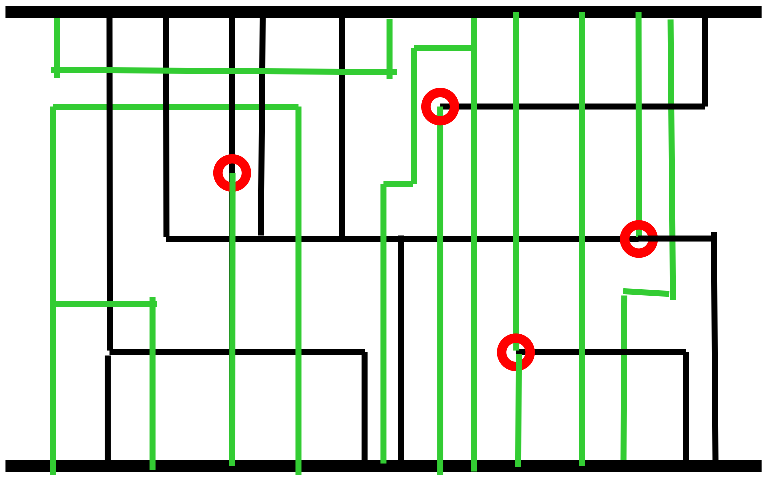
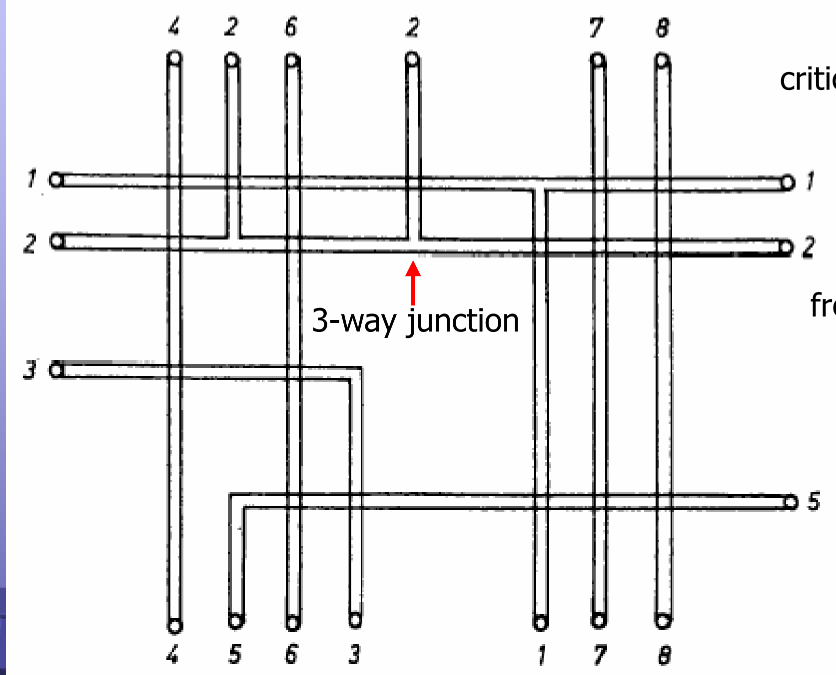
Another Example

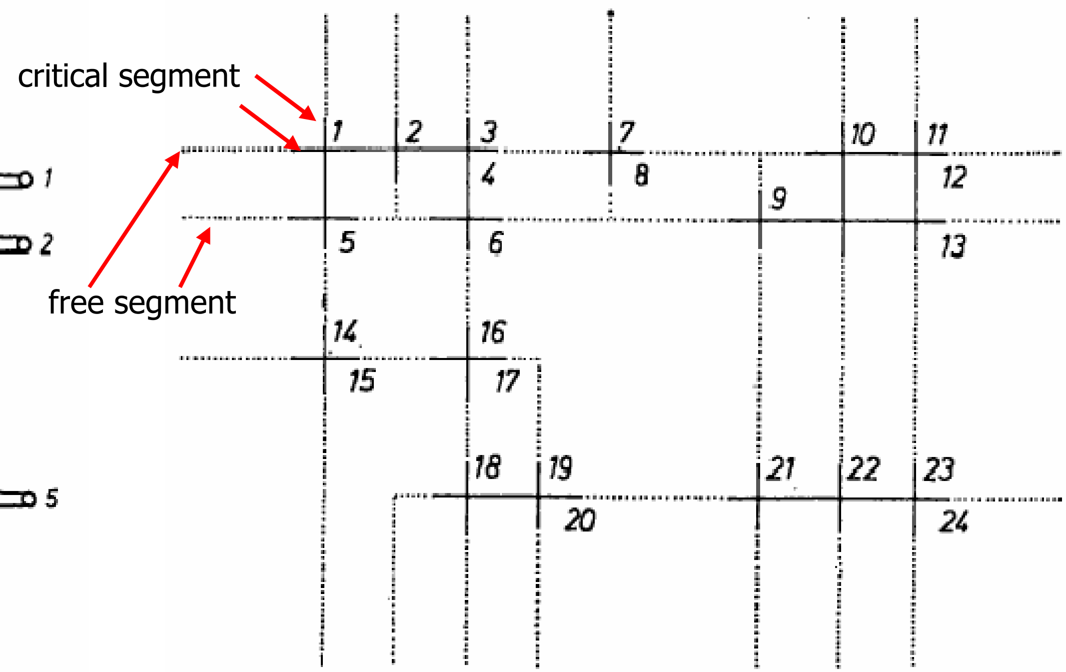


10 nets



Via minimization





transient routing of 8 nets,
one net is a 4-pin net (2),
one a 3-pin net (1),
all others are 2-pin nets

24 critical and some free segments
of the transient routing

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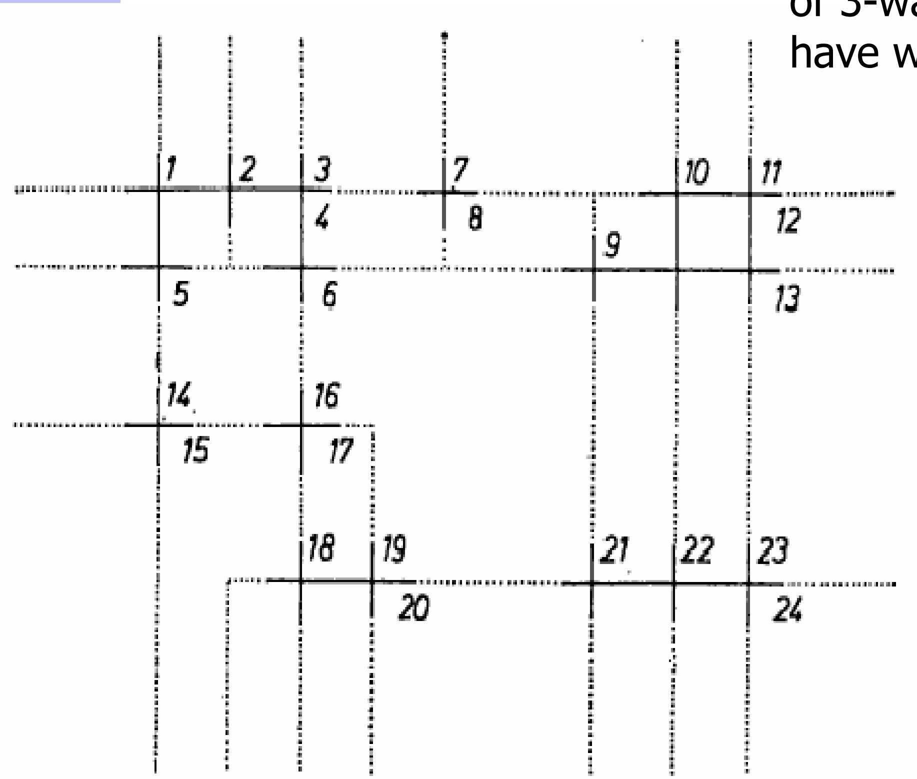
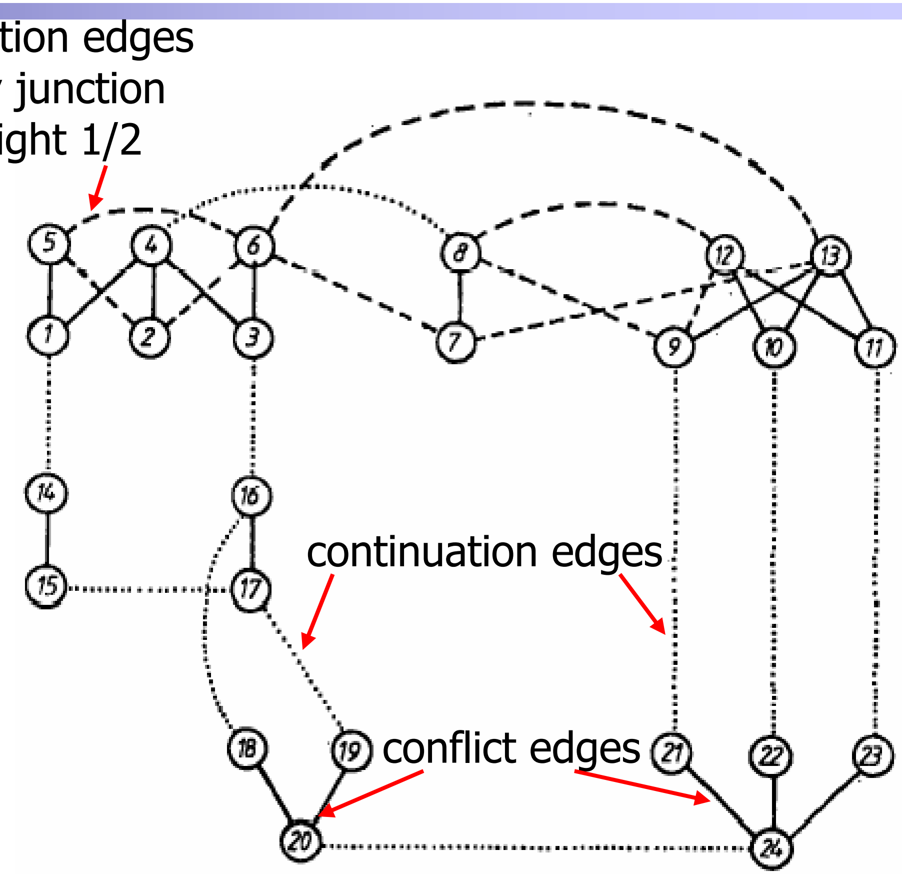
Constructing the layout graph





24 critical segments
of the transient routing

layout graph G = (V, E), E = A ∩ B

A = conflict edges

B = continuation edges

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Via Minimization and Max Cut

Result of this construction:

Finding, among all cuts in the layout graph G that contain all conflict edges, a cut C such that the sum of the weights of its continuation edges is as small as possible, yields the smallest number of vias with which a transient layout can be assigned to two layers.

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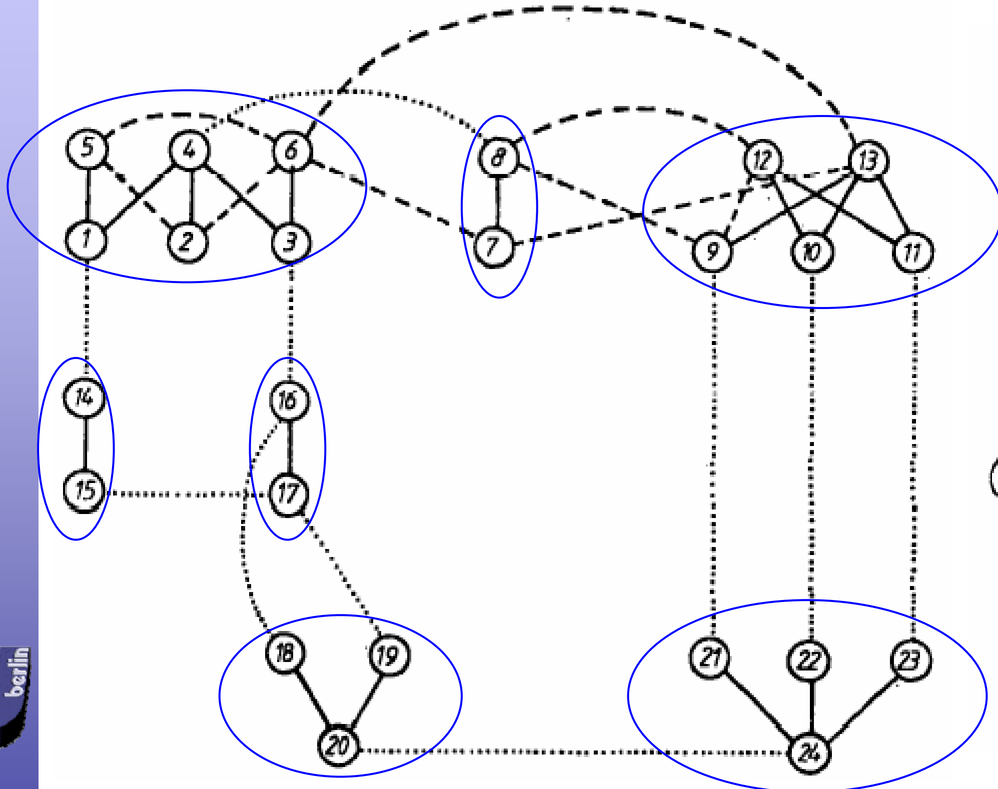
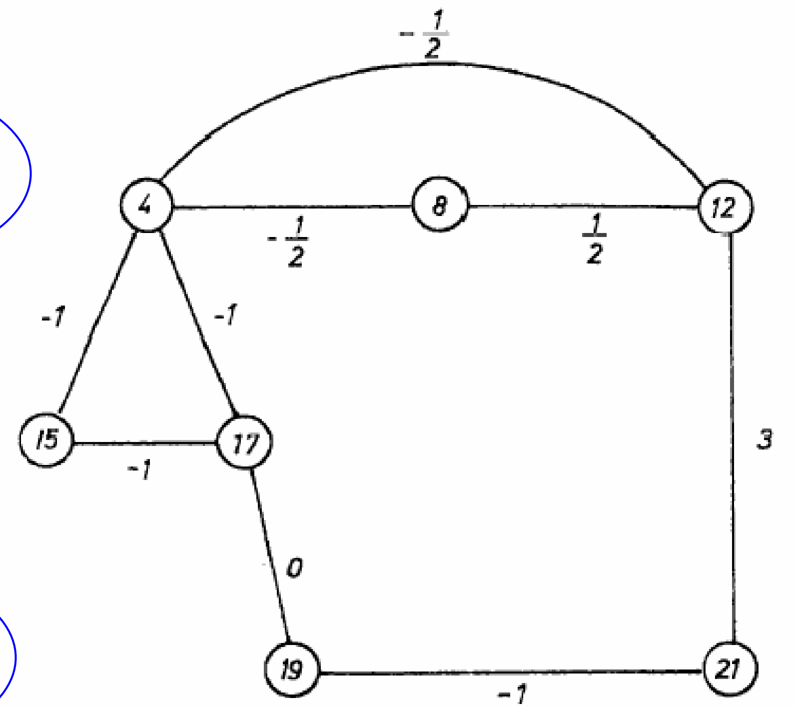
Another reduction step





layout graph G = (V, E), E = A ∩ B

A = conflict edges

B = continuation edges

reduced layout graph R = (W, F)

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Observation

- Any cut of maximum weight in the reduced layout graph R yields a cut in the layout graph G that contains all conflict edges such that the sum of the weights of its continuation edges is as small as possible.
- This construction produces a minimal number of vias.

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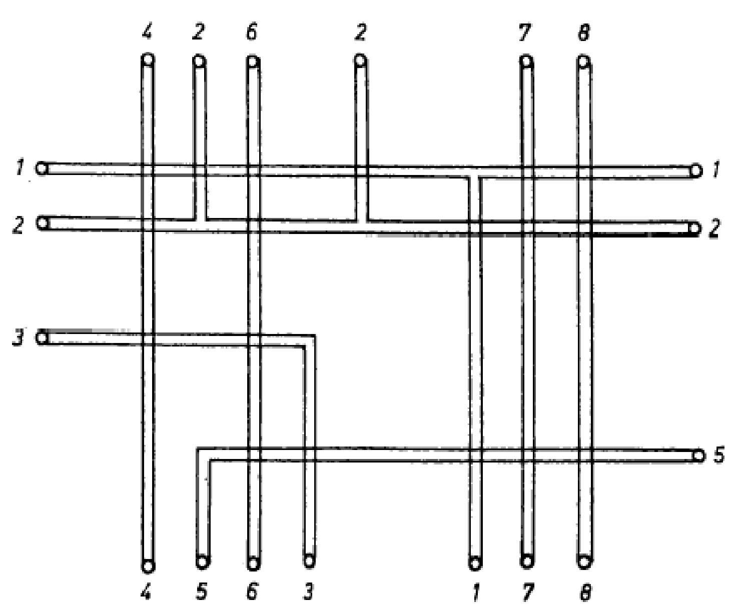
The Max-Cut Problem

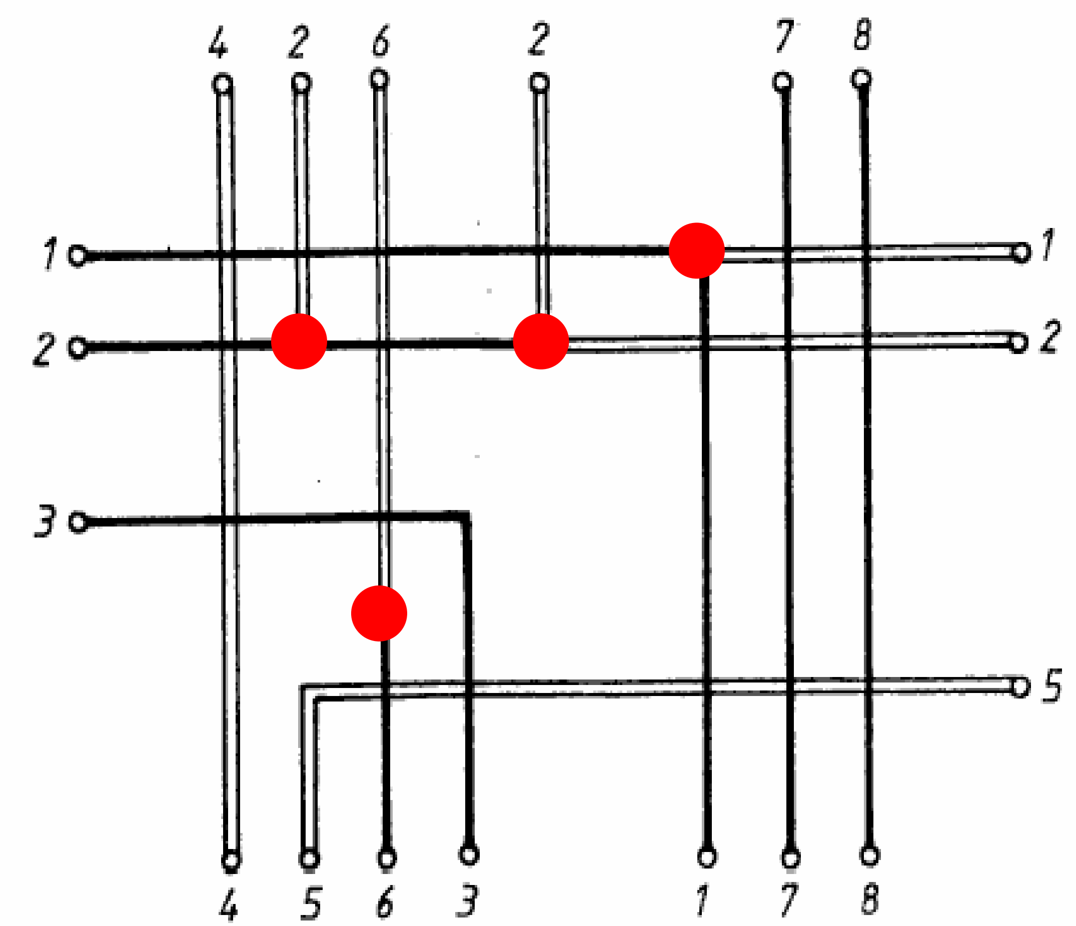
Given a graph with edge weights, find a partition of the nodes into two parts so that the sum of the weights of the edges linking the two parts is as large as possible.

Compare to min-cut problem!

$$\max \{c(\delta(W)) \mid W \text{ subset of } V\},\ G=(V,E) \text{ a graph}$$

Optimal Layout has 4 vias





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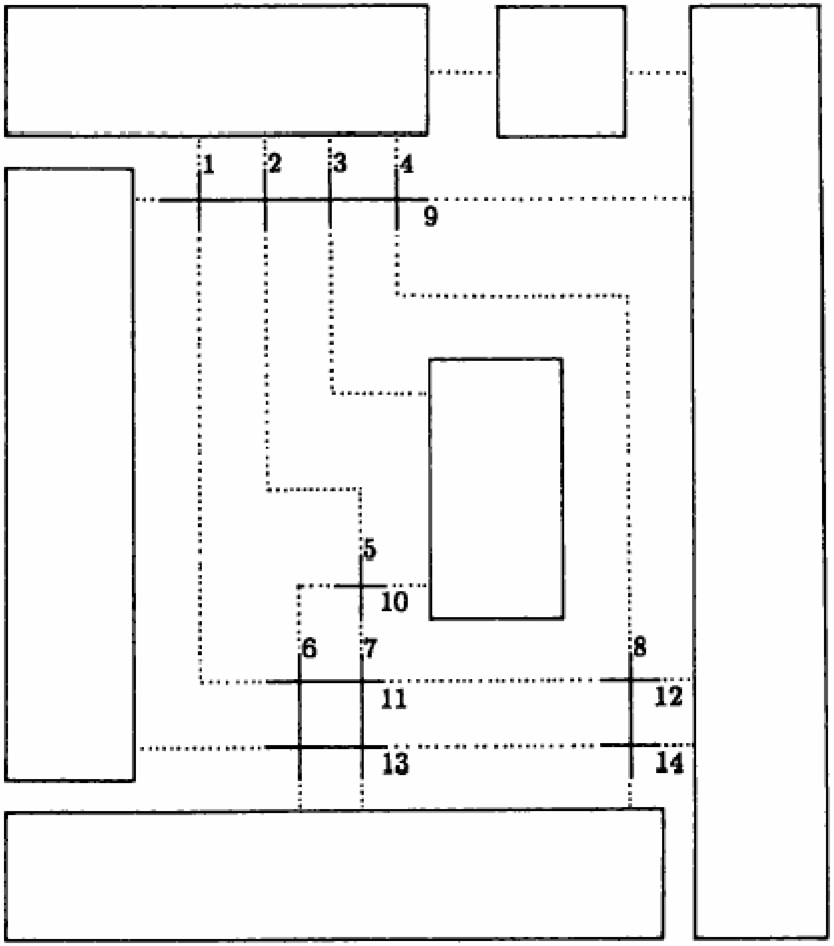
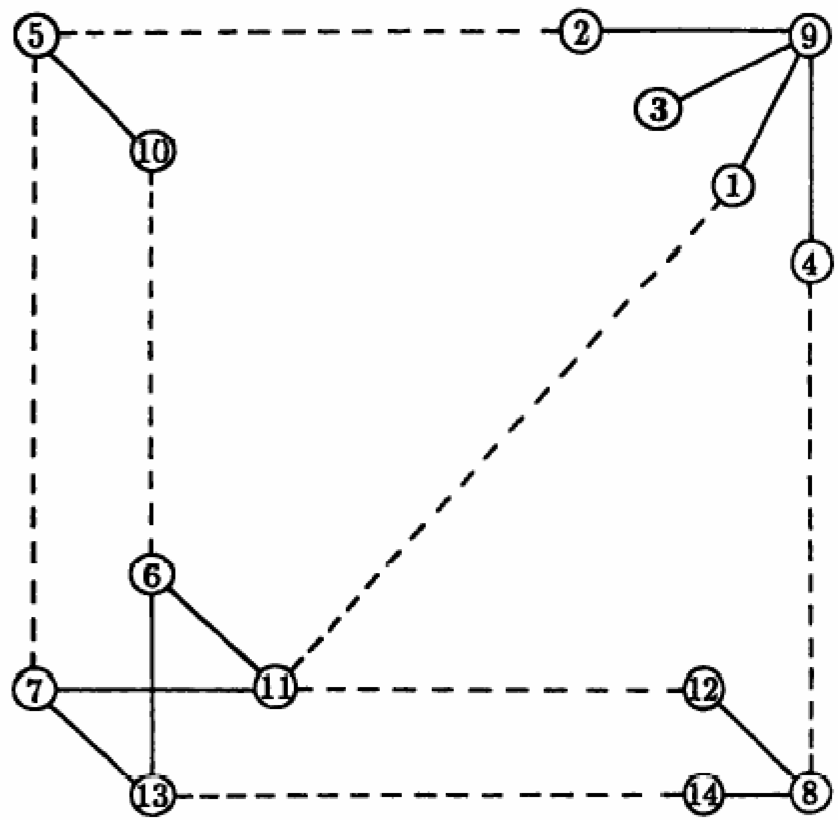
Another example





Printed Circuit Board transient routing with critical segments

Barahona, Grötschel, Jünger, Reinelt

layout graph G = (V, E), E = A ∩ B

A = conflict edges = solid lines

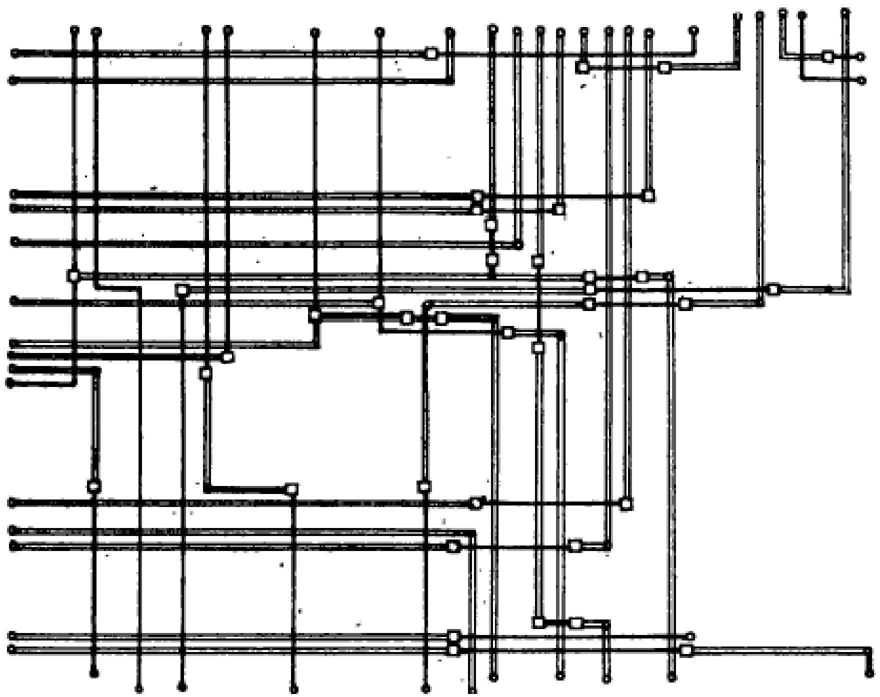
B = continuation edges = dashed lines

Some examples: Siemens printed circuit boards

Table 1

	C1	C2	C3	C4	C5
nodes in reduced layout graph	828	980	1327	1202	1366
edges in reduced layout graph	1445	1775	2480	2234	2606
vias in original design	421	434	683	650	782
via minimization with preassignments	302	376	563	504	645
via minimization without preassignments	272	347	513	475	610

A small cut-out of the optimal solution of a real Siemens PCB

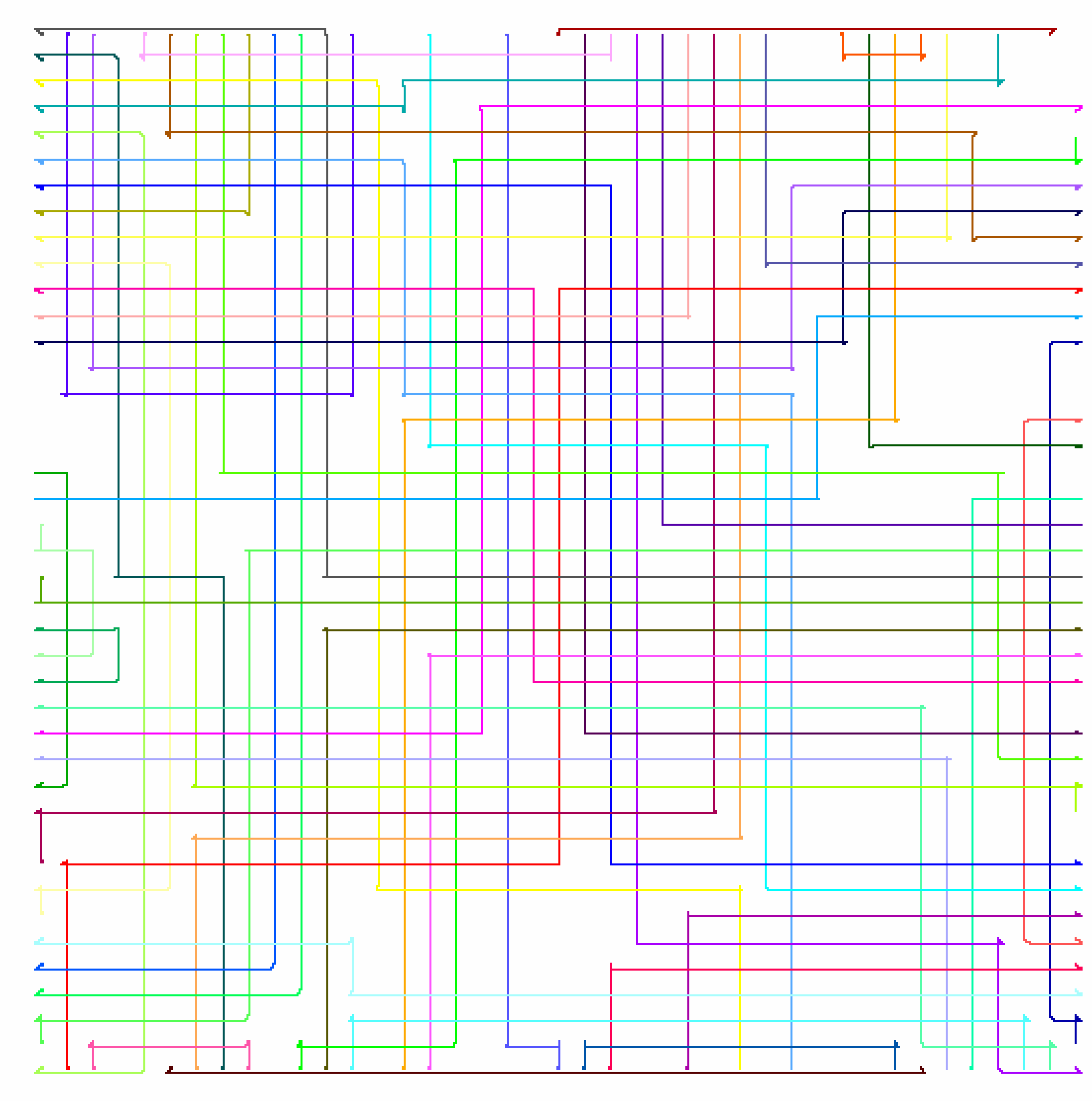


Grötschel, Jünger, Reinelt

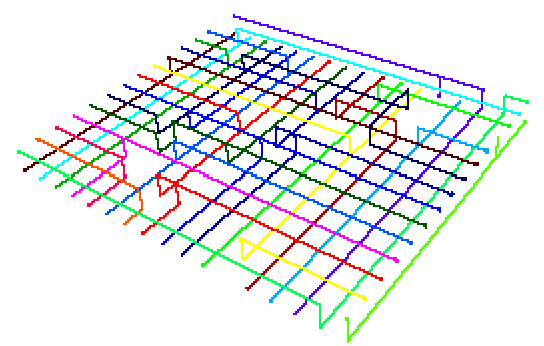
Thorsten Koch PhD Thesis

- new model
- any number of layers
- combines two objectives:
  wiring length & number of vias

Dissertation Thorsten Koch



optimal solution of a track routing problem with simultaneous via minimization



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3. Via Minimization
4. The Max-Cut and the Chinese Postman Problem
5. Optimization Problems in PCB Assembly (Petra Bauer)

The Max-Cut problem

Given a graph with edge weights, find a partition of the nodes into two parts so that the sum of the weights of the edges linking the two parts is as large as possible.

Compare to min-cut problem!

$\max \{c(\delta(W)) \mid W \text{ subset of } V\}$, $G=(V,E)$ a graph

Martin Grötschel

Eulerian Subgraphs and the Chinese Postman Problem

Given a graph G=(V,E) with edge weights c(e), eεE. A subset F of E is called Eulerian if all nodes in H=(V,F) have even degree. (H does not have to be connected.) Find an Eulerian edge set (or subgraph) of maximum weight.

- Kwan, M. K. "Graphic Programming Using Odd or Even Points." *Chinese Math.* **1**, 273-277, 1962.
- Alan Goldman first coined the name 'Chinese Postman Problem' for this problem, as it was originally studied by the Chinese mathematician Mei Ko Kuan.
- Edmonds, J. and Johnson, E. L. "Matching, Euler Tours, and the Chinese Postman." *Math. Programm.* **5**, 88-124, 1973.

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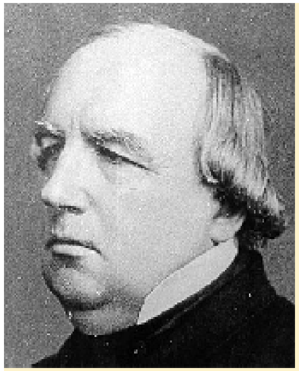
Three well known mathematical members of the BBAW

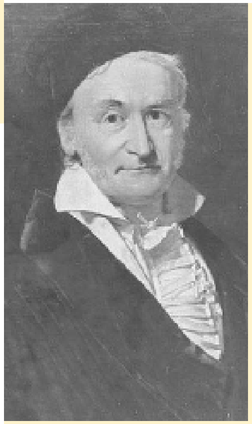
Karl Weierstrass * 31.10.1815 (Ostenfelde/Westf.) - † 19.02.1897 (Berlin)



Mitgliedschaft(en):

19.11.1856 Ordentliches Mitglied

Karl Friedrich Gauss * 30.04.1777 (Braunschweig) - † 23.02.1855 (Göttingen)

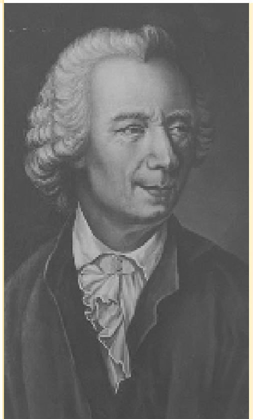


Mitgliedschaft(en):

18.07.1810 Auswärtiges Mitglied

Änderung/Austritt/Ausschluss:
11.11.1824

Leonhard Euler * 15.04.1707 (Basel) - † 18.09.1783 (Petersburg)



Mitgliedschaft(en):

1741 Ordentliches Mitglied

Änderung/Austritt/Ausschluss:
29.05.1766

29.05.1766 Auswärtiges Mitglied

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The Origin of Graph Theory

SOLUTIO PROBLEMATIS
AD GEOMETRIAM SITUS PERTINENTIS

Commentatio 53 indicis ENESTROEMIANI
Commentarii academiae scientiarum Petropolitanae 8 (1736), 1741, p. 128—140

Euler

[Link to paper]

This is not Königsberg but another example from Euler's article.

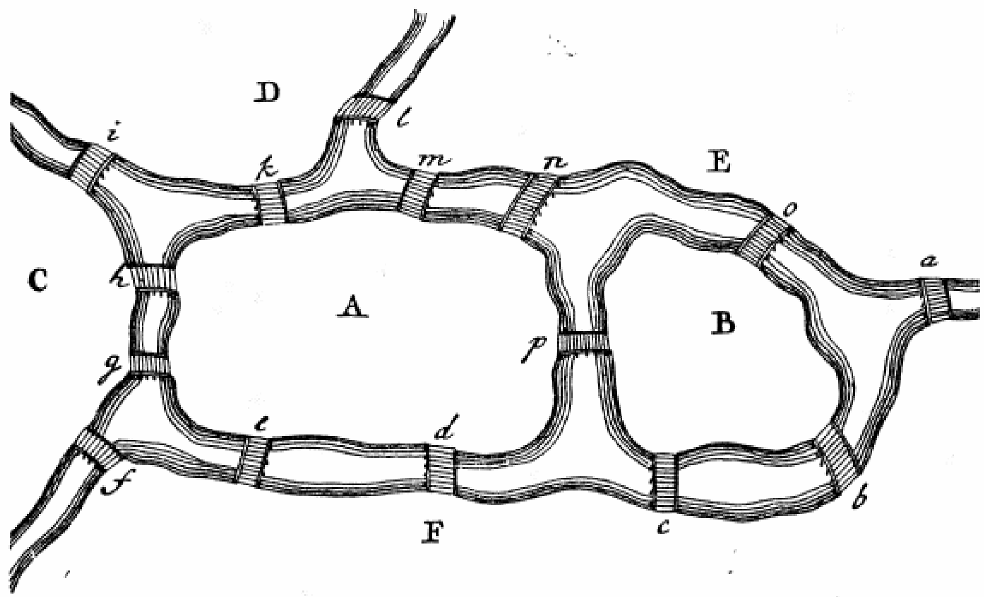

Fig. 3.

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Let us look at the Euler paper

- Euler introduces the symbols A,B,... (for what we call nodes nowadays) and a, b, … (for edges), and the notation a = AB for edges.
- He introduces the notation ACDBAC for paths and defines path length.
- He discusses notational difficulties with parallel edges.
- …sequentes observationes in medium protulero…
(…a few preliminary observations…)
- Euler states the First Theorem in Graph Theory:
- Sum of node degrees = twice the number of edges.
- In the second side remark Euler mentions the
- Second Theorem in Graph Theory:
- The number of odd nodes is even.
- Euler's Theorem
Si fuerint plures duabus regiones, ad quas ducentium pontium numerus est impar, tum certo affirmari potest talem transitum non dari

The Chinese Postman and the Max-Cut Problem

Given a graph G=(V,E) with edge weights c(e), eεE. A subset F of E is called Eulerian if all nodes in H=(V,F) have even degree. (H does not have to be connected.) Find an Eulerian edge set (or subgraph) of maximum weight.

Given a graph G=(V,E) with edge weights c(e), eεE. Find a partition of the nodes into two parts so that the sum of the weights of the edges linking the two parts is as large as possible.

Literature

- Euler L., Solutio Problematis ad Geometrica Situs Pertinentis, Commentarii academiae scientarum Petropolitanae 8, 1736, pp 128-140.
- Hierholzer C., Über die Moglichkeit, einen Linienzug ohne Wiederholung und ohne Unterbrechung zu umfahren, Mathematische Annalen VI, 1873, pp 30-32.
- Guan M., Graphic Programming Using Odd and Even Points, Chinese Mathematics 1, 1962, pp 273-277. (Meigu Guan or Kwan Mei-Ko)
- Kwan, M., An improved graphic method for solving the transportation problem. Sci. Sin. 13, 173-183 (1964).
- Edmonds J. and Johnson E.L., Matching, Euler Tours and the Chinese Postman Problem, Math. Prog. 5, 1973, pp 88-124.
- Papadimitriou C. H., On the Complexity of Edge Traversing, Journal of the Association for Computing Machinery, Vol. 23, no 3, July 1976, pp 544-554.

Martin Grötschel

- **Guan, Meigu**
  Graph theory in China. (English), in:

  Capobianco, Michael F. (ed.), Guan, Mei Gu (ed.); Hsu, D.Frank (ed.); Tian, Feng (ed.),
  Graph theory and its applications: East and West. Proceedings of the first China-USA international conference, held in Jinan, China, June 9-20, 1986. New York: New York Academy of Sciences,. Ann. N. Y. Acad. Sci. 576, 203-218 (1989)

from Schrijver's book

[1960] M.-g. Guan, Graphic programming using odd or even points [in Chinese], *Acta Mathematica Sinica* 10 (1960) 263–266 [English translation: *Chinese Mathematics* 1 (1962) 273–277]. [486–487, 501, 519]

[1979] M.-g. Guan, A brief survey of research works on network theory by Chinese mathematicians, [in: *Proceedings of the First Franco-Southeast Asian Mathematical Conference* Vol. II (Singapore, 1979)] *Southeast Asian Bulletin of Mathematics* (1979) Special Issue b, 264–267. [902]

[1984] M. Guan, On the windy postman problem, *Discrete Applied Mathematics* 9 (1984) 41–46. [518]

[1985] M. Guan, W.R. Pulleyblank, Eulerian orientations and circulations, *SIAM Journal on Algebraic and Discrete Methods* 6 (1985) 657–664. [518]

Who gave the name?

Call a walk $C = (v_0, e_1, v_1, \ldots, e_t, v_t)$ in a graph G a *Chinese postman tour* if $v_t = v_0$ and each edge of G occurs at least once in C. The *Chinese postman problem*, first studied by Guan [1960] (and named by Edmonds [1965e]), is:

(29.4) given: a connected graph $G = (V, E)$ and a length function $l \in \mathbb{Q}_+^E$,
find: a shortest Chinese postman tour C.

By Euler's theorem, if each vertex has even degree, there is an Eulerian tour, that is, a walk traversing each edge *exactly* once. So in that case, any Eulerian tour is a shortest Chinese postman tour.

[1965e] J. Edmonds, The Chinese postman's problem, *Bulletin of the Operations Research Society of America* 13 (1965) B-73. [486–487, 519]

Meigu Guan (Kwan Mei-Ko)

- worked in China before moving to Australia in 1995. He completed his tertiary education at the East China Normal University, Shanghai in 1957 and worked in Shandong Teachers' University from 1957 to 1990. He became a full professor in 1980. In Australia, Meigu worked at the Transport Research Centre in RMIT from 1995 to 1998 as a Senior Research Fellow. Throughout his career, Meigu Guan has accomplished much theoretical and applied research in operations research and he is well-published. The well-known "Chinese Postman Problem" was proposed and first investigated by him.
- Contact
  Prof. Meigu Guan
  The Transport Research Centre
  Royal Melbourne Institute of Technology
  Melbourne VIC 3000 Australia
  (Info from Dec 1998)

2. Problema autem hoc, quod mihi satis notum esse perhibebatur, erat sequens: Regiomonti in Borussia esse insulam *A*, *der Kneiphof* dictam, fluviumque eam cingentem in duos dividi ramos, quemadmodum ex figura (Fig. 1) videre licet; ramos vero huius fluvii septem instructos esse pontibus *a*, *b*, *c*, *d*, *e*, *f* et *g*. Circa hos pontes iam ista proponebatur quaestio, num quis cursum ita instituere queat, ut per singulos pontes semel et non plus quam semel transeat. Hocque fieri posse, mihi dictum est, alios negare alios dubitare; neminem vero affirmare. Ego ex hoc mihi sequens maxime generale formavi problema: quaecunque sit fluvii figura et distributio in ramos atque quicunque fuerit numerus pontium, invenire, utrum per singulos pontes semel tantum transiri queat an vero secus.

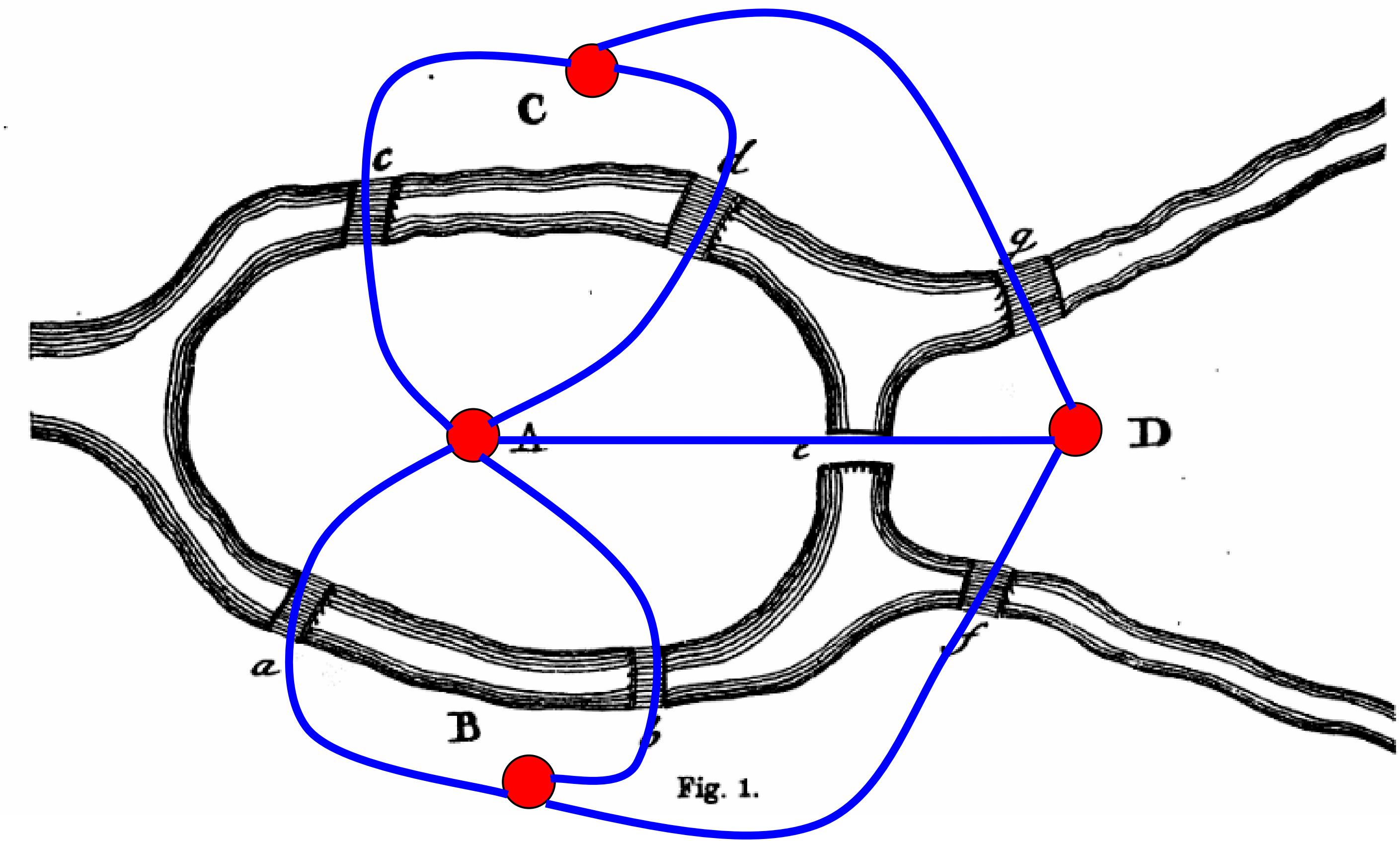
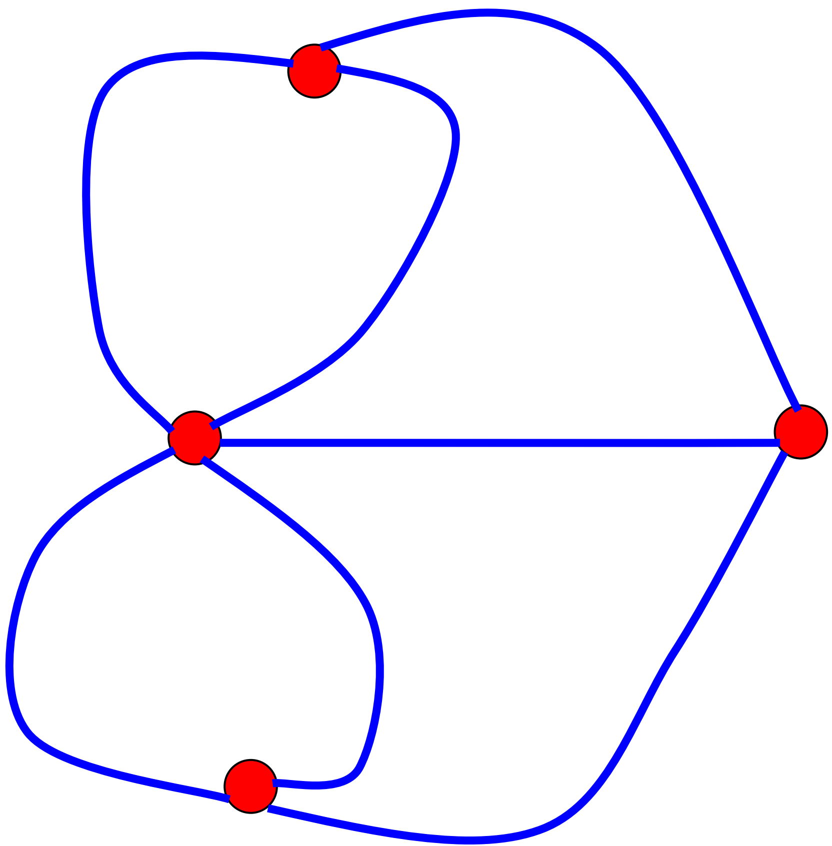


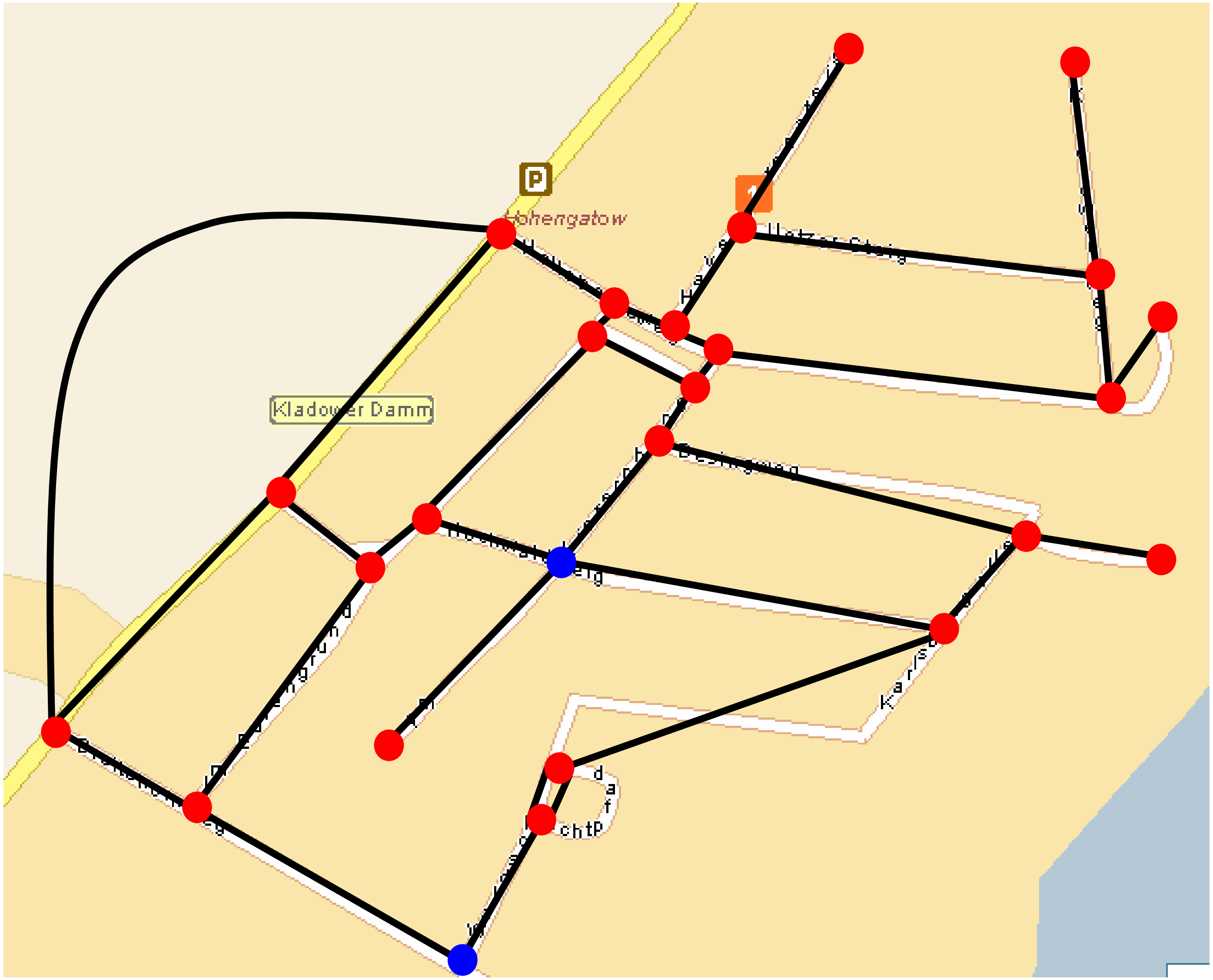
Fig. 1.

Euler's Theorem (1736)

A graph has a (connected) Eulerian Tour if and only if, it is connected and every node has even degree.



Berlin-Hohengatow



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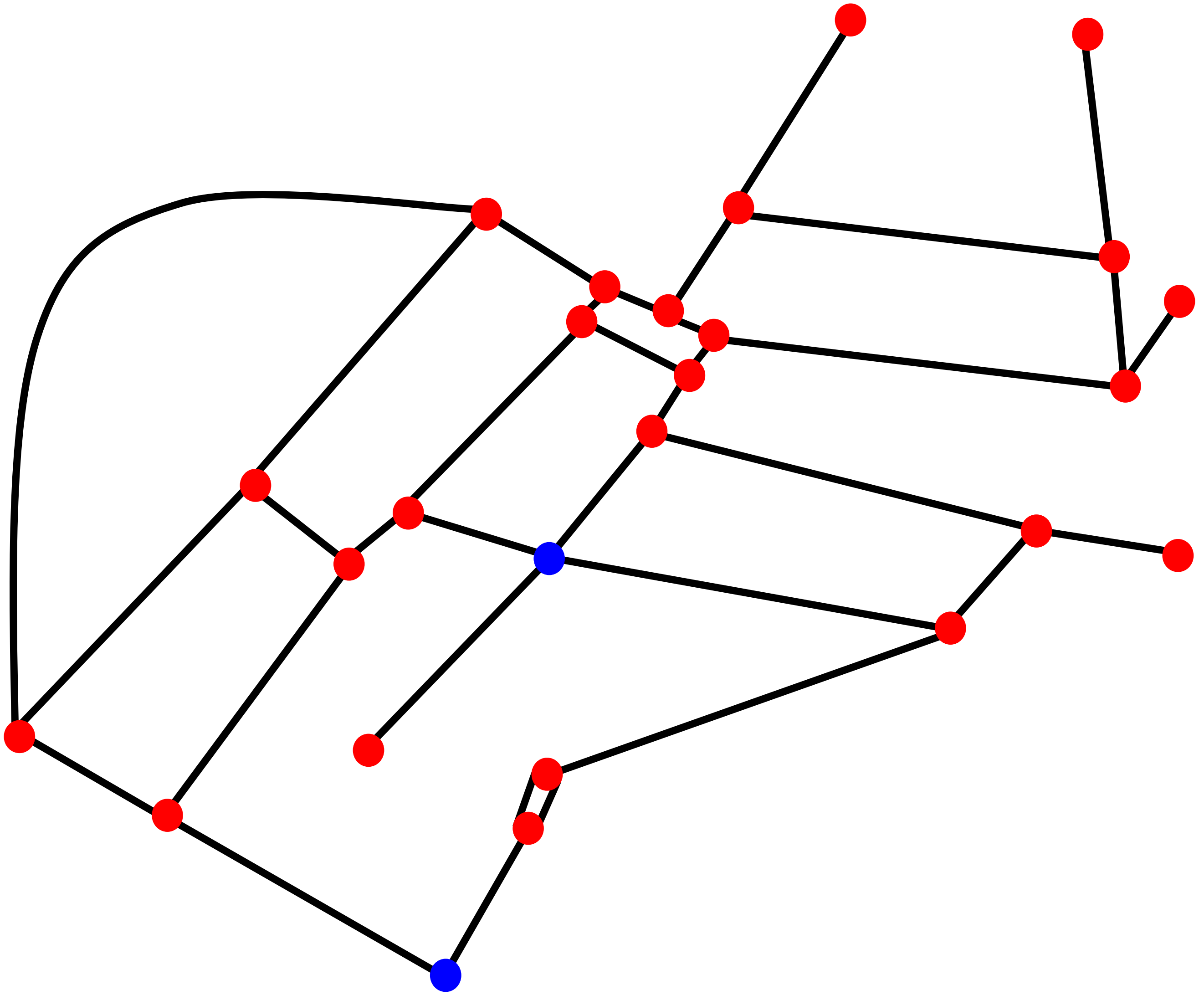
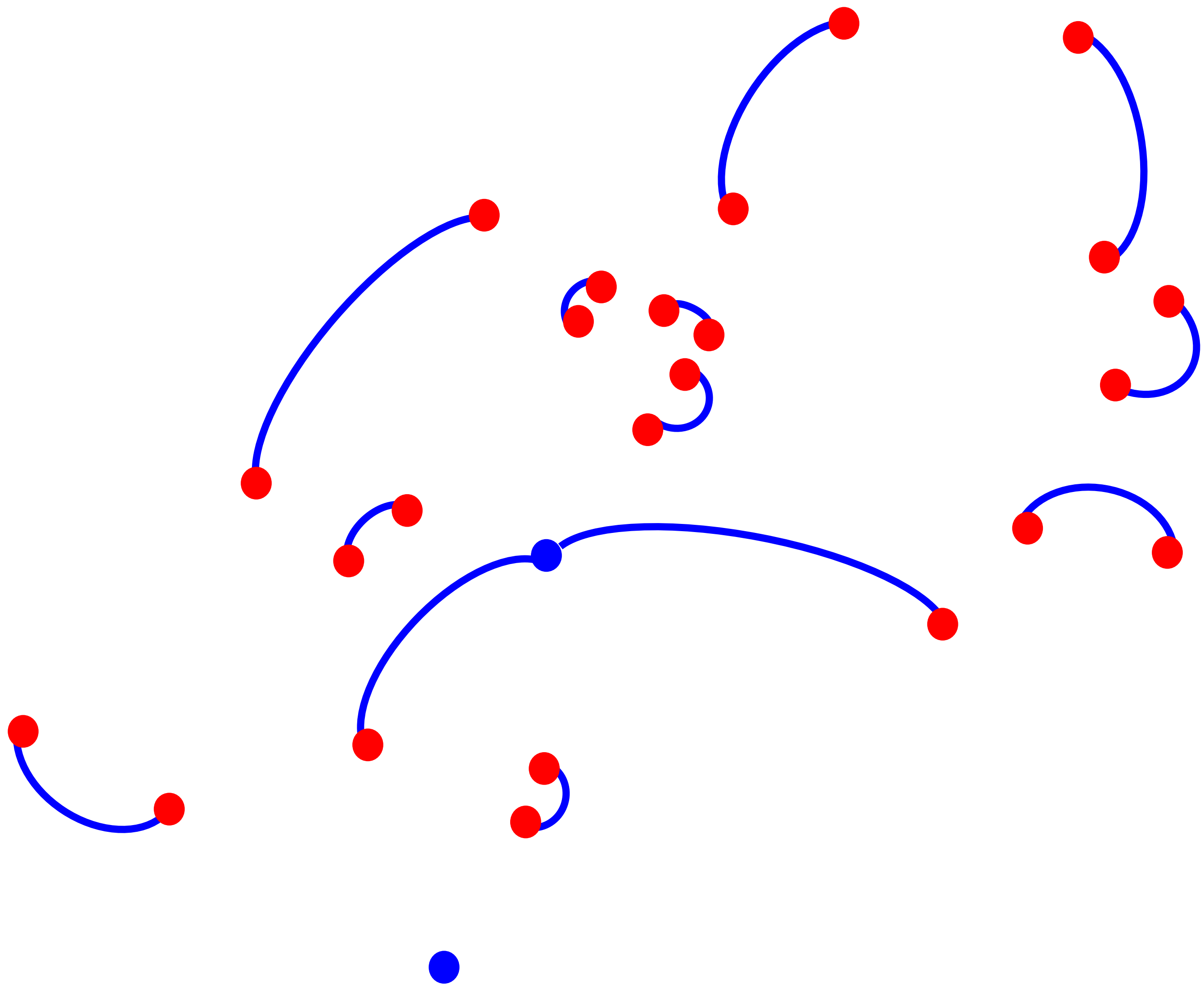
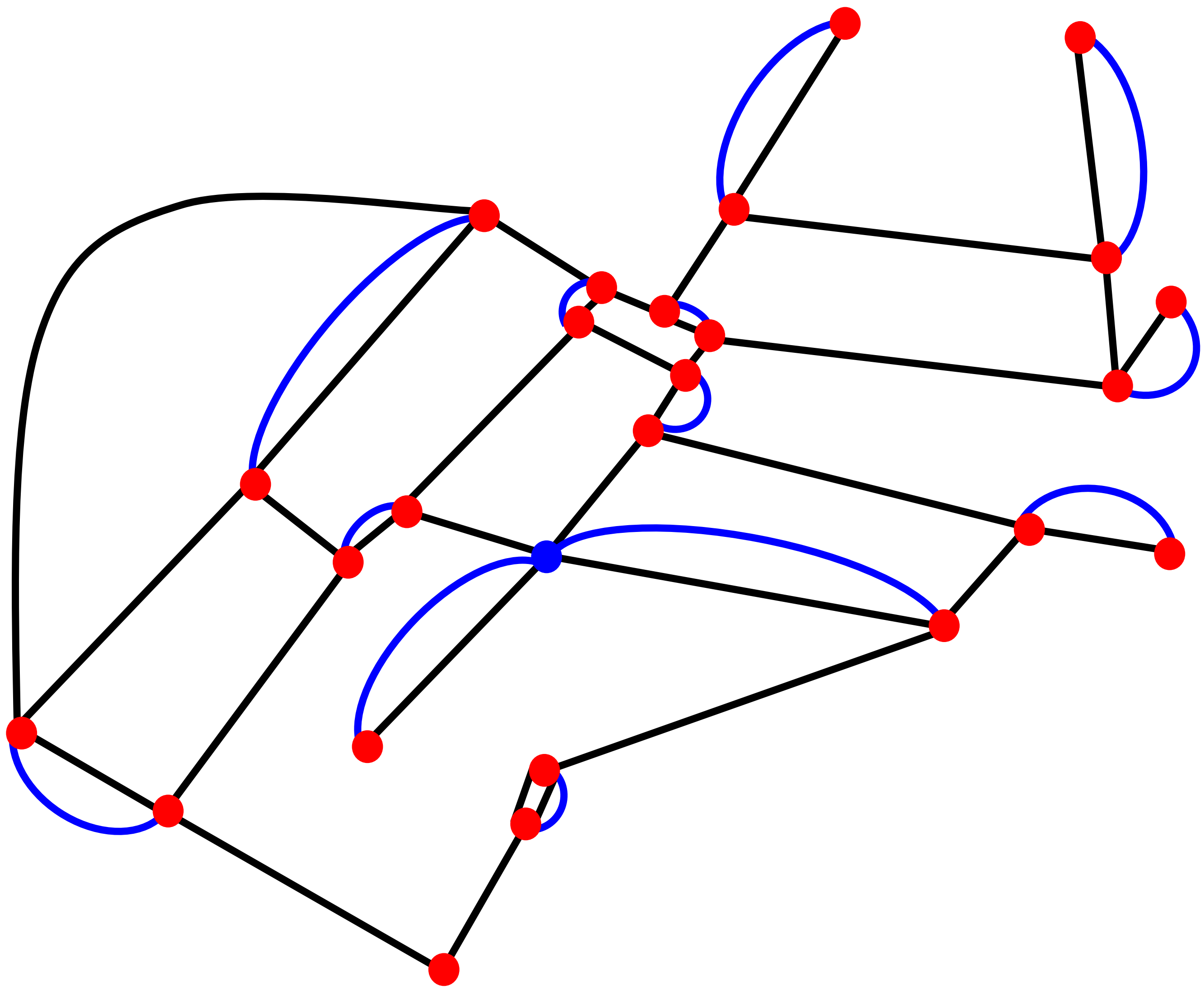
Hohengatow-Graph



Hohengatow-Graph



Hohengatow-Graph



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Exercises

- Formulate different versions of the Chinese postman problem.
- Explore the relations between the Chinese postman problem and the max-cut problem.
- Under which additional assumption can one transform one problem into the other and vice versa?
- Provide IP formulations of the two problems.

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Optimization Problems in Printed Circuit Board Assembly

Petra Bauer
Siemens AG, Munich, Germany

CORPORATE TECHNOLOGY

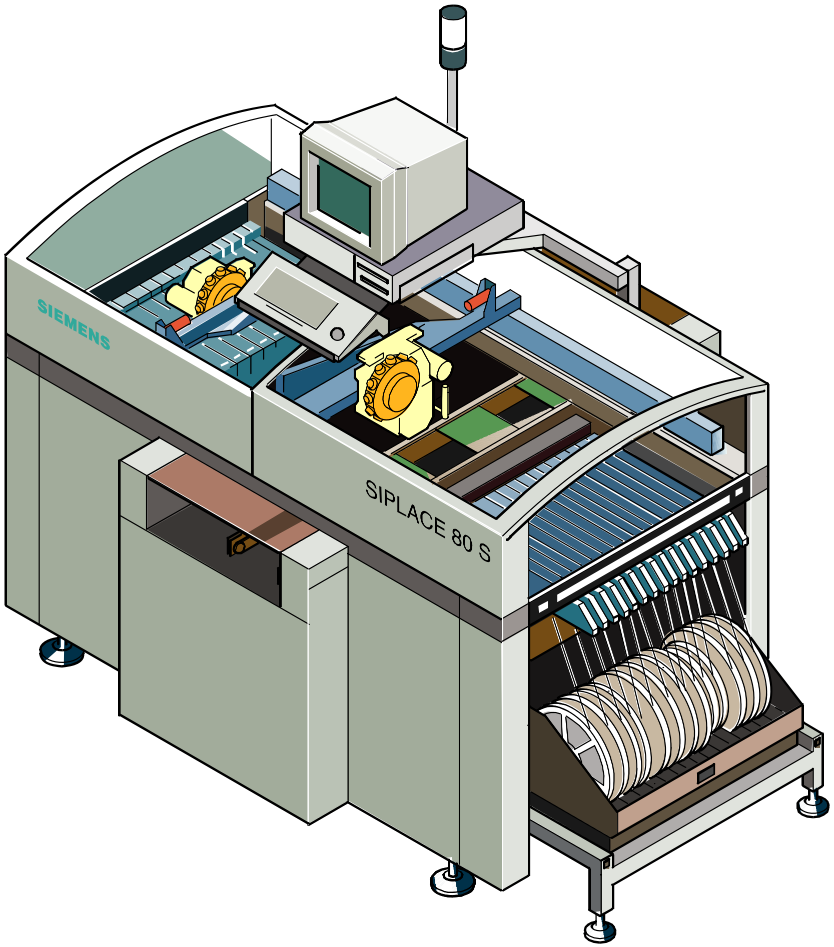
CT 6 S&E
Software & Engineering
Discrete Optimization

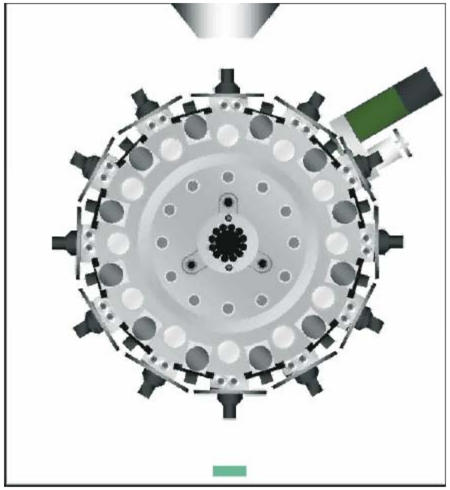
Overview

- **SIPLACE Placement Machines**
  - Automata and Performance (Examples)
  - Terminology
- **Optimization Problems / Overview**
  - Line Balancing and its Subproblems
  - Planning Problems
- **A Closer Look at Two Problems**
  - Split Component Types: Assignment of Placements to Potential Heads
  - The Nozzle Selection Problem for Revolver Heads
    - Integer Programming Formulation
    - An Efficient Exact Algorithm for Graphs with COP

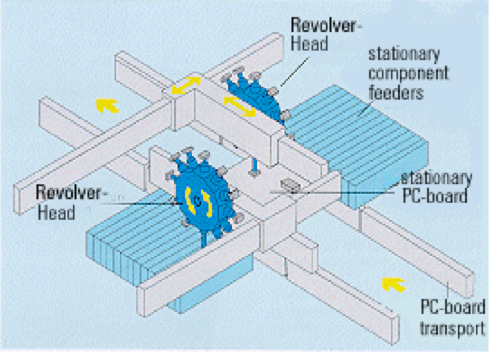
CORPORATE TECHNOLOGY

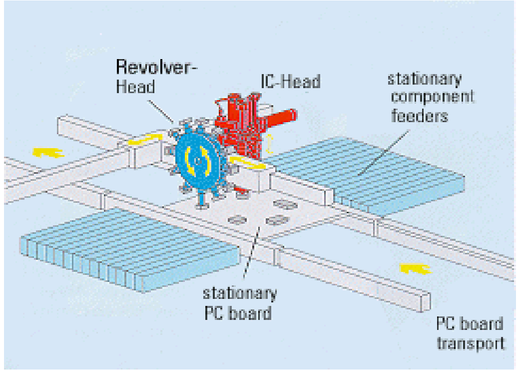
SIPLACE Placement Machines





Automata and Performance (Examples)





SIPLACE HS-60

4x12 60000 comp/h (16.6 comp/sec)

SILPACE F

12 or 6 nozzle revolver head

IC 1800 comp/h (0.5 comp/sec)

SIPLACE S-27 HM

12/12 26500 comp/h (7.5 comp/sec)

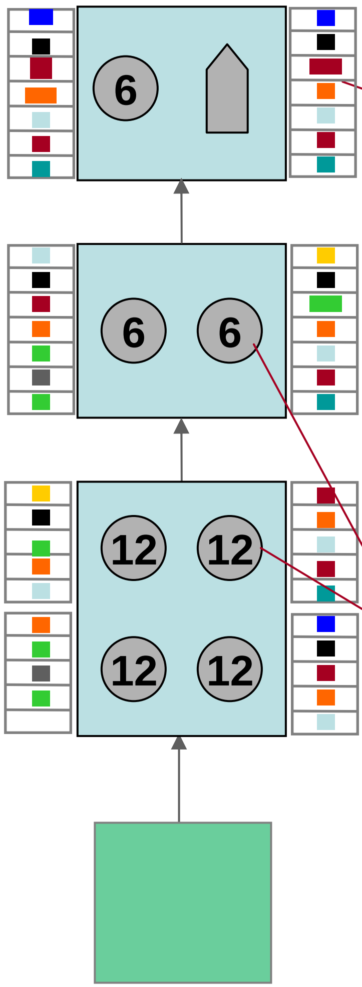
6/6 17000 comp/h (4.7 comp/sec)

and more …

component ranges:	12 nozzle head	0.6 x 0.3 (0201) – 18.7 x 18.7 mm²
	6 nozzle head	1.6 x 0.8 (0603) – 32 x 32 mm²
	Twin head	

Terminology

placement line



component setup

feeder area, feeder tables

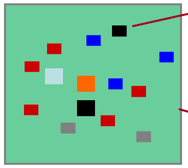
cycle: pick-up and placement of up to 12 (6) components

set of jobs

board type	number
LP1	150
LP5	20
LP3	900
LP7	70
⋮	⋮

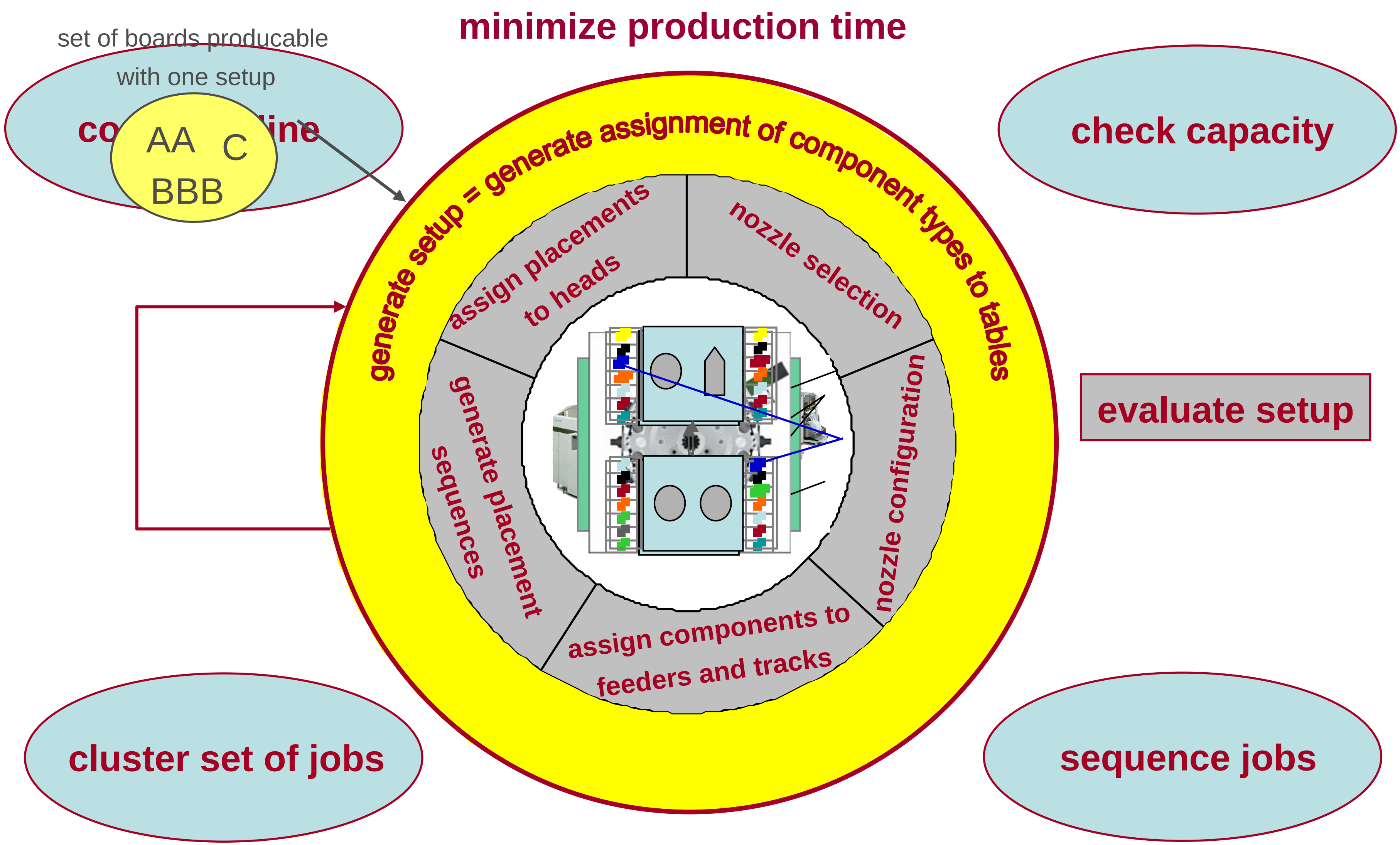
job

placement position



board

Optimization Problems / Overview

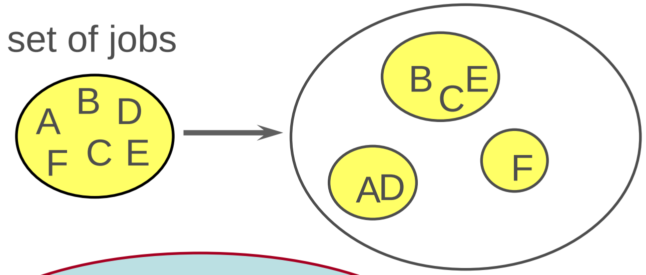


Optimization Problems / Overview

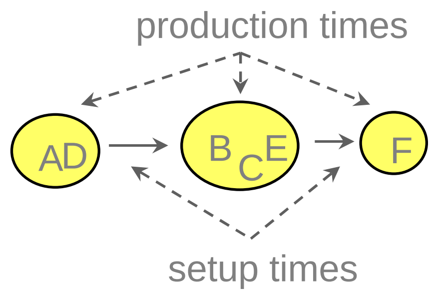
configure line

- automata
- placement heads
- productivity-lifts

check capacity



cluster set of jobs



sequence jobs

Line Balancing

generate setup

evaluate =

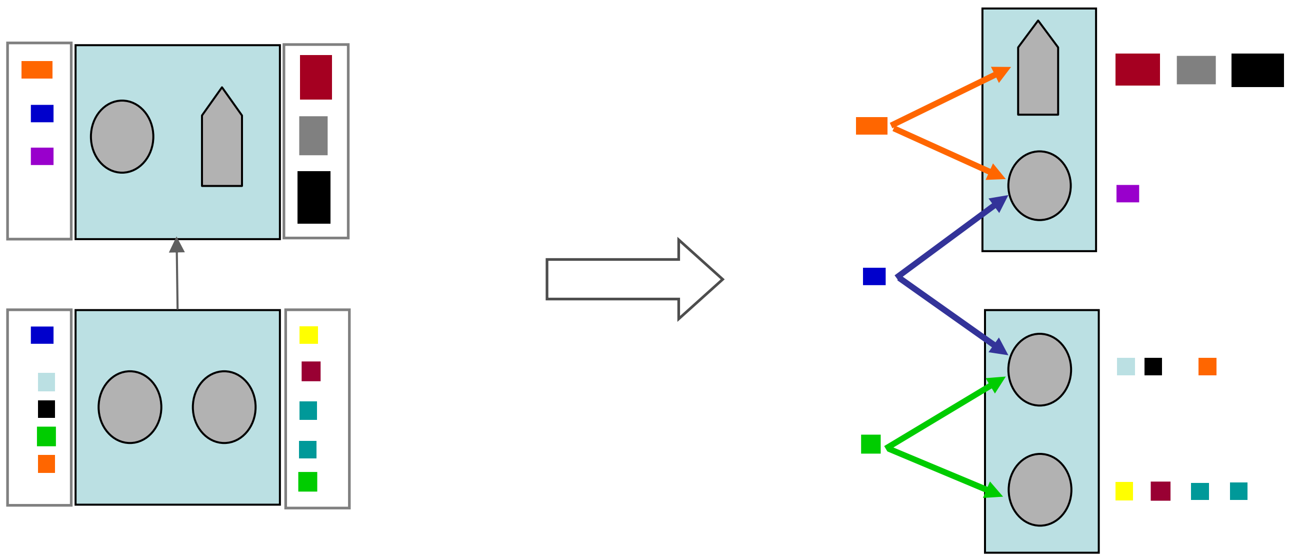
approximate production time

- **assign placements to heads in case of alternatives**
- **for each head: select nozzles = minimize number of cycles**
- **generate placement sequences**

Assign Placements to Heads in Case of Alternatives

Given: assignment of component types to feeder tables.

If there are alternative heads for a component type, we have to determine how many components each possible head has to place.



Linear Program / Heuristic

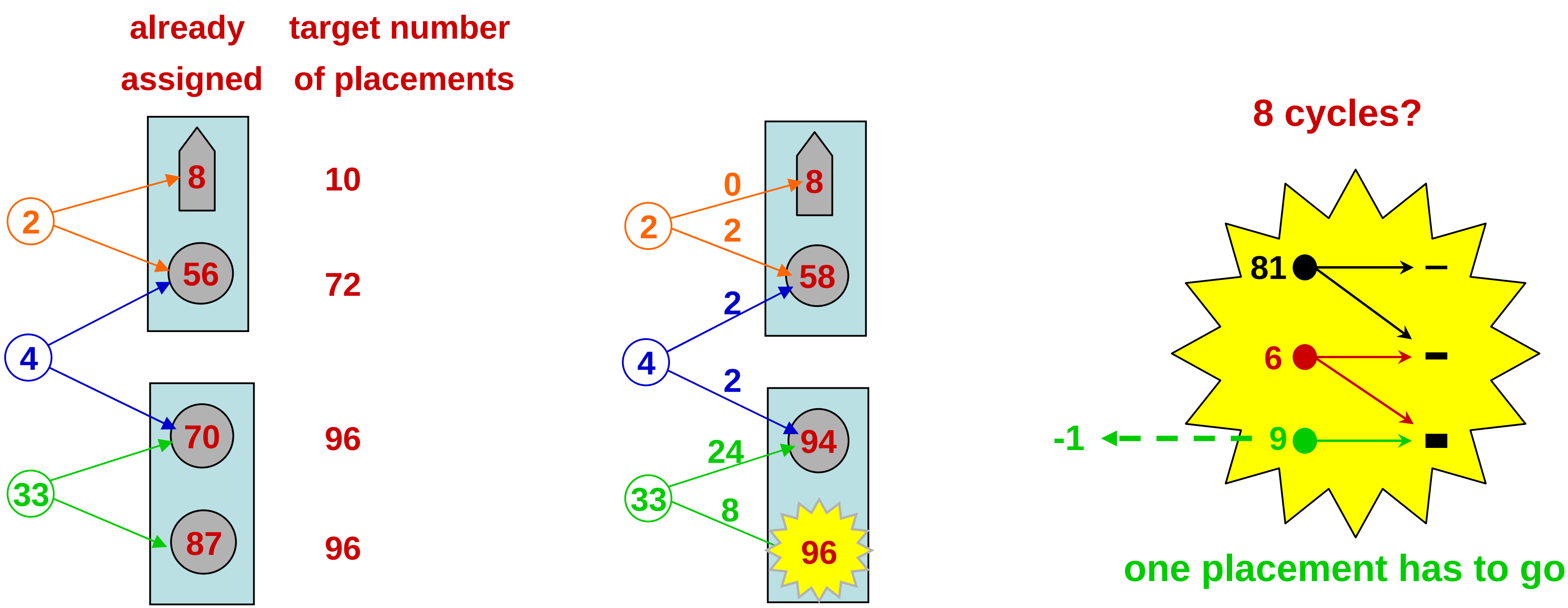
Determine "good" values for the number of cycles/placements of each head.

Flow Algorithm

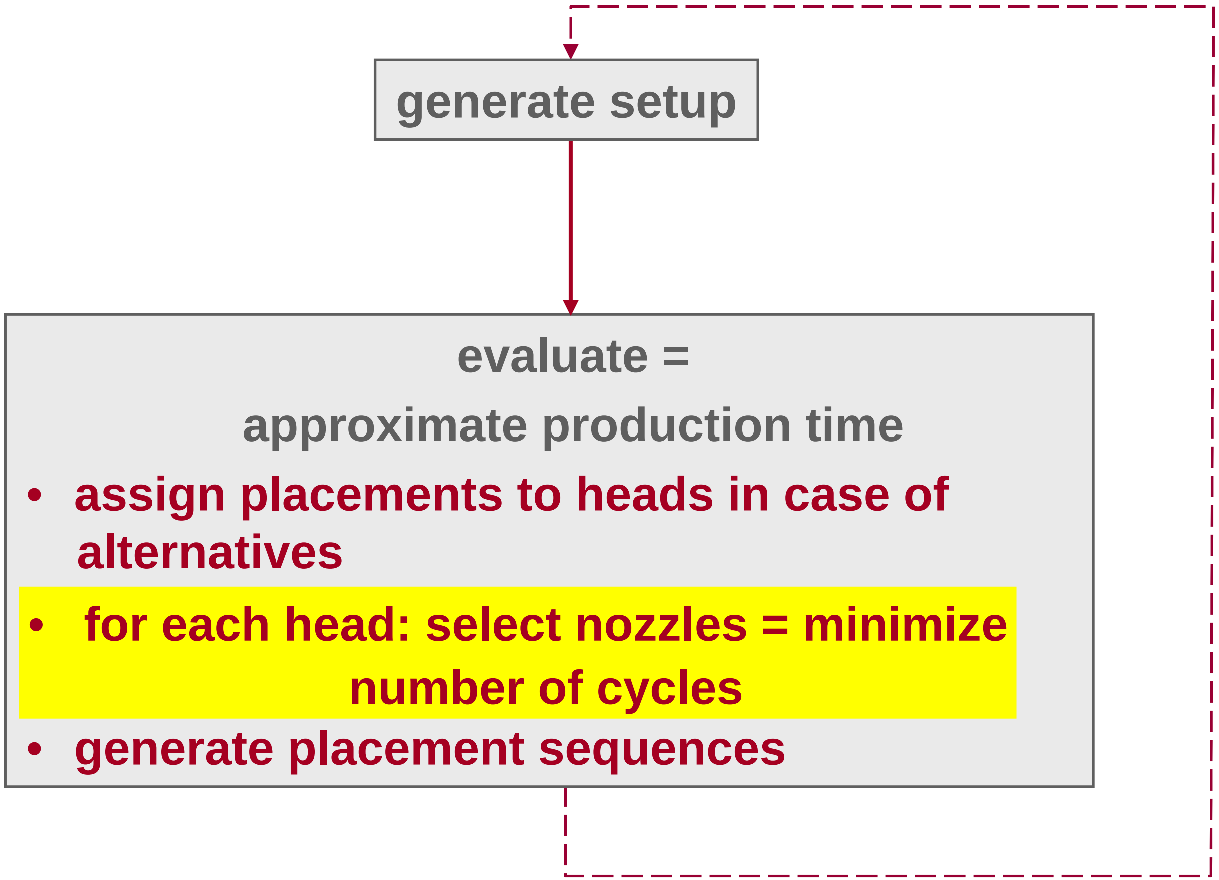
Distribute workload for the component types with alternatives considering the values generated by the preceding step.

Nozzle Selection

Solve the nozzle selection problems for the prescribed values. If for a nozzle selection problem there is no solution, suggest components to be redistributed.



Line Balancing



The Nozzle Selection Problem for Revolver Heads

Choose the right nozzles in order to minimize the number of cycles

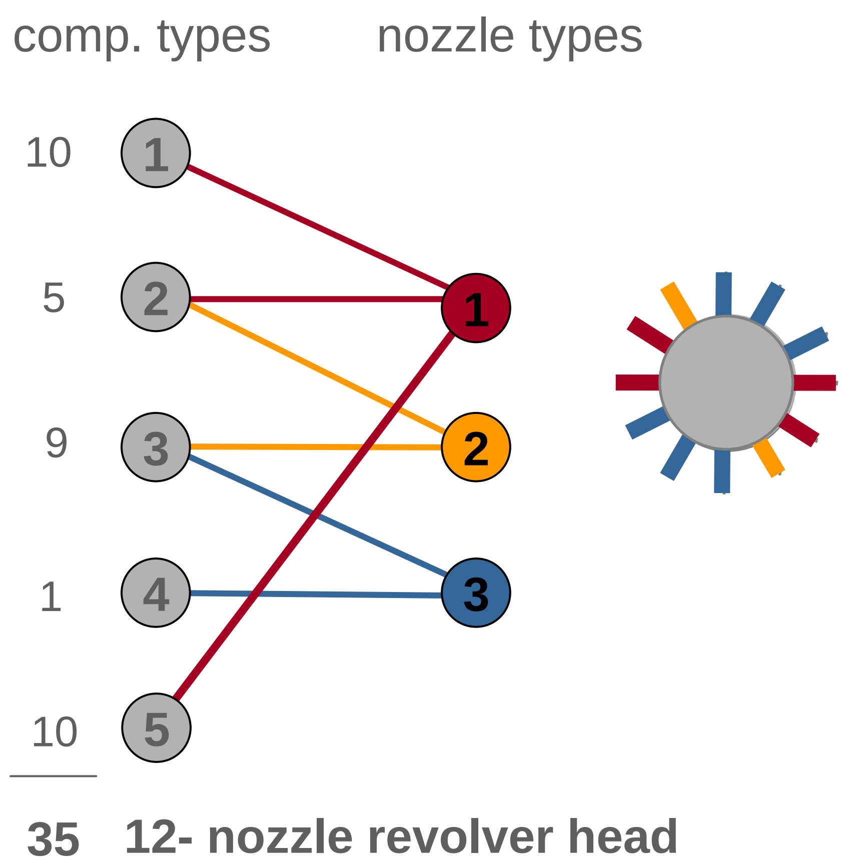
Given:

- **number of segments**
- **workload =**
  list of (comp. type, number)
- **possible component type - nozzle type assignments**

Problem:

Minimize the number of cycles necessary by optimally

- **choosing the nozzles**
- **assigning components to nozzle types**



Is it possible to place all components with 3 cycles? How?

Integer Programming Formulation

Given z. Is there a solution to

$$\sum_{p:(c,p)\in E} x(c,p) = n(c) \qquad \text{for all component types } c$$

$$\sum_{c:(c,p)\in E} x(c,p) \le z \cdot y(p) \qquad \text{for all nozzle types } p$$

$$\sum_{p} y(p) \le s$$

x,y integer

Let n be the total number of components and s the number of segments.

Observation: Usually there is a solution for z within $\lceil n/s \rceil$ and $\lceil n/s \rceil + 2$.

The Nozzle Selection Problem for Revolver Heads

Complexity

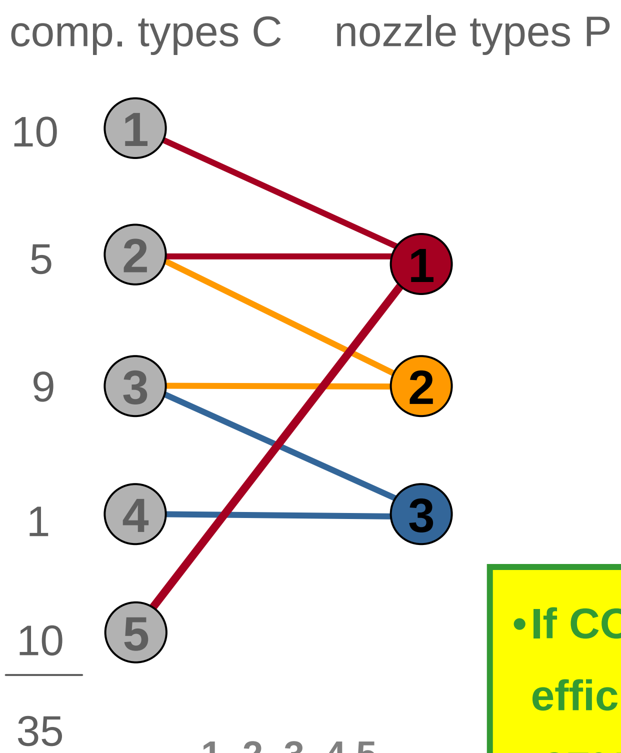
In general, the nozzle selection problem is **NP-hard** (e.g., 3-dimensional matching can be reduced to the nozzle selection problem).

Integer Programming Approach

- "Is there a solution with z cycles?“ can be stated as an integer linear program and can be solved by a cutting plane approach (specialized and general cuts).
- empirical observation: usually there is a solution where the number of cycles is not more than 10% off the trivial lower bound ⇒ try increasing values of z (10% usually means one or two cycles).

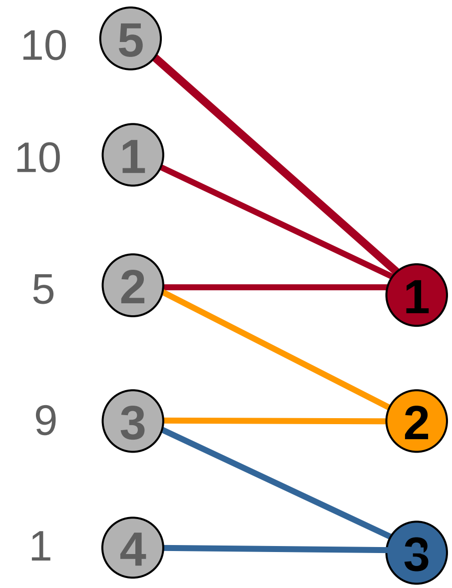
But: the nozzle selection problem has to be solved thousands of times within the line balancing procedure ⇒ **running time per instance required to be less than 5 msec**

Consecutive Ones Property (COP)



	1	2	3	4	5
1	1	1	0	0	1
2	0	1	1	0	0
3	0	0	1	1	0

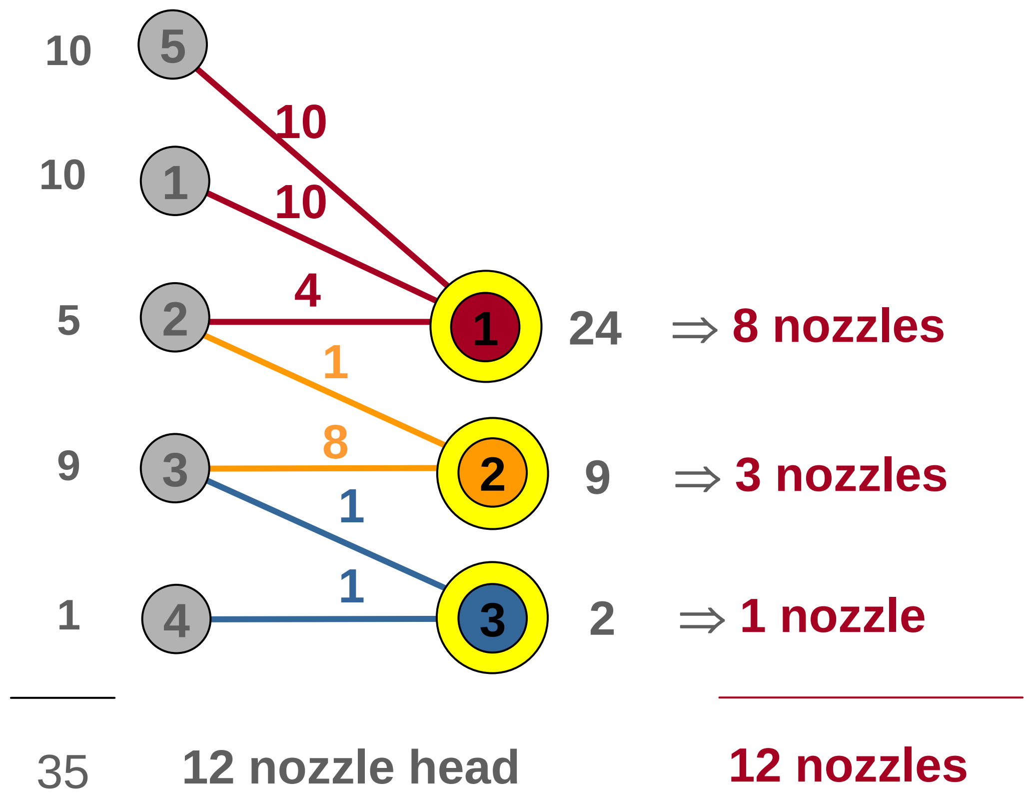
- If COP is given, there is an efficient algorithm (O(ICI•IPI)).
- 97% of our real world data satisfy COP.
- COP can be tested in linear time (O(E)).



	5	1	2	3	4
1	1	1	1	0	0
2	0	0	1	1	0
3	0	0	0	1	1

An Efficient Exact Algorithm for Graphs with COP

R. Enders



$\lceil 35/12 \rceil = 3$ → **Is it possible to set all components with 3 cycles?**

03M2 Lecture
Printed Circuit Board Production: Some Issues

The End

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